Two-Round Threshold Signature from Algebraic One-More Learning with Errors

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Our Lattice-based Threshold Signature Scheme

- Signing Protocol

  👉 **2-Round with Offline-Online Efficiency**

- Security

  👉 **New Assumption** : **Algebraic One-More MLWE**

- Efficiency

  Signature Size ≈ 11 KB,

  Online Communication Cost ≈ 14 KB

Background

T-out-of-N Threshold Signatures (Key Generation)

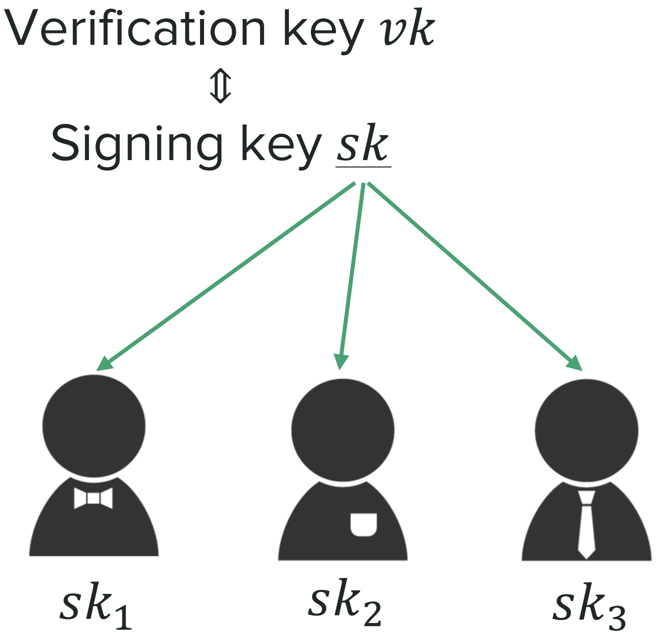
Verification key vk

$\Updownarrow$

Signing key sk

※2-out-of-3

T-out-of-N Threshold Signatures (Key Generation)



Verification key vk

$\Updownarrow$

Signing key $\underline{sk}$

sk_1 sk_2 sk_3

- T or more key shares reconstruct sk

※2-out-of-3

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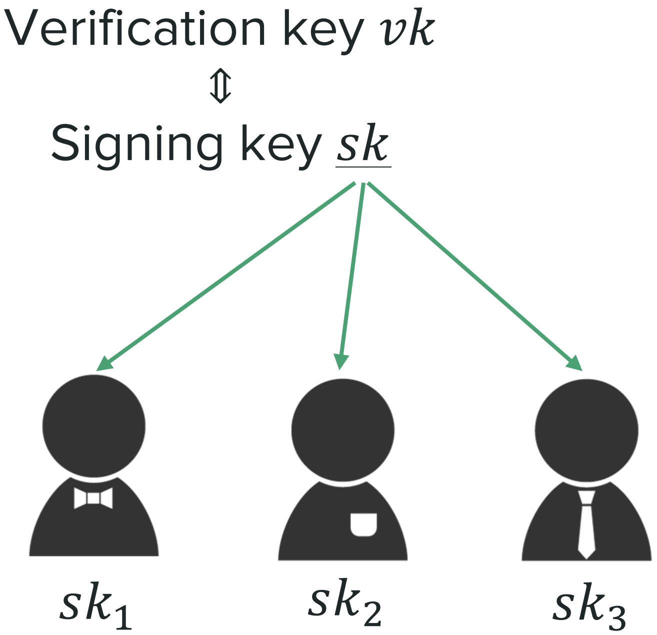

※2-out-of-3

- T or more key shares reconstruct sk

- No user knows sk
- Less than T key shares leak no information about sk

※We assume that a trusted party executes distributed key generation as well as [BCK+22,dPKM+24] etc.

T-out-of-N Threshold Signatures (Signing)

※2-out-of-3

T-out-of-N Threshold Signatures (Signing)

“**Multi-Round**” Signing Protocol

General Procedure:

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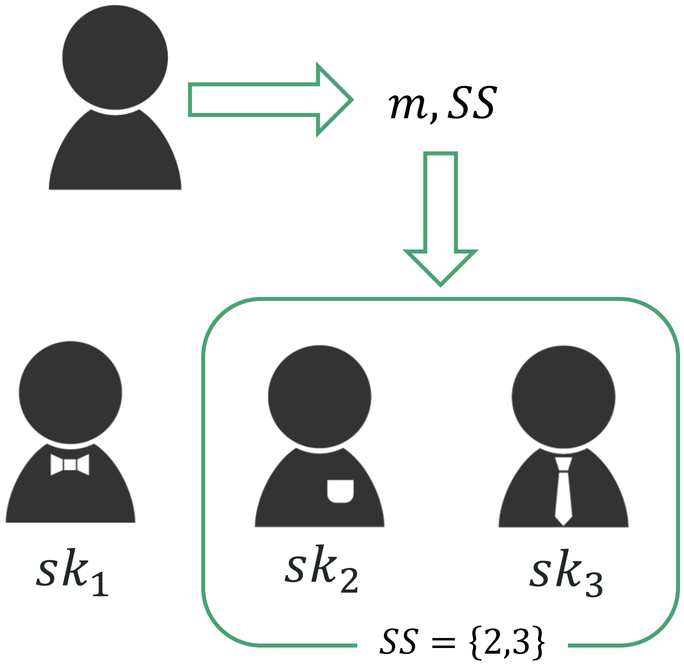

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1. One decides message m and signer set SS

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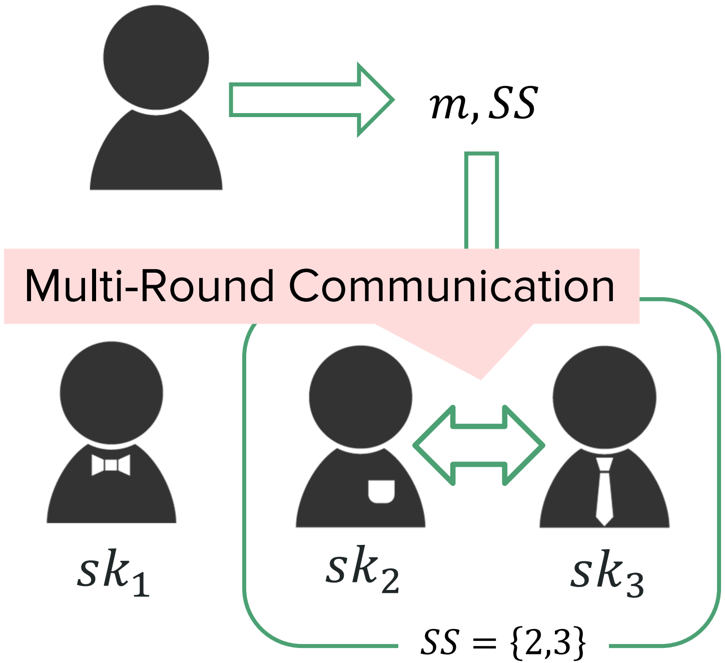

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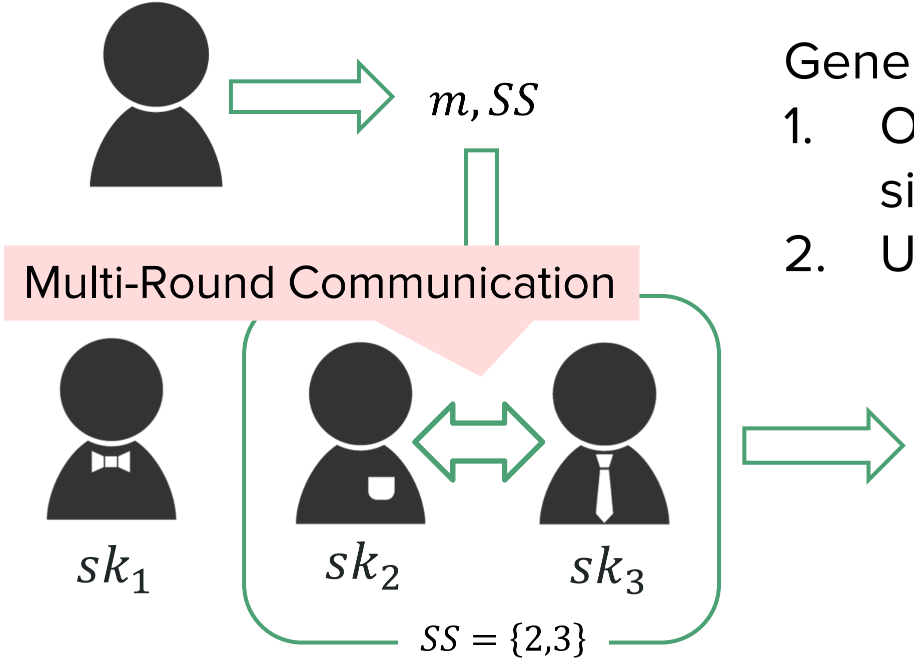

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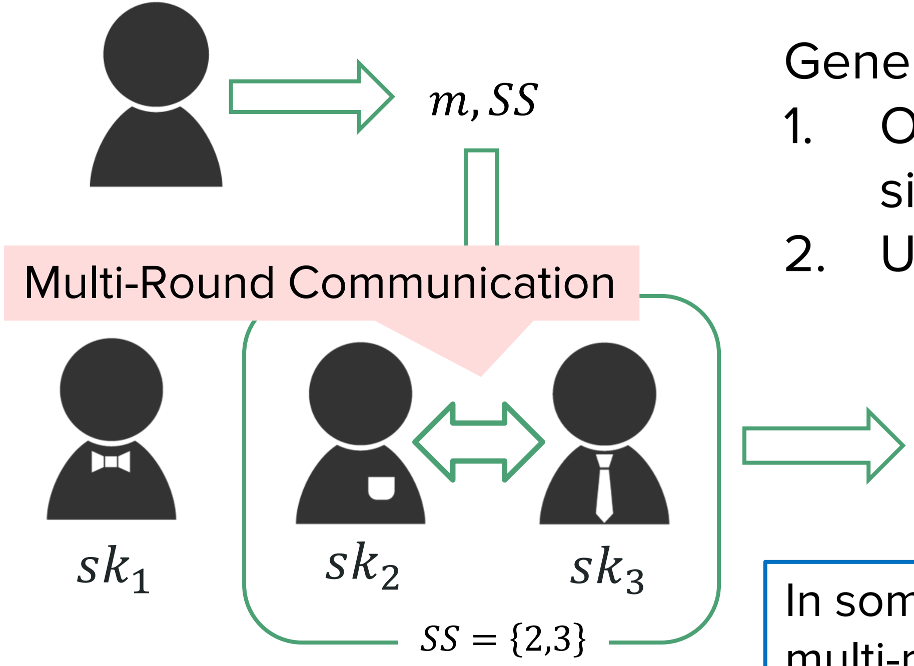

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"Multi-Round" Signing Protocol

General Procedure:

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2. Users in SS execute signing protocol

In some restricted environments, multi-round is performance bottleneck 😞

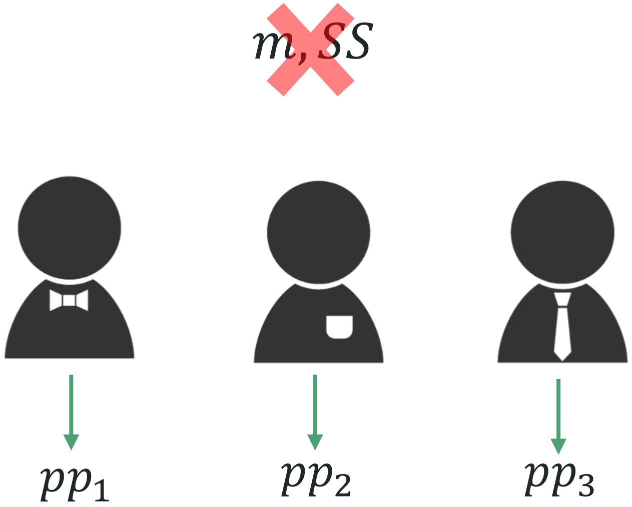
Nice Property: Offline-Online Efficiency

First Round: Pre-processing Phase

Second Round: Signing Phase

Nice Property: Offline-Online Efficiency

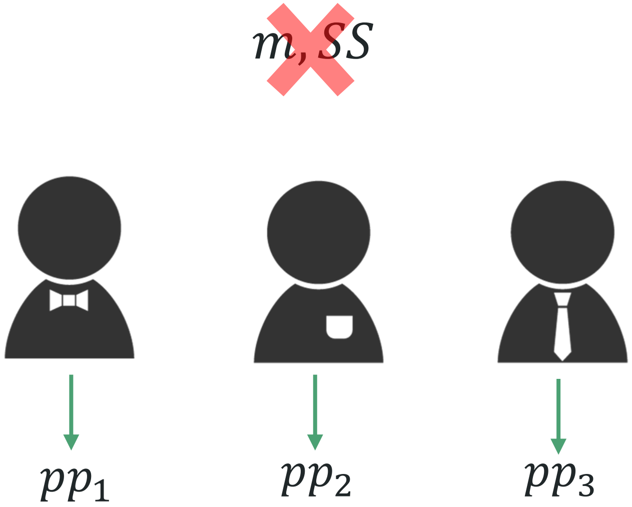
First Round: Pre-processing Phase



Second Round: Signing Phase

Nice Property: Offline-Online Efficiency

First Round: Pre-processing Phase



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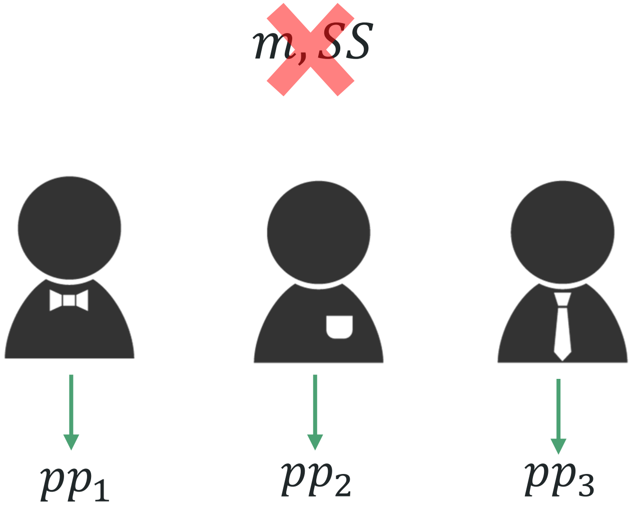

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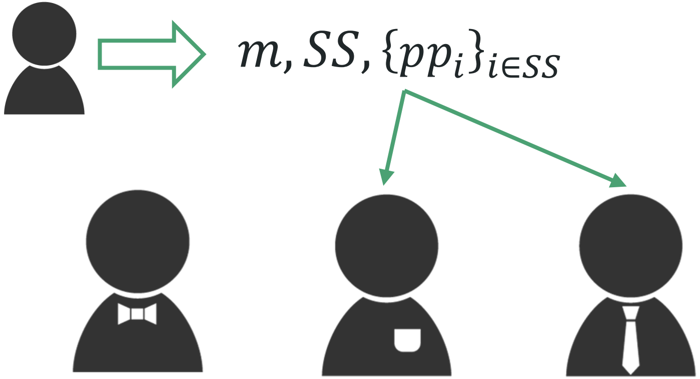

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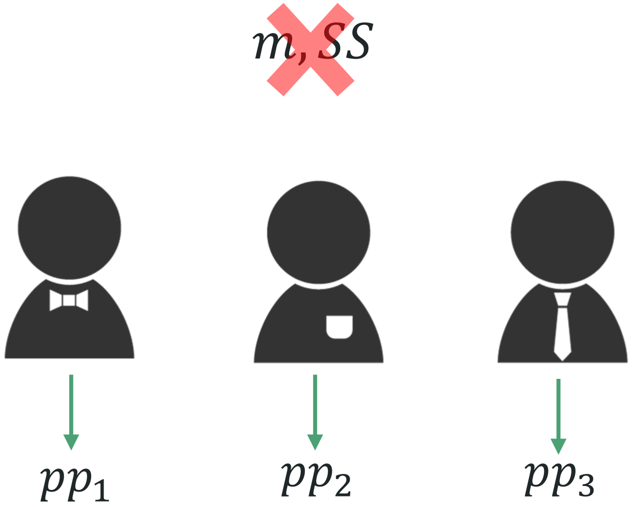

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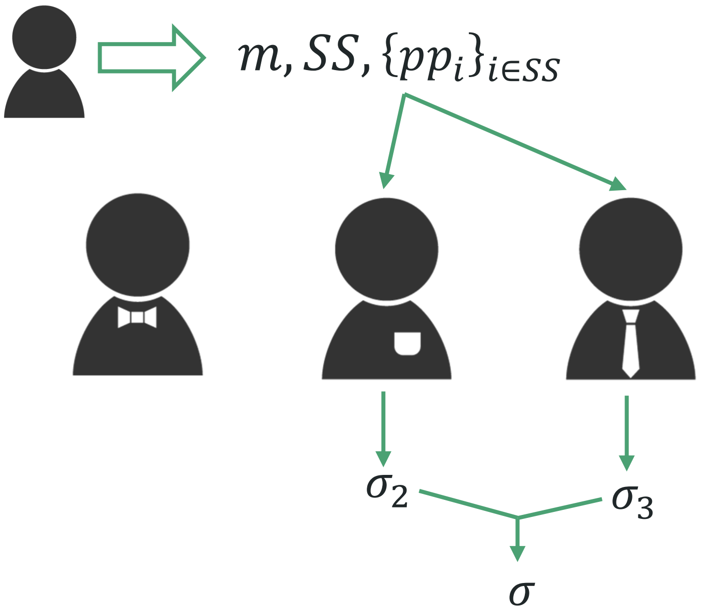

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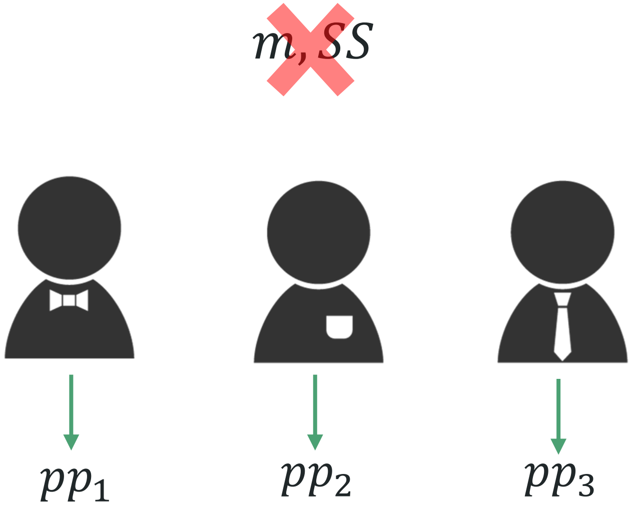

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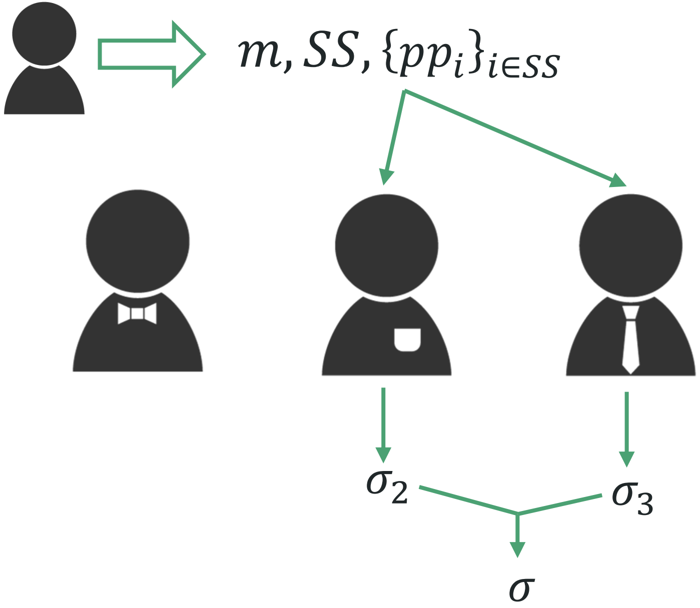

Non-interactive!

Recent Breakthrough

Previous: [BKP13], [BGG+18], [ASY22], [GKS23].

They rely on heavy tools like FHE and HTDC.

Very Recent:

Practical Lattice-based TS : **Threshold Raccoon (TRaccoon)** [EC:dPKM+24]

- w/o heavy tools
- |Sig| ≈ 13 KB, Comm. Cost ≈ 40 KB

TRaccoon [EC:dPKM+24]

Lattice-based TS based on **3-Round** DL-based TS Sparkle [CKM23]

TRaccoon [EC:dPKM+24]

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Classical Setting

Sparkle [CKM23]:

- ➢ Schnorr signature
  ⇒ Discrete Log (DL)
  ⇒ Built from Fiat-Shamir Transform
- ➢ 3-round signing protocol

TRaccoon [EC:dPKM+24]

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Lattice Setting

TRaccoon:

- Raccoon signature (Dilithium-like)
  ⇒ MLWE and MSIS
  ⇒ Built from Fiat-Shamir Transform

TRaccoon [EC:dPKM+24]

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  ⇒ **Masking Technique**

Lattice specific technique

Open Problem

3-round

Classical Setting

Sparkle [CKM23]

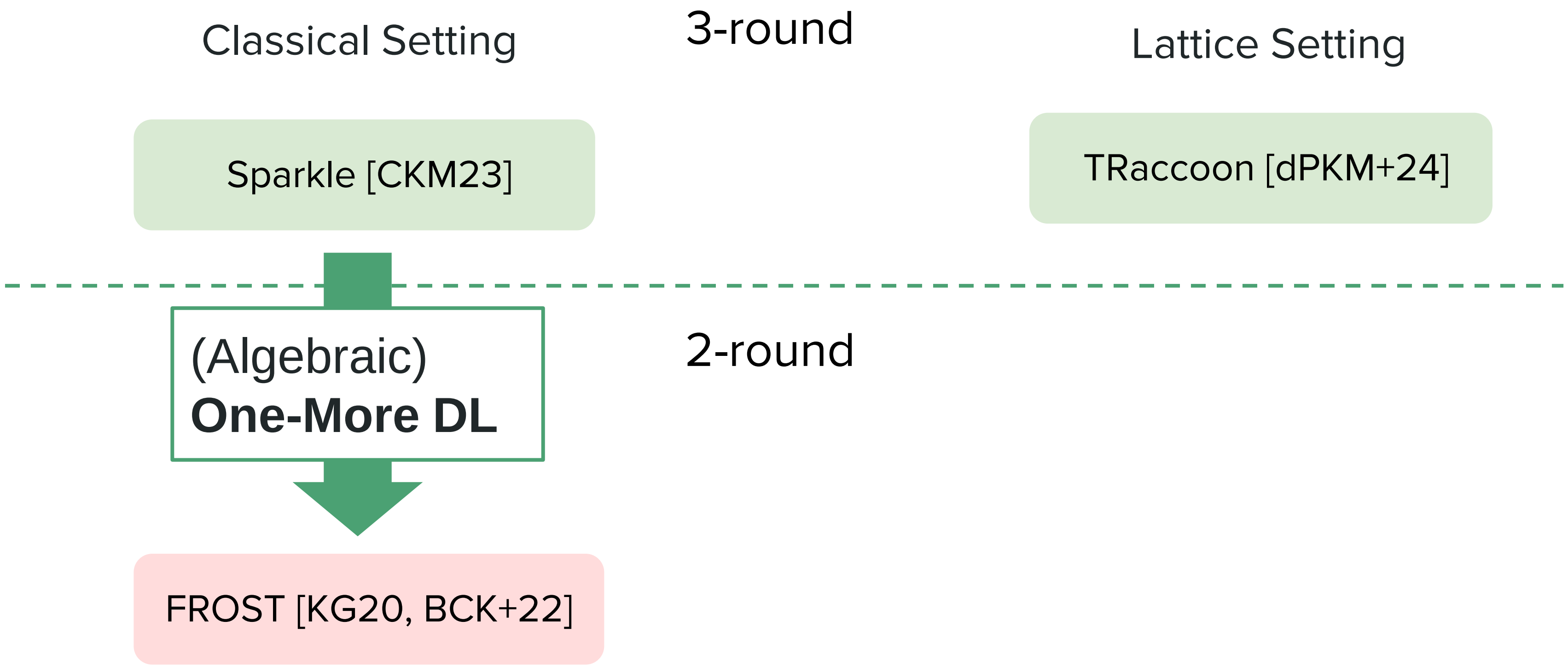
Lattice Setting

TRaccoon [dPKM+24]

Open Problem

	Classical Setting	Lattice Setting
3-round	Sparkle [CKM23]	TRaccoon [dPKM+24]
2-round	FROST [KG20, BCK+22]	

Open Problem



Classical Setting

3-round

Lattice Setting

Sparkle [CKM23]

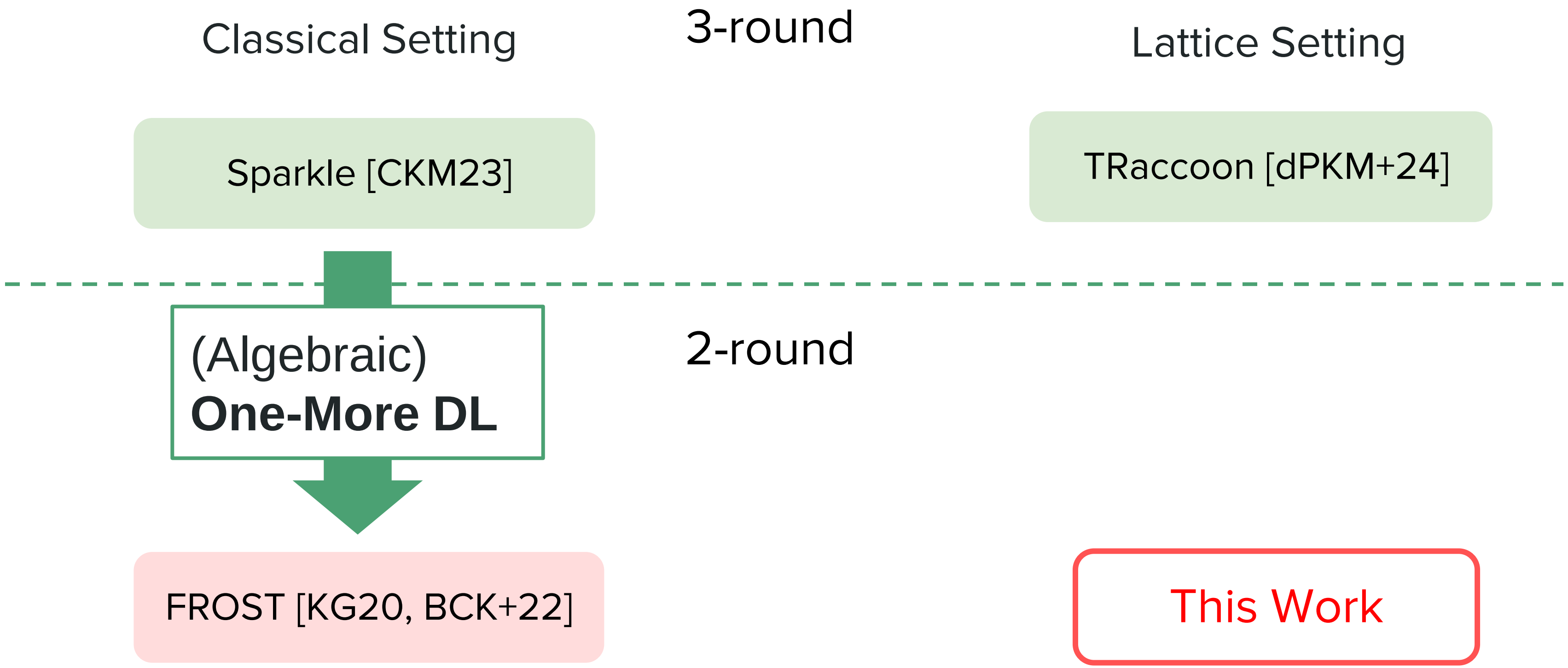
TRaccoon [dPKM+24]

(Algebraic)
One-More DL

2-round

FROST [KG20, BCK+22]

Open Problem



Classical Setting

3-round

Lattice Setting

Sparkle [CKM23]

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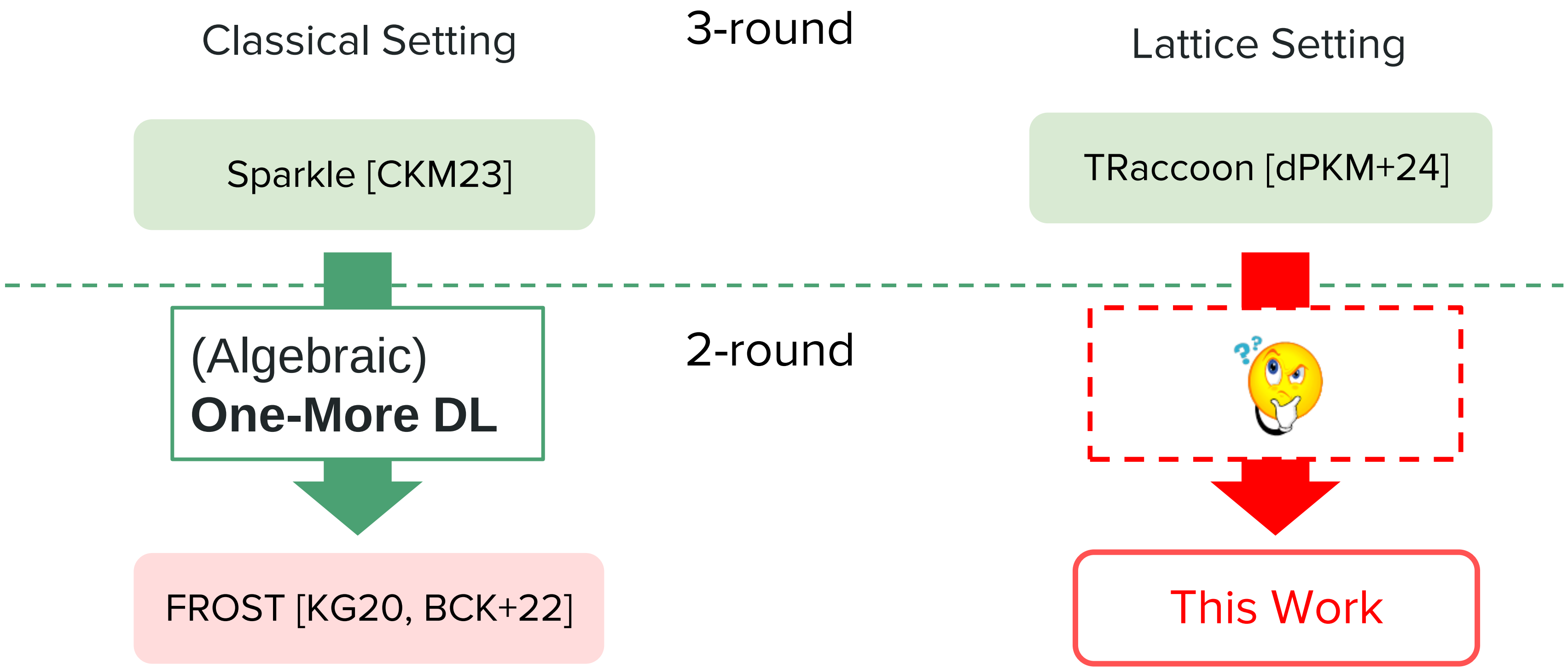
This Work

Open Problem



Classical Setting

3-round

Lattice Setting

Sparkle [CKM23]

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This Work

Introducing Algebraic One-More MLWE

One-More DL [BNPS02, BNPS03, BMV08]

Instance: $g, X_0, X_1, \dots, X_Q$

$X_i = g^{x_i},\ x_i \in \mathbb{Z}_p$

$\boldsymbol{x} = (x_0, x_1, \dots, x_Q)$

$g, X_0, X_1, \dots, X_Q$

Adversary $\mathcal{A}$

Challenger C

One-More DL [BNPS02, BNPS03, BMV08]

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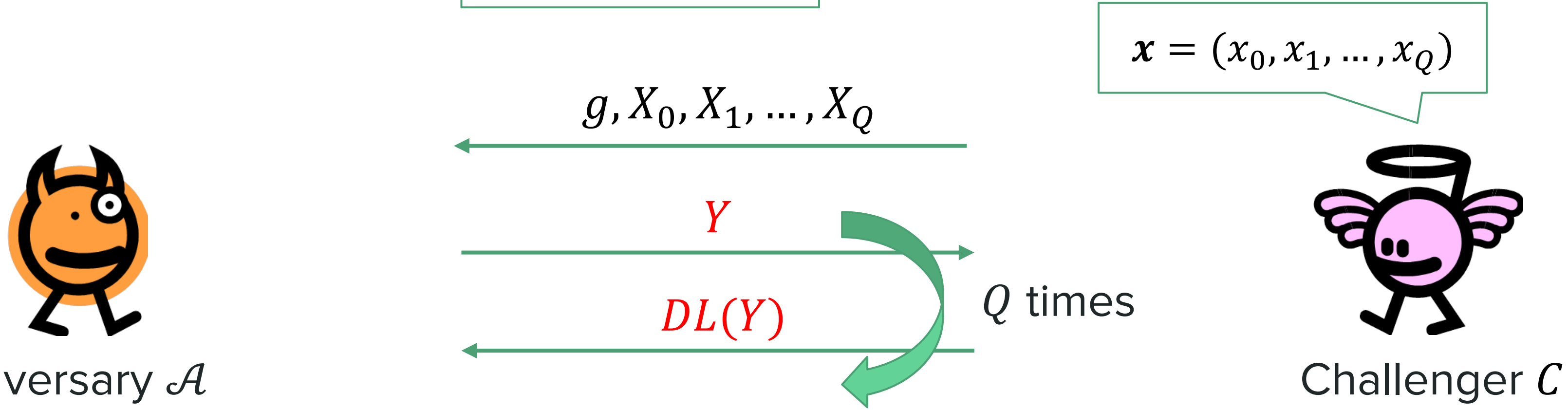

$\boldsymbol{x} = (x_0, x_1, \dots, x_Q)$

$g, X_0, X_1, \dots, X_Q$

Y

$DL(Y)$

Q times

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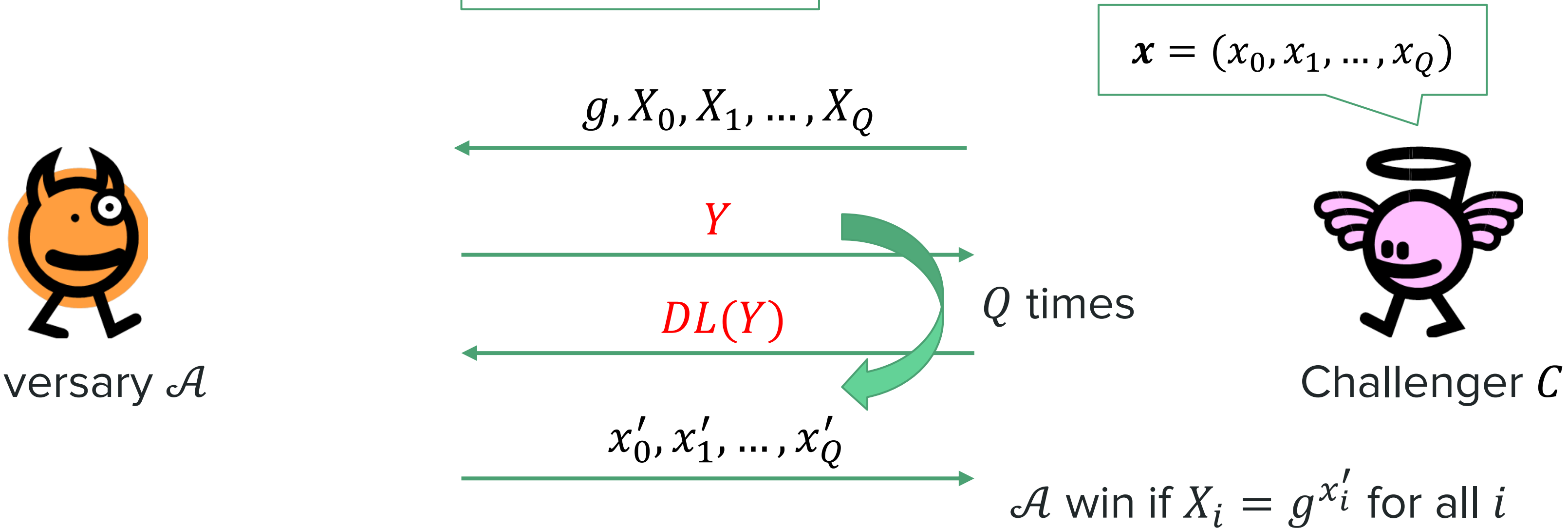

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$\mathcal{A}$ win if $X_i = g^{x'_i}$ for all i

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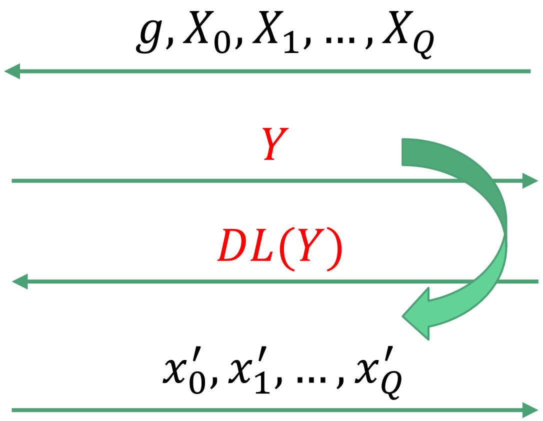

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$DL(Y)$

Q times

$x'_0, x'_1, \dots, x'_Q$

Challenger C

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C ***has to solve*** $DL(Y)$ to answer queries ⇒ Unfalsifiable

Algebraic One-More DL [NRS21]

$\Rightarrow$ $\mathcal{A}$ is allowed to make only **algebraic queries.**

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$$\xleftarrow{\quad g, X_0, X_1, \dots, X_Q \quad}$$

Adversary $\mathcal{A}$

Challenger $\mathcal{C}$

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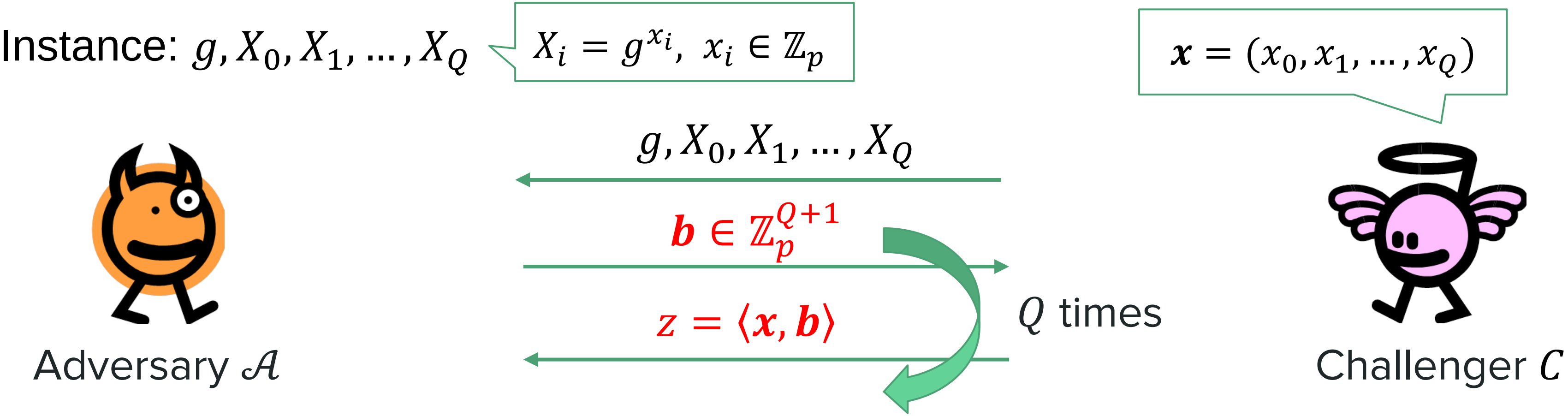

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$g, X_0, X_1, \dots, X_Q$

$\boldsymbol{b} \in \mathbb{Z}_p^{Q+1}$

$z = \langle \boldsymbol{x}, \boldsymbol{b} \rangle$

Q times

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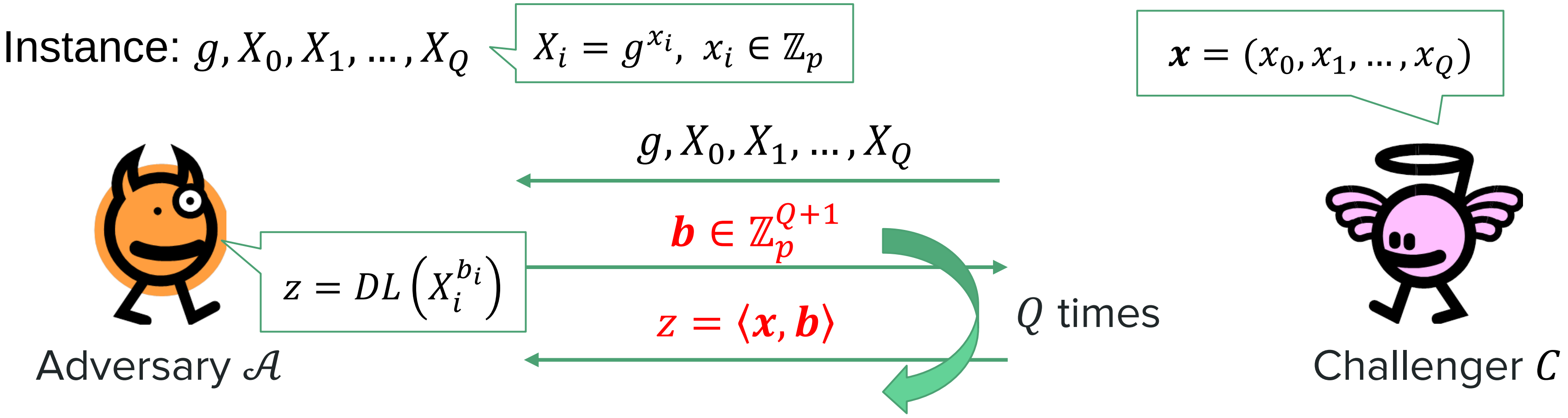

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$z = DL\left(X_i^{b_i}\right)$

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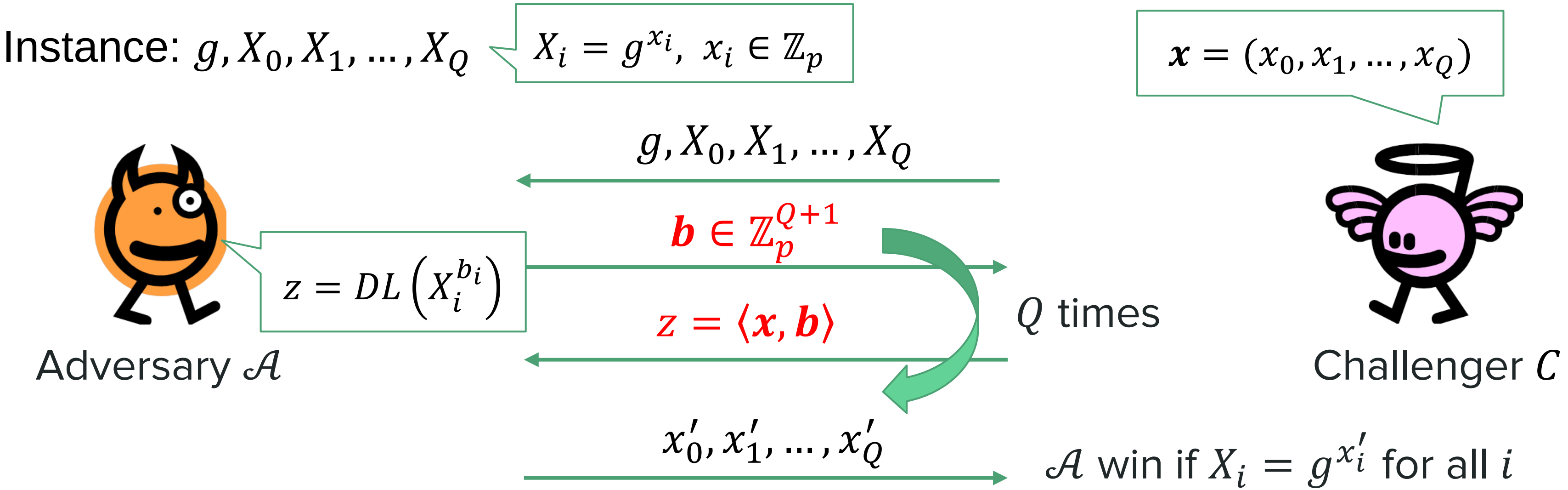

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$x'_0, x'_1, \dots, x'_Q$

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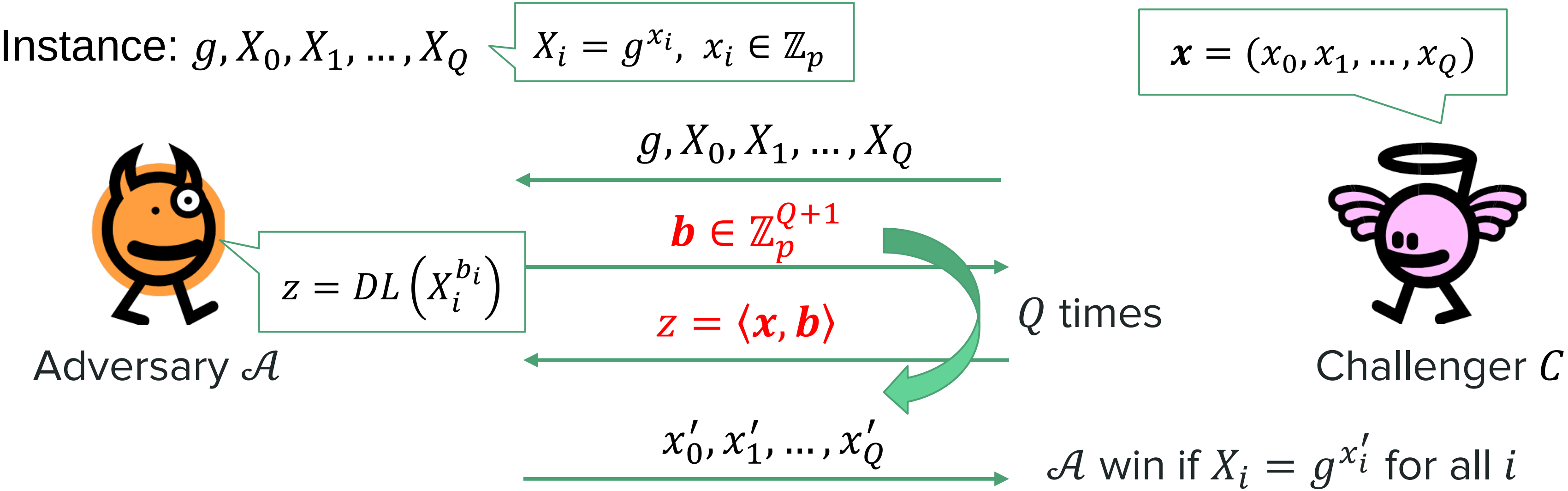

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$x_0', x_1', \dots, x_Q'$

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C ***can compute*** $z = \langle \boldsymbol{x}, \boldsymbol{b} \rangle$ ***efficiently*** to answer queries ⇒ Falsifiable

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⇒ $\mathcal{A}$ is allowed to make only **algebraic queries.**

Instance: $g, X_0, X_1, \ldots, X_Q$

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$\boldsymbol{x} = (x_0, x_1, \ldots, x_Q)$

$g, X_0, X_1, \ldots, X_Q$

AOM-DL ⇒ 2-Round TS FROST

Lattice-based variant?

C

$\mathcal{A}$ win if $X_i = g^{x_i}$ for all i

C ***can compute*** $z = \langle \boldsymbol{x}, \boldsymbol{b} \rangle$ ***efficiently*** to answer queries ⇒ Falsifiable

Naive Attempt

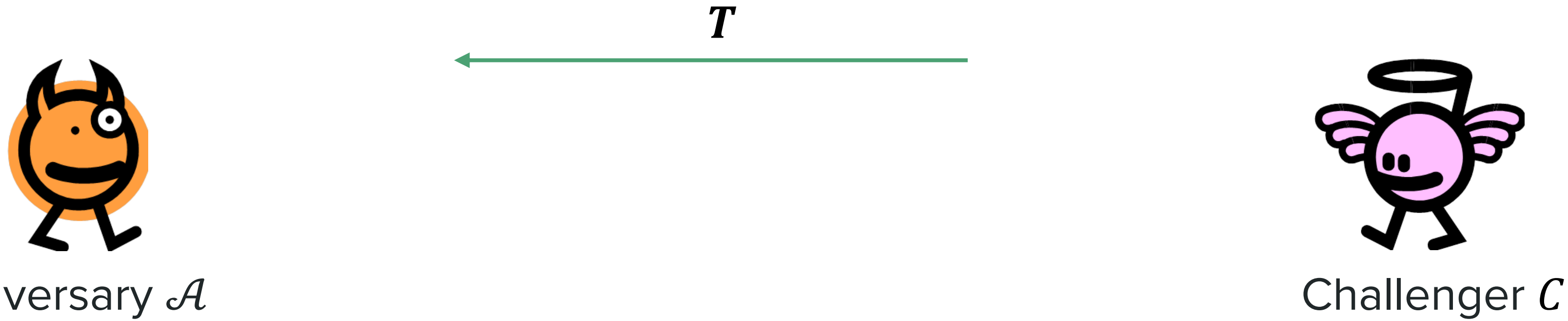
Instance: $\boldsymbol{A} \in \mathcal{R}_q^{k\times\ell}, \boldsymbol{T} = [\boldsymbol{t}_0\boldsymbol{t}_1 \cdots \boldsymbol{t}_Q] \in \mathcal{R}_q^{k\times(Q+1)}$

$$\boxed{\boldsymbol{t}_i} = \boxed{\boldsymbol{A}' \mid \boldsymbol{I}}\,\boxed{\boldsymbol{s}_i}$$

$\boldsymbol{s}_i$ is short

$\boldsymbol{S} = [\boldsymbol{s}_0\boldsymbol{s}_1 \cdots \boldsymbol{s}_Q]$

$\boldsymbol{T} = \boldsymbol{A}\boldsymbol{S}$



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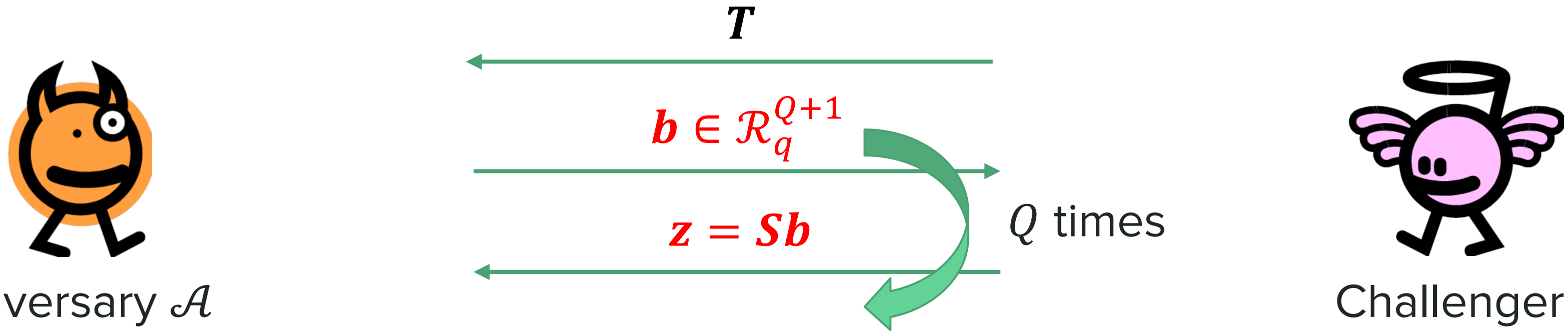

$\boldsymbol{T}$
$\boldsymbol{b} \in \mathcal{R}_q^{Q+1}$
$\boldsymbol{z} = \boldsymbol{S}\boldsymbol{b}$
Q times

Adversary $\mathcal{A}$

Challenger C

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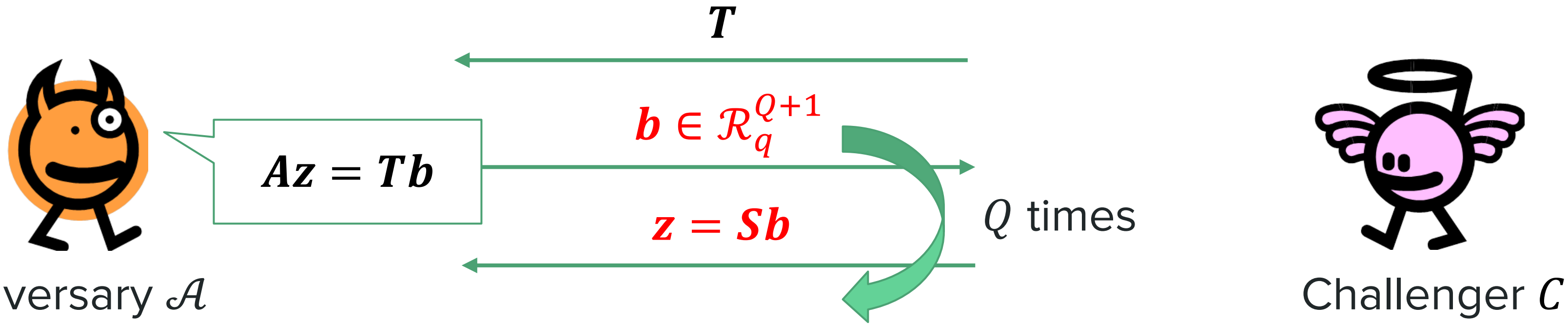

$\boldsymbol{T}$

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Q times

$\boldsymbol{A}\boldsymbol{z} = \boldsymbol{T}\boldsymbol{b}$

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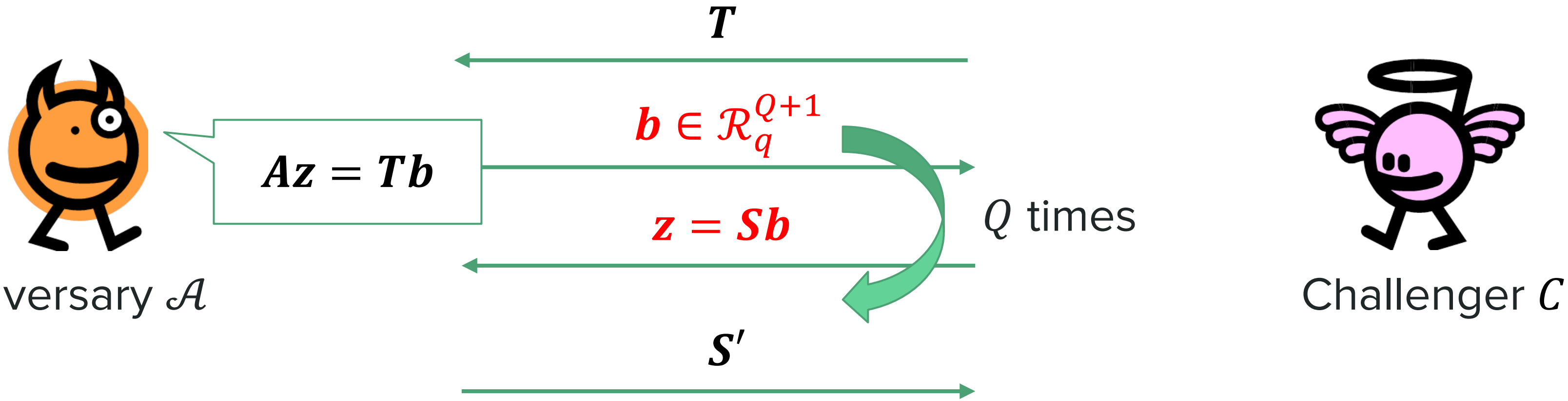
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$\boldsymbol{z} = \boldsymbol{S}\boldsymbol{b}$
Q times
$\boldsymbol{S}'$
Adversary $\mathcal{A}$
Challenger C

$\mathcal{A}$ wins if $\boldsymbol{T} = \boldsymbol{A}\boldsymbol{S}'$ and $\boldsymbol{S}'$ is short

Naive Attempt

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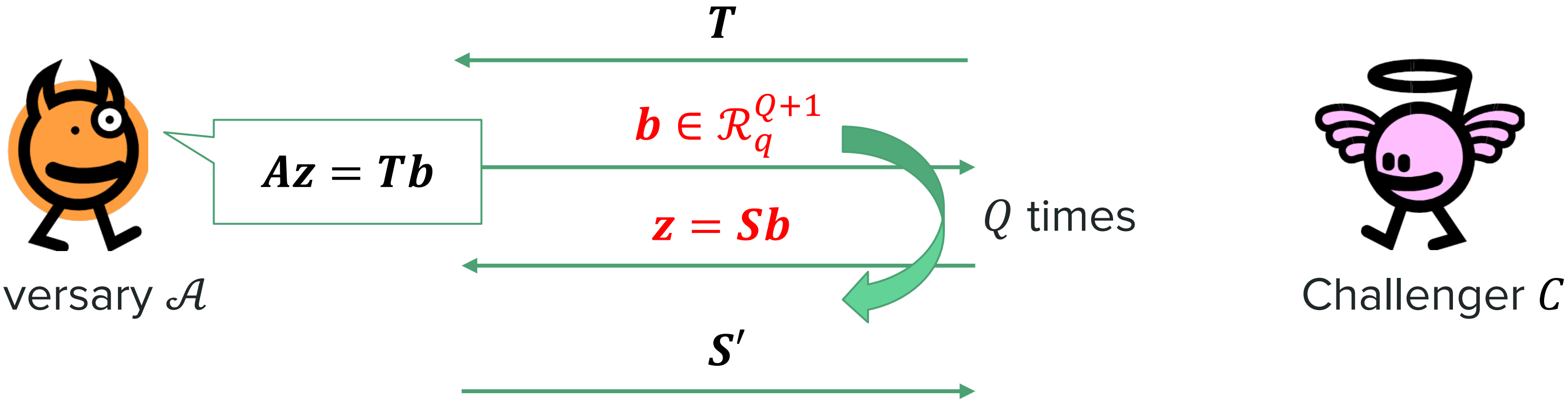

$\boldsymbol{T}$

$\boldsymbol{b} \in \mathcal{R}_q^{Q+1}$

$\boldsymbol{A}\boldsymbol{z} = \boldsymbol{T}\boldsymbol{b}$

$\boldsymbol{z} = \boldsymbol{S}\boldsymbol{b}$

Q times

$\boldsymbol{S}'$

Adversary $\mathcal{A}$

Challenger C

$\mathcal{A}$ wins if $\boldsymbol{T} = \boldsymbol{A}\boldsymbol{S}'$ and $\boldsymbol{S}'$ is short

Is this problem hard?

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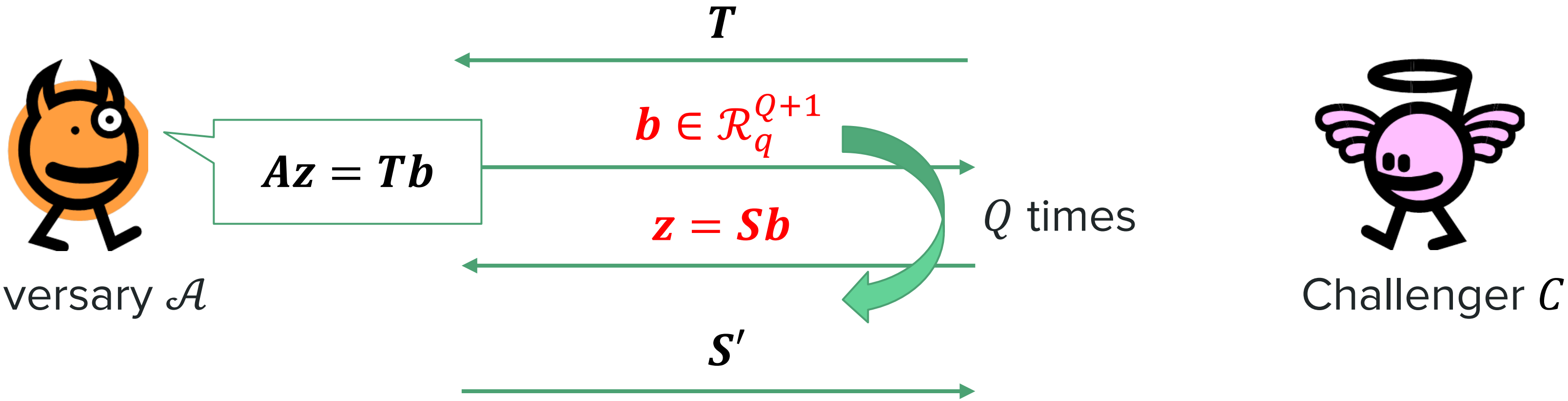
$\boldsymbol{S} = [\boldsymbol{s}_0 \boldsymbol{s}_1 \cdots \boldsymbol{s}_Q]$

$\boldsymbol{T} = \boldsymbol{A}\boldsymbol{S}$



$\boldsymbol{T}$

$\boldsymbol{A}\boldsymbol{z} = \boldsymbol{T}\boldsymbol{b}$

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$\mathcal{A}$ wins if $\boldsymbol{T} = \boldsymbol{A}\boldsymbol{S}'$ and $\boldsymbol{S}'$ is short

Is this problem hard?

No! We have attacks against this problem.

Insecure Example: Large Algebraic Query

Assume
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- $B \ll$ modulus q
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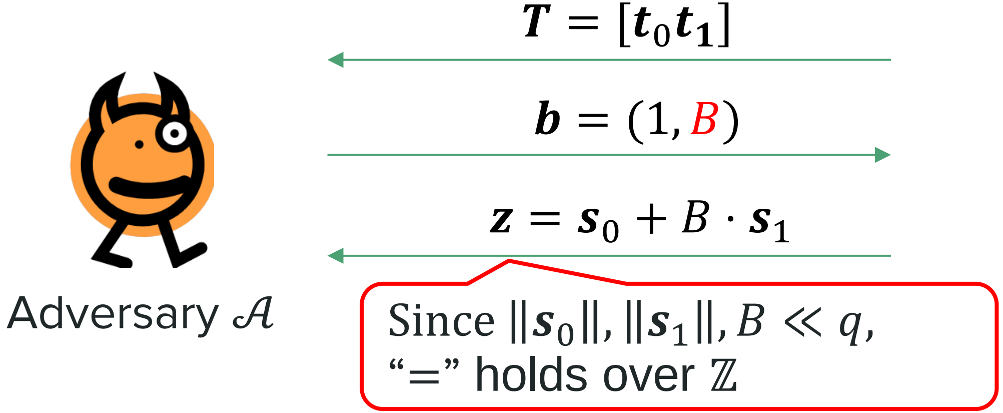

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Since $\|\boldsymbol{s}_0\|, \|\boldsymbol{s}_1\|, B \ll q$,
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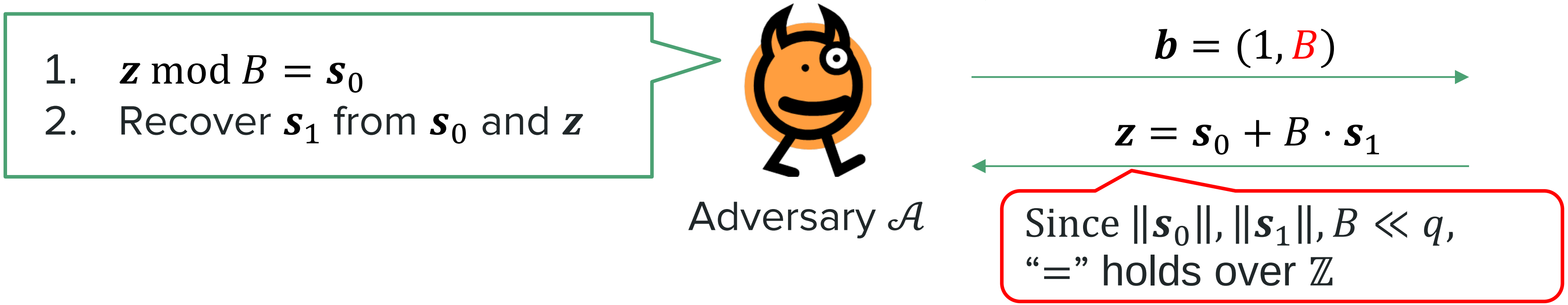

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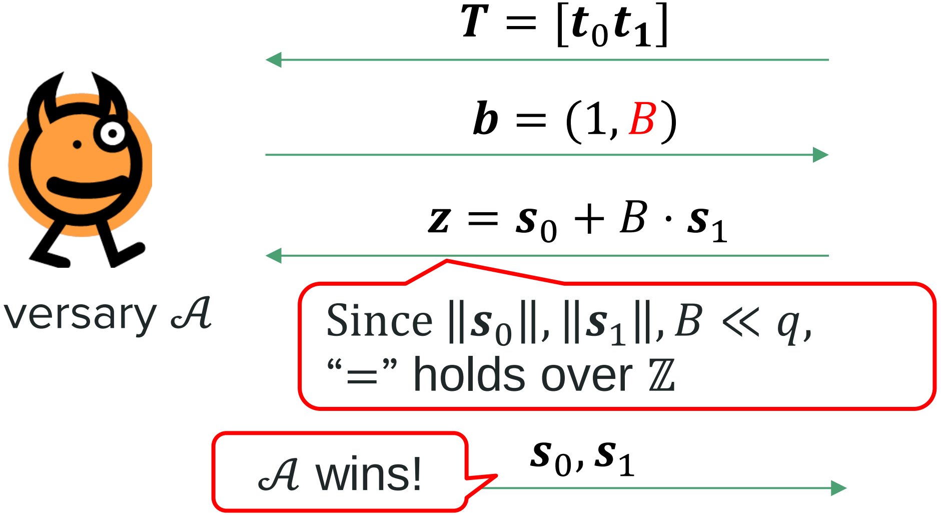

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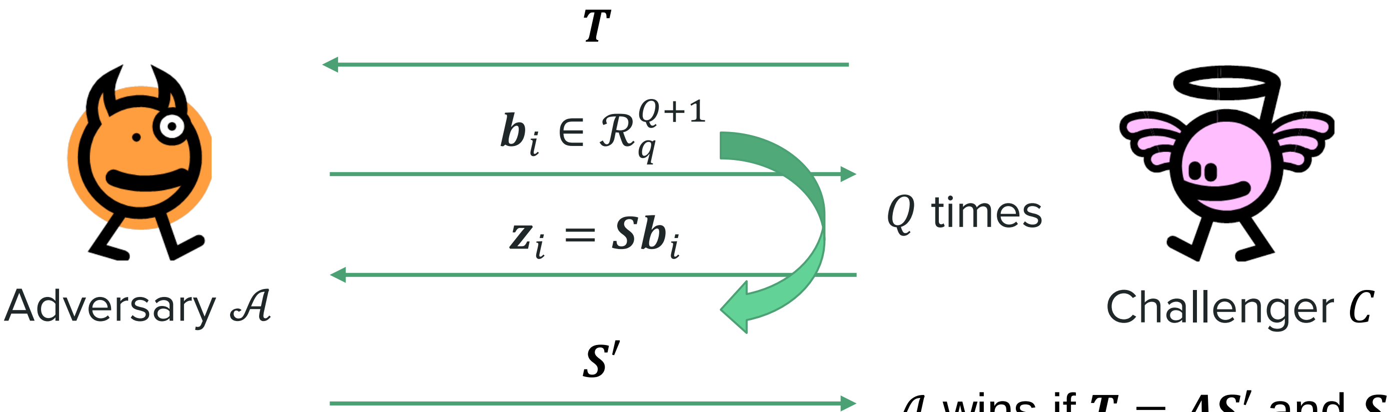

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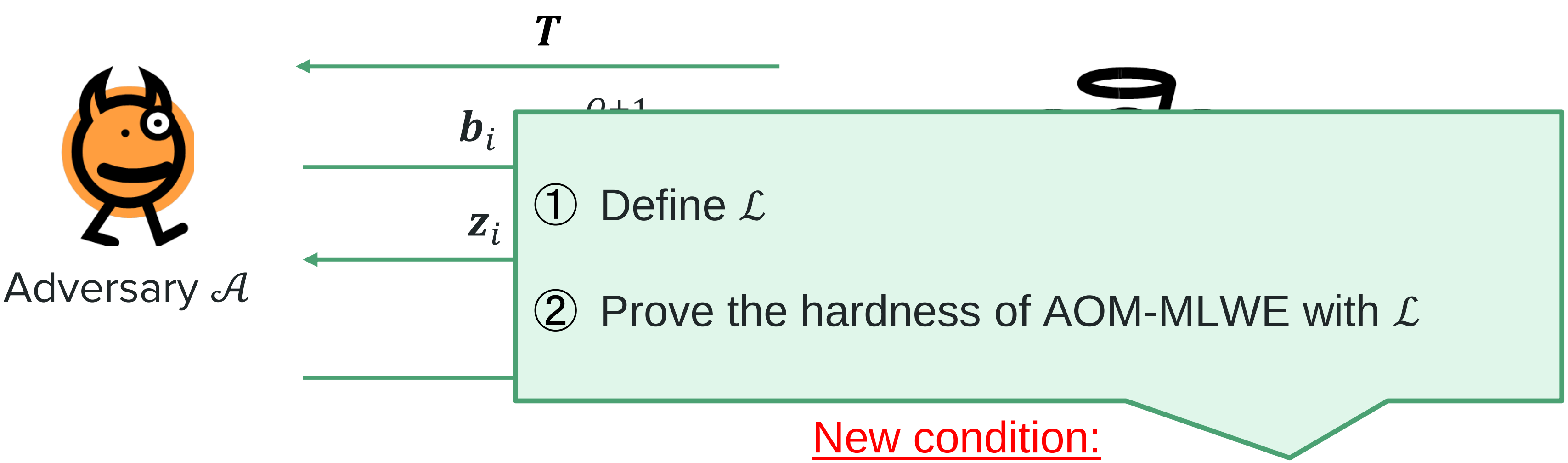

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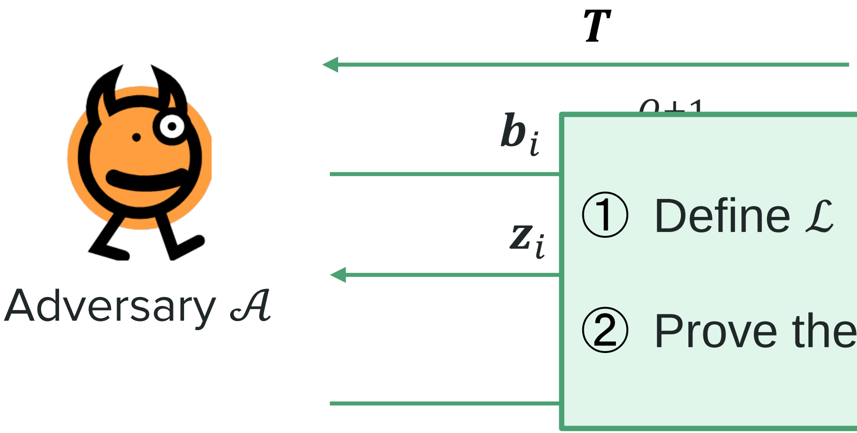

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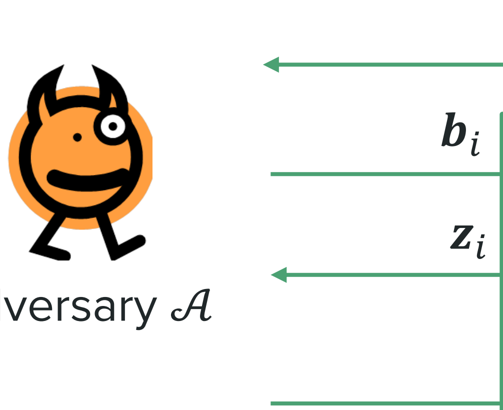

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How do we establish the hardness?

We *heuristically* establish it in ***two steps***:

1. "**Selective**" AOM-MLWE with specific $\mathcal{L}$ is hard under standard assumptions.
2. Practical cryptanalysis against **adaptive** adversary.

Step 1: Hardness of Selective AOM-MLWE with $\mathcal{L}$

What is sel-AOMMLWE?

$\mathcal{A}$ has to output a query matrix $\mathcal{B} \in \mathcal{L}$ at the beginning of the game.

Why selective?

Previous insecure example induces **a statistical attack**.

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We showed that

The **sel**-AOM-LWE with certain $\mathcal{L}$ is **hard under MLWE + MSIS.**

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Consider a generic attack for certain $\mathcal{L}$.

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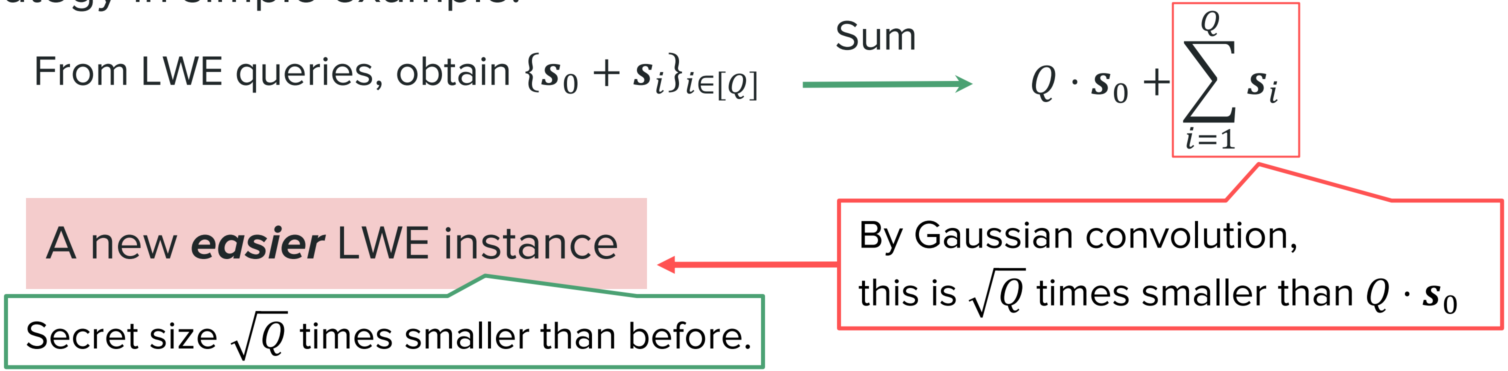

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Generalize this strategy to all accepted queries in $\mathcal{L}$.

Heuristically show that for $\mathcal{A}$ following this strategy

An adaptive $\mathcal{A}$ is *no stronger* than a selective $\mathcal{A}$.

Two-Round Threshold Raccoon

Construction

Construct by combining FROST + TRaccoon

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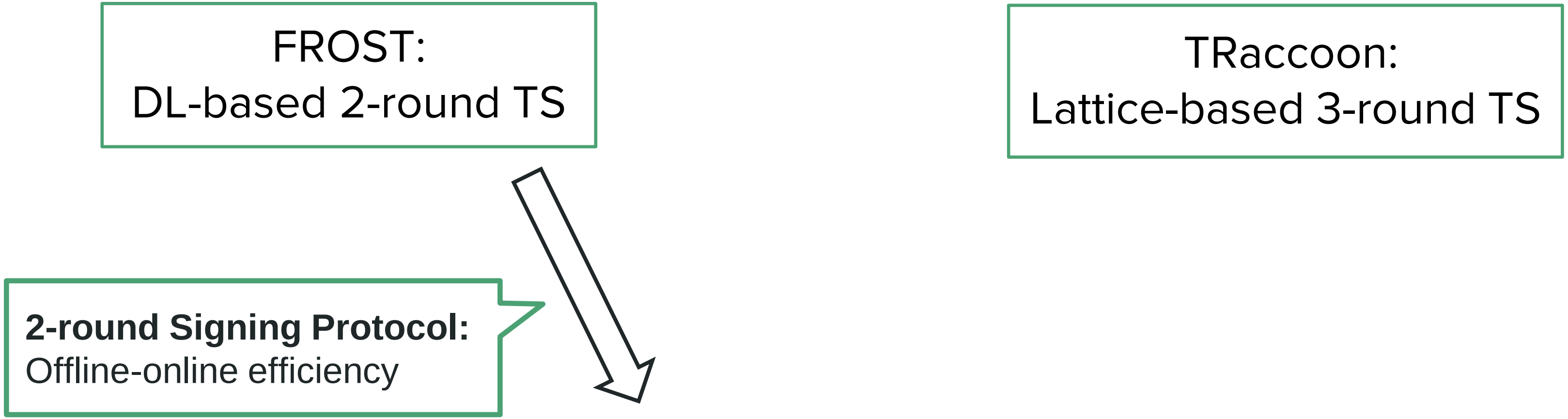

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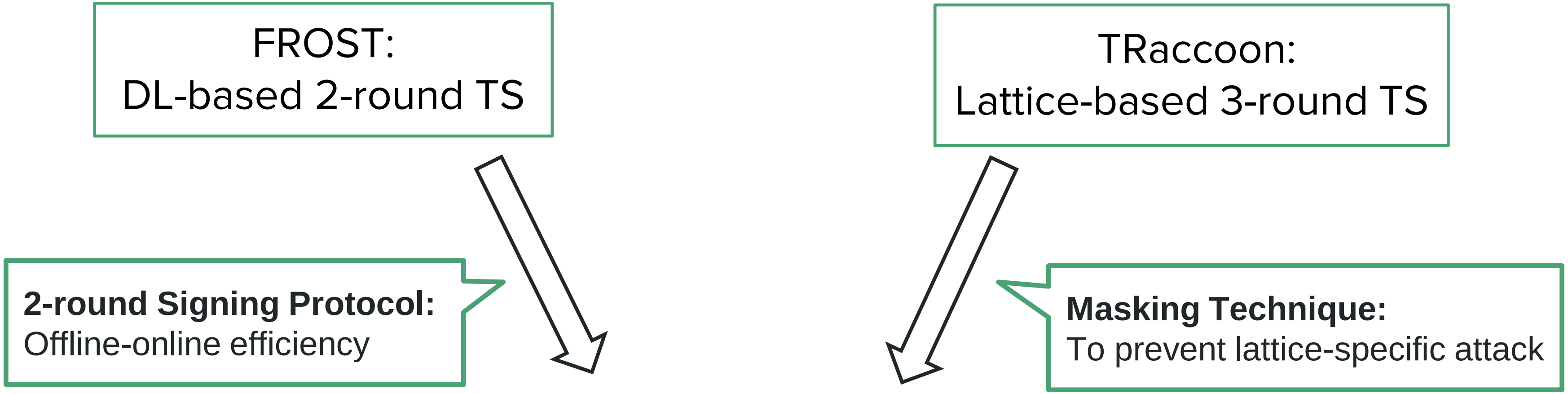

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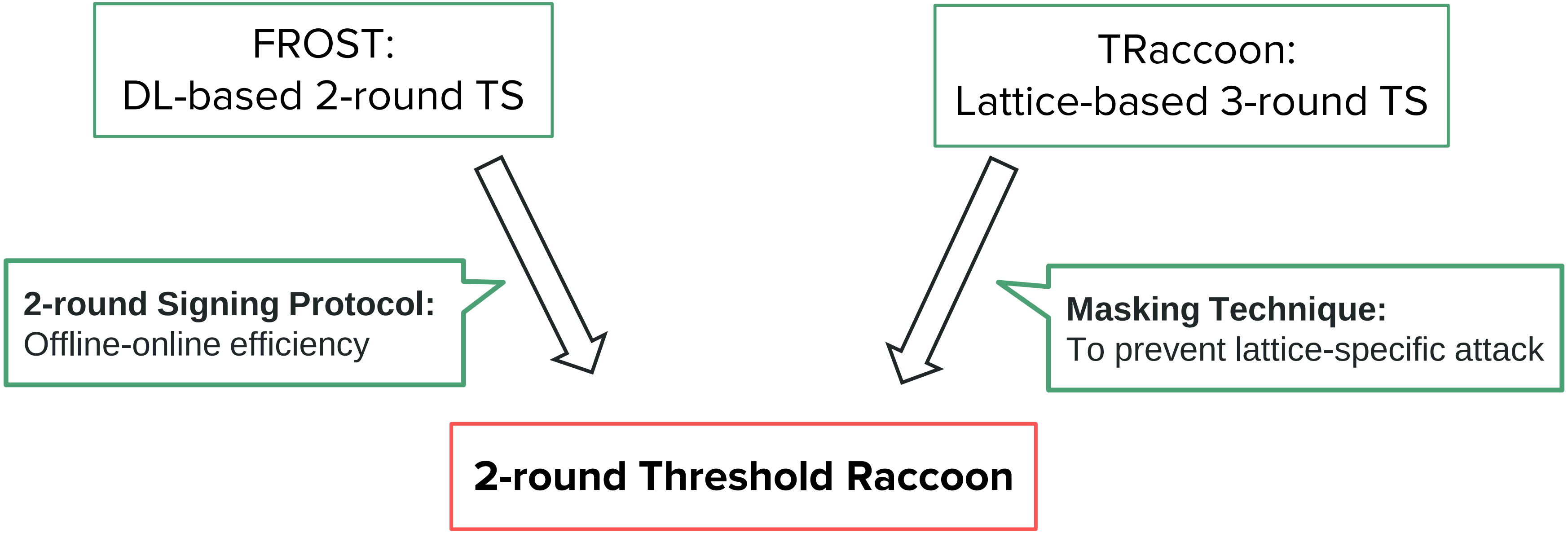

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Our ALC :

Row vector

$$\mathcal{L}_{\mathsf{TS}} = \left\{ \begin{bmatrix} 1 & \\ & \mathbf{P}_{\text{row}} \end{bmatrix} \begin{bmatrix} \mathbf{c}_1^\top & \mathbf{c}_2^\top & \cdots & \mathbf{c}_{Q'}^\top \\ \mathbf{B}_1 & & & \\ & \mathbf{B}_2 & & \\ & & \ddots & \\ & & & \mathbf{B}_{Q'} \end{bmatrix} \cdot \mathbf{P}_{\text{column}} \subset \mathcal{R}_q^{Q\times(Q-1)} \;\middle|\; \begin{array}{l} \forall i \in [Q'], (\mathbf{c}_i, \mathbf{B}_i) \in \mathcal{C}_{\mathsf{TS}} \times \mathcal{B}_{\mathsf{TS}}, \\ (\mathbf{P}_{\text{row}}, \mathbf{P}_{\text{column}}) \in \mathcal{P}_{Q-1}^2 \end{array} \right\}$$

Block diagonal

Performances

Under $T \leq 1024$ setting, for 128-bit security,

Scheme	|vk|	|Sig|	Online Comm./User	Offline Comm./User
3-round	3.9 KB	12.7 KB	40.8 KB	-
2-round	5.5 KB	10.8 KB	14.1 KB	262 KB

Almost the same

Efficient!

Overhead

Thank You!

Important Future Work:

- ➢ To prove **the hardness of adaptive AOM-MLWE.**

Concurrent Works:

- ➢ "Adaptively Secure 5 Round Threshold Signatures from MLWE/MSIS and DL with Rewinding" [C: KTR24] (Next Talk!!)
- ➢ "Flood and submerse: Verifiable short secret sharing and application to robust threshold signatures on lattices" [C: EPN24] (Talk was this morning!!)
- ➢ "Partially Non-Interactive Two-Round Lattice-Based Threshold Signatures" [Eprint:CATZ24]

Recent Related Works:

- ➢ "Ringtail: Practical Two-Round Threshold Signatures from Learning with Errors" [Eprint:BKL+24]
  (※ partially offline-online efficient)