Laconic Function Evaluation and ABE for RAMs from (Ring-)LWE

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Crypto 2024

Laconic Function Evaluation (LFE)

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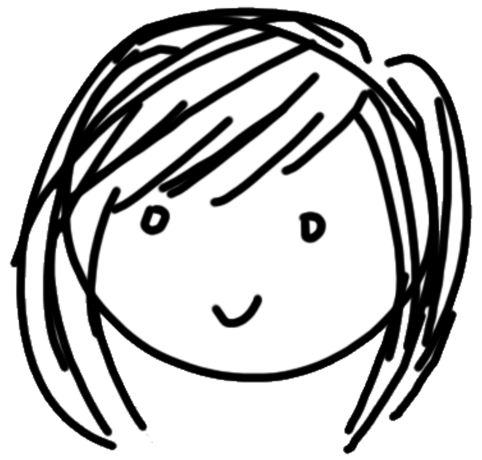
C



x

Laconic Function Evaluation (LFE)

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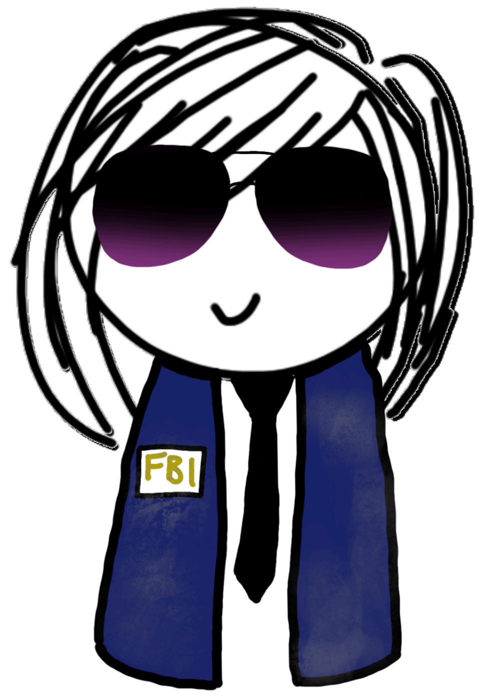

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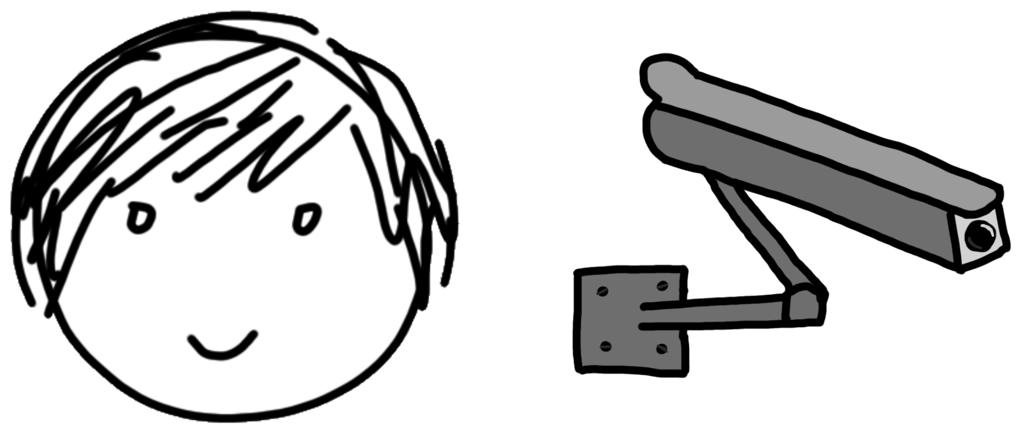
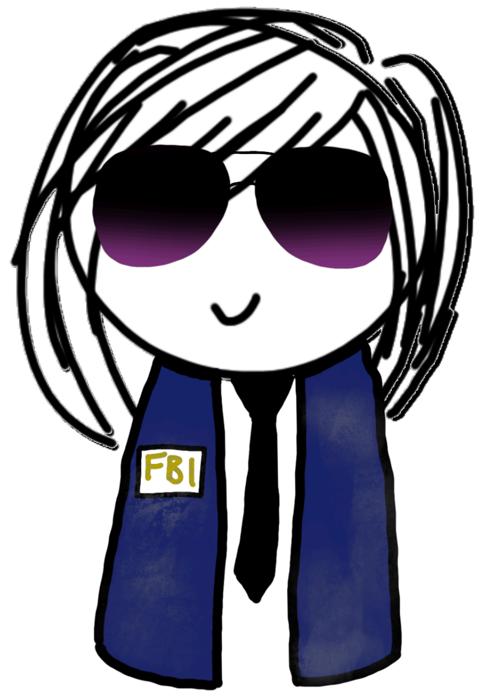


Laconic Function Evaluation (LFE)

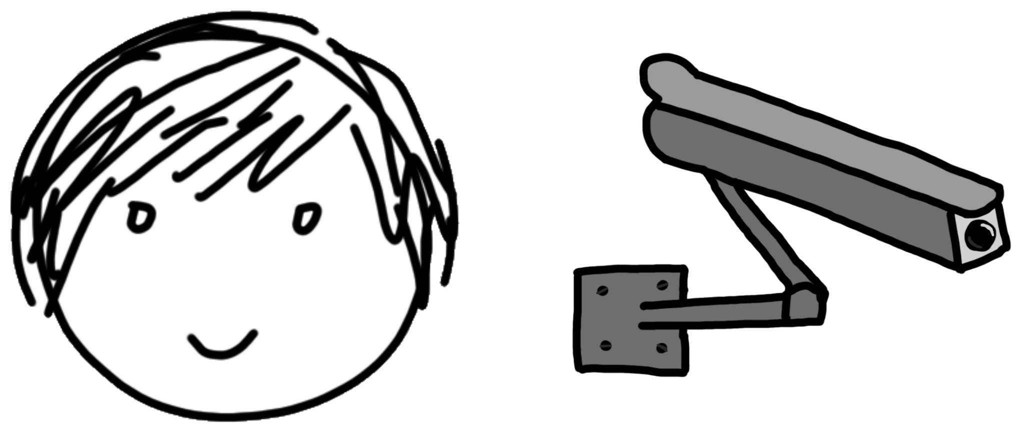
* in CRS model, CRS hidden

C



$\mathrm{dig} = \mathrm{Hash}(C)$ →

x

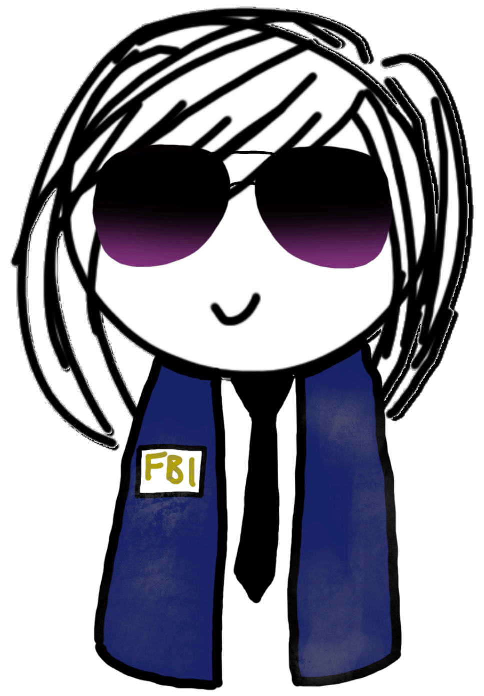


Laconic Function Evaluation (LFE)

* in CRS model, CRS hidden

digest very short compared to $|C|$

C



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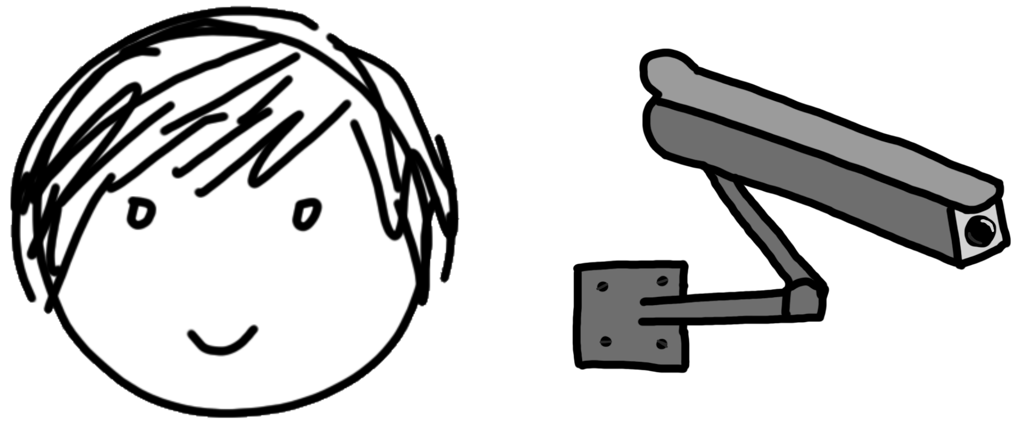

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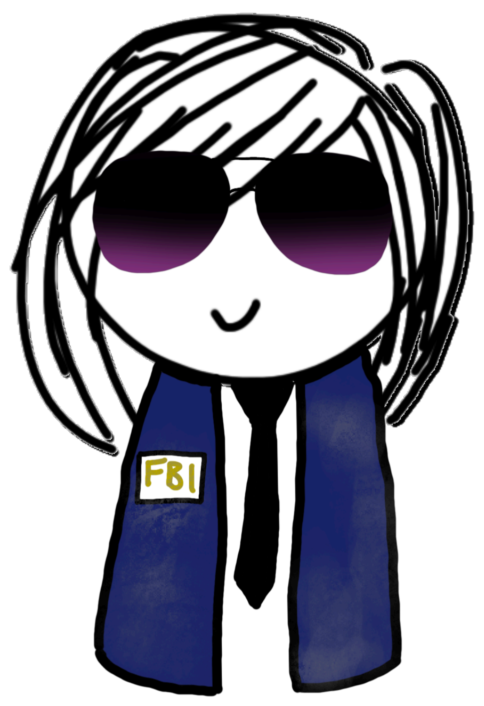

$\mathsf{dig} = \mathsf{Hash}(C)$

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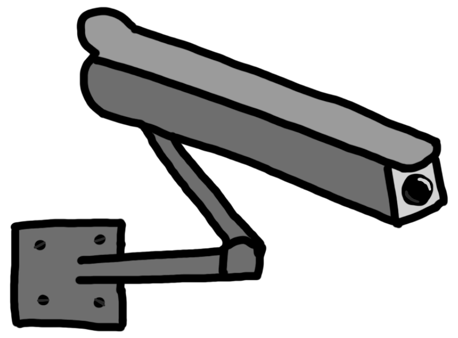

$\mathsf{ct} \leftarrow \mathsf{Enc}(\mathsf{dig}, x)$

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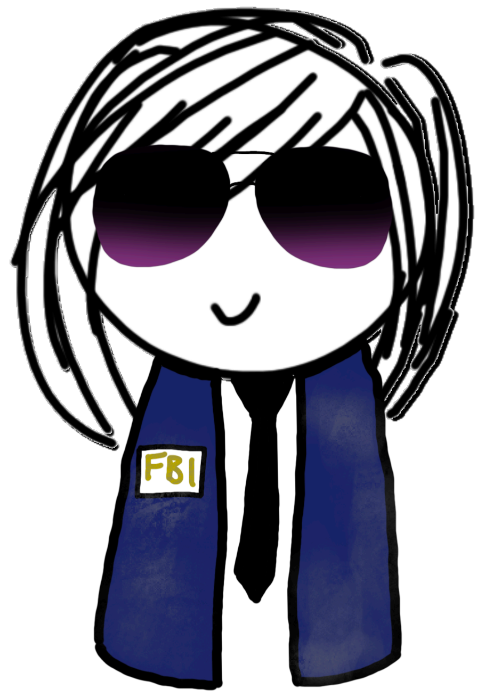
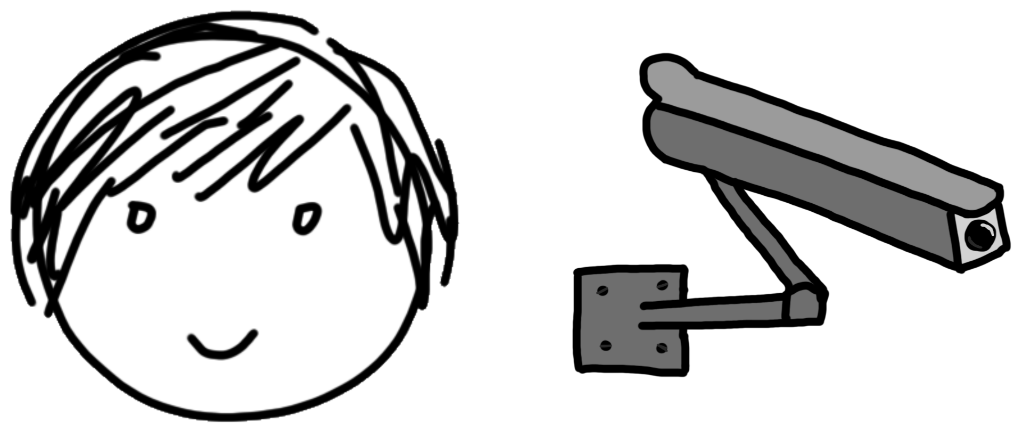


$\mathsf{dig} = \mathsf{Hash}(C)$ →

← ct

x



$\mathsf{ct} \leftarrow \mathsf{Enc}(\mathsf{dig}, x)$

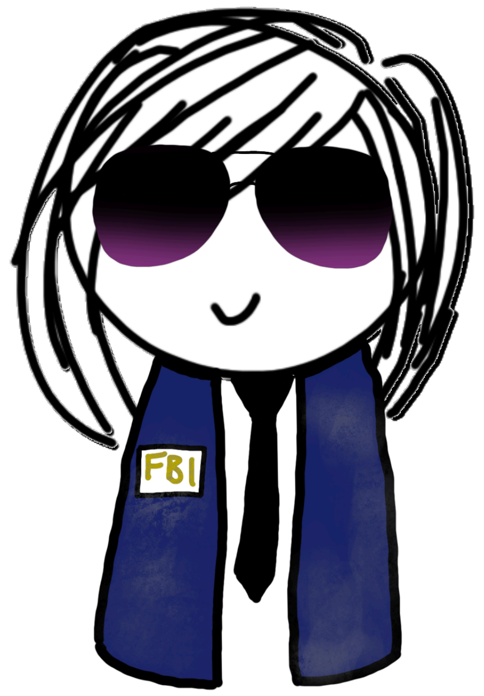
$\mathsf{Dec}(C, \mathsf{ct}) = C(x)$

Laconic Function Evaluation (LFE)

* in CRS model, CRS hidden

digest very short compared to $|C|$

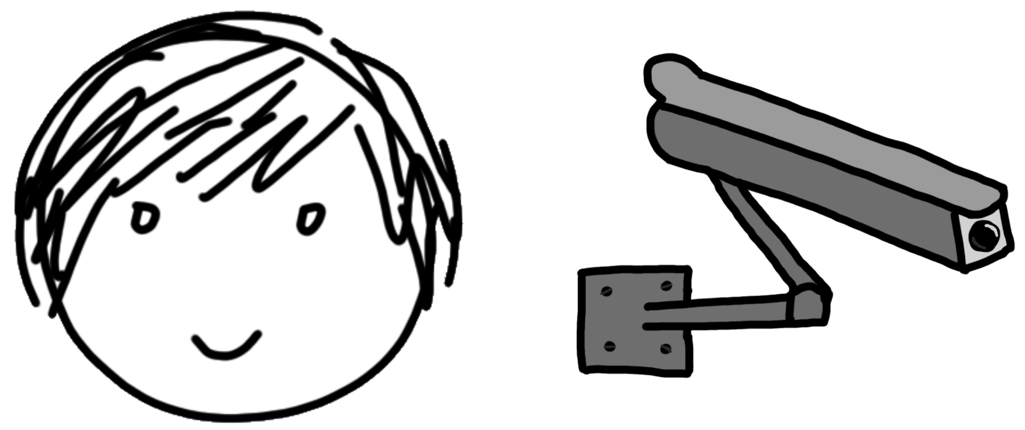
C



$\mathsf{dig} = \mathsf{Hash}(C)$

ct

x



$\mathsf{ct} \leftarrow \mathsf{Enc}(\mathsf{dig}, x)$

$\mathsf{Dec}(C, \mathsf{ct}) = C(x)$

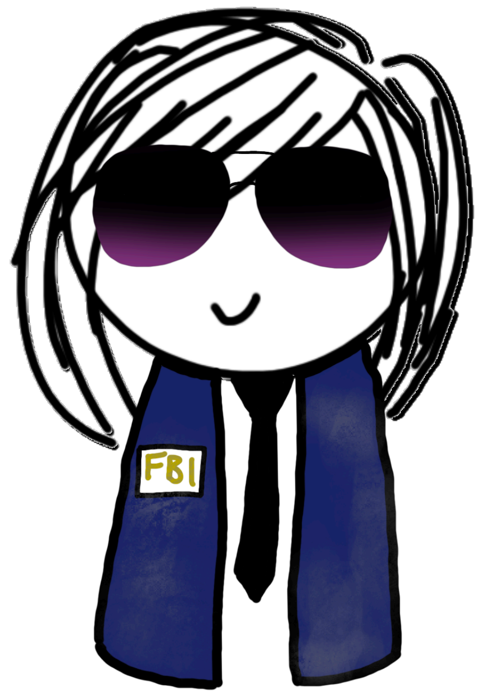
Security: Server learns nothing more than $C(x)$

Laconic Function Evaluation (LFE)

* in CRS model, CRS hidden

digest very short compared to $|C|$

C



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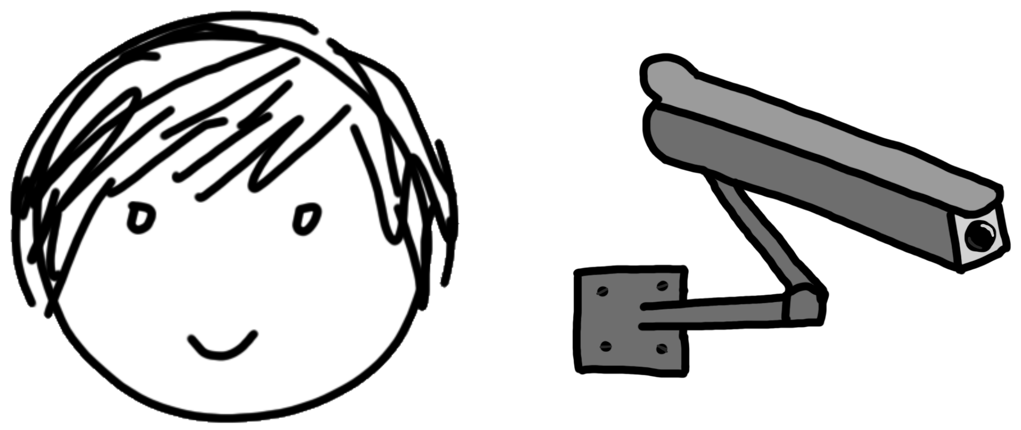

$\text{ct} \leftarrow \text{Enc}(\text{dig}, x)$

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Security: Server learns nothing more than $C(x)$

Like FHE: 2-round 2PC where Server does the computational work

Laconic Function Evaluation (LFE)

* in CRS model, CRS hidden

digest very short compared to $|C|$

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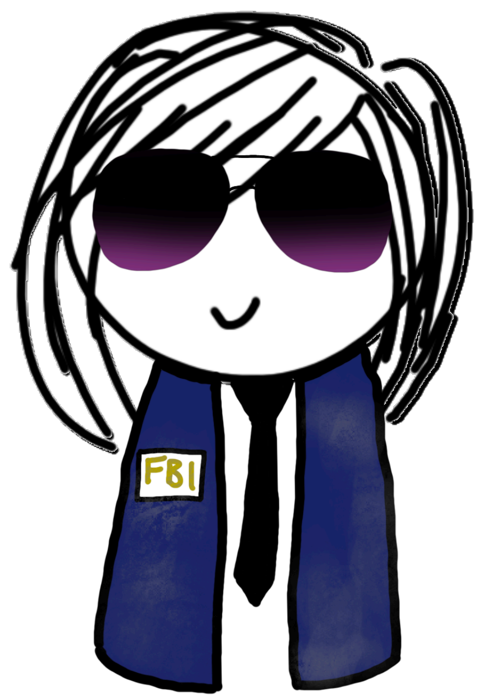
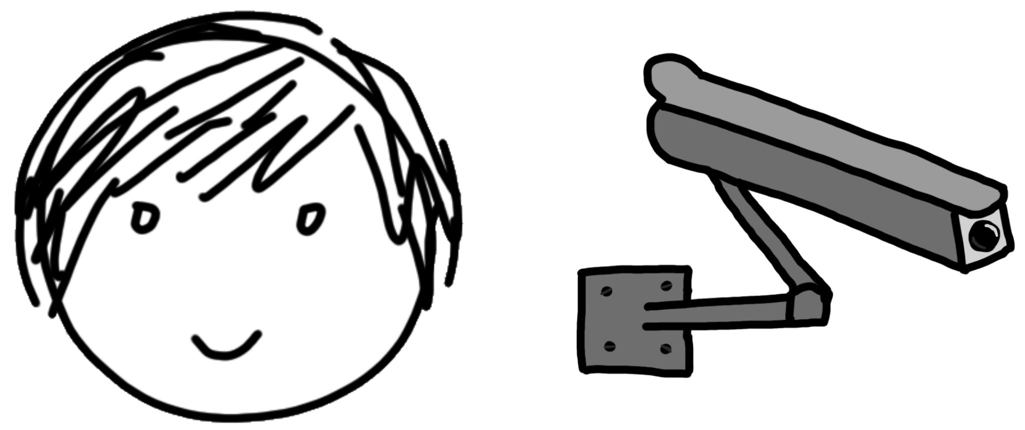


$\text{dig} = \text{Hash}(C)$ →

← ct

x



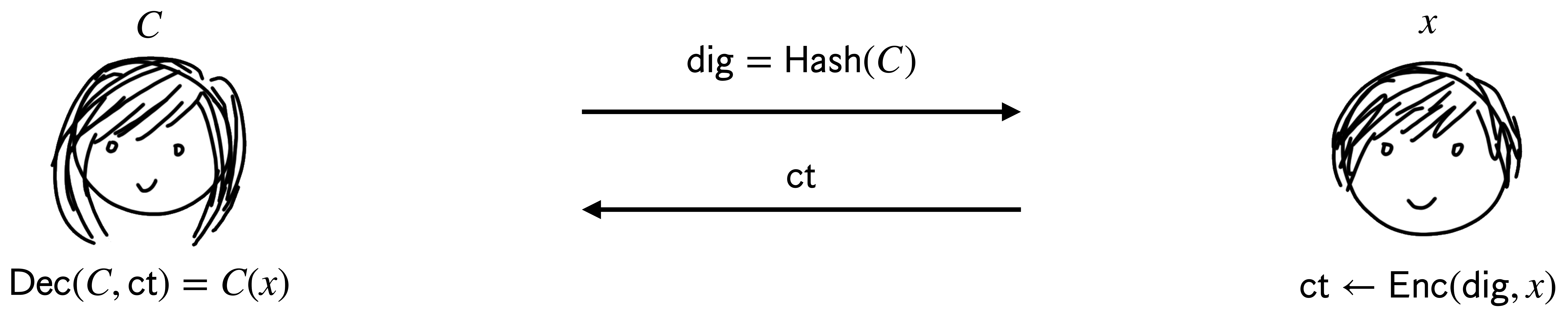
$\text{ct} \leftarrow \text{Enc}(\text{dig}, x)$

$\text{Dec}(C, \text{ct}) = C(x)$

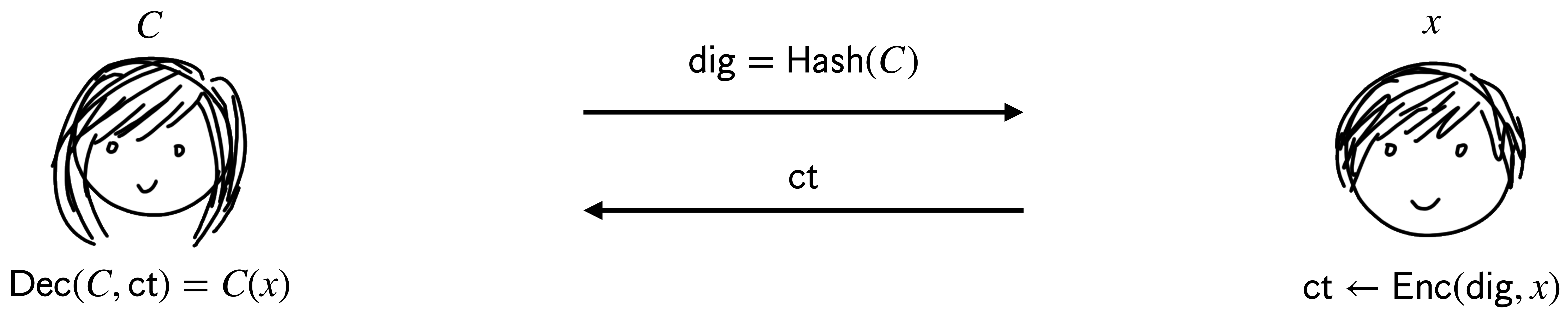
Security: Server learns nothing more than $C(x)$

Like FHE: 2-round 2PC where Server does the computational work
But "flipped": Server learns the output (instead of Client)

Laconic Function Evaluation (LFE)



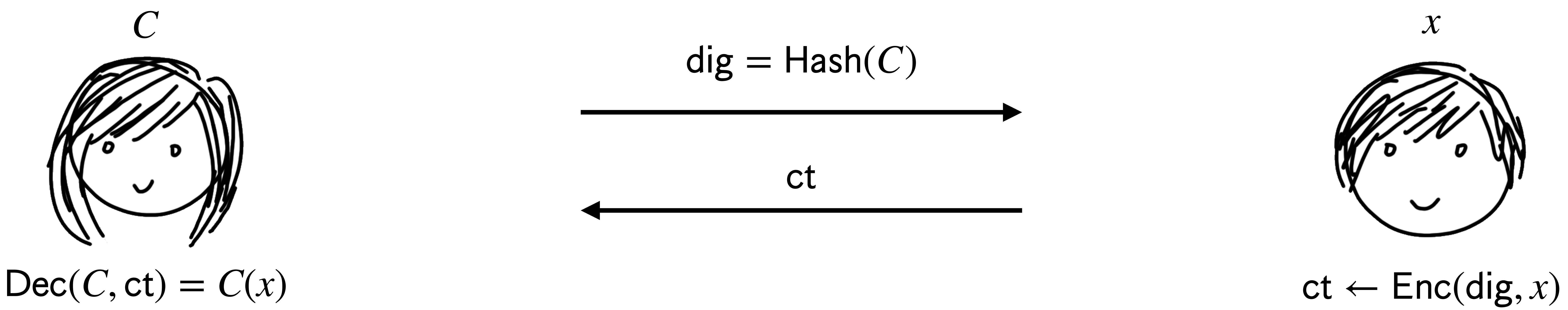
Laconic Function Evaluation (LFE)



Prior work:

- [Quach-Wee-Wichs'17]: LFE for circuits from LWE

Laconic Function Evaluation (LFE)



Prior work:
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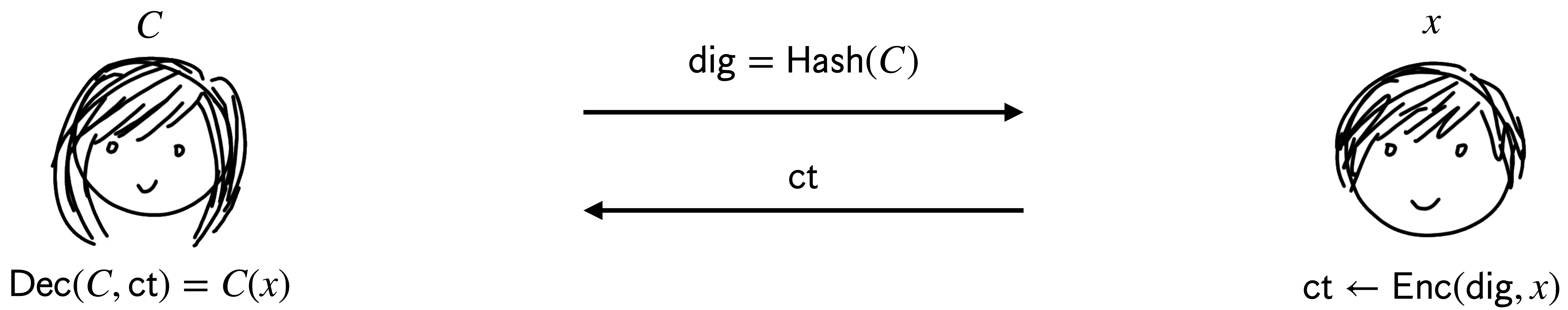
Laconic Function Evaluation (LFE)



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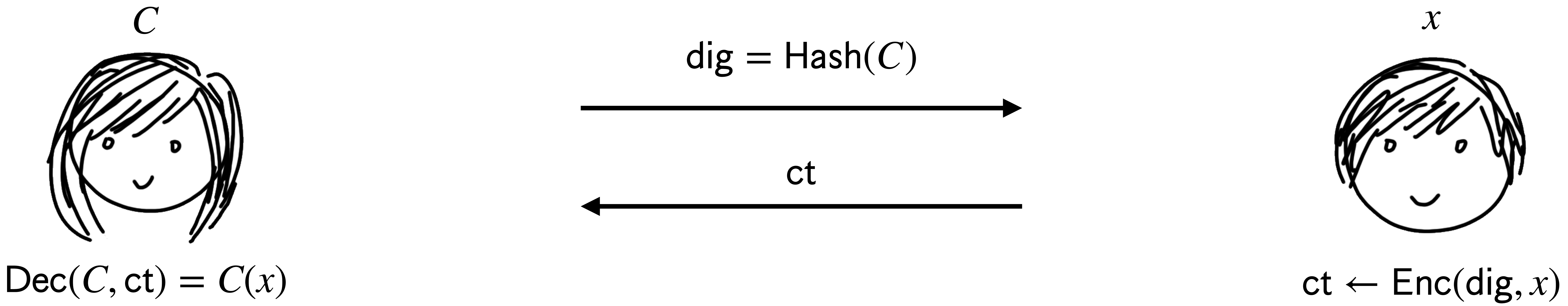
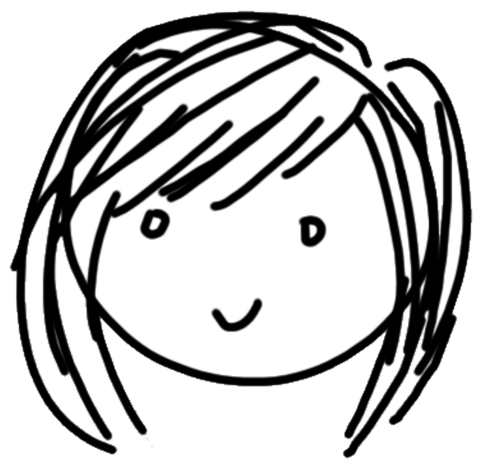
Problem: Server computation is at least linear in inputs!

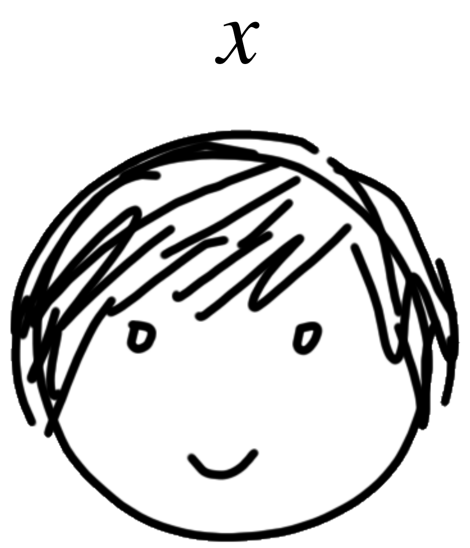
Laconic Function Evaluation (LFE)



Prior work:
- [Quach-Wee-Wichs'17]: LFE for circuits from LWE
- [Döttling-Gajland-Malavolta'23]: LFE for TM from iO + SSB
- [Dong-Hao-**M**-Wichs'24]: LFE for RAM from RingLWE (+iO)

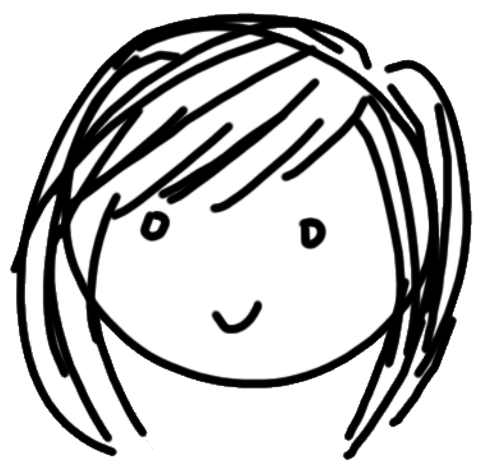
LFE for RAMs





LFE for RAMs

Goal: output RAM computation $f_{\mathsf{DB}}(x)$

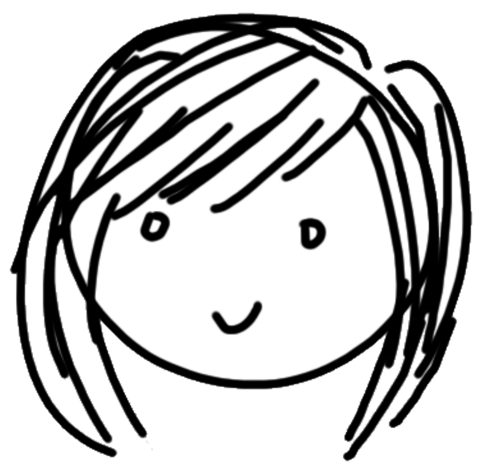


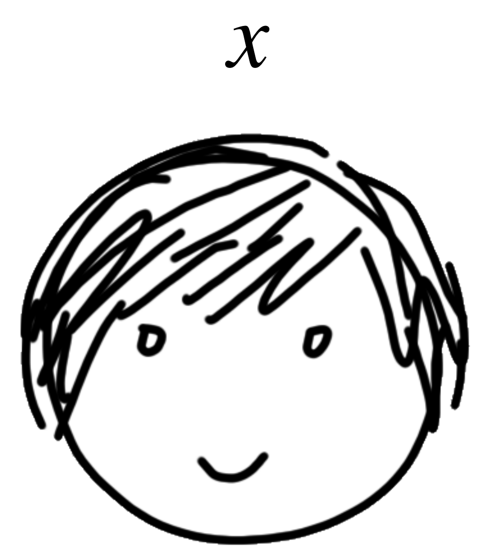
x

LFE for RAMs

Some fixed RAM program (e.g. universal)

Goal: output RAM computation $f_{\mathsf{DB}}(x)$





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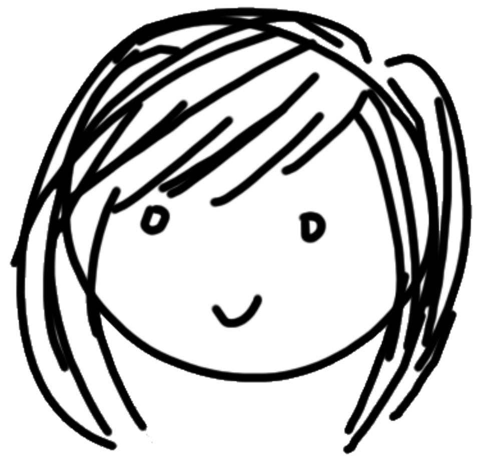
Goal: output RAM computation $f_{\mathsf{DB}}(x)$

$f_{\mathsf{DB}}(x)$ has RAM runtime T



x

Server holds some arbitrary public database

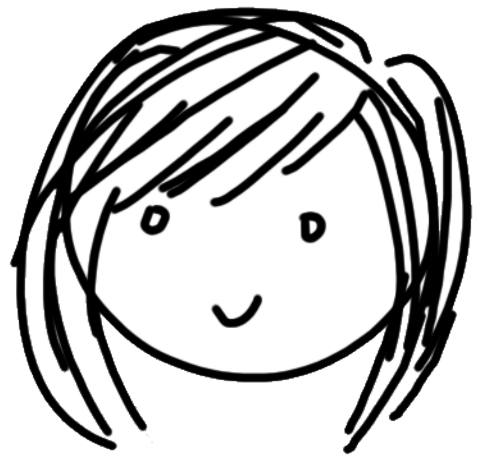
LFE for RAMs

Some fixed RAM program (e.g. universal)

Goal: output RAM computation $f_{\text{DB}}(x)$

$f_{\text{DB}}(x)$ has RAM runtime T

DB



x

LFE for RAMs

Server holds some arbitrary public database

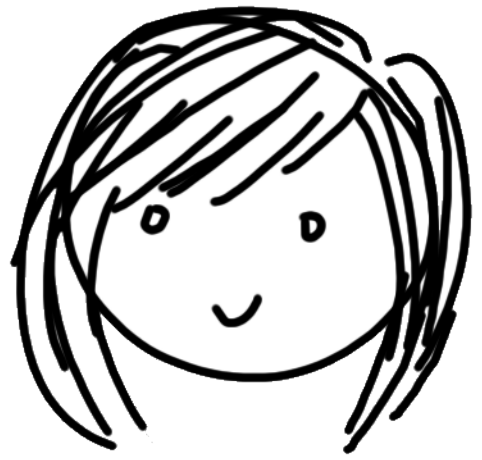
And preprocesses it

Some fixed RAM program (e.g. universal)

Goal: output RAM computation $f_{\mathsf{DB}}(x)$

$f_{\mathsf{DB}}(x)$ has RAM runtime T

$\mathsf{DB} \xrightarrow{\mathsf{Prep}} \widetilde{\mathsf{DB}}$



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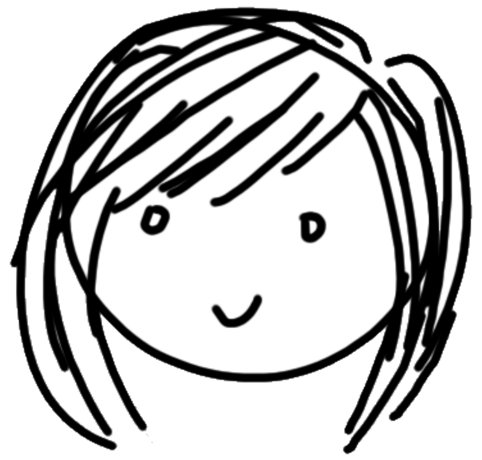

$\mathsf{dig} = \mathsf{Hash}(\widetilde{\mathsf{DB}})$ →

x

LFE for RAMs

Server holds some arbitrary public database

And preprocesses it

Some fixed RAM program (e.g. universal)

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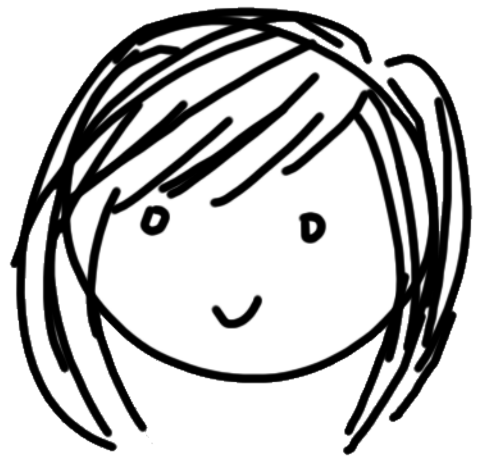
$f_{\mathsf{DB}}(x)$ has RAM runtime T

$\mathsf{DB} \xrightarrow{\mathsf{Prep}} \widetilde{\mathsf{DB}}$



x

$\mathsf{dig} = \mathsf{Hash}(\widetilde{\mathsf{DB}})$ →

← ct

$\mathsf{ct} \leftarrow \mathsf{Enc}(\mathsf{dig}, x)$

LFE for RAMs

Server holds some arbitrary public database

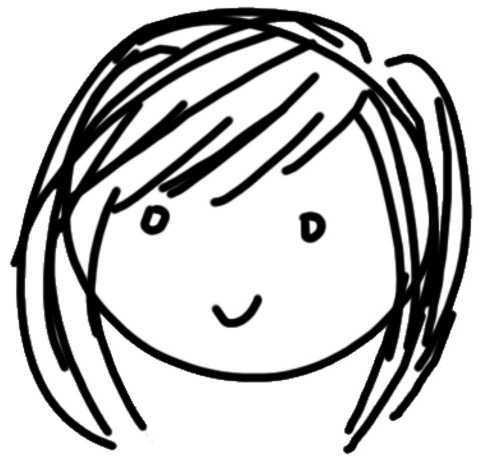
And preprocesses it

Some fixed RAM program (e.g. universal)

Goal: output RAM computation $f_{\mathsf{DB}}(x)$

$f_{\mathsf{DB}}(x)$ has RAM runtime T

$\mathsf{DB} \xrightarrow{\mathsf{Prep}} \widetilde{\mathsf{DB}}$



$\mathsf{Dec}(\widetilde{\mathsf{DB}}, \mathsf{ct}) = f_{\mathsf{DB}}(x)$

$\mathsf{dig} = \mathsf{Hash}(\widetilde{\mathsf{DB}})$ →

← ct

x

$\mathsf{ct} \leftarrow \mathsf{Enc}(\mathsf{dig}, x)$

LFE for RAMs

Server holds some arbitrary public database

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Goal: output RAM computation $f_{\mathsf{DB}}(x)$

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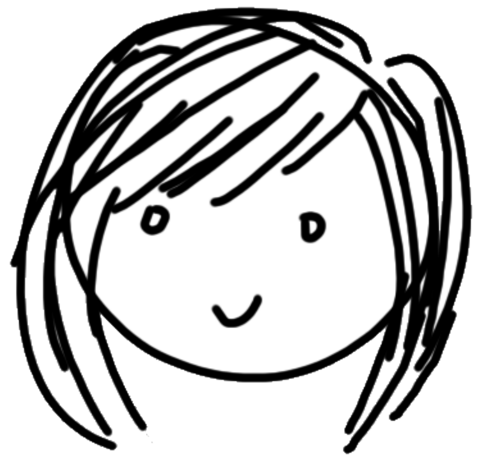

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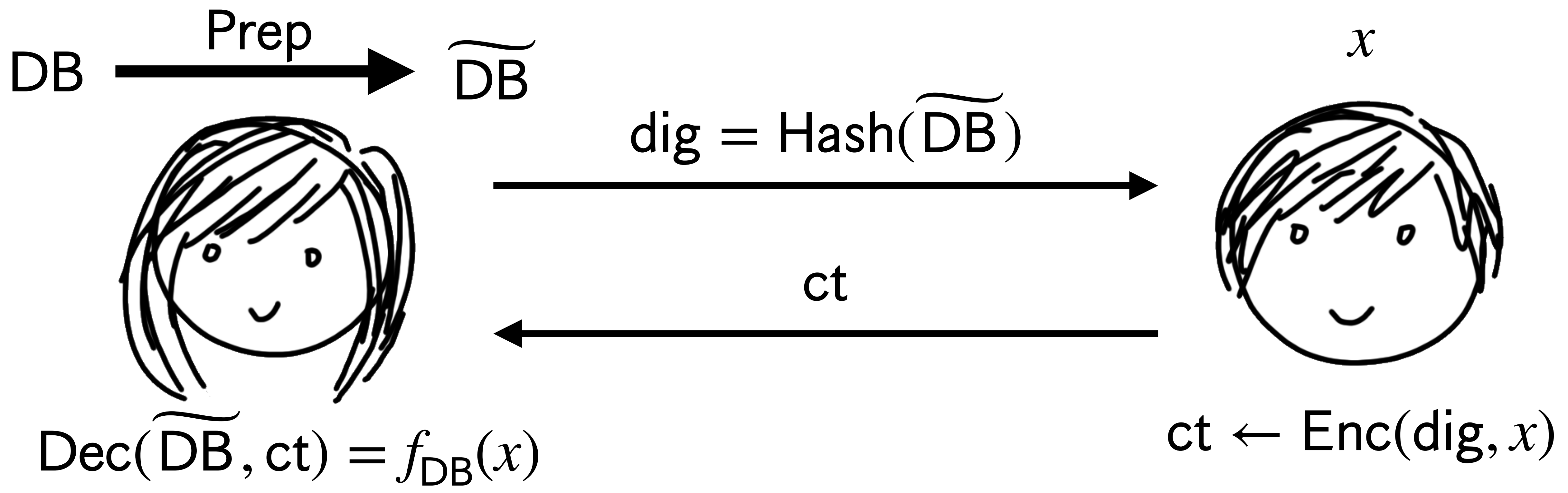
← ct

$\mathsf{Dec}(\widetilde{\mathsf{DB}}, \mathsf{ct}) = f_{\mathsf{DB}}(x)$

$\mathsf{ct} \leftarrow \mathsf{Enc}(\mathsf{dig}, x)$

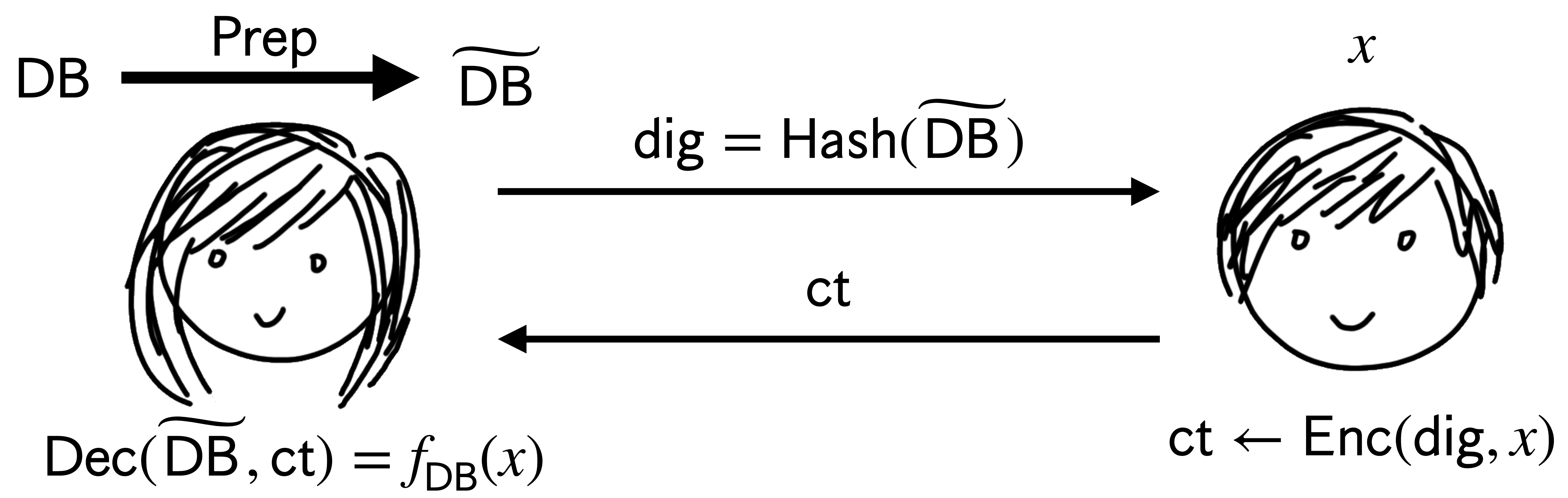
Want Dec to run in time $\approx T$

LFE for RAMs



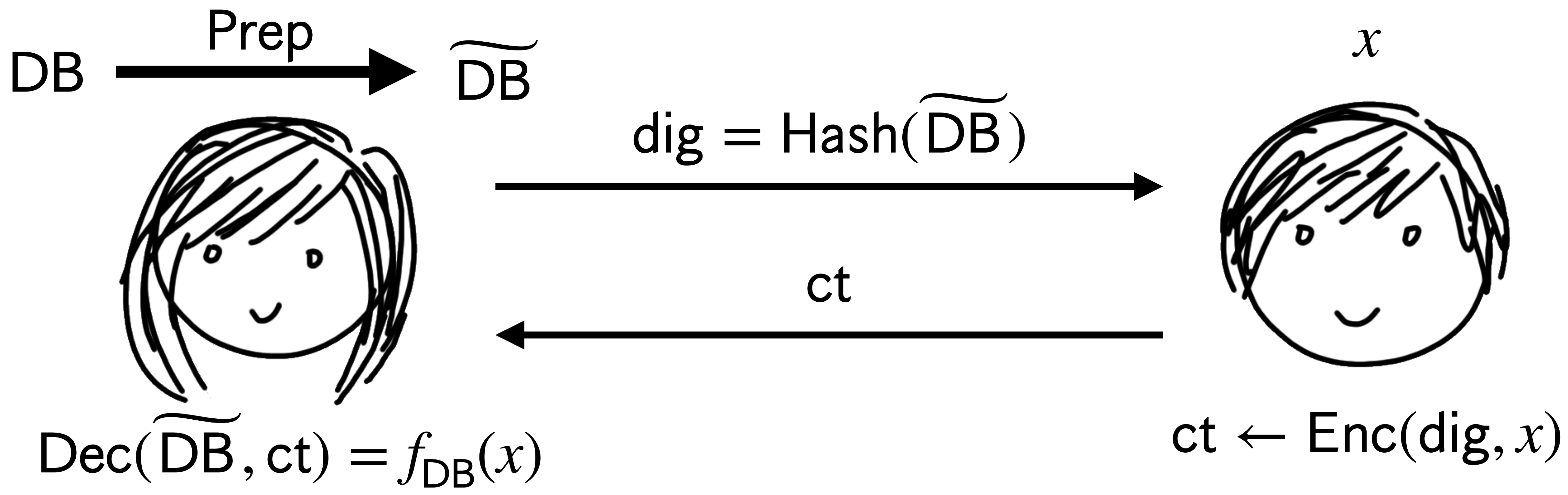
	[DH**M**W'24] (weak efficiency)	[DH**M**W'24] (strong efficiency)
Prep:		
Enc:		
Dec:		
Assumption:		

LFE for RAMs



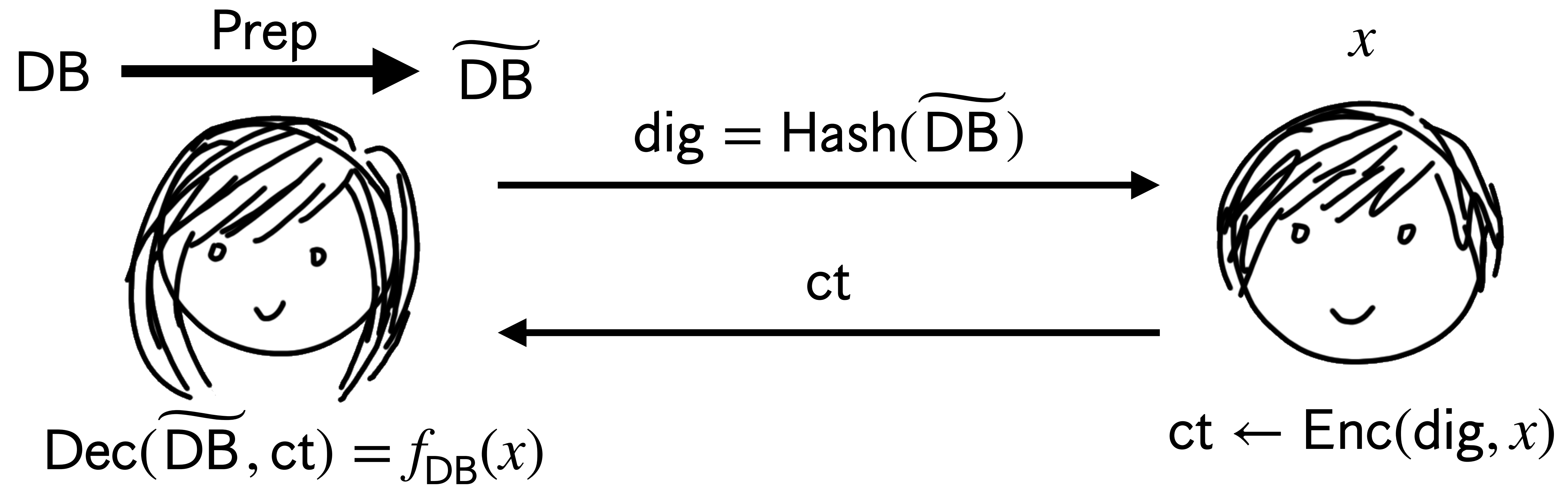
	[DH**M**W'24] (weak efficiency)	[DH**M**W'24] (strong efficiency)
Prep:	$\lvert \mathsf{DB} \rvert^{1+\varepsilon}$	$\lvert \mathsf{DB} \rvert^{1+\varepsilon}$
Enc:		
Dec:		
Assumption:		

LFE for RAMs



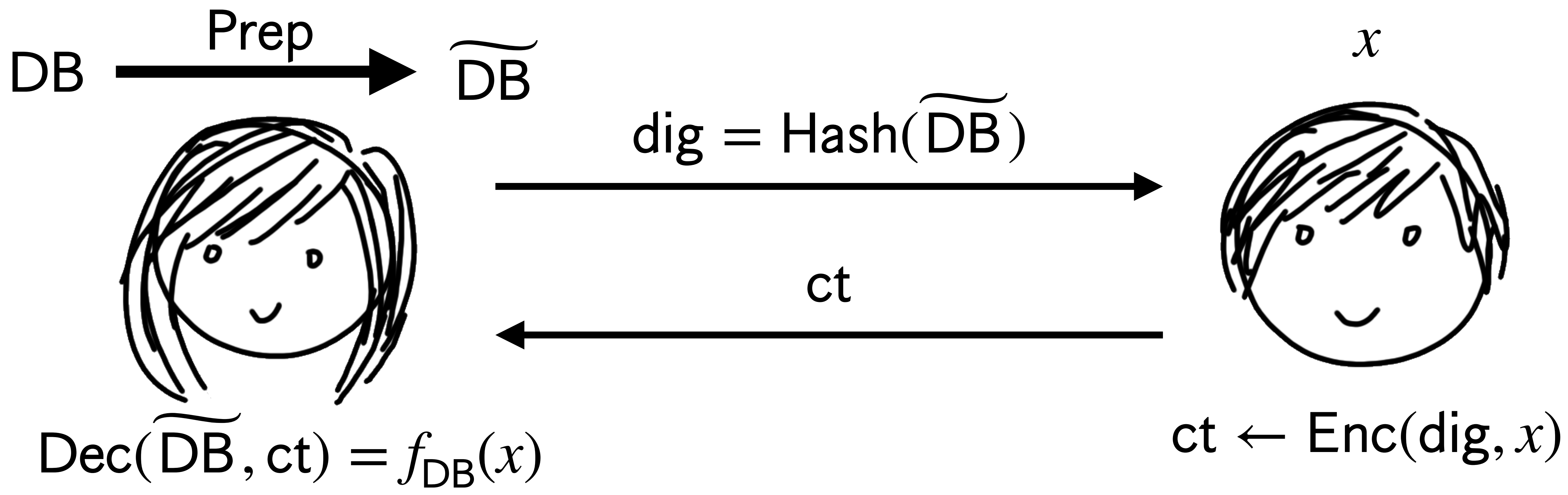
	[DH**M**W'24] (weak efficiency)	[DH**M**W'24] (strong efficiency)
Prep:	$\lvert \mathsf{DB} \rvert^{1+\varepsilon}$	$\lvert \mathsf{DB} \rvert^{1+\varepsilon}$
Enc:	$\lvert x \rvert + T$	$\lvert x \rvert$
Dec:		
Assumption:		

LFE for RAMs



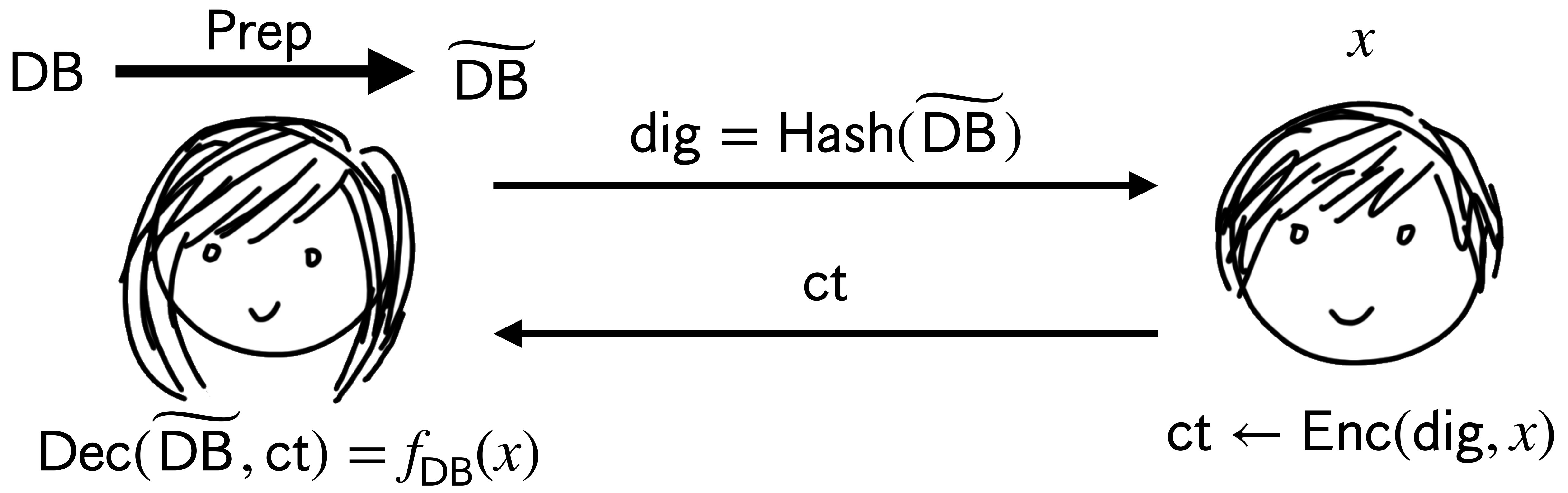
	[DH**M**W'24] (weak efficiency)	[DH**M**W'24] (strong efficiency)
Prep:	$\lvert \mathsf{DB} \rvert^{1+\varepsilon}$	$\lvert \mathsf{DB} \rvert^{1+\varepsilon}$
Enc:	$\lvert x \rvert + T$	$\lvert x \rvert$
Dec:	T	T
Assumption:		

LFE for RAMs



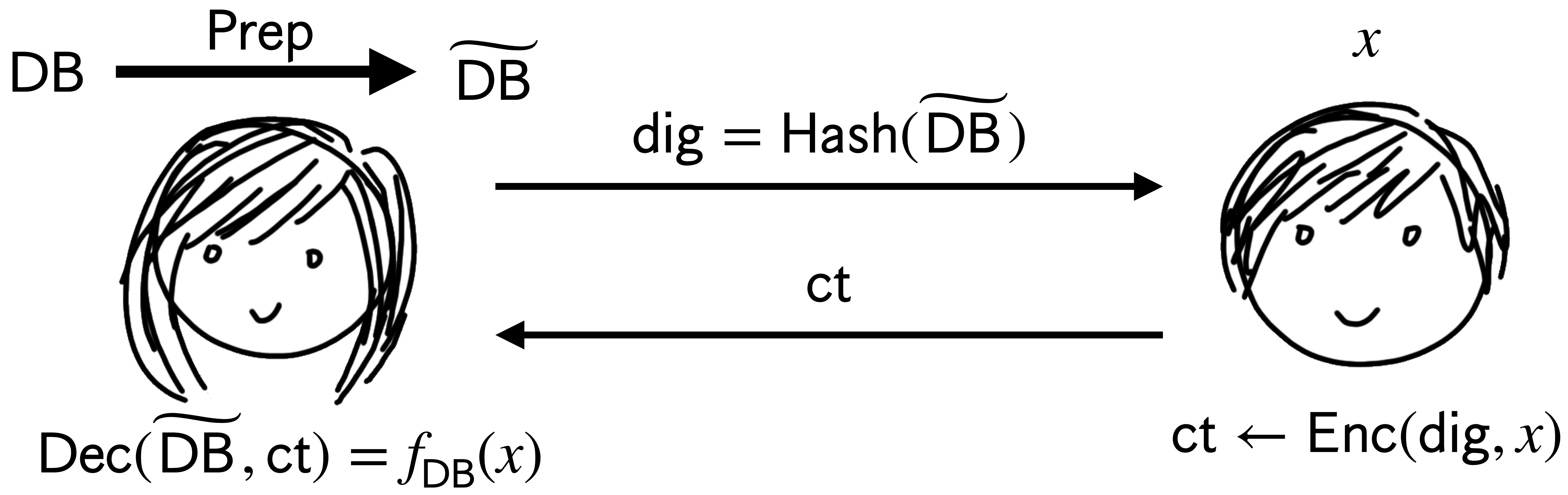
	[DHMW'24] (weak efficiency)	[DHMW'24] (strong efficiency)
Prep:	$\lvert \mathsf{DB} \rvert^{1+\varepsilon}$	$\lvert \mathsf{DB} \rvert^{1+\varepsilon}$
Enc:	$\lvert x \rvert + T$	$\lvert x \rvert$
Dec:	T	T
Assumption:	RingLWE	RingLWE + iO

LFE for RAMs



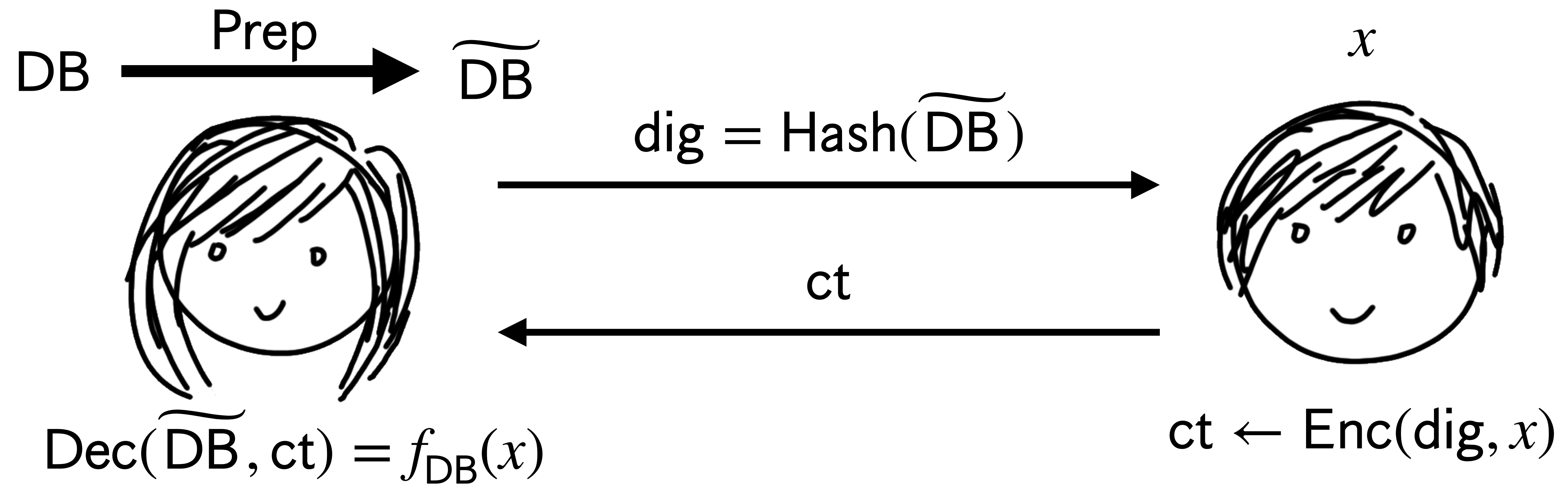
	[DH**M**W'24] (weak efficiency)	[DH**M**W'24] (strong efficiency)	This work
Prep:	$\lvert \mathsf{DB} \rvert^{1+\varepsilon}$	$\lvert \mathsf{DB} \rvert^{1+\varepsilon}$	
Enc:	$\lvert x \rvert + T$	$\lvert x \rvert$	
Dec:	T	T	
Assumption:	RingLWE	RingLWE + iO	

LFE for RAMs



	[DHMW'24] (weak efficiency)	[DHMW'24] (strong efficiency)	This work
Prep:	$\lvert \mathsf{DB} \rvert^{1+\varepsilon}$	$\lvert \mathsf{DB} \rvert^{1+\varepsilon}$	$s^{1+o(1)}$
Enc:	$\lvert x \rvert + T$	$\lvert x \rvert$	
Dec:	T	T	
Assumption:	RingLWE	RingLWE + iO	

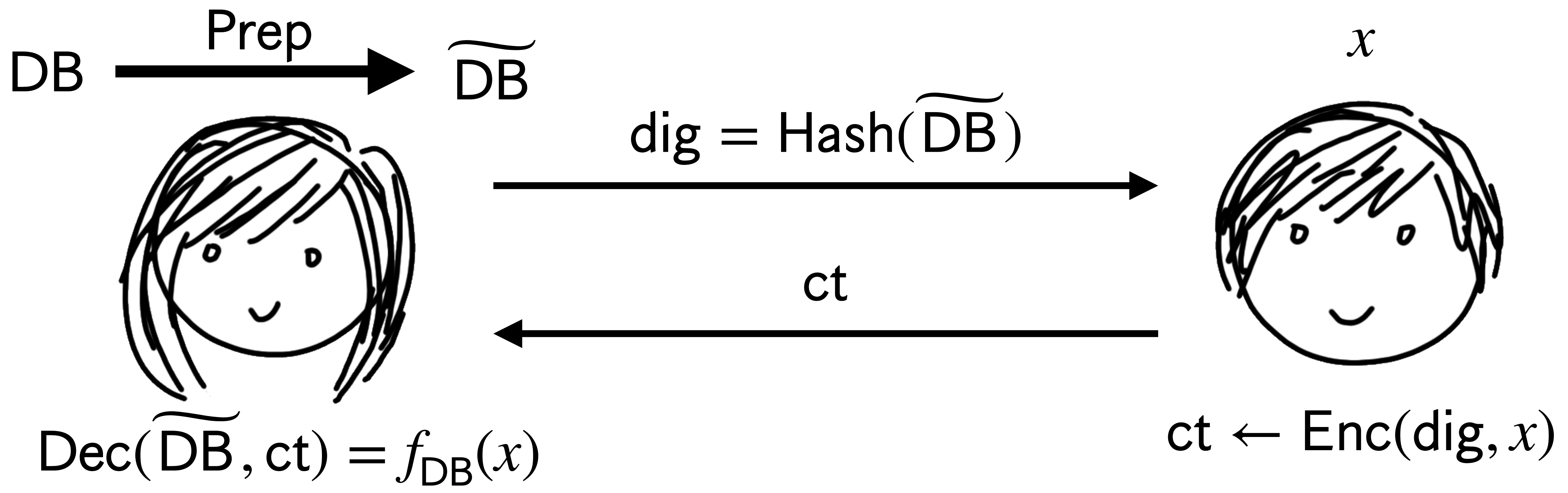
LFE for RAMs



s = ***circuit size*** of $f_{\text{DB}} \approx T \cdot |\text{DB}|$

	[DH**M**W'24] (weak efficiency)	[DH**M**W'24] (strong efficiency)	This work
Prep:	$\|\text{DB}\|^{1+\varepsilon}$	$\|\text{DB}\|^{1+\varepsilon}$	$s^{1+o(1)}$
Enc:	$\|x\| + T$	$\|x\|$	
Dec:	T	T	
Assumption:	RingLWE	RingLWE + iO	

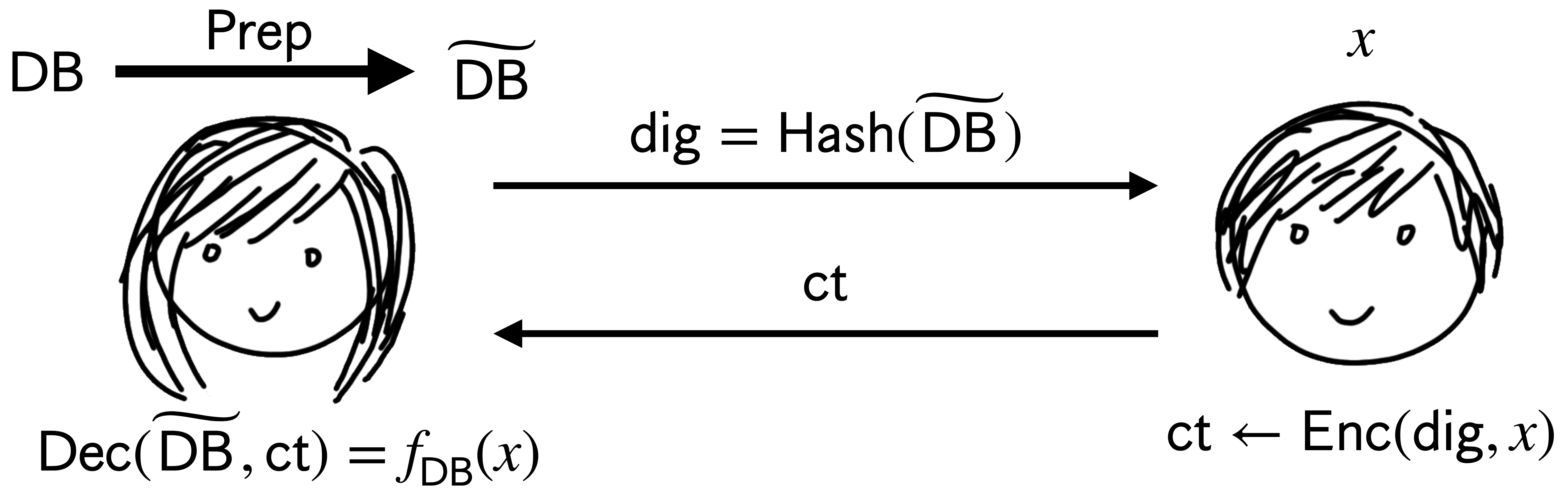
LFE for RAMs



s = ***circuit size*** of $f_{\text{DB}} \approx T \cdot |\text{DB}|$

	[DH**M**W'24] (weak efficiency)	[DH**M**W'24] (strong efficiency)	This work
Prep:	$\|\text{DB}\|^{1+\varepsilon}$	$\|\text{DB}\|^{1+\varepsilon}$	$s^{1+o(1)}$
Enc:	$\|x\| + T$	$\|x\|$	$\|x\|^{1+o(1)}$
Dec:	T	T	$T^{1+o(1)}$
Assumption:	RingLWE	RingLWE + iO	RingLWE (+ circular security)

LFE for RAMs



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	[DHMW'24] (weak efficiency)	[DHMW'24] (strong efficiency)	This work
Prep:	$\|\text{DB}\|^{1+\varepsilon}$	$\|\text{DB}\|^{1+\varepsilon}$	$s^{1+o(1)}$
Enc:	$\|x\| + T$	$\|x\|$	$\|x\|^{1+o(1)}$
Dec:	T	T	$T^{1+o(1)}$
Assumption:	RingLWE	RingLWE + iO	RingLWE (+ circular security)

About as efficient online, worse offline

LFE Construction Template

[Quach-Wee-Wichs'17]

Attribute-based LFE

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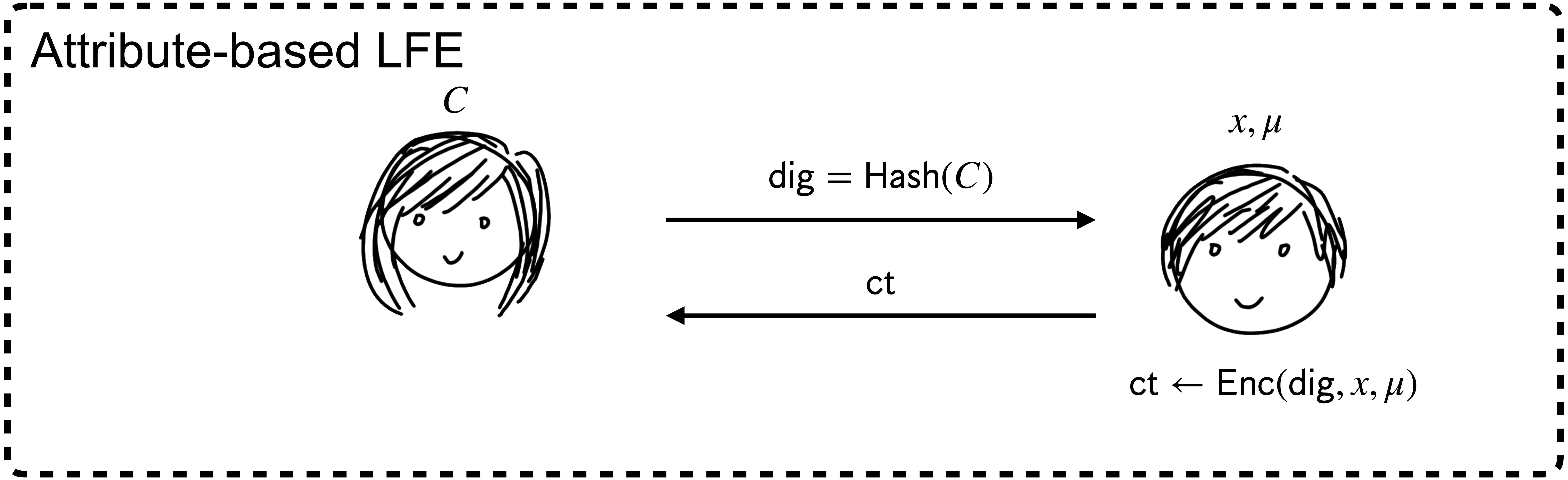
LFE Construction Template

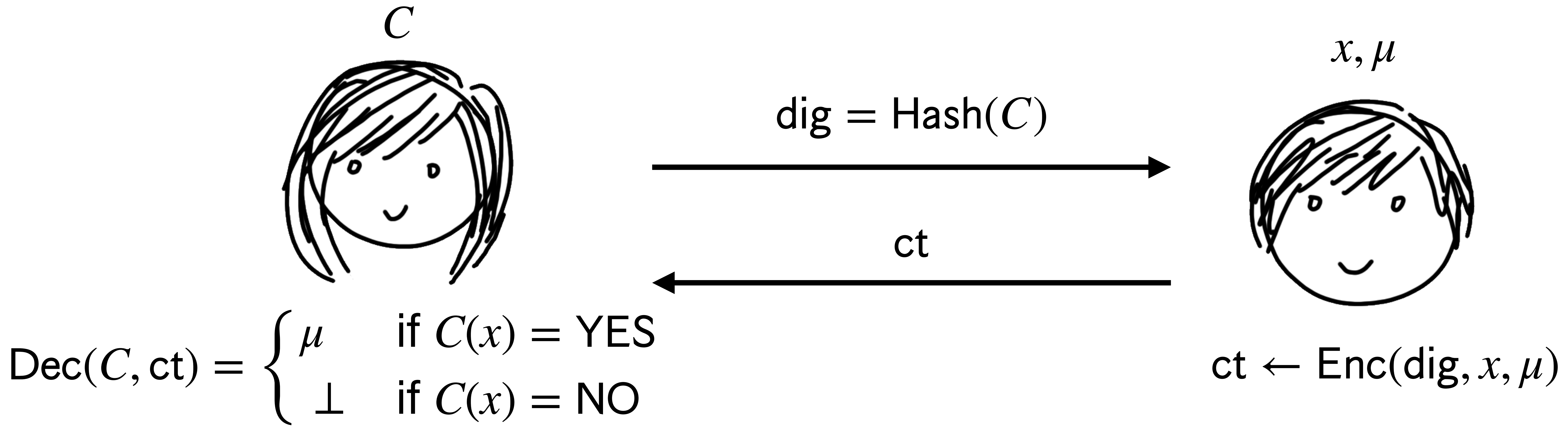
[Quach-Wee-Wichs'17]



LFE Construction Template

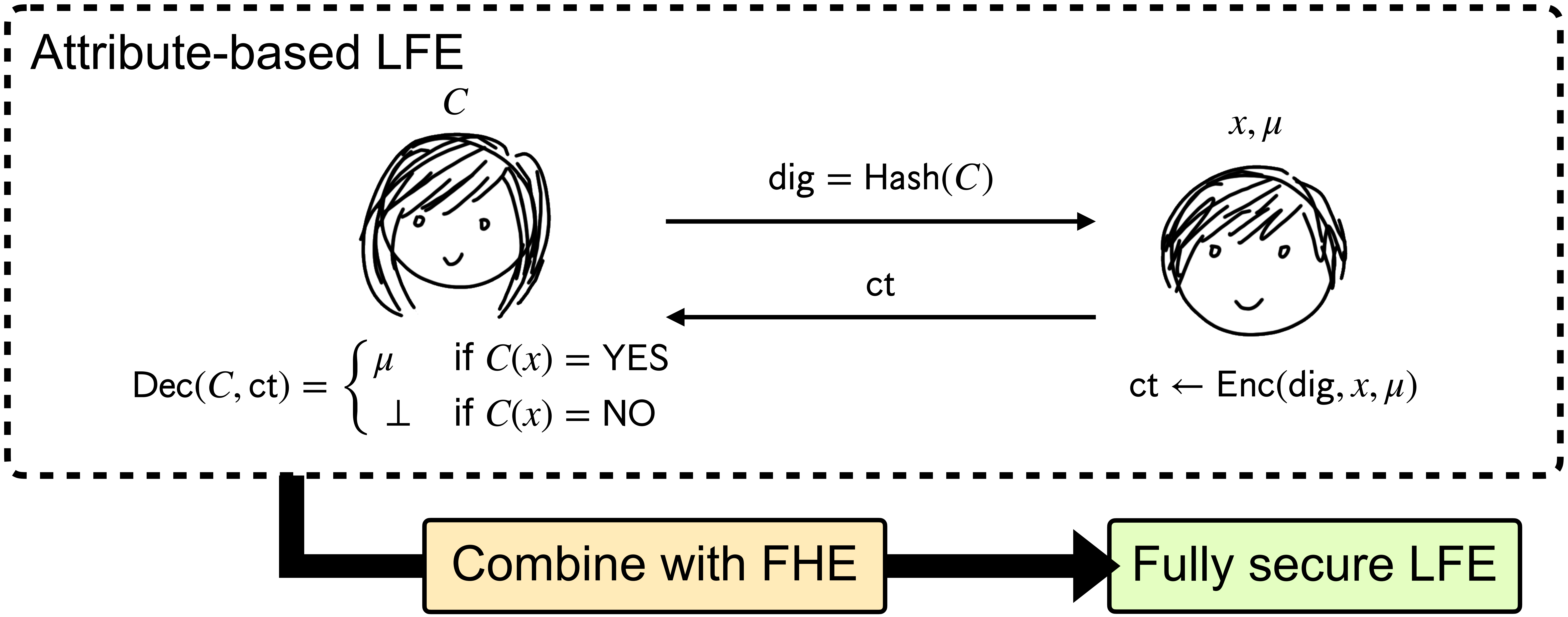
[Quach-Wee-Wichs'17]

Attribute-based LFE



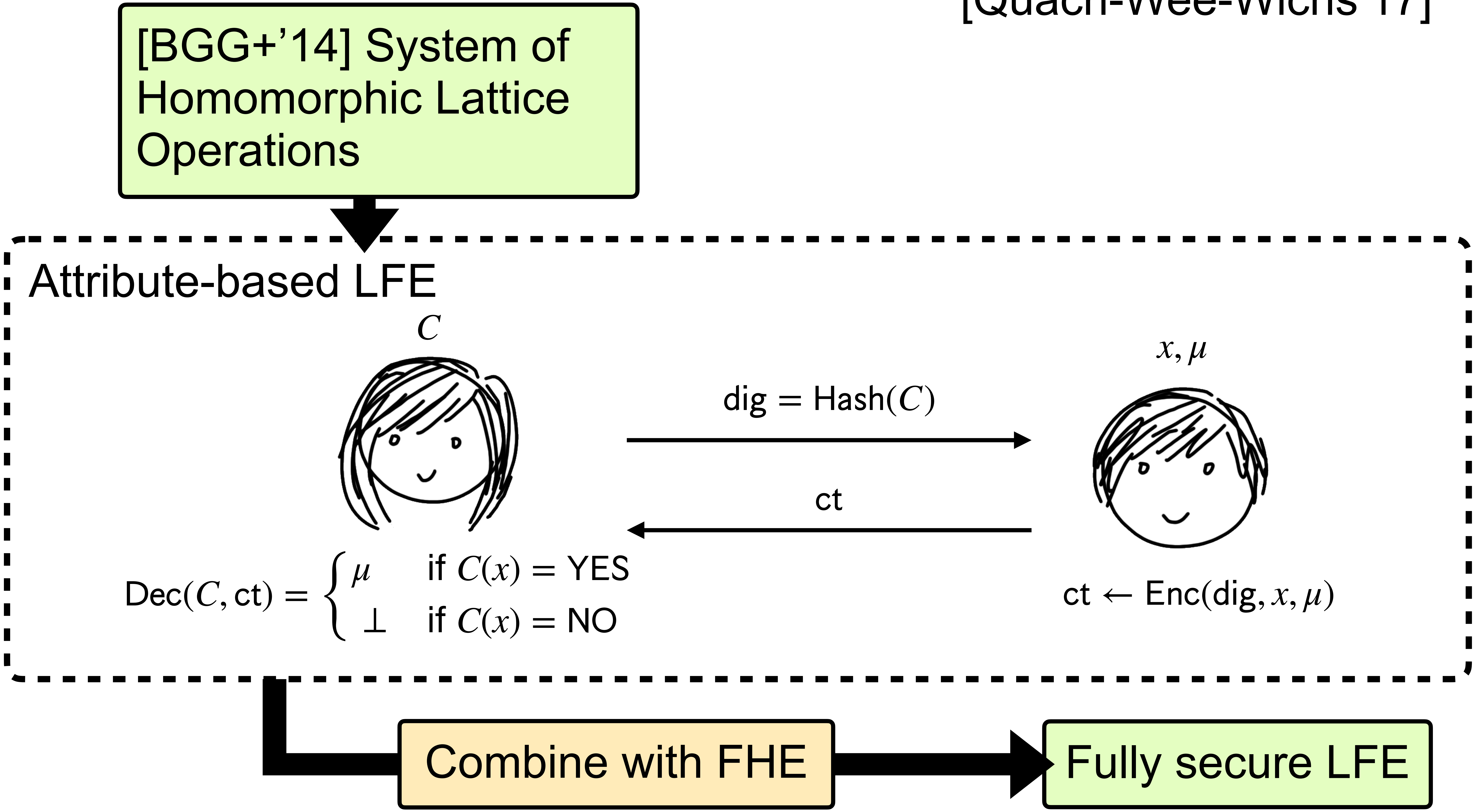
LFE Construction Template

[Quach-Wee-Wichs'17]



LFE Construction Template

[Quach-Wee-Wichs'17]



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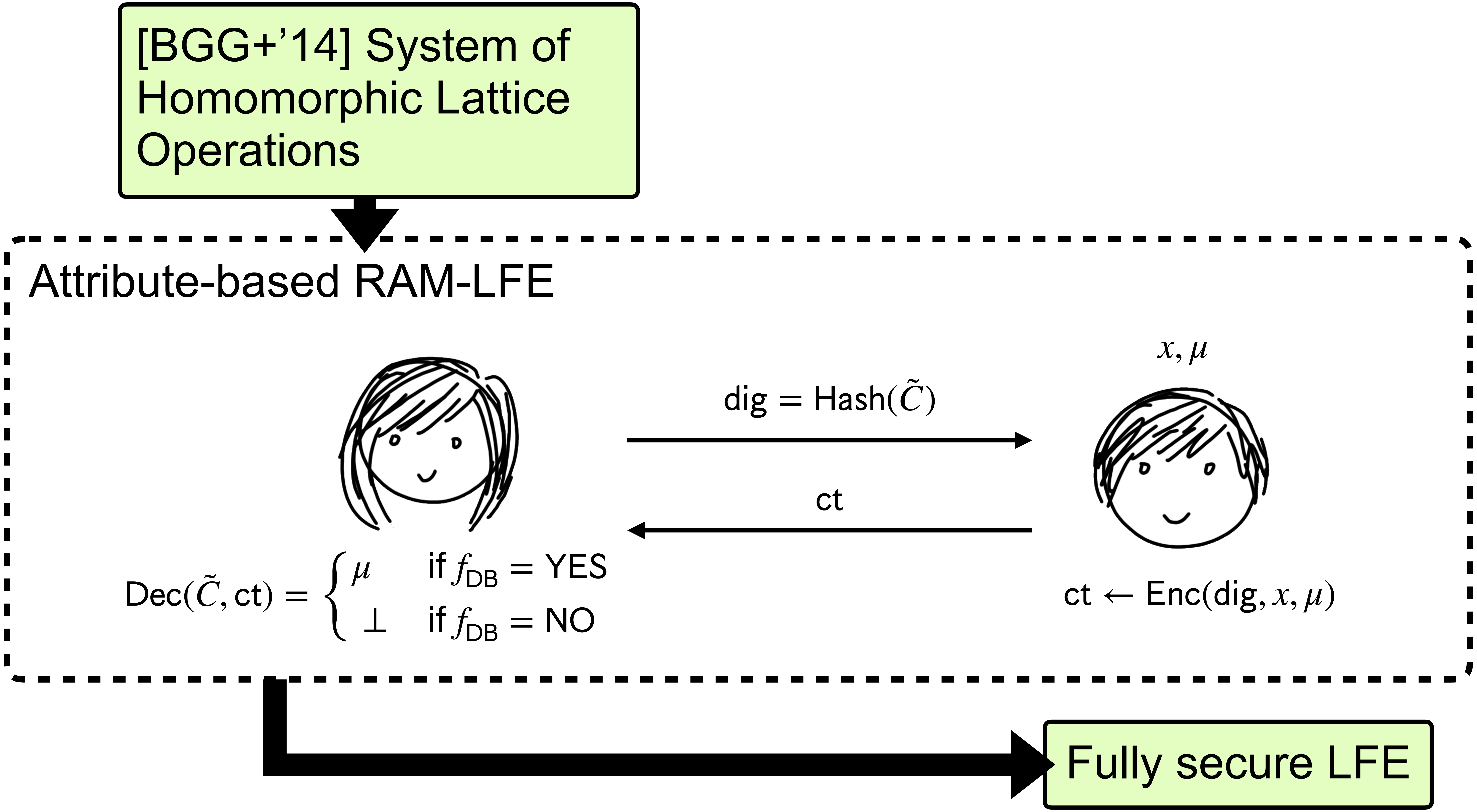

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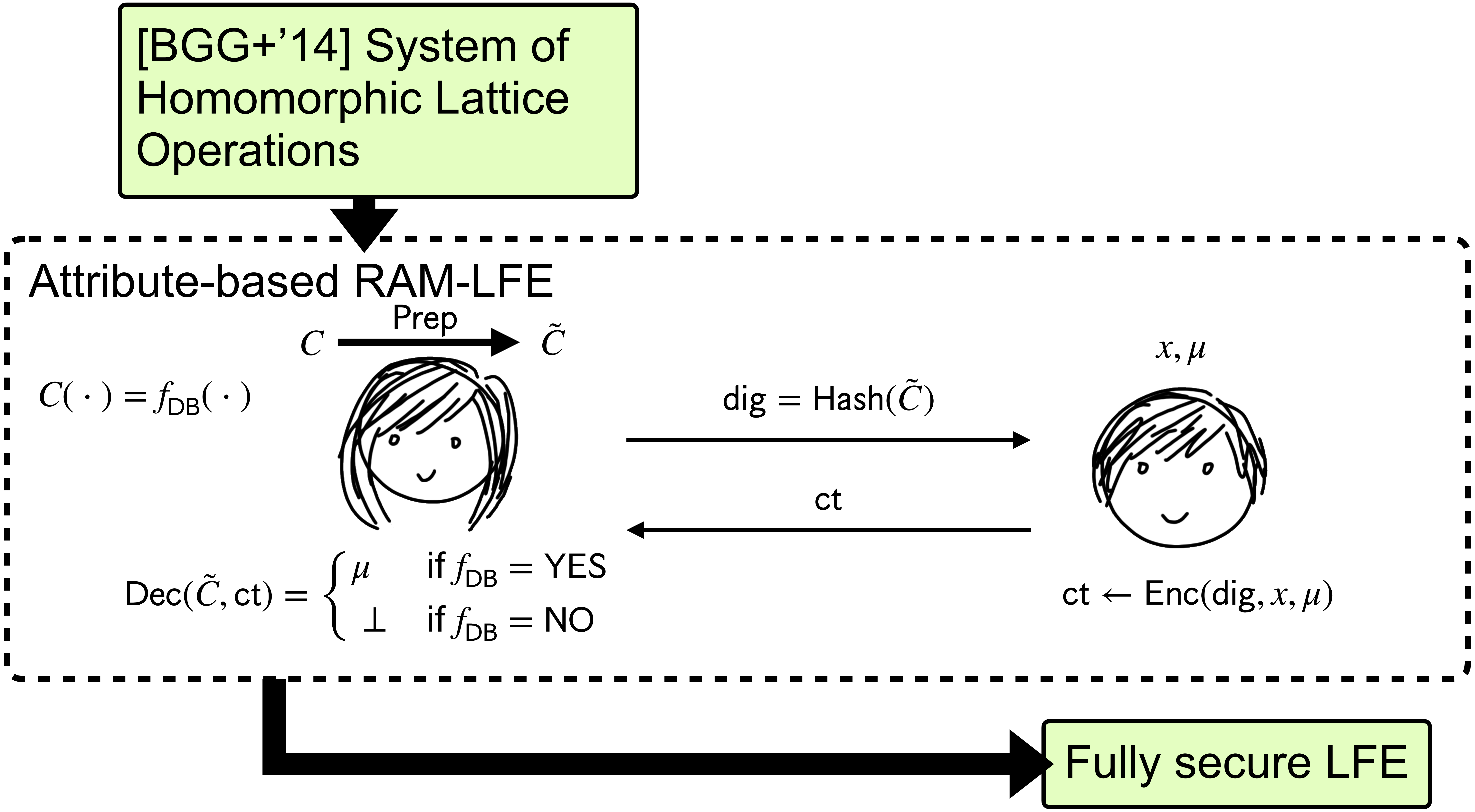

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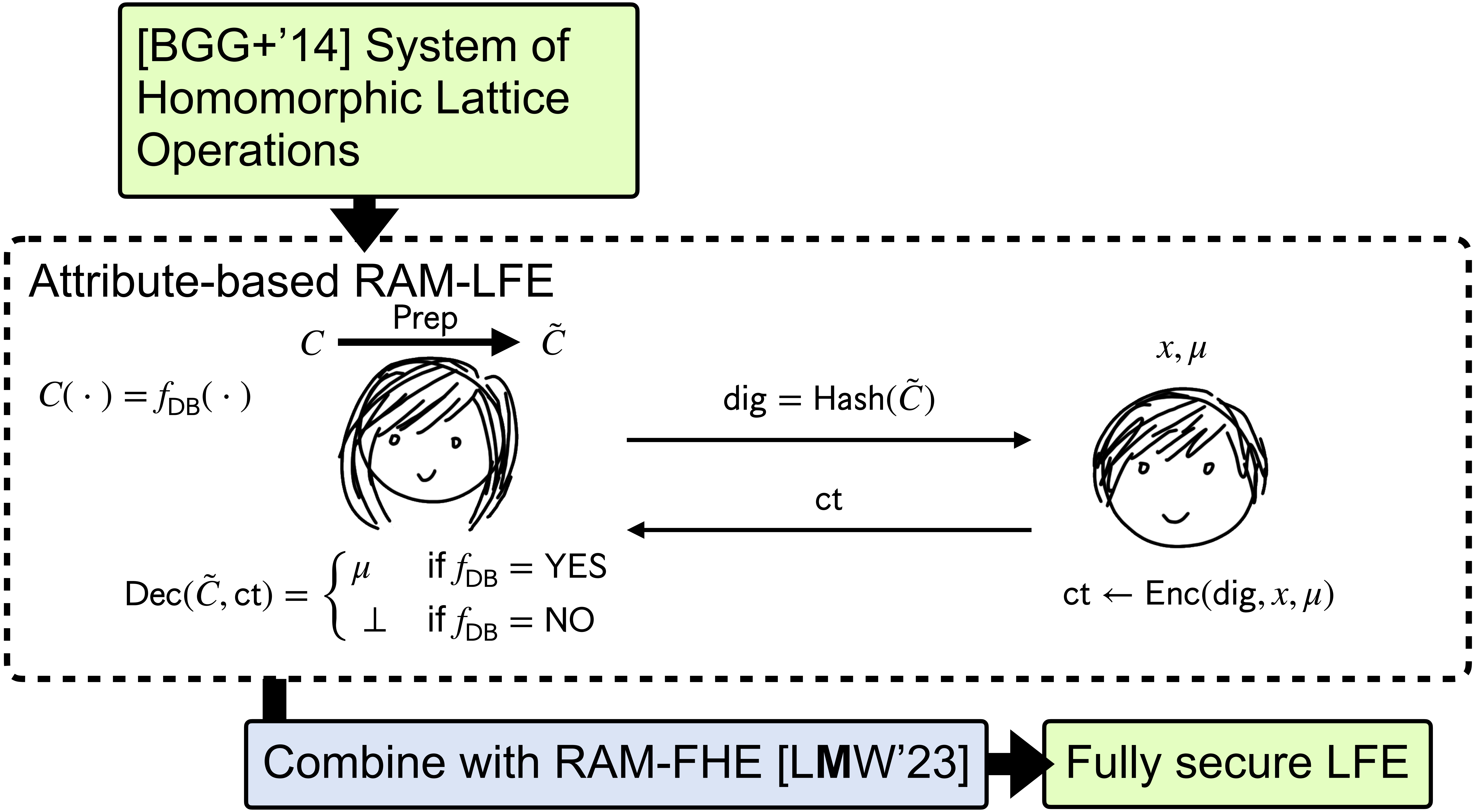
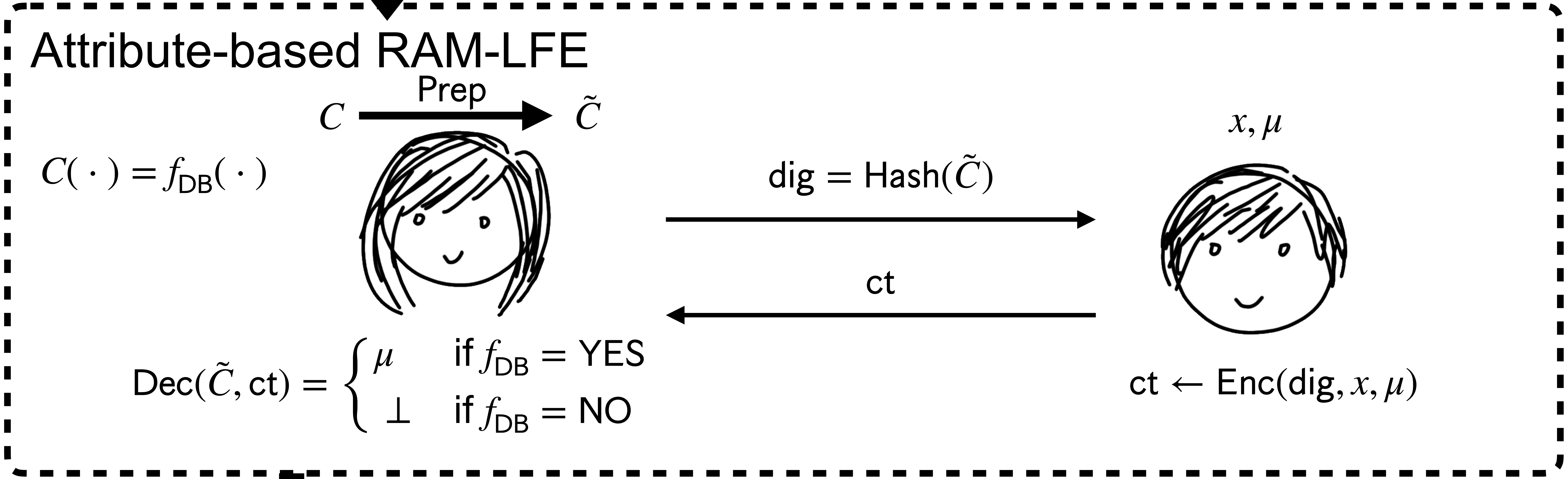


LFE Construction Template

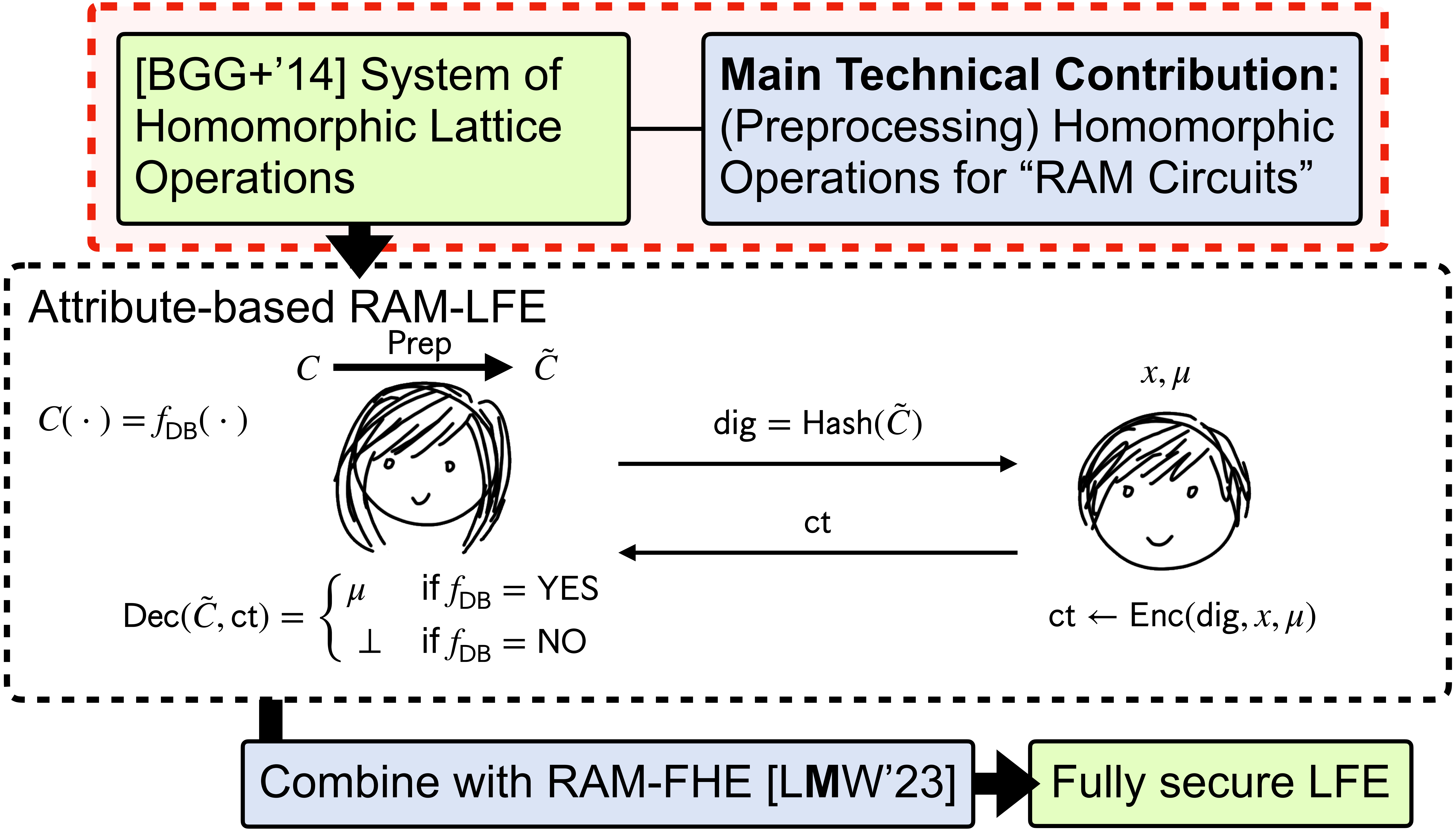
[BGG+'14] System of Homomorphic Lattice Operations

Main Technical Contribution: (Preprocessing) Homomorphic Operations for "RAM Circuits"



Combine with RAM-FHE [L**M**W'23] → Fully secure LFE

LFE Construction Template



Homomorphic Operations

[BGG+'14]

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Goal: Given encodings of an input x want to get an encoding of the output $f(x)$

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$$\mathbf{A}_1 \quad \cdots \quad \mathbf{A}_N$$

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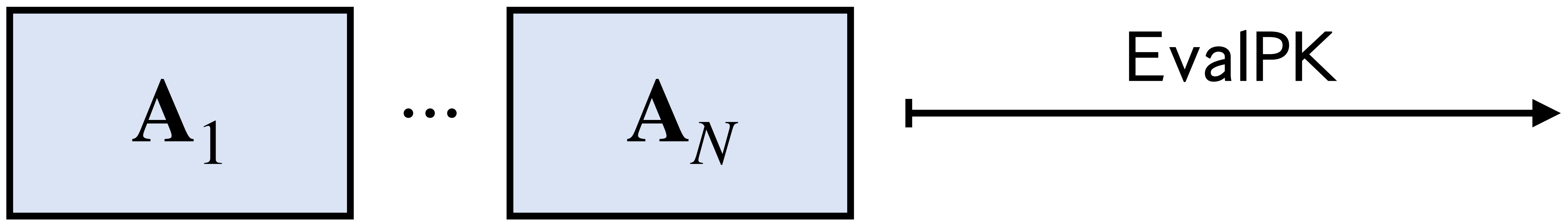

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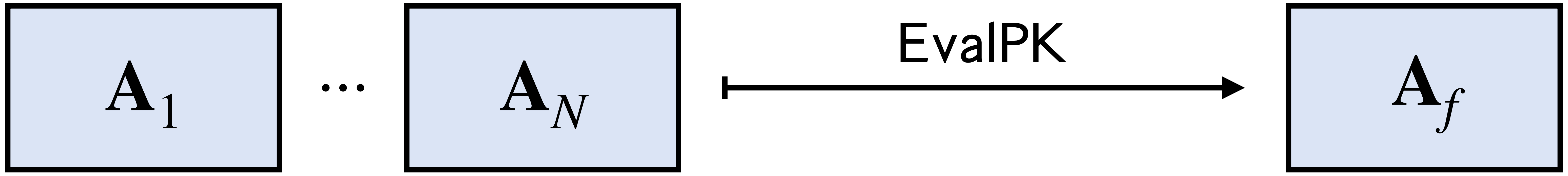

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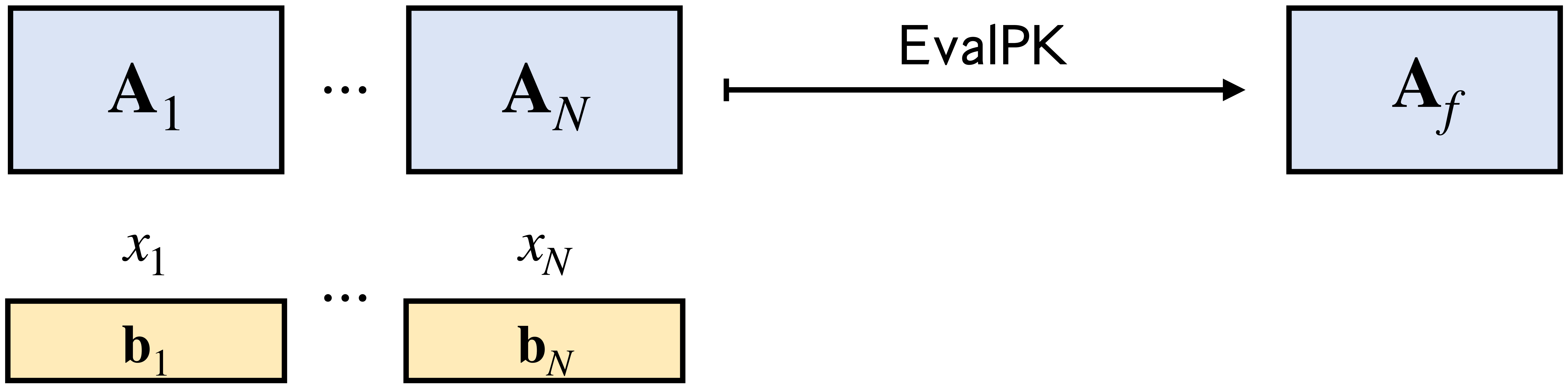

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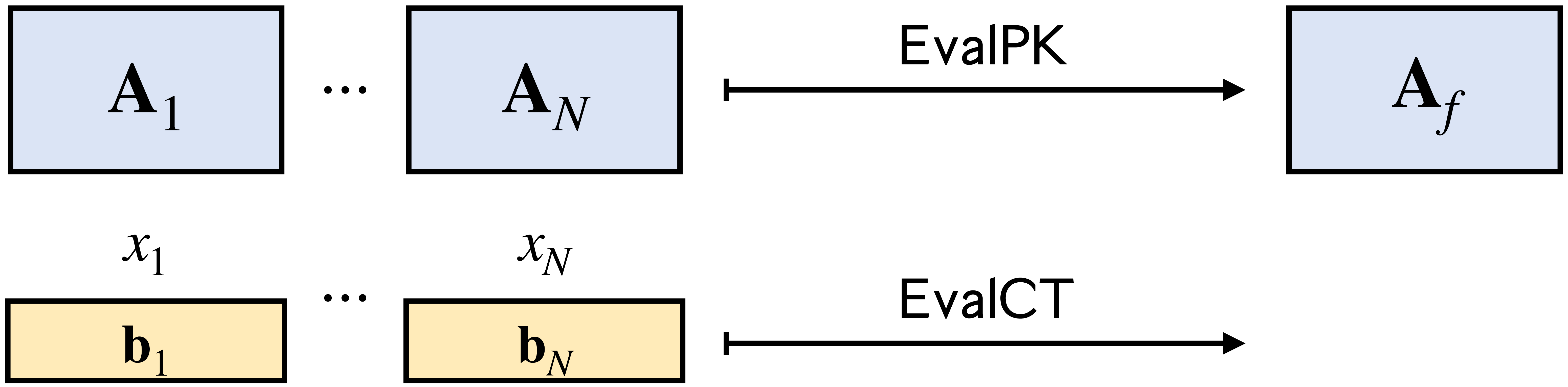

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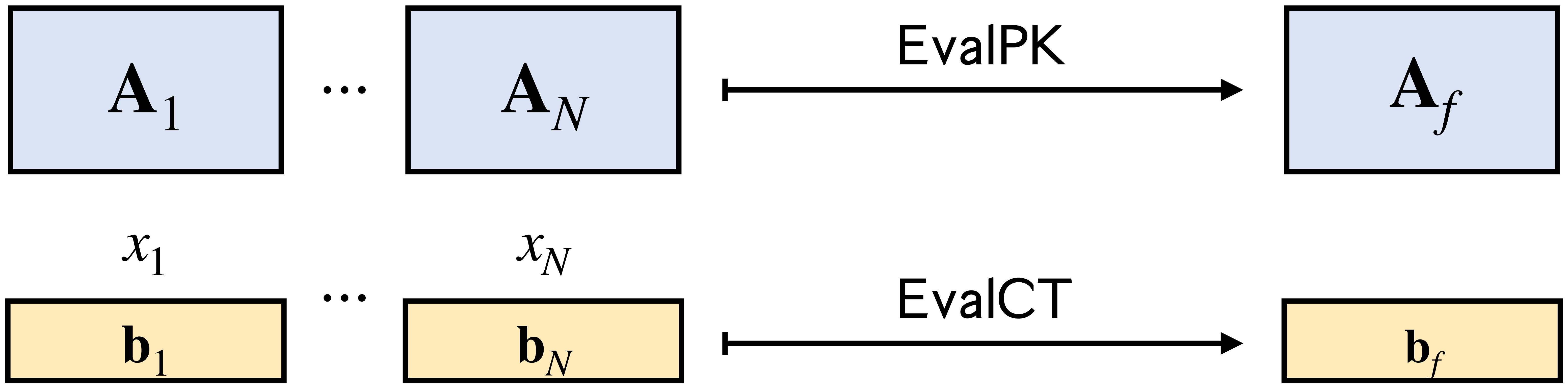

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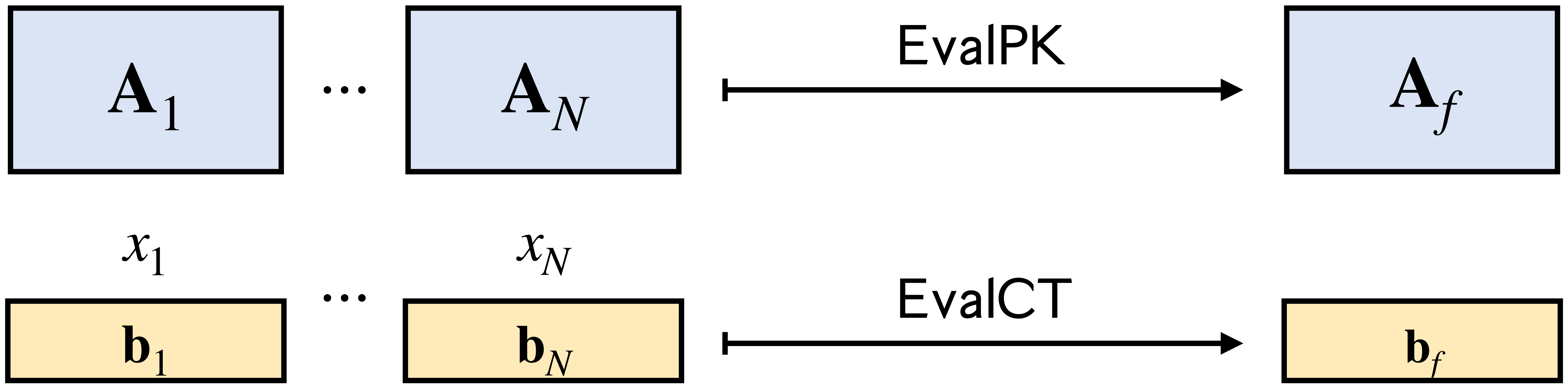

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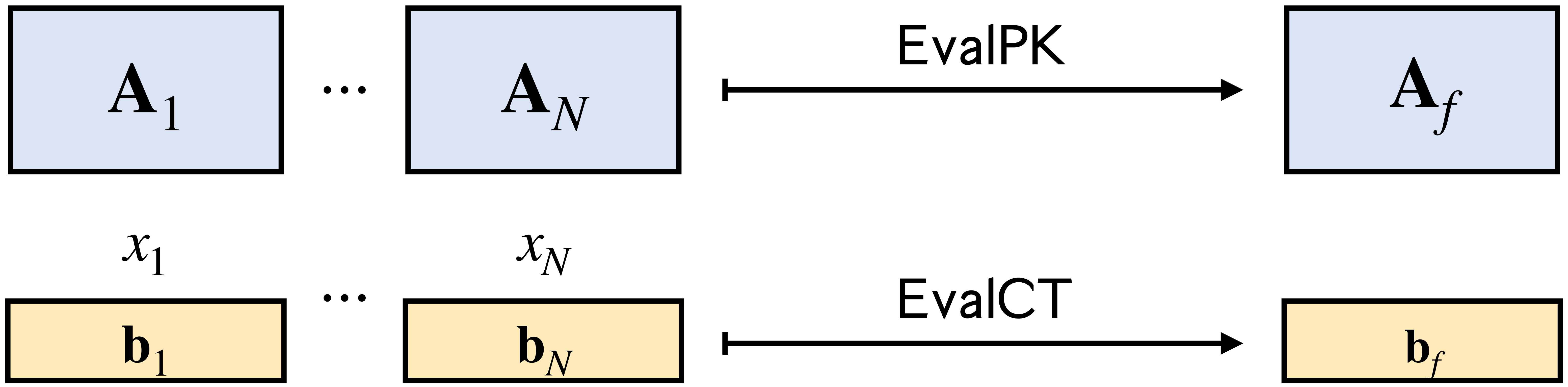

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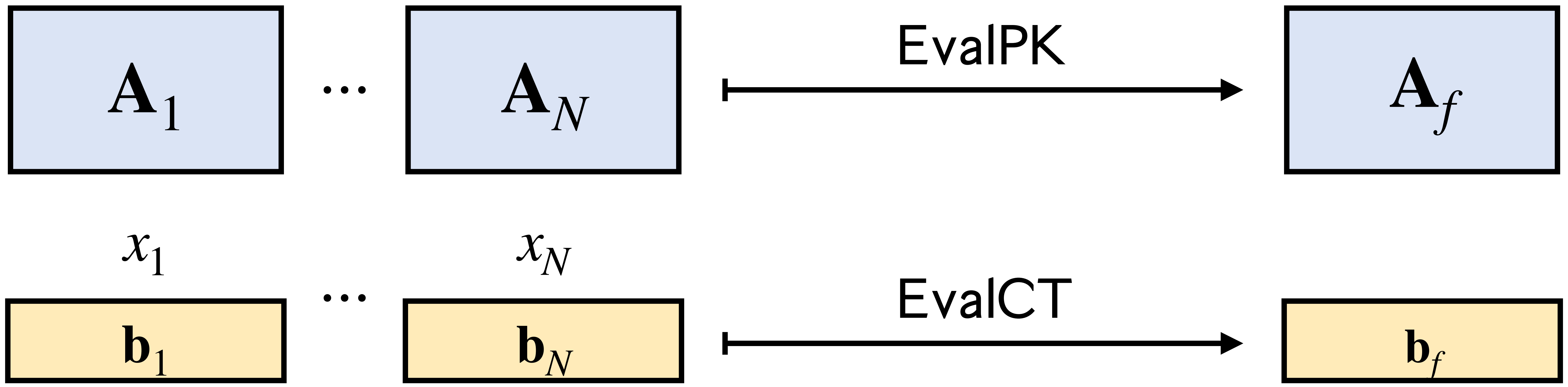

Error grows with depth of f

$\mathbf{b}_i \approx \mathbf{s}^\top(\mathbf{A}_i - x_i\mathbf{G})$

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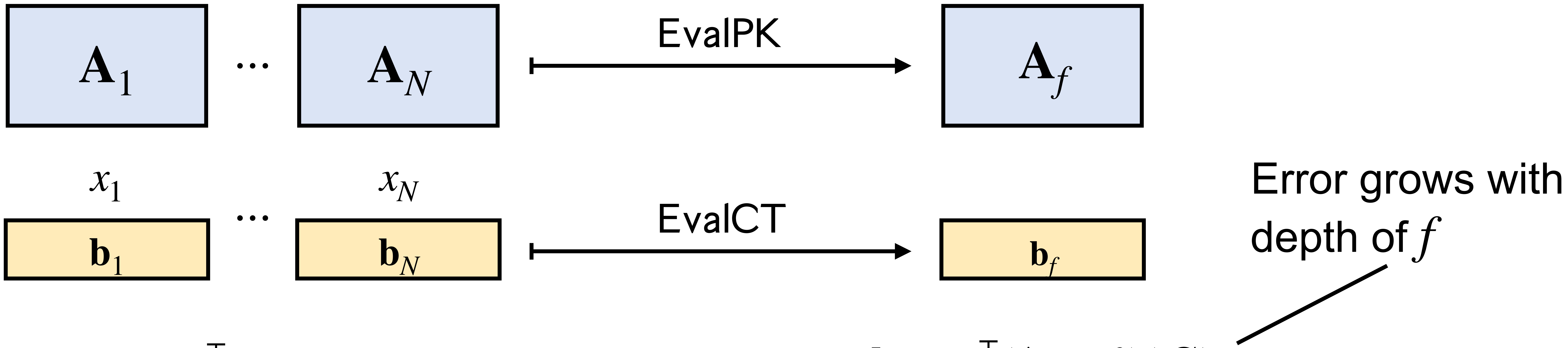

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[BGG+'14]: EvalPK, EvalCT run in time proportional to the circuit size of f

RAM Circuits

Boolean circuits + two new gates:

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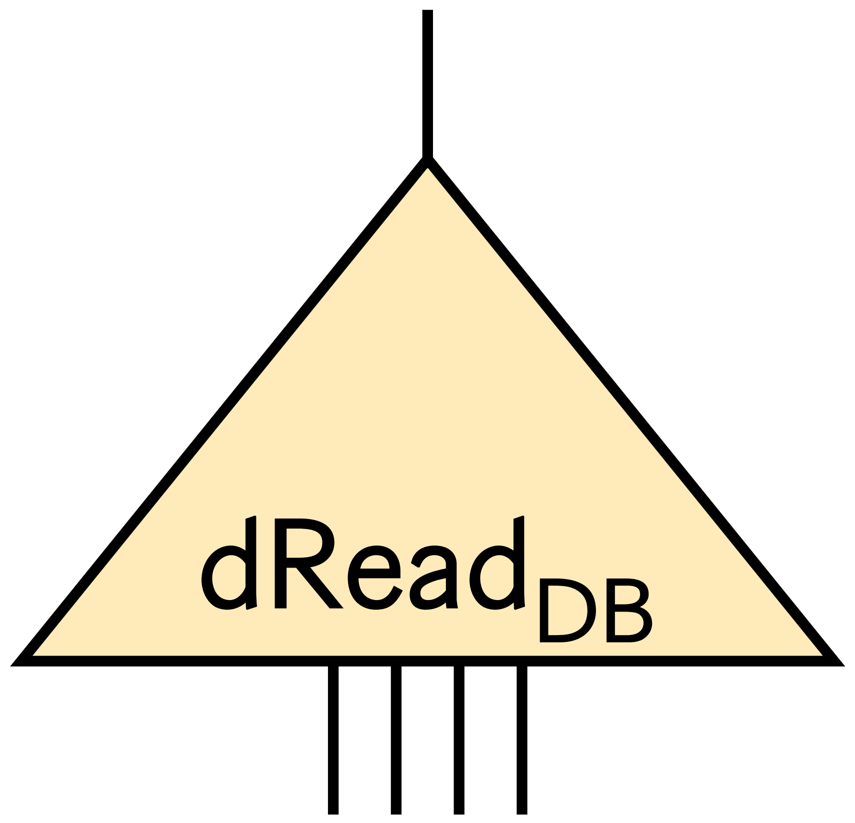
Data-Read Gates: For any DB $\in \{0,1\}^L$



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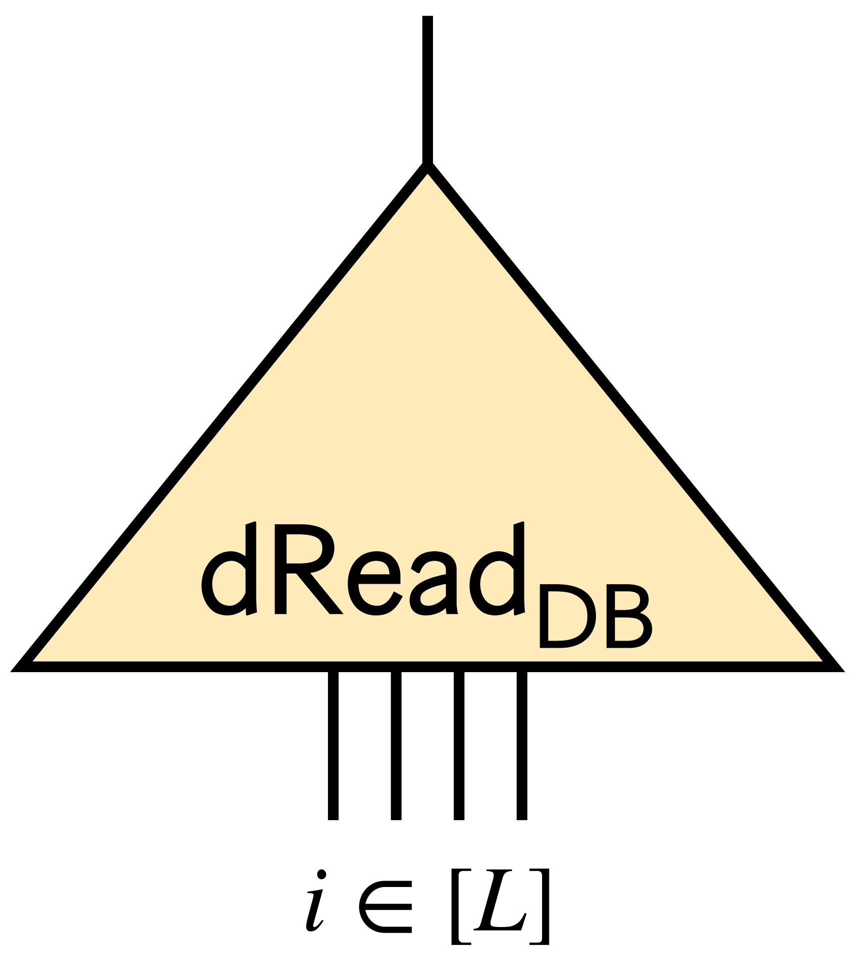

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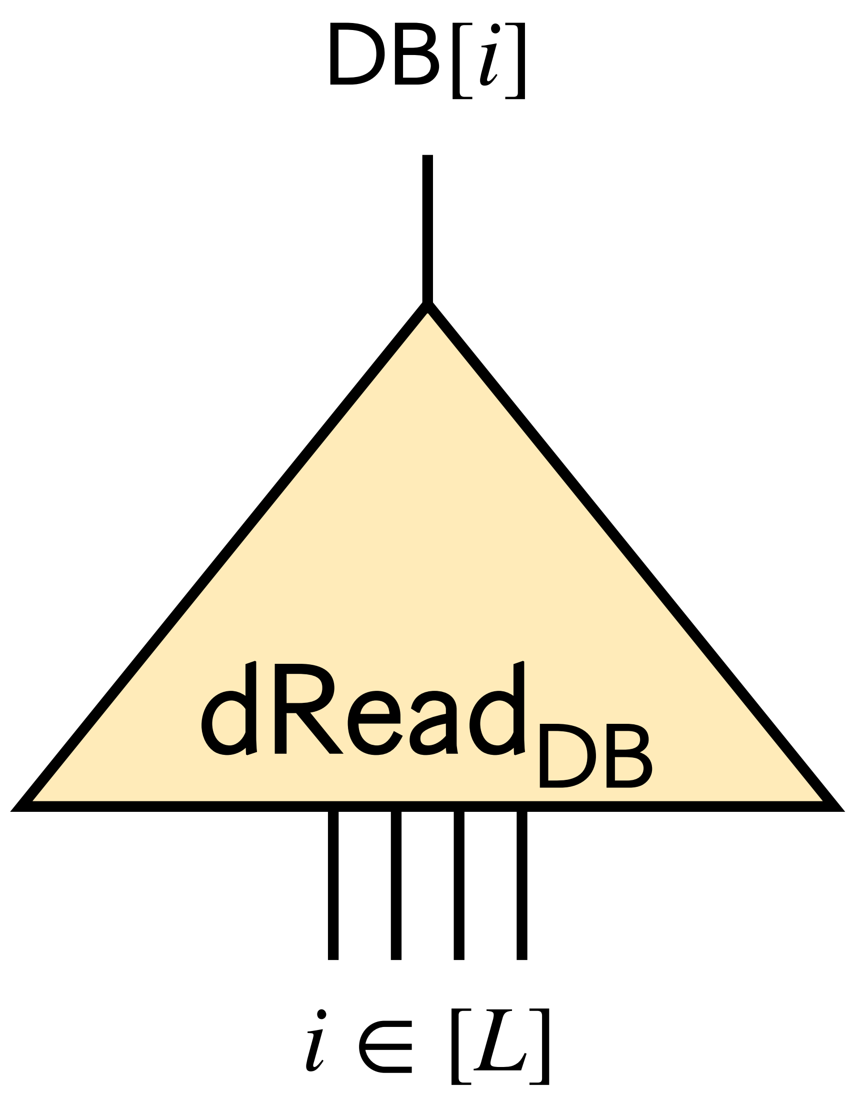

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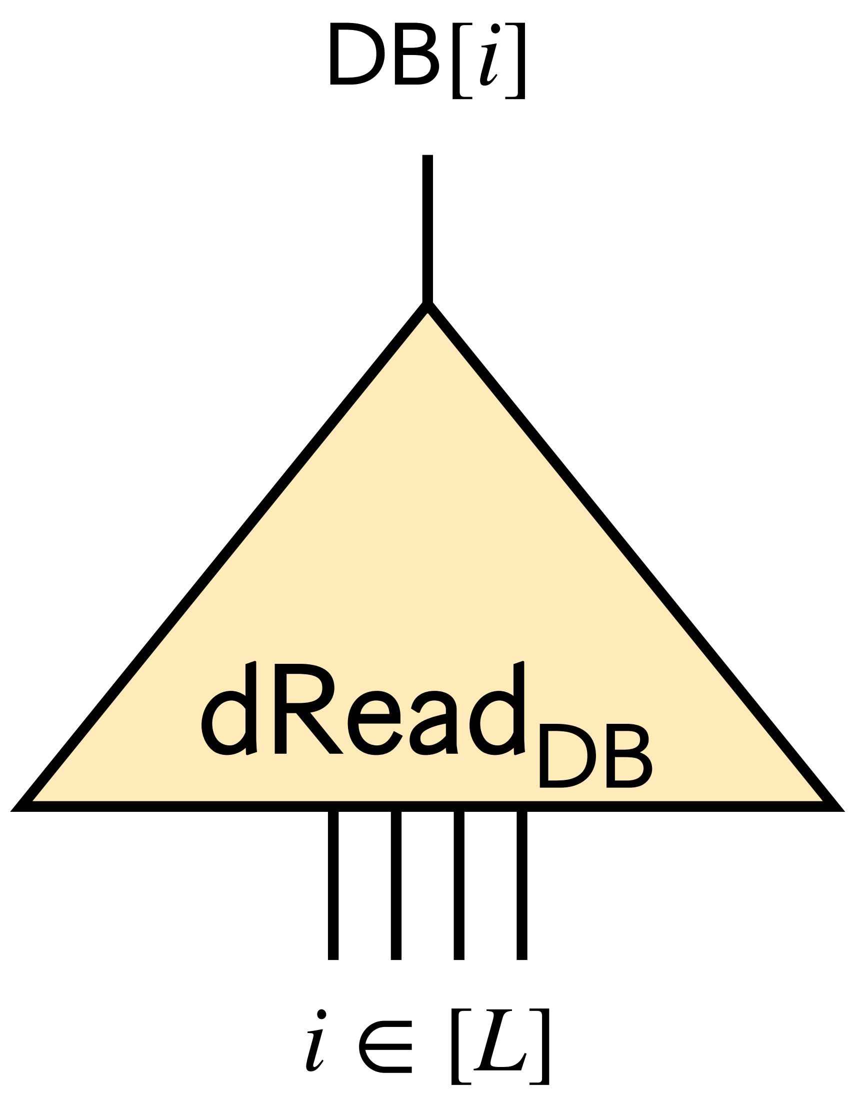
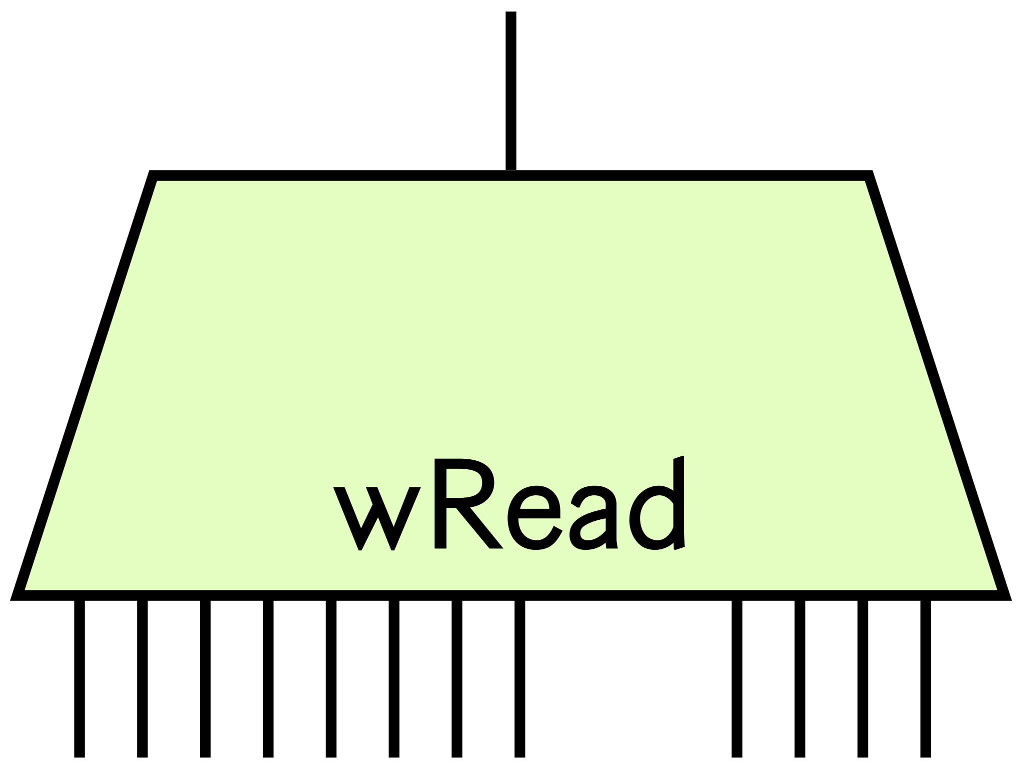


Wire-Read Gates:



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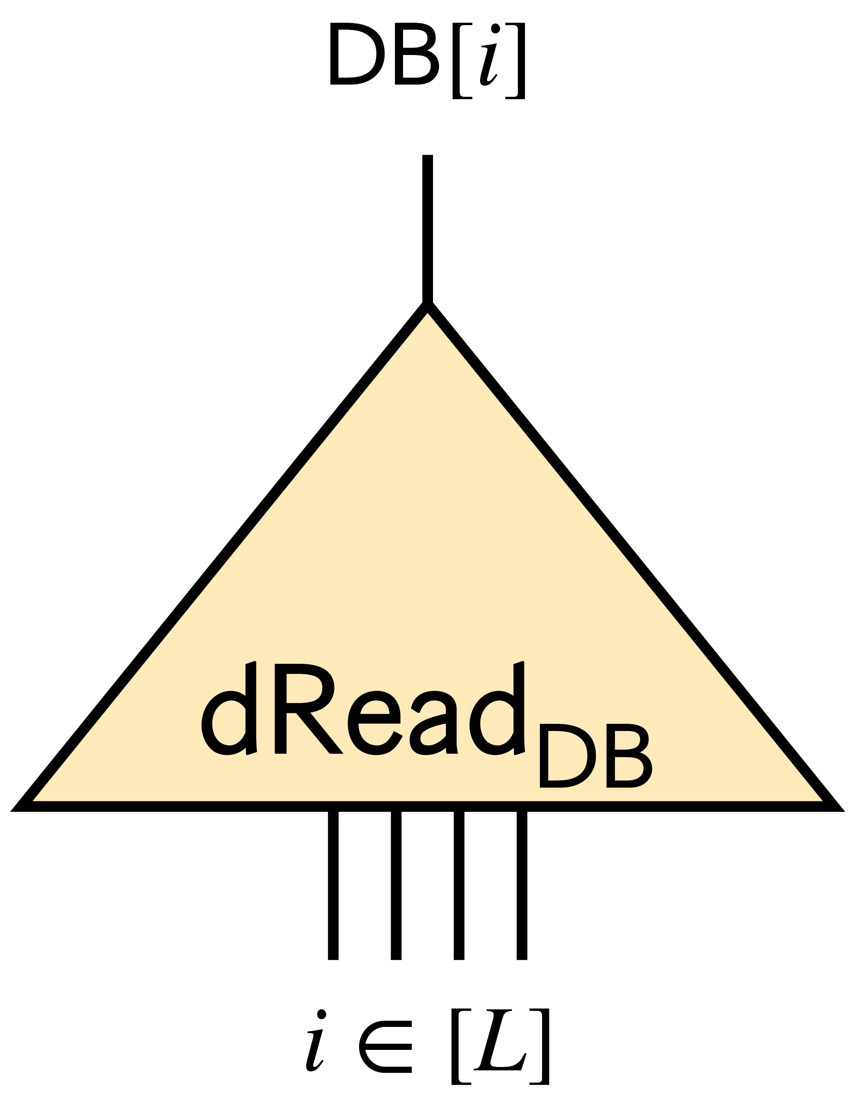

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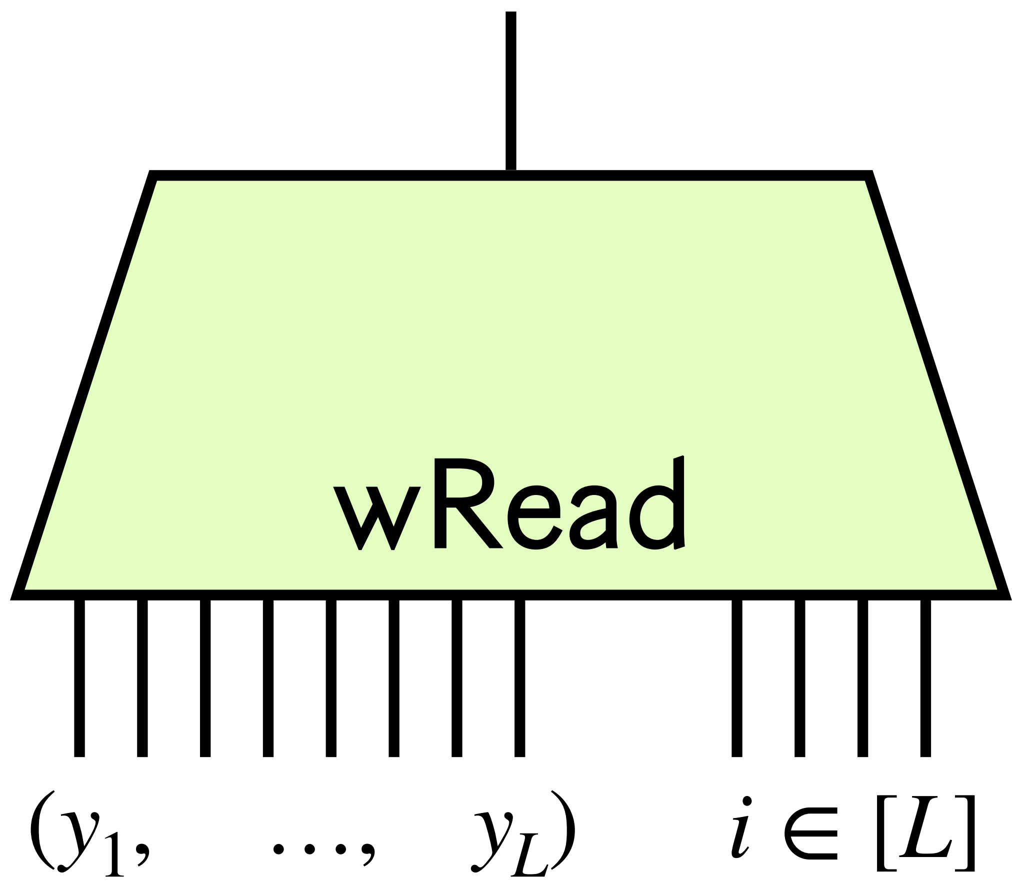

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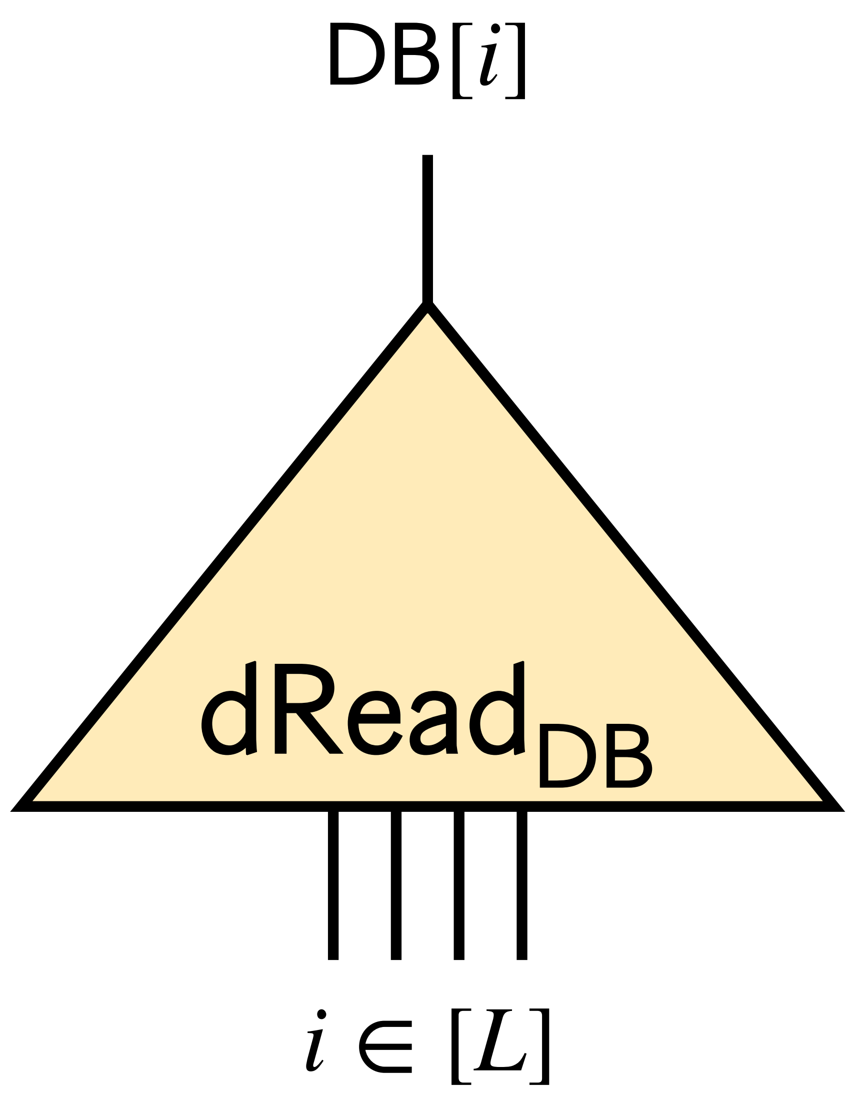

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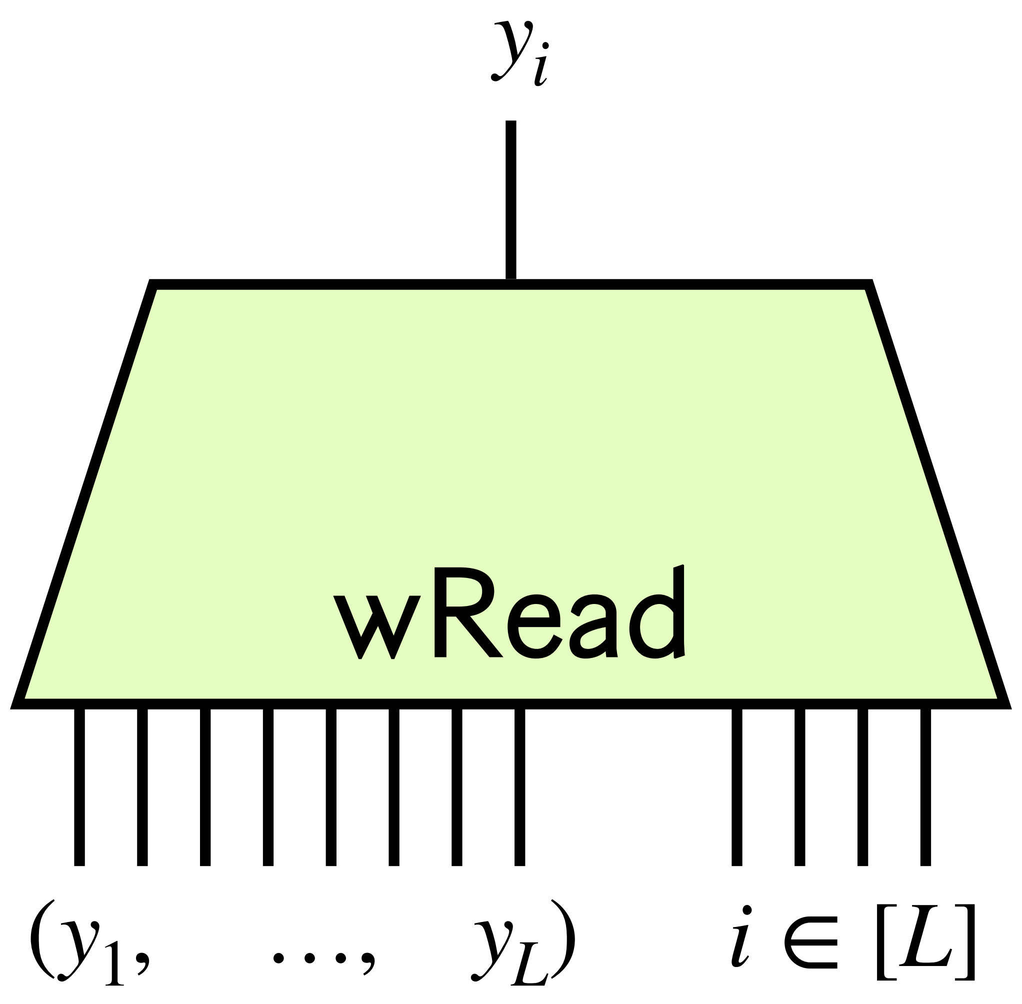

RAM Circuits

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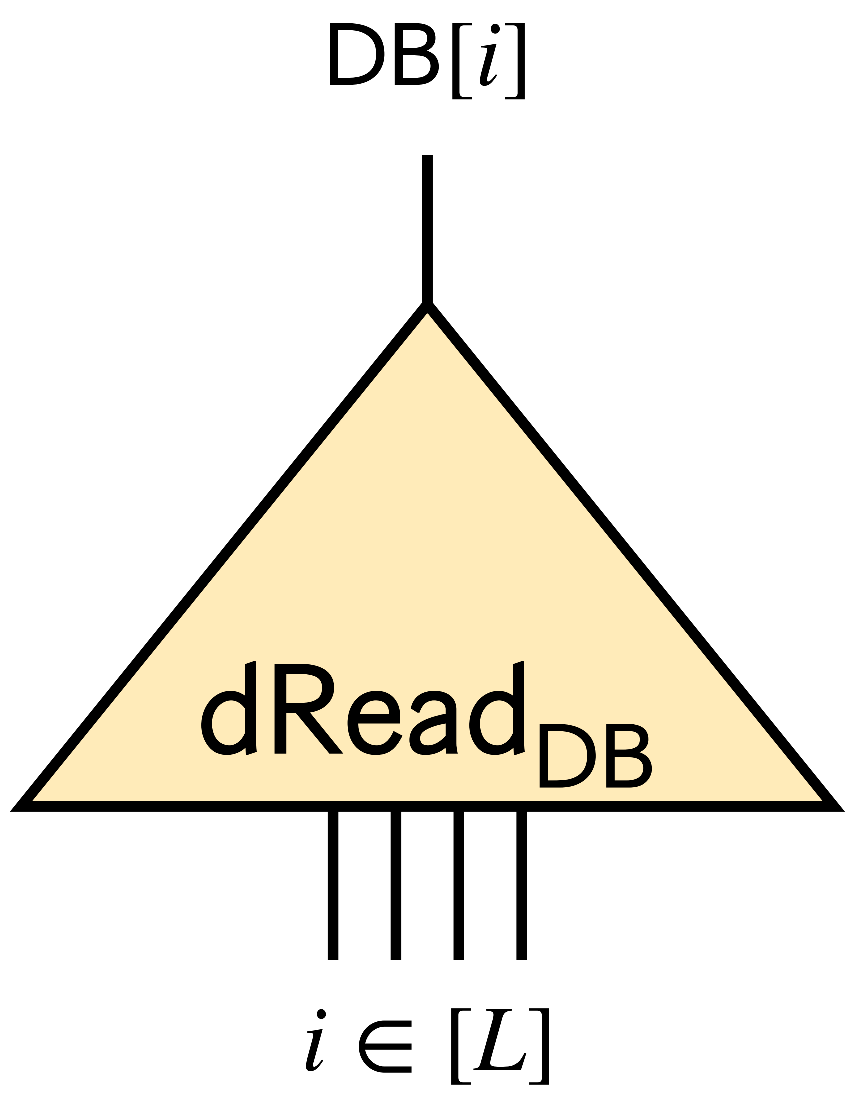
Efficient access to a hardcoded database



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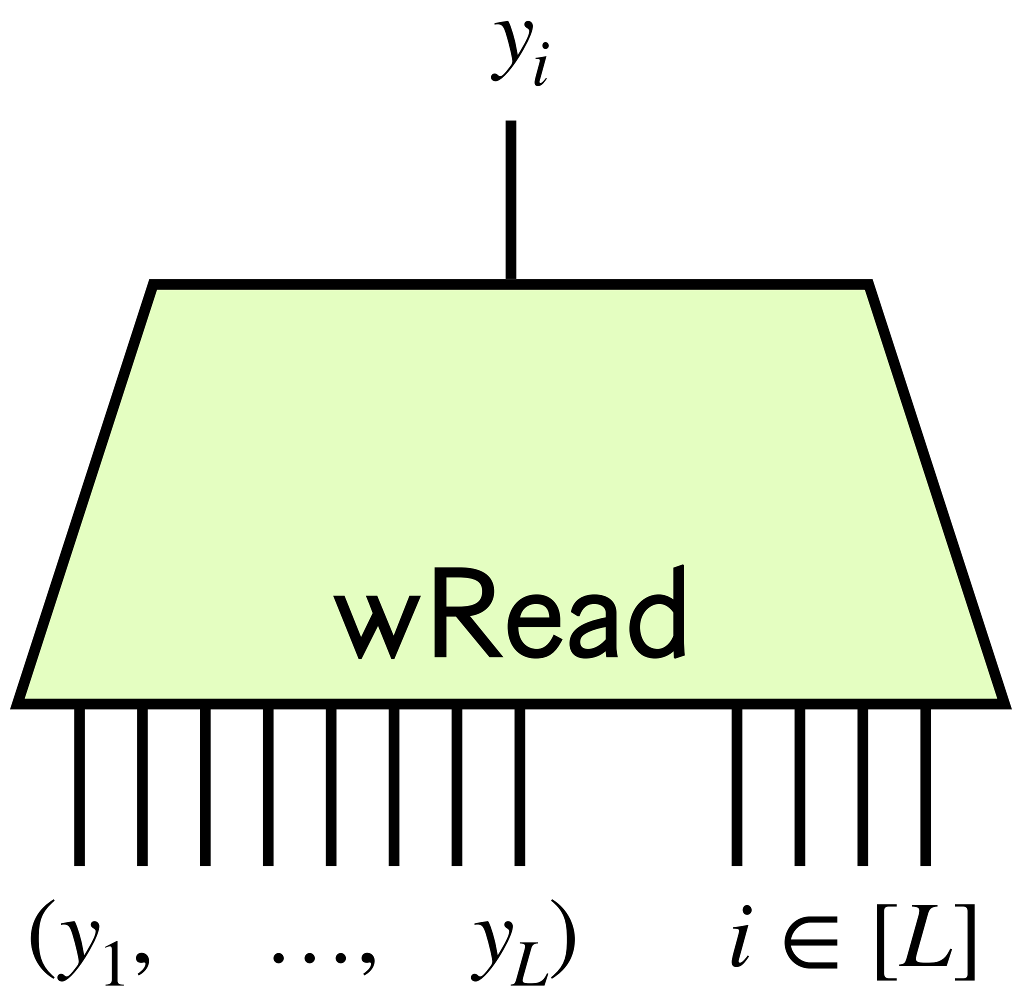

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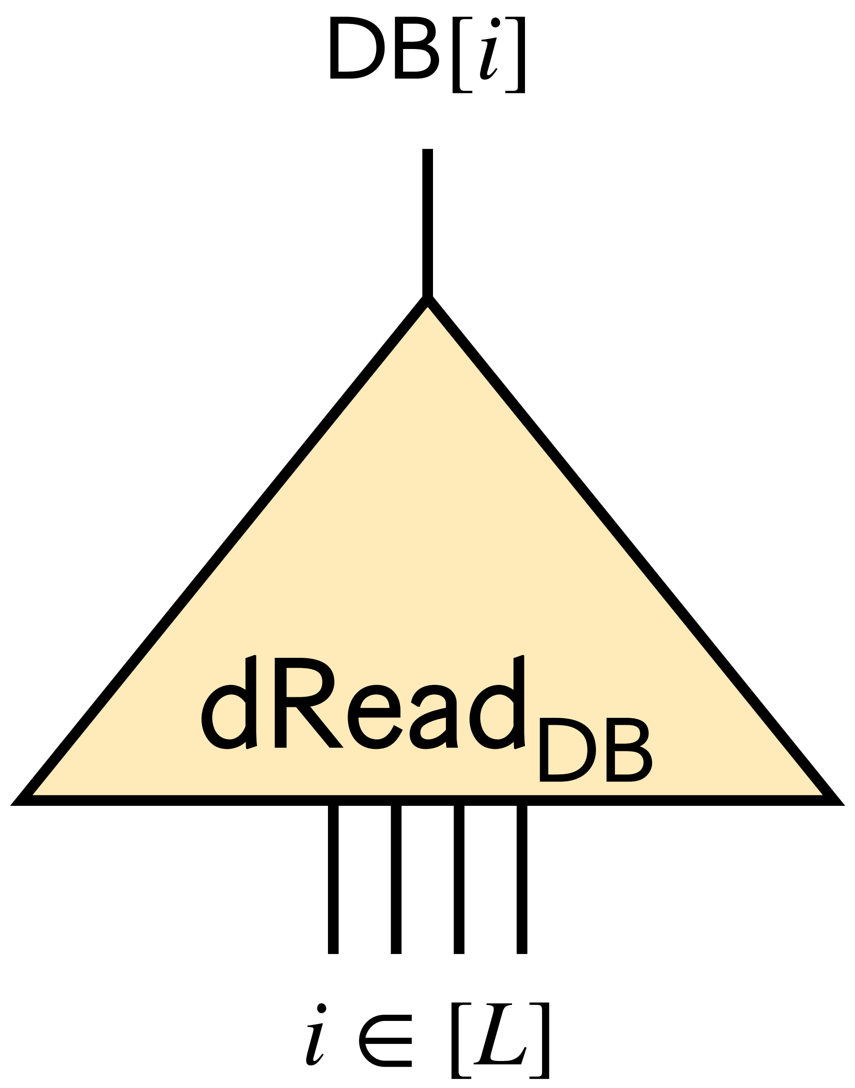

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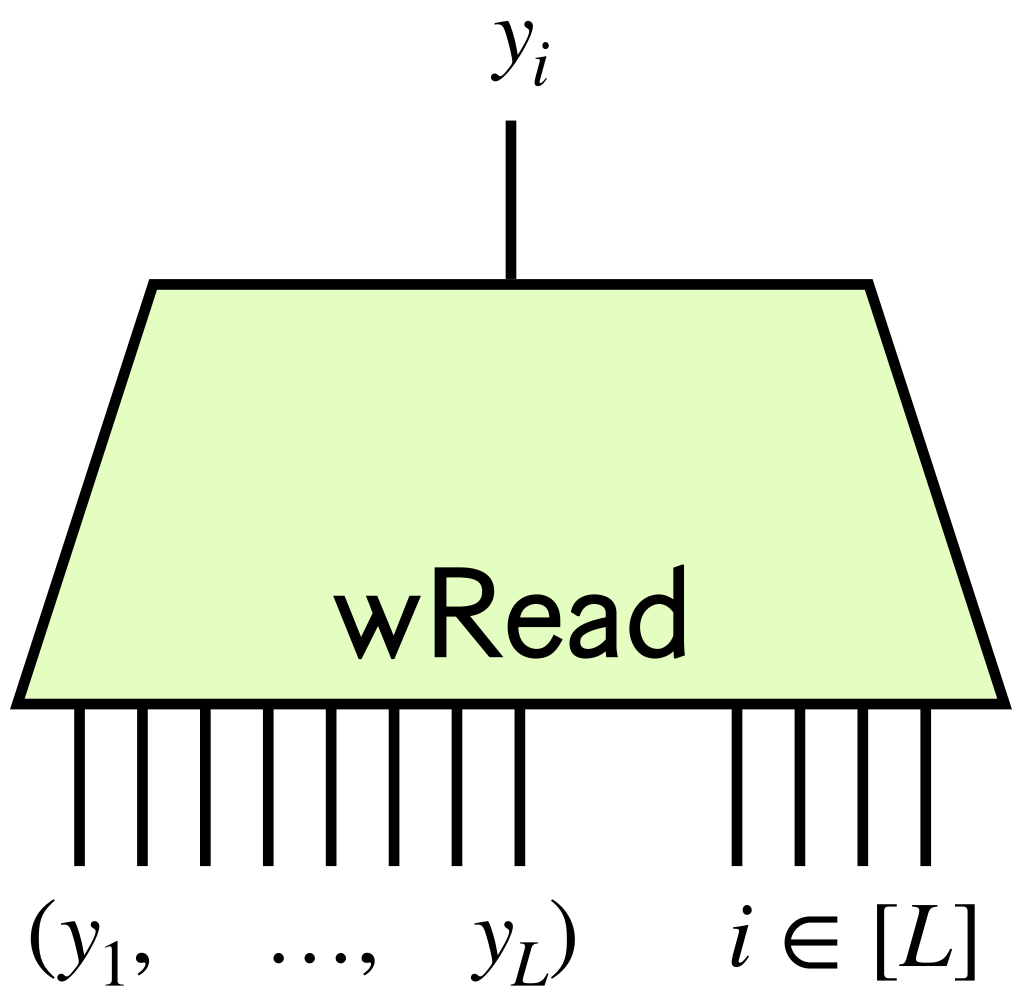

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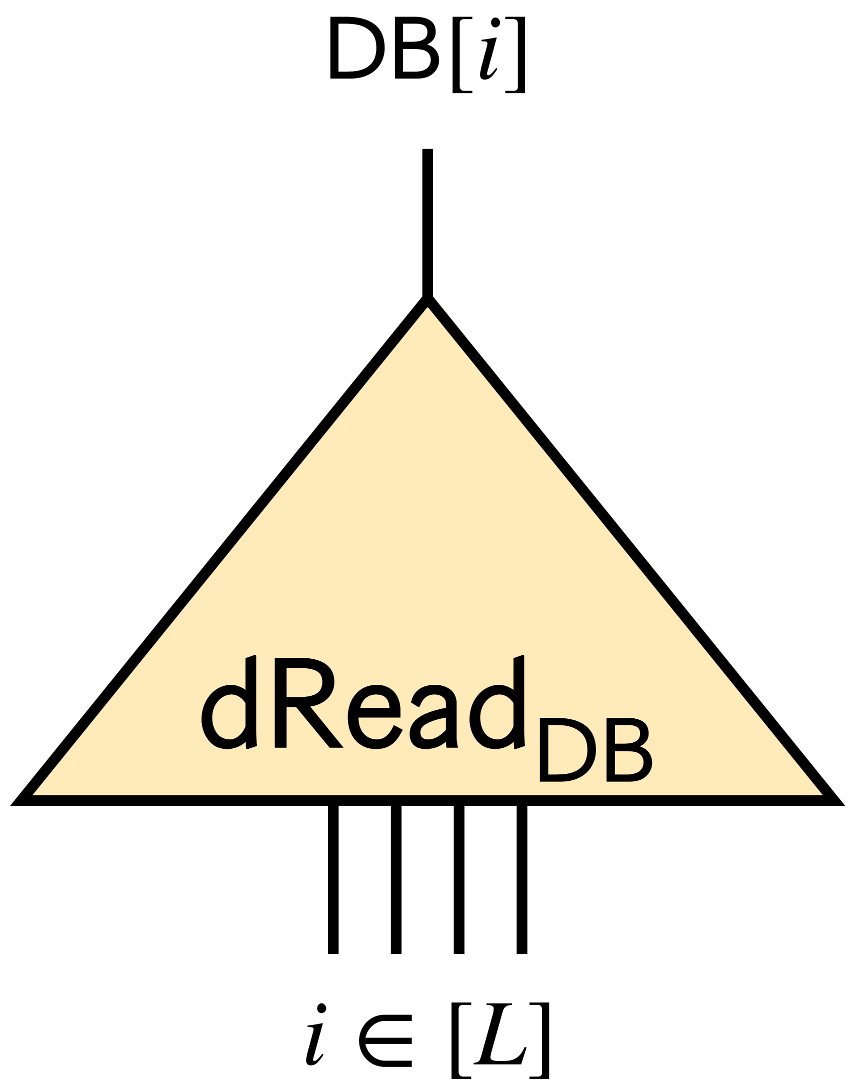

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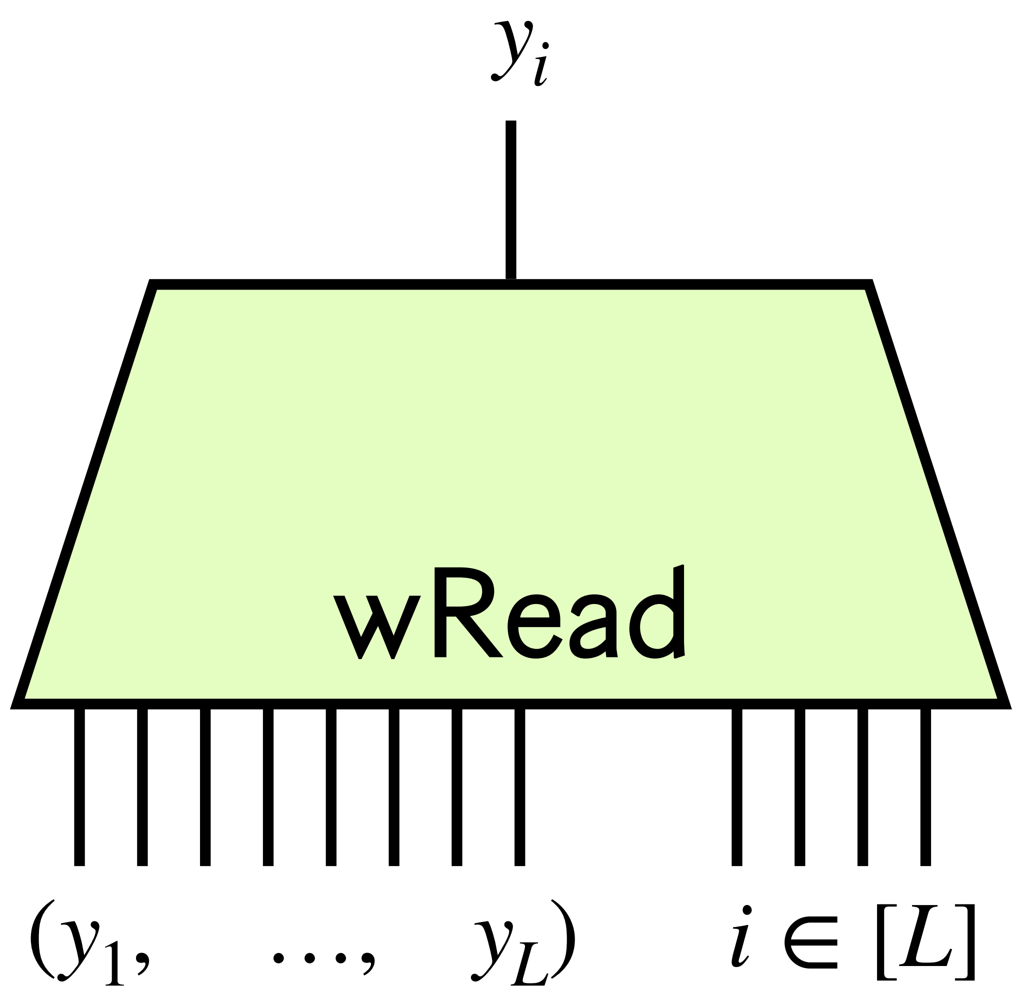

Efficient access to input/working memory

Fixed circuit topology $\Longrightarrow$ write locations must be fixed in advance

RAM Circuits

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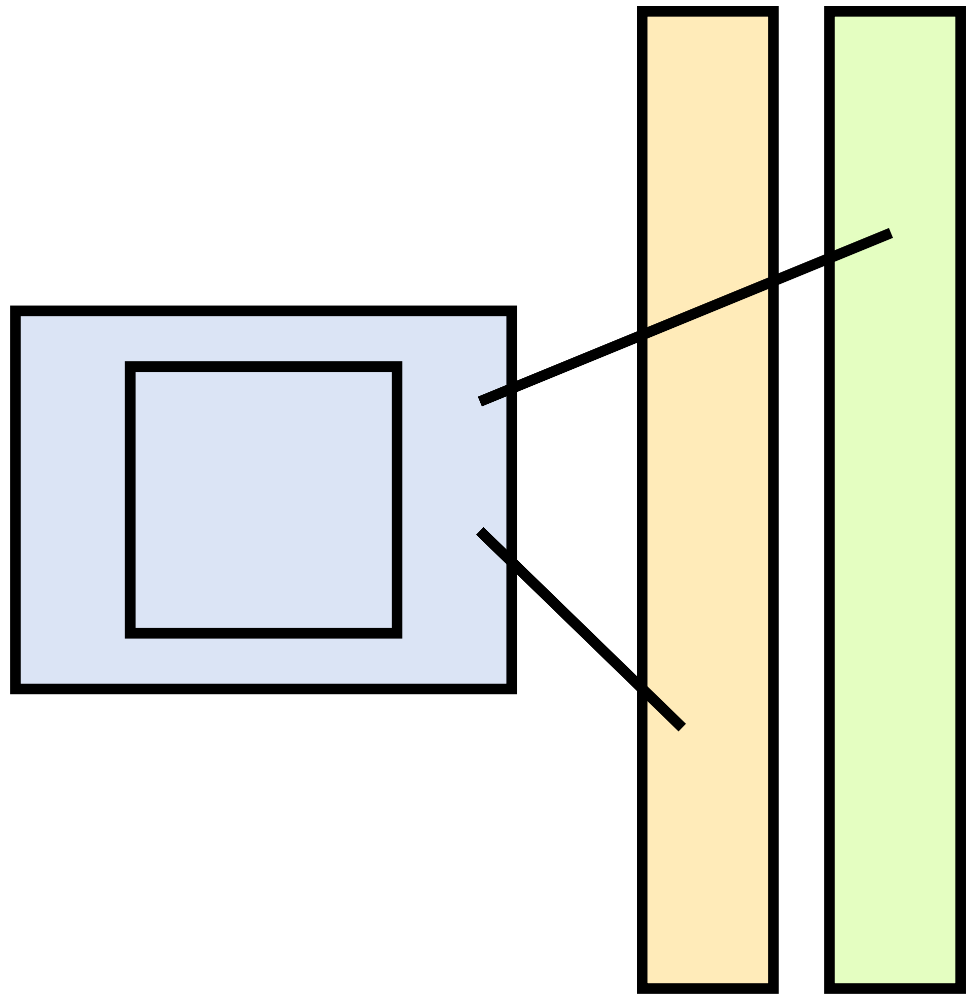

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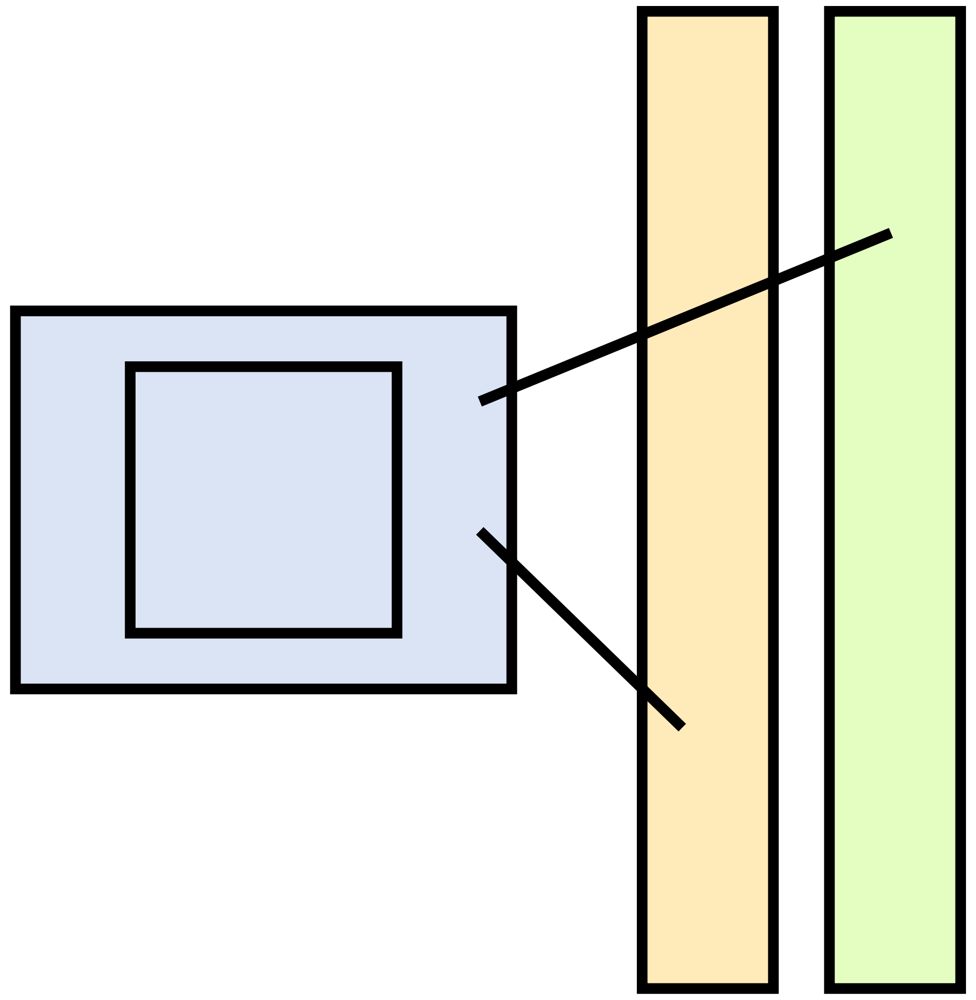

Lemma: A RAM program f_{DB} of runtime T can be represented by a RAM circuit of size $\tilde{O}(\max(T, N))$ for inputs of size N

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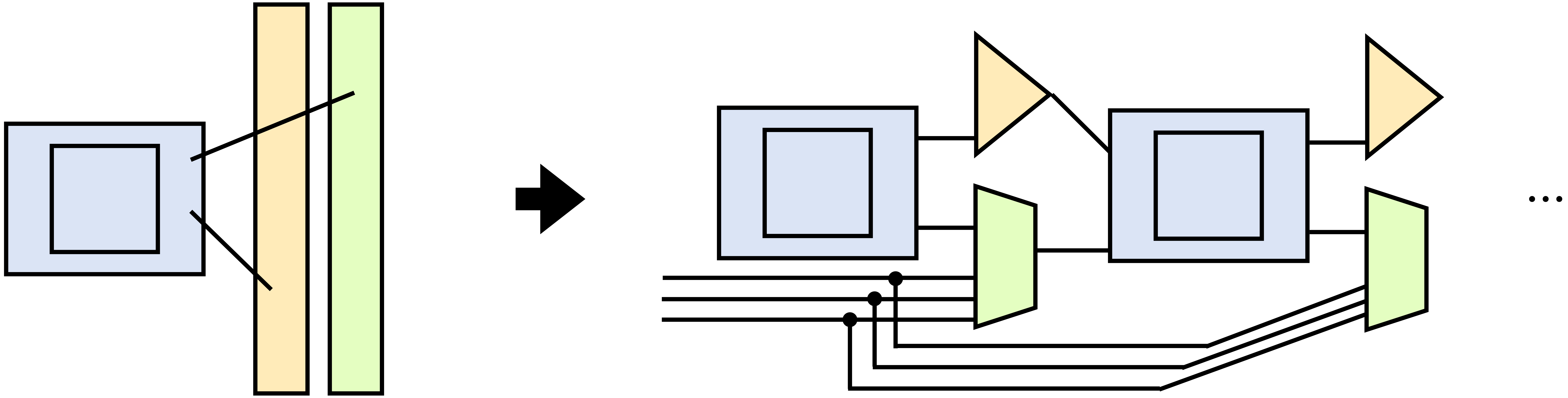

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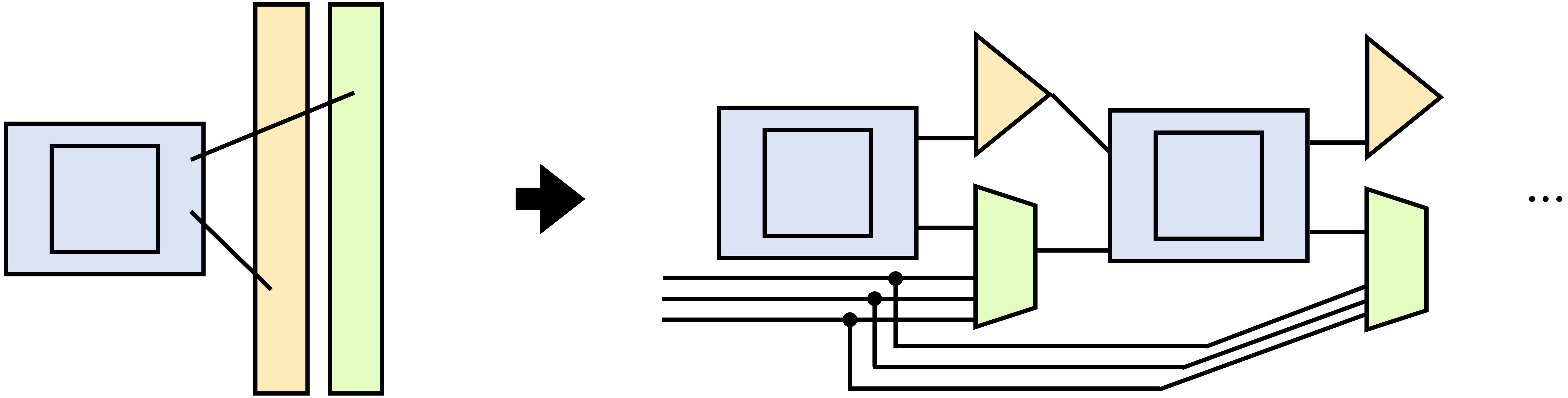

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Note: The transformation yields RAM circuit of depth $\tilde{O}(T)$, but could do better if f_{DB} is parallelizable

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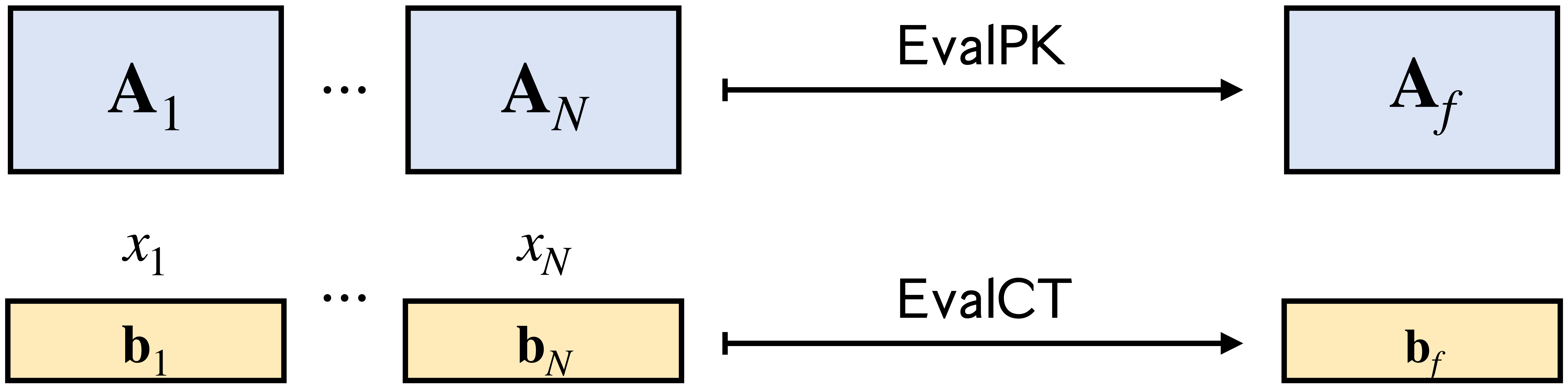

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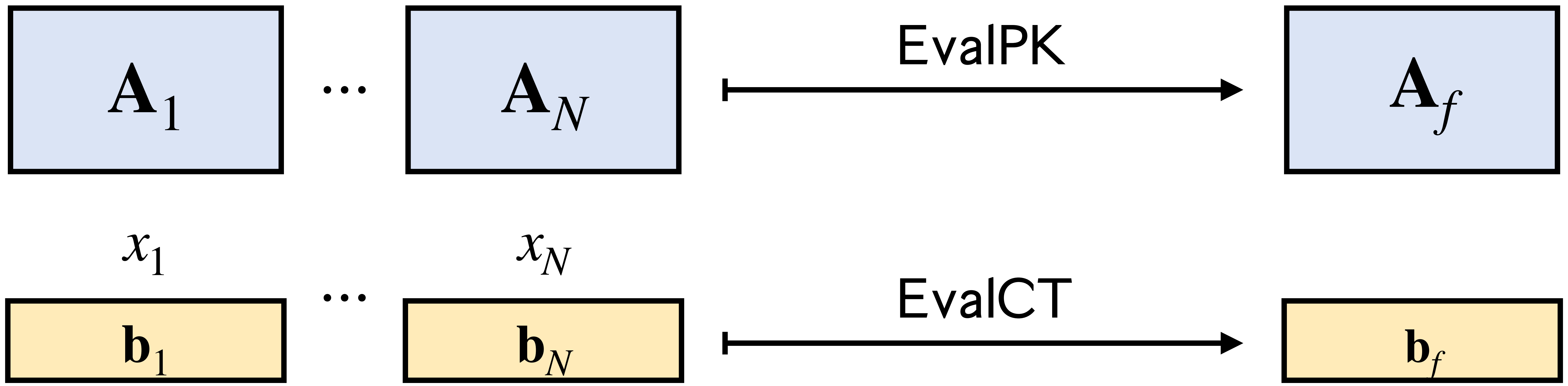

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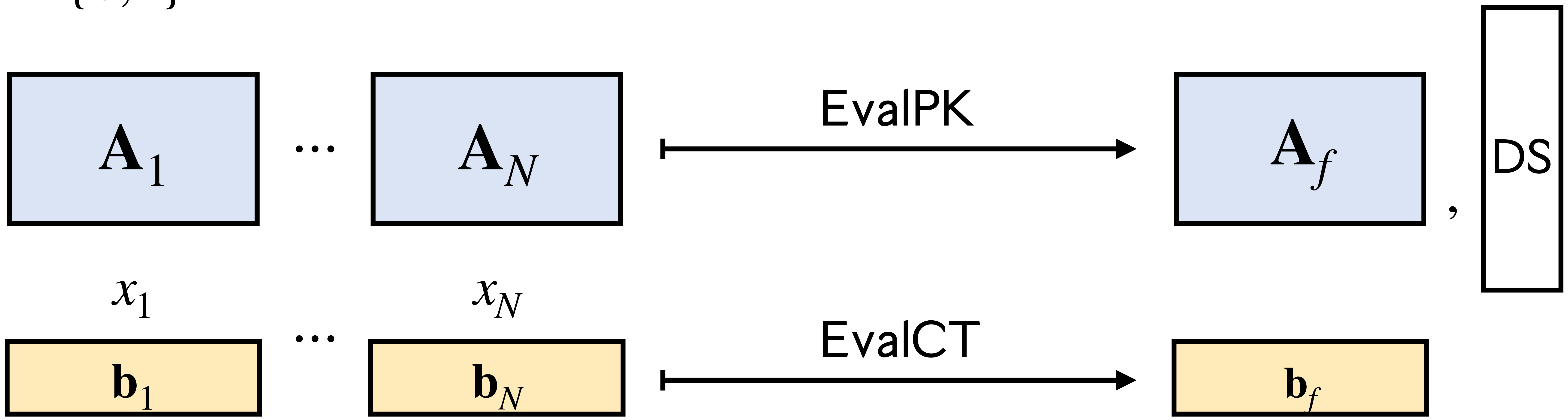

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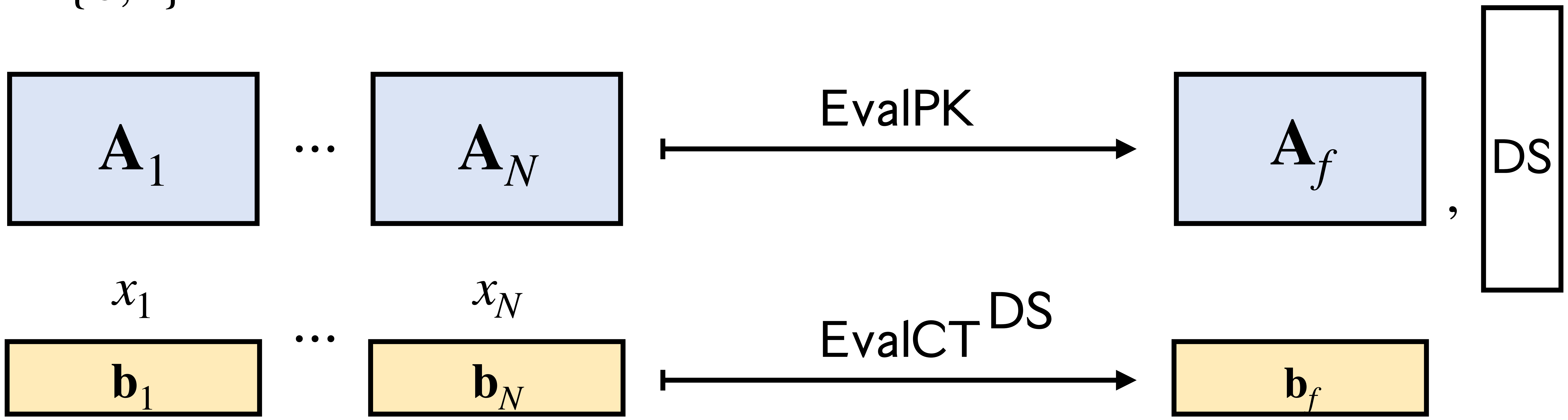

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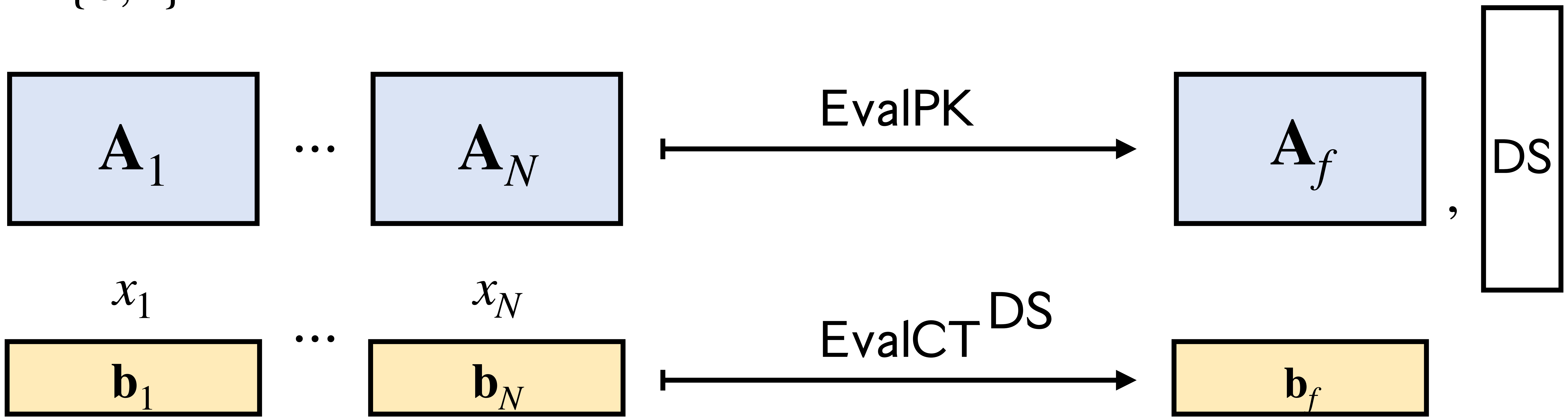

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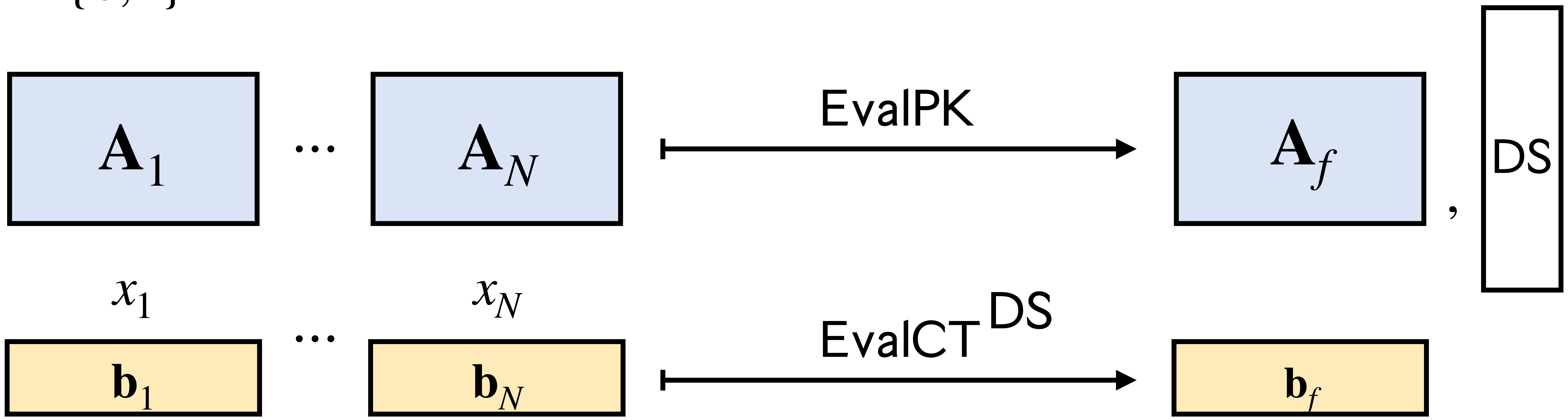

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- EvalPK runs in time proportional to boolean circuit size of f

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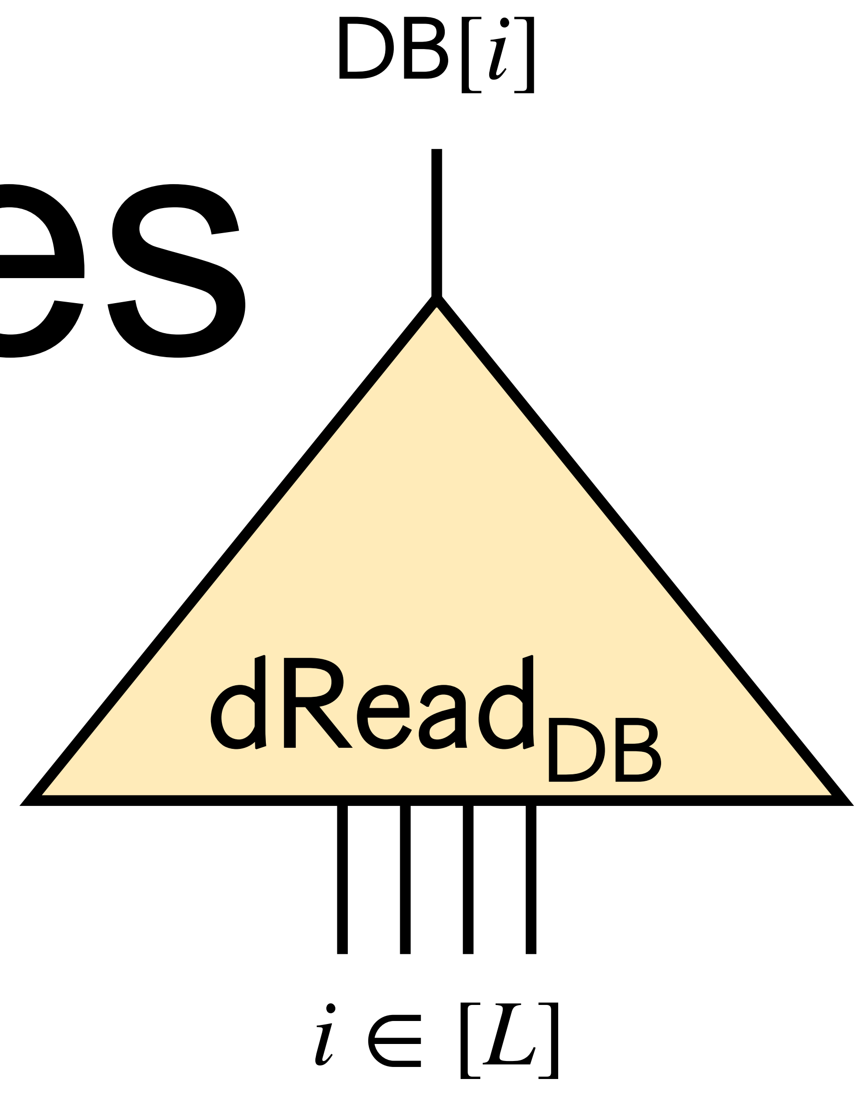

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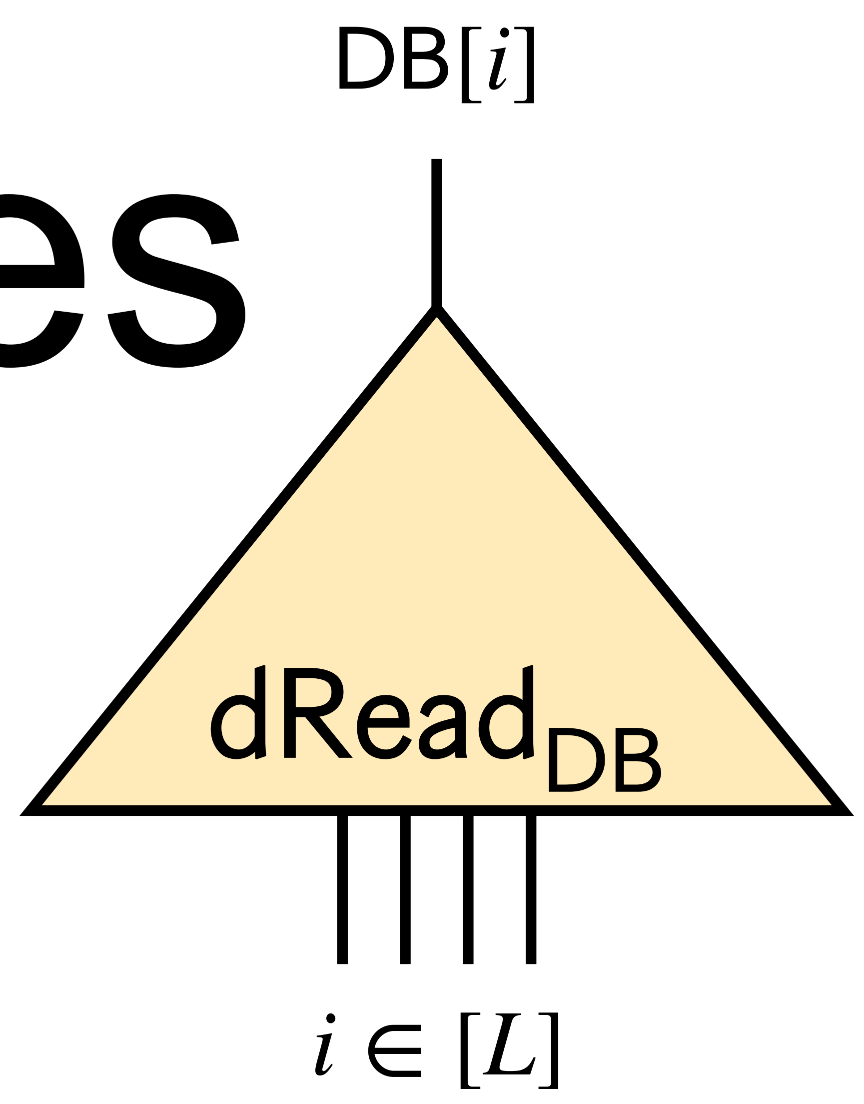
Goal: Apply [BGG+'14] operations to the function $\mathsf{dRead}_{\mathsf{DB}}$



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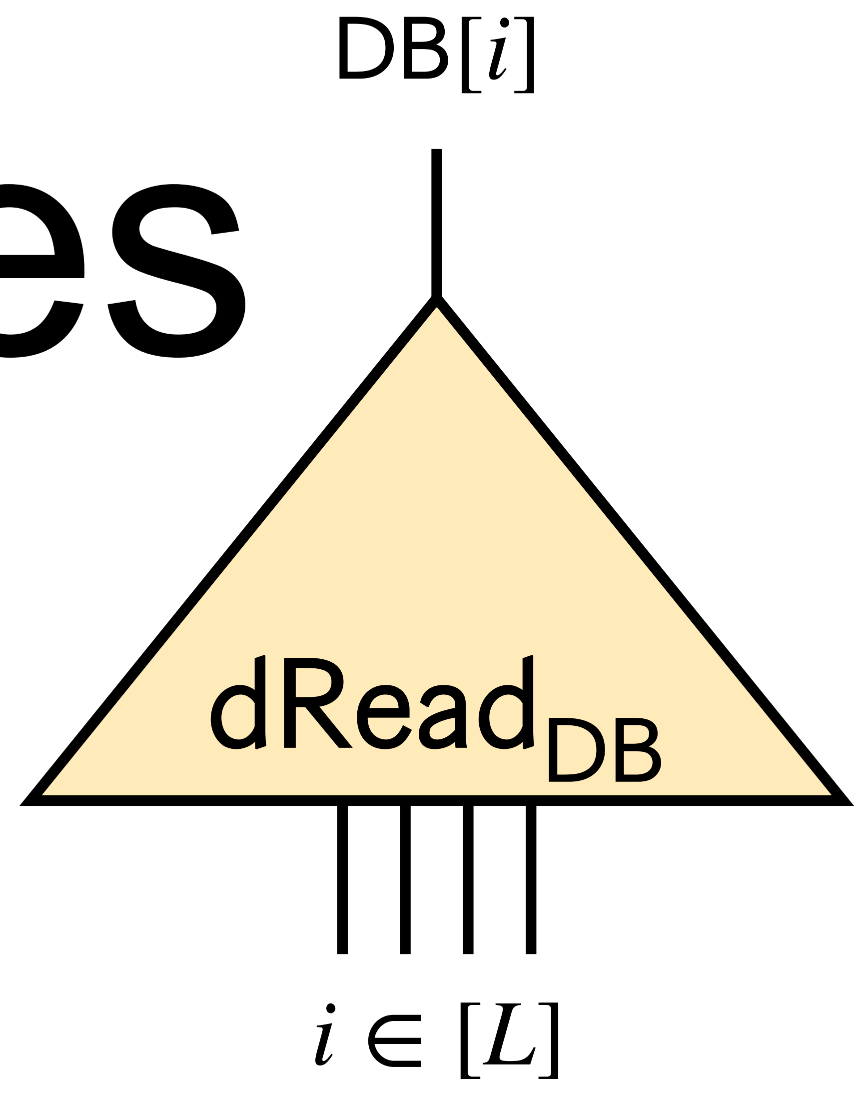
- But somehow speed up EvalCT



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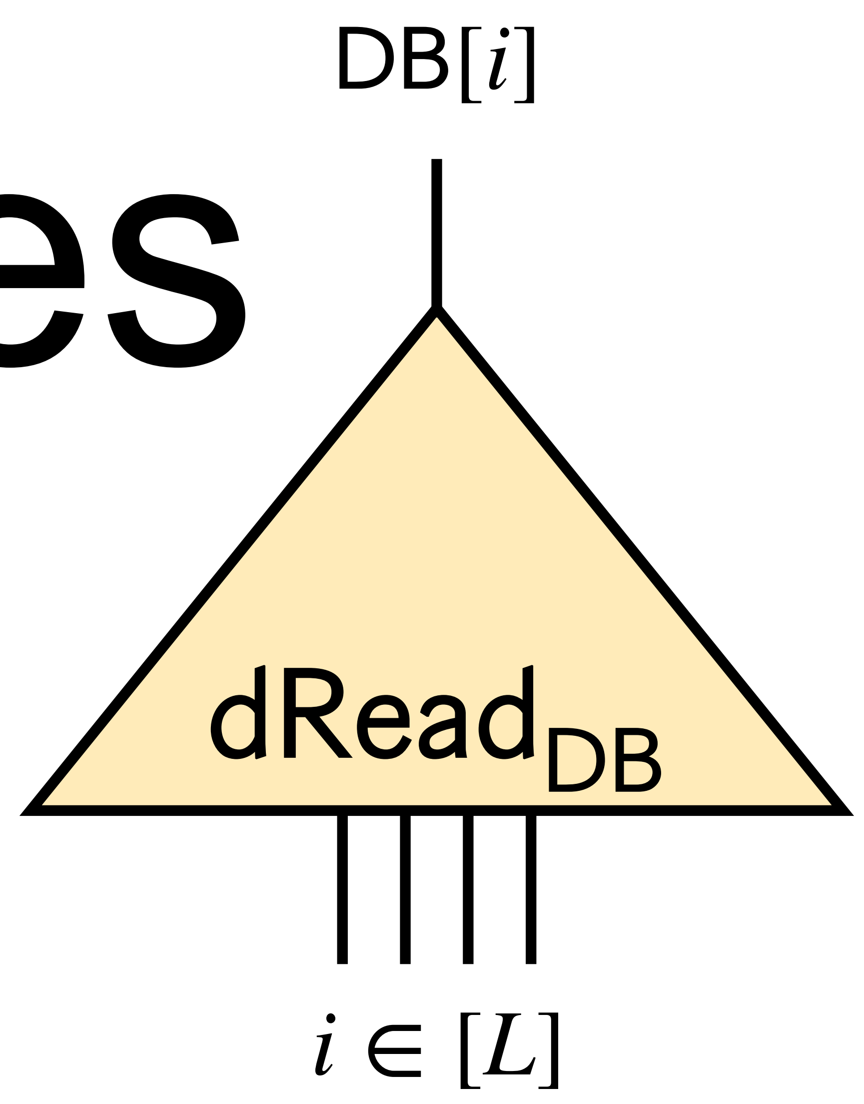
Crucial Fact: [BGG+'14] operations are **linear**



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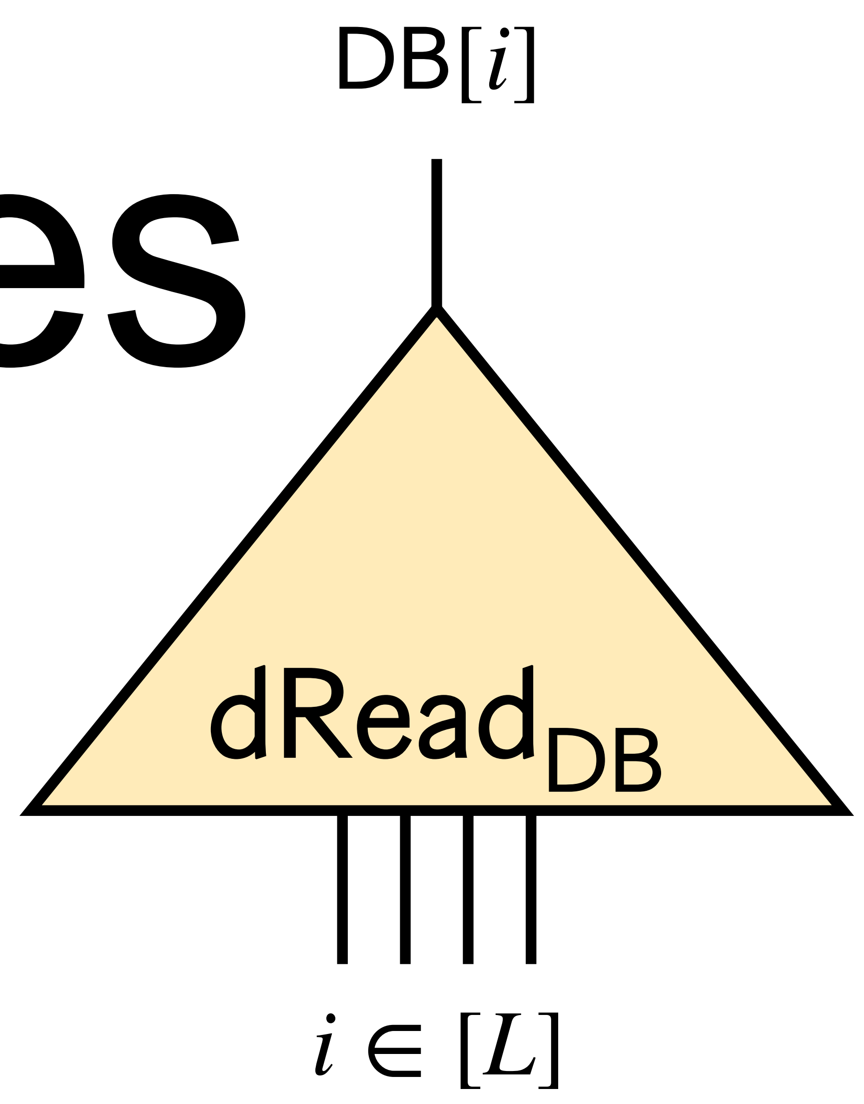

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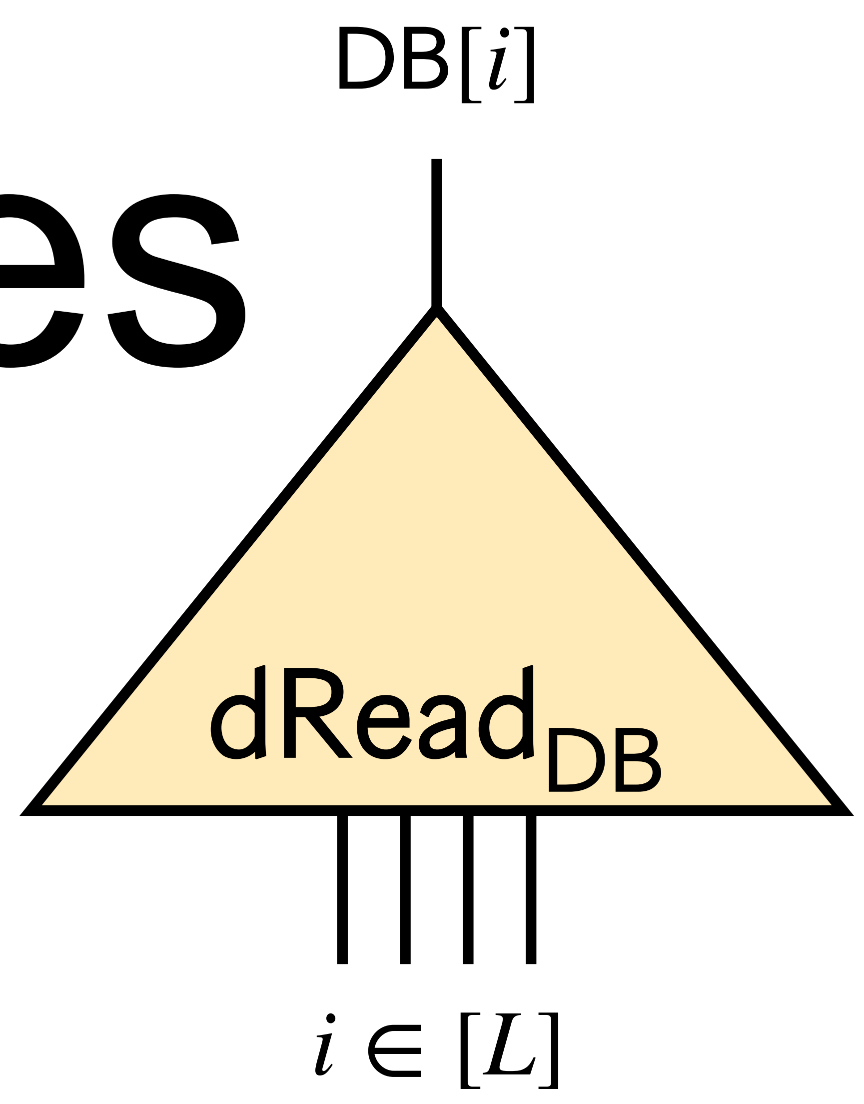

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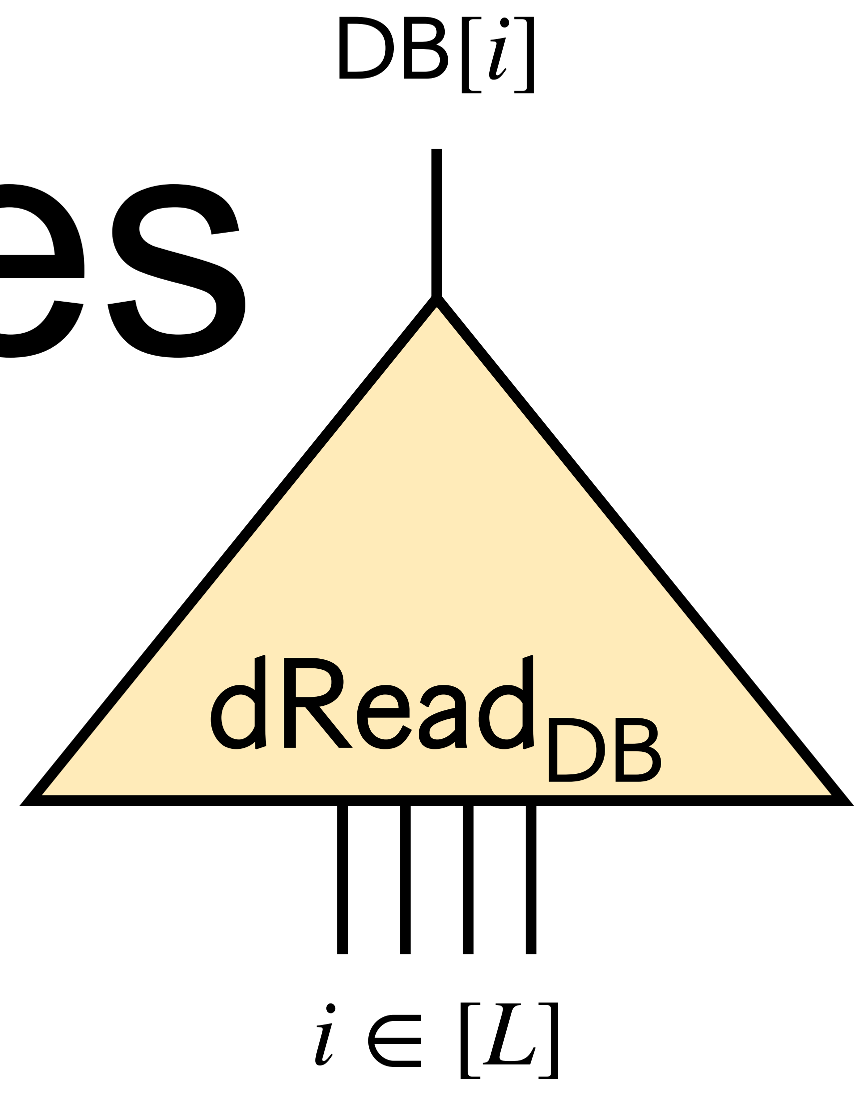
Handling Data-Read Gates



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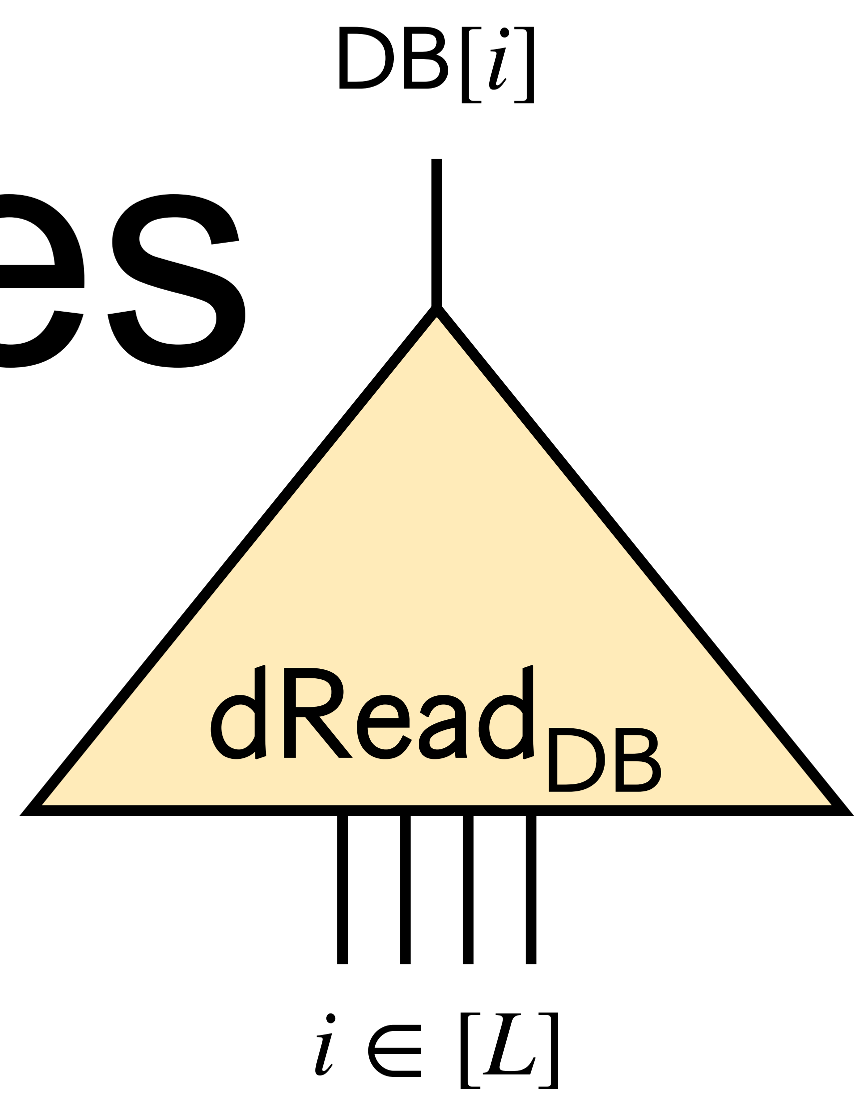
Depends on $\mathbf{A}$ and i

Handling Data-Read Gates



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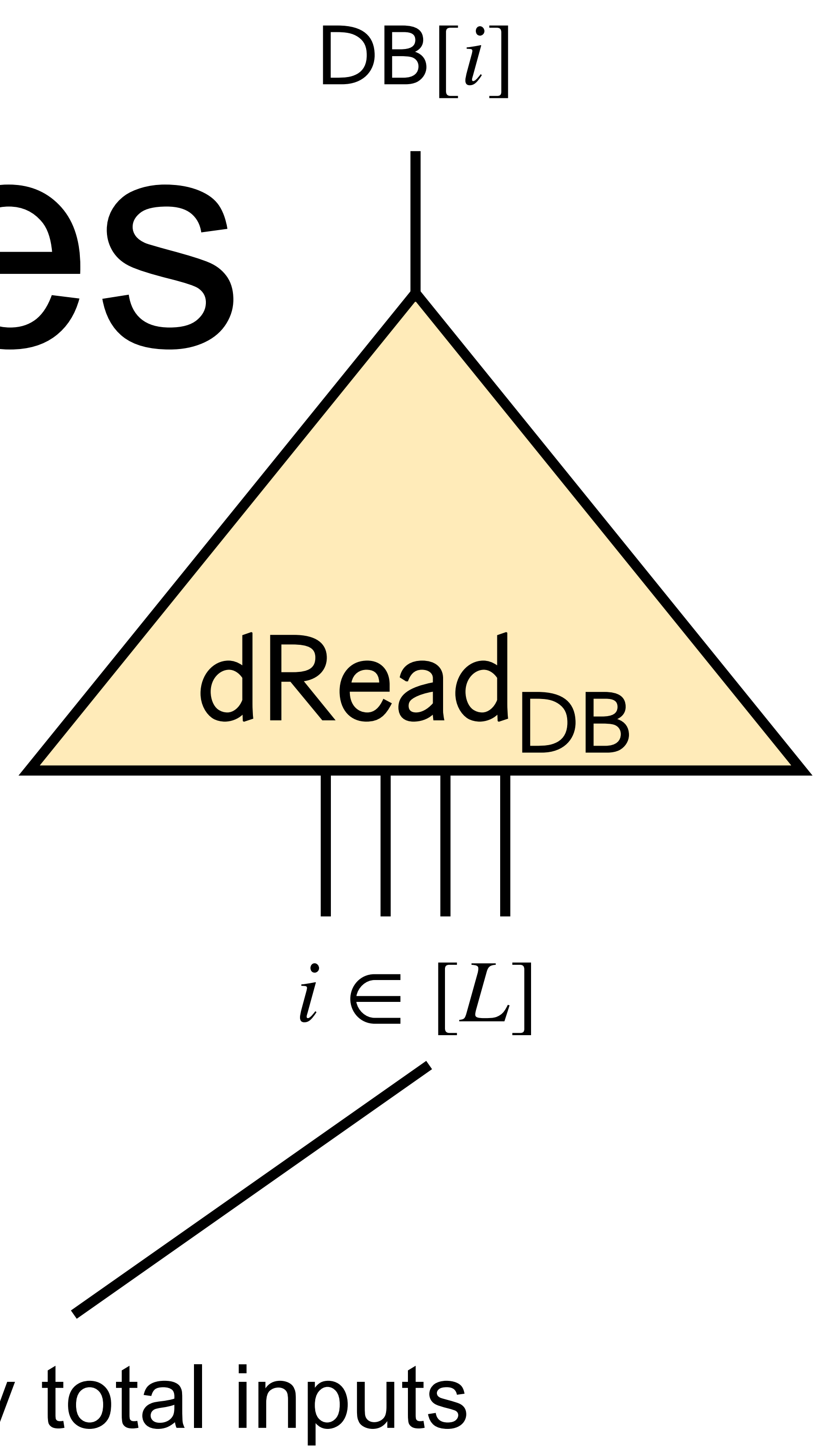
Idea: During preprocessing, just compute all $\mathbf{H}_{\text{dRead}_{\text{DB}},\mathbf{A},i}$ in advance for each $i \in [L]$

Handling Data-Read Gates



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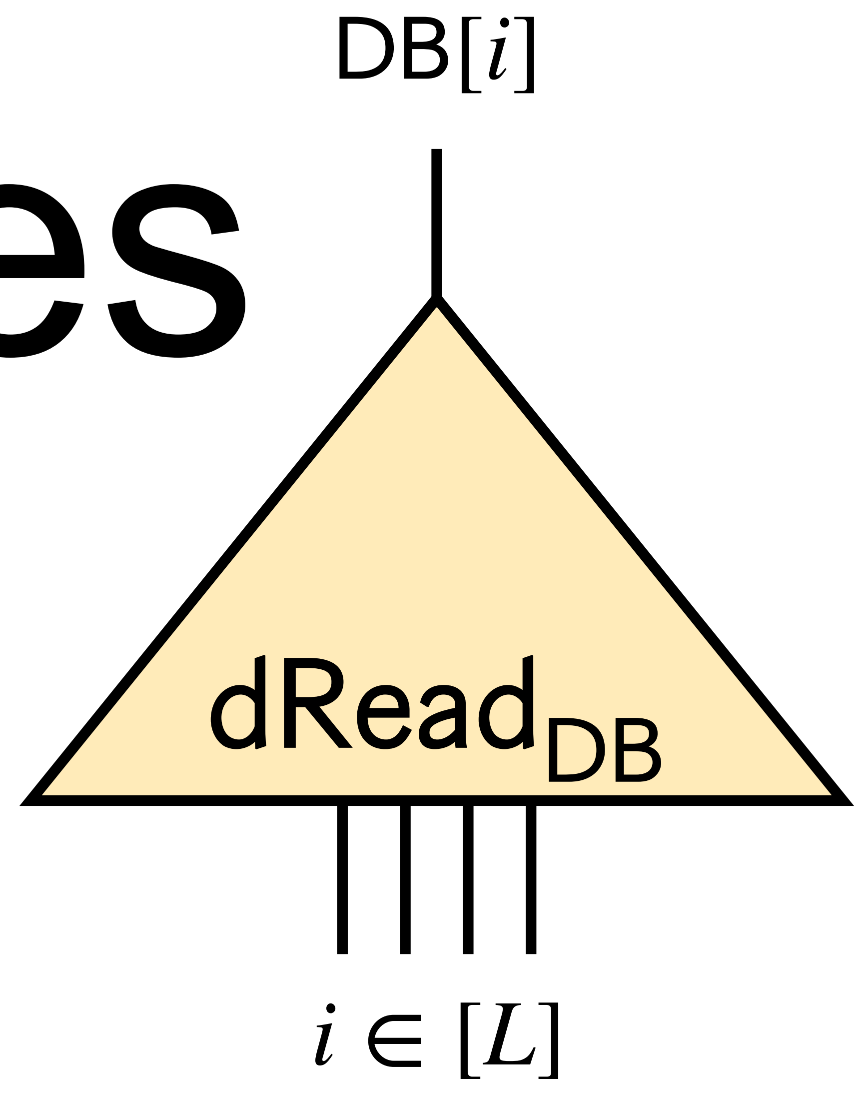
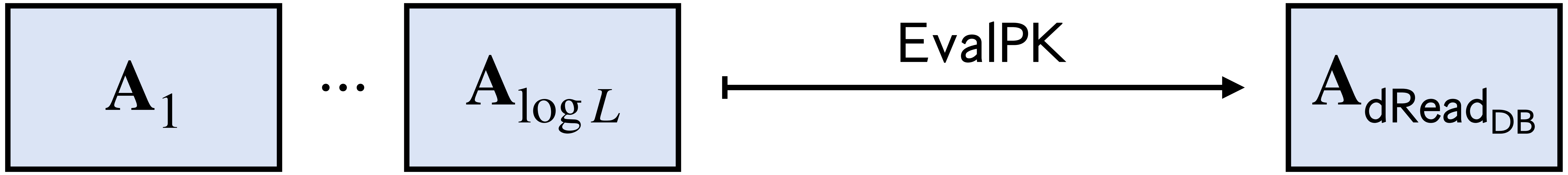
Idea: During preprocessing, just compute all $\mathbf{H}_{\mathsf{dRead}_{\mathsf{DB}},\mathbf{A},i}$ in advance for each $i \in [L]$

Handling Data-Read Gates





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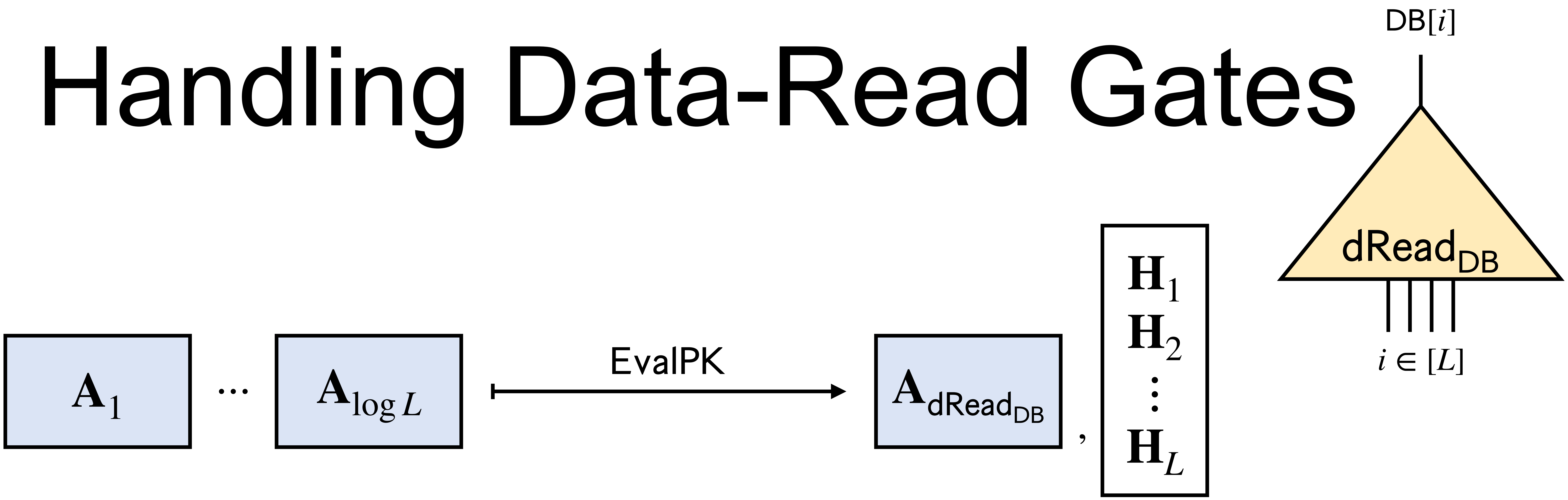

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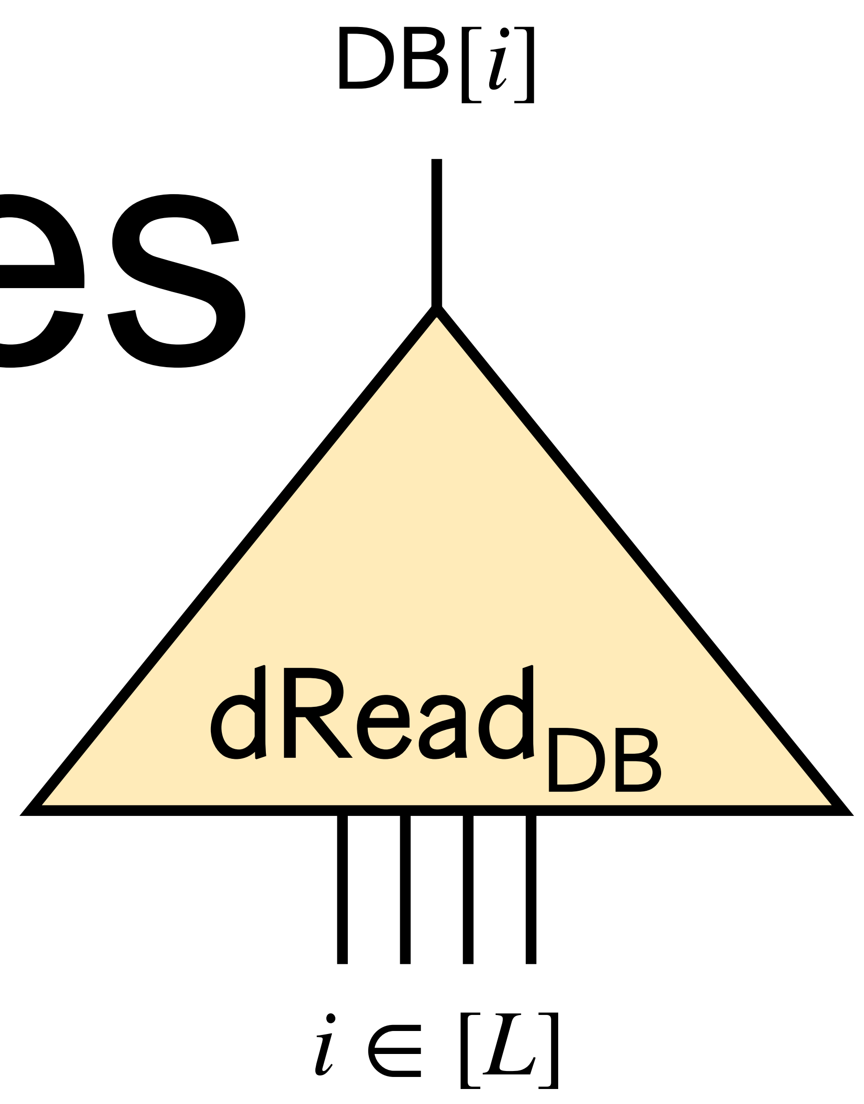

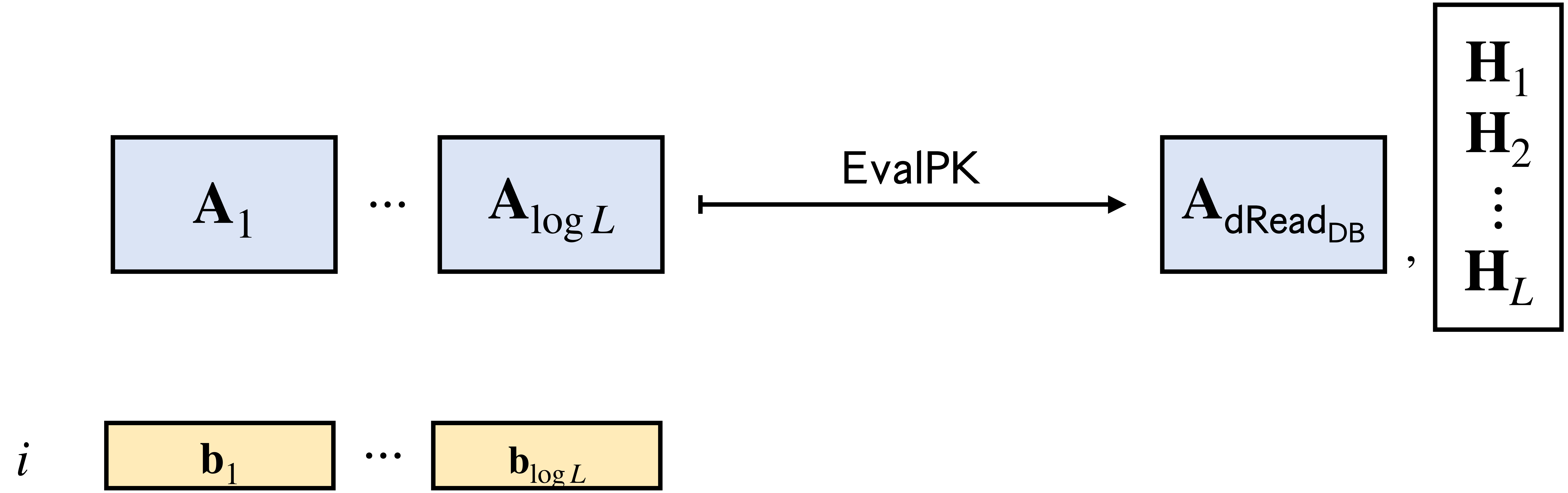

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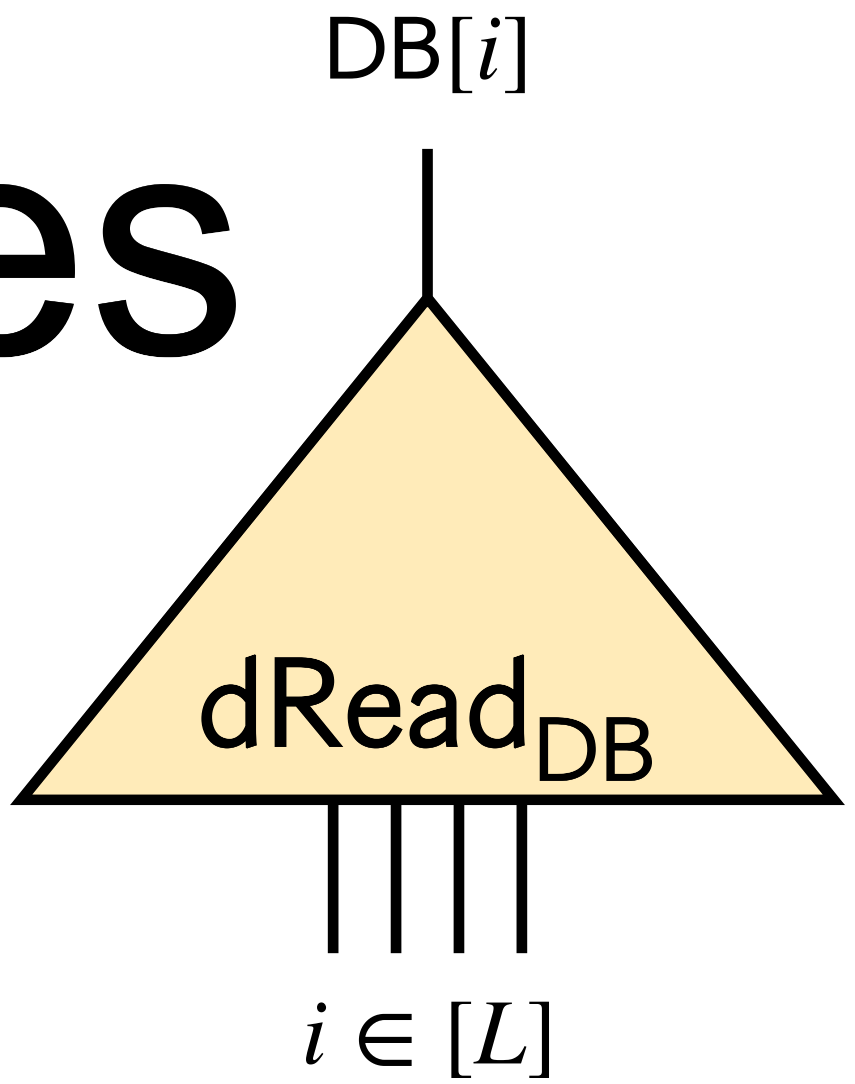

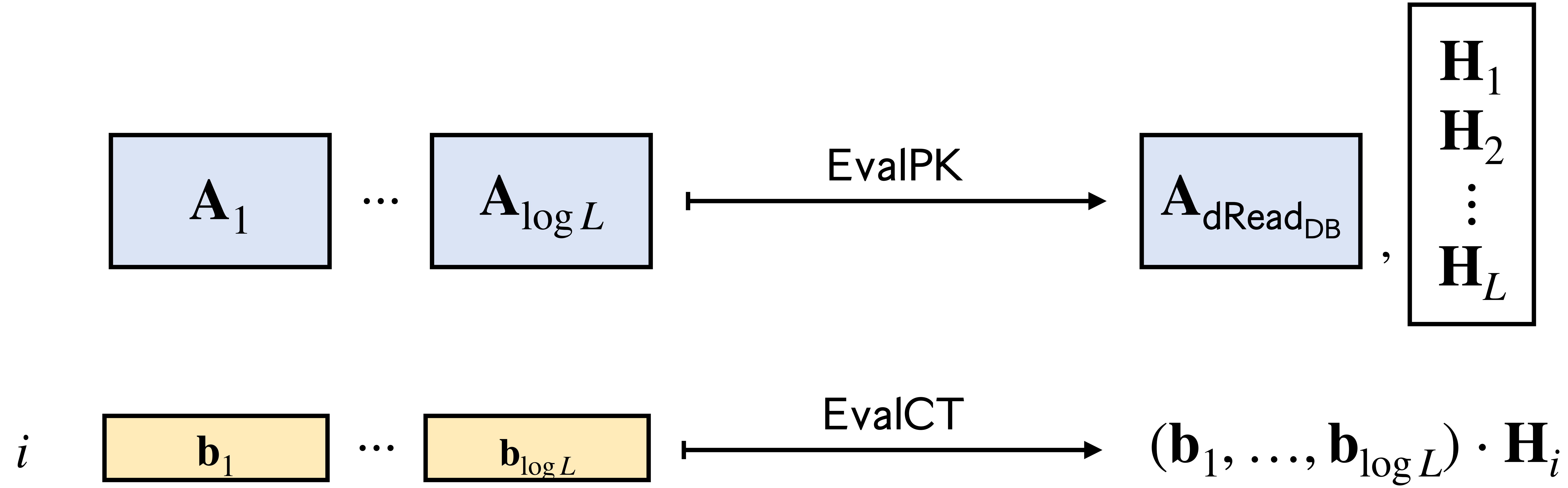

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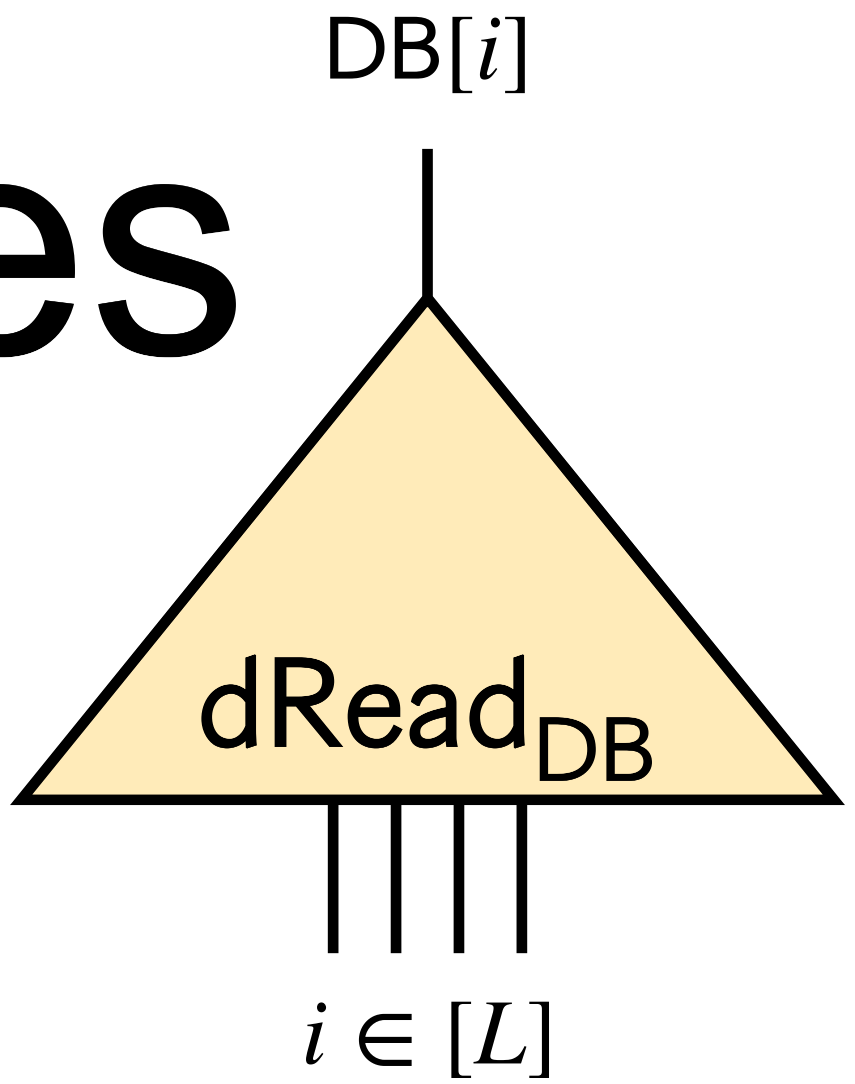
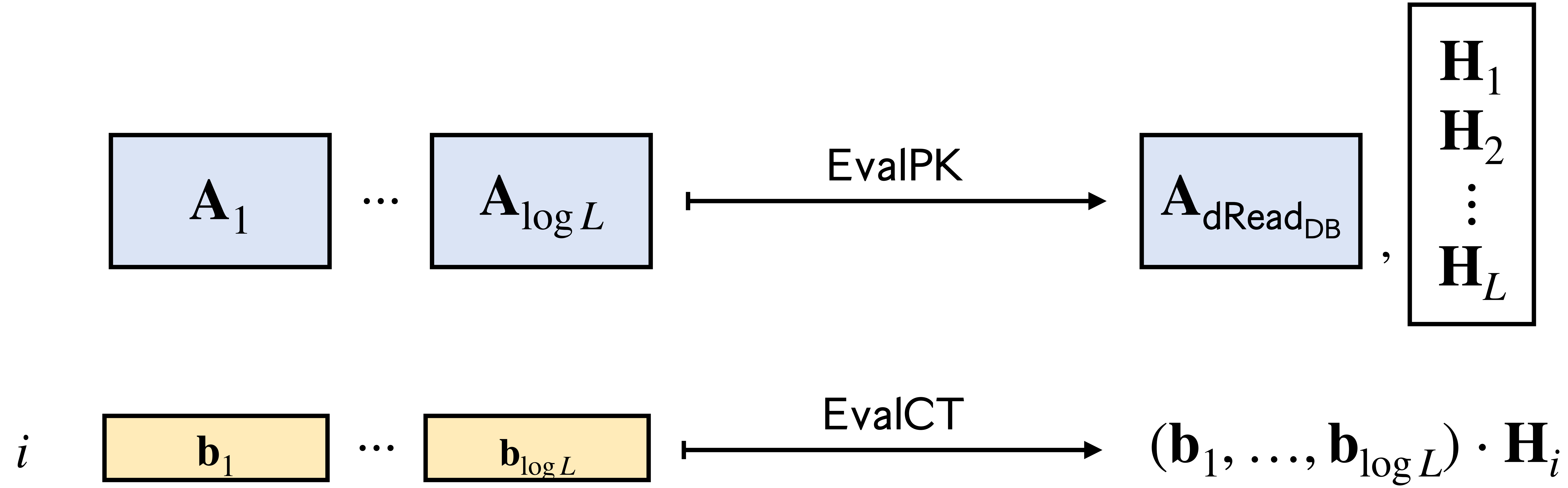
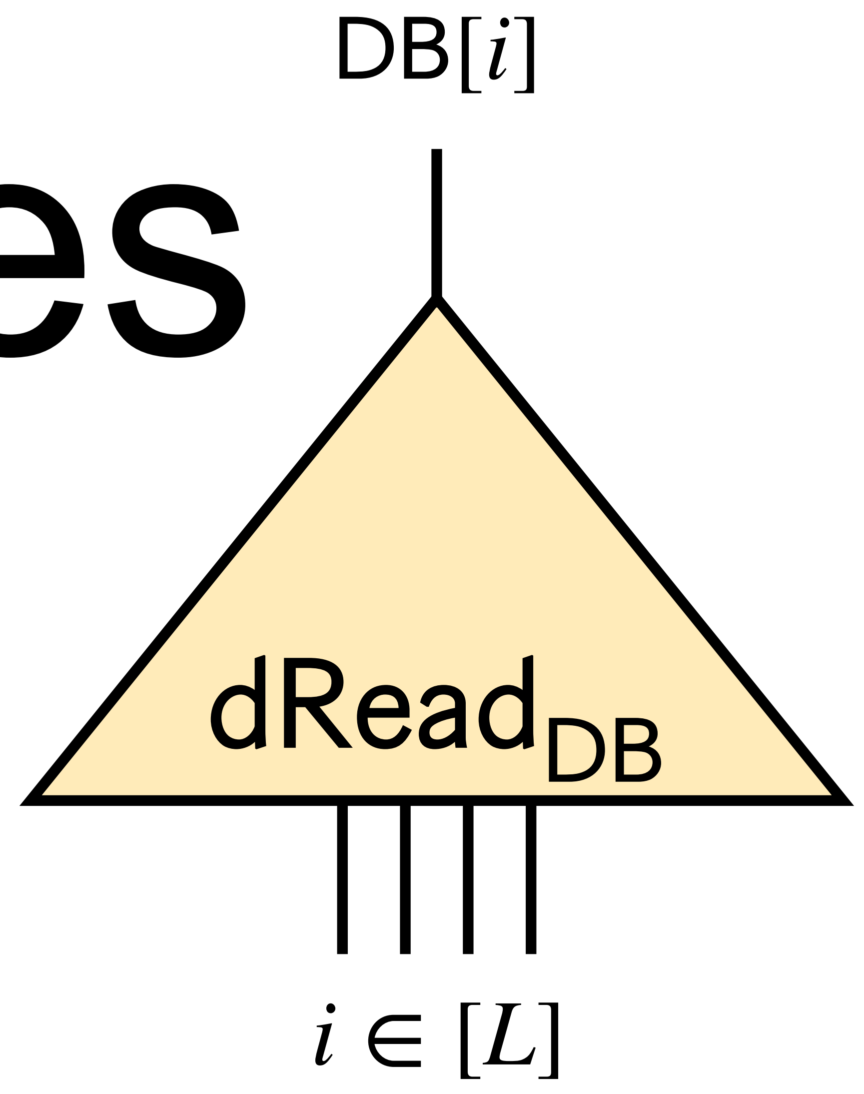
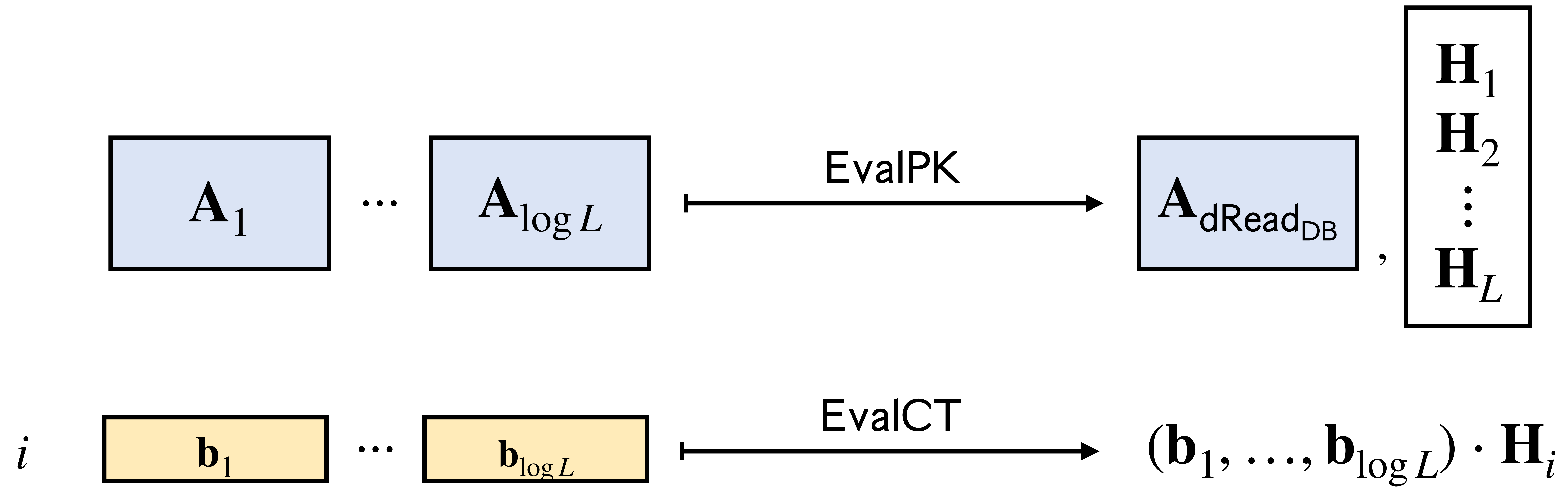




There are L many $\mathbf{H}$ matrices and each takes $\tilde{O}(L)$ time to compute
$\Longrightarrow$ EvalPK runs in time $\tilde{O}(L^2)$

Handling Data-Read Gates





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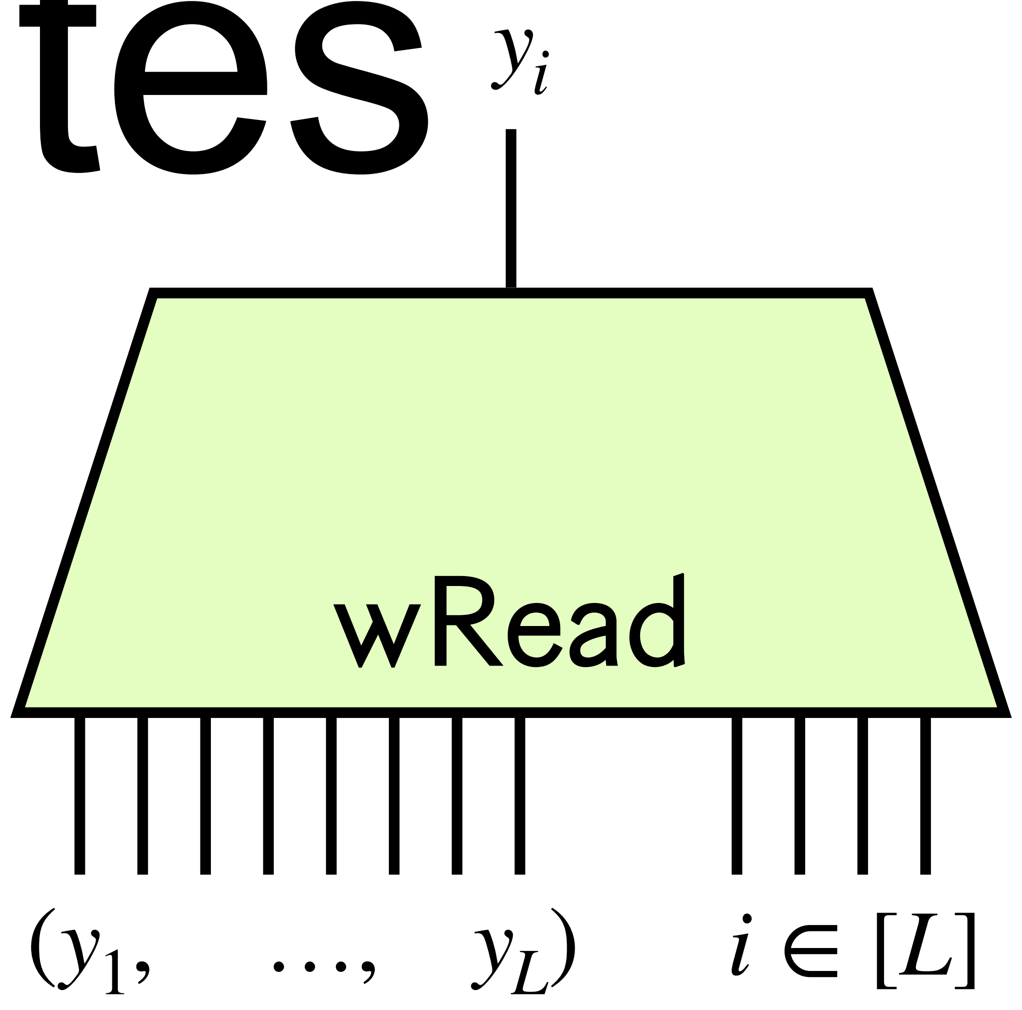
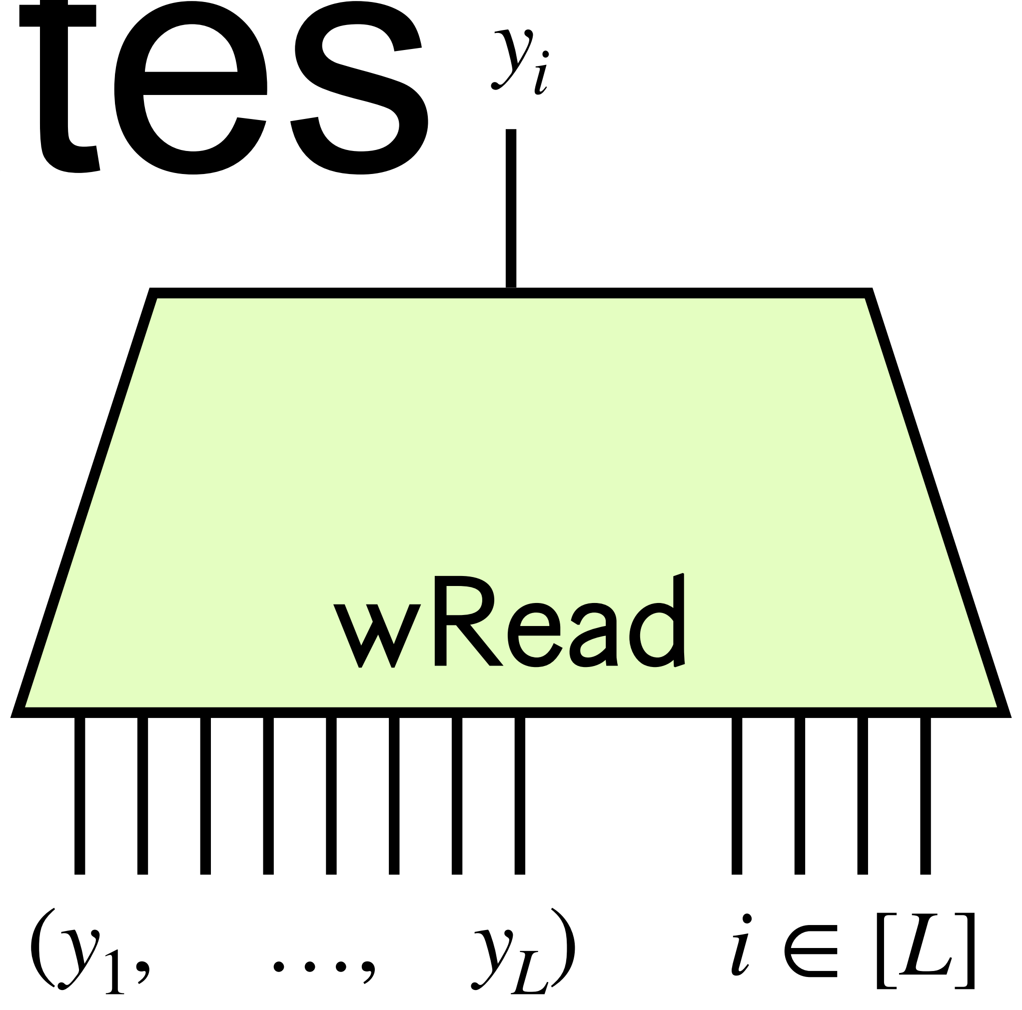
Can get just $\tilde{O}(L)$ using a recursive data structure that lets you compute $\mathbf{H}$ on the fly

Handling Wire-Read Gates

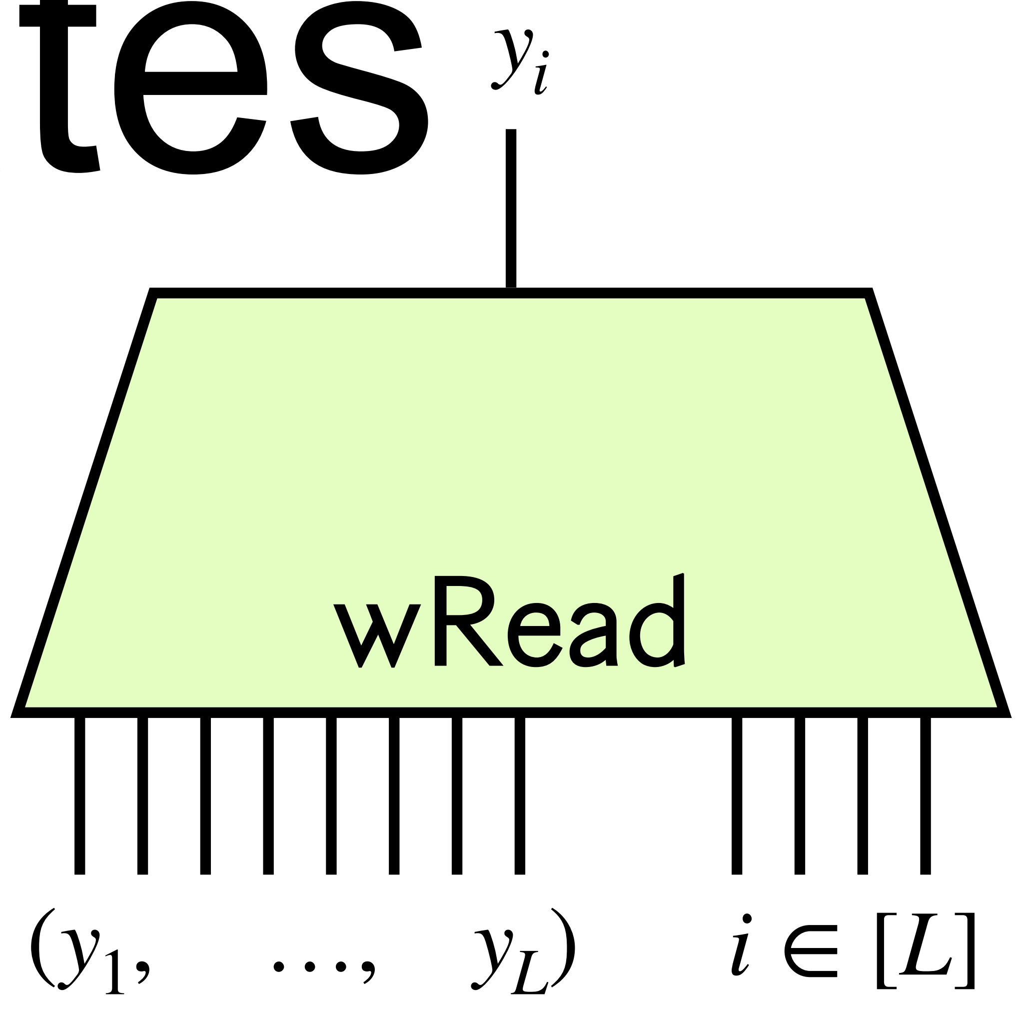


Handling Wire-Read Gates

Problem: wRead has exponentially many inputs!



Handling Wire-Read Gates



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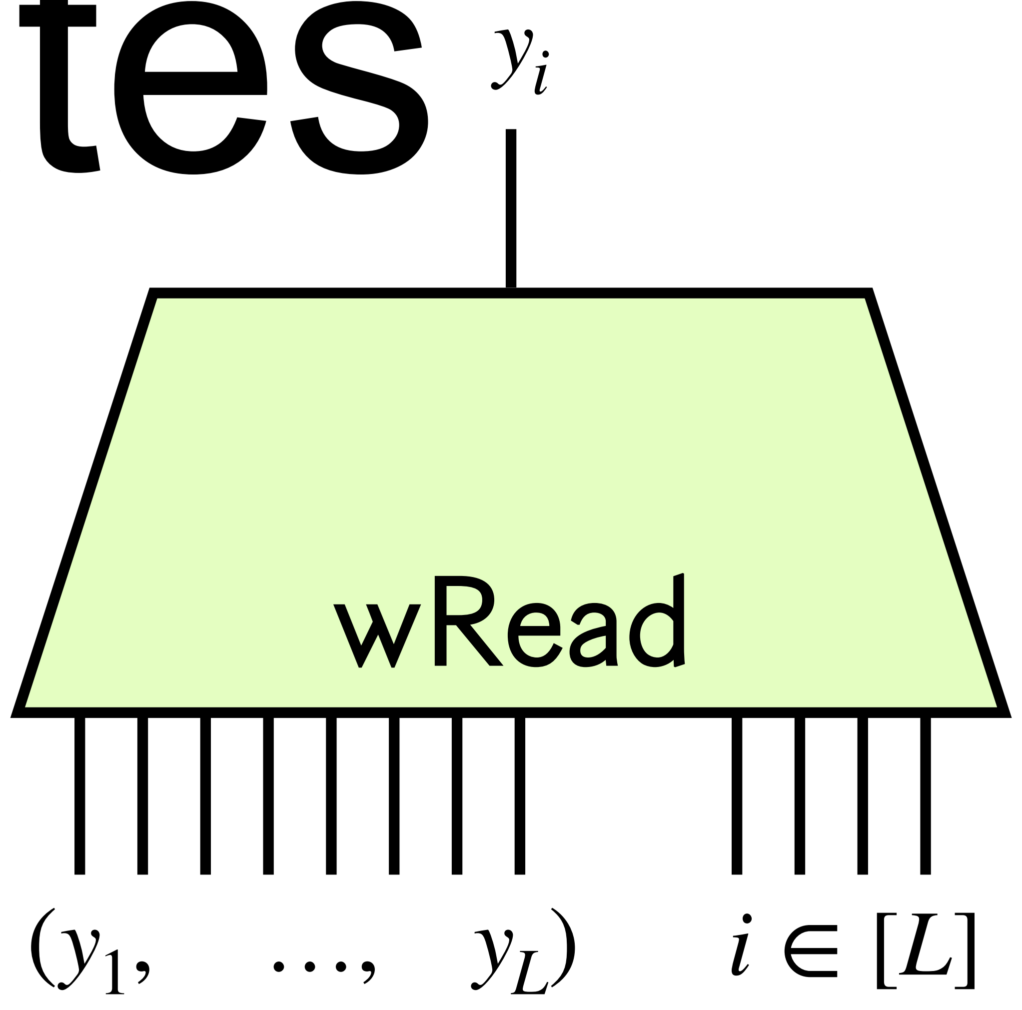
$$\mathrm{wRead}(y, i) = \sum_{j=1}^{L} y_i \cdot \mathbb{I}(i = j)$$

Handling Wire-Read Gates



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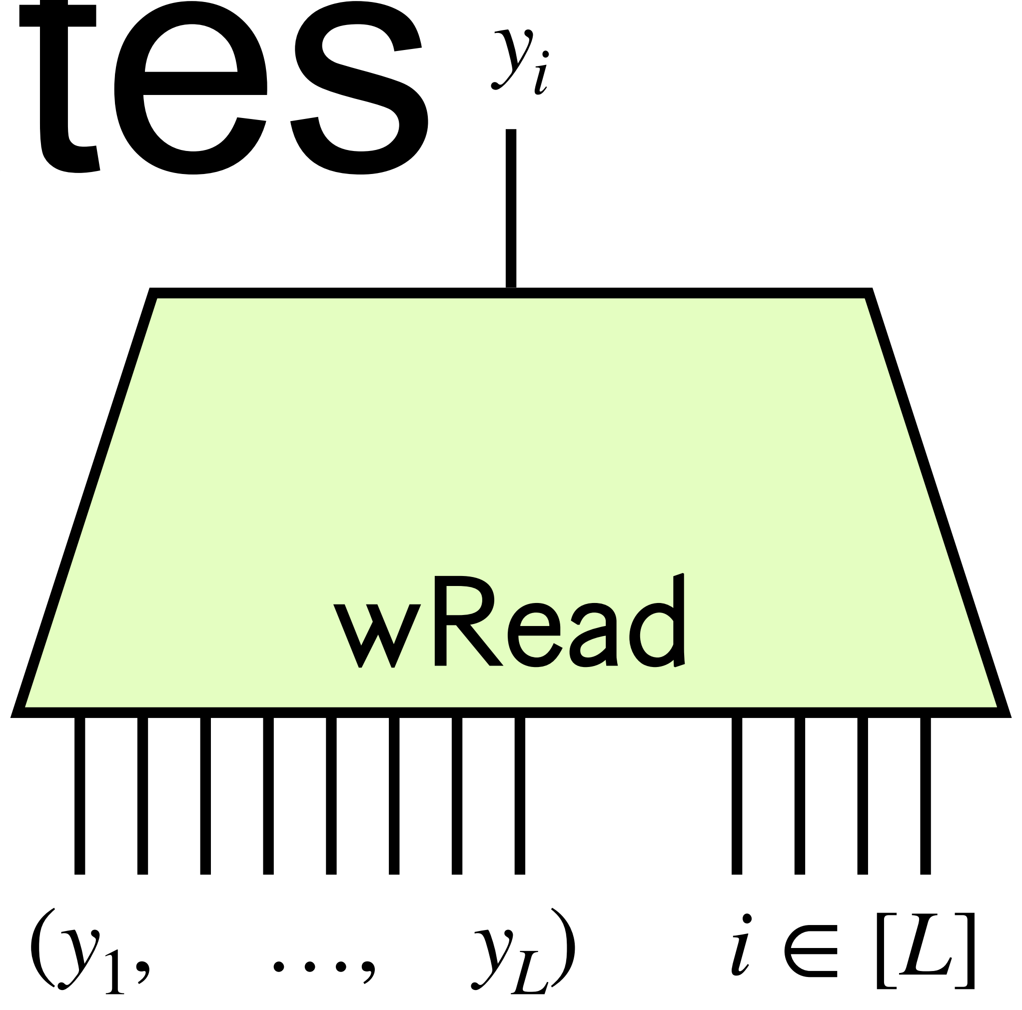
Handling Wire-Read Gates



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Saving Grace: For a given i, $\mathbf{b}_{\text{wRead}}$ only depends on one location of y

Can precompute some data structure in time $\tilde{O}(L)$ over all i's that allows computing $\mathbf{H}_{y,i}$ on the fly

Removing depth dependence

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Prior work: [HLL'23] show how to bootstrap homomorphic operations to eliminate error growth assuming circular security

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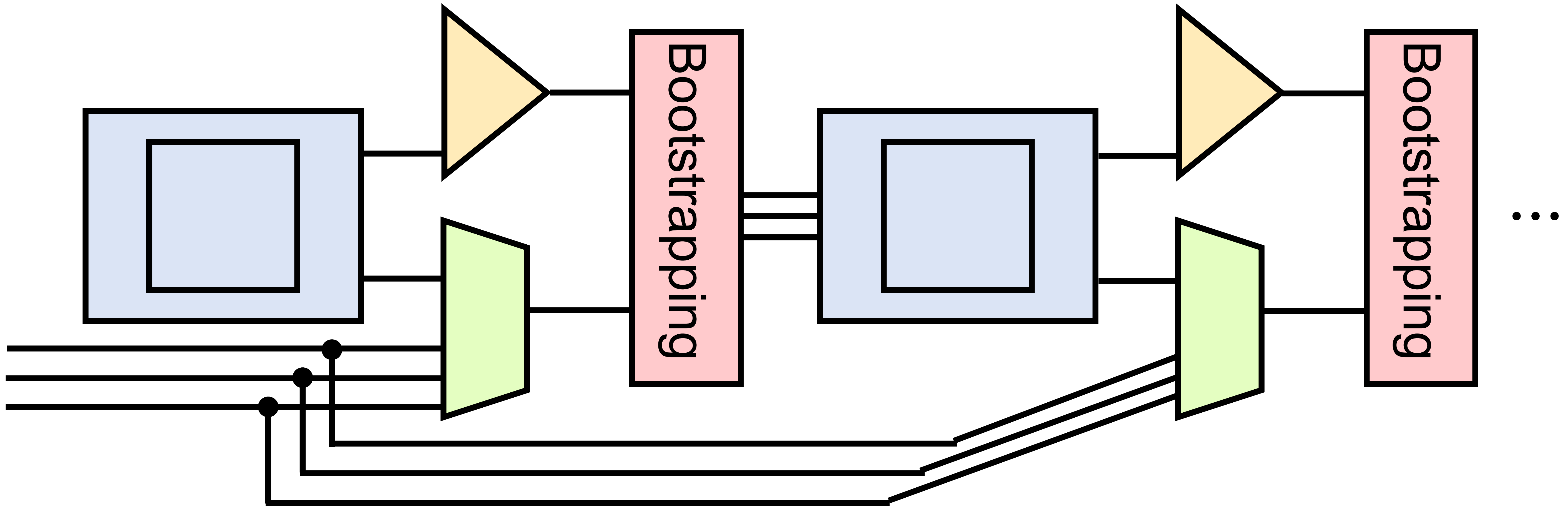

Additional Result: ABE

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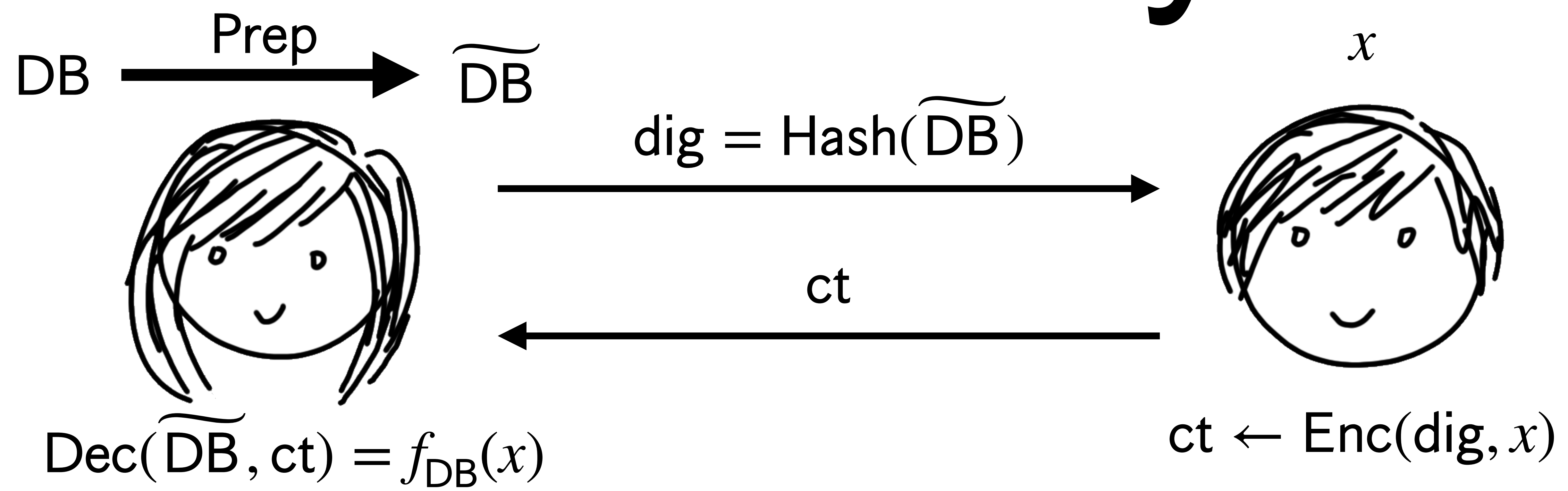
- Our preprocessing preserves linearity and hence lattice trapdoors $\Longrightarrow$ [BGG+'14] ABE construction goes through
- As in [HLL'23] cannot remove depth dependence without stronger assumptions
  - Still captures parallel RAM computation

Summary



Result: We build LFE for RAM programs from RingLWE + circular security

- Prep runtime scales with **circuit size** of RAM program
- Enc runtime slightly superlinear in input size
- Dec runtime slightly superlinear in RAM runtime

Result: We build ABE for RAM circuits of bounded depth from LWE