



MAX PLANCK INSTITUTE
FOR SECURITY AND PRIVACY

Formally verifying Kyber

Episode V: Machine-checked IND-CCA Security and Correctness of ML-KEM in EasyCrypt

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 - Optimized & verified implementations

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 - No analysis of cryptographic properties in EasyCrypt
 - Targeted round 3 Kyber, not ML-KEM

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Motivation: objectives

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Derive that our implementations are an IND-CCA secure KEM at the assembly level

- Kyber 2017 [BDK⁺18]
 - Public key compression invalidated assumption in security proof
 - Tweaked FO transform [FO99, FO13] broke (security) proof in QROM

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- KyberSlash: timing side-channel found when using DIV instruction [BBB⁺24]

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- Functional correctness proofs

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- Security proofs
- Functional correctness proofs
- Link the two:
 - Ensure the functional specification matches the one in the security proof
 - Derive that our implementations are secure

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 - compression
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Concrete security bound for ML-KEM that considers low-level details

- Lattice-based PKE scheme based on Module-LWE [[BDK⁺18](#)]

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- Several optimizations: parameter choice + **public-key and ciphertext compression**
- Correctness and security implications:
 - Correctness bound is hard to compute: only heuristic results
 - Public-key compression not contemplated in security proof
 - Tweaked FO transform: ciphertext hashing

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- Tweaked FO transform still in use: ciphertext hashing

Algorithm 8 KYBER.CCAKEM.Enc(pk)

Input: Public key $pk \in \mathcal{B}^{12 \cdot k \cdot n/8 + 32}$
Output: Ciphertext $c \in \mathcal{B}^{d_u \cdot k \cdot n/8 + d_v \cdot n/8}$
Output: Shared key $K \in \mathcal{B}^*$

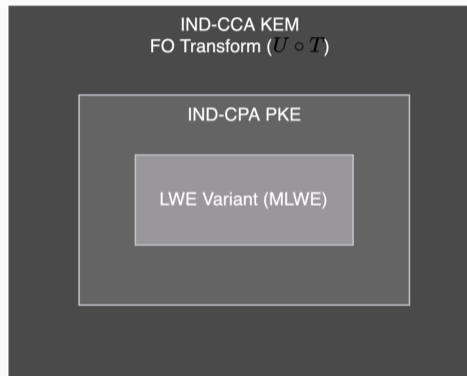
- 1: $m \leftarrow \mathcal{B}^{32}$
- 2: $m \leftarrow H(m)$ ←
- 3: $(\bar{K}, r) := G(m \| H(pk))$ ←
- 4: $c := \text{KYBER.CPAPKE.Enc}(pk, m, r)$
- 5: $K := \text{KDF}(\bar{K} \| H(c))$ ←
- 6: **return** (c, K)

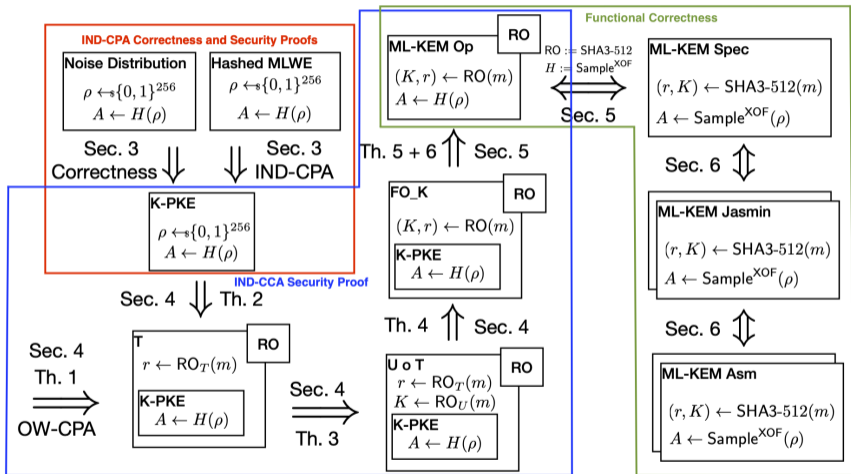
Algorithm 9 KYBER.CCAKEM.Dec(c, sk)

Input: Ciphertext $c \in \mathcal{B}^{d_u \cdot k \cdot n/8 + d_v \cdot n/8}$
Input: Secret key $sk \in \mathcal{B}^{24 \cdot k \cdot n/8 + 96}$
Output: Shared key $K \in \mathcal{B}^*$

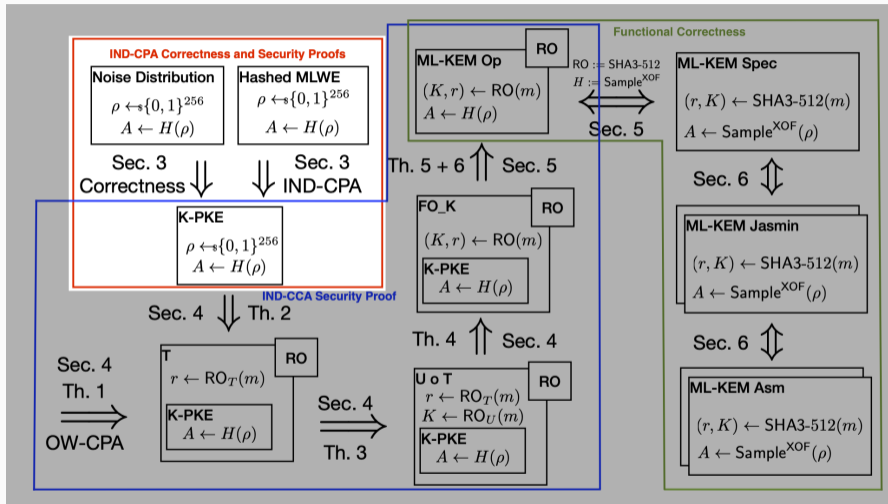
- 1: $pk := sk + 12 \cdot k \cdot n/8$
- 2: $h := sk + 24 \cdot k \cdot n/8 + 32 \in \mathcal{B}^{32}$
- 3: $z := sk + 24 \cdot k \cdot n/8 + 64$
- 4: $m' := \text{KYBER.CPAPKE.Dec}(sk, c)$
- 5: $(\bar{K}', r') := G(m' \| h)$
- 6: $c' := \text{KYBER.CPAPKE.Enc}(pk, m', r')$
- 7: **if** $c = c'$ **then**
- 8: **return** $K := \text{KDF}(\bar{K}' \| H(c))$ ←
- 9: **else**
- 10: **return** $K := \text{KDF}(z \| H(c))$ ←
- 11: **end if**
- 12: **return** K

- Formerly CRYSTALS-Kyber
- Typical design of post-quantum KEMs
- IND-CPA PKE scheme from variant of LWE
- IND-CCA KEM using FO transform [FO99, FO13, HHK17]

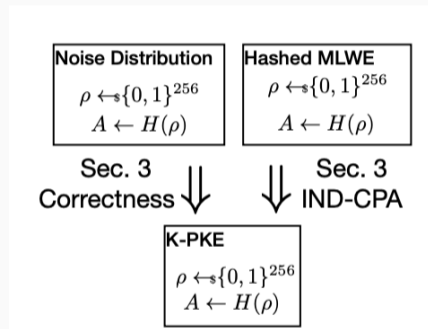




Overview: IND-CPA construction

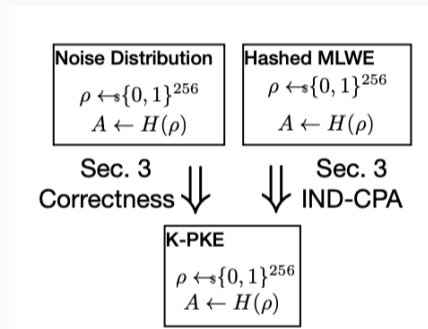


- IND-CPA security and correctness proof



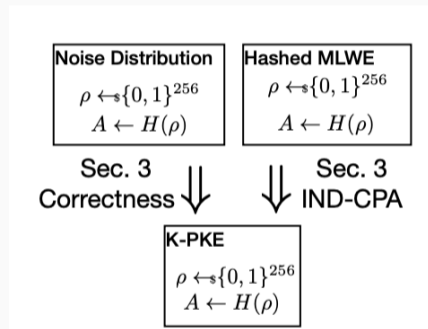
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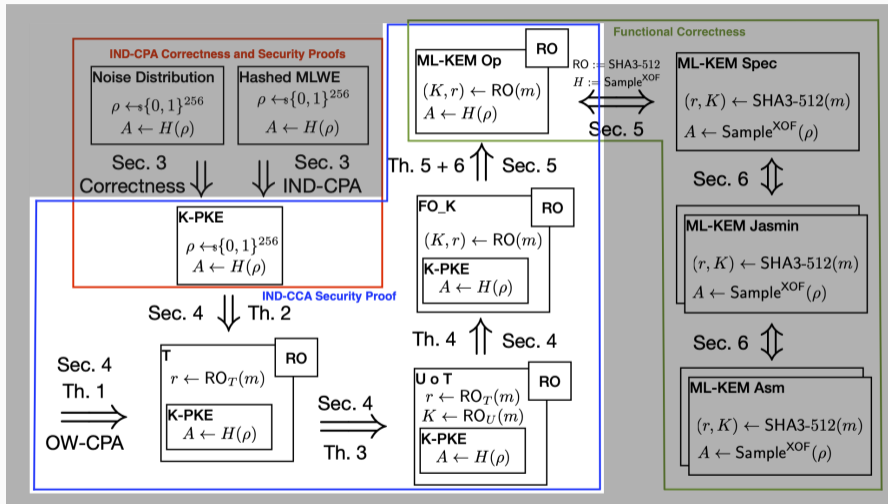


Overview: IND-CPA construction

- IND-CPA security and correctness proof
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- Security proof under a variant of MLWE: Hashed MLWE
 - Replace sampling of matrix A with deterministic procedure H
- Correctness proof sets upper bound for a decryption failure

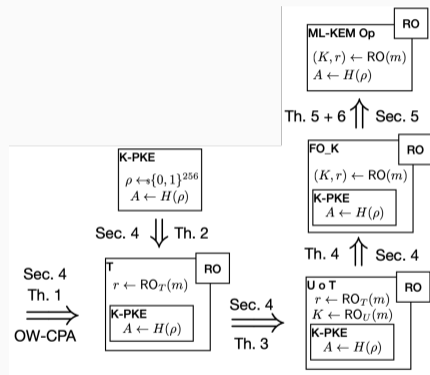


Overview: IND-CCA security



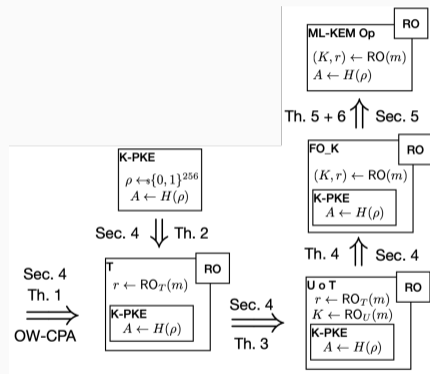
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- Machine-checked security proofs for ML-KEM



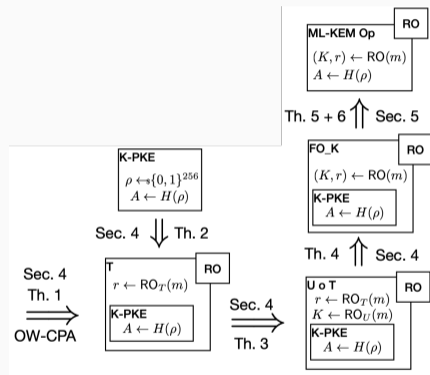
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 - Reuse K-PKE construction from earlier
 - Instantiate with concrete parameters (ML-KEM-768 in our case)

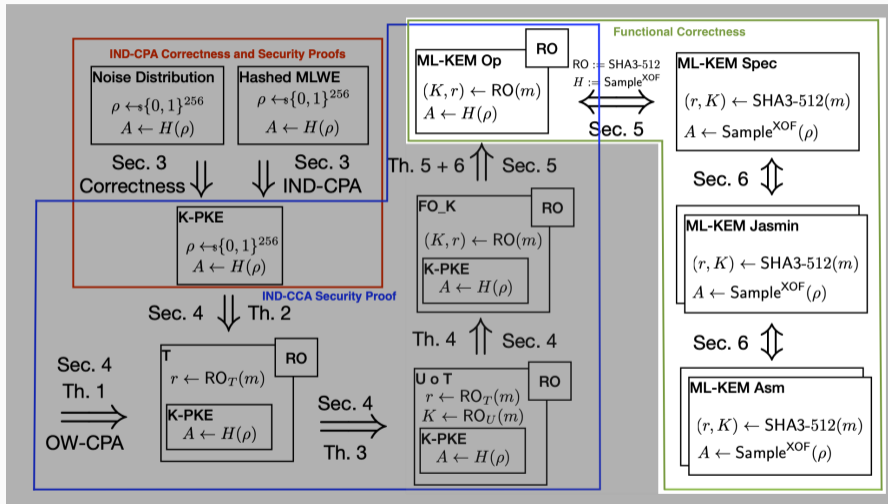


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- IND-CCA security $ML\text{-KEM}_{op}$ follows from instantiating FO_k transform:
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- IND-CCA security of FO_k derived from proof that shows security of the composition of T and U transforms

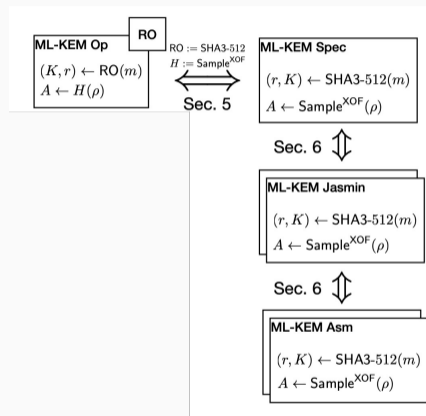


Overview: Implementation correctness



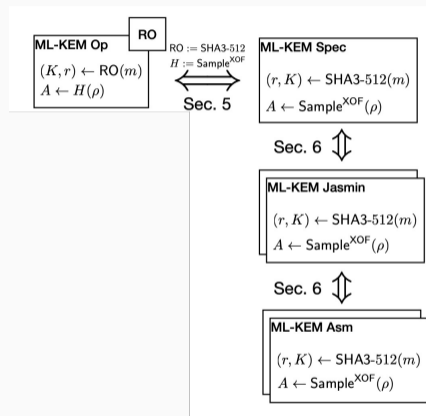
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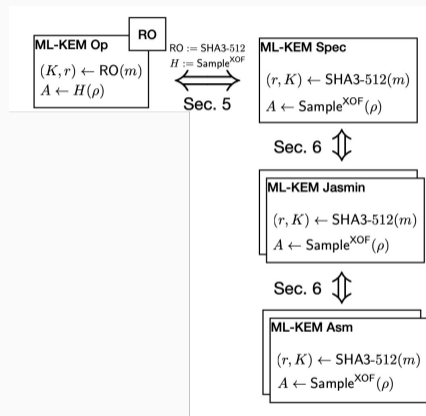
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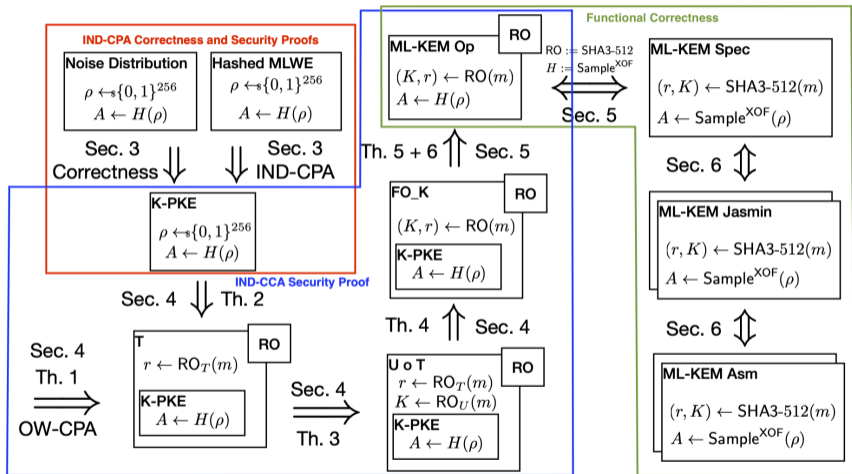
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 - Constant-time code via the type system
- Gap between security proof and assembly implementation:
 - Hash function (SHA3-512) is not a Random Oracle





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 - Games, theorems and proofs match papers
 - Abstract proofs (e.g. FO transform) can be instantiated with concrete schemes/parameters
 - Supports specifications, security proofs, implementations, functional correctness
- Drawbacks:
 - Proofs are not automatic and require significant effort
 - Theorems can be hard to read

- Large Trusted Code Base (TCB):
 - EasyCrypt (not formally verified)
 - EasyCrypt proof statements and specifications¹
 - SMT solvers
- Classical security proof only: no security proof against quantum adversaries

¹Machine-readable standards could provide a solution to the latter (future work)

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- Formally verify other primitives: ML-DSA, SLH-DSA, FrodoKEM, etc
- Industry adoption of formally verified implementations



<https://formosa-crypto.org>

- **High-assurance Kyber:**
 - Episode IV: <https://eprint.iacr.org/2023/215>
 - Episode V: <https://eprint.iacr.org/2024/843>
- **EasyCrypt specifications:** <https://github.com/formosa-crypto/crypto-specs>
- **Libjade:** <https://github.com/formosa-crypto/libjade>

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