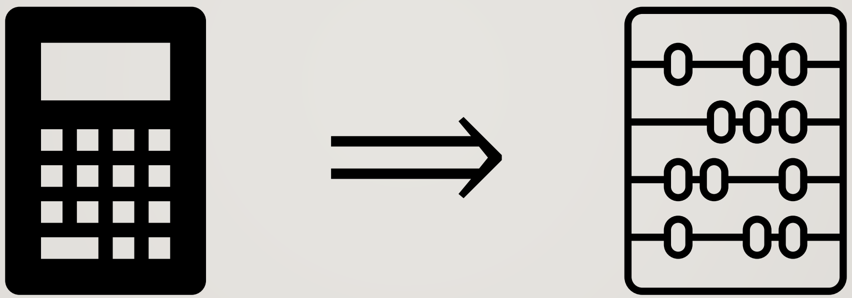
Robust Additive Randomized Encodings From IO And Pseudo-non-linear-codes

NIR BITANSKY   SAPIR FREIZEIT

Basic Paradigm In Secure Computation

[Yao82, BMR90, IK00, ...]

Reduce general secure computation to secure computation of simple functions.



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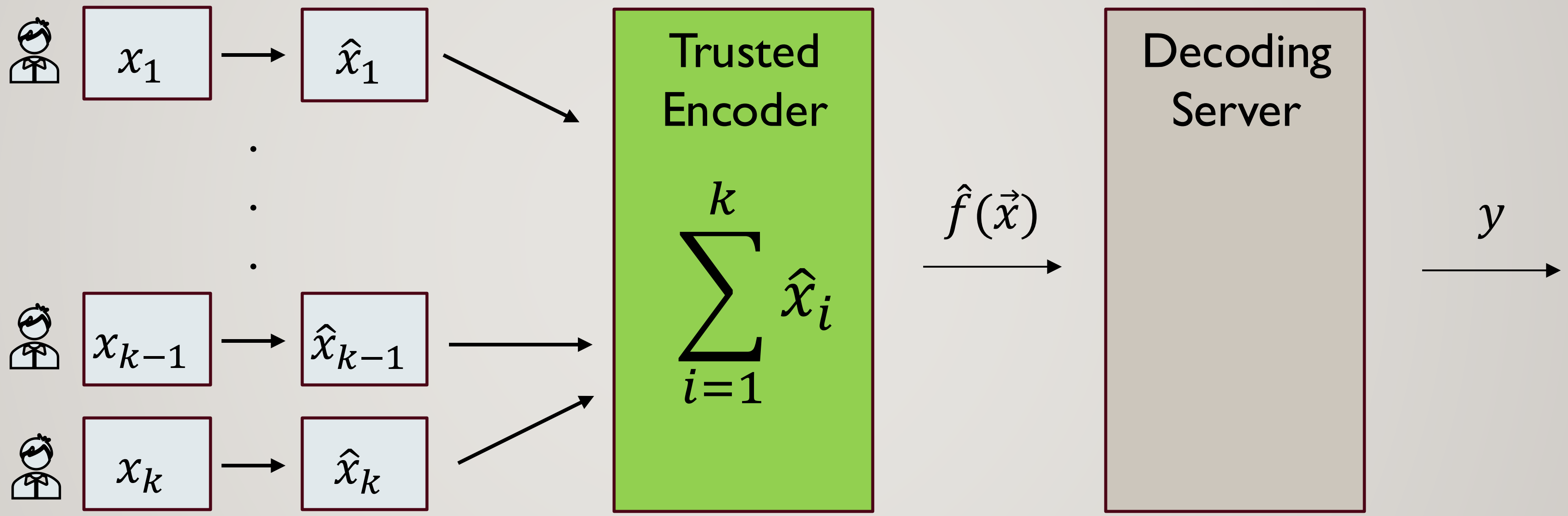

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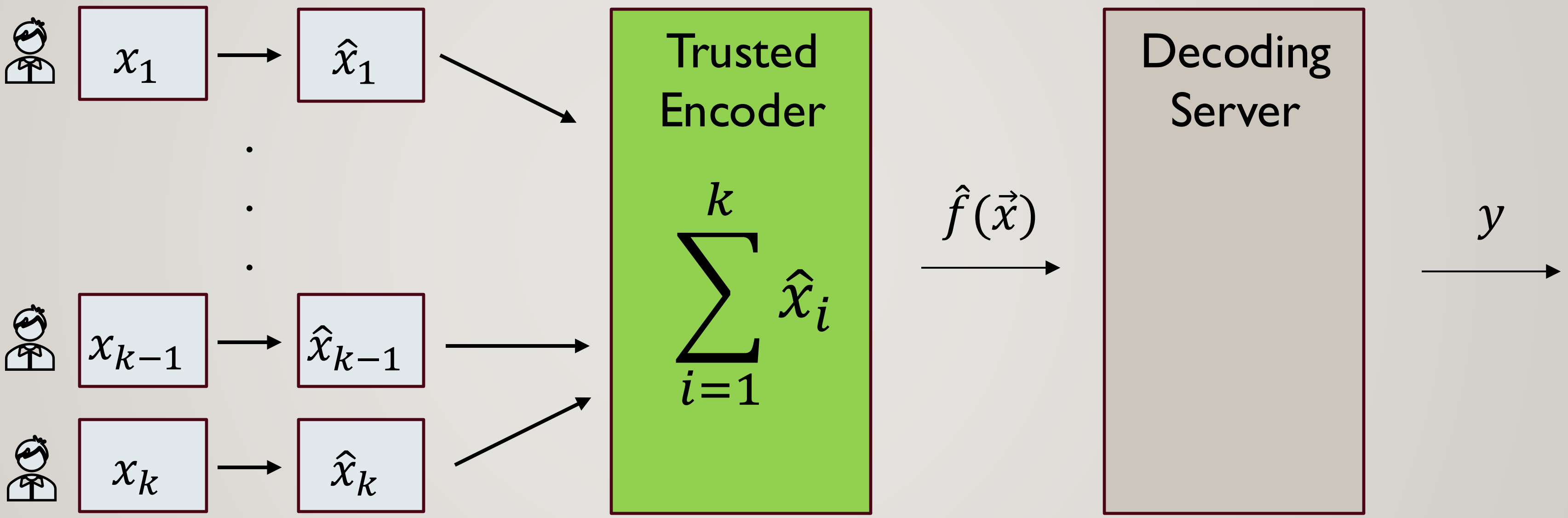

Correctness
$y=f(\vec{x})$

Security
Decoder learns nothing, but $f(\vec{x})$

*The summation is over some abelian group

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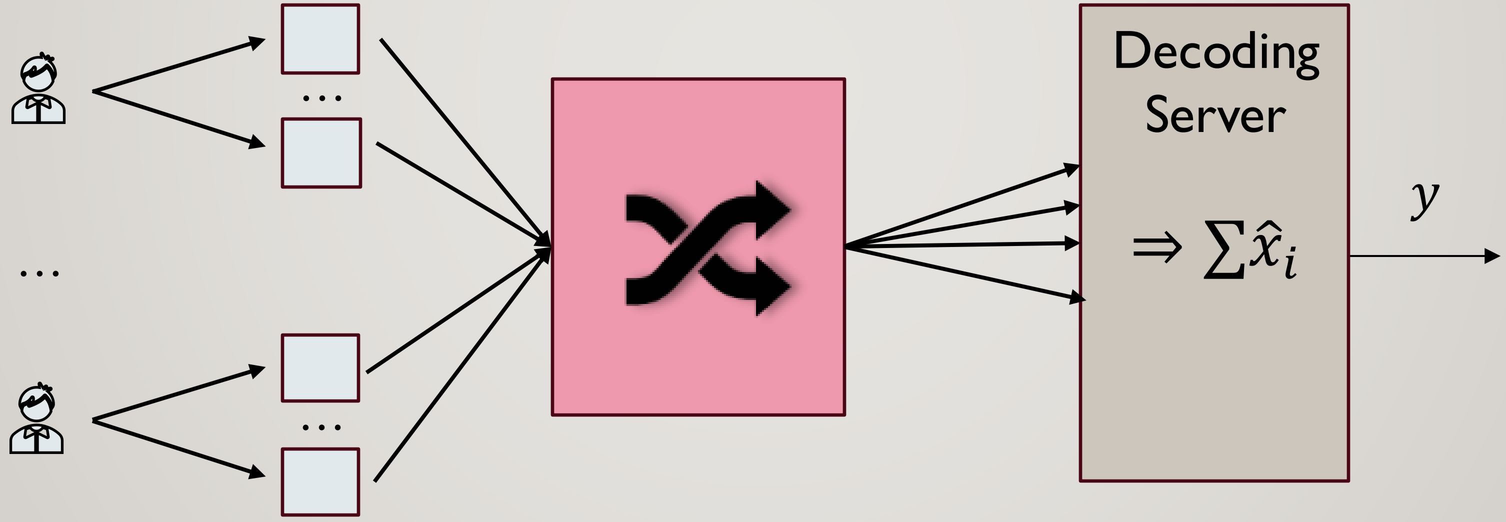

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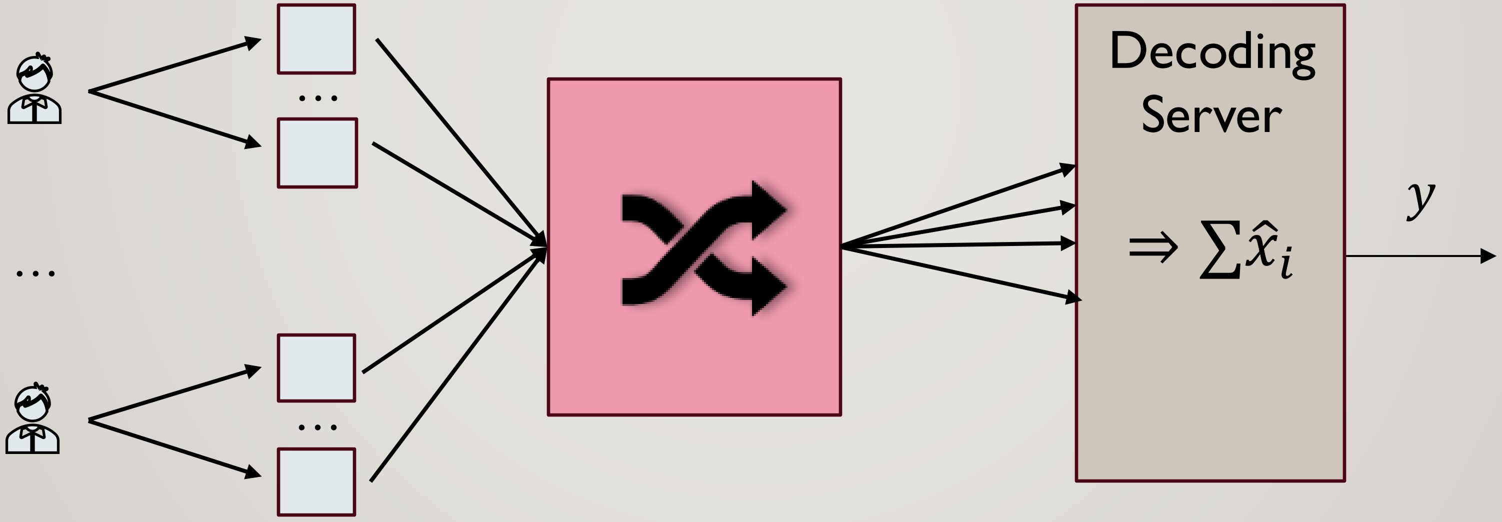
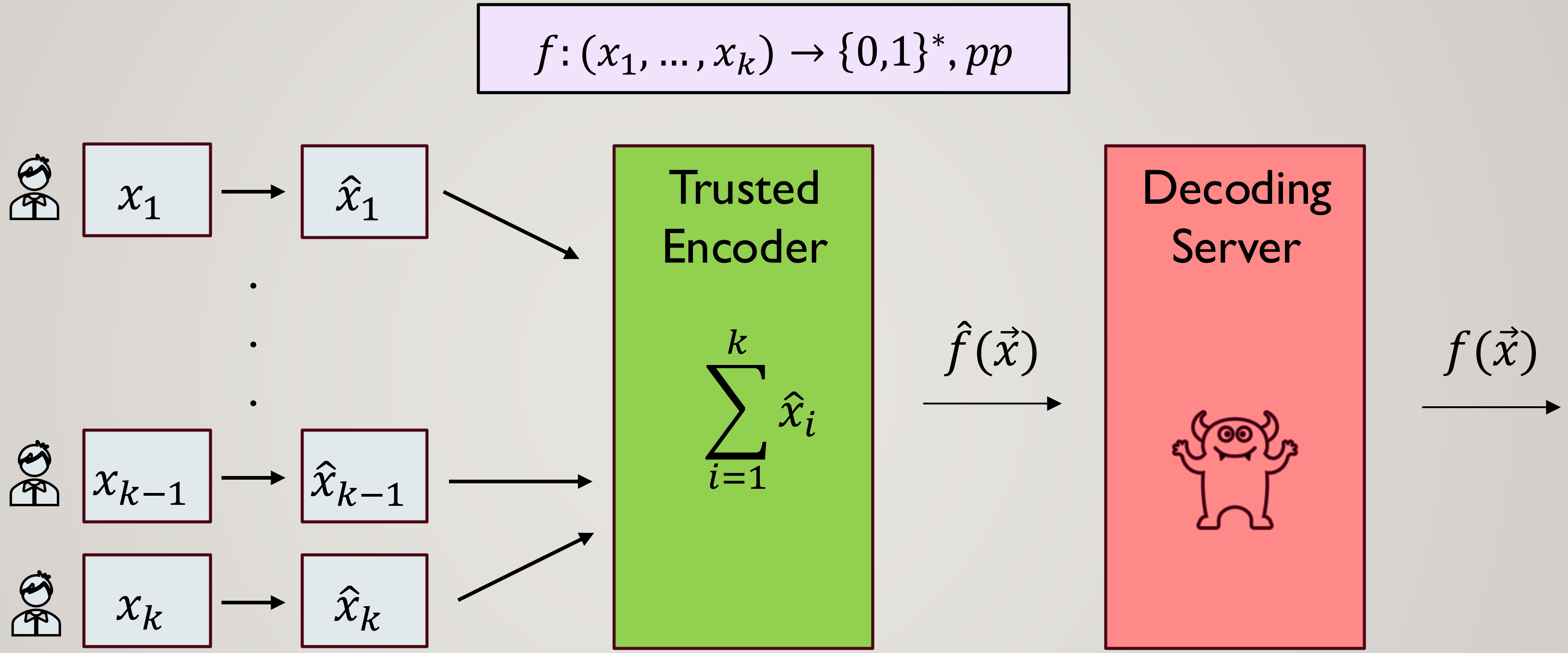


- Yields NI-MPC (in the shuffle model) w/o correlated-randomness, nor public-key-infrastructure.

ARE Security



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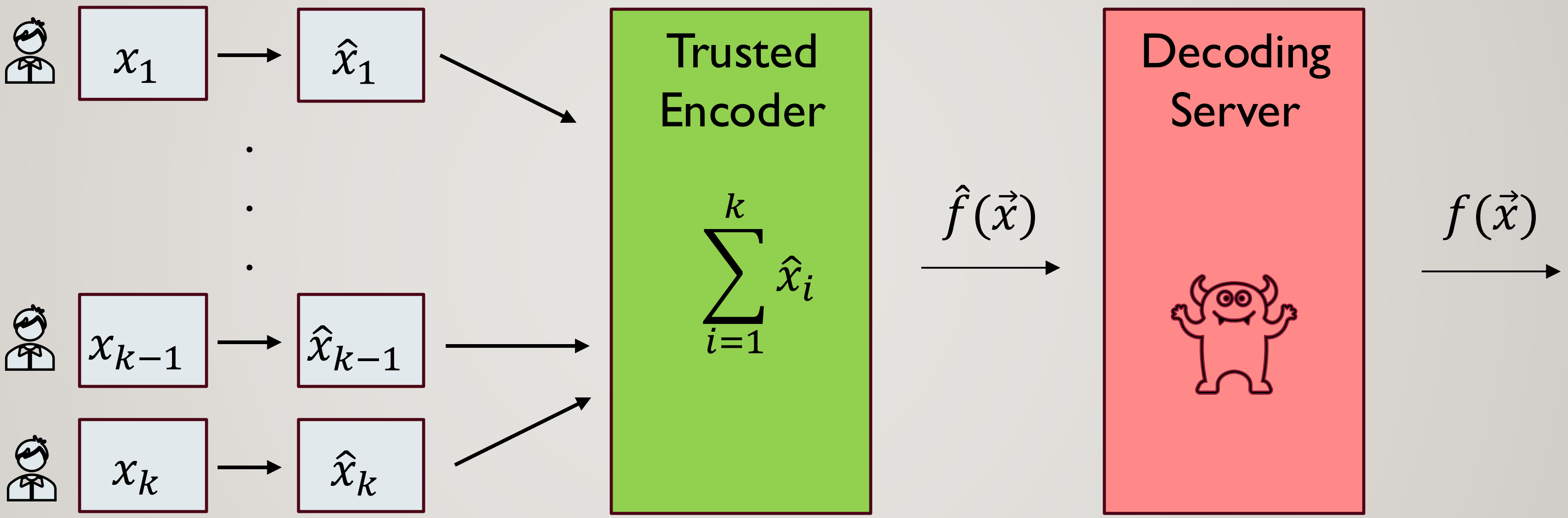

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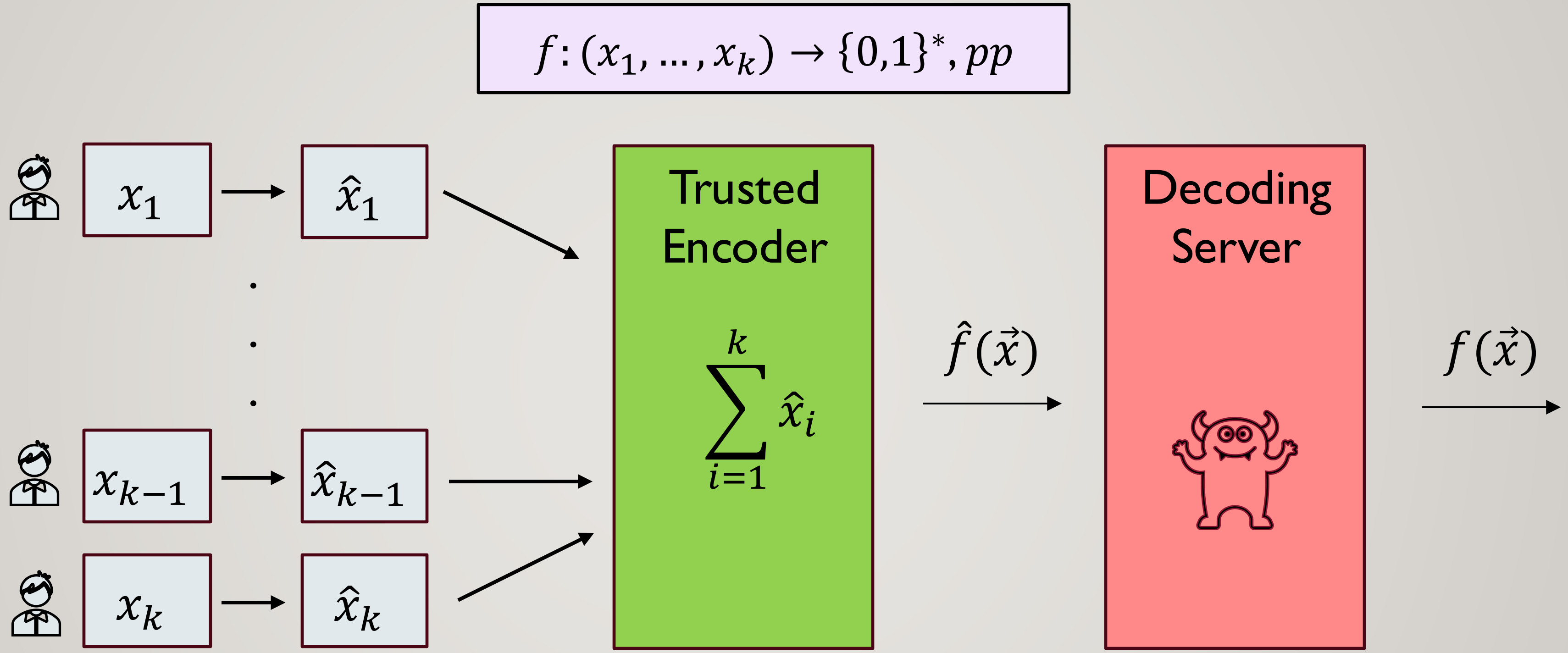

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- Intuitively: decoder's view $\left(\hat{f}(\vec{x}), pp\right)$ can be recovered from $f(\vec{x})$.
- Simulation: $Sim(f(\vec{x})) \approx \left(\hat{f}(\vec{x}), pp\right)$ (perfect / statistical / computational).

Our Focus: **Robust** ARE (RARE) [HIKR23]

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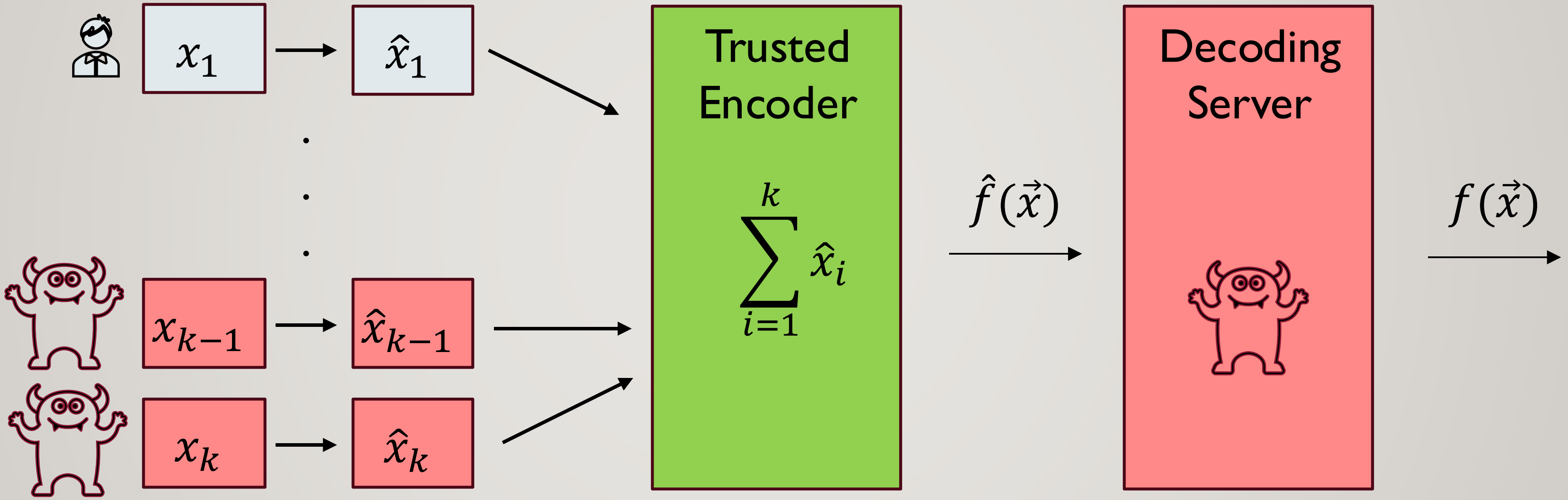

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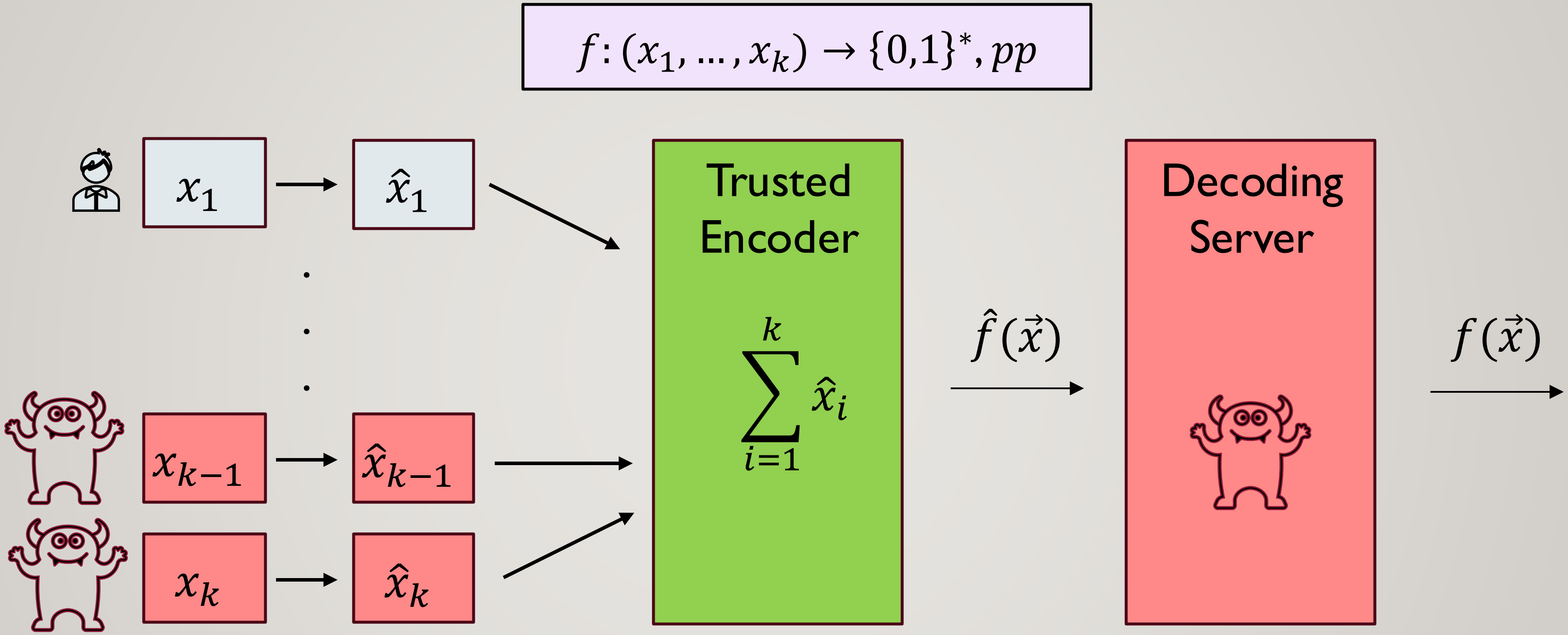

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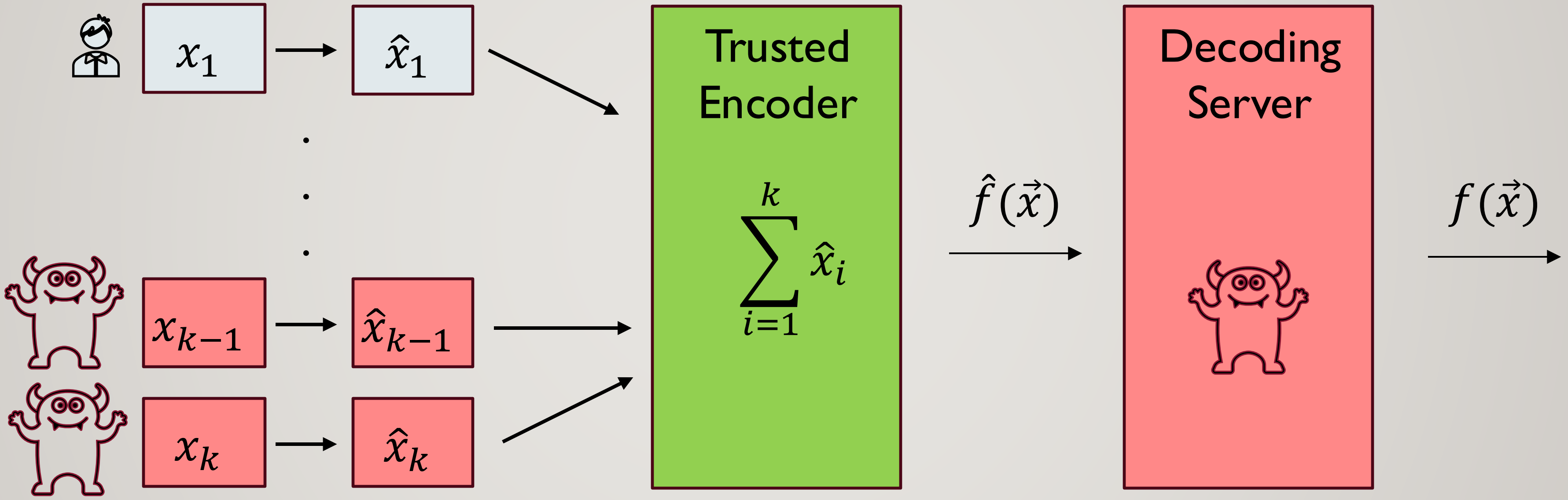

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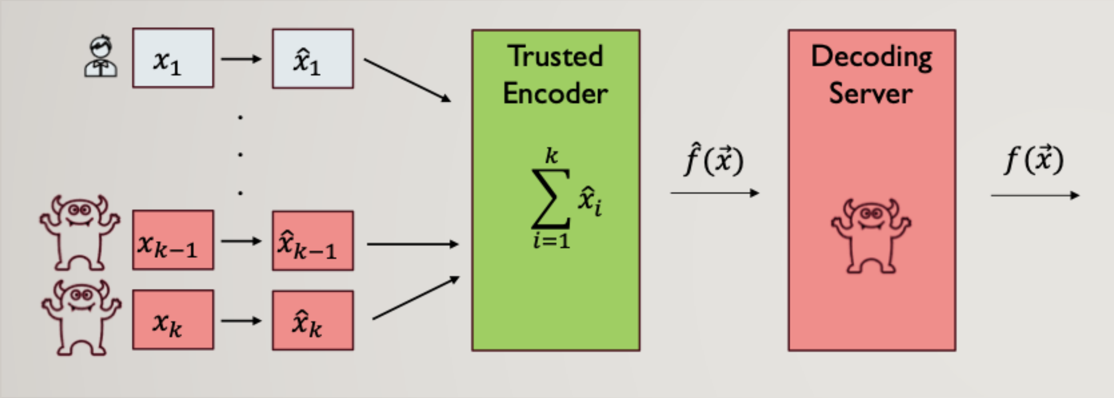
- Inevitable attack: **residual function** of honest parties H: $f_{x_H}(x_C) := f(x_H, x_C)$.
- VBB simulation security, $Sim^{f_{x_H}}(f(x)) \approx (\hat{f}(x), pp)$

RARE implies Obfuscation



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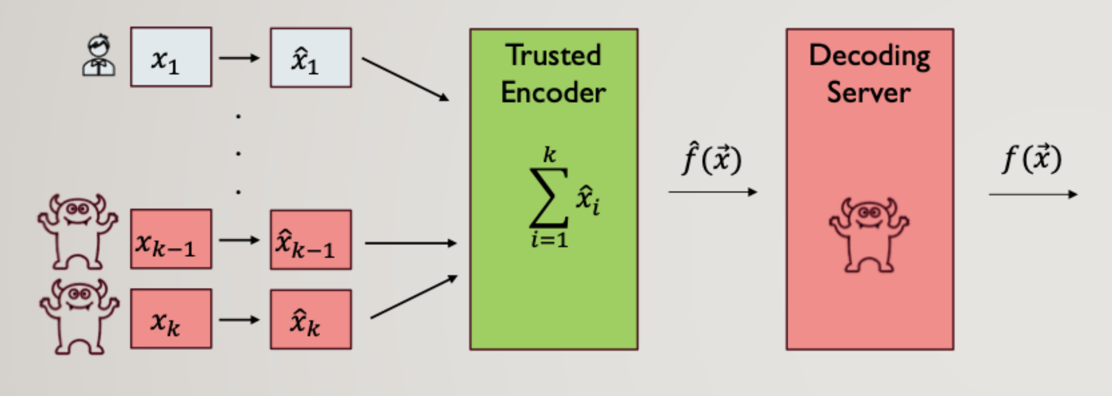

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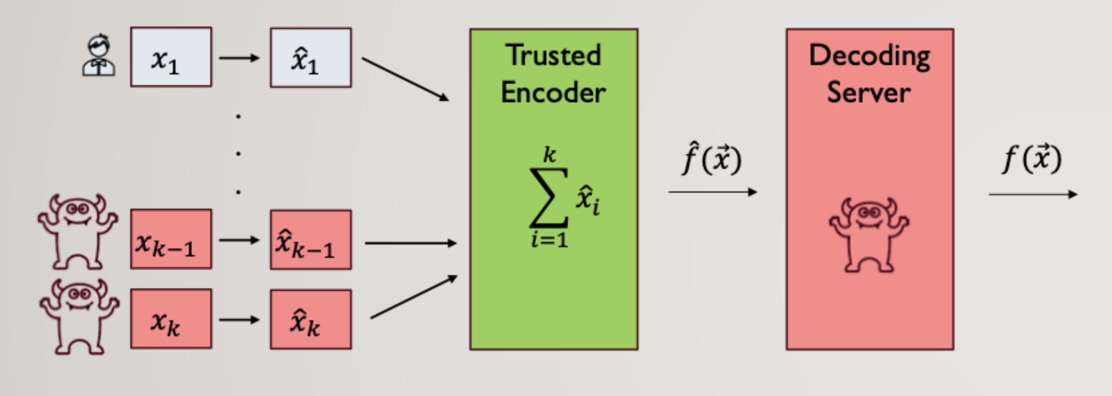

<u>Our focus</u>: **Indistinguishability** security:

For every $\vec{x}_H, \vec{w}_H$, with $f_{\vec{x}_H} \equiv f_{\vec{w}_H}$,

$$\left(pp, \hat{f}(\vec{x}_H)\right) \approx_c \left(pp, \hat{f}(\vec{w}_H)\right)$$

Implies iO

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Open question:

Can we construct **indistinguishability-based R**ARE for all efficient functions from IO and standard cryptographic assumptions? (in the plain model)

Our Results

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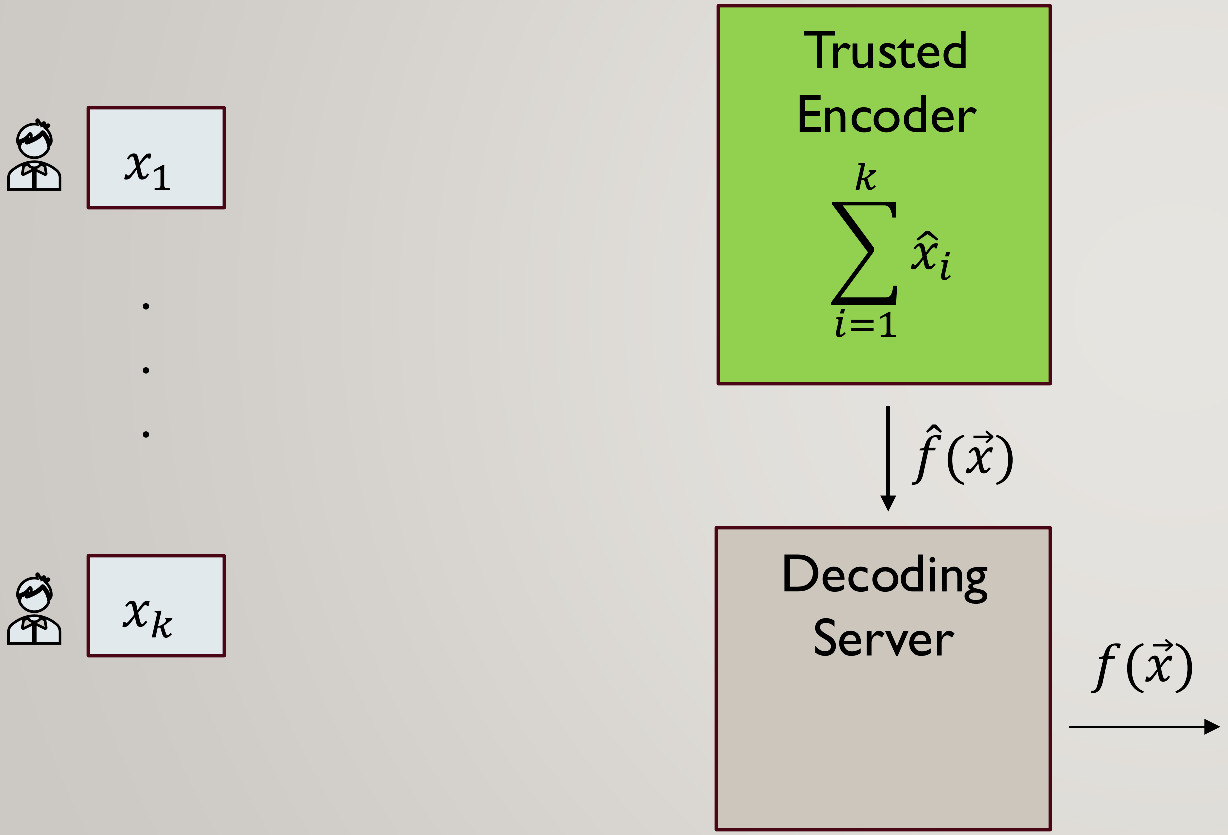
3. Our **RARE** is **succinct** (more in next slide).

Succinct RARE



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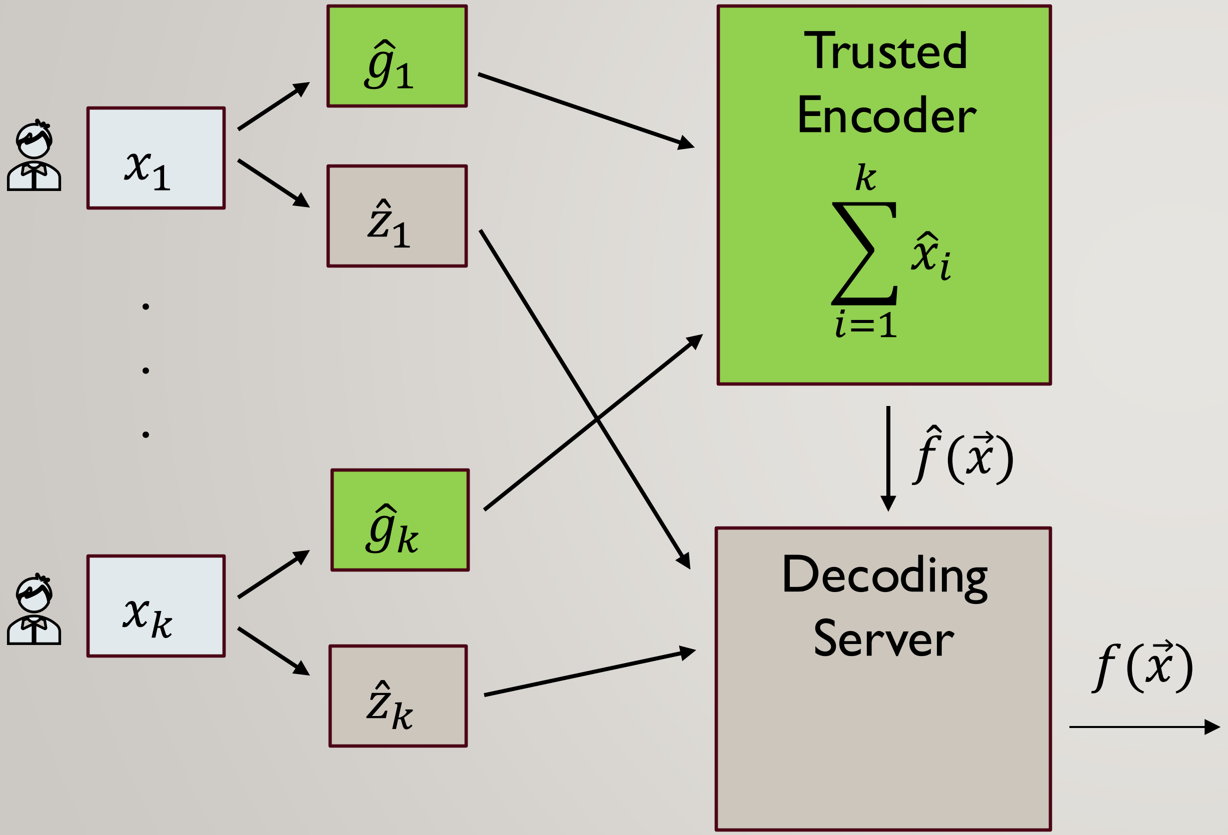

Split parties' encodings $\hat{x}_i = (\hat{z}_i, \hat{g}_i)$.

- $\hat{g}_i$ group element, $\hat{z}_i$ non-interactive part.

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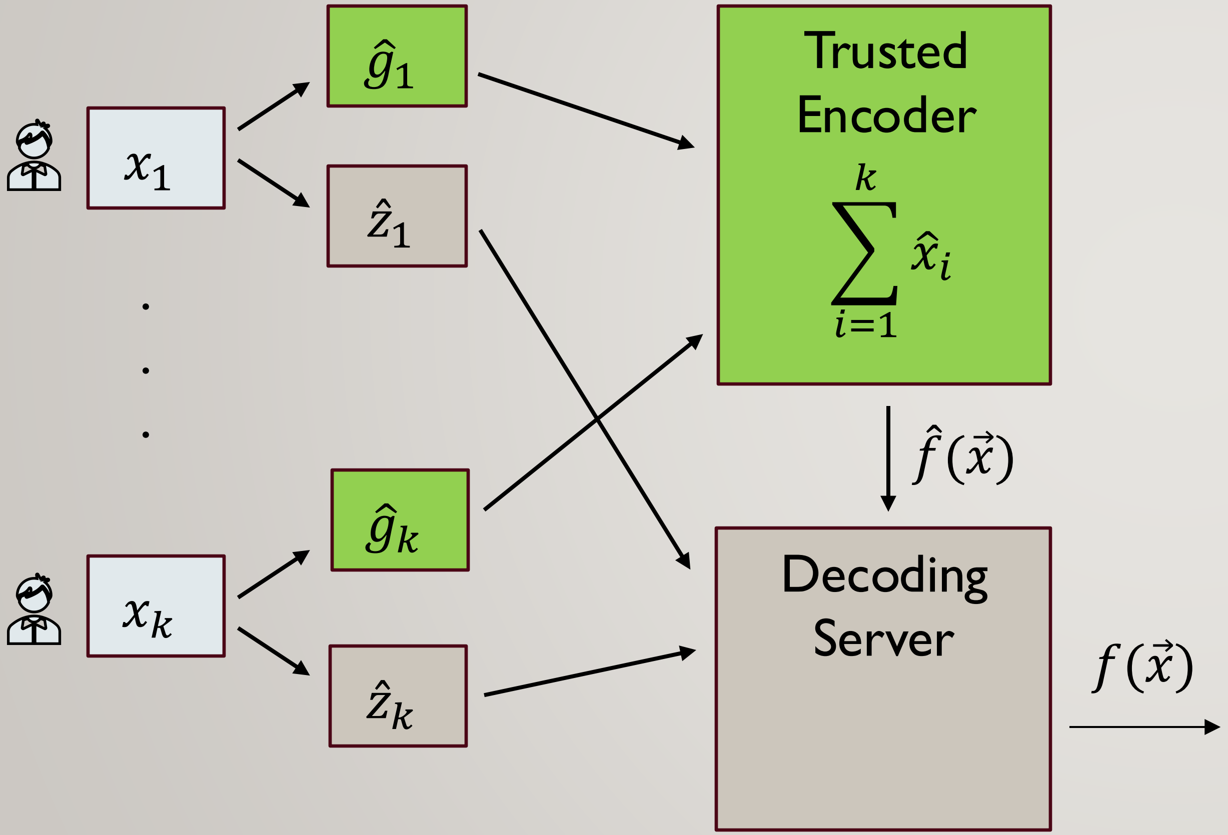

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  - $|\hat{g}_i| \sim$ **security parameter**.
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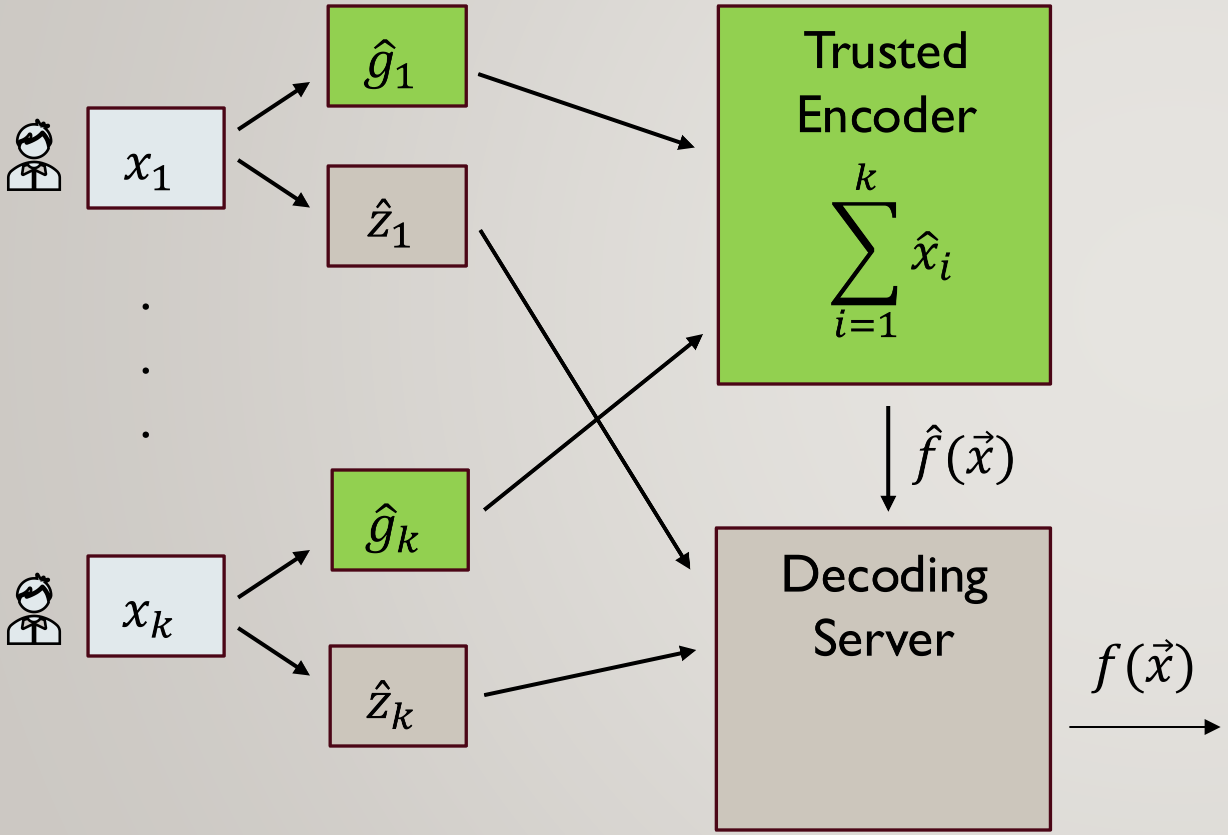

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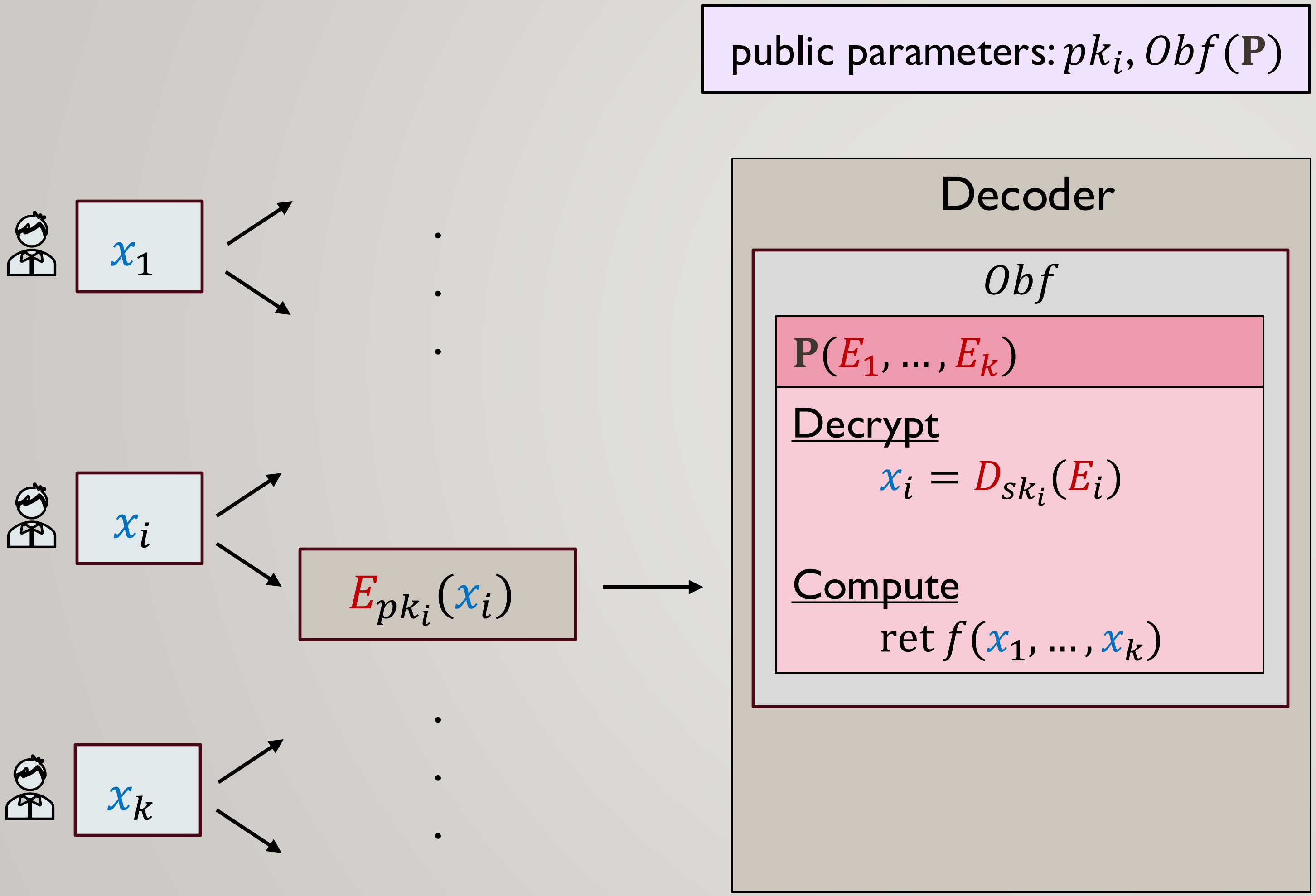
- Independent of $k, |f|$
- Trusted computation is minimal.

And now, the construction

Warmup: Ideal Obfuscation



Problem:
Can change subsets of the honest parties' inputs.

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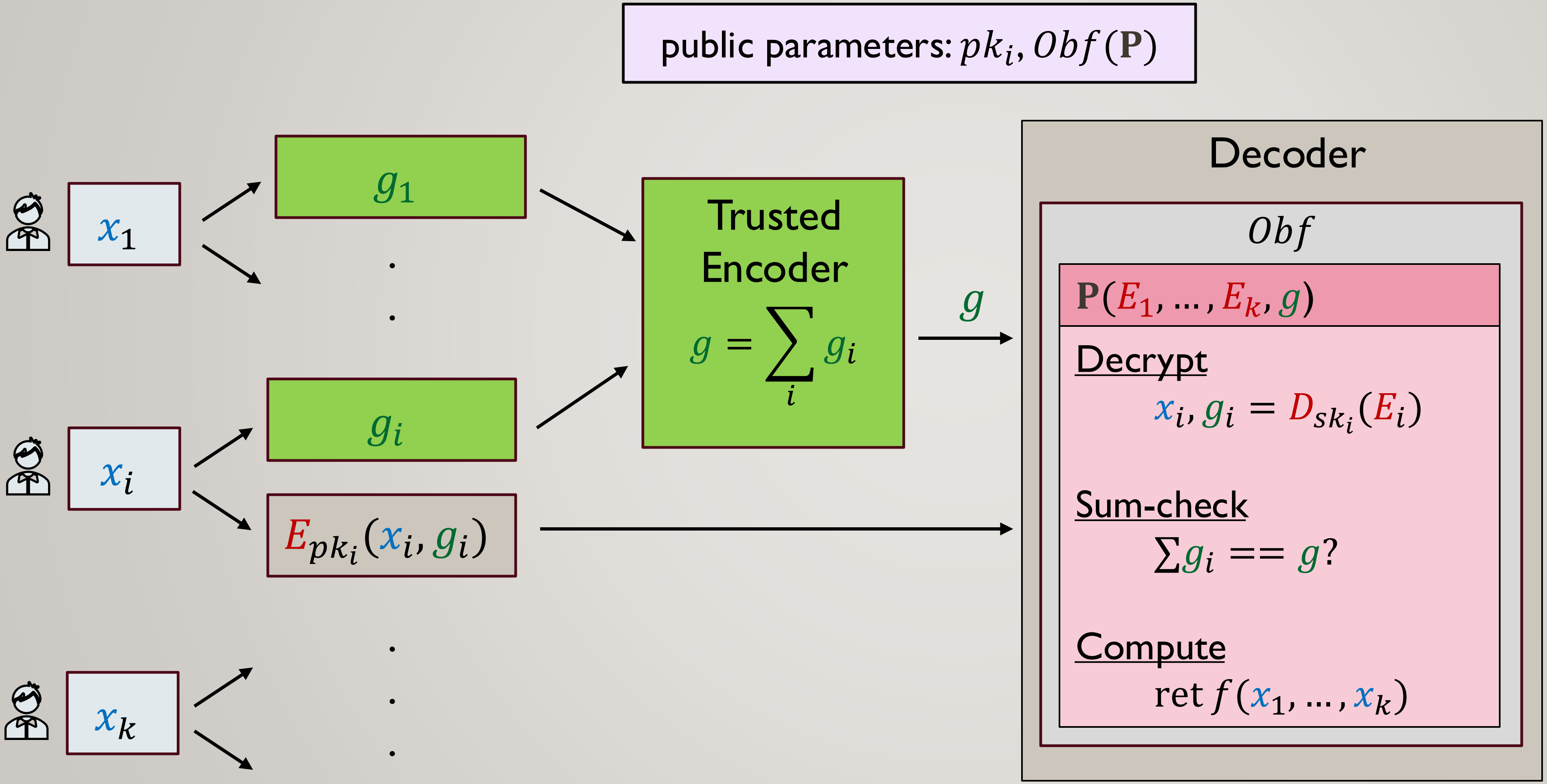

- Idea: "glue" honest parties together.
- Add additive elements g_i.

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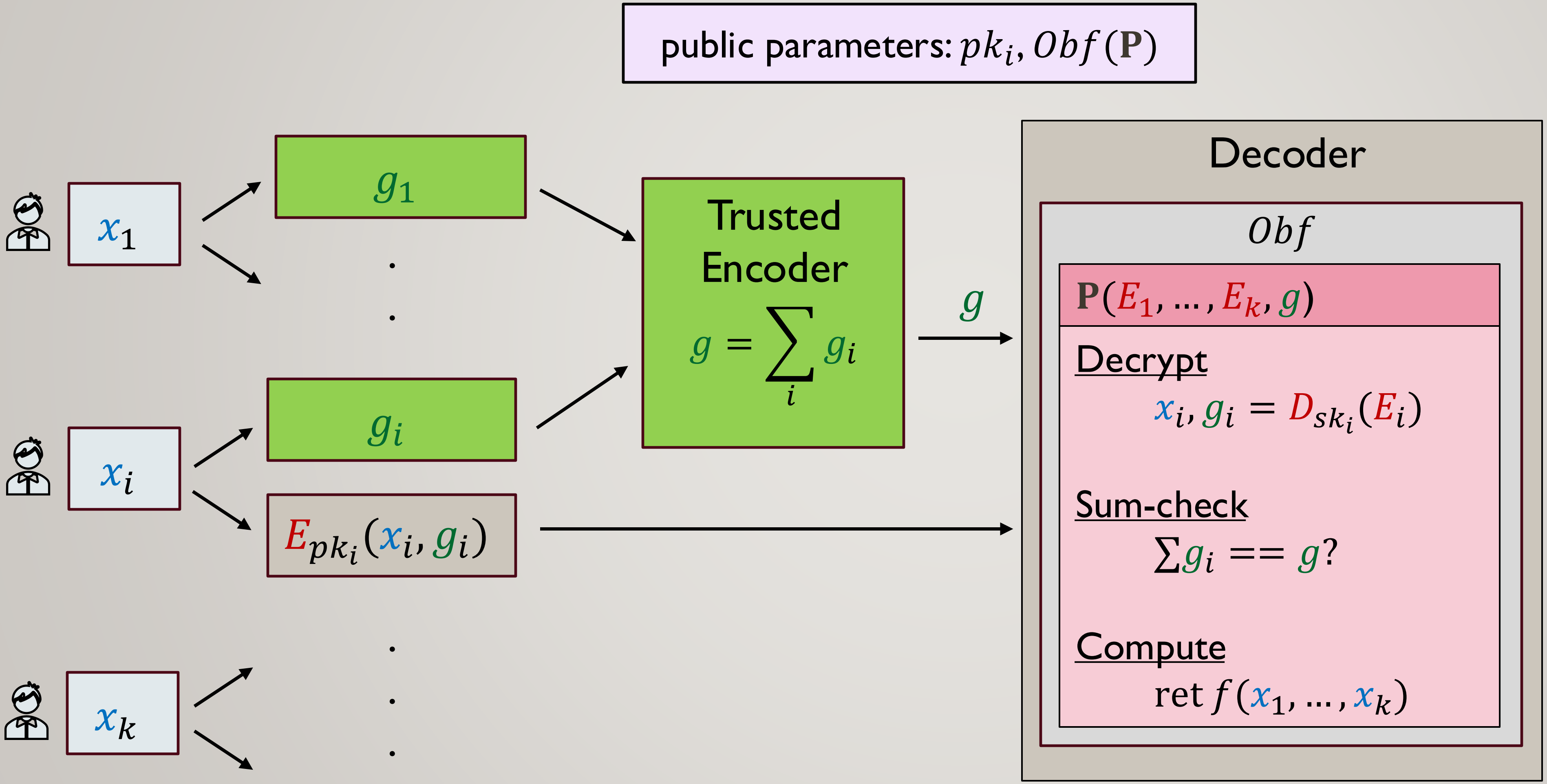

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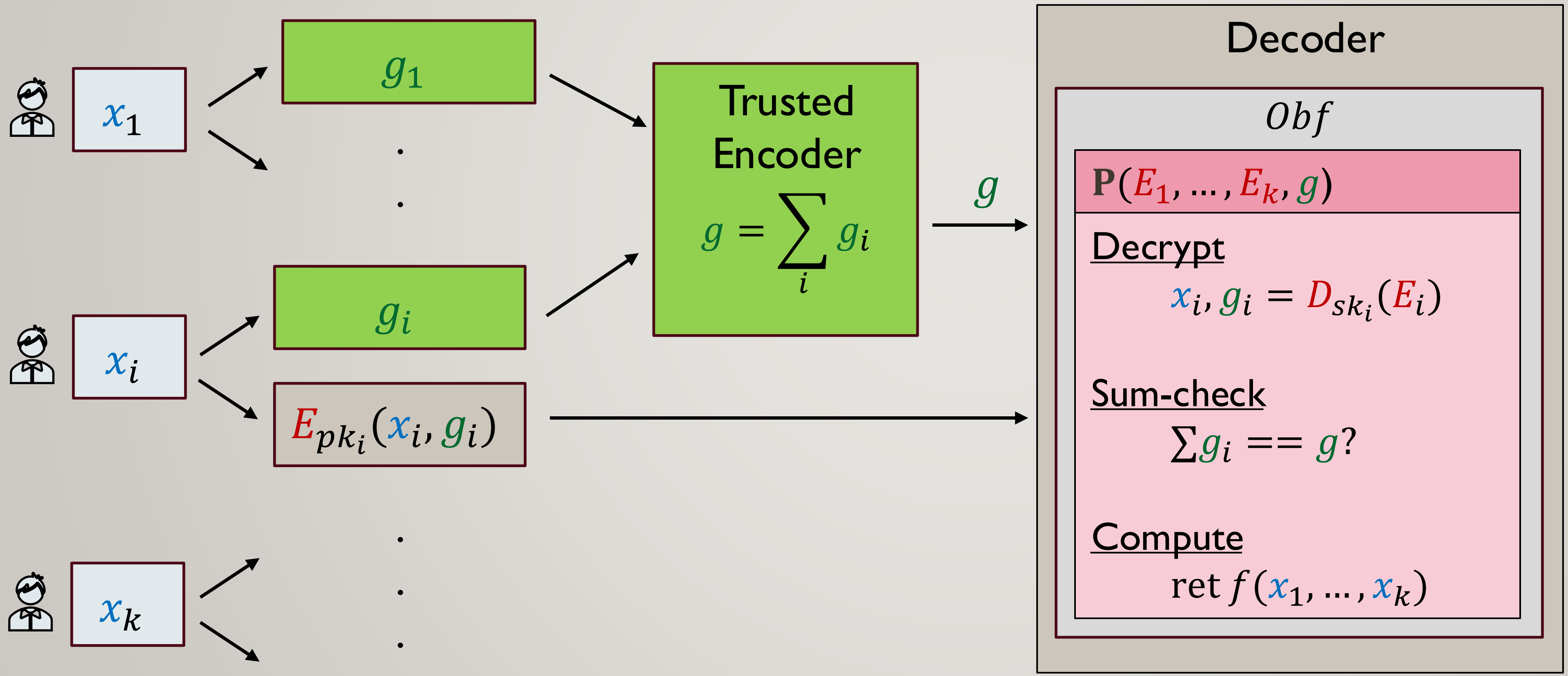
public parameters: $pk_i, Obf(\mathbf{P})$



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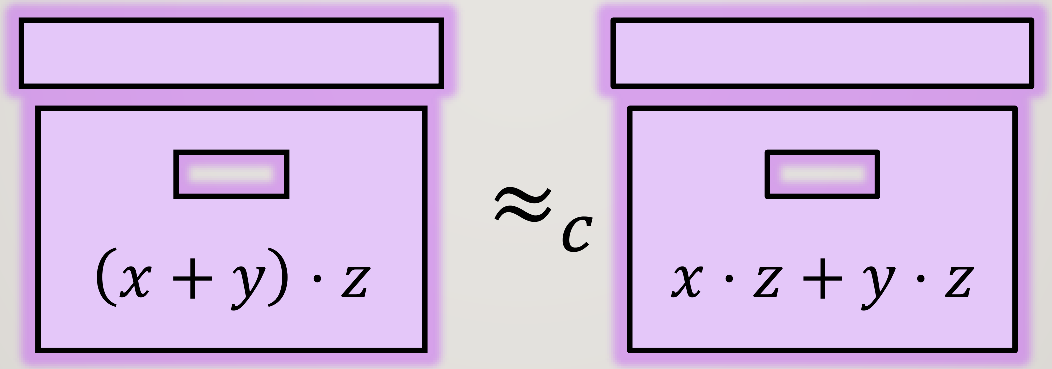
- Add additive elements g_i.
- Non-malleable encryption (CCA2-secure).
- Secure in the ideal-obf model.

Moving To iO

$$f_1 \equiv f_2 \Longrightarrow iO(f_1) \approx_c iO(f_2)$$



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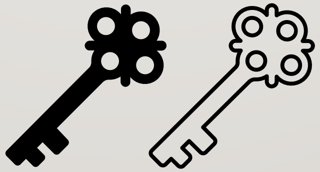
- Solution: Naor-Yung **double** encryption with a Statistically-Simulation-Sound NIZK [Sah99].

$$E(m) = (E_{left}(m), E_{right}(m), \Pi_{NIZK})$$



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Problem:

If $\exists$ **subset** $F \subsetneq H$: $f(\vec{a}_F, \cdot) \not\equiv f(\vec{b}_F, \cdot)$

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different functionalities (might be hard to find).

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3. Can construct from wither LWE or DDH

More details in the paper…

Future Direction - ARE

Improving **R**ARE
1. Simpler **public parameters**? no setup at all?
2. Assumptions **lighter then iO** for limited classes of functions.

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