Polymath:
Groth16 Is Not The Limit

Helger Lipmaa, University of Tartu, Estonia

Crypto 2024 Presentation

ZK-SNARKs

Computation: f
Public input (statement) $\mathbb{X}$
Private input (witness) $\mathbb{W}$

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Proof π that $f(\mathbb{X}, \mathbb{W}) = 1$

- Completeness
- Knowledge-soundness
- Zero-knowledge
- Succinct arguments

Landscape

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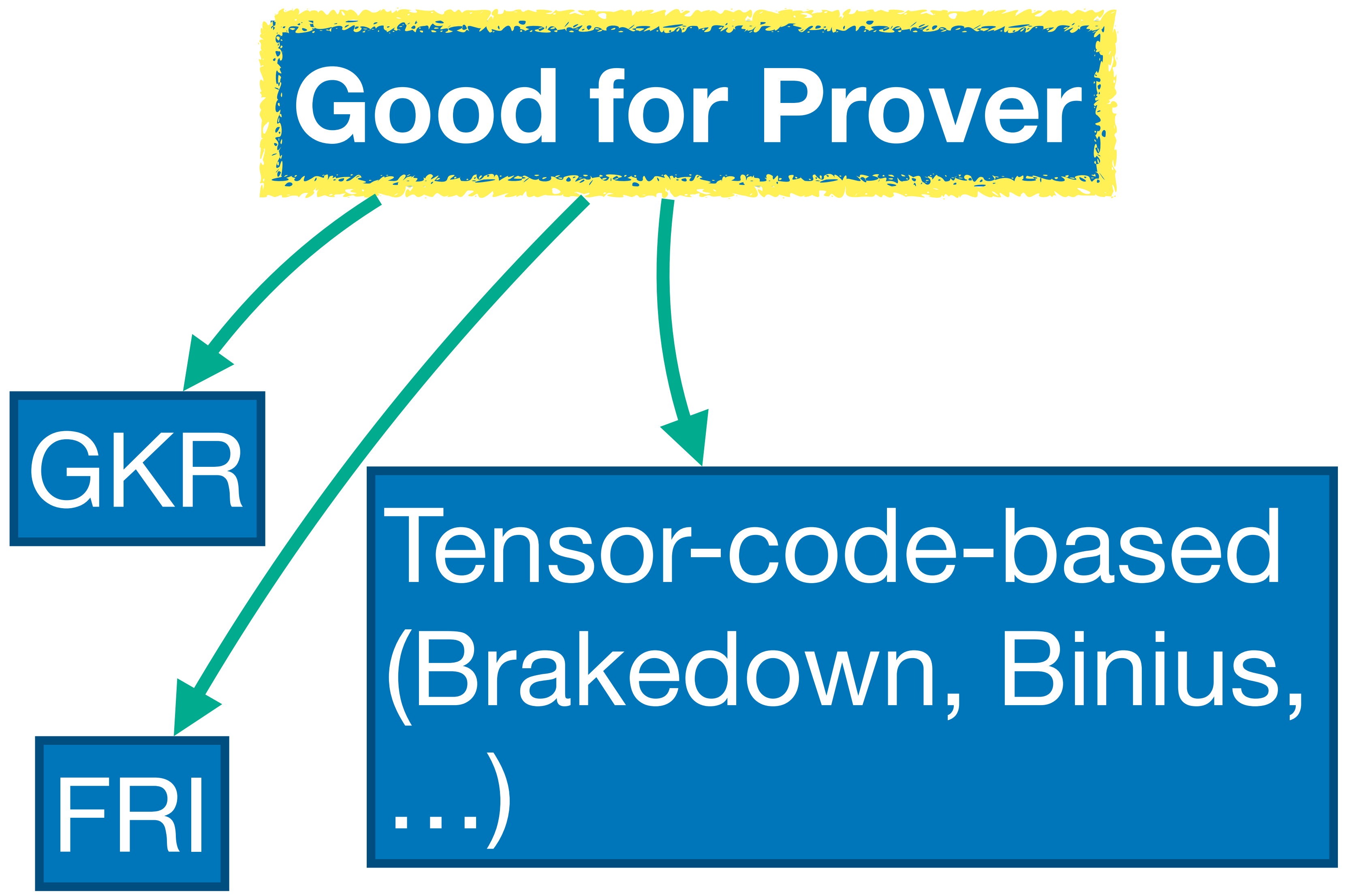

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FRI
Tensor-code-based (Brakedown, Binius, …)
Good for Verifier
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Plonk
Groth16

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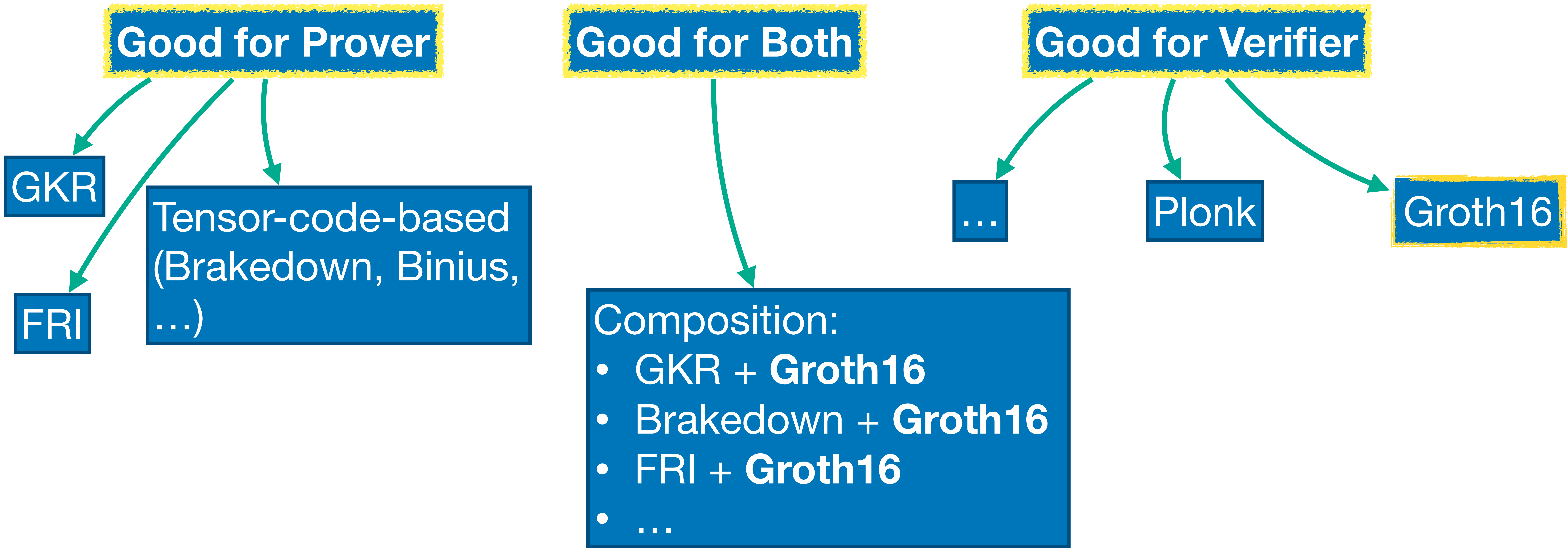
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Landscape

Huge progress in zk-SNARK land in last 5 years
Groth16 still lands supreme after 8 years
- Shortest argument
- Fastest verifier

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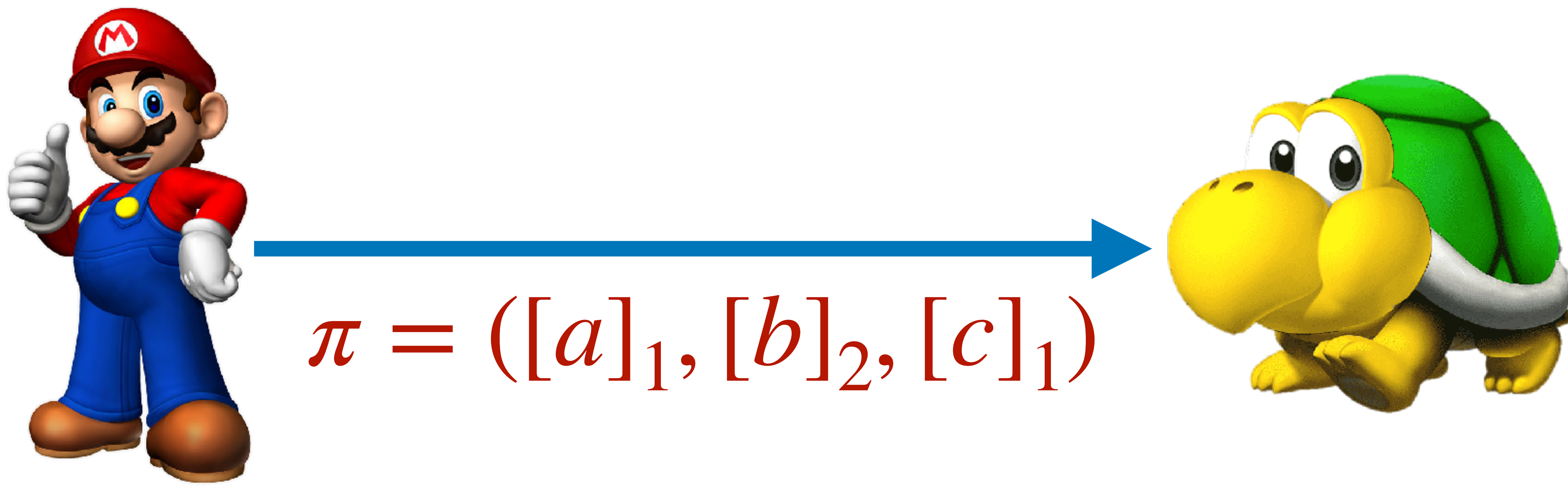
- Verifier executes three pairings and $|\mathbb{X}|$ group ops

On Optimality



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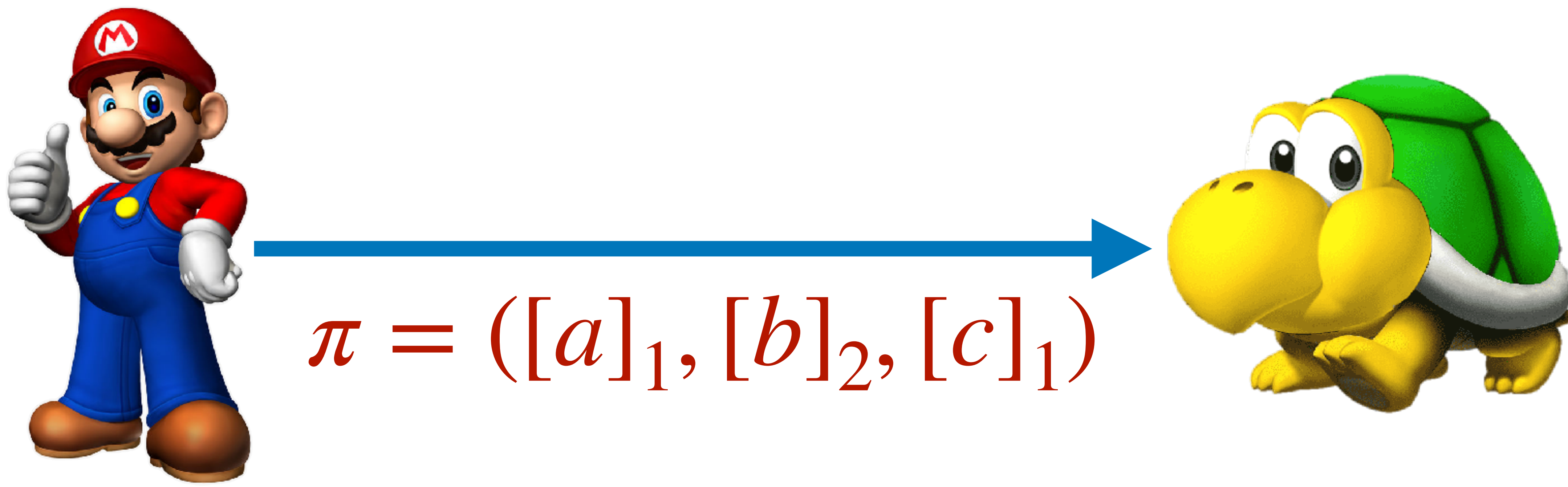

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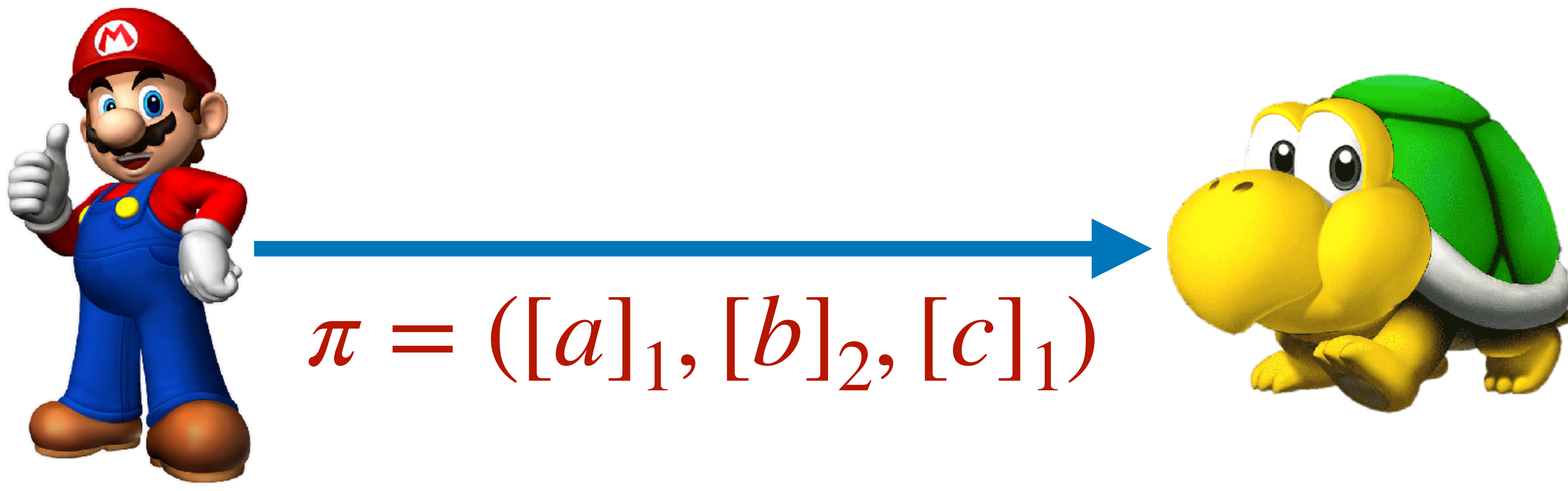

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- It talks about **#group elements**, not **bit-length**

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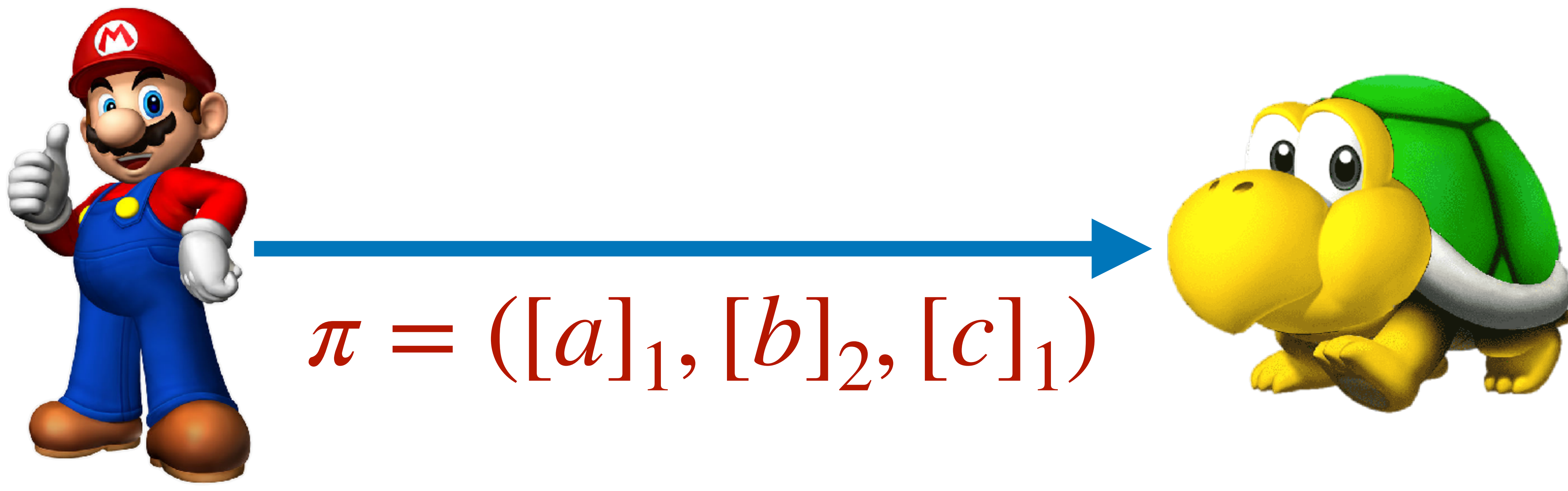

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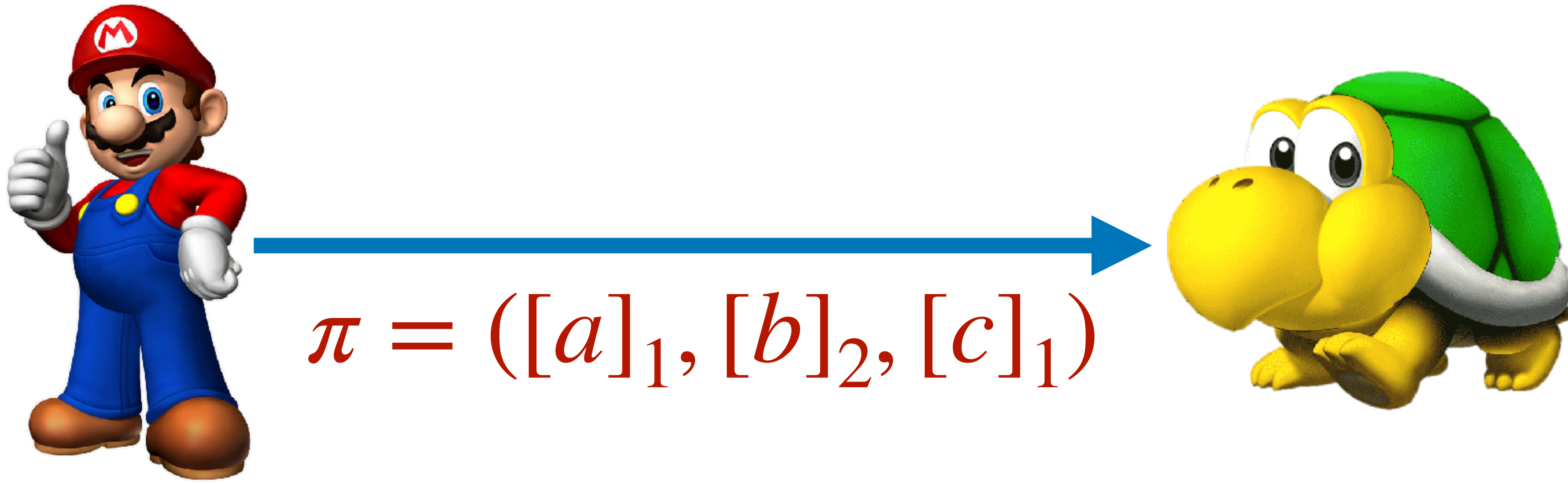

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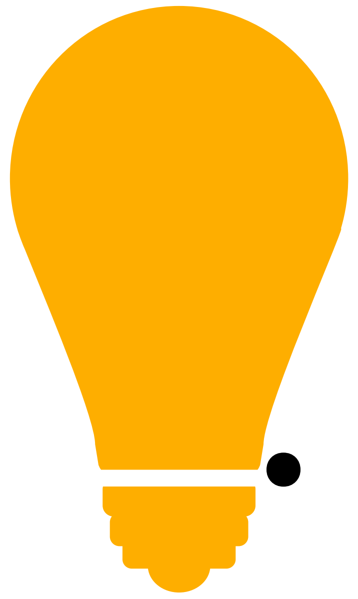

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Problem:

- we still have $[b]_1$ in the argument!
- $\ell([b]_2) < \ell([b]_1) + \ell(\bar{b}) + \ell([h]_1)$ in 128-bit level

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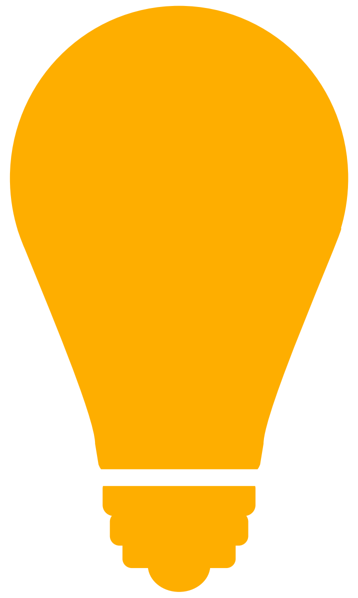

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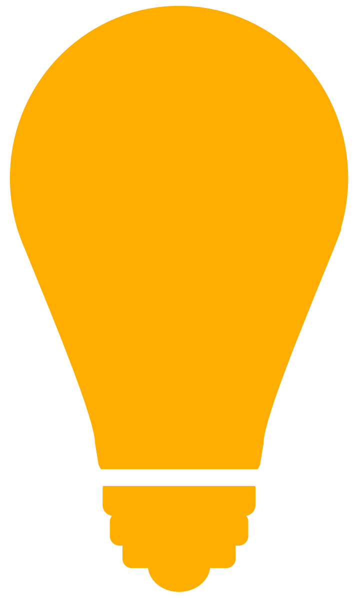

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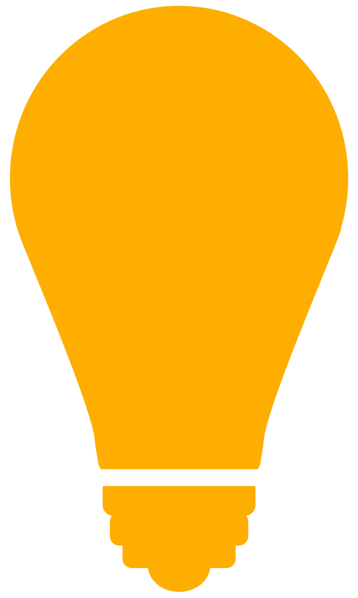

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Problem:
- Groth16 has five trapdoors, KZG is univariate
- Not clear how to use KZG

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Problem:

- even after exhaustive search, the exponents i are quite large
- KZG prover time Ω(polynomial degree)
  - => Results in high prover complexity

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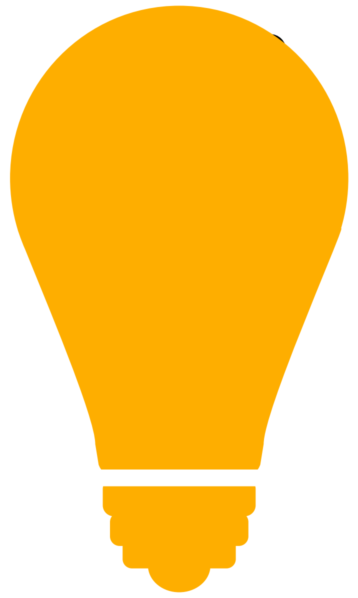

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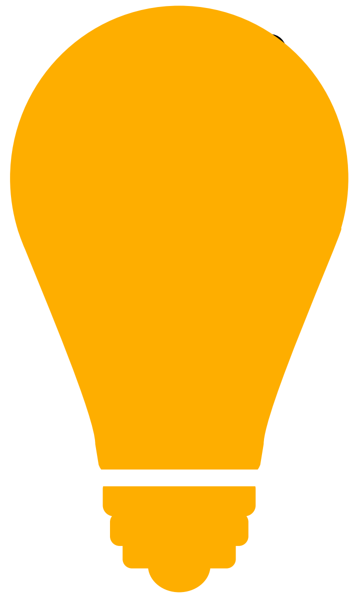

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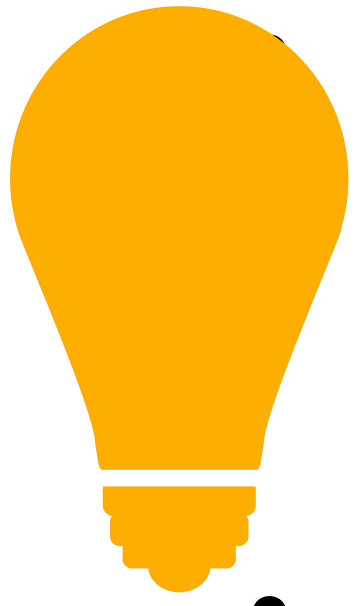

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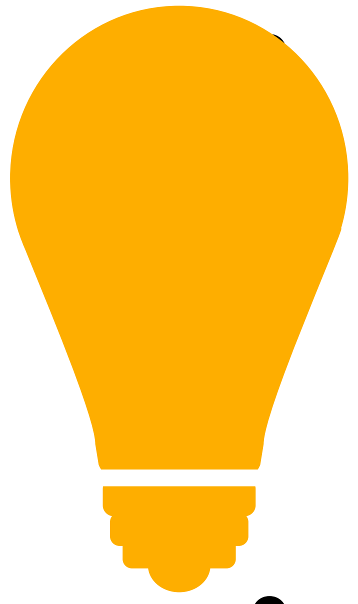

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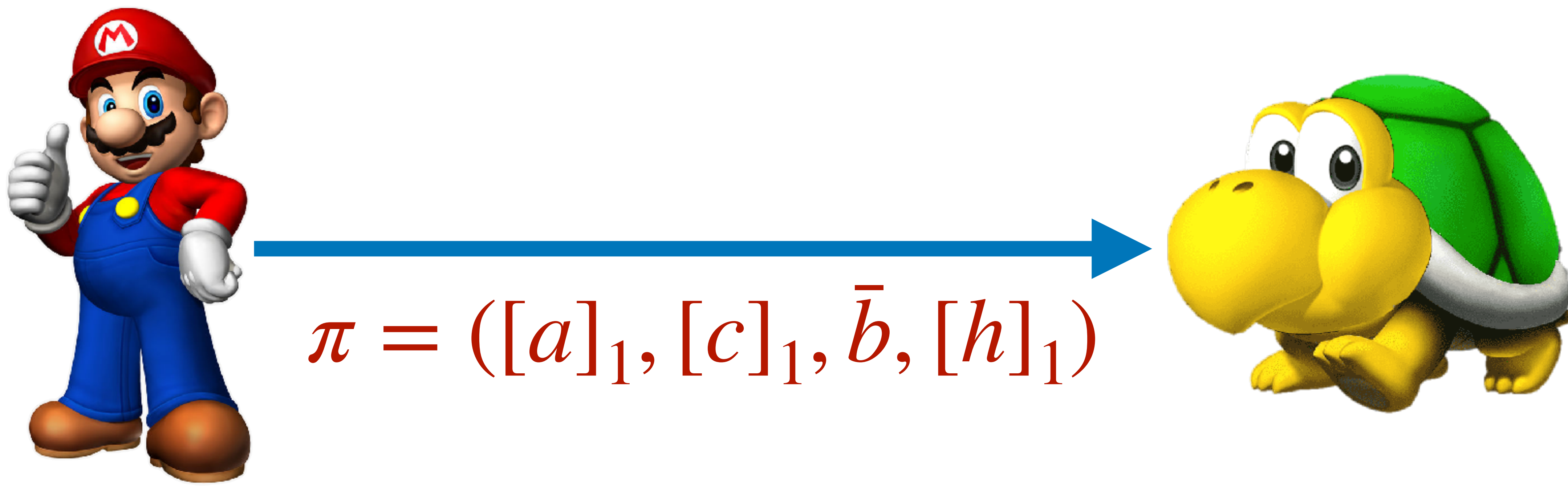

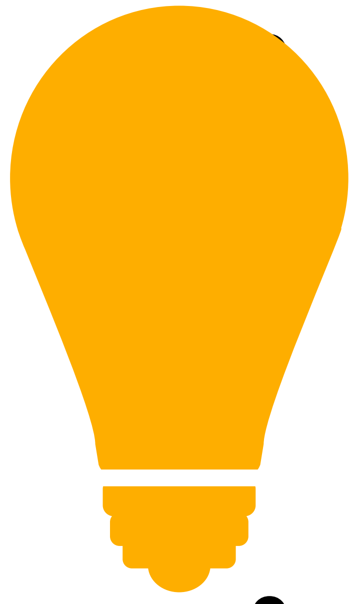

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  - In Polymath, V interpolates a polynomial in $\mathbb{F}_p$ of degree $|\mathbb{X}|$
    - Instead of doing $|\mathbb{X}|$-long MSM in Groth16

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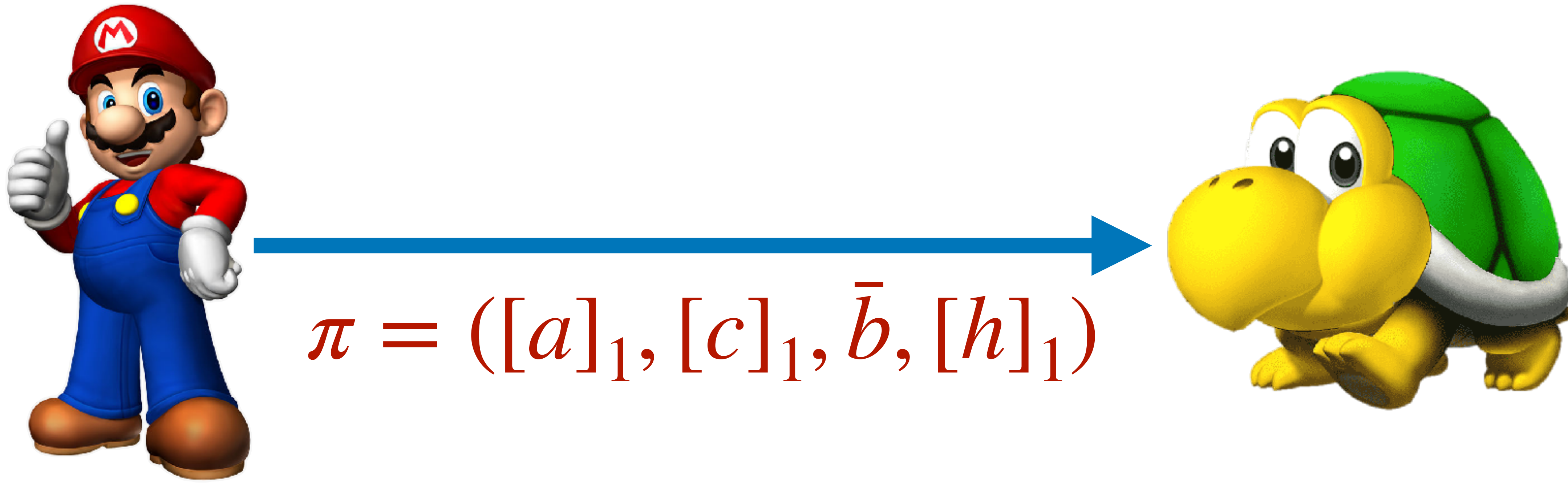

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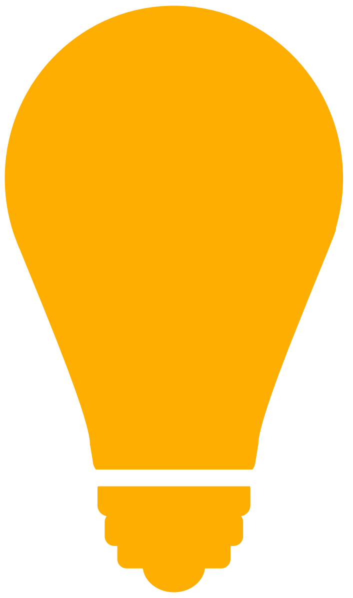

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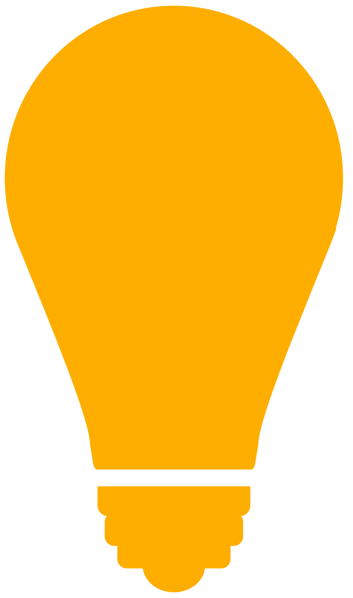

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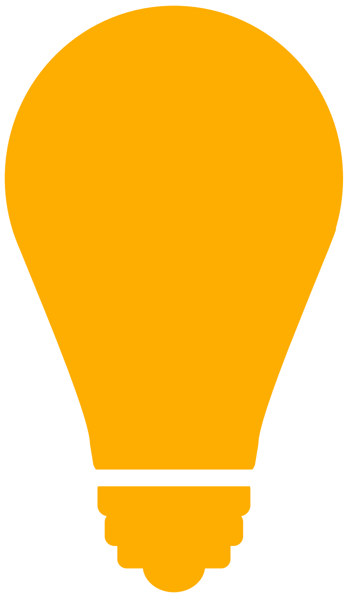

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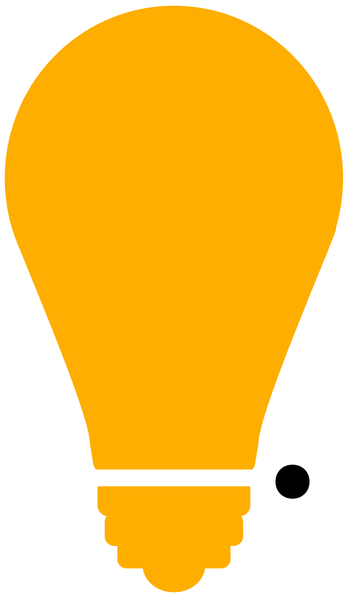

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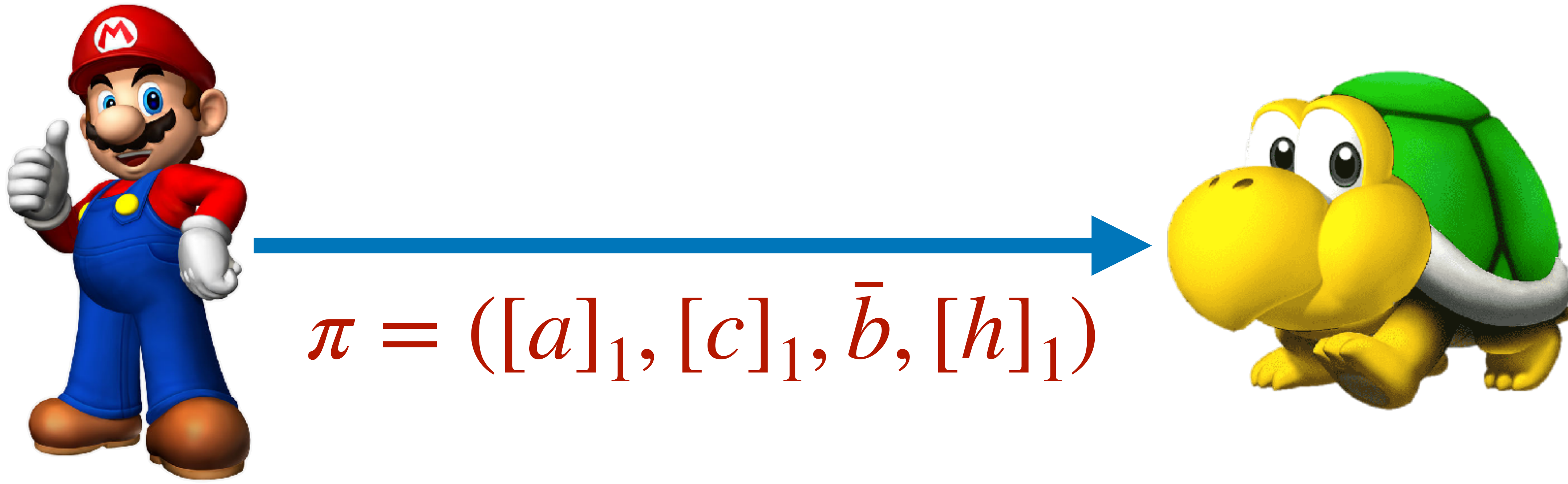

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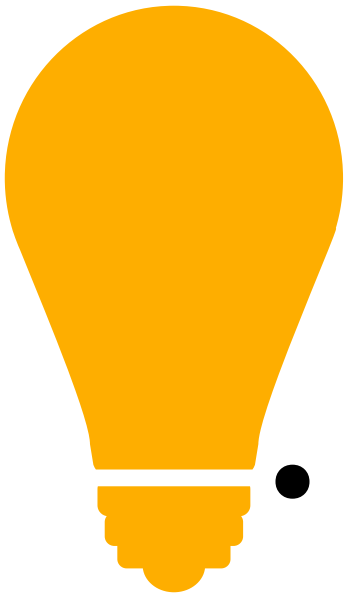

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Problem:
- SRS is circuit-dependent
- It does not contain enough elements to compute $[h]_1$

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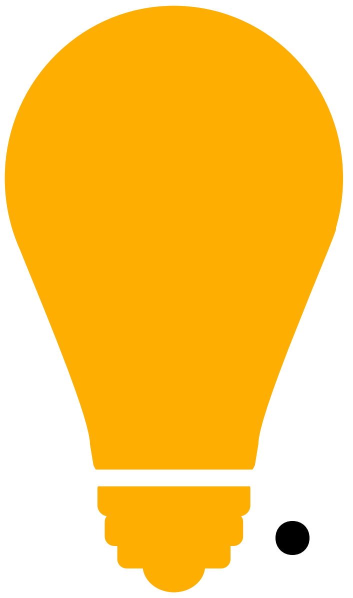

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Part of Polymath's proof is machine-checked

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