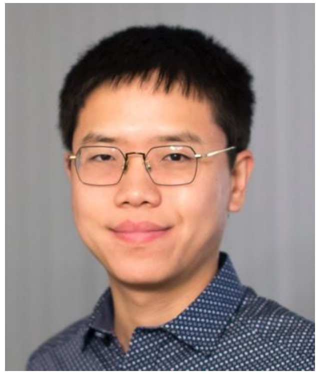
Adaptively Secure BLS Threshold Signatures from DDH and co-CDH



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Boneh-Lynn-Sacham (BLS) Signatures

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Bilinear pairing based signature scheme

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Key generation

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Key generation

$\mathrm{sk} \coloneqq s \leftarrow \mathbb{F}$

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$$\mathrm{sk} \coloneqq s \leftarrow \mathbb{F}$$
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Signing

Boneh-Lynn-Sacham (BLS) Signatures

Bilinear pairing based signature scheme

Key generation

$$\mathrm{sk} := s \leftarrow \mathbb{F}$$
$$\mathrm{pk} := g^s$$

Signing

$$\sigma := \mathrm{H}(m)^s$$

Boneh-Lynn-Sacham (BLS) Signatures

Bilinear pairing based signature scheme

Key generation

$\mathrm{sk} \coloneqq s \leftarrow \mathbb{F}$
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Verification

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$$e\big(\mathrm{pk}, \mathrm{H}(m)\big) = e(g, \sigma)$$

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$$e\big(\mathrm{pk}, \mathrm{H}(m)\big) = e(g, \sigma)$$

Correctness:

- LHS: $e\big(\mathrm{pk}, \mathrm{H}(m)\big) = e\big(g, \mathrm{H}(m)\big)^s$
- RHS: $e(g, \sigma) = e\big(g, \mathrm{H}(m)\big)^s$

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Security:

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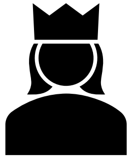
Security:

- Hardness of CDH in the random oracle model (ROM)

Background

(n, t) Threshold Secret Sharing

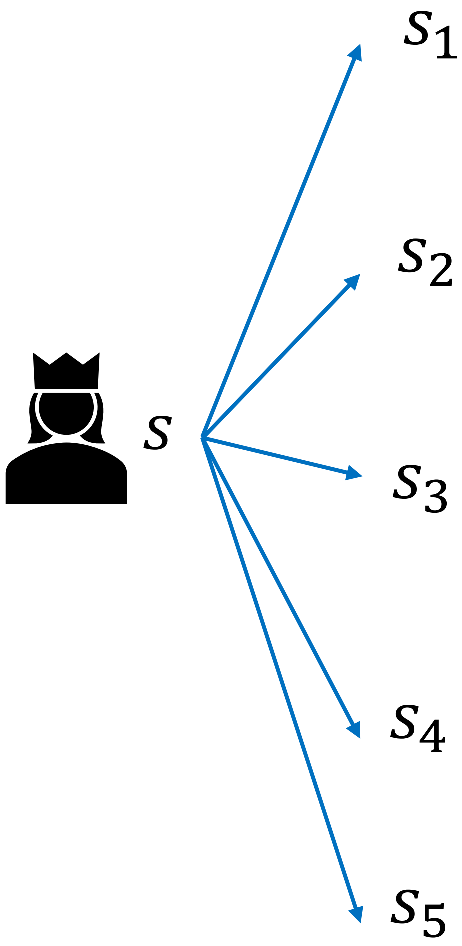
(n, t) Threshold Secret Sharing



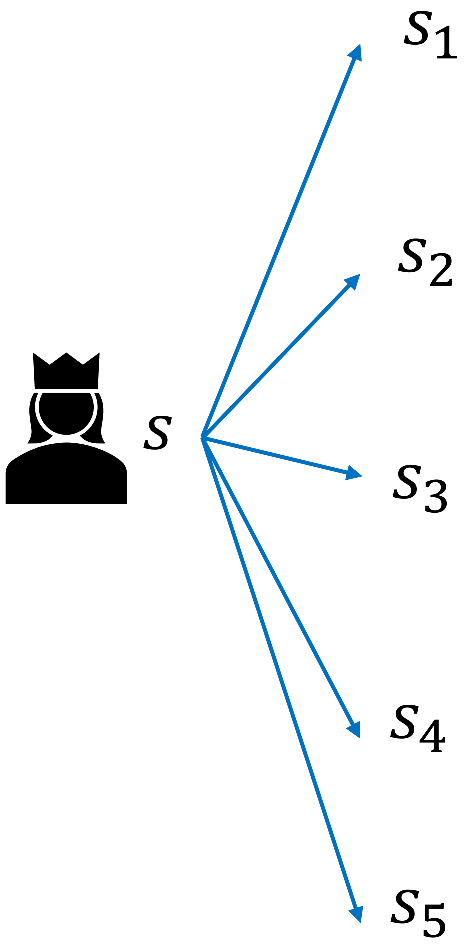
s

(n, t) Threshold Secret Sharing

- A mechanism to share a secret s into n shares

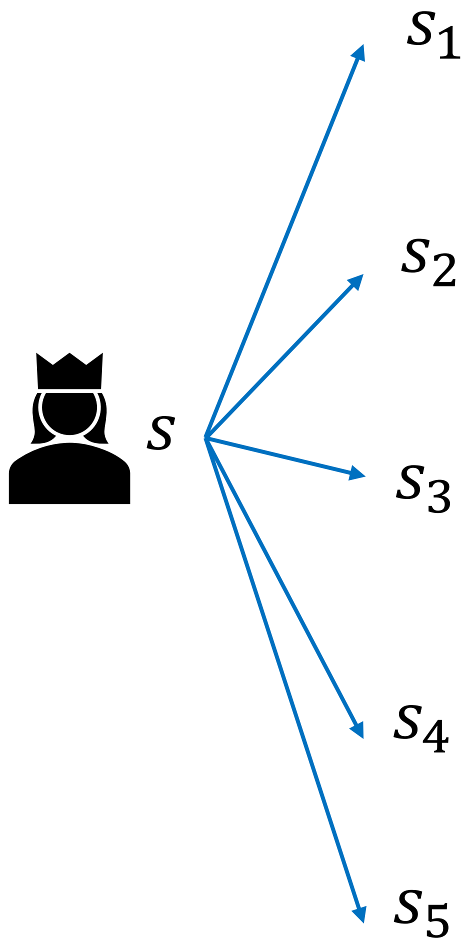


(n, t) Threshold Secret Sharing



- A mechanism to share a secret s into n shares
- Any subset of $t + 1$ shares reveal the secret

(n, t) Threshold Secret Sharing



- A mechanism to share a secret s into n shares
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- Any subset of t or less shares reveal nothing about s

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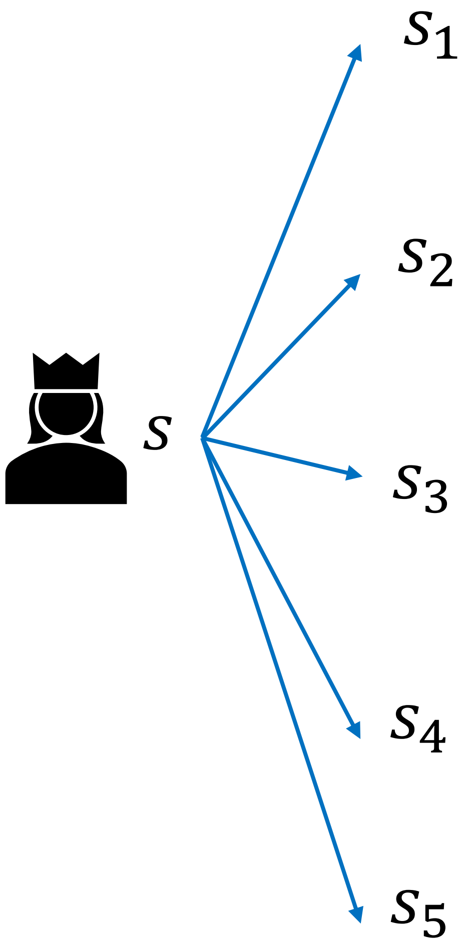

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- An example of $(5,3)$ threshold secret sharing scheme

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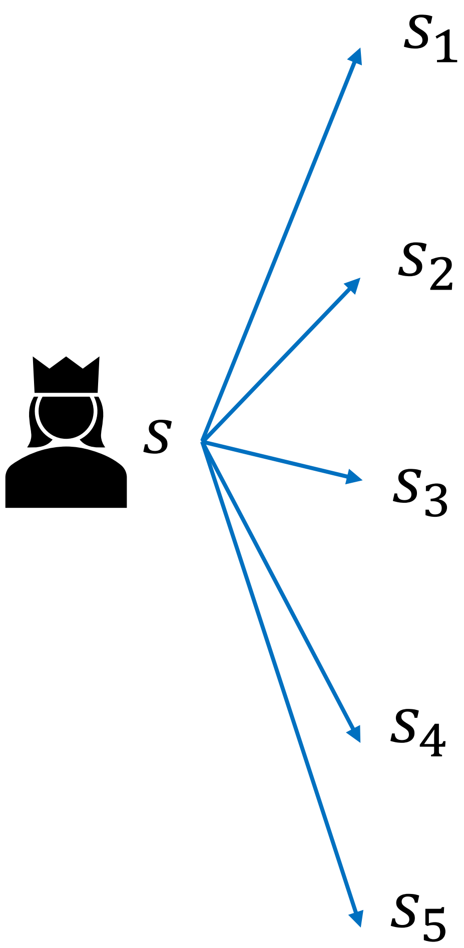

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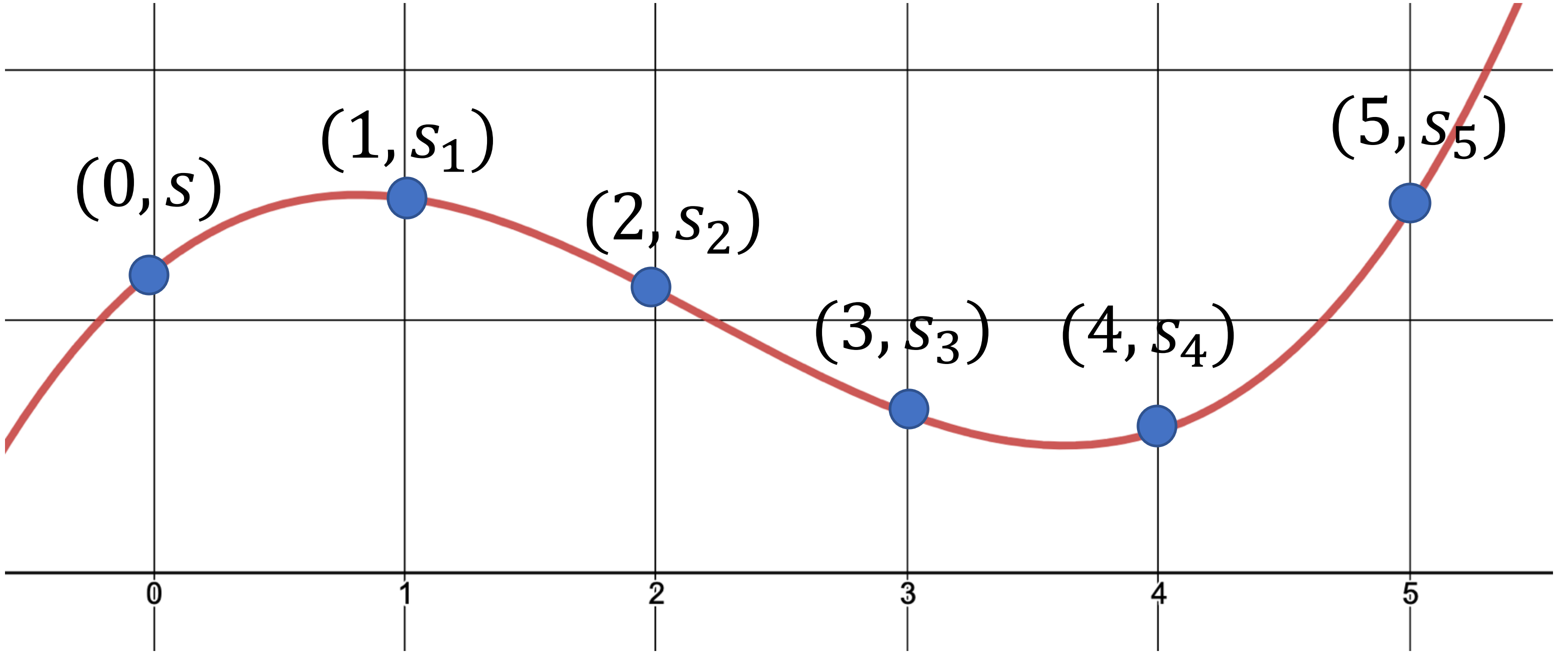

BLS Threshold signature [Boldyreva’03]

BLS Threshold signature [Boldyreva’03]: Key Generation

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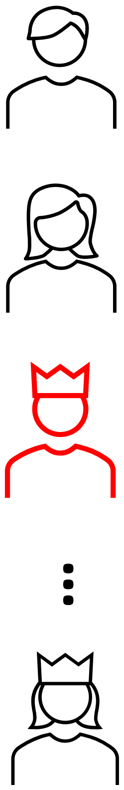

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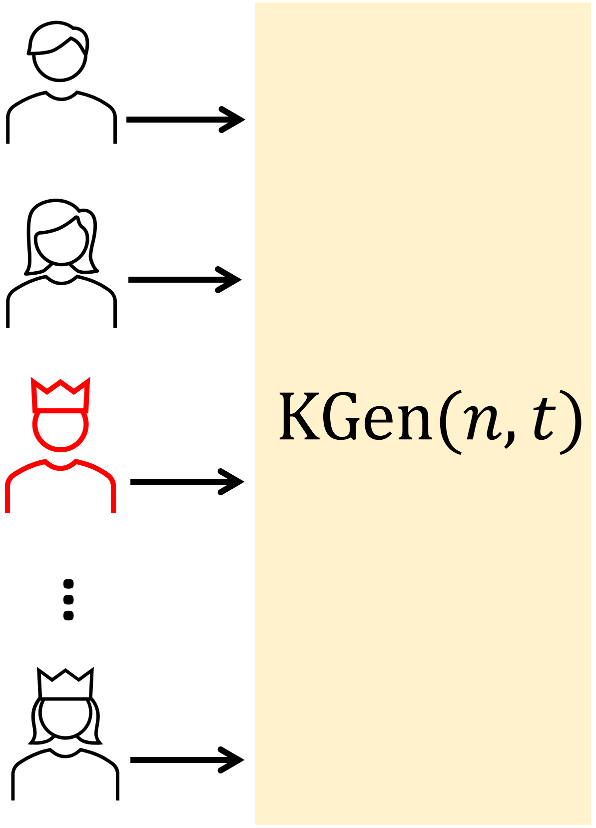

$\mathrm{KGen}(n, t)$

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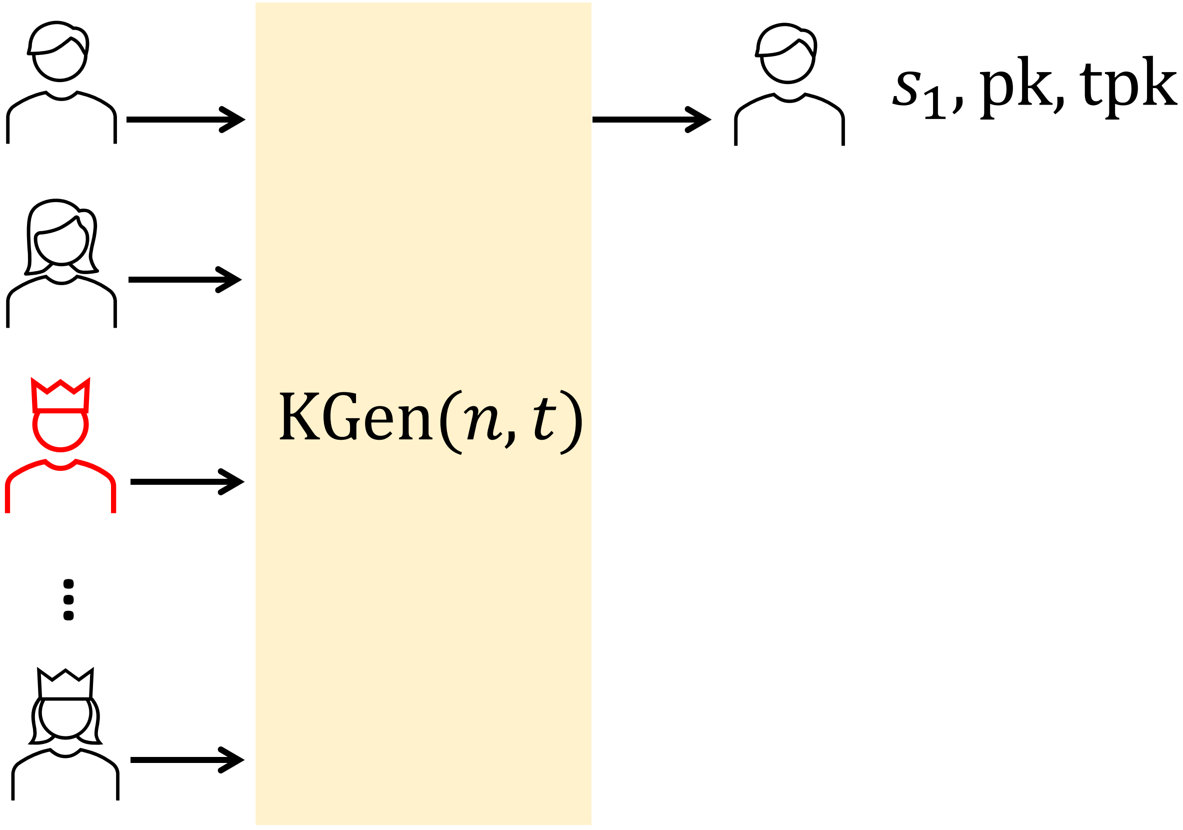

$\mathrm{KGen}(n, t)$

$s_1, \mathrm{pk}, \mathrm{tpk}$

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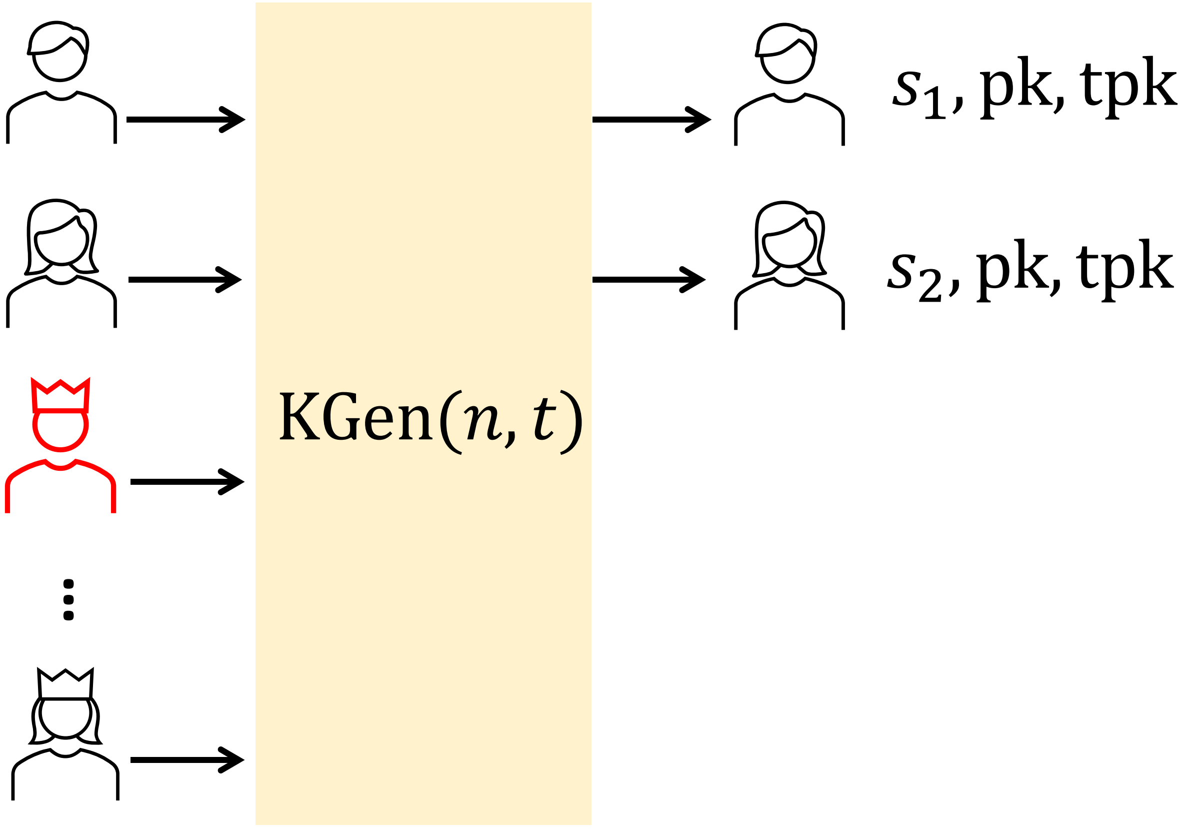

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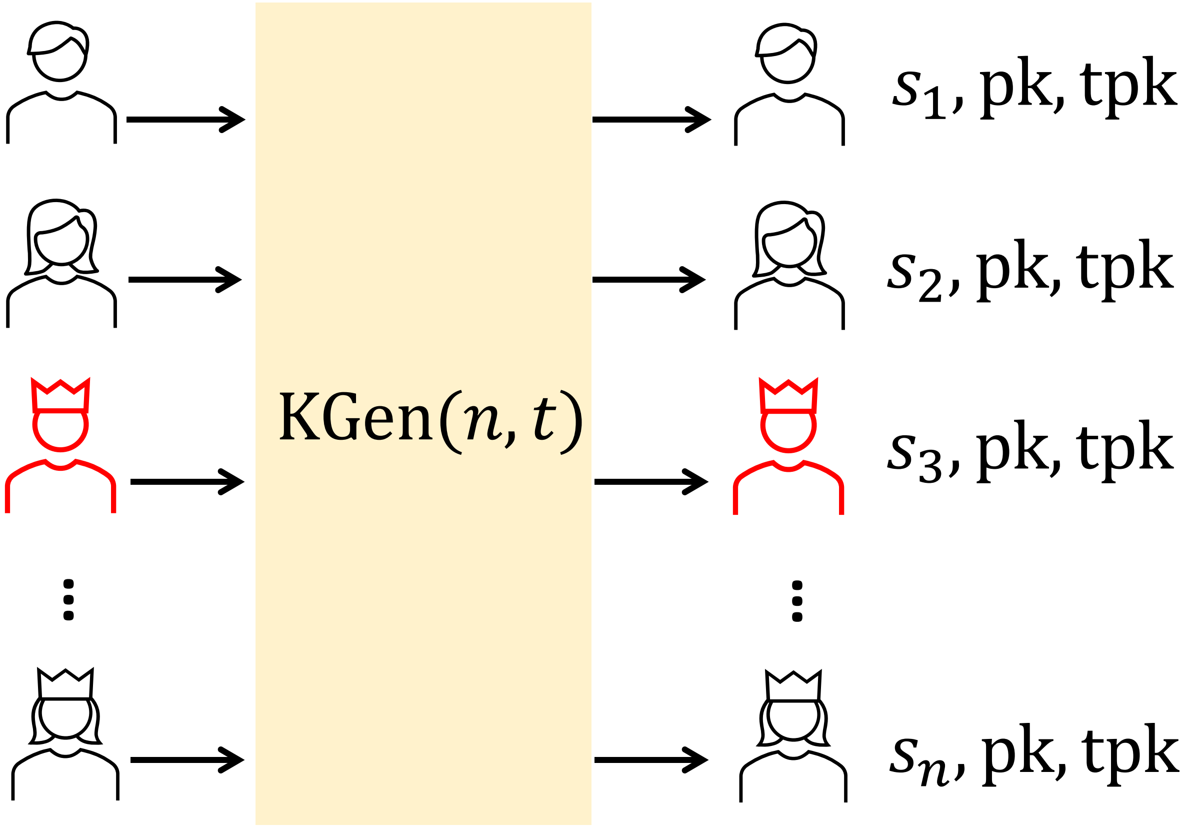

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$s_1, \mathrm{pk}, \mathrm{tpk}$

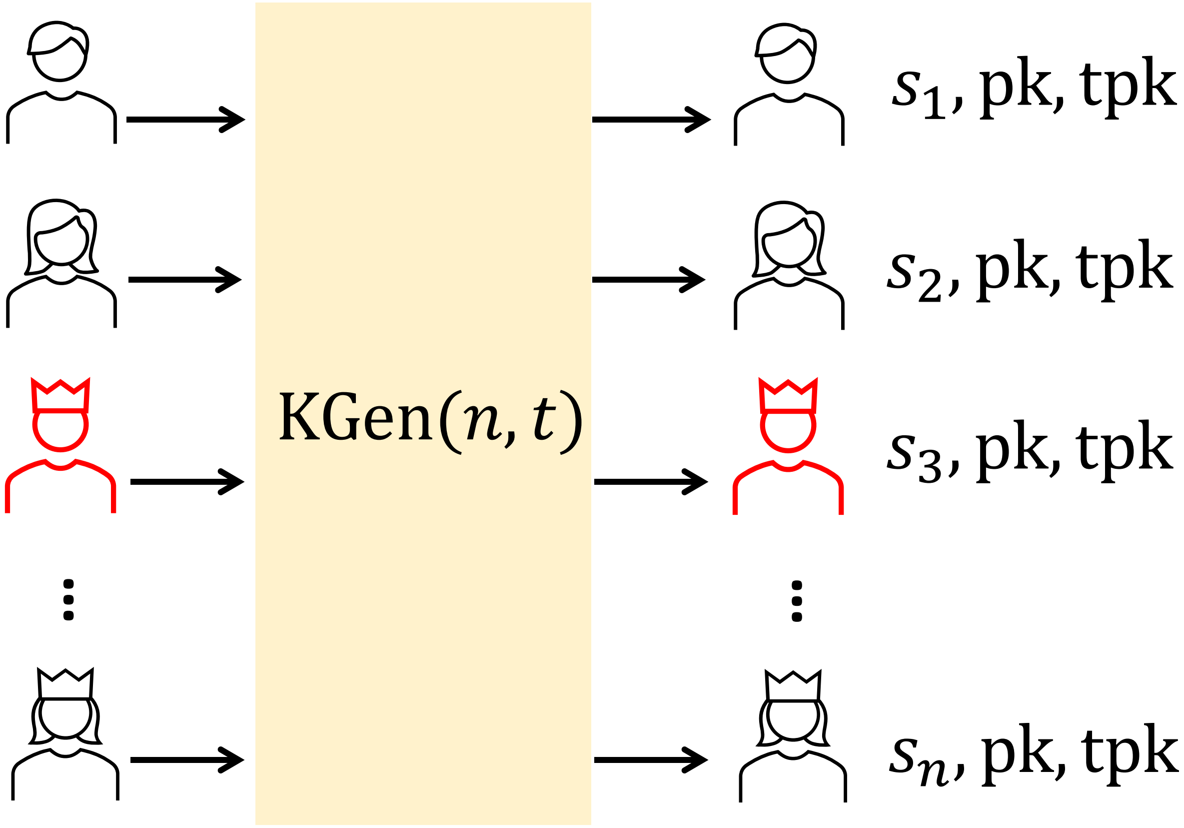
$s_2, \mathrm{pk}, \mathrm{tpk}$

$s_3, \mathrm{pk}, \mathrm{tpk}$

⋮

$s_n, \mathrm{pk}, \mathrm{tpk}$

BLS Threshold signature [Boldyreva'03]: Key Generation



$\mathrm{KGen}(n, t)$

$s_1, \mathrm{pk}, \mathrm{tpk}$

$s_2, \mathrm{pk}, \mathrm{tpk}$

$s_3, \mathrm{pk}, \mathrm{tpk}$

$s_n, \mathrm{pk}, \mathrm{tpk}$

$$\{s_1, \dots, s_n\} \leftarrow \mathrm{Share}(s)$$
$$\mathrm{pk} = g^s$$
$$\mathrm{tpk} = \{g^{s_1}, g^{s_2}, \dots, g^{s_n}\}$$

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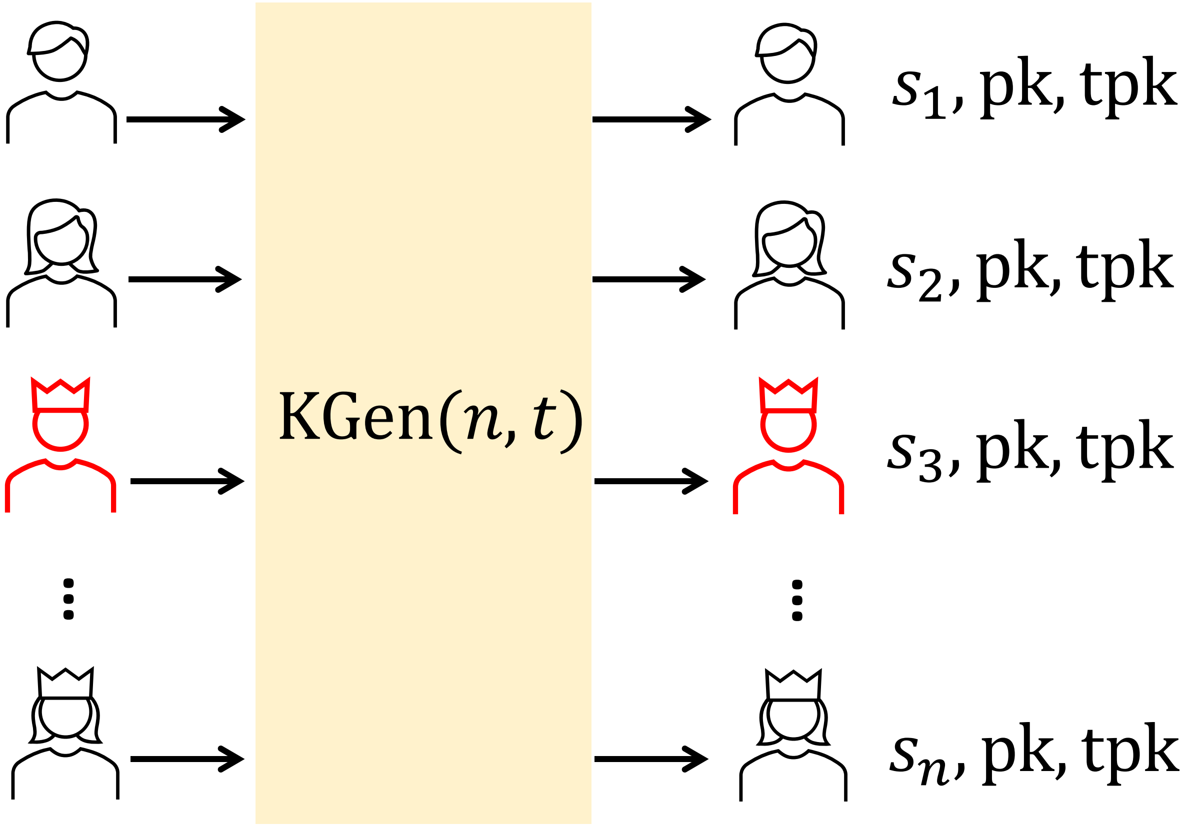

KGen(n, t)

s_1, pk, tpk

s_2, pk, tpk

s_3, pk, tpk

s_n, pk, tpk

$$\{s_1, \dots, s_n\} \leftarrow \text{Share}(s)$$
$$\text{pk} = g^s$$
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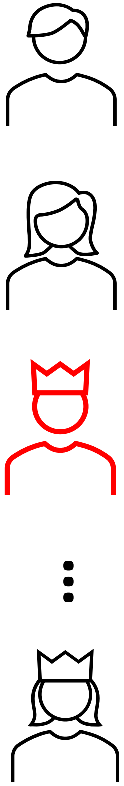
Can also use Distributed Key Generation (DKG).
[GJKR-JoC'07, DYXMKR-SP'22, DXKR-USENIX'23]

BLS Threshold signature: Signing



s_1

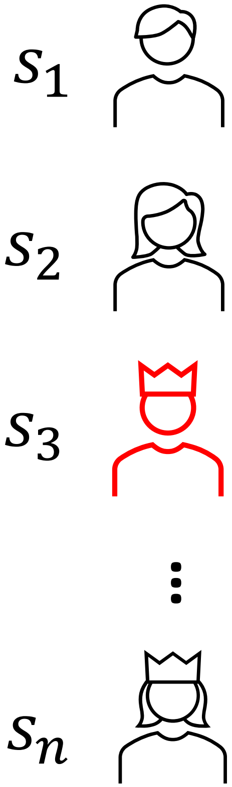
s_2

s_3

⋮

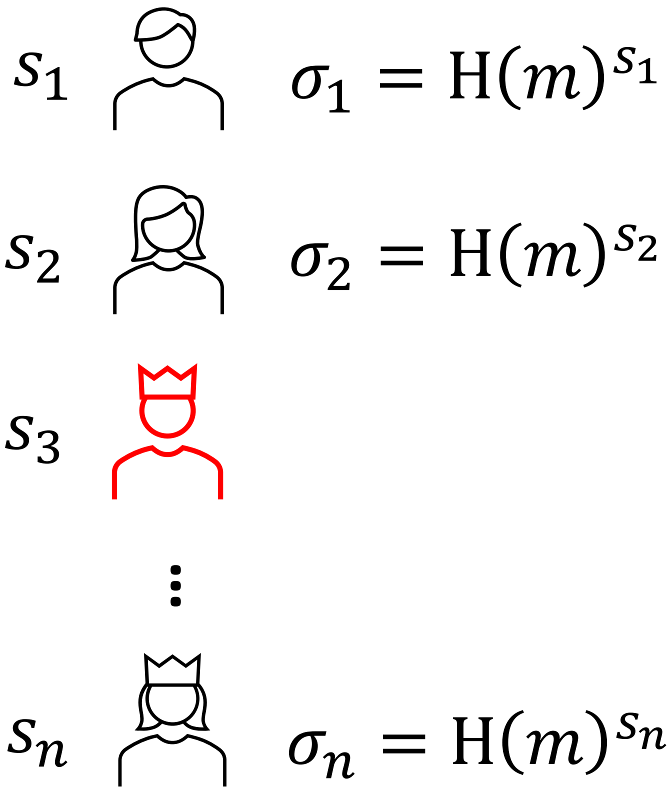
s_n

BLS Threshold signature: Signing



$$\sigma_1 = \mathrm{H}(m)^{s_1}$$

BLS Threshold signature: Signing



s_1 $\sigma_1 = \mathrm{H}(m)^{s_1}$

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⋮

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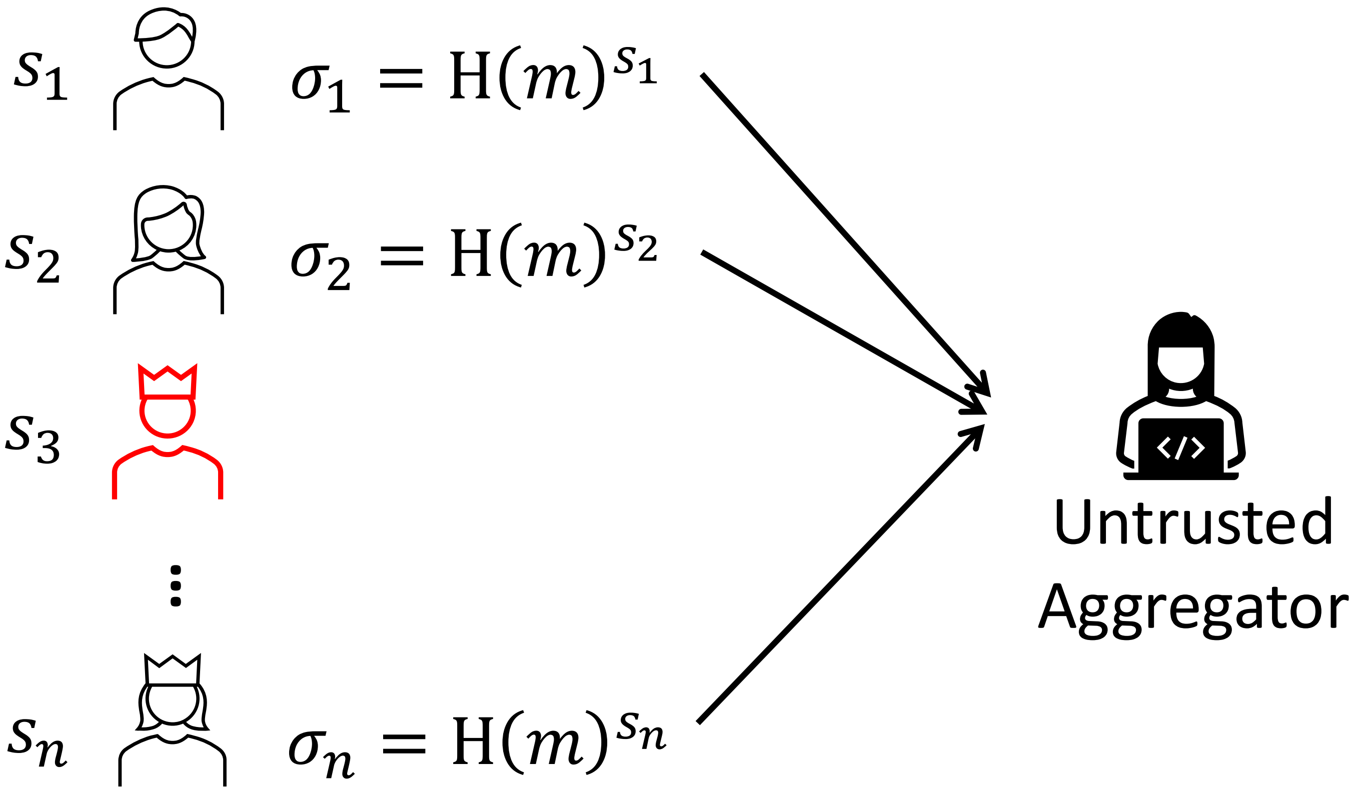

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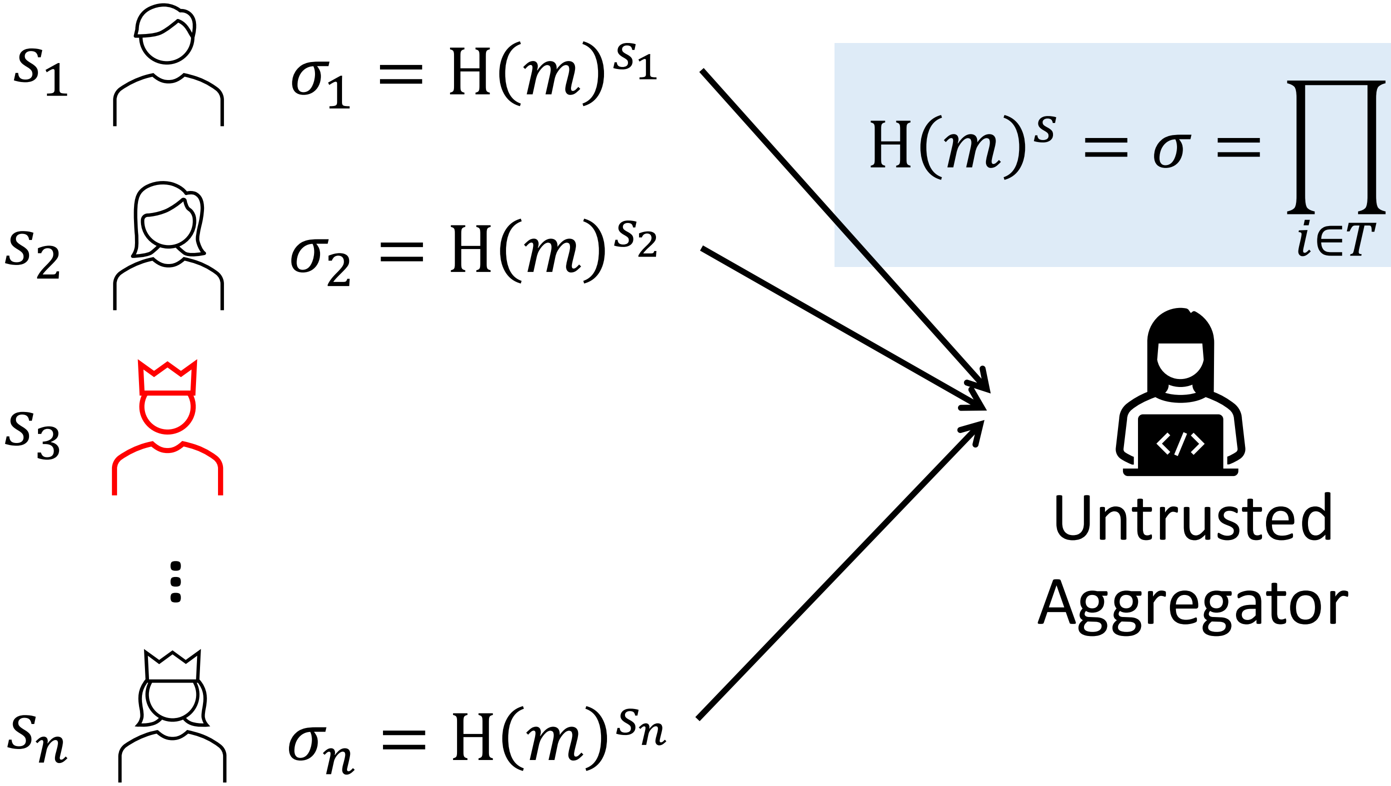
Untrusted Aggregator

BLS Threshold signature: Signing

λ_i are the Lagrange coefficients.



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s_n $\quad \sigma_n = \mathrm{H}(m)^{s_n}$

$$\mathrm{H}(m)^s = \sigma = \prod_{i \in T} \sigma_i^{\lambda_i}$$

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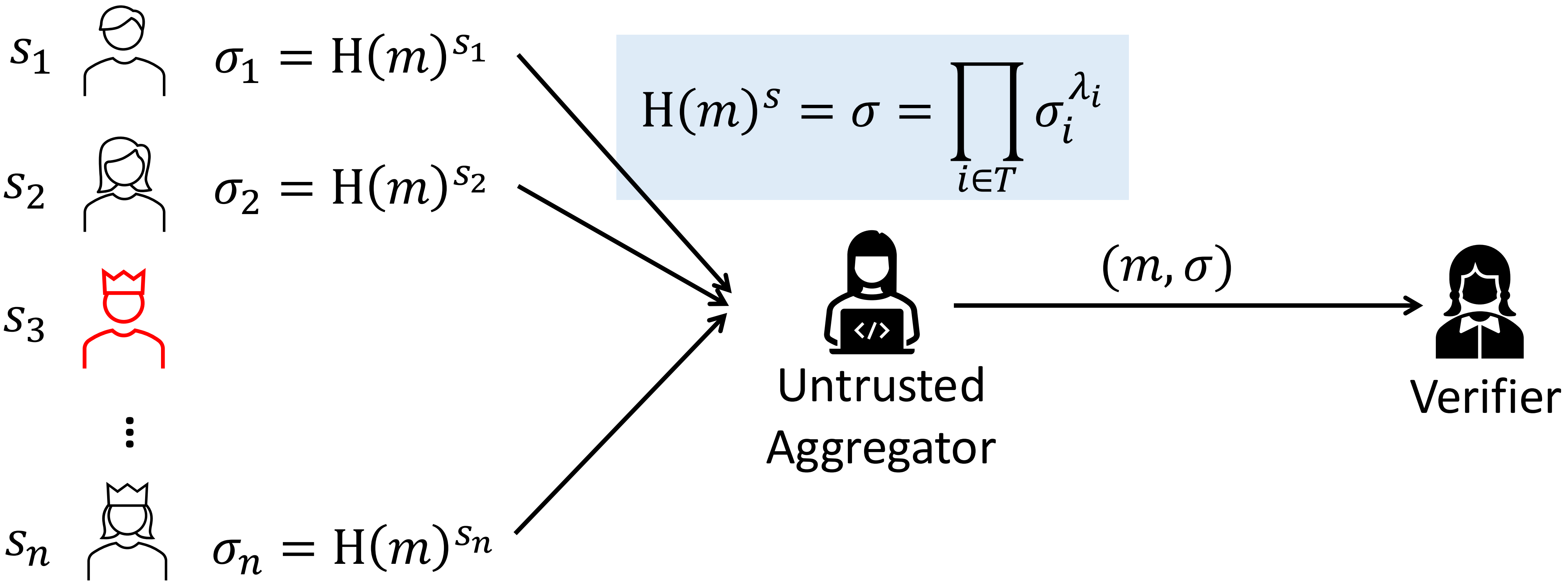

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(m, σ)

Untrusted Aggregator

Verifier

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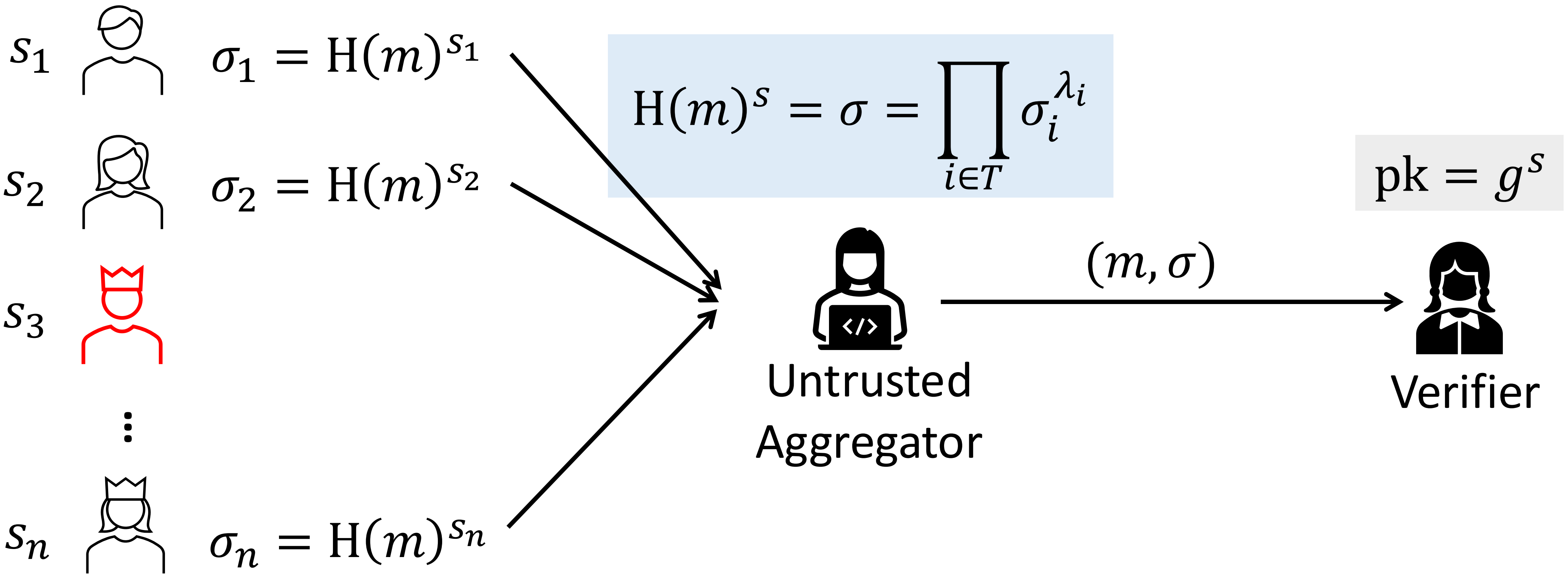

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Untrusted Aggregator

(m, σ)

$\mathrm{pk} = g^s$

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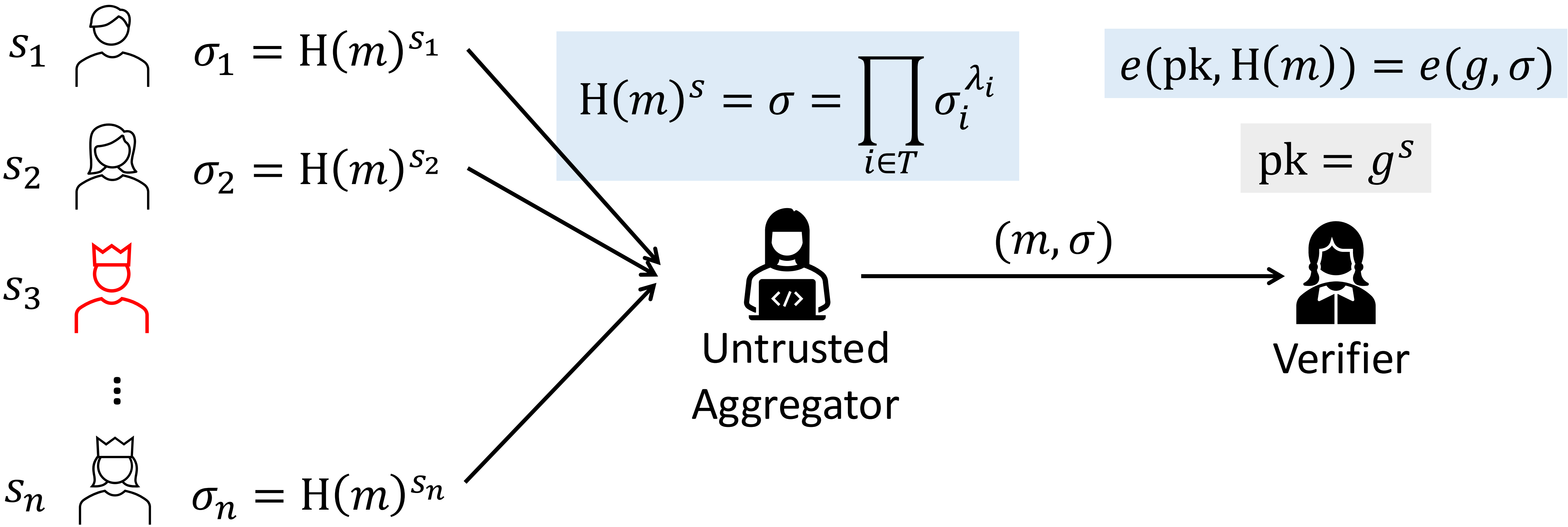

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$$e(\mathrm{pk}, \mathrm{H}(m)) = e(g, \sigma)$$

$$\mathrm{pk} = g^s$$

(m, σ)

Untrusted Aggregator

Verifier

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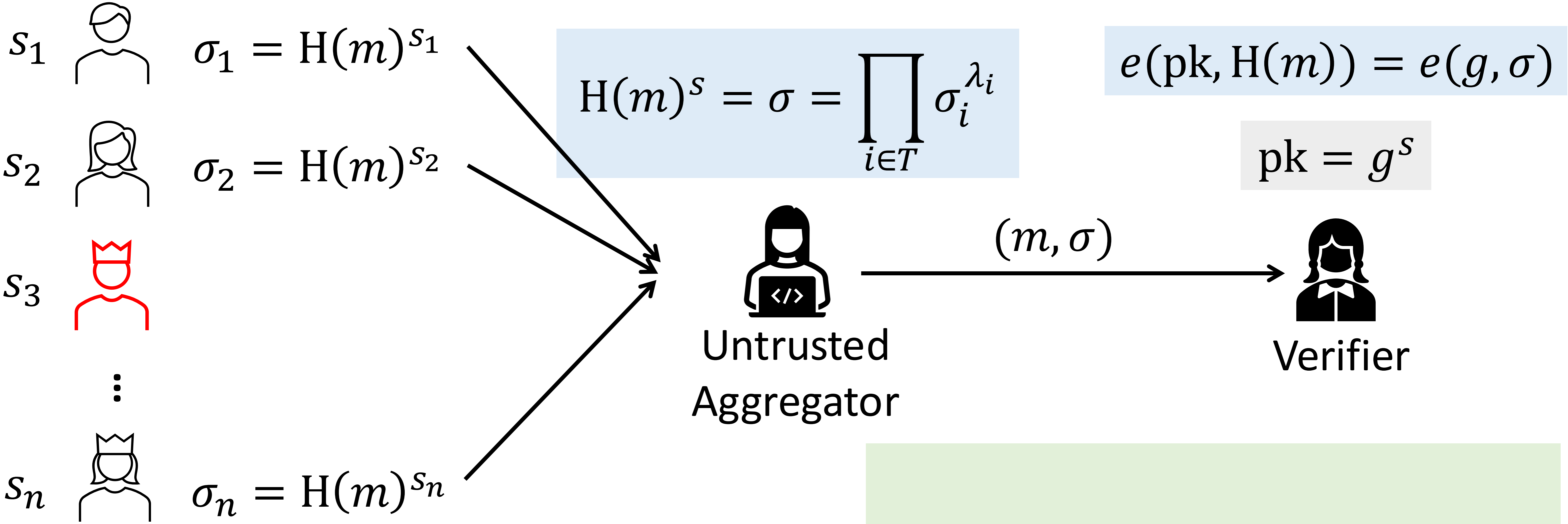

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(m, σ)

Untrusted Aggregator

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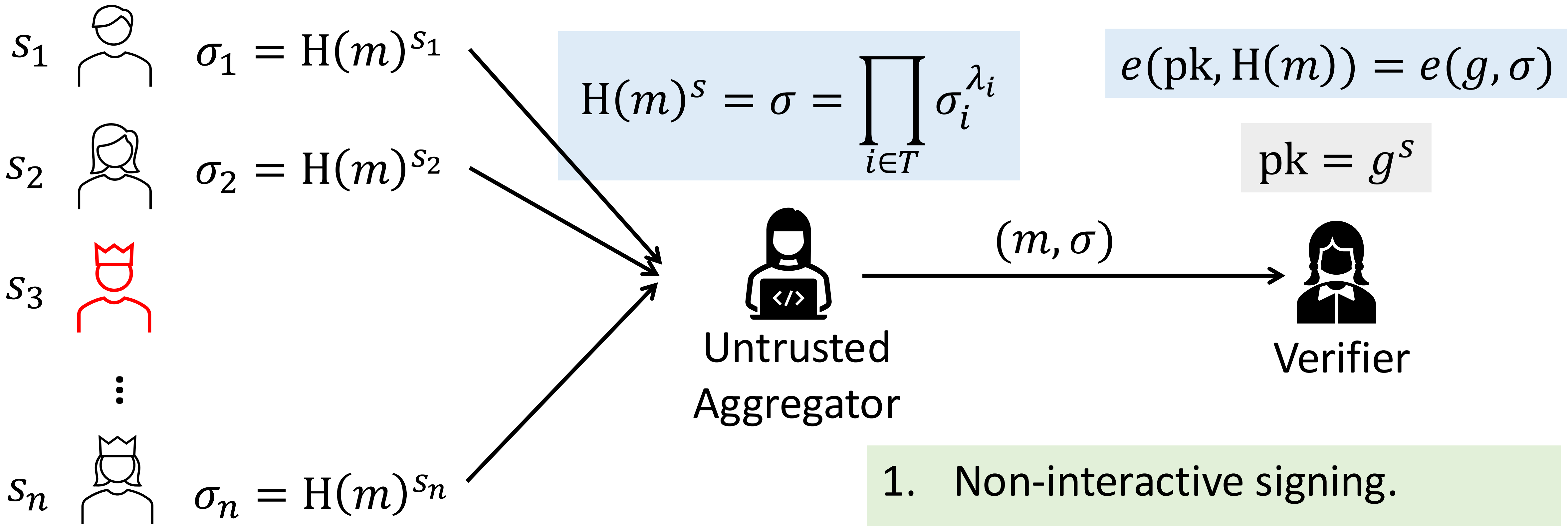

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(m, σ)

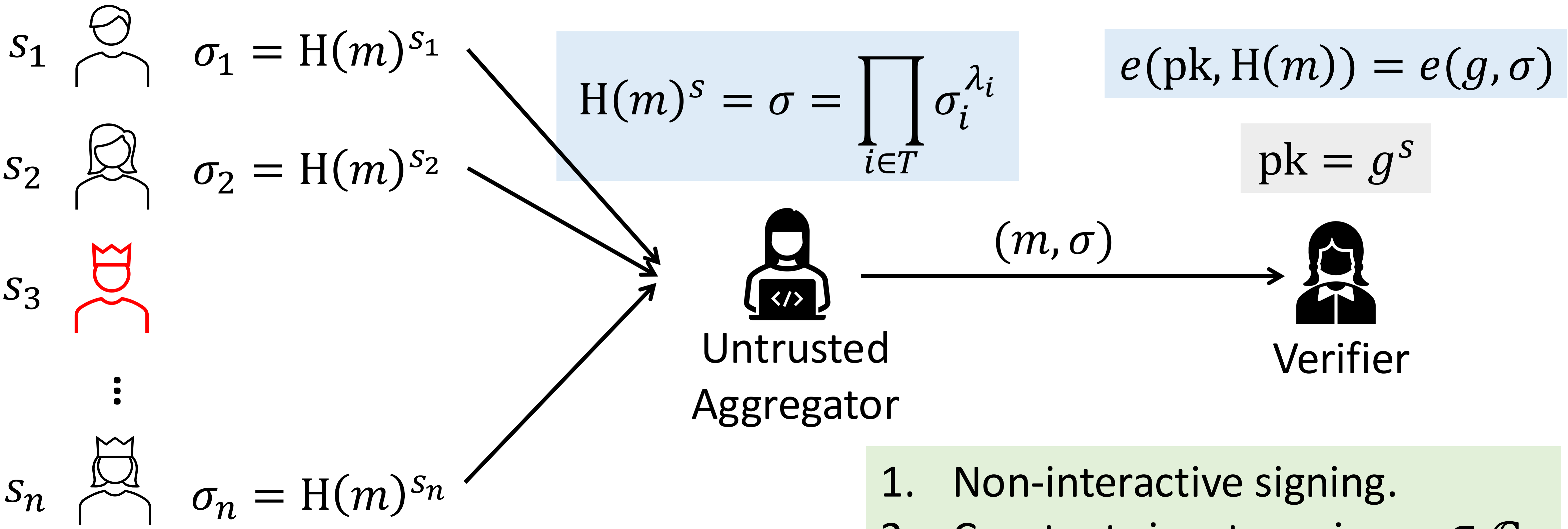
Untrusted Aggregator

Verifier

1. Non-interactive signing.

BLS Threshold signature: Signing

λ_i are the Lagrange coefficients.



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(m, σ)

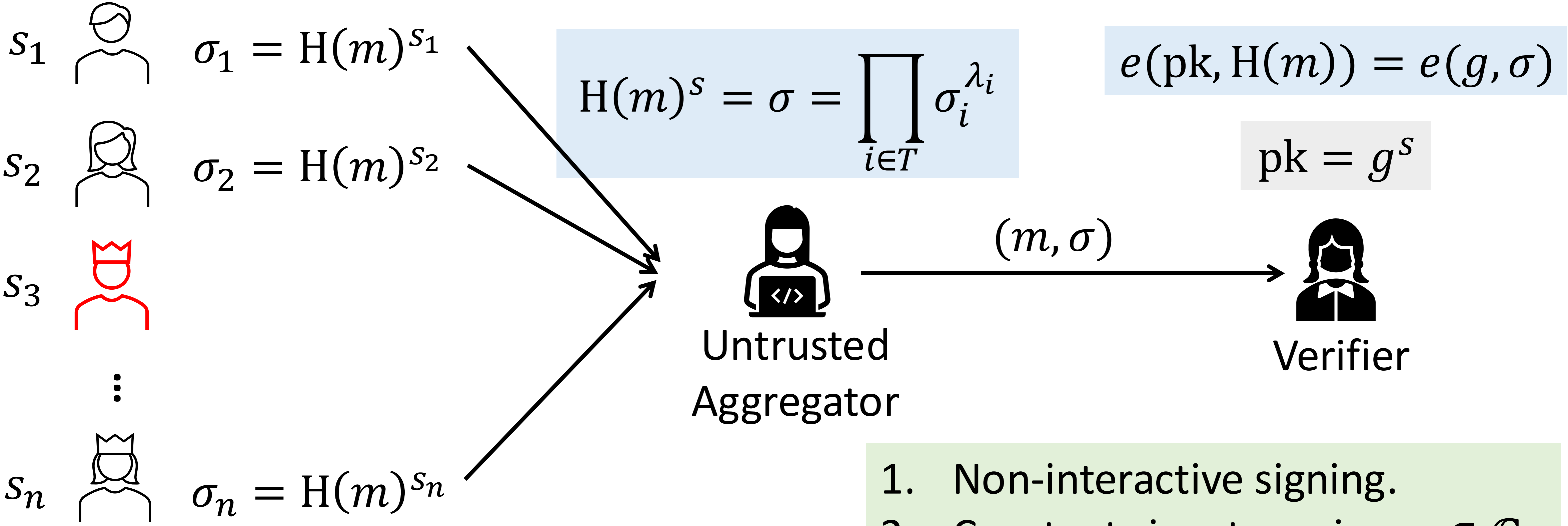
Untrusted Aggregator

Verifier

1. Non-interactive signing.
2. Constant signature size: $\sigma \in \mathbb{G}$.

BLS Threshold signature: Signing

λ_i are the Lagrange coefficients.



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$\mathrm{pk} = g^s$

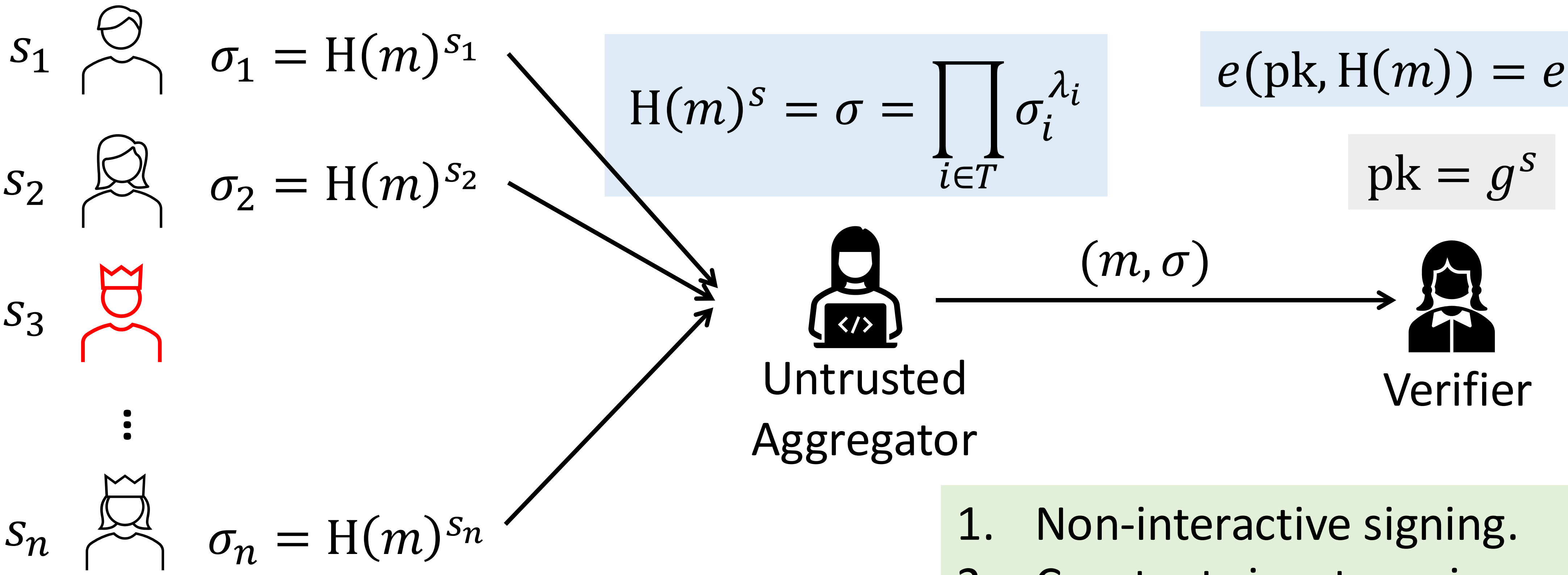
(m, σ)

Untrusted Aggregator

Verifier

1. Non-interactive signing.
2. Constant signature size: $\sigma \in \mathbb{G}$.
3. Verification cost: two pairings.

BLS Threshold signature: Signing



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(m, σ)

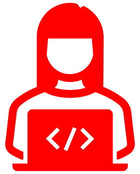
Untrusted Aggregator

Verifier

1. Non-interactive signing.
2. Constant signature size: $\sigma \in \mathbb{G}$.
3. Verification cost: two pairings.
4. Unique signatures.

Static vs Adaptive Security

Static vs Adaptive Security



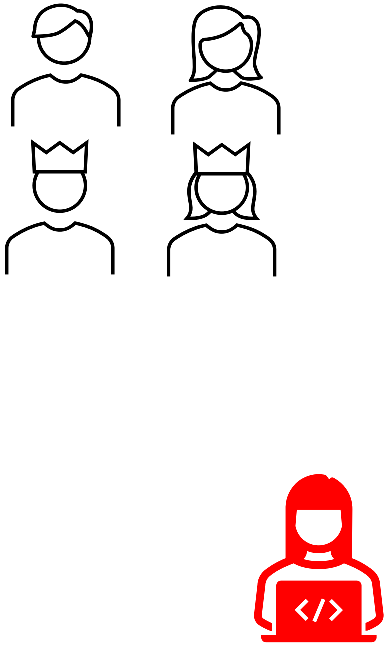
Static $\mathcal{A}_{\mathrm{th}}$

Static vs Adaptive Security

Static $\mathcal{A}_{\text{th}}$

Needs to decide who to corrupt before the protocol begins.

Static vs Adaptive Security



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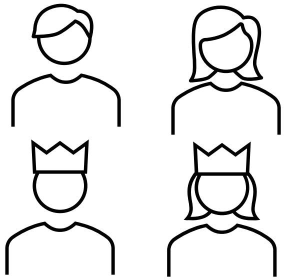

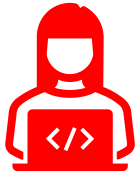

$\mathcal{C} \subset [n]$

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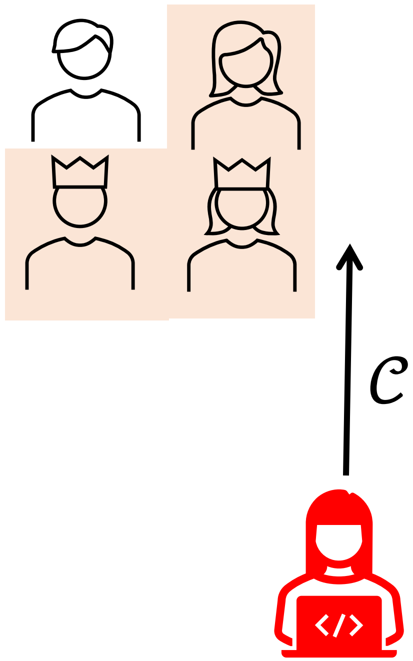

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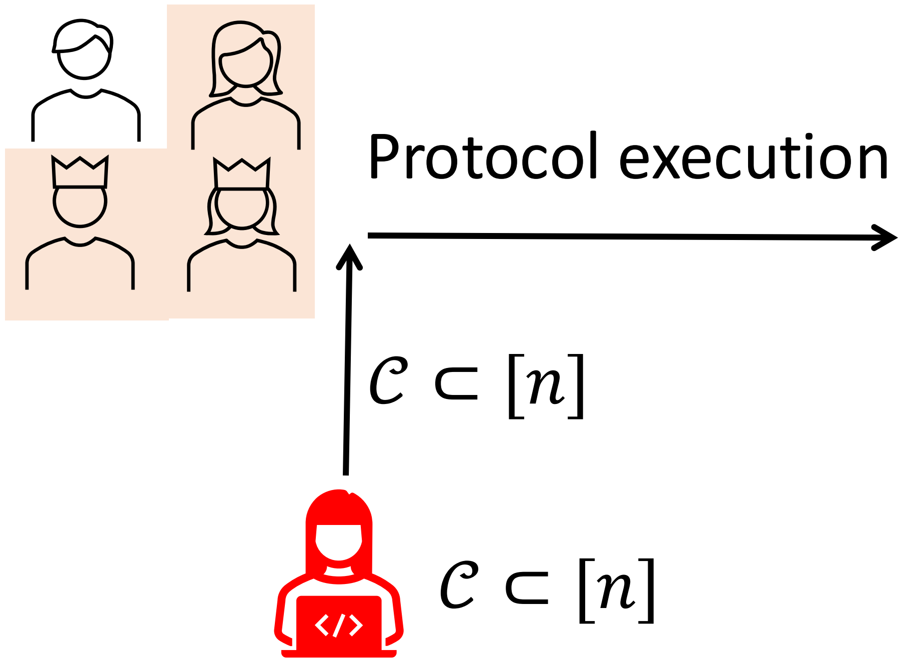

Protocol execution

$\mathcal{C} \subset [n]$

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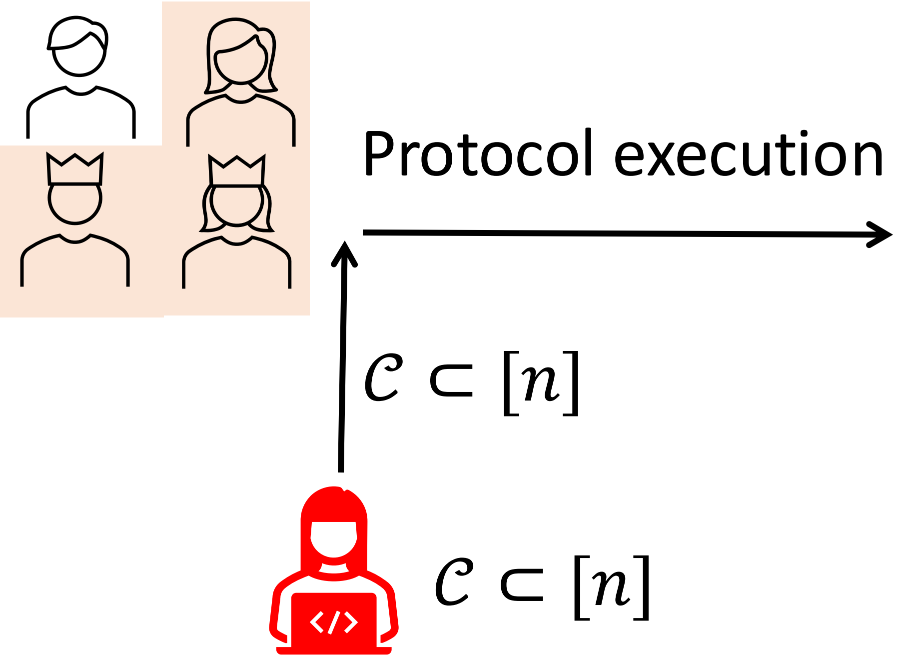

Protocol execution

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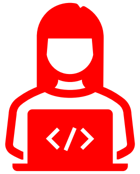
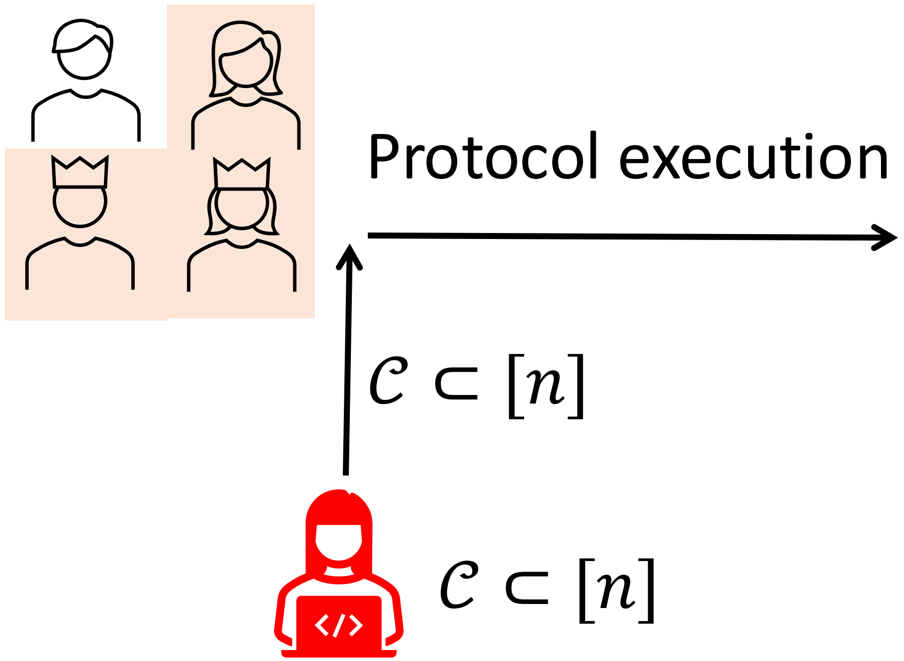


Adaptive $\mathcal{A}_{\text{th}}$

Needs to decide who to corrupt before the protocol begins.

Static vs Adaptive Security



Protocol execution

$\mathcal{C} \subset [n]$

$\mathcal{C} \subset [n]$

Static $\mathcal{A}_{\text{th}}$

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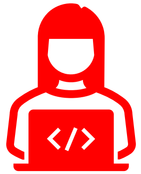
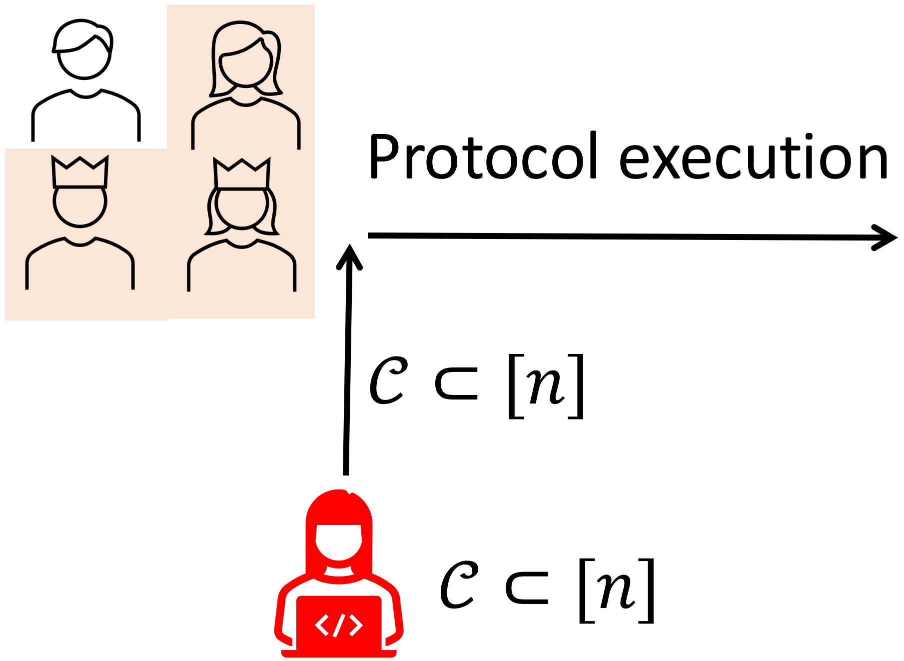


Adaptive $\mathcal{A}_{\text{th}}$

No restriction corruption timing

Static vs Adaptive Security



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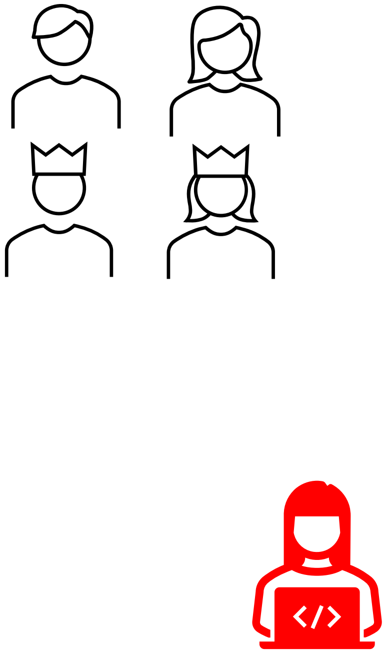

Adaptive $\mathcal{A}_{\text{th}}$

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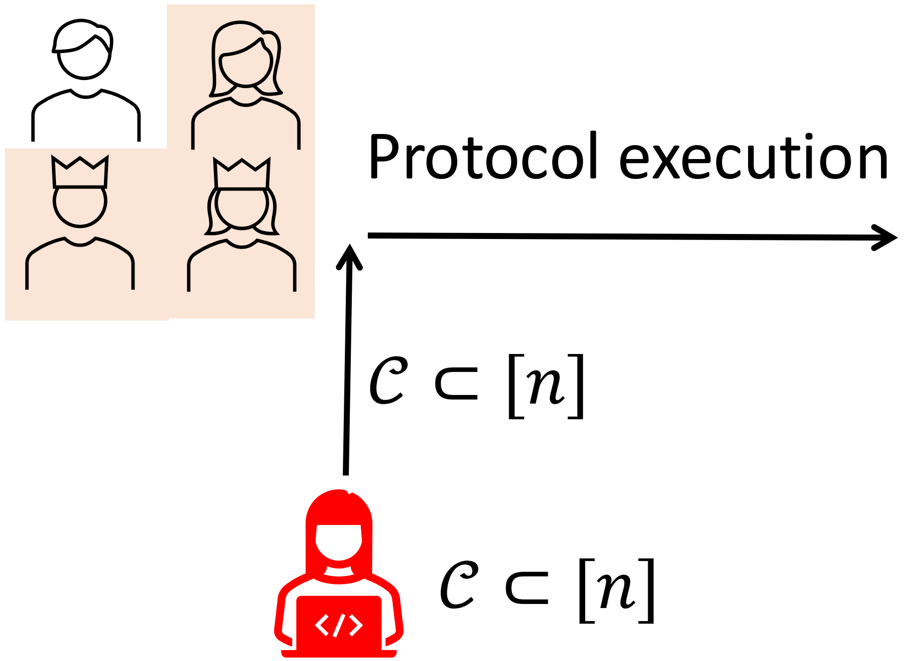
Static vs Adaptive Security



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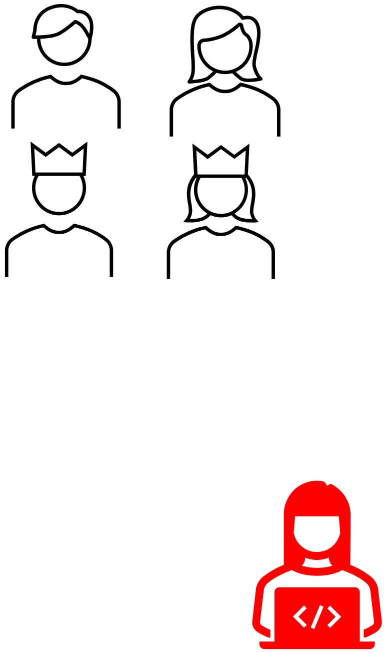
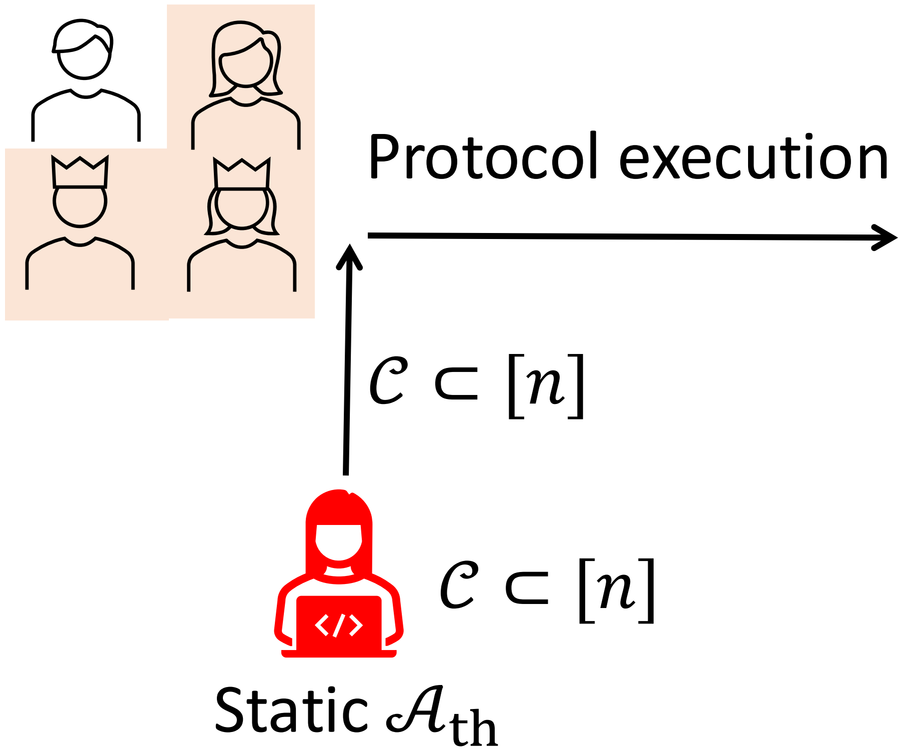


$\mathcal{C} = \{3\}$

Adaptive $\mathcal{A}_{\text{th}}$

No restriction corruption timing

Static vs Adaptive Security



Protocol execution

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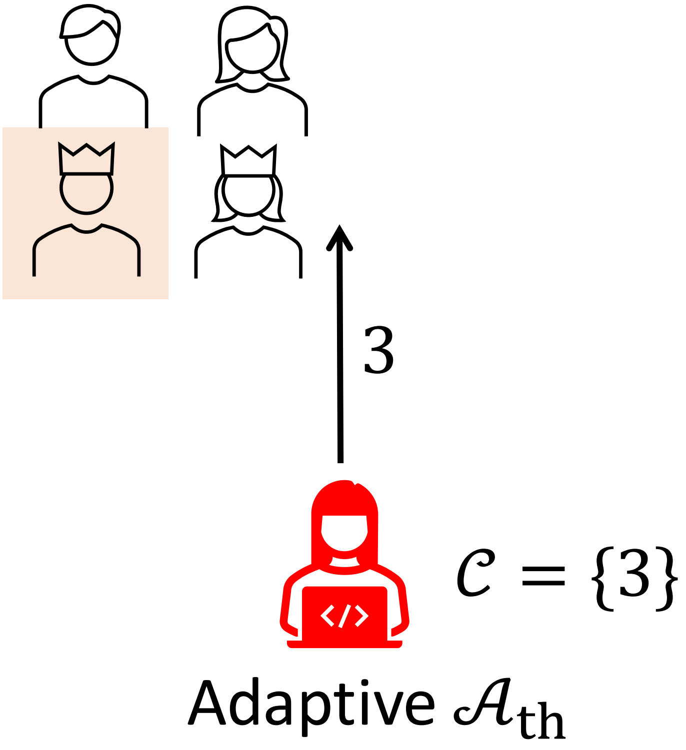


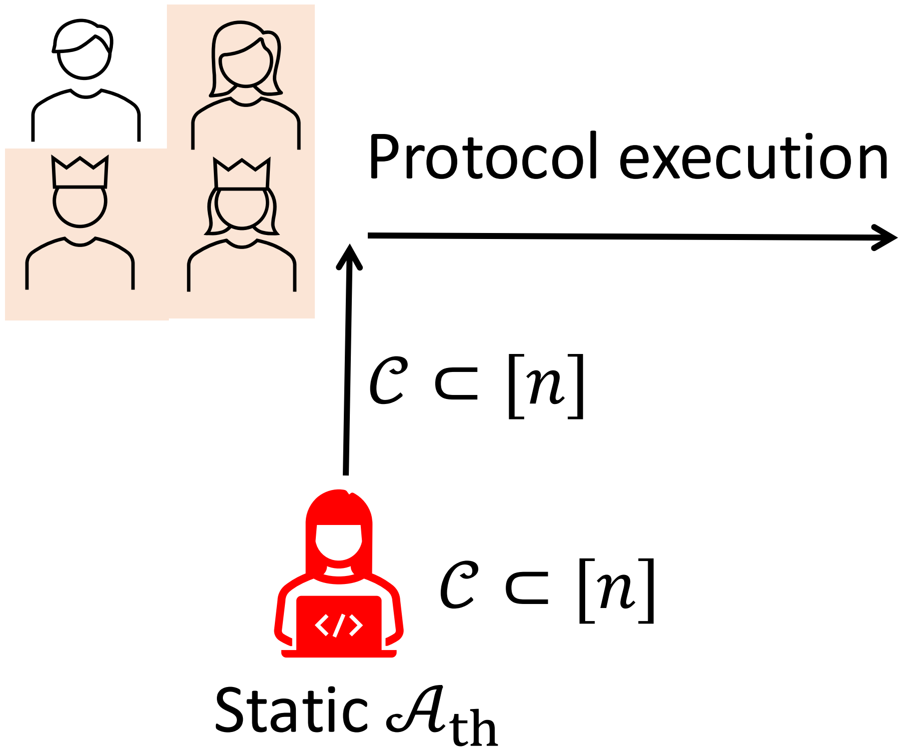
3

$\mathcal{C} = \{3\}$

Adaptive $\mathcal{A}_{\text{th}}$

No restriction corruption timing

Static vs Adaptive Security


Protocol execution

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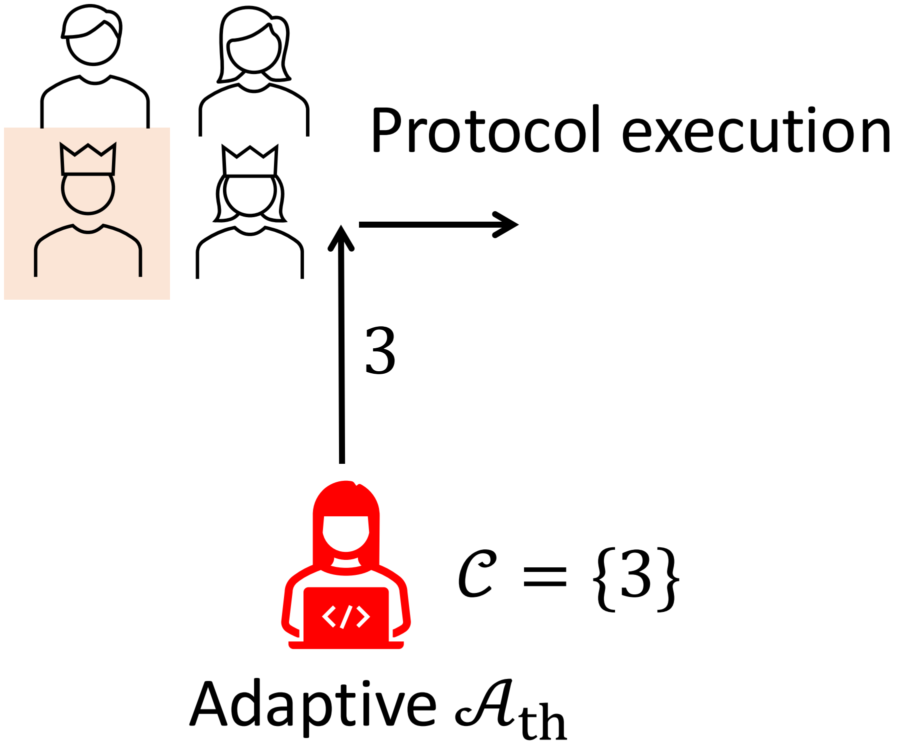
Protocol execution

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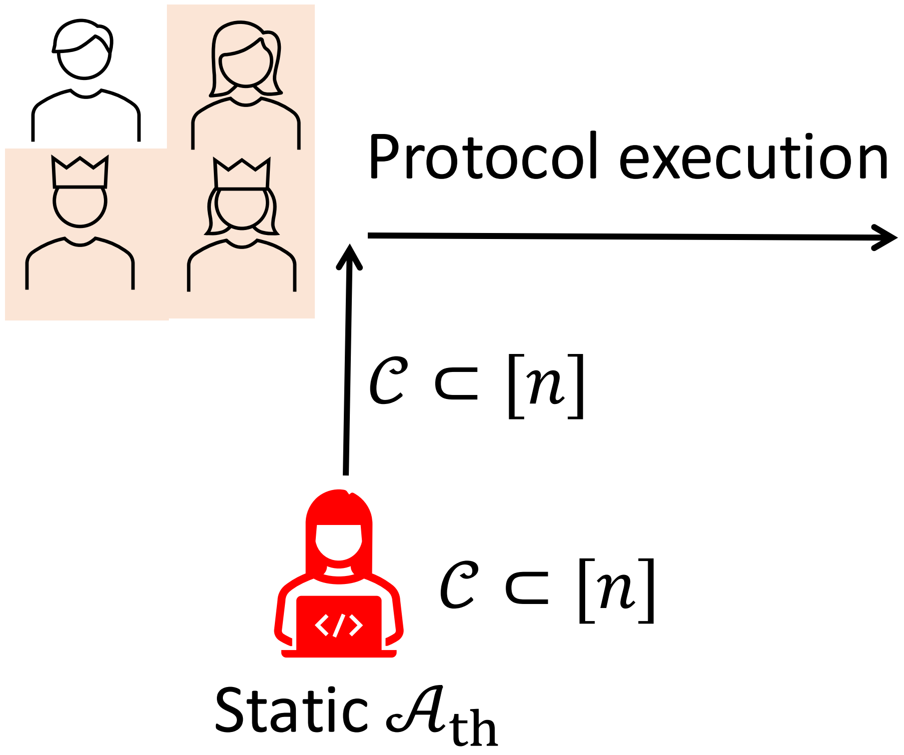
Static vs Adaptive Security



Protocol execution

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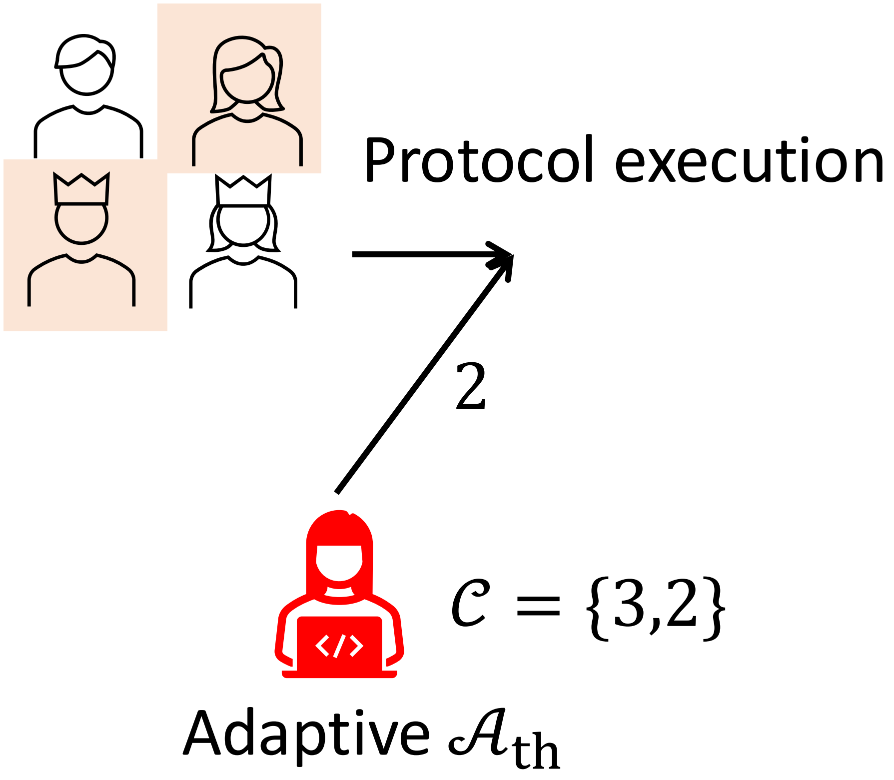


Protocol execution

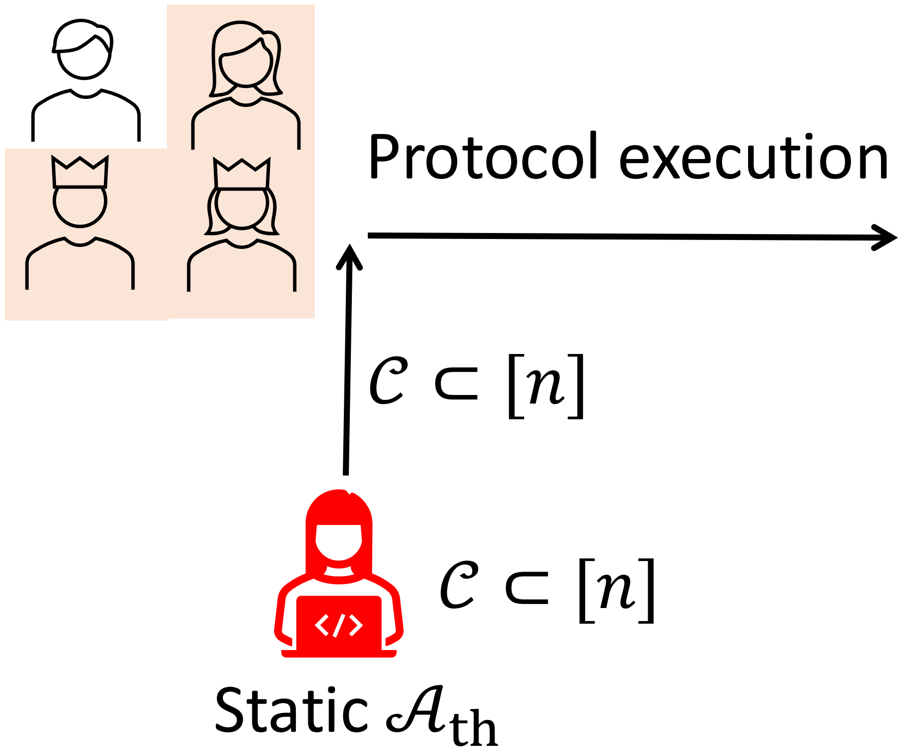
2

$\mathcal{C} = \{3,2\}$

Adaptive $\mathcal{A}_{\text{th}}$

No restriction corruption timing

Static vs Adaptive Security



Protocol execution

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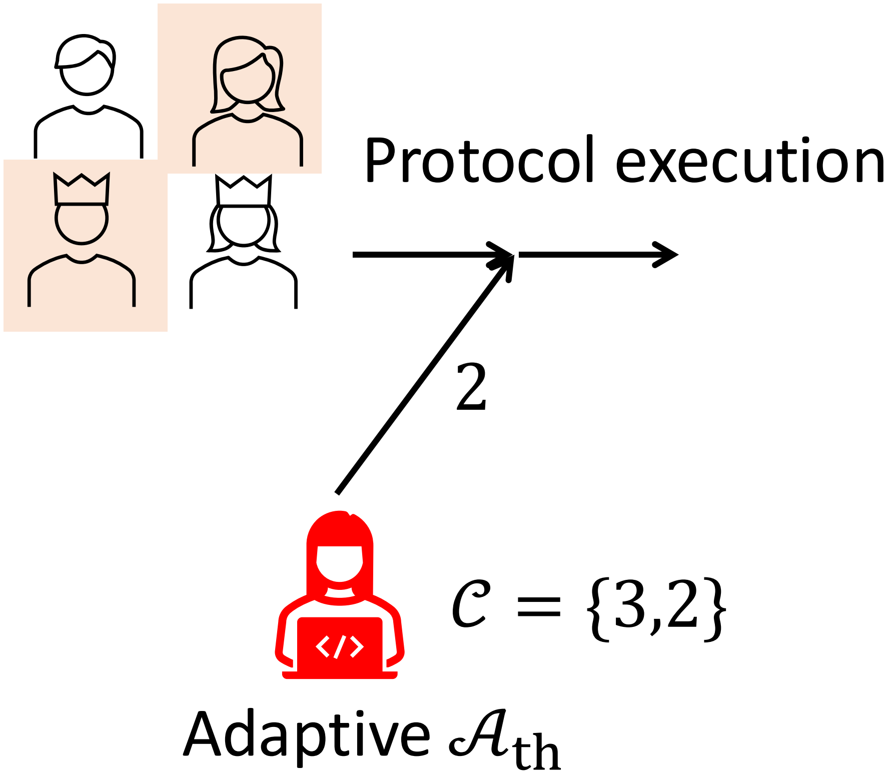

Protocol execution

2

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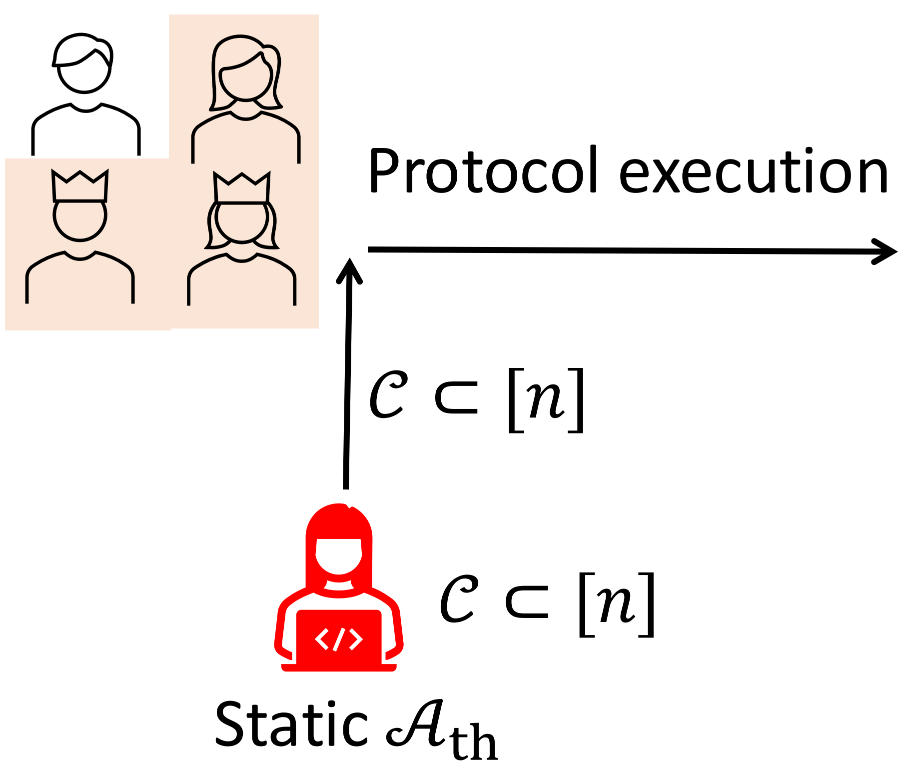
No restriction corruption timing

Static vs Adaptive Security



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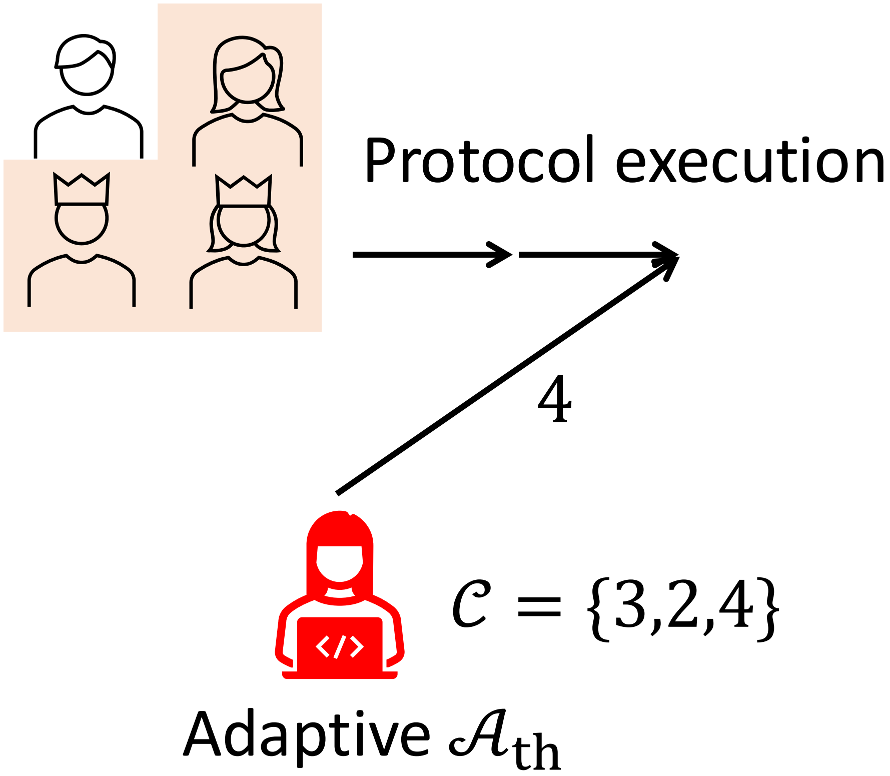

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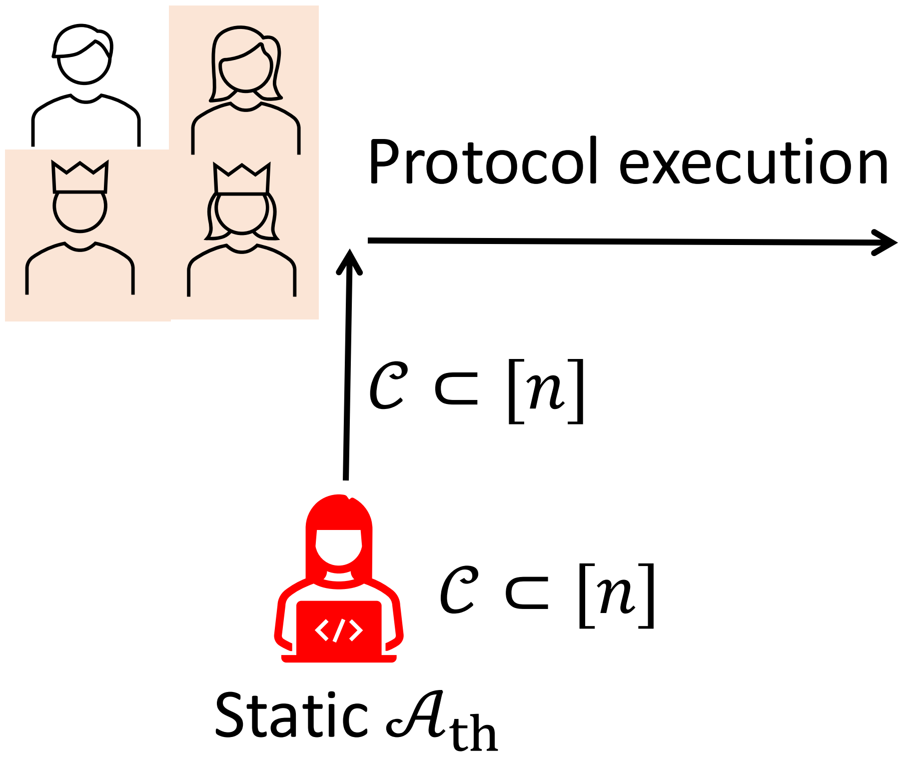

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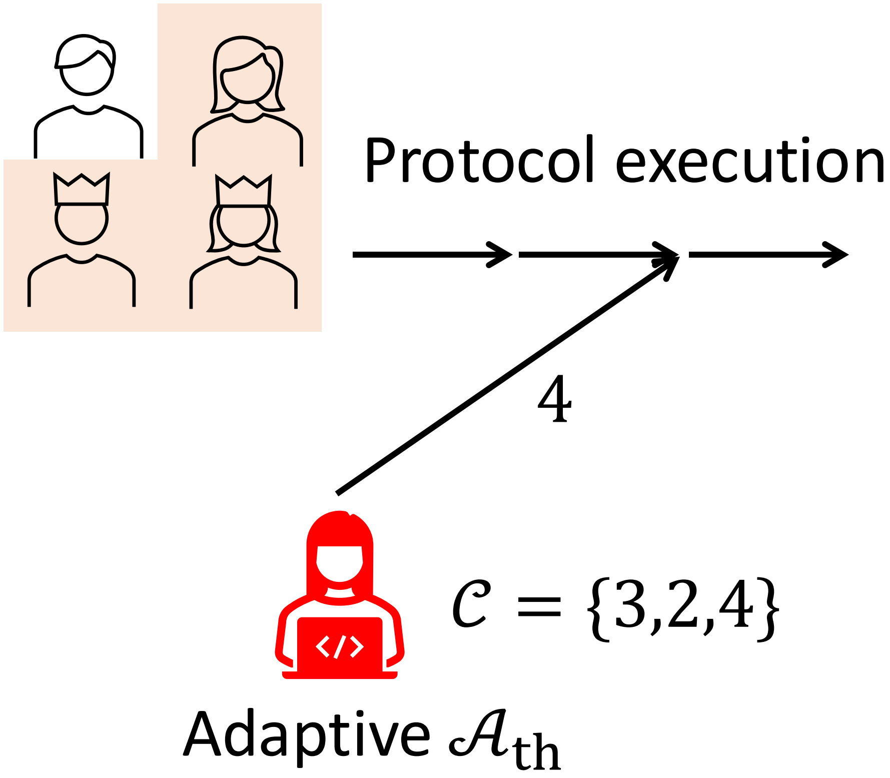


Protocol execution

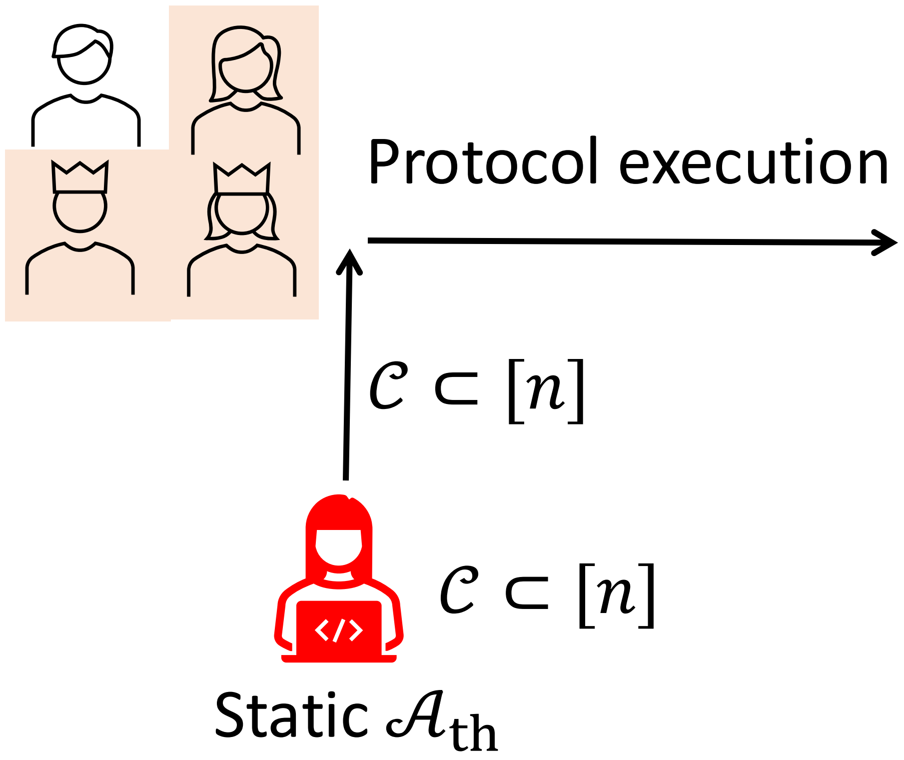
4

$\mathcal{C} = \{3,2,4\}$

Adaptive $\mathcal{A}_{\text{th}}$

No restriction corruption timing

Static vs Adaptive Security



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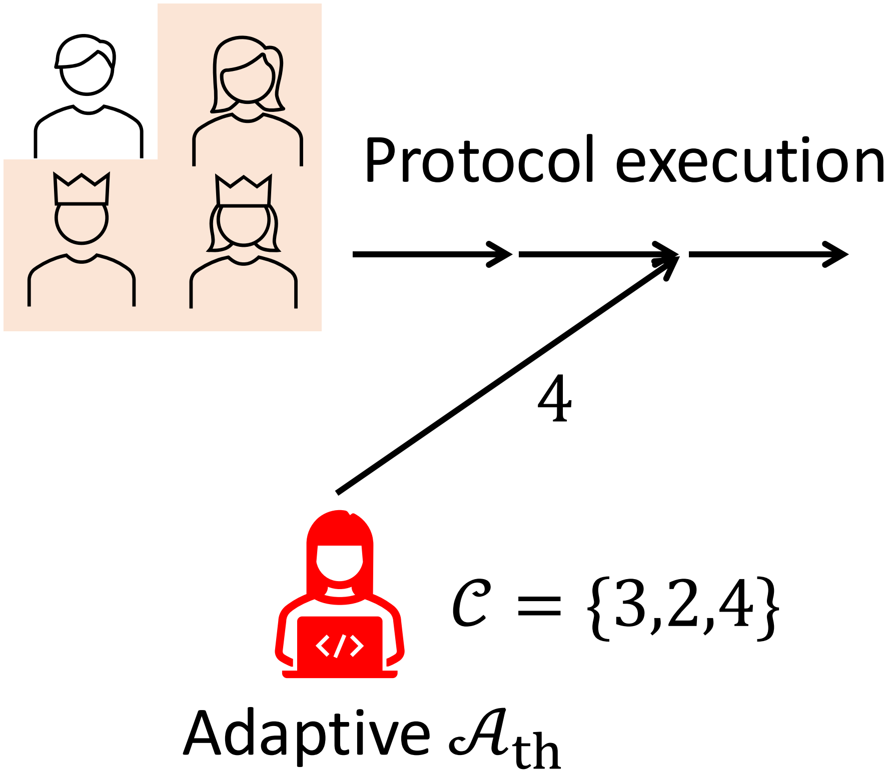

Protocol execution

4

$\mathcal{C} = \{3,2,4\}$

Adaptive $\mathcal{A}_{\text{th}}$

No restriction corruption timing

Adaptive corruption is more natural

Why Static Security?

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Has to do with known techniques of security reduction.

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- Minor modification to [Boldyreva03] while ensuring compatibility

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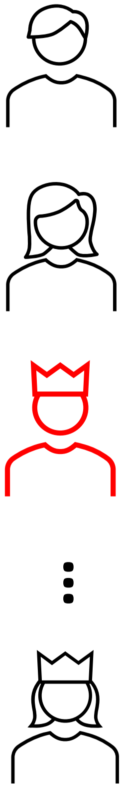
Our Approach:

- Minor modification to [Boldyreva03] while ensuring compatibility
- New proof techniques

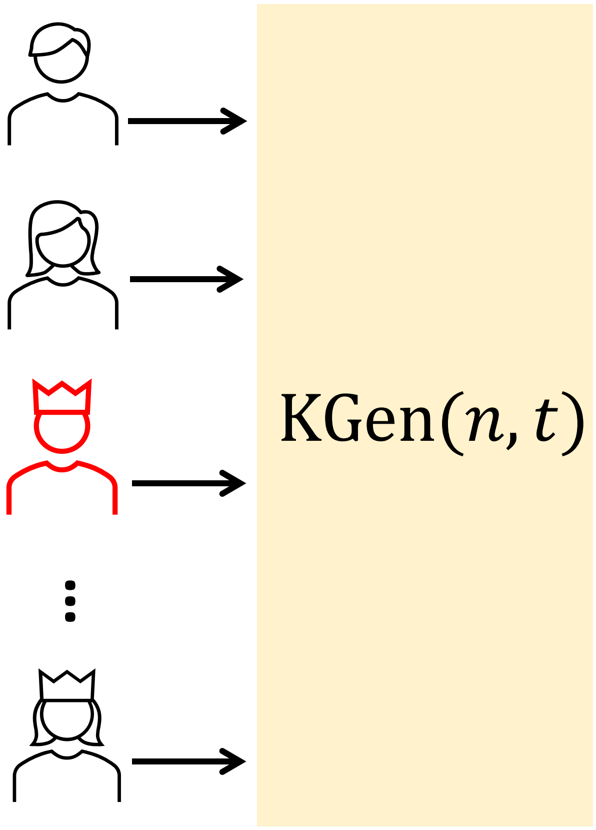
Our Protocol with Idealized Key Generation

Our Approach: Key Generation

Our Approach: Key Generation

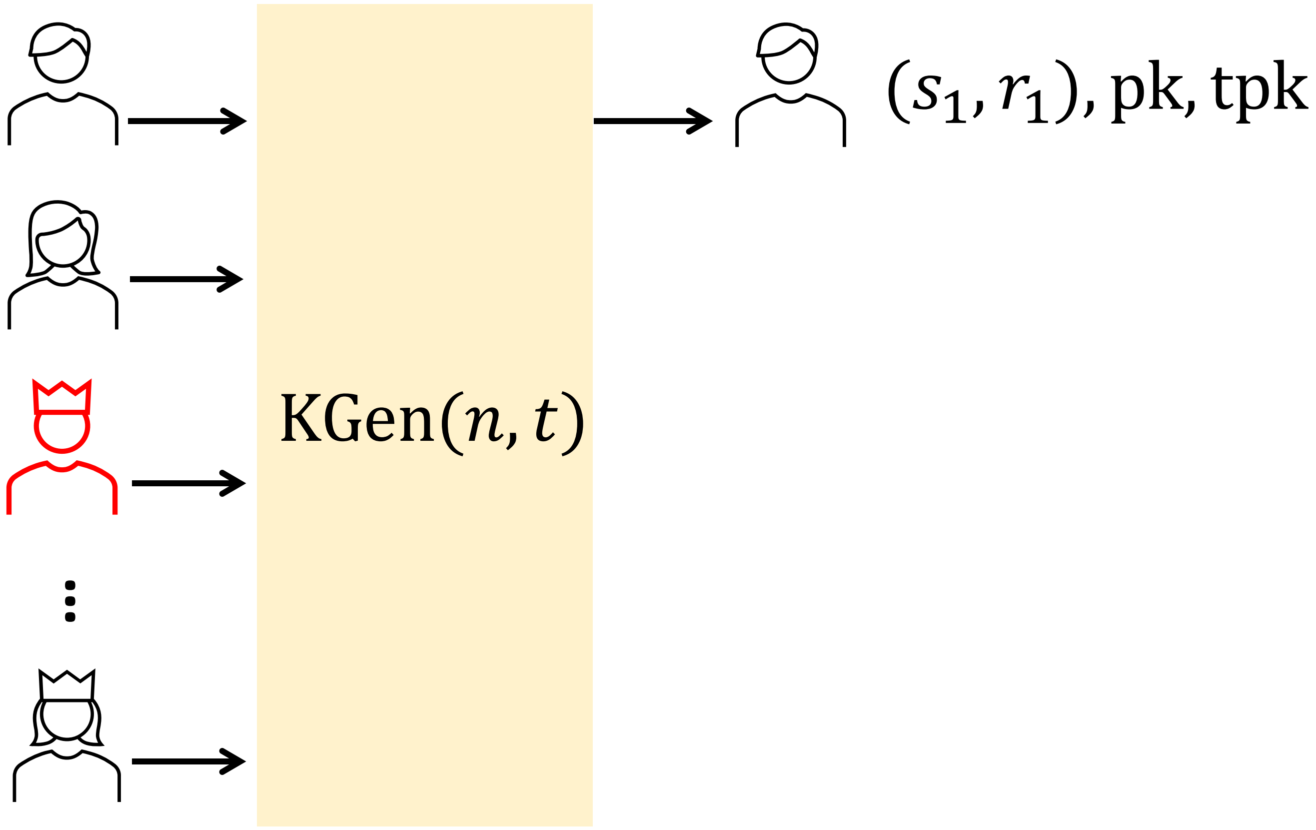


Our Approach: Key Generation



$\mathrm{KGen}(n, t)$

Our Approach: Key Generation



$\mathrm{KGen}(n, t)$

$(s_1, r_1), \mathrm{pk}, \mathrm{tpk}$

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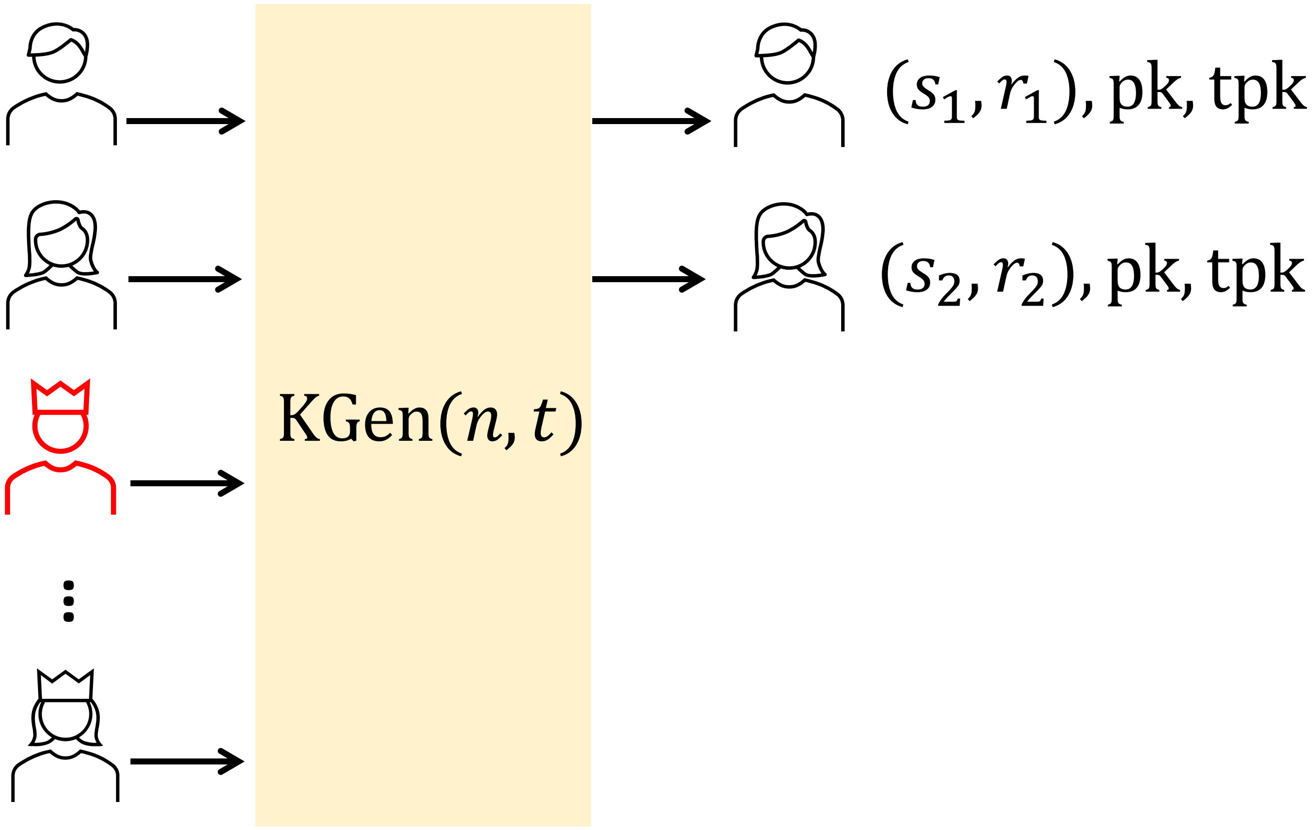

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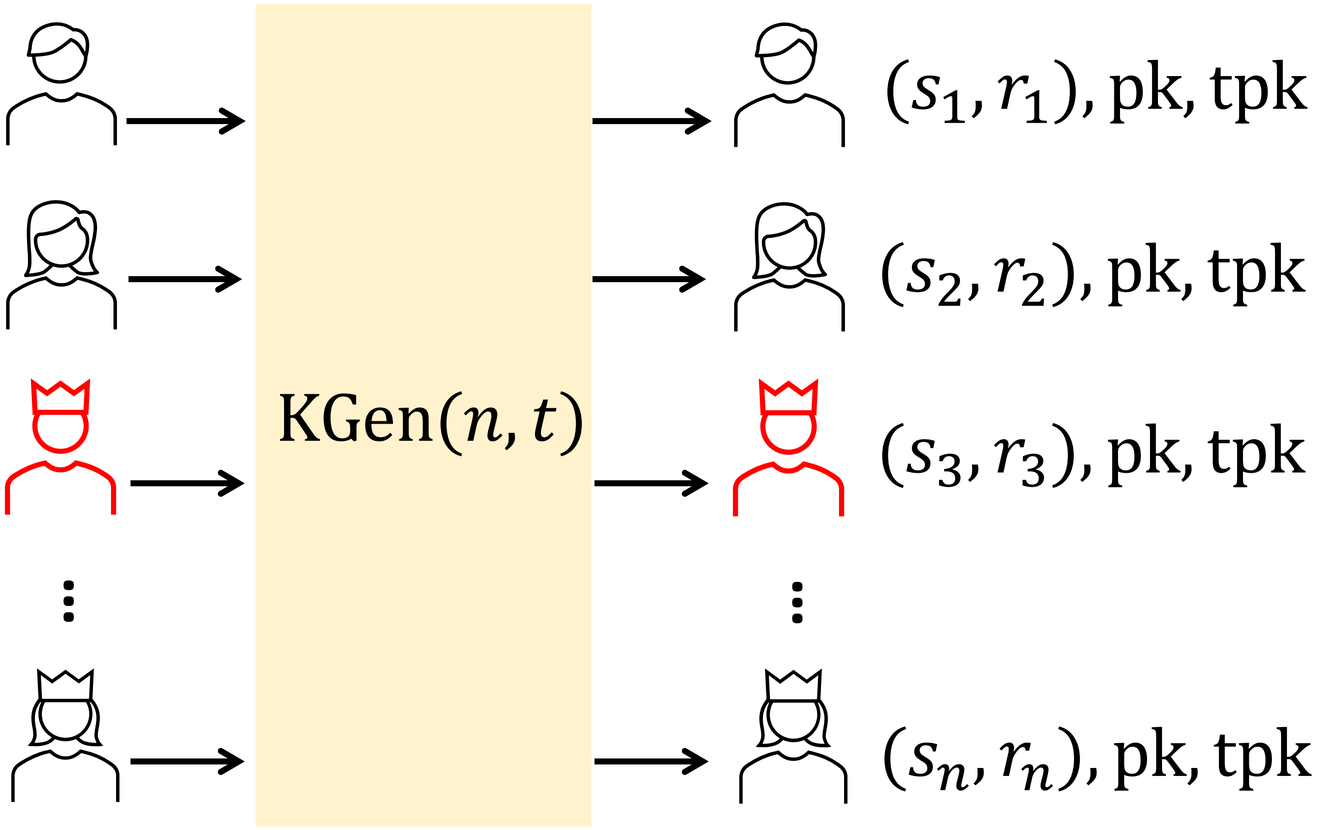

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$(s_2, r_2), \mathrm{pk}, \mathrm{tpk}$

$(s_3, r_3), \mathrm{pk}, \mathrm{tpk}$

$(s_n, r_n), \mathrm{pk}, \mathrm{tpk}$

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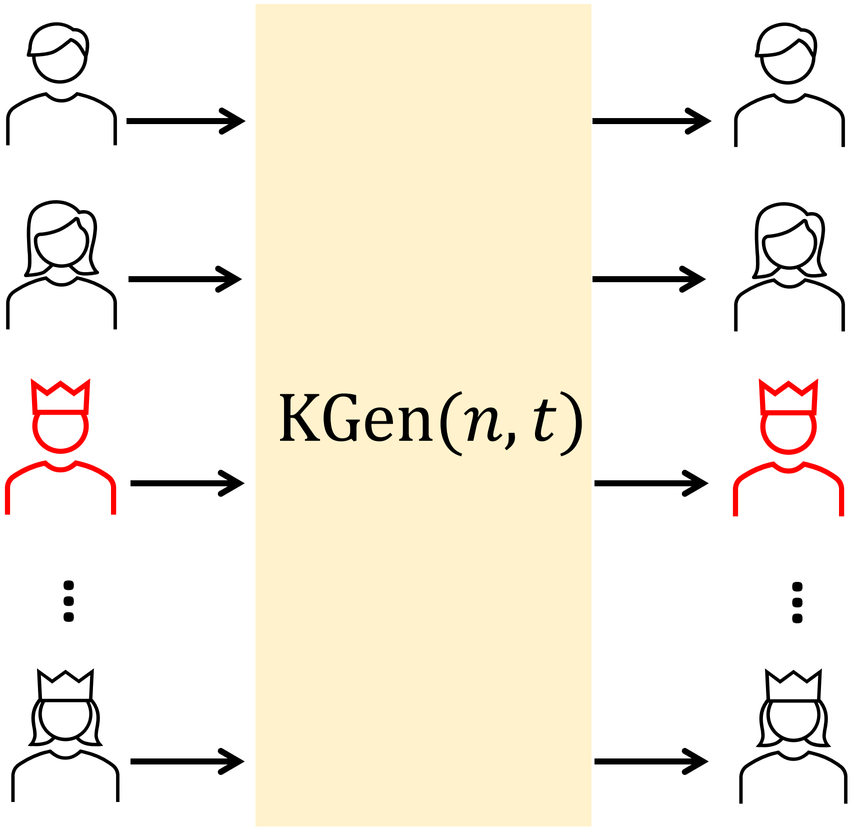

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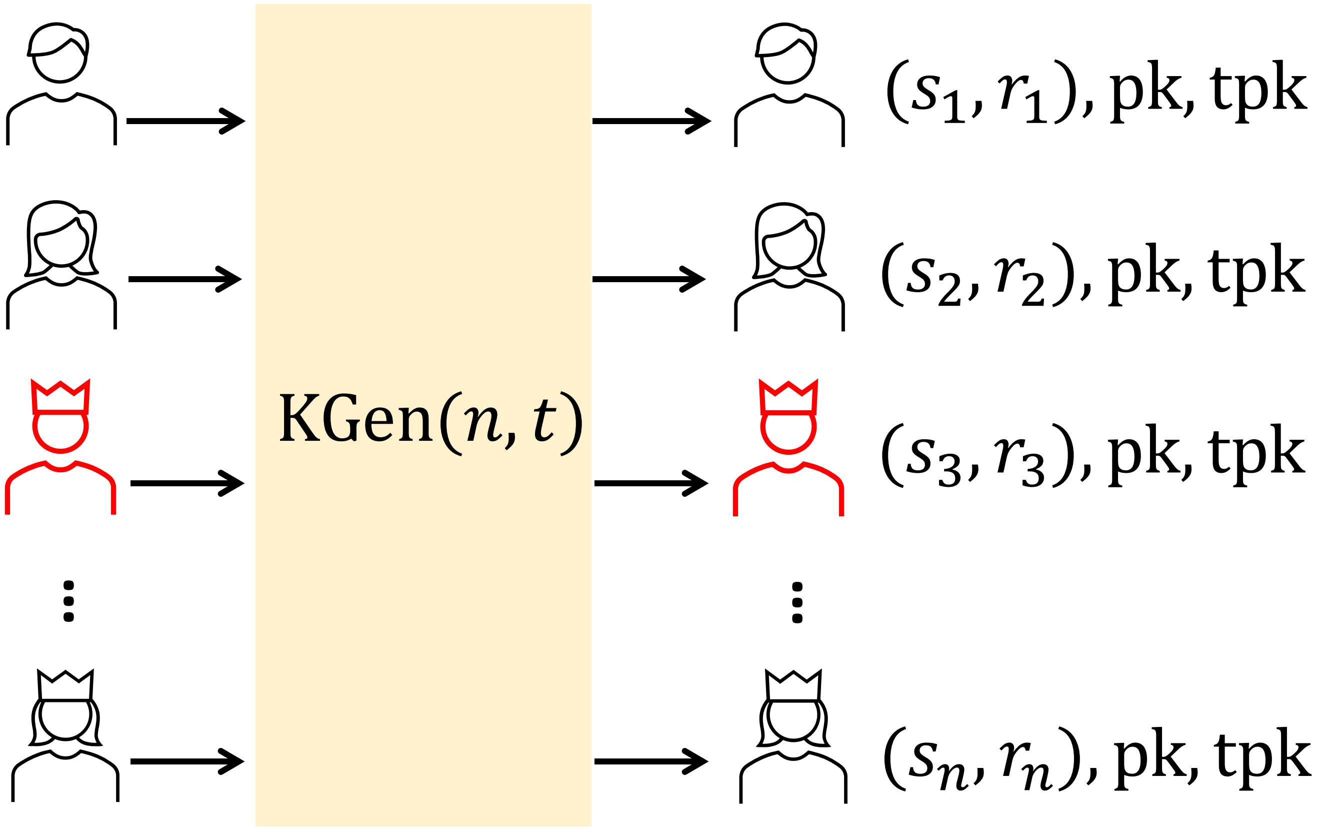
$\{s_1, \dots, s_n\} \leftarrow \mathrm{Share}(s)$

Our Approach: Key Generation



$\mathrm{KGen}(n, t)$

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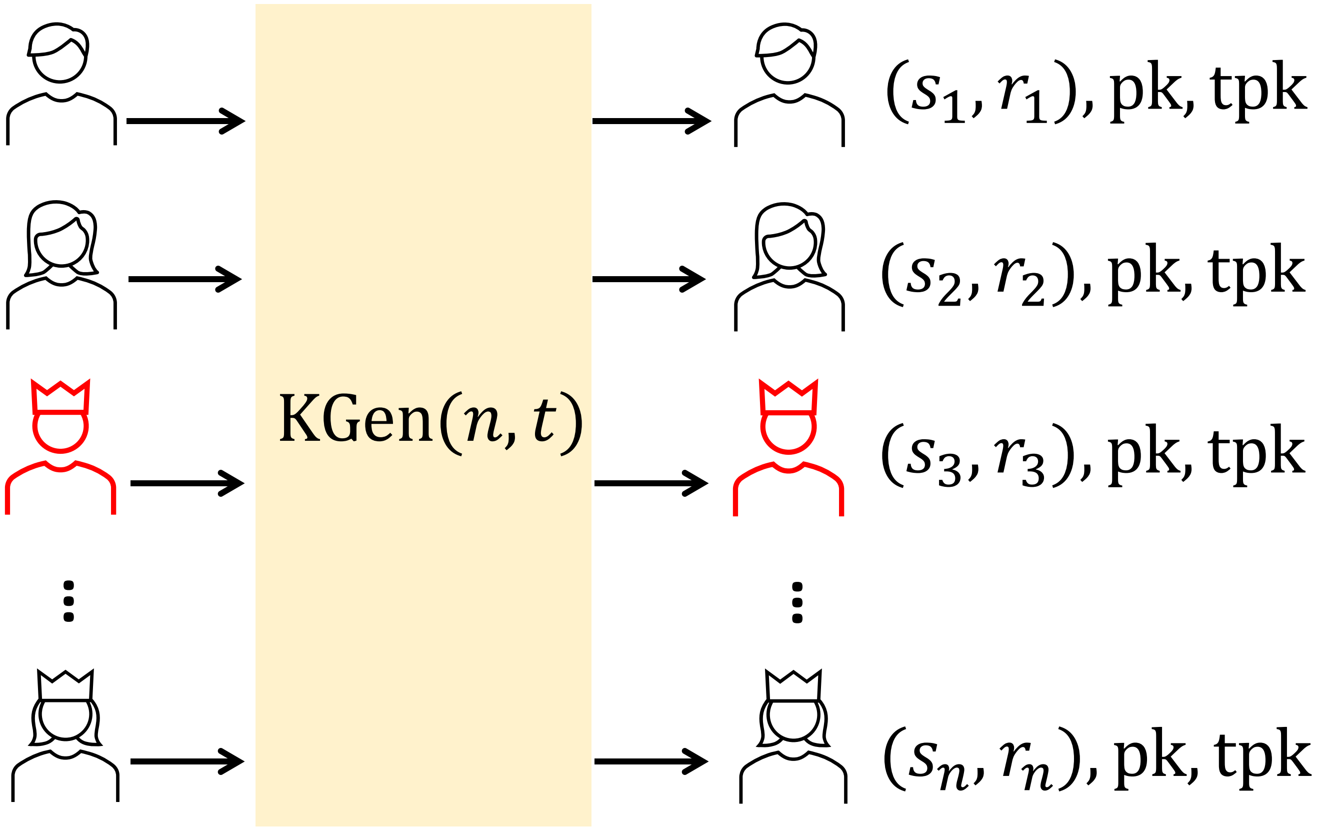

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$(s_n, r_n), \mathrm{pk}, \mathrm{tpk}$

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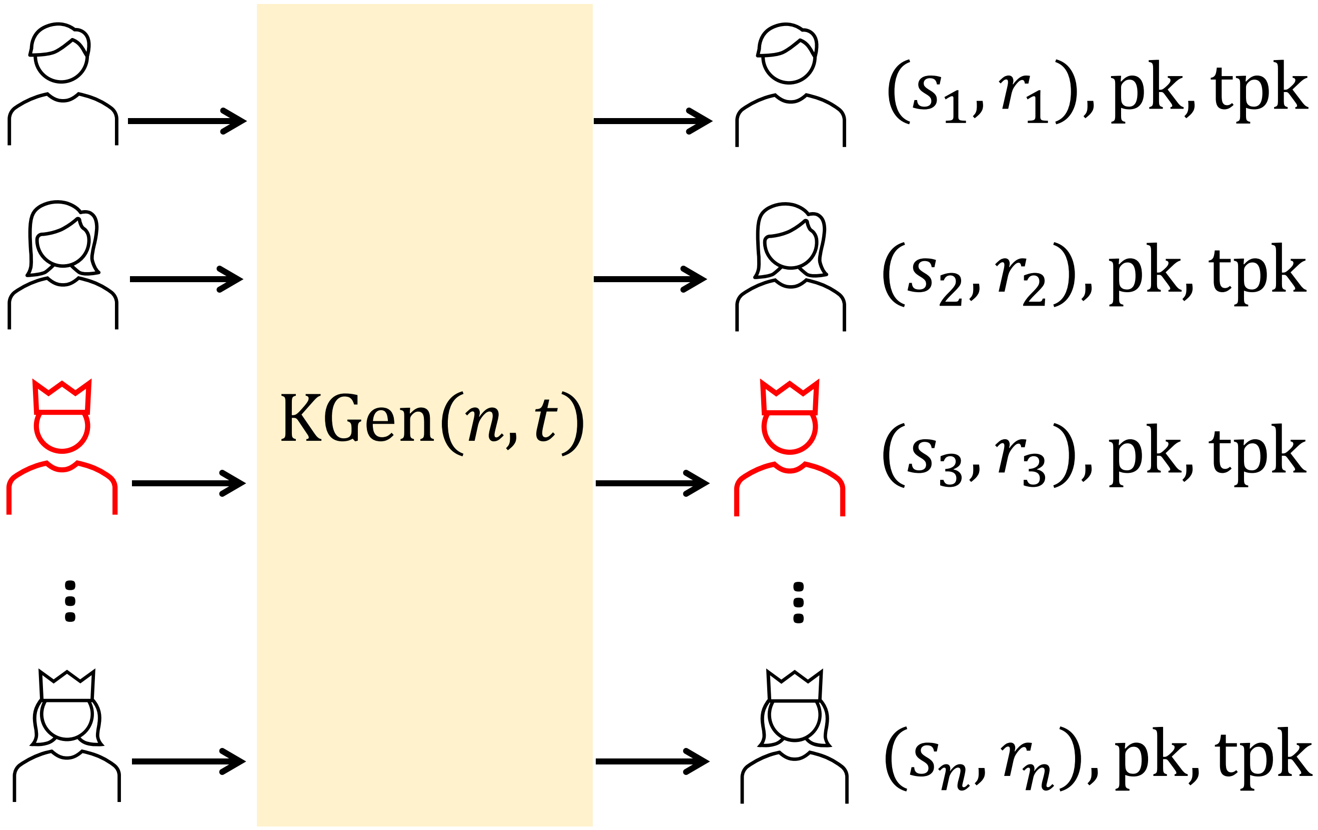
$$\mathrm{pk} = g^s$$

Our Approach: Key Generation



$\mathrm{KGen}(n, t)$

(s_1, r_1), pk, tpk

(s_2, r_2), pk, tpk

(s_3, r_3), pk, tpk

⋮

(s_n, r_n), pk, tpk

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$$\mathrm{pk} = g^s$$
$$\mathrm{tpk} = \{g^{s_1}h^{r_1} \ldots, g^{s_n}h^{r_n}\}$$

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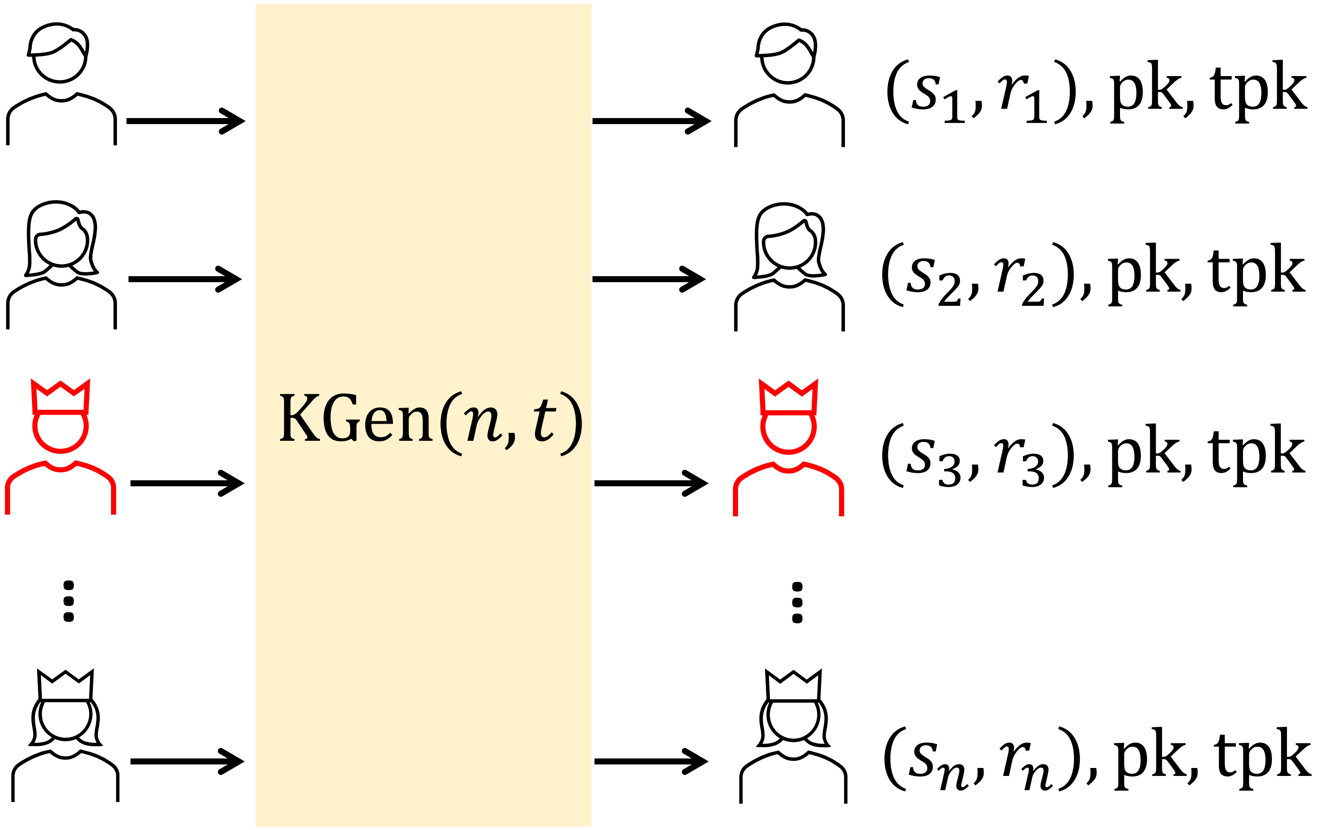

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$(s_n, r_n), \mathrm{pk}, \mathrm{tpk}$

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$$g, h \in \mathbb{G}$$

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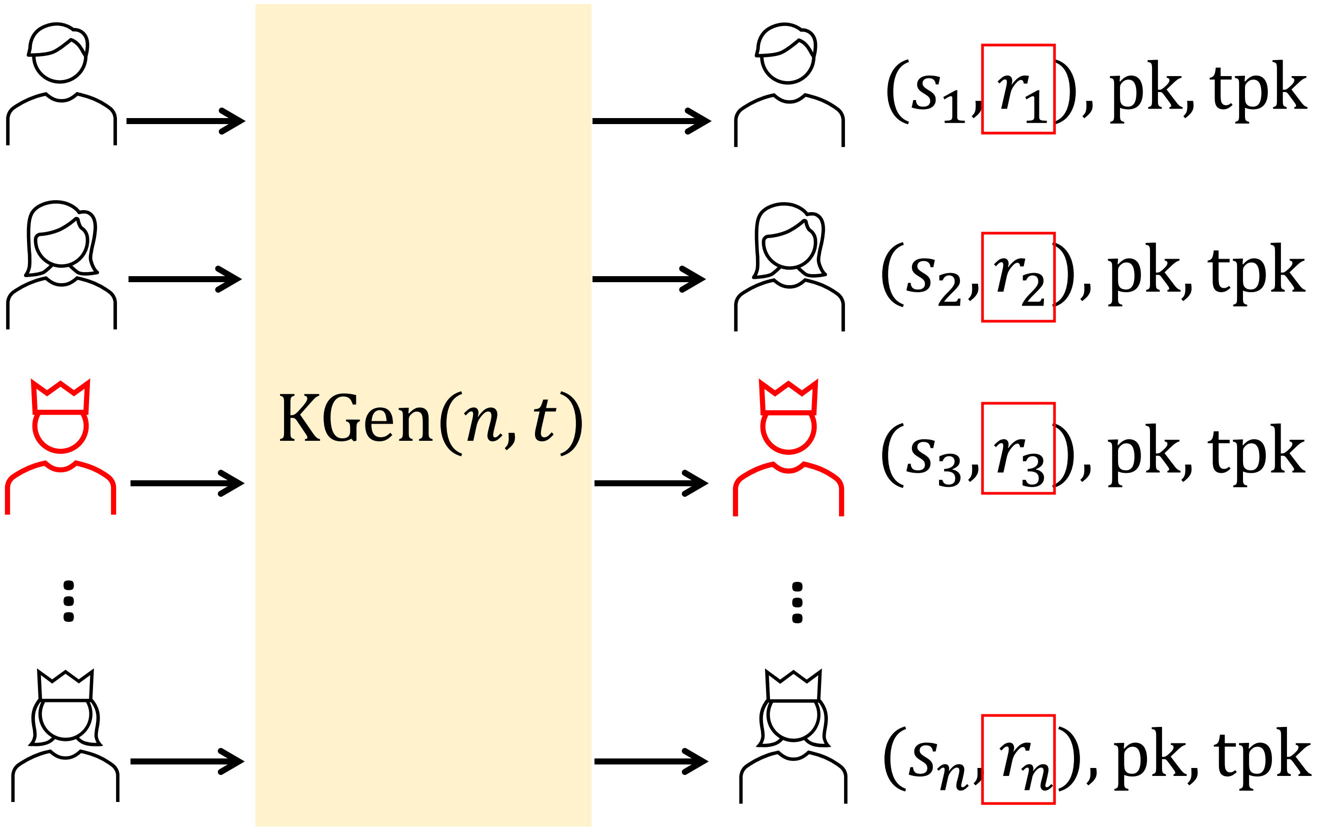

$\mathrm{KGen}(n, t)$

$(s_1, \boxed{r_1}), \mathrm{pk}, \mathrm{tpk}$

$(s_2, \boxed{r_2}), \mathrm{pk}, \mathrm{tpk}$

$(s_3, \boxed{r_3}), \mathrm{pk}, \mathrm{tpk}$

⋮

$(s_n, \boxed{r_n}), \mathrm{pk}, \mathrm{tpk}$

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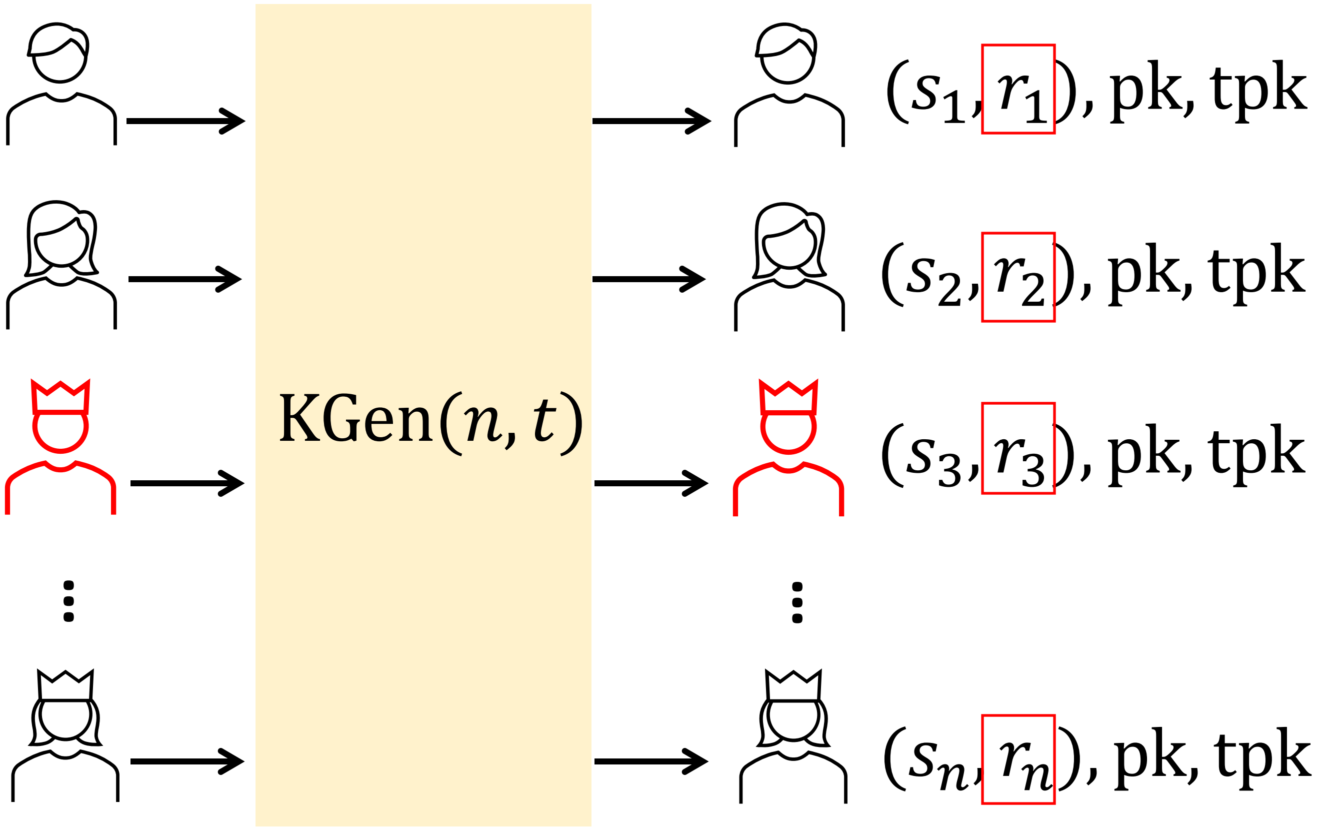

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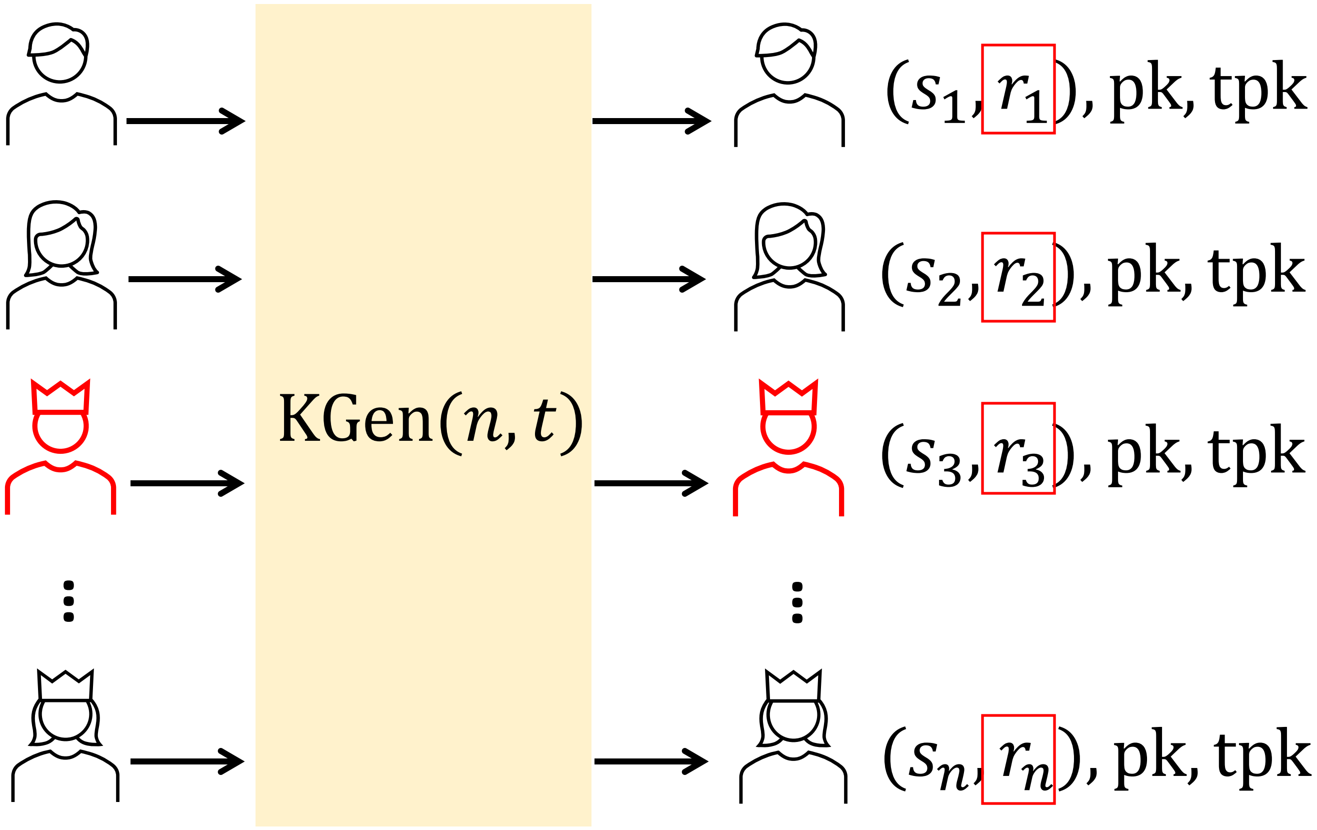

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$(s_3, \boxed{r_3}), \mathrm{pk}, \mathrm{tpk}$

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$$g, h \in \mathbb{G}$$

We describe a Distributed Key Generation (DKG) in the paper.

Our Approach: Signing

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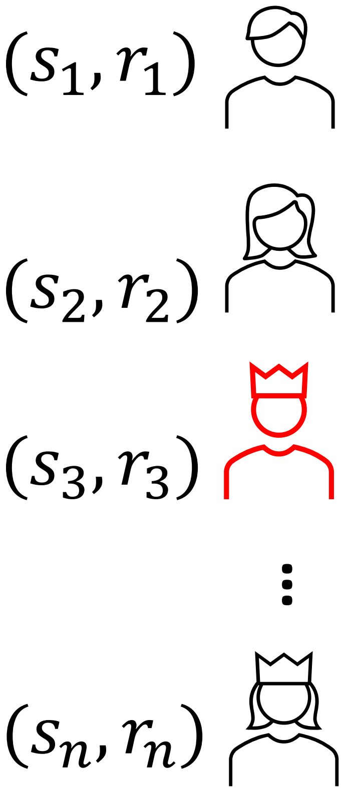


(s_1, r_1)

(s_2, r_2)

(s_3, r_3)

$\vdots$

(s_n, r_n)

Our Approach: Signing

(s_1, r_1) $\sigma_1 = \mathrm{H}(m)^{s_1}\widehat{\mathrm{H}}(m)^{r_1}$

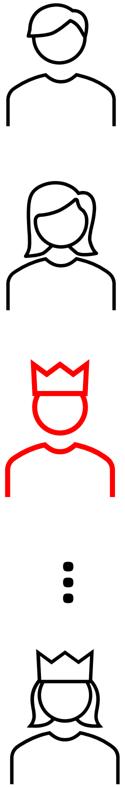
(s_2, r_2)

(s_3, r_3)

⋮

(s_n, r_n)

Our Approach: Signing



(s_1, r_1) $\sigma_1 = \mathrm{H}(m)^{s_1}\widehat{\mathrm{H}}(m)^{r_1}$

(s_2, r_2)

(s_3, r_3)

⋮

(s_n, r_n)

Our Approach: Signing

(s_1, r_1) $\sigma_1 = \mathrm{H}(m)^{s_1}\widehat{\mathrm{H}}(m)^{r_1}$

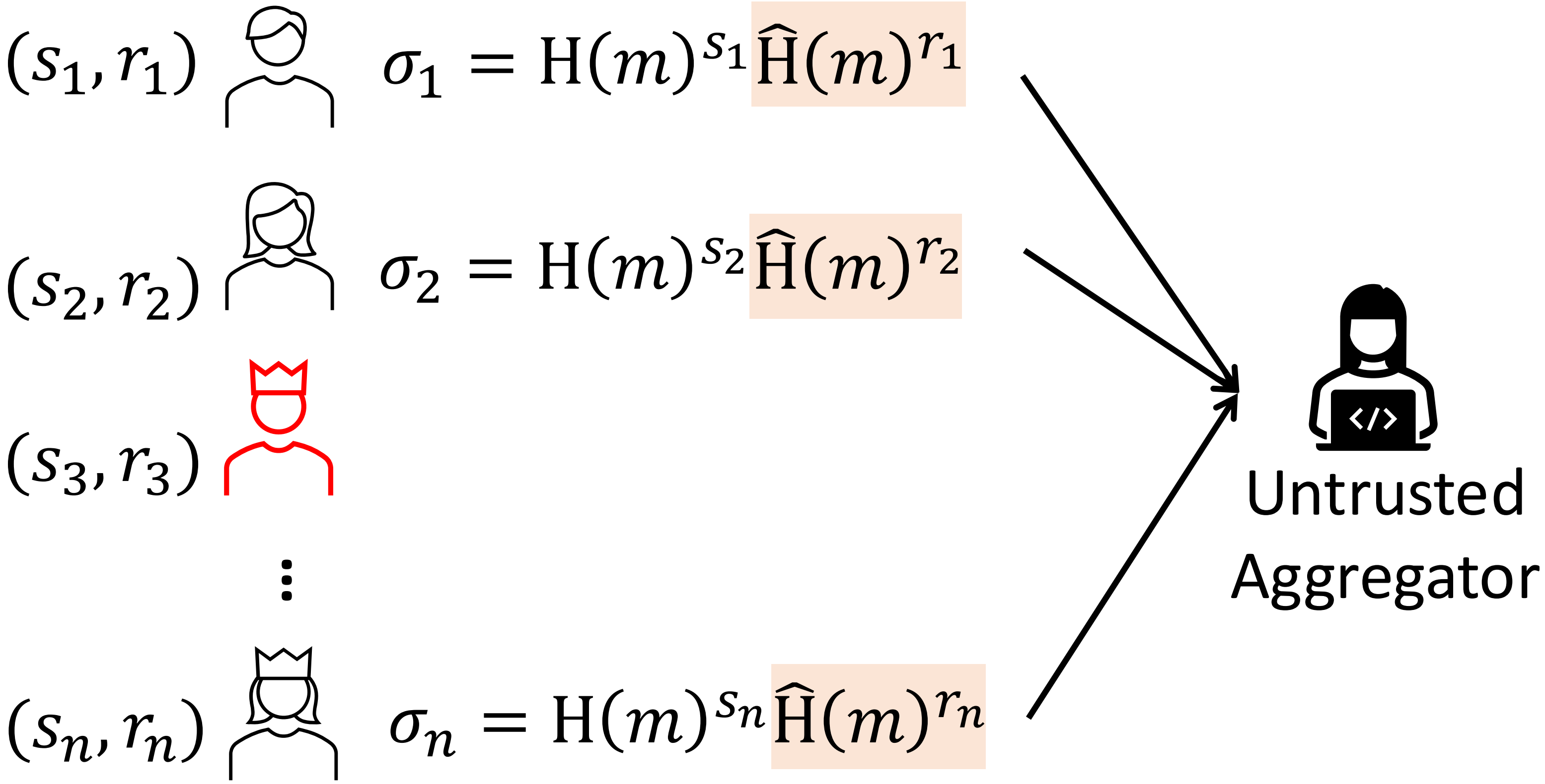
(s_2, r_2) $\sigma_2 = \mathrm{H}(m)^{s_2}\widehat{\mathrm{H}}(m)^{r_2}$

(s_3, r_3)

⋮

(s_n, r_n) $\sigma_n = \mathrm{H}(m)^{s_n}\widehat{\mathrm{H}}(m)^{r_n}$

Our Approach: Signing



(s_1, r_1) $\sigma_1 = \mathrm{H}(m)^{s_1}\widehat{\mathrm{H}}(m)^{r_1}$

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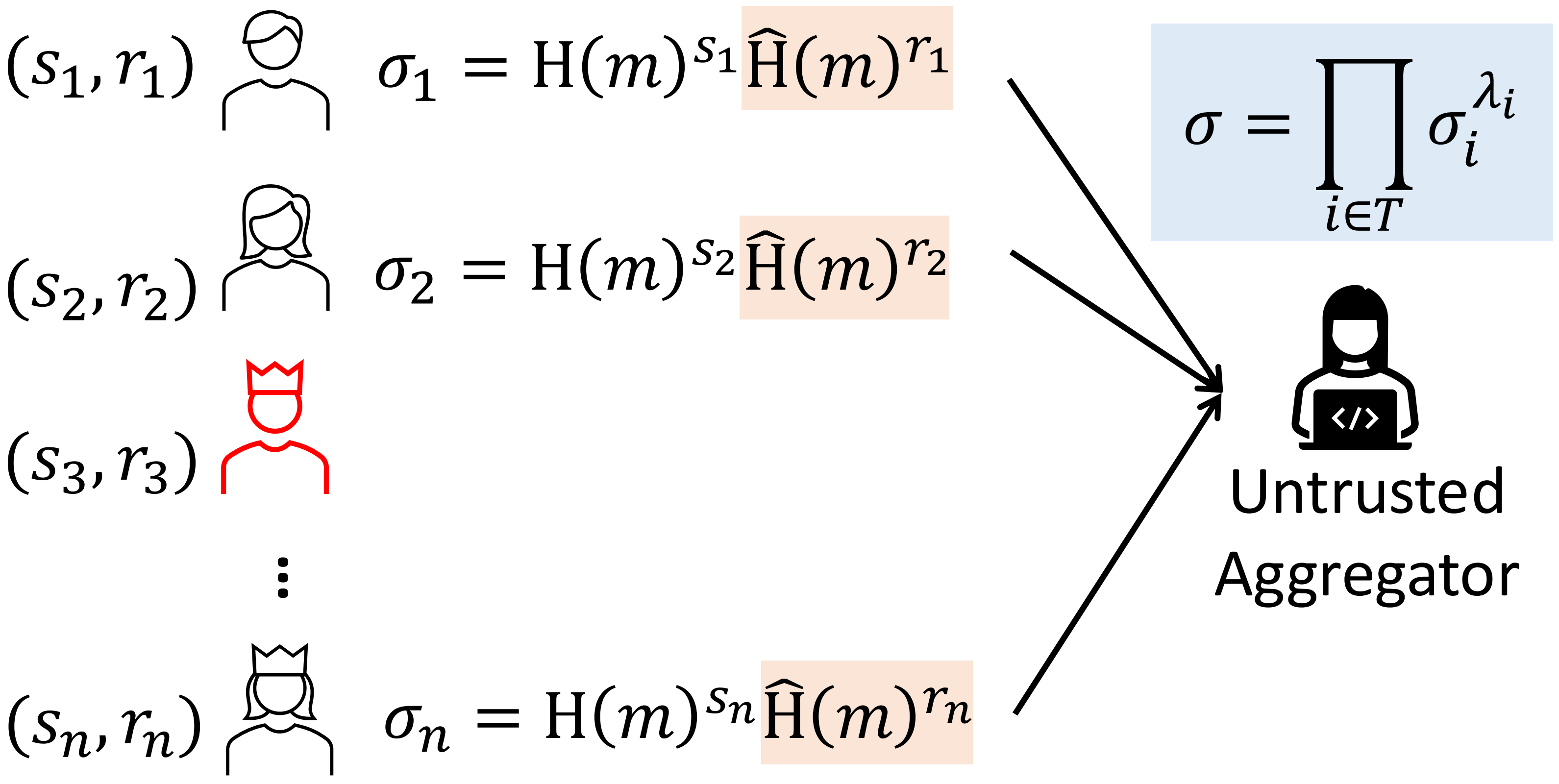
(s_3, r_3)

⋮

(s_n, r_n) $\sigma_n = \mathrm{H}(m)^{s_n}\widehat{\mathrm{H}}(m)^{r_n}$

Untrusted Aggregator

Our Approach: Signing



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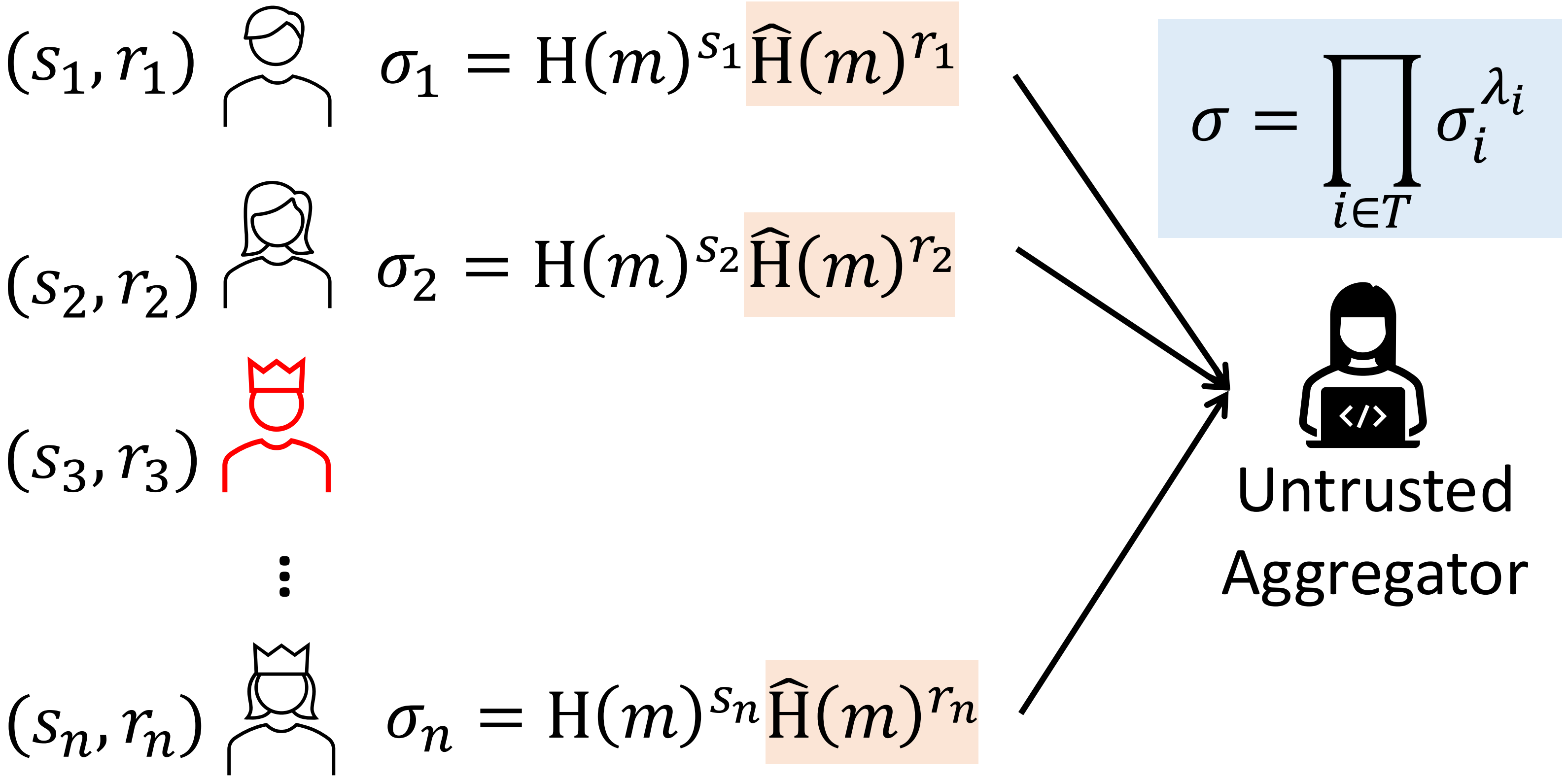
$$\sigma = \prod_{i \in T} \sigma_i^{\lambda_i}$$

Untrusted Aggregator

Our Approach: Signing

λ_i are the Lagrange coefficients.



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Untrusted Aggregator

Our Approach: Signing

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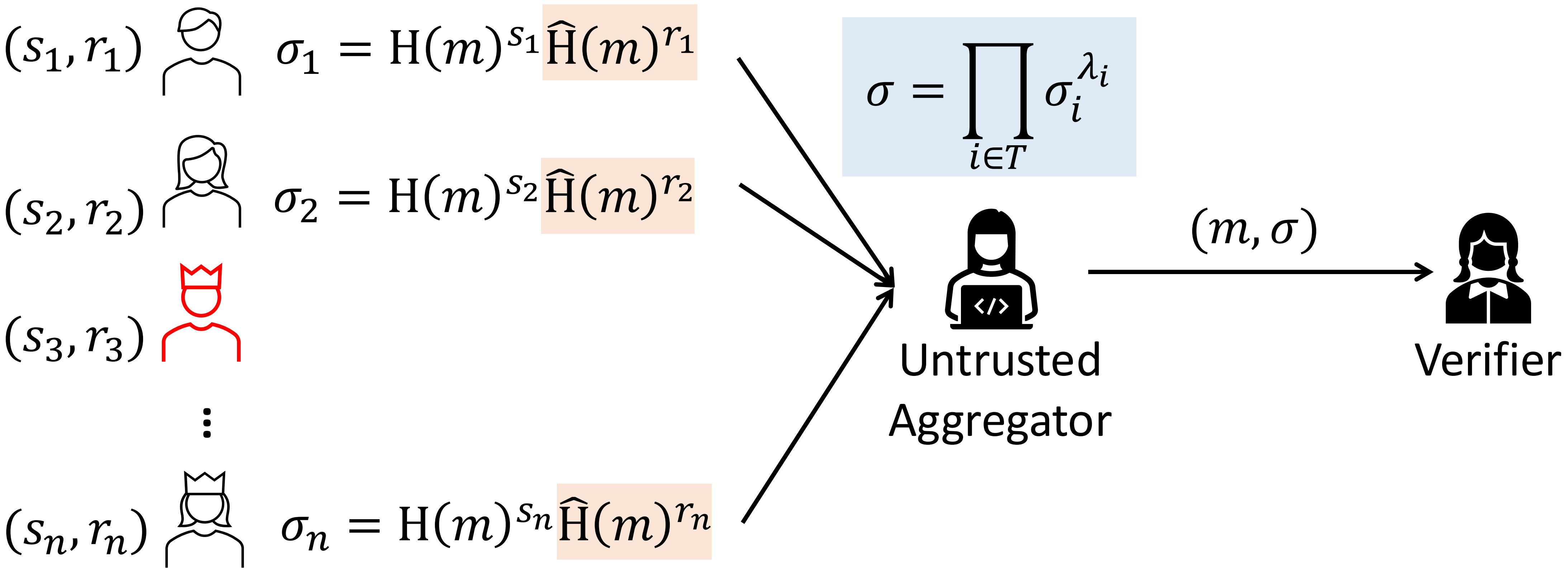

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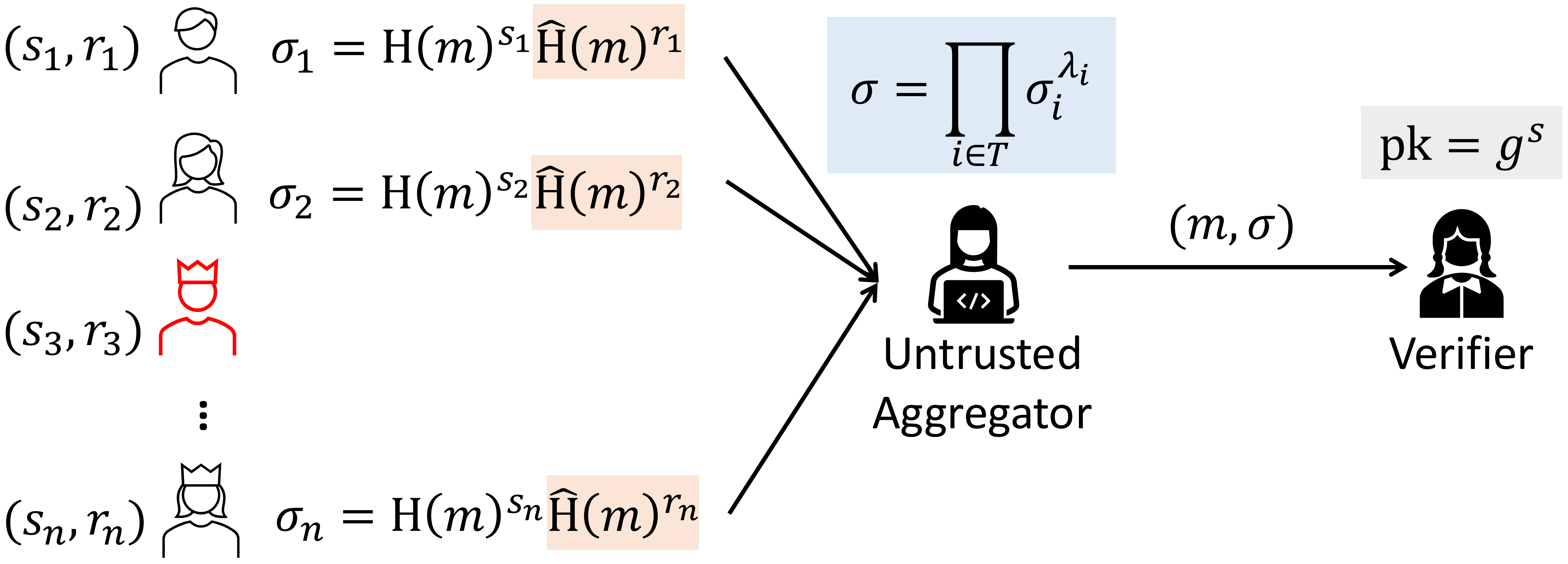
(m, σ)

Untrusted Aggregator

Verifier

Our Approach: Signing

λ_i are the Lagrange coefficients.



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$\mathrm{pk} = g^s$

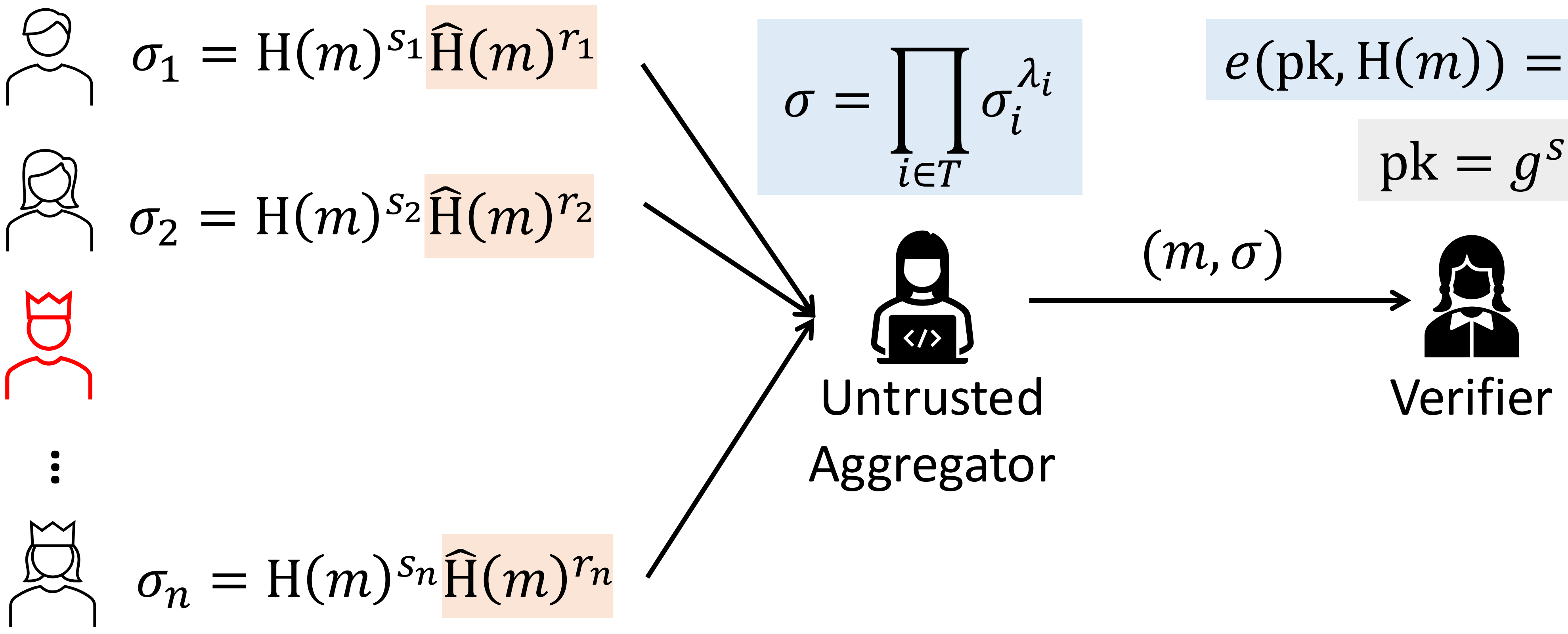
(m, σ)

Untrusted Aggregator

Verifier

Our Approach: Signing

λ_i are the Lagrange coefficients.



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Untrusted Aggregator

(m, σ)

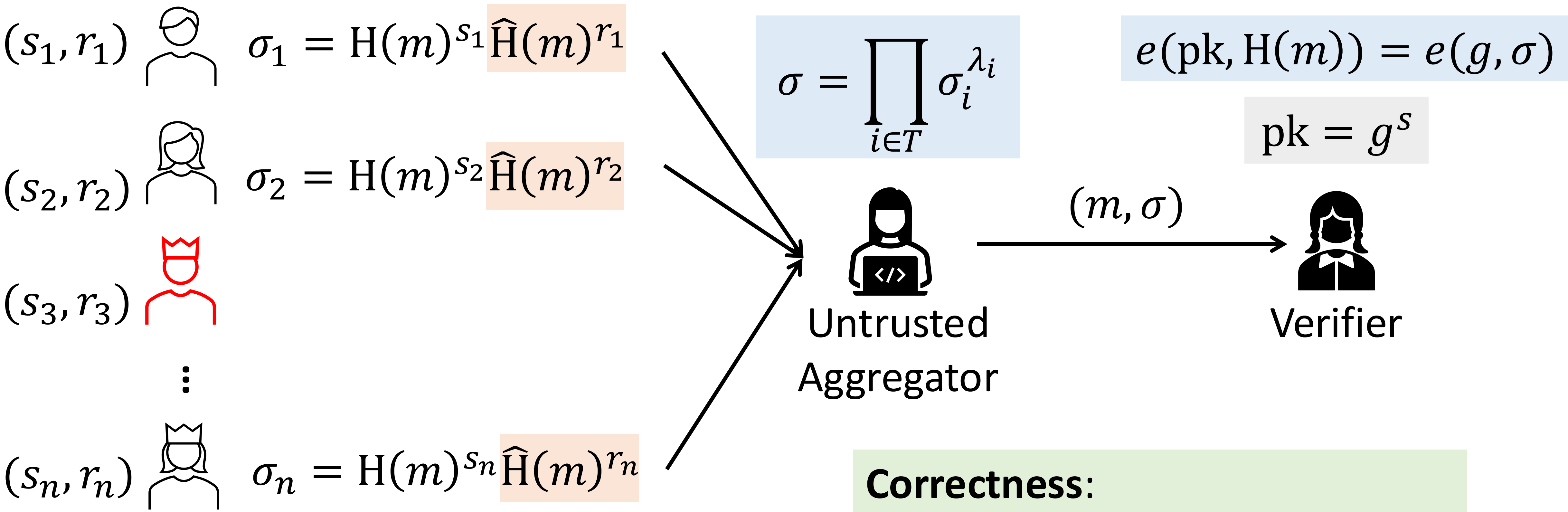
$$e(\mathrm{pk}, \mathrm{H}(m)) = e(g, \sigma)$$

$$\mathrm{pk} = g^s$$

Verifier

Our Approach: Signing

λ_i are the Lagrange coefficients.



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$$\sigma = \prod_{i \in T} \sigma_i^{\lambda_i}$$

$$e(\mathrm{pk}, \mathrm{H}(m)) = e(g, \sigma)$$

$$\mathrm{pk} = g^s$$

(m, σ)

Untrusted Aggregator

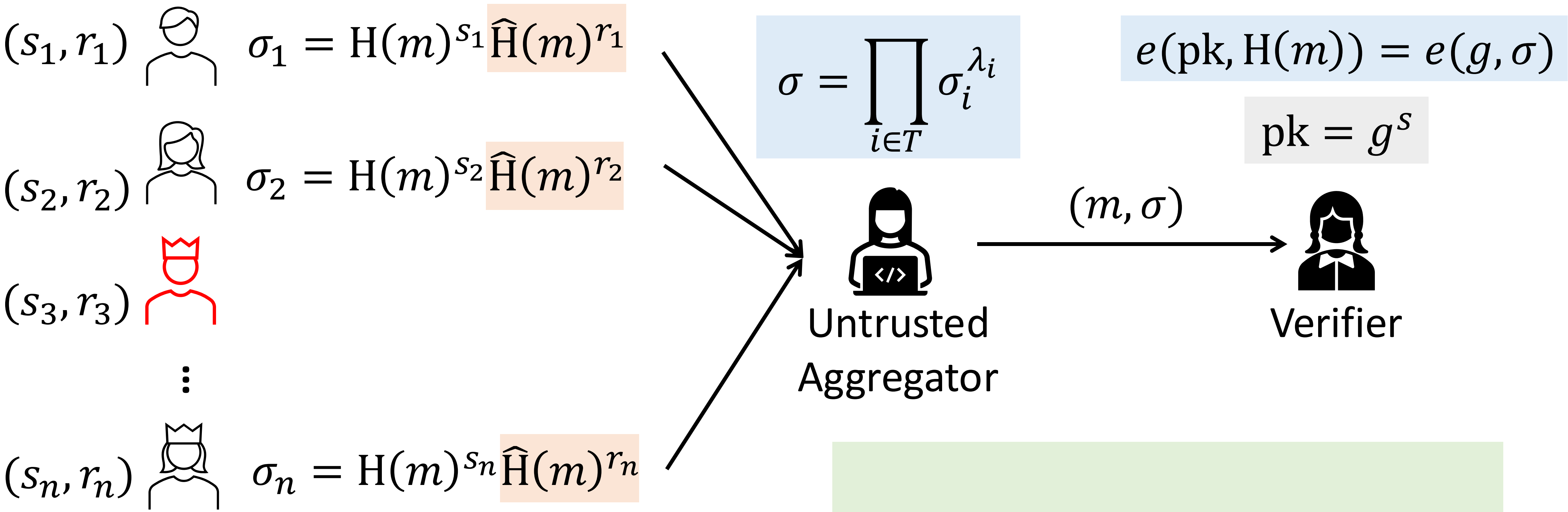
Verifier

Correctness:

$$\sigma = \mathrm{H}(m)^s\widehat{\mathrm{H}}(m)^{r=0} = \mathrm{H}(m)^s$$

Our Approach: Properties

λ_i are the Lagrange coefficients.



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$$e(\mathrm{pk}, \mathrm{H}(m)) = e(g, \sigma)$$

$$\mathrm{pk} = g^s$$

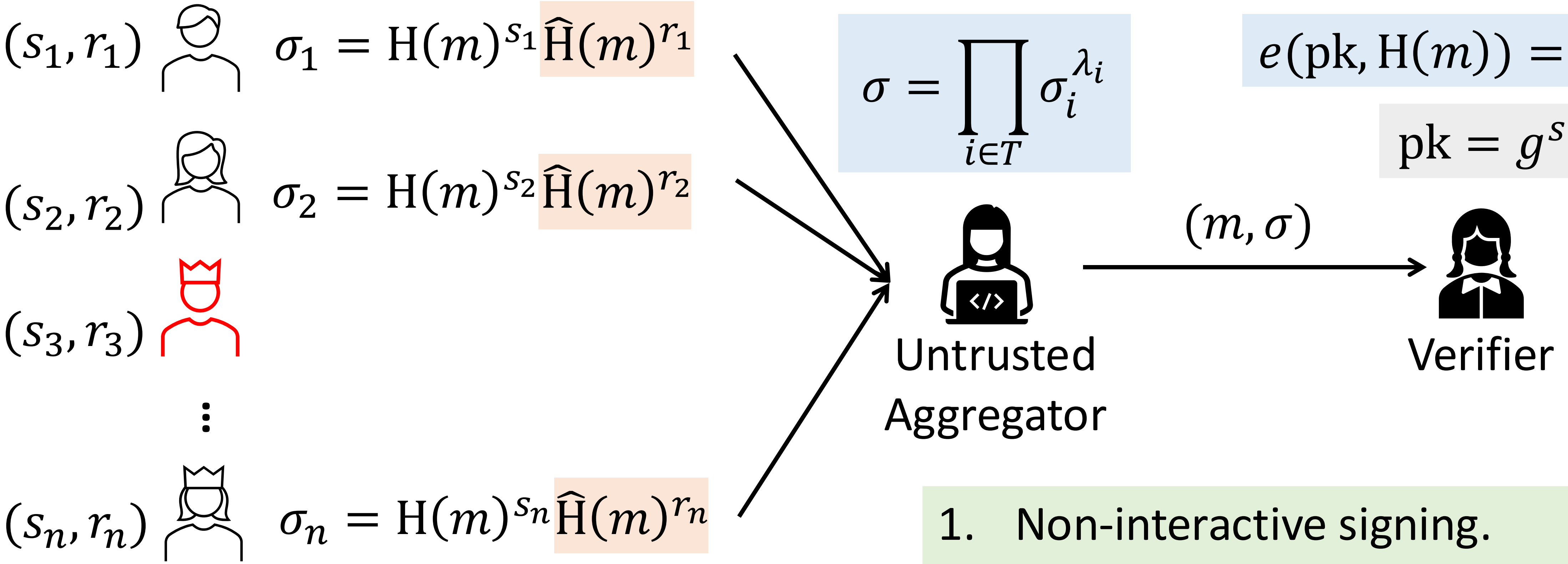
(m, σ)

Untrusted Aggregator

Verifier

Our Approach: Properties

λ_i are the Lagrange coefficients.



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$$\sigma = \prod_{i \in T} \sigma_i^{\lambda_i}$$

(m, σ)

Untrusted Aggregator

Verifier

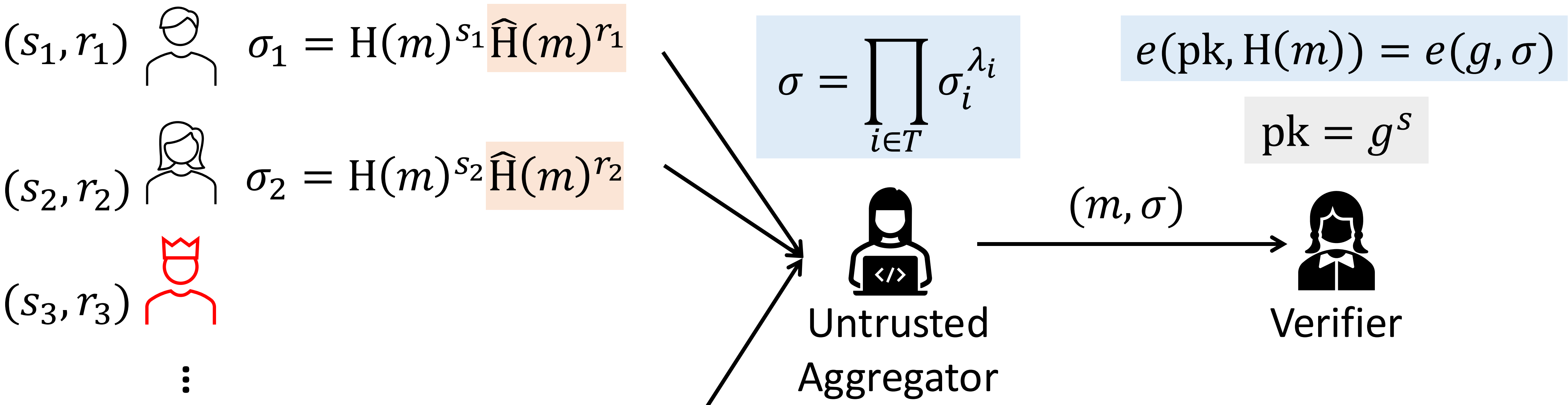
$$e(\mathrm{pk}, \mathrm{H}(m)) = e(g, \sigma)$$

$$\mathrm{pk} = g^s$$

1. Non-interactive signing.

Our Approach: Properties

λ_i are the Lagrange coefficients.



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$$\sigma = \prod_{i \in T} \sigma_i^{\lambda_i}$$

$$e(\mathrm{pk}, \mathrm{H}(m)) = e(g, \sigma)$$

$$\mathrm{pk} = g^s$$

(m, σ)

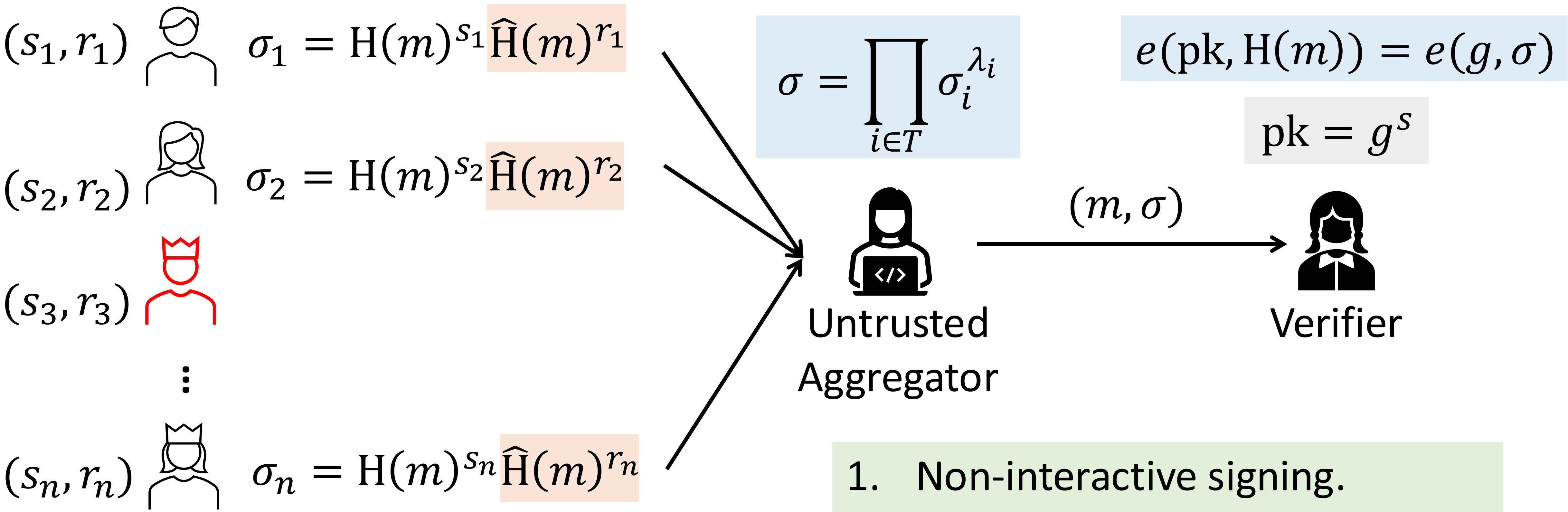
Untrusted Aggregator

Verifier

1. Non-interactive signing.
2. Constant signature size: $\sigma \in \mathbb{G}$.

Our Approach: Properties

λ_i are the Lagrange coefficients.



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$$e(\mathrm{pk}, \mathrm{H}(m)) = e(g, \sigma)$$

$$\mathrm{pk} = g^s$$

(m, σ)

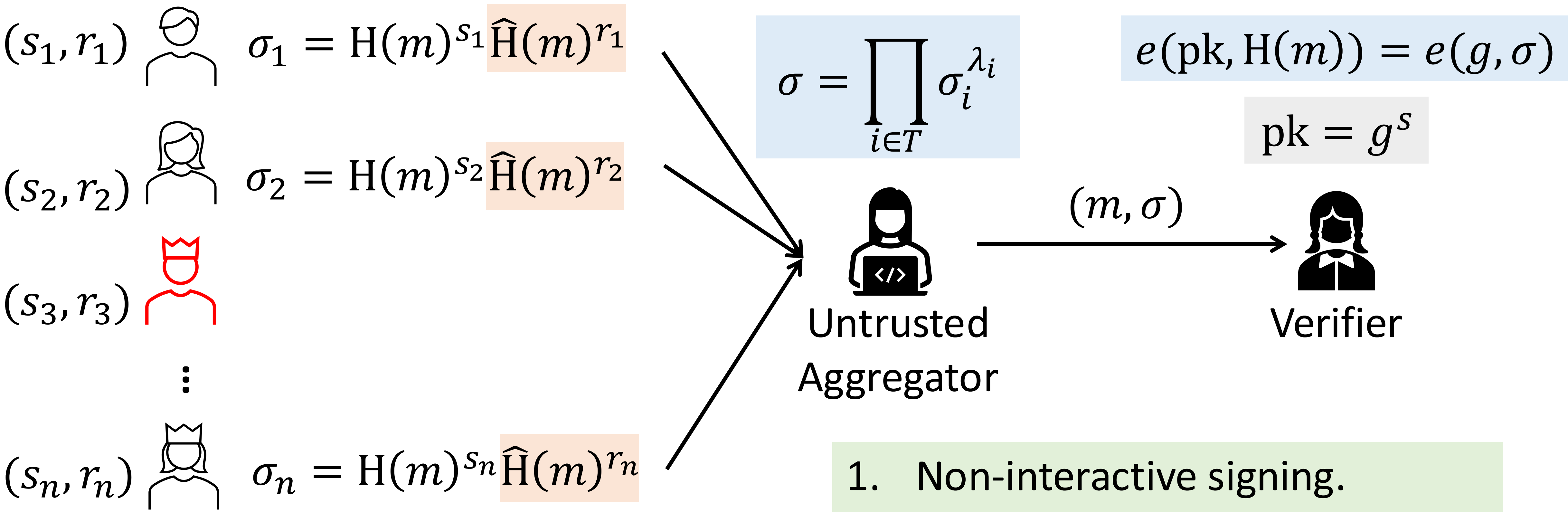
Untrusted Aggregator

Verifier

1. Non-interactive signing.
2. Constant signature size: $\sigma \in \mathbb{G}$.
3. Verification cost: two pairings.

Our Approach: Properties

λ_i are the Lagrange coefficients.



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(s_3, r_3)

⋮

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$$\sigma = \prod_{i \in T} \sigma_i^{\lambda_i}$$

$$e(\mathrm{pk}, \mathrm{H}(m)) = e(g, \sigma)$$

$$\mathrm{pk} = g^s$$

(m, σ)

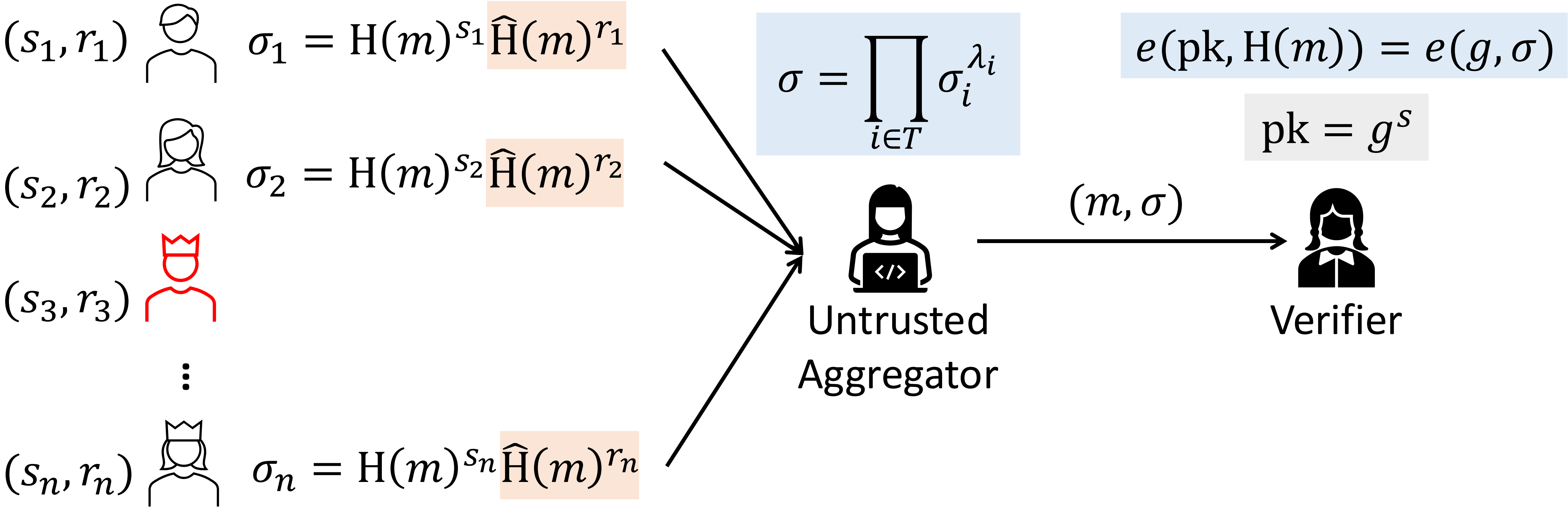
Untrusted Aggregator

Verifier

1. Non-interactive signing.
2. Constant signature size: $\sigma \in \mathbb{G}$.
3. Verification cost: two pairings.
4. Unique signature.

Our Approach: Main Idea

λ_i are the Lagrange coefficients.



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(s_n, r_n) $\sigma_n = \mathrm{H}(m)^{s_n}\widehat{\mathrm{H}}(m)^{r_n}$

$$\sigma = \prod_{i \in T} \sigma_i^{\lambda_i}$$

Untrusted Aggregator

(m, σ)

$$e(\mathrm{pk}, \mathrm{H}(m)) = e(g, \sigma)$$

$$\mathrm{pk} = g^s$$

Verifier

NOTE: We use Σ-protocol to verify partial signatures.

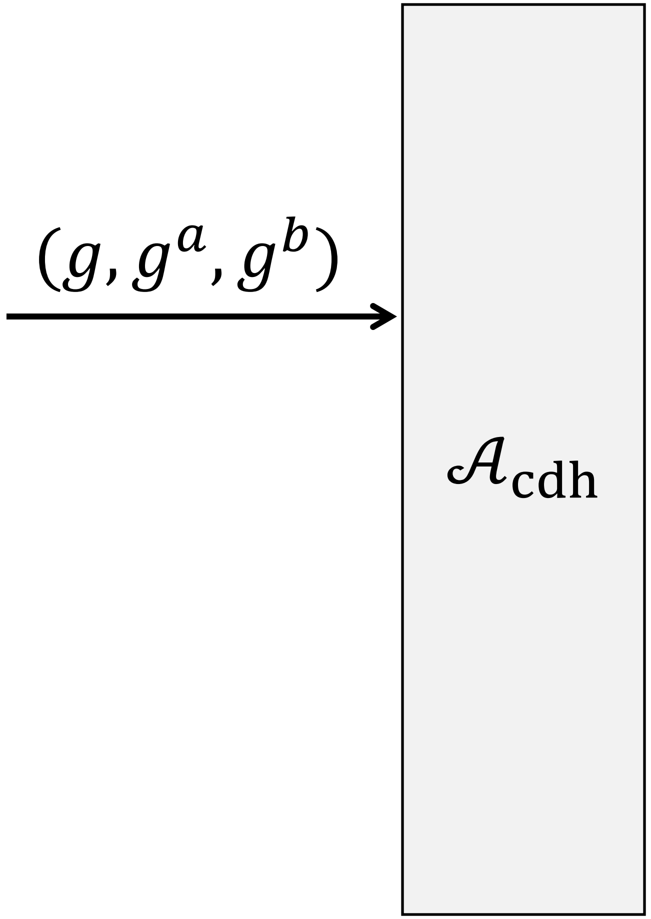
Proof Technique

Proving Security of a Signature Scheme

Proving Security of a Signature Scheme

$\mathcal{A}_{\mathrm{cdh}}$

Proving Security of a Signature Scheme



(g, g^a, g^b)

$\mathcal{A}_{\mathrm{cdh}}$

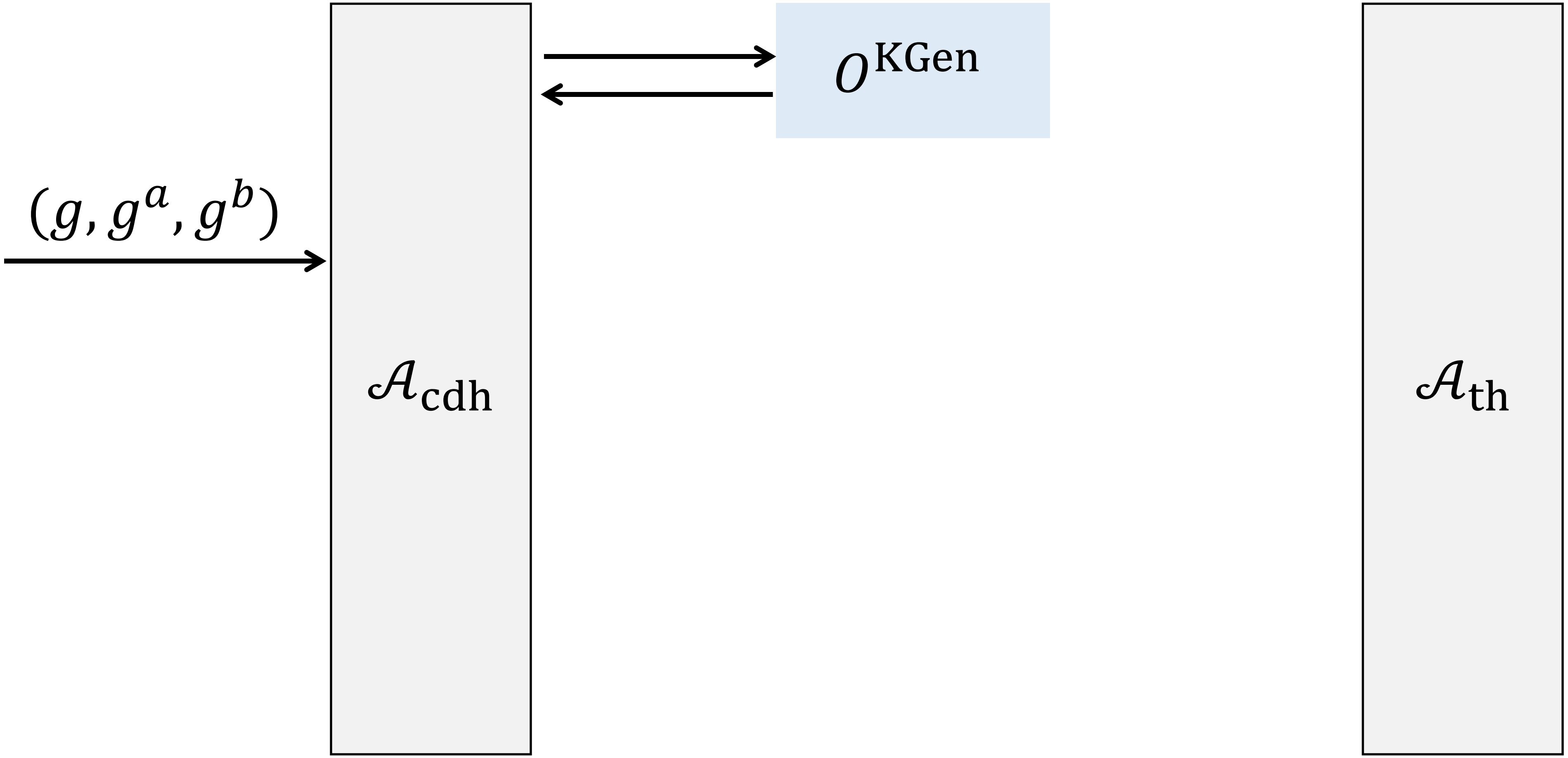
Proving Security of a Signature Scheme

(g, g^a, g^b)

$\mathcal{A}_{\text{cdh}}$

$\mathcal{A}_{\text{th}}$

Proving Security of a Signature Scheme



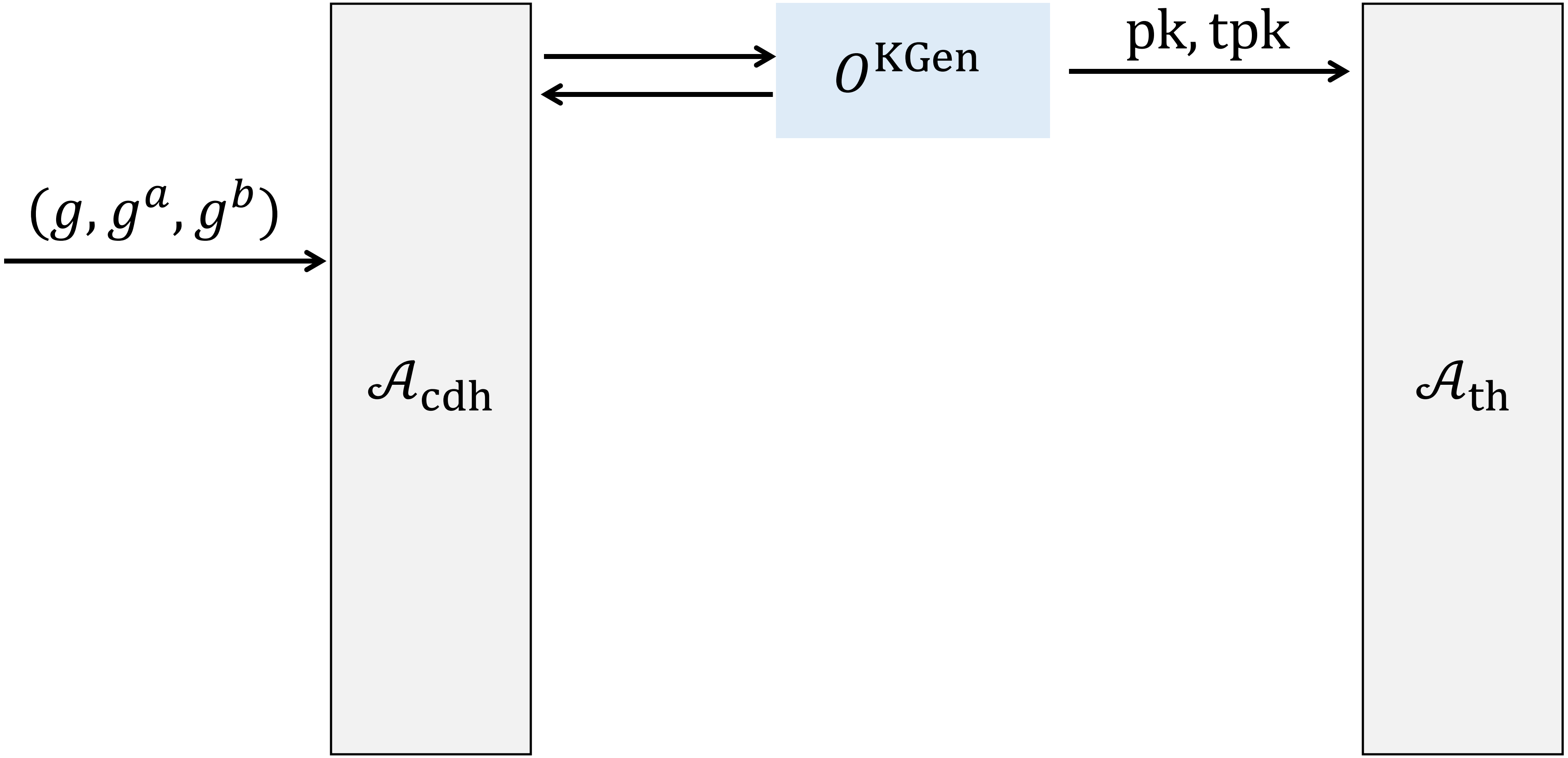
O^{KGen}

(g, g^a, g^b)

$\mathcal{A}_{\text{cdh}}$

$\mathcal{A}_{\text{th}}$

Proving Security of a Signature Scheme



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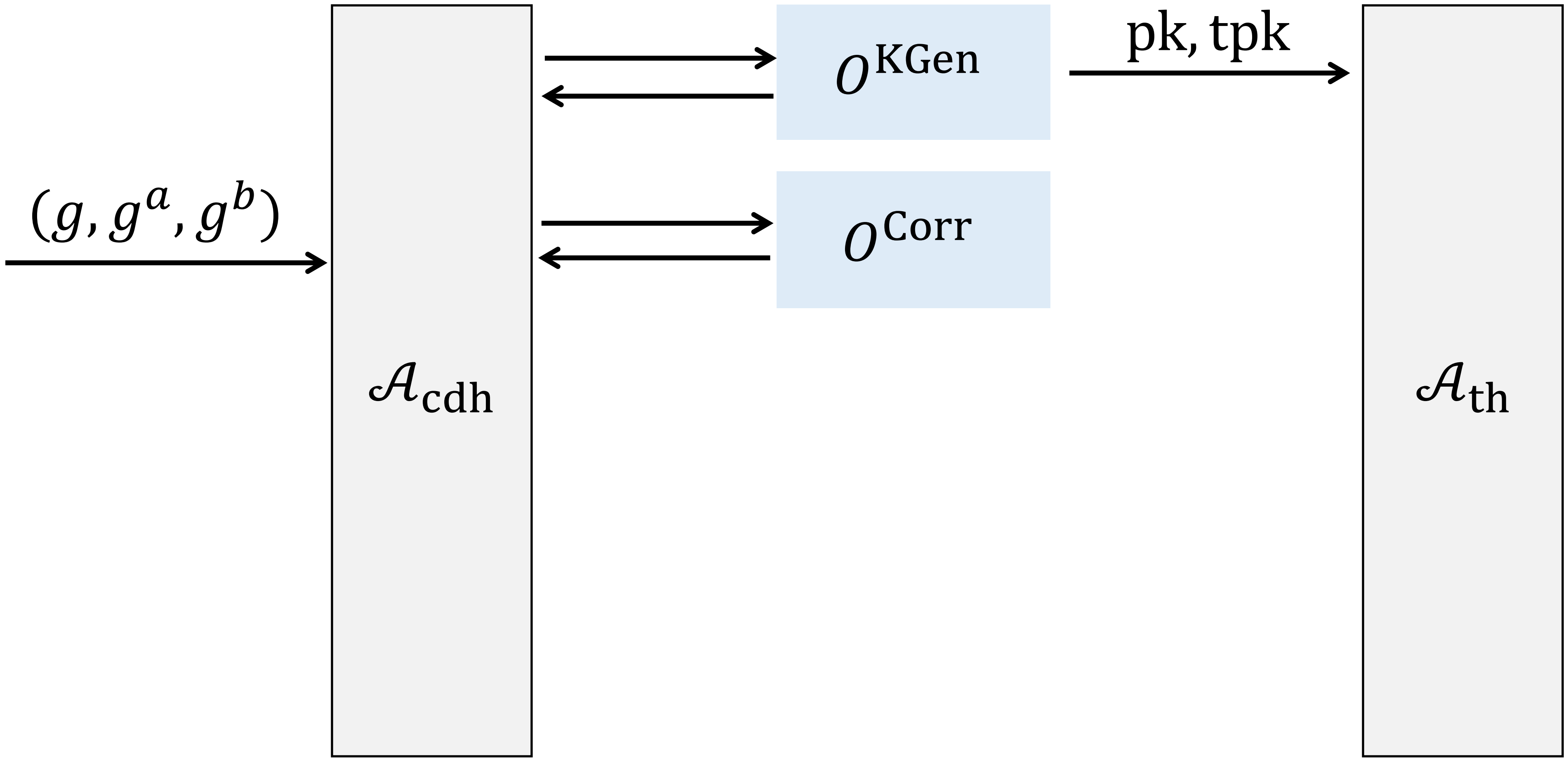


(g, g^a, g^b)

$\mathcal{A}_{\mathrm{cdh}}$

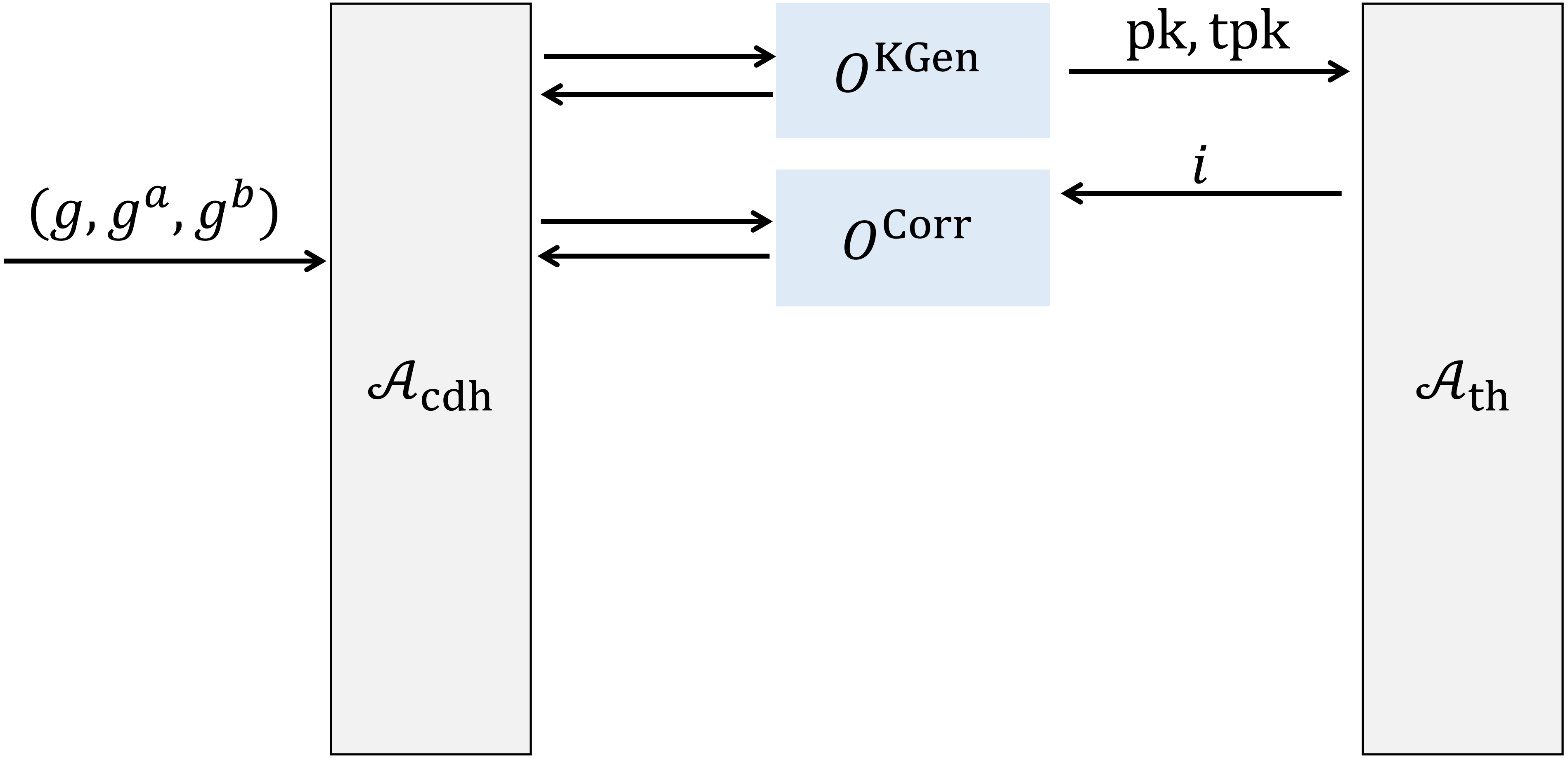
O^{KGen}

O^{Corr}

$\mathrm{pk}, \mathrm{tpk}$

$\mathcal{A}_{\mathrm{th}}$

Proving Security of a Signature Scheme



(g, g^a, g^b)

$\mathcal{A}_{\mathrm{cdh}}$

O^{KGen}

O^{Corr}

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i

$\mathcal{A}_{\mathrm{th}}$

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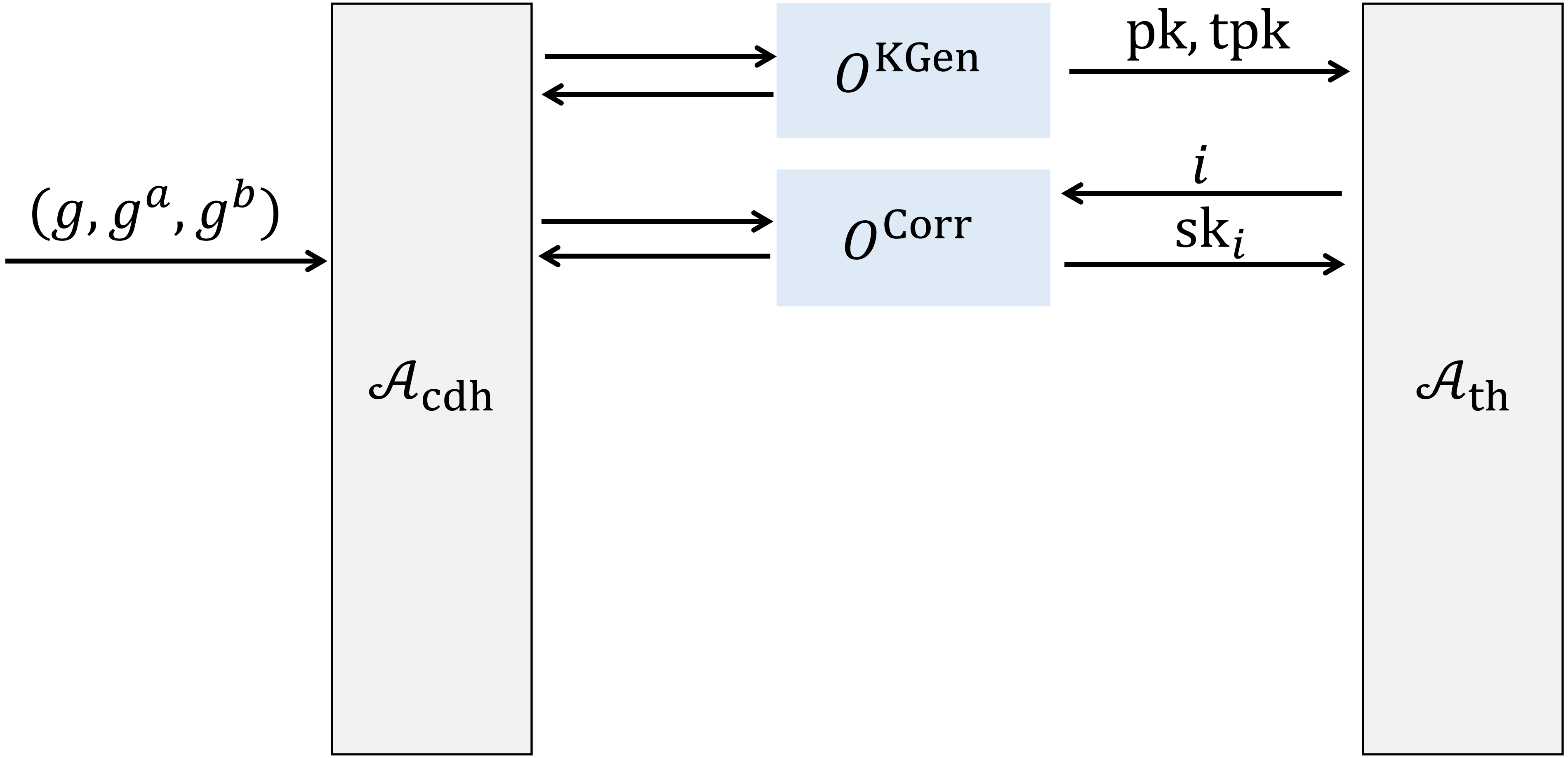

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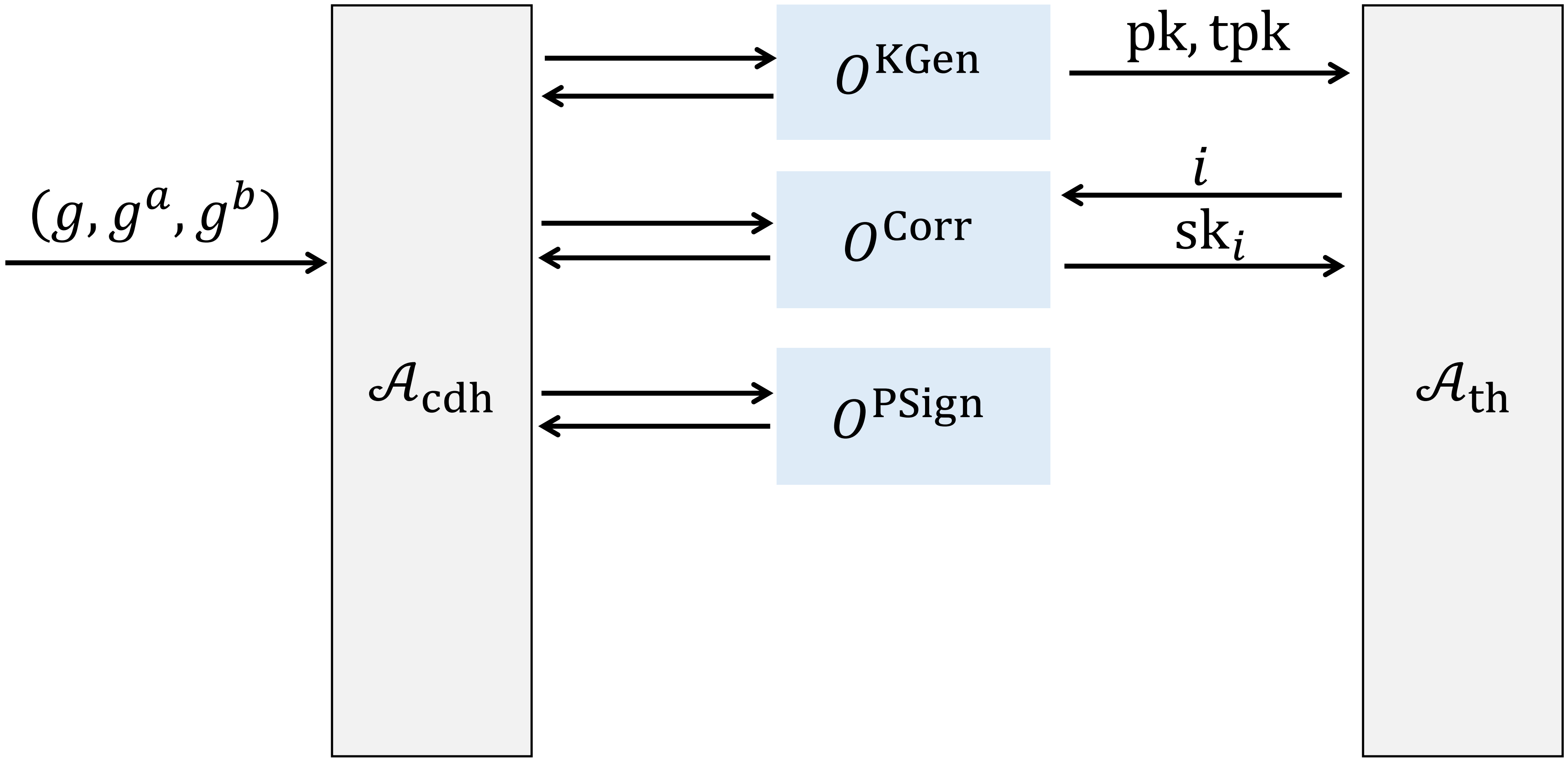

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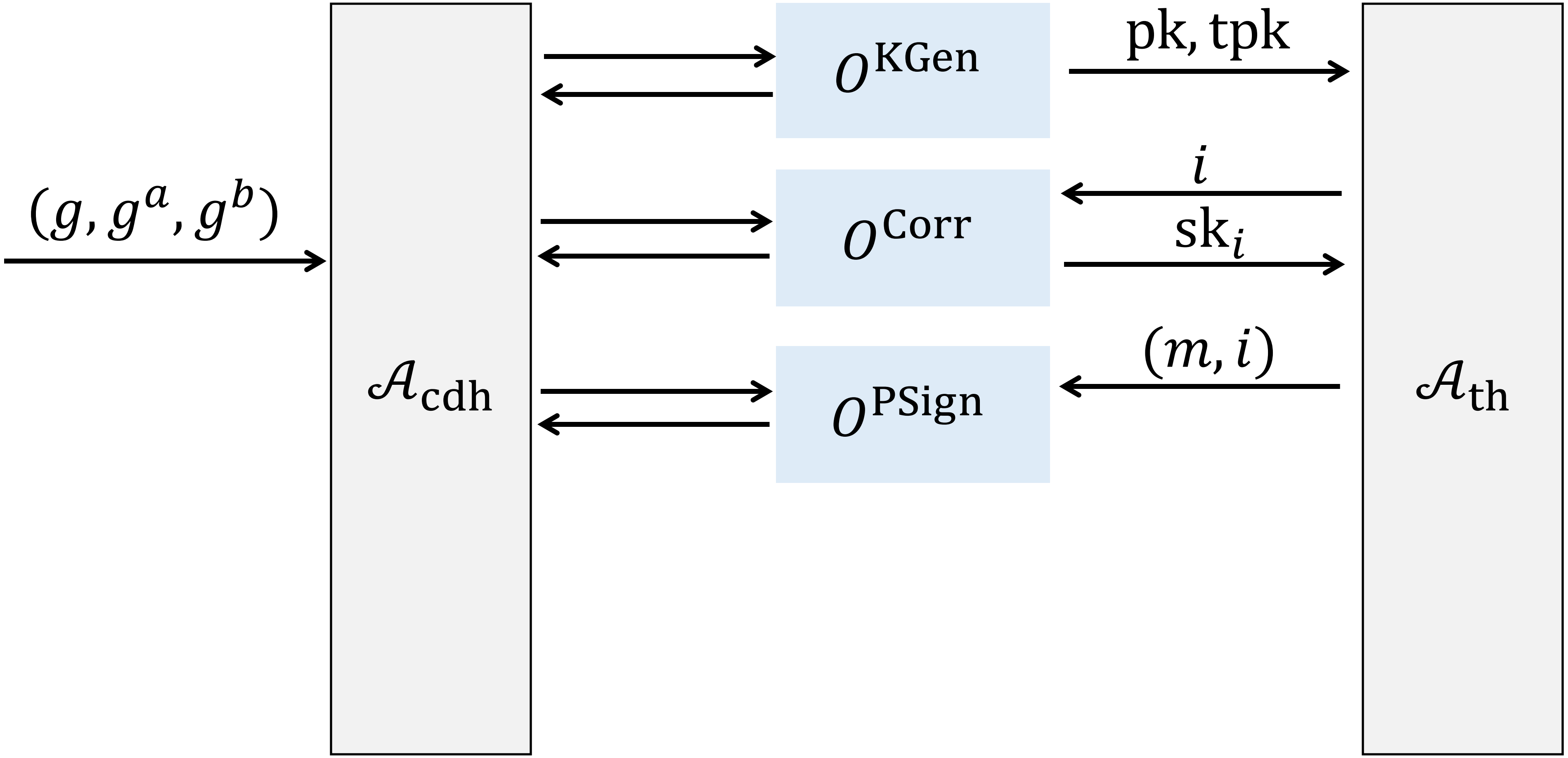


(g, g^a, g^b)

$\mathcal{A}_{\mathrm{cdh}}$

$\mathcal{O}^{\mathrm{KGen}}$

$\mathcal{O}^{\mathrm{Corr}}$

$\mathcal{O}^{\mathrm{PSign}}$

$\mathrm{pk}, \mathrm{tpk}$

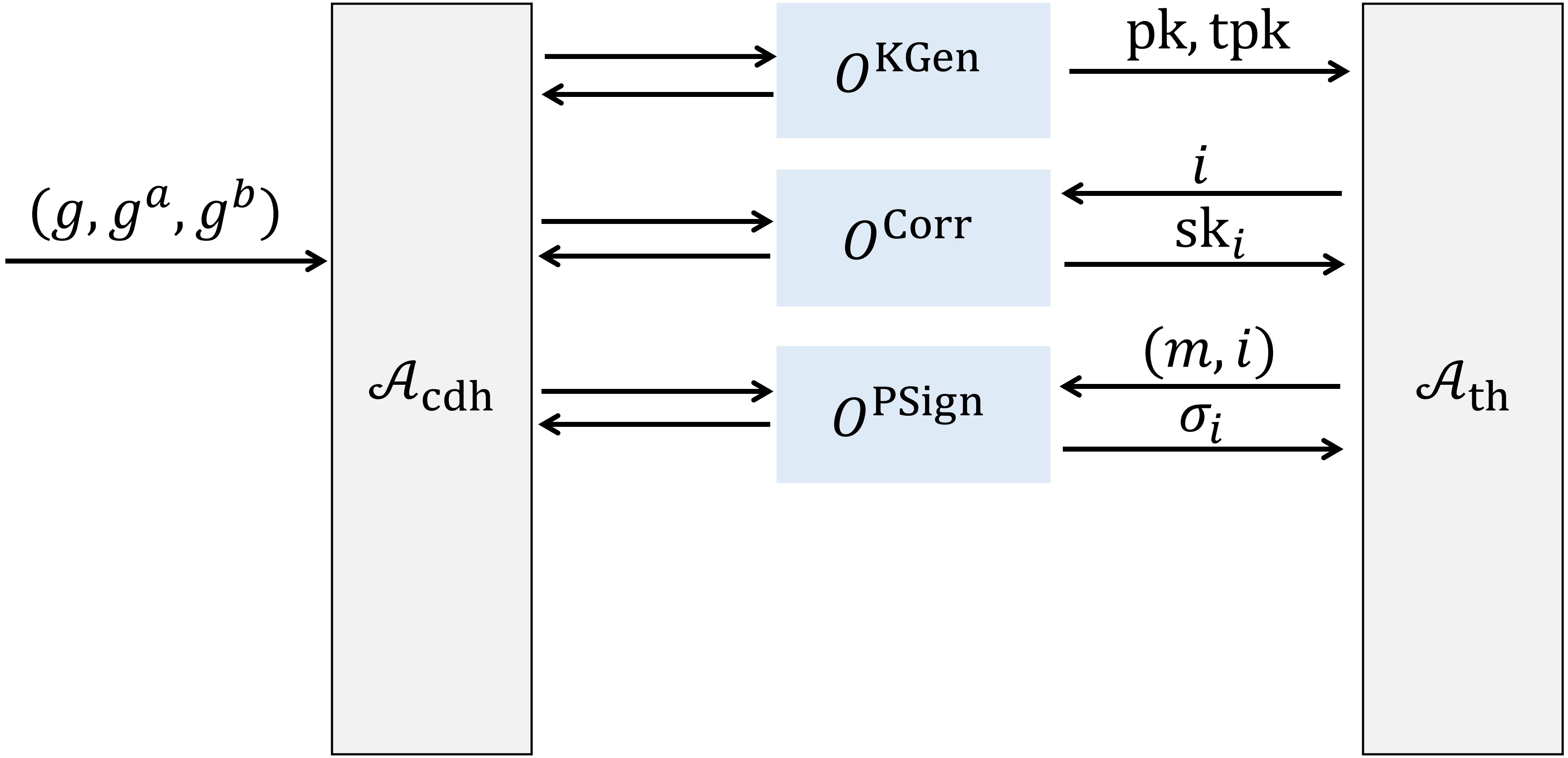
i

sk_i

(m, i)

$\mathcal{A}_{\mathrm{th}}$

Proving Security of a Signature Scheme



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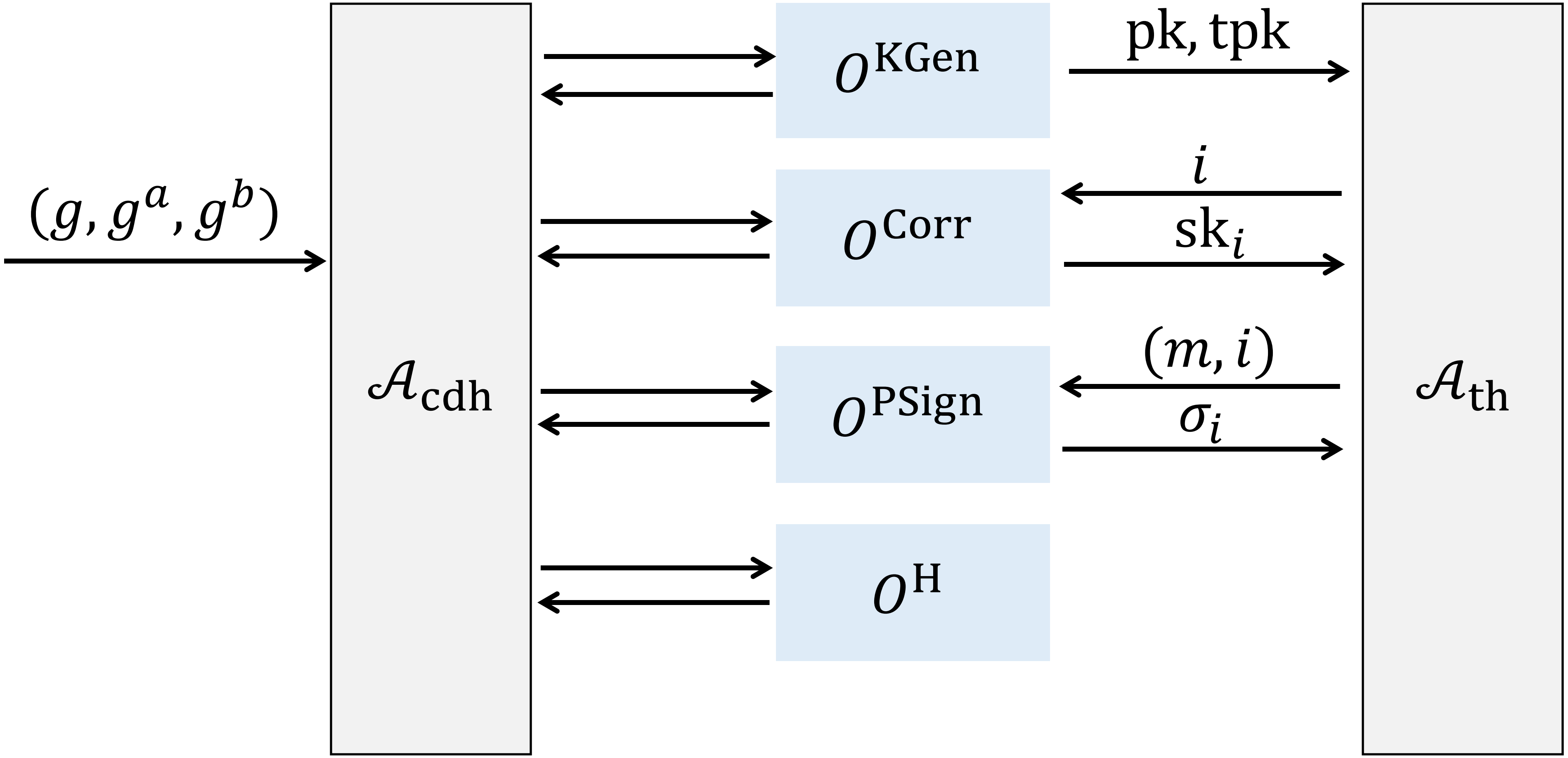


(g, g^a, g^b)

$\mathcal{A}_{\mathrm{cdh}}$

O^{KGen}

O^{Corr}

O^{PSign}

O^{H}

$\mathrm{pk}, \mathrm{tpk}$

i

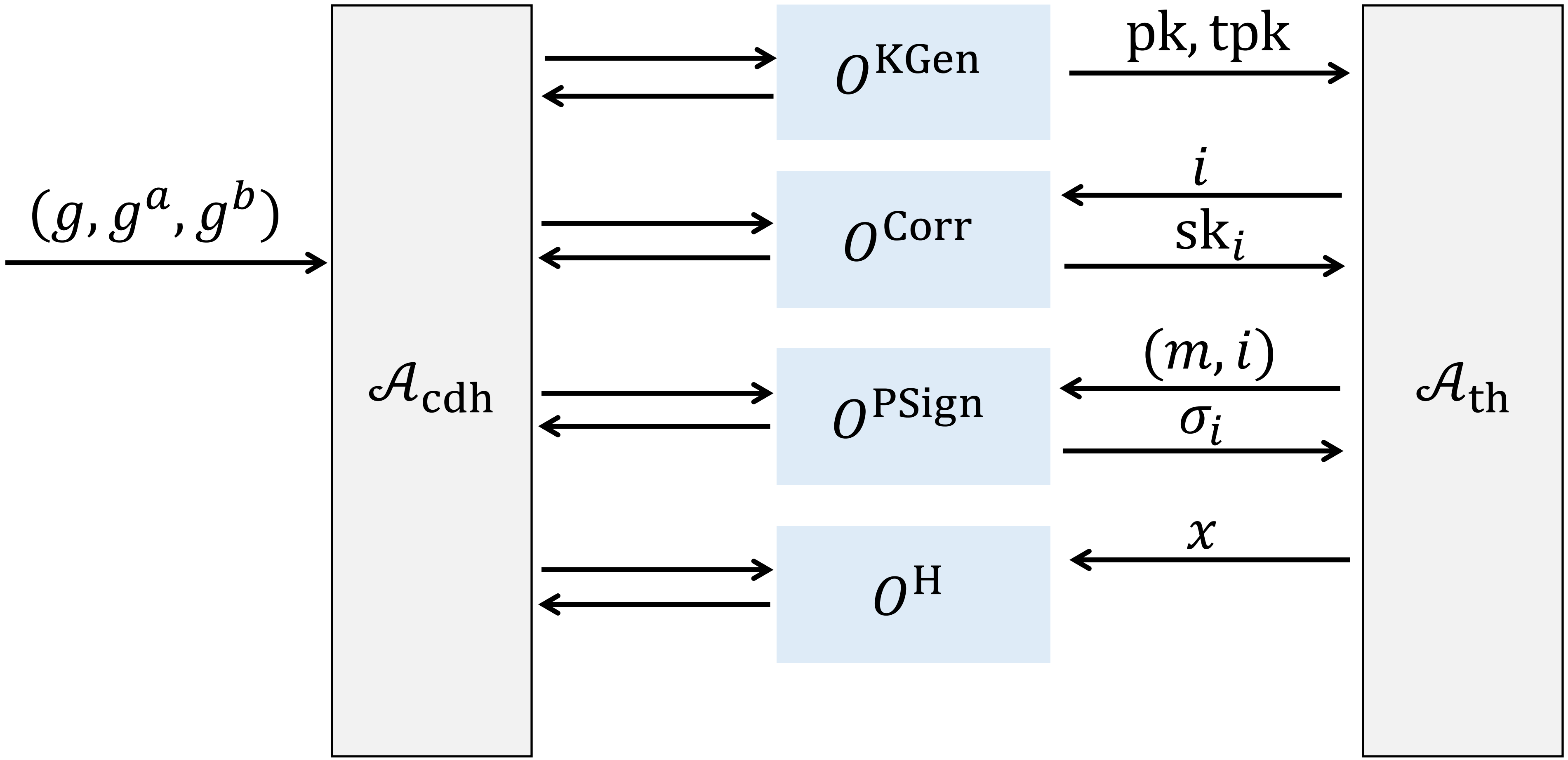
sk_i

(m, i)

σ_i

$\mathcal{A}_{\mathrm{th}}$

Proving Security of a Signature Scheme



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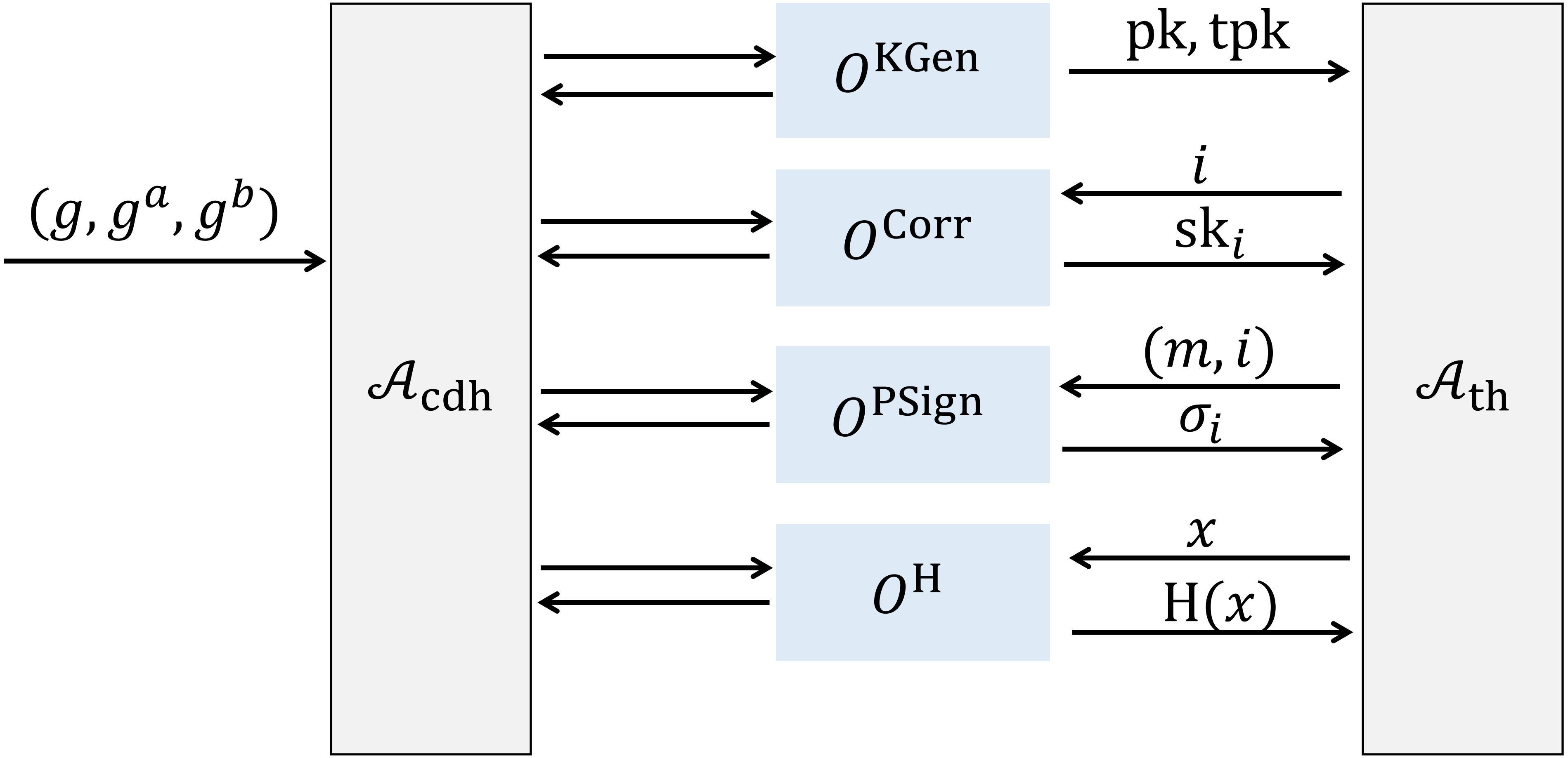


(g, g^a, g^b)

$\mathcal{A}_{\mathrm{cdh}}$

O^{KGen}

$\mathrm{pk}, \mathrm{tpk}$

i

sk_i

O^{Corr}

(m, i)

σ_i

O^{PSign}

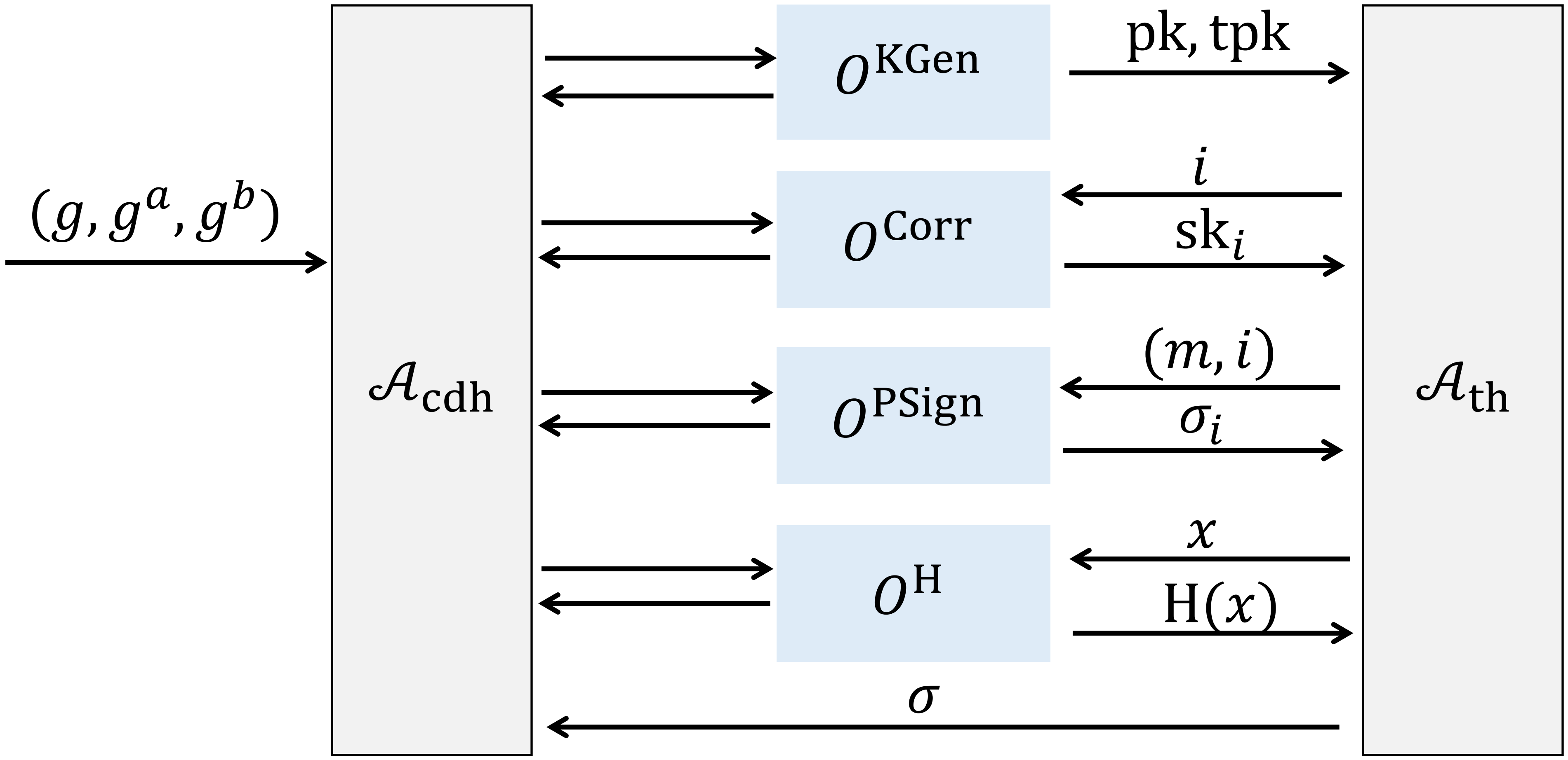
x

$\mathrm{H}(x)$

O^{H}

$\mathcal{A}_{\mathrm{th}}$

Proving Security of a Signature Scheme



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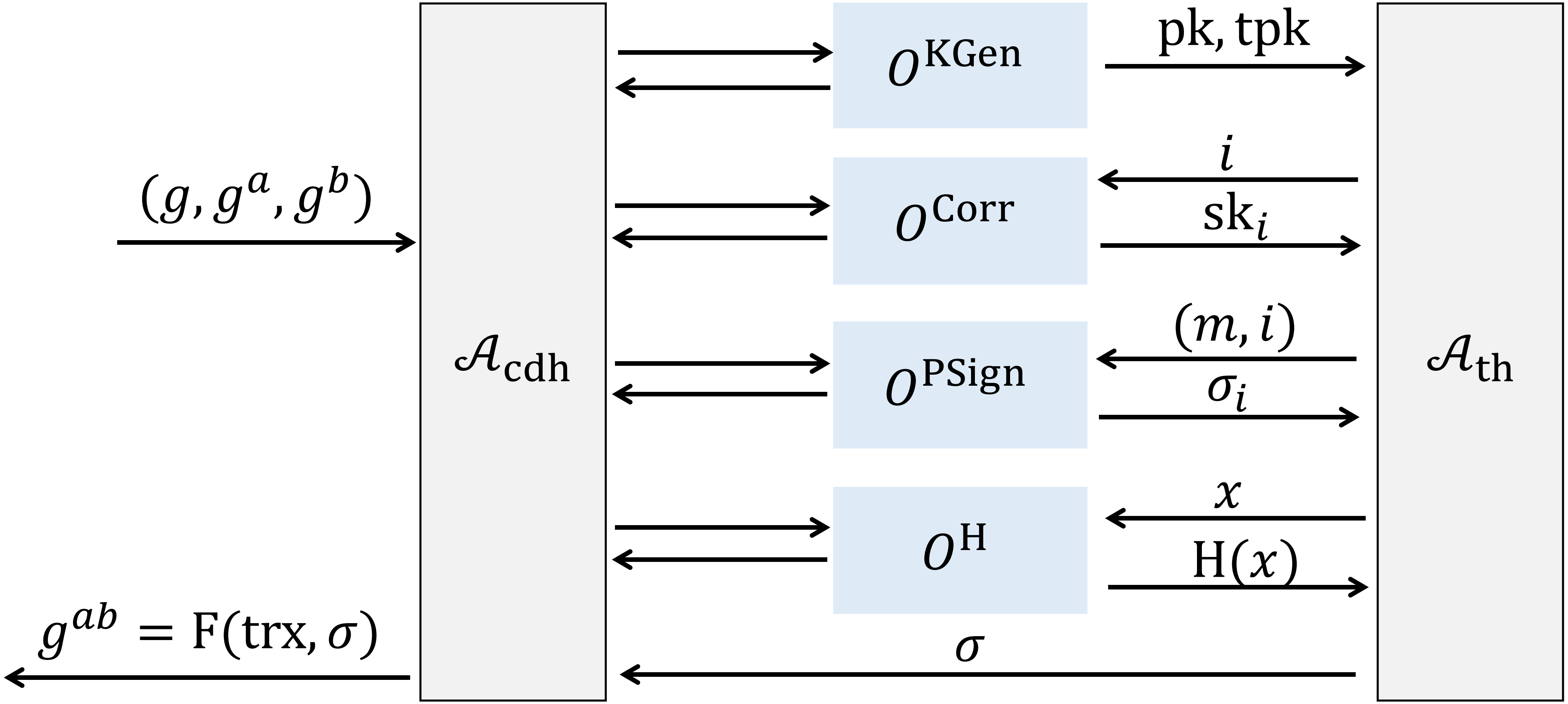

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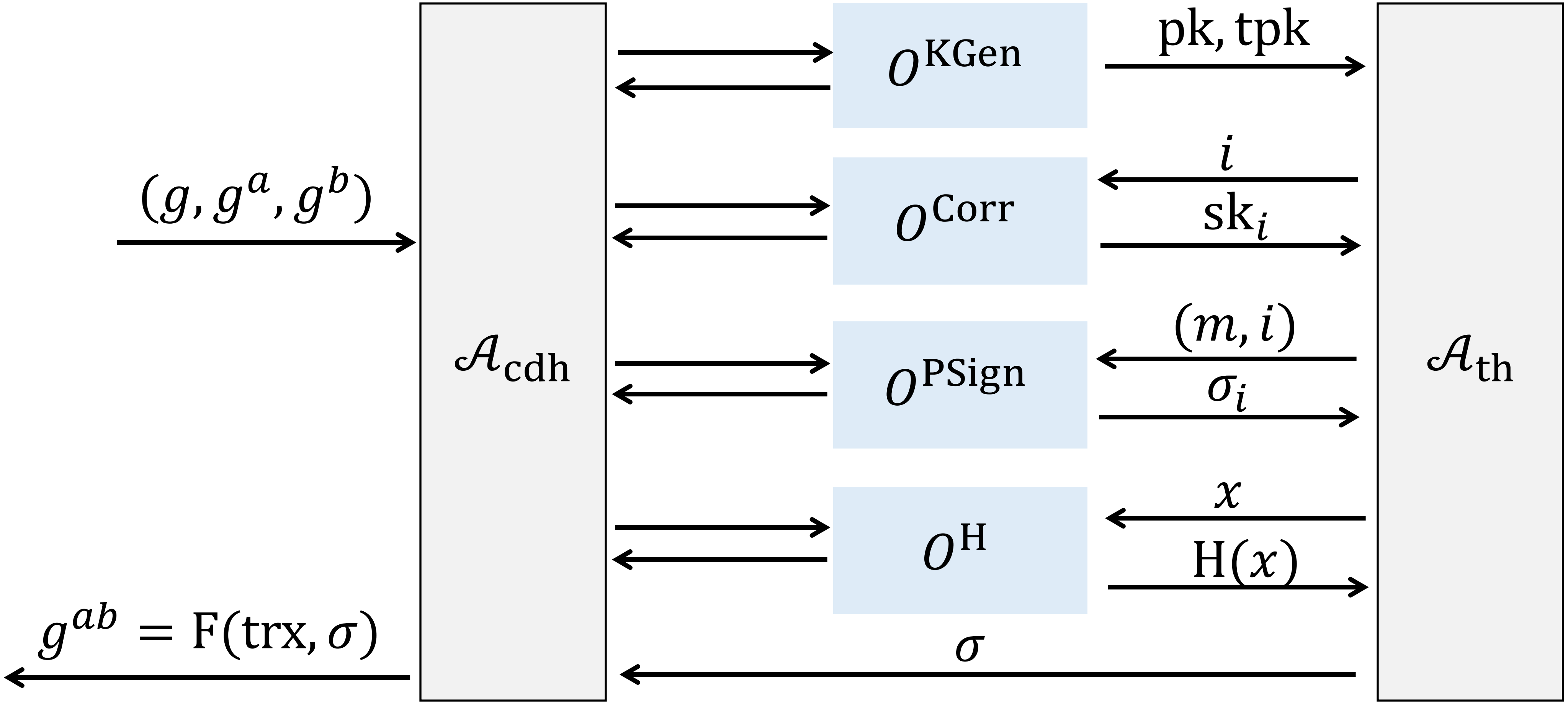


O^{Corr} is the trickiest to simulate. Next, we will see why.

Existing Proof Techniques: Breaking CDH

Existing Proof Techniques: Breaking CDH

$\mathcal{A}_{\mathrm{cdh}}$

$\mathcal{A}_{\mathrm{th}}$

Existing Proof Techniques: Breaking CDH

$\mathcal{A}_{\mathrm{cdh}}$

(g, g^{a}, g^{b})

$\mathcal{A}_{\mathrm{th}}$

Existing Proof Techniques: Breaking CDH

$\mathcal{A}_{\mathrm{cdh}}$

$\mathcal{A}_{\mathrm{th}}$

$\xrightarrow{(g, g^{a}, g^{b})}$

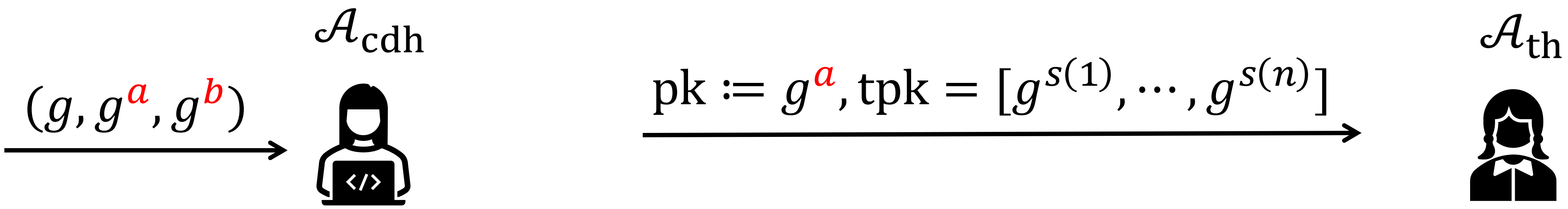
$s(x) \coloneqq a + a_1 x + \cdots + a_t x^t$

Existing Proof Techniques: Breaking CDH

$$s(x) \coloneqq a + a_1 x + \cdots + a_t x^t$$

$$\mathrm{sk} \coloneqq a = s(0); \quad \mathrm{sk}_i = s(i)$$

Existing Proof Techniques: Breaking CDH



$\mathcal{A}_{\text{cdh}}$

(g, g^{a}, g^{b})

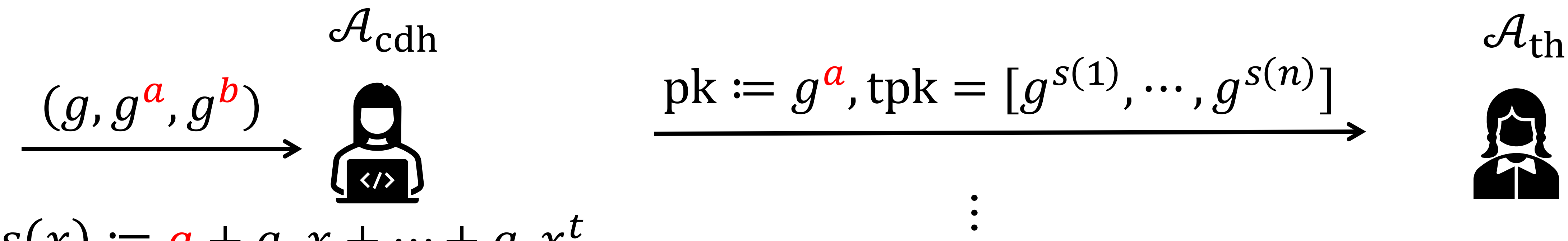
$\text{pk} := g^{a}, \text{tpk} = [g^{s(1)}, \cdots, g^{s(n)}]$

$\mathcal{A}_{\text{th}}$

$$s(x) := a + a_1 x + \cdots + a_t x^t$$

$$\text{sk} := a = s(0); \quad \text{sk}_i = s(i)$$

Existing Proof Techniques: Breaking CDH



$\mathcal{A}_{\text{cdh}}$

$(g, g^{a}, g^{b}) \longrightarrow$

$\text{pk} \coloneqq g^{a}, \text{tpk} = [g^{s(1)}, \cdots, g^{s(n)}] \longrightarrow$

$\vdots$

$\mathcal{A}_{\text{th}}$

$$s(x) \coloneqq a + a_1 x + \cdots + a_t x^t$$

$$\text{sk} \coloneqq a = s(0); \quad \text{sk}_i = s(i)$$

Existing Proof Techniques: Breaking CDH

$\mathcal{A}_{\text{cdh}}$

$$\xrightarrow{(g, g^{a}, g^{b})}$$

$$s(x) := a + a_1 x + \cdots + a_t x^t$$

$$\text{sk} := a = s(0); \quad \text{sk}_i = s(i)$$

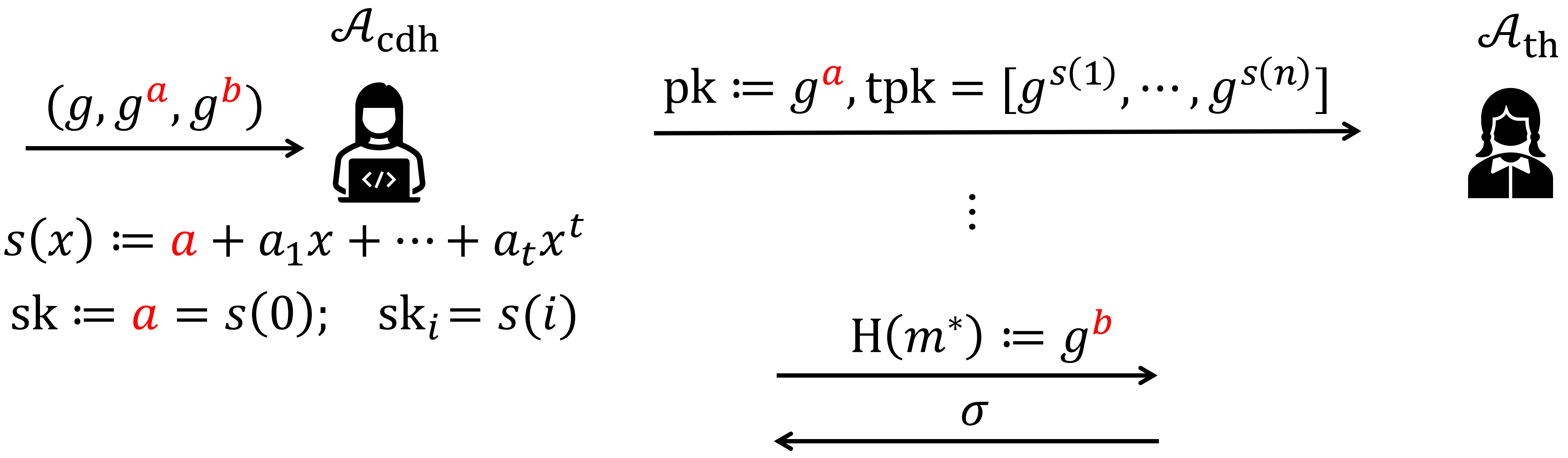
$\mathcal{A}_{\text{th}}$

$$\xrightarrow{\text{pk} := g^{a}, \text{tpk} = [g^{s(1)}, \cdots, g^{s(n)}]}$$

$$\vdots$$

$$\xrightarrow{\text{H}(m^*) := g^{b}}$$

Existing Proof Techniques: Breaking CDH



$\mathcal{A}_{\text{cdh}}$

$\mathcal{A}_{\text{th}}$

$$\xrightarrow{(g, g^{a}, g^{b})}$$

$$s(x) \coloneqq a + a_1 x + \cdots + a_t x^t$$

$$\text{sk} \coloneqq a = s(0); \quad \text{sk}_i = s(i)$$

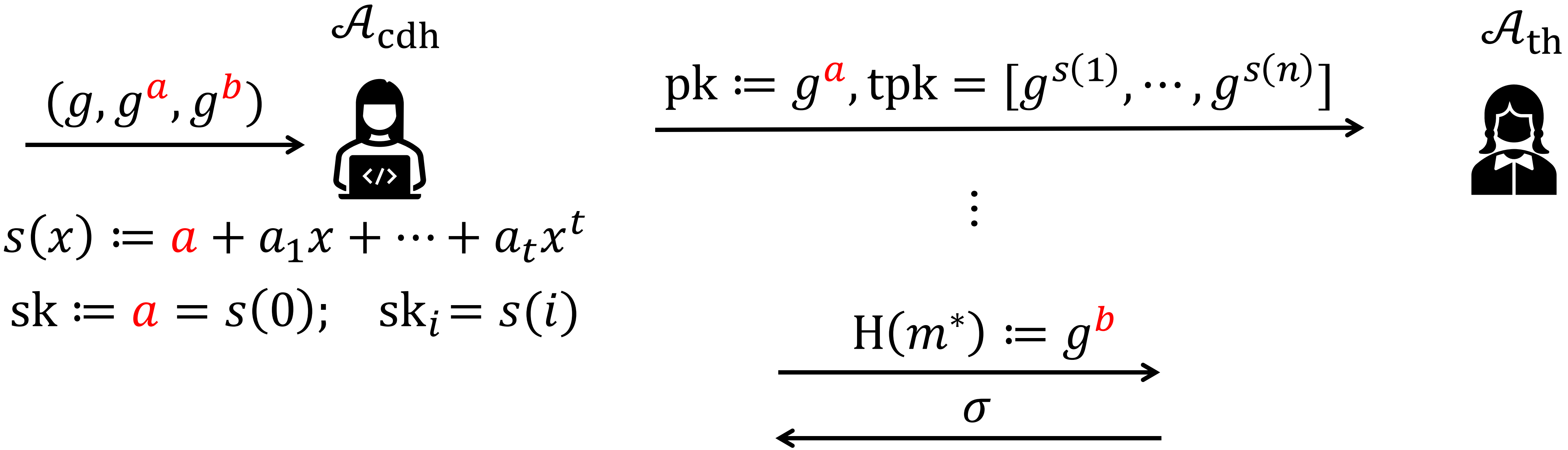
$$\xrightarrow{\text{pk} \coloneqq g^{a}, \text{tpk} = [g^{s(1)}, \cdots, g^{s(n)}]}$$

$$\vdots$$

$$\xrightarrow{\text{H}(m^*) \coloneqq g^{b}}$$

$$\xleftarrow{\sigma}$$

Existing Proof Techniques: Breaking CDH



$\mathcal{A}_{\text{cdh}}$

$\xrightarrow{(g, g^{a}, g^{b})}$

$$s(x) \coloneqq a + a_1 x + \cdots + a_t x^t$$

$$\text{sk} \coloneqq a = s(0); \quad \text{sk}_i = s(i)$$

$\mathcal{A}_{\text{th}}$

$\xrightarrow{\text{pk} \coloneqq g^{a}, \text{tpk} = [g^{s(1)}, \cdots, g^{s(n)}]}$

$\vdots$

$\xrightarrow{\mathrm{H}(m^*) \coloneqq g^{b}}$

$\xleftarrow{\sigma}$

Existing Proof Techniques: Breaking CDH

$\mathcal{A}_{\text{cdh}}$

$$\xrightarrow{(g, g^{a}, g^{b})}$$

$$s(x) \coloneqq a + a_1 x + \cdots + a_t x^t$$

$$\text{sk} \coloneqq a = s(0); \quad \text{sk}_i = s(i)$$

$$\xrightarrow{\text{pk} \coloneqq g^{a}, \text{tpk} = [g^{s(1)}, \dots, g^{s(n)}]}$$

$\mathcal{A}_{\text{th}}$

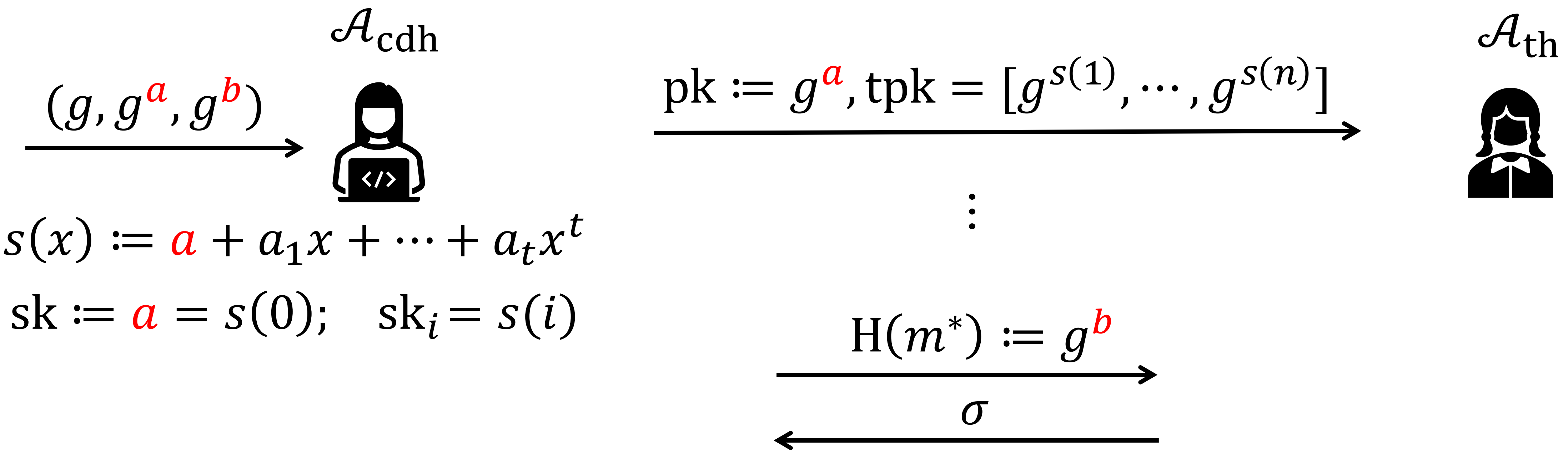
$$\vdots$$

$$\xrightarrow{\text{H}(m^*) \coloneqq g^{b}}$$

$$\xleftarrow{\sigma}$$

$$e\big(\text{pk}, \text{H}(m^*)\big) = e(g, \sigma)$$

Existing Proof Techniques: Breaking CDH



$\mathcal{A}_{\text{cdh}}$

$\mathcal{A}_{\text{th}}$

(g, g^{a}, g^{b})

$s(x) \coloneqq a + a_1 x + \cdots + a_t x^t$

$\text{sk} \coloneqq a = s(0); \quad \text{sk}_i = s(i)$

$\text{pk} \coloneqq g^{a}, \text{tpk} = [g^{s(1)}, \cdots, g^{s(n)}]$

$\vdots$

$\text{H}(m^*) \coloneqq g^{b}$

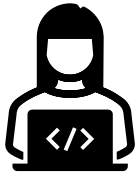
σ

$$e\big(\text{pk}, \text{H}(m^*)\big) = e(g, \sigma)$$
$$\Rightarrow e\big(g^{a}, g^{b}\big) = e(g, \sigma) \Rightarrow \sigma = g^{ab}$$

Existing Proof Techniques: Breaking CDH

$\mathcal{A}_{\text{cdh}}$

$$(g, g^{a}, g^{b}) \xrightarrow{\qquad\qquad}$$



$$s(x) \coloneqq a + a_1 x + \cdots + a_t x^t$$

$$\text{sk} \coloneqq a = s(0); \quad \text{sk}_i = s(i)$$

$\mathcal{A}_{\text{th}}$

$$\xrightarrow{\text{pk} \coloneqq g^{a}, \text{tpk} = [g^{s(1)}, \cdots, g^{s(n)}]}$$

$$\vdots$$

$$\xrightarrow{\mathrm{H}(m^*) \coloneqq g^{b}}$$

$$\xleftarrow{\sigma}$$

$$e\big(\text{pk}, \mathrm{H}(m^*)\big) = e(g, \sigma)$$
$$\Rightarrow e\big(g^{a}, g^{b}\big) = e(g, \sigma) \Rightarrow \sigma = g^{ab}$$

How does $\mathcal{A}_{\text{cdh}}$ compute $\text{tpk} = [g^{s(1)}, \cdots, g^{s(n)}]$?

Existing Proof Techniques: Breaking CDH

$\mathcal{A}_{\text{cdh}}$

$$\xrightarrow{(g, g^{a}, g^{b})}$$

$$s(x) \coloneqq a + a_1 x + \cdots + a_t x^t$$

$$\text{sk} \coloneqq a = s(0); \quad \text{sk}_i = s(i)$$

$\mathcal{A}_{\text{th}}$

$$\xrightarrow{\text{pk} \coloneqq g^{a}, \text{tpk} = [g^{s(1)}, \cdots, g^{s(n)}]}$$

$$\vdots$$

$$\xrightarrow{\text{H}(m^*) \coloneqq g^{b}}$$

$$\xleftarrow{\sigma}$$

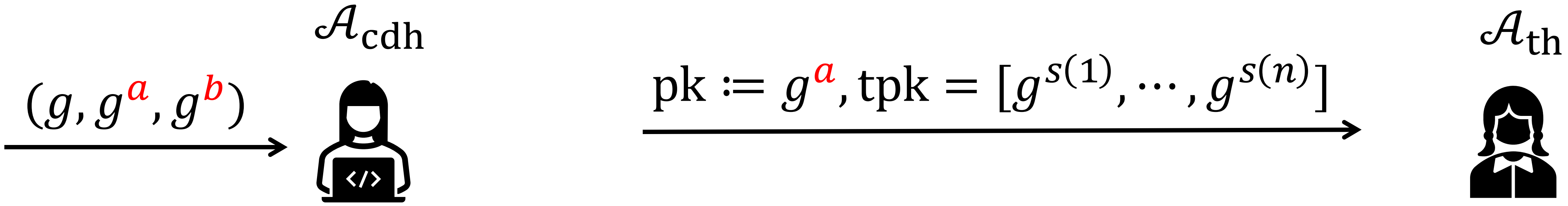
$$e\big(\text{pk}, \text{H}(m^*)\big) = e(g, \sigma)$$
$$\Rightarrow e\big(g^{a}, g^{b}\big) = e(g, \sigma) \Rightarrow \sigma = g^{ab}$$

How does $\mathcal{A}_{\text{cdh}}$ compute $\text{tpk} = [g^{s(1)}, \cdots, g^{s(n)}]$?

How does $\mathcal{A}_{\text{cdh}}$ respond to corruption queries?

Existing Proof Techniques: Corruption queries

Existing Proof Techniques: Corruption queries



$\mathcal{A}_{\text{cdh}}$

(g, g^a, g^b)

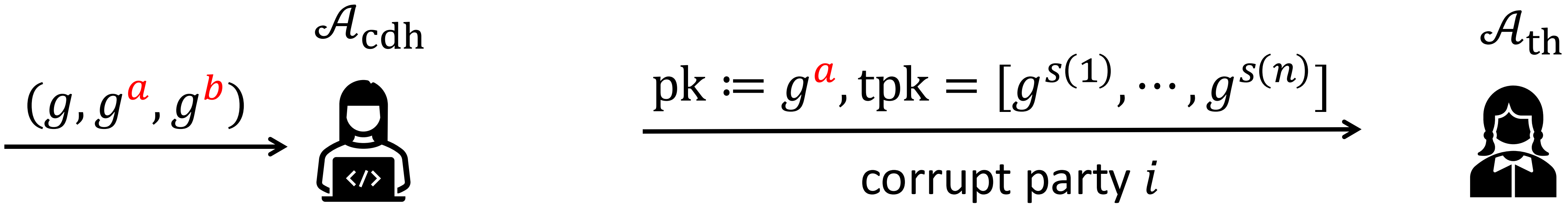
$\text{pk} \coloneqq g^a, \text{tpk} = [g^{s(1)}, \cdots, g^{s(n)}]$

$\mathcal{A}_{\text{th}}$

$$s(x) \coloneqq a + a_1 x + \cdots + a_t x^t$$

$$\text{sk} \coloneqq a = s(0); \quad \text{sk}_i = s(i)$$

Existing Proof Techniques: Corruption queries



$\mathcal{A}_{\text{cdh}}$

(g, g^{a}, g^{b})

$\text{pk} \coloneqq g^{a}, \text{tpk} = [g^{s(1)}, \cdots, g^{s(n)}]$

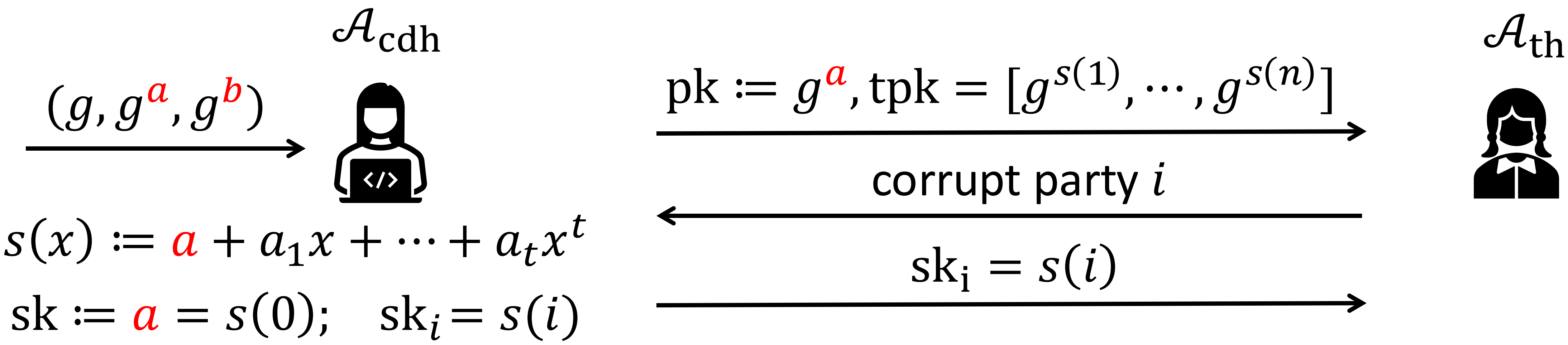
corrupt party i

$\mathcal{A}_{\text{th}}$

$$s(x) \coloneqq a + a_1 x + \cdots + a_t x^t$$

$$\text{sk} \coloneqq a = s(0); \quad \text{sk}_i = s(i)$$

Existing Proof Techniques: Corruption queries



$\mathcal{A}_{\text{cdh}}$

$\xrightarrow{(g, g^{a}, g^{b})}$

$s(x) \coloneqq a + a_1 x + \cdots + a_t x^t$

$\text{sk} \coloneqq a = s(0); \quad \text{sk}_i = s(i)$

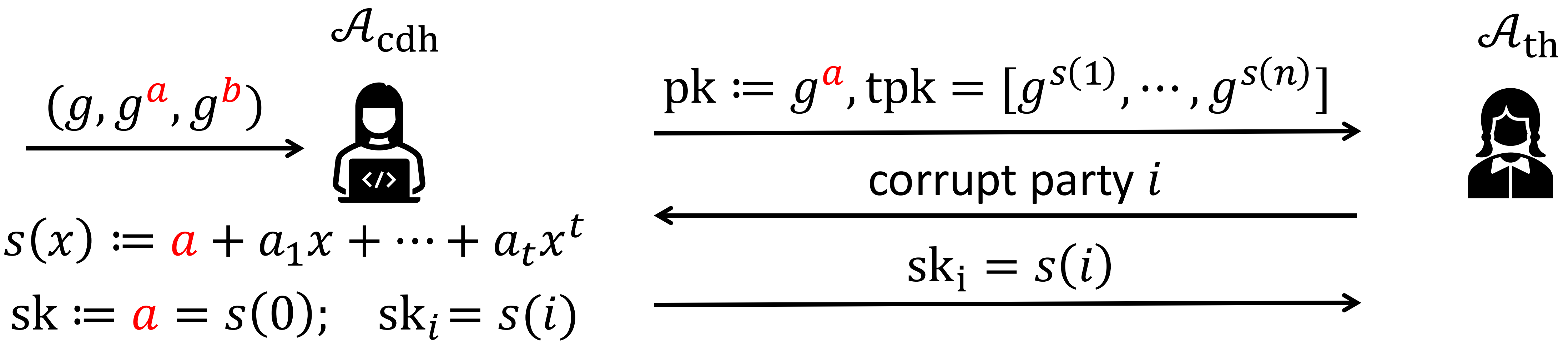
$\mathcal{A}_{\text{th}}$

$\xrightarrow{\text{pk} \coloneqq g^{a}, \text{tpk} = [g^{s(1)}, \dots, g^{s(n)}]}$

$\xleftarrow{\text{corrupt party } i}$

$\xrightarrow{\text{sk}_\text{i} = s(i)}$

Existing Proof Techniques: Corruption queries



$\mathcal{A}_{\text{cdh}}$

(g, g^{a}, g^{b})

$s(x) \coloneqq a + a_1 x + \cdots + a_t x^t$

$\text{sk} \coloneqq a = s(0); \quad \text{sk}_i = s(i)$

$\mathcal{A}_{\text{th}}$

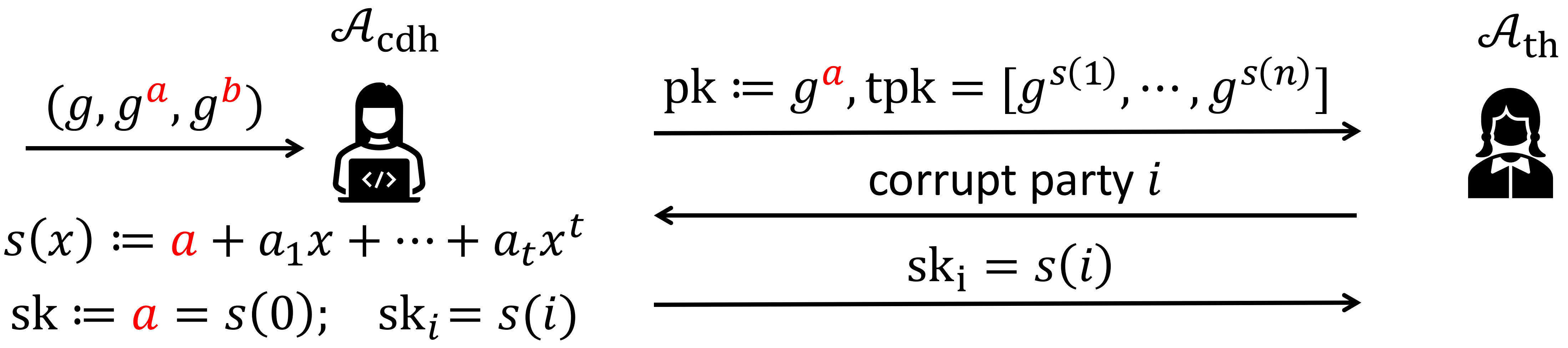
$\text{pk} \coloneqq g^{a}, \text{tpk} = [g^{s(1)}, \cdots, g^{s(n)}]$

corrupt party i

$\text{sk}_\text{i} = s(i)$

How?

Existing Proof Techniques: Corruption queries



$\mathcal{A}_{\text{cdh}}$

$\xrightarrow{(g, g^{a}, g^{b})}$

$s(x) \coloneqq a + a_1 x + \cdots + a_t x^t$

$\text{sk} \coloneqq a = s(0); \quad \text{sk}_i = s(i)$

$\mathcal{A}_{\text{th}}$

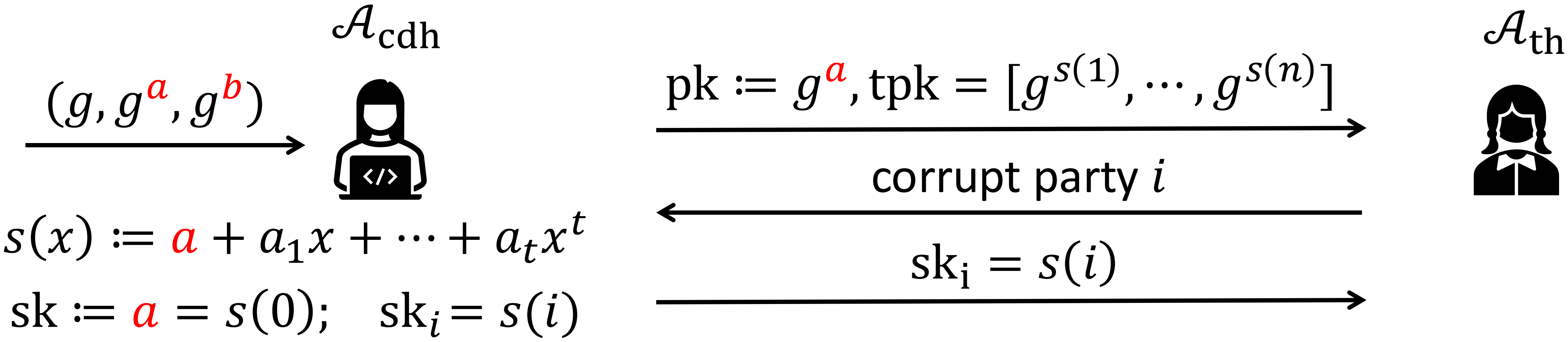
$\xrightarrow{\text{pk} \coloneqq g^{a}, \text{tpk} = [g^{s(1)}, \cdots, g^{s(n)}]}$

$\xleftarrow{\text{corrupt party } i}$

$\xrightarrow{\text{sk}_\text{i} = s(i)}$

How? $\mathcal{A}_{\text{cdh}}$ does not know $s(x)$

Existing Proof Techniques: Corruption queries



$\mathcal{A}_{\mathrm{cdh}}$

(g, g^{a}, g^{b})

$s(x) := a + a_1 x + \cdots + a_t x^t$

$\mathrm{sk} := a = s(0); \quad \mathrm{sk}_i = s(i)$

$\mathcal{A}_{\mathrm{th}}$

$\mathrm{pk} := g^{a}, \mathrm{tpk} = [g^{s(1)}, \cdots, g^{s(n)}]$

corrupt party i

$\mathrm{sk_i} = s(i)$

How? $\mathcal{A}_{\mathrm{cdh}}$ does not know $s(x)$

This is why we need to restrict $\mathcal{A}_{\mathrm{th}}$ to be static.

Existing Proof Techniques: Static Security

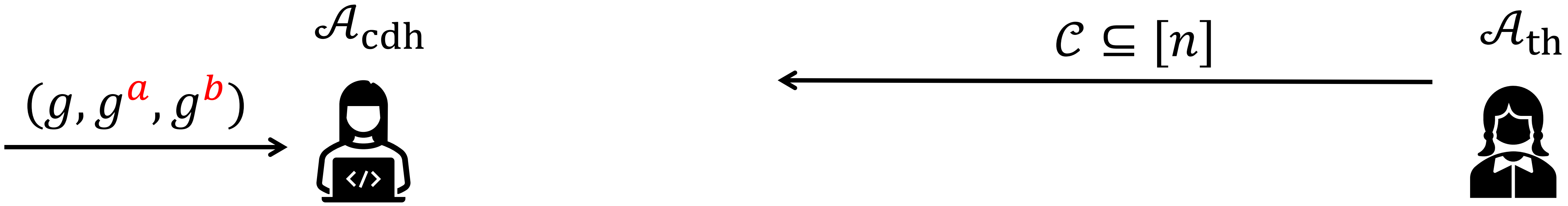
Existing Proof Techniques: Static Security

$\mathcal{A}_{\mathrm{cdh}}$

$\mathcal{A}_{\mathrm{th}}$

$(g, g^{a}, g^{b}) \longrightarrow$

Existing Proof Techniques: Static Security



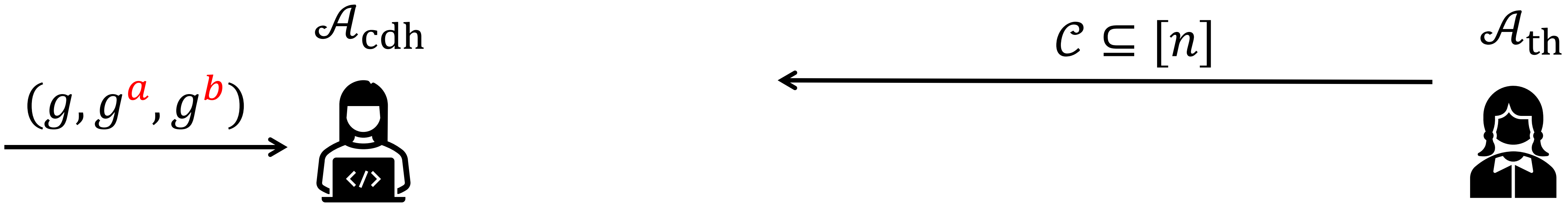
(g, g^{a}, g^{b})

$\mathcal{A}_{\text{cdh}}$

$\mathcal{C} \subseteq [n]$

$\mathcal{A}_{\text{th}}$

Existing Proof Techniques: Static Security



$\mathcal{A}_{\mathrm{cdh}}$

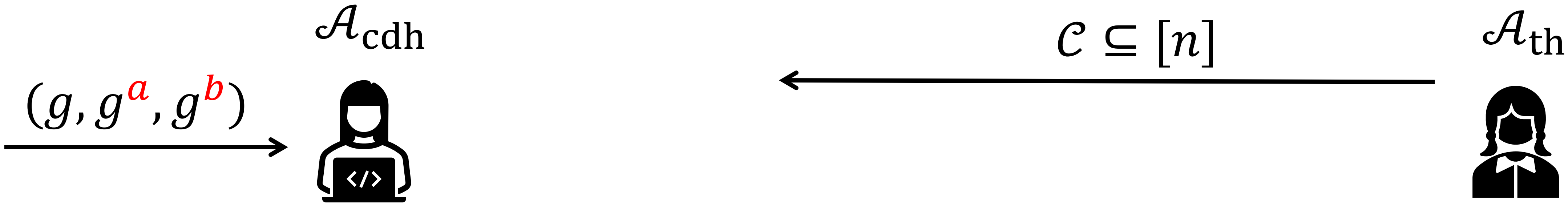
(g, g^{a}, g^{b})

$\mathcal{C} \subseteq [n]$

$\mathcal{A}_{\mathrm{th}}$

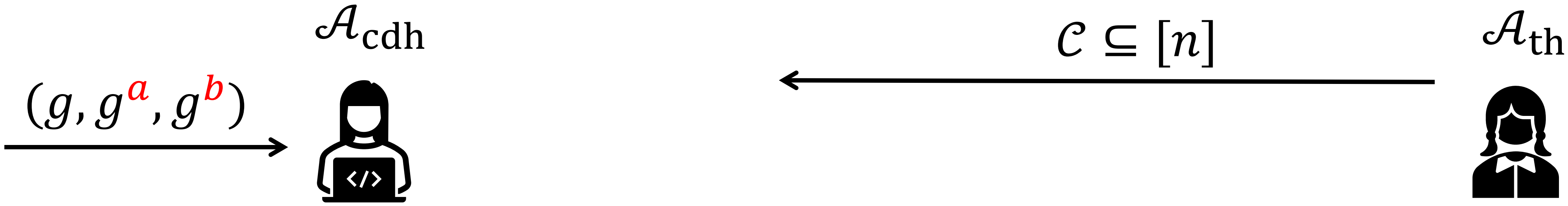
- Let $\mathcal{C} = \{1,2, \ldots, t\}$

Existing Proof Techniques: Static Security



- Let $\mathcal{C} = \{1,2, \dots, t\}$
- Sample $s(1), \dots, s(t) \leftarrow \mathbb{F}$

Existing Proof Techniques: Static Security



$\mathcal{A}_{\text{cdh}}$

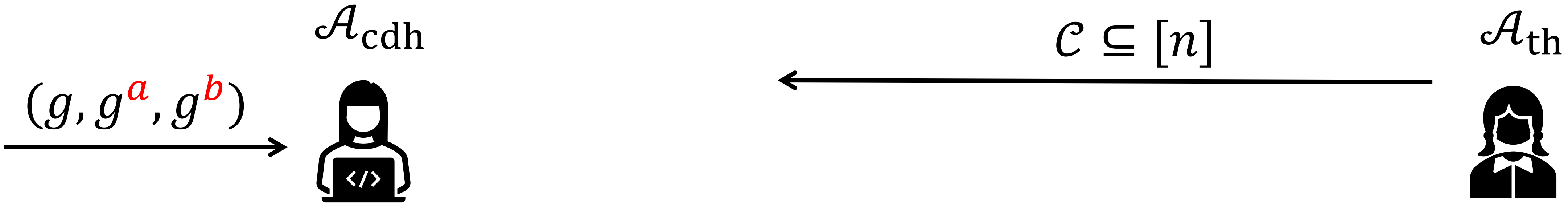
(g, g^{a}, g^{b})

$\mathcal{C} \subseteq [n]$

$\mathcal{A}_{\text{th}}$

- Let $\mathcal{C} = \{1,2,\dots,t\}$
- Sample $s(1),\dots,s(t) \leftarrow \mathbb{F}$
- Let $s(0) = a$

Existing Proof Techniques: Static Security



$\mathcal{A}_{\text{cdh}}$

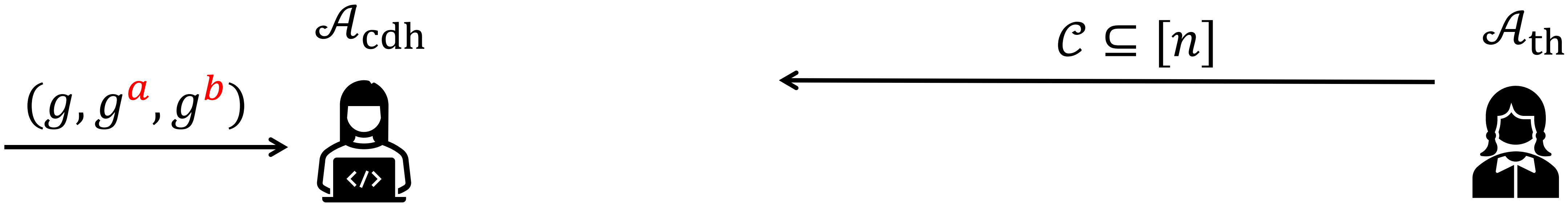
(g, g^{a}, g^{b})

$\mathcal{C} \subseteq [n]$

$\mathcal{A}_{\text{th}}$

- Let $\mathcal{C} = \{1,2, \ldots, t\}$
- Sample $s(1), \ldots, s(t) \leftarrow \mathbb{F}$
- Let $s(0) = a$
- Interpolate $\{g^{a}, g^{s(1)}, \cdots, g^{s(t)}\}$

Existing Proof Techniques: Static Security



$\mathcal{A}_{\text{cdh}}$

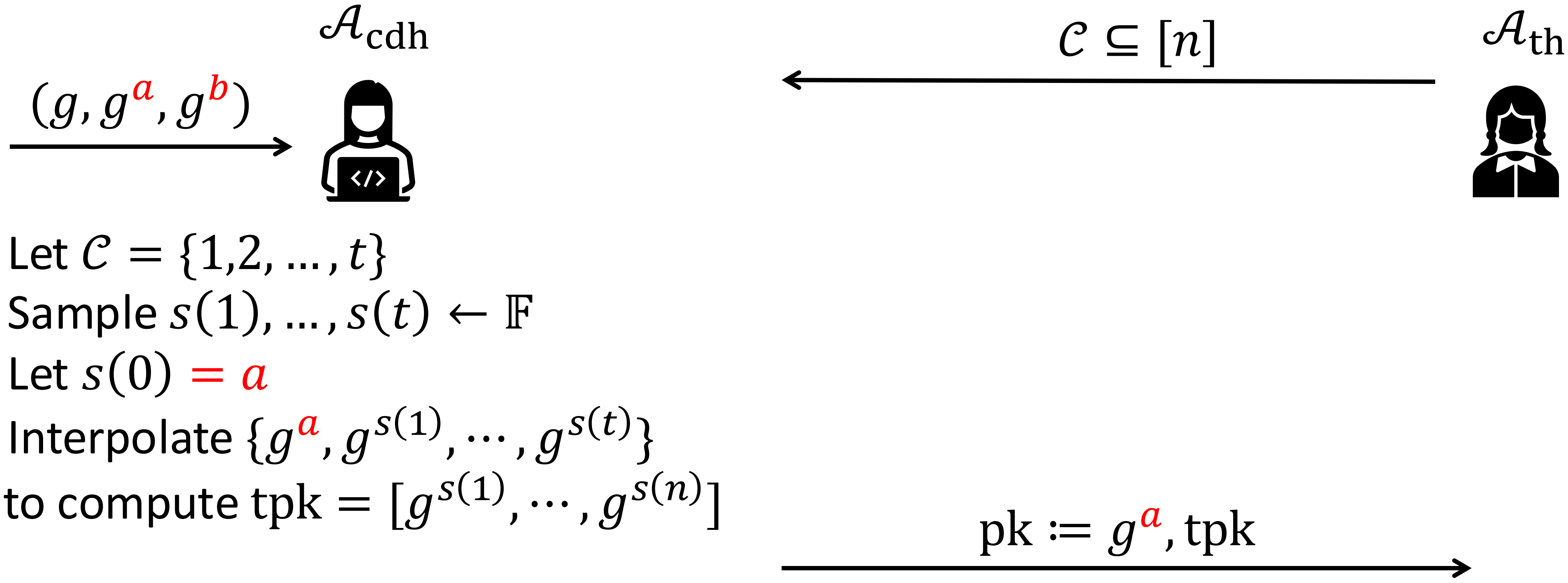
(g, g^{a}, g^{b})

$\mathcal{C} \subseteq [n]$

$\mathcal{A}_{\text{th}}$

- Let $\mathcal{C} = \{1,2, \dots, t\}$
- Sample $s(1), \dots, s(t) \leftarrow \mathbb{F}$
- Let $s(0) = a$
- Interpolate $\{g^{a}, g^{s(1)}, \cdots, g^{s(t)}\}$ to compute $\text{tpk} = [g^{s(1)}, \cdots, g^{s(n)}]$

Existing Proof Techniques: Static Security



$\mathcal{A}_{\text{cdh}}$

(g, g^{a}, g^{b})

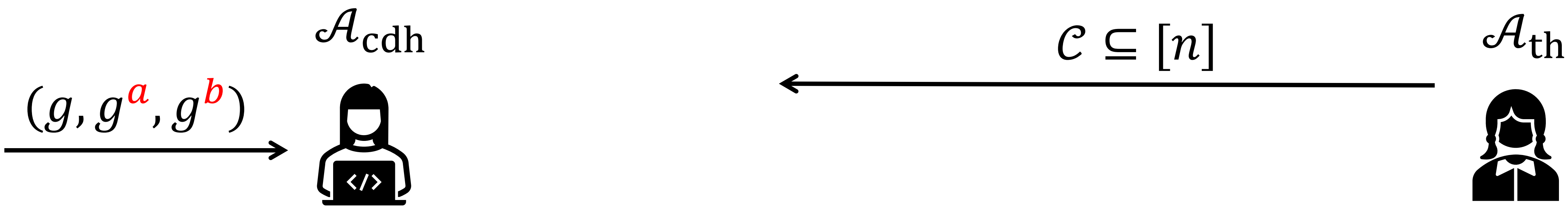
$\mathcal{C} \subseteq [n]$

$\mathcal{A}_{\text{th}}$

- Let $\mathcal{C} = \{1,2,\ldots,t\}$
- Sample $s(1),\ldots,s(t) \leftarrow \mathbb{F}$
- Let $s(0) = a$
- Interpolate $\{g^{a}, g^{s(1)}, \cdots, g^{s(t)}\}$ to compute $\text{tpk} = [g^{s(1)}, \cdots, g^{s(n)}]$

$\text{pk} \coloneqq g^{a}, \text{tpk}$

Existing Proof Techniques: Static Security



$\mathcal{A}_{\text{cdh}}$

$(g, g^{a}, g^{b}) \longrightarrow$

$\xleftarrow{\mathcal{C} \subseteq [n]}$

$\mathcal{A}_{\text{th}}$

- Let $\mathcal{C} = \{1,2, \ldots, t\}$
- Sample $s(1), \ldots, s(t) \leftarrow \mathbb{F}$
- Let $s(0) = a$
- Interpolate $\{g^{a}, g^{s(1)}, \cdots, g^{s(t)}\}$ to compute $\text{tpk} = [g^{s(1)}, \cdots, g^{s(n)}]$

$\xrightarrow{\text{pk} := g^{a}, \text{tpk}}$

$\xrightarrow{\{\text{sk}_i = s(i)\}_{i \in \mathcal{C}}}$

Existing Proof Techniques: Static Security

$\mathcal{A}_{\text{cdh}}$

$\xrightarrow{(g, g^a, g^b)}$

$\xleftarrow{\mathcal{C} \subseteq [n]}$

$\mathcal{A}_{\text{th}}$

- Let $\mathcal{C} = \{1,2, \ldots, t\}$
- Sample $s(1), \ldots, s(t) \leftarrow \mathbb{F}$
- Let $s(0) = a$
- Interpolate $\{g^a, g^{s(1)}, \cdots, g^{s(t)}\}$ to compute $\text{tpk} = [g^{s(1)}, \cdots, g^{s(n)}]$

$\xrightarrow{\text{pk} := g^a, \text{tpk}}$

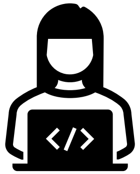
$\xrightarrow{\{\text{sk}_i = s(i)\}_{i \in \mathcal{C}}}$

$\vdots$

Our Proof: Rigged Public Key

Our Proof: Rigged Public Key

$\mathcal{A}_{\mathrm{cdh}}$



$\mathcal{A}_{\mathrm{th}}$

Our Proof: Rigged Public Key

$\mathcal{A}_{\text{cdh}}$

(g, g^{a}, g^{b})

$\mathcal{A}_{\text{th}}$

Our Proof: Rigged Public Key

$\mathcal{A}_{\text{cdh}}$

(g, g^{a}, g^{b})

$h := g^{a}$

$\mathcal{A}_{\text{th}}$

Our Proof: Rigged Public Key

$\mathcal{A}_{\text{cdh}}$

$(g, g^{a}, g^{b}) \longrightarrow$

$s(x) \coloneqq s + s_1 x + \cdots + s_t x^t$

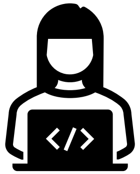
$h \coloneqq g^{a}$

$\mathcal{A}_{\text{th}}$

Our Proof: Rigged Public Key

$\mathcal{A}_{\text{cdh}}$

$$\xrightarrow{(g, g^{a}, g^{b})}$$



$$s(x) \coloneqq s + s_1 x + \cdots + s_t x^t$$
$$r(x) \coloneqq r + r_1 x + \cdots + r_t x^t$$

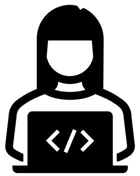
$$\boxed{h \coloneqq g^{a}}$$

$\mathcal{A}_{\text{th}}$

Our Proof: Rigged Public Key

$\mathcal{A}_{\text{cdh}}$

$$\xrightarrow{(g, g^{a}, g^{b})}$$



$$s(x) \coloneqq s + s_1 x + \cdots + s_t x^t$$

$$r(x) \coloneqq r + r_1 x + \cdots + r_t x^t$$

$$\boxed{r \neq 0}$$

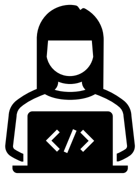
$$\boxed{h \coloneqq g^{a}}$$

$\mathcal{A}_{\text{th}}$

Our Proof: Rigged Public Key

$\mathcal{A}_{\text{cdh}}$

$$\xrightarrow{(g, g^{a}, g^{b})}$$



$\boxed{h := g^{a}}$

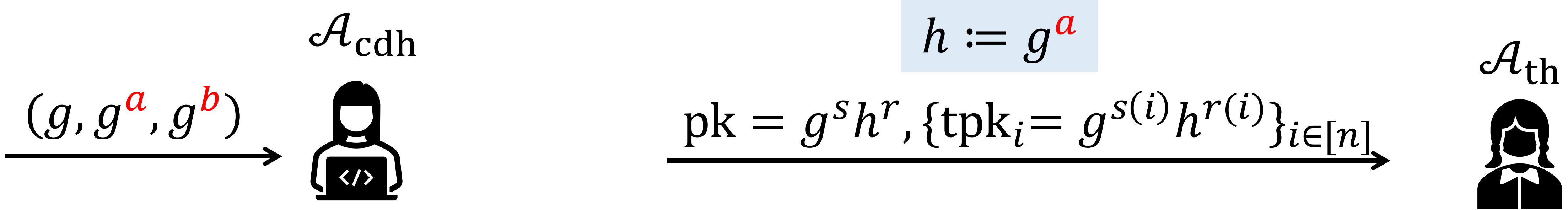
$\mathcal{A}_{\text{th}}$

$$s(x) := s + s_1 x + \cdots + s_t x^t$$
$$r(x) := r + r_1 x + \cdots + r_t x^t$$

$$\boxed{r \neq 0}$$

$$\text{sk} := (s, r);\ \ \text{sk}_i = (s(i), r(i))$$

Our Proof: Rigged Public Key



$\mathcal{A}_{\text{cdh}}$

$$(g, g^{a}, g^{b}) \longrightarrow$$

$$h \coloneqq g^{a}$$

$$\xrightarrow{\text{pk} = g^{s}h^{r}, \{\text{tpk}_i = g^{s(i)}h^{r(i)}\}_{i\in[n]}}$$

$\mathcal{A}_{\text{th}}$

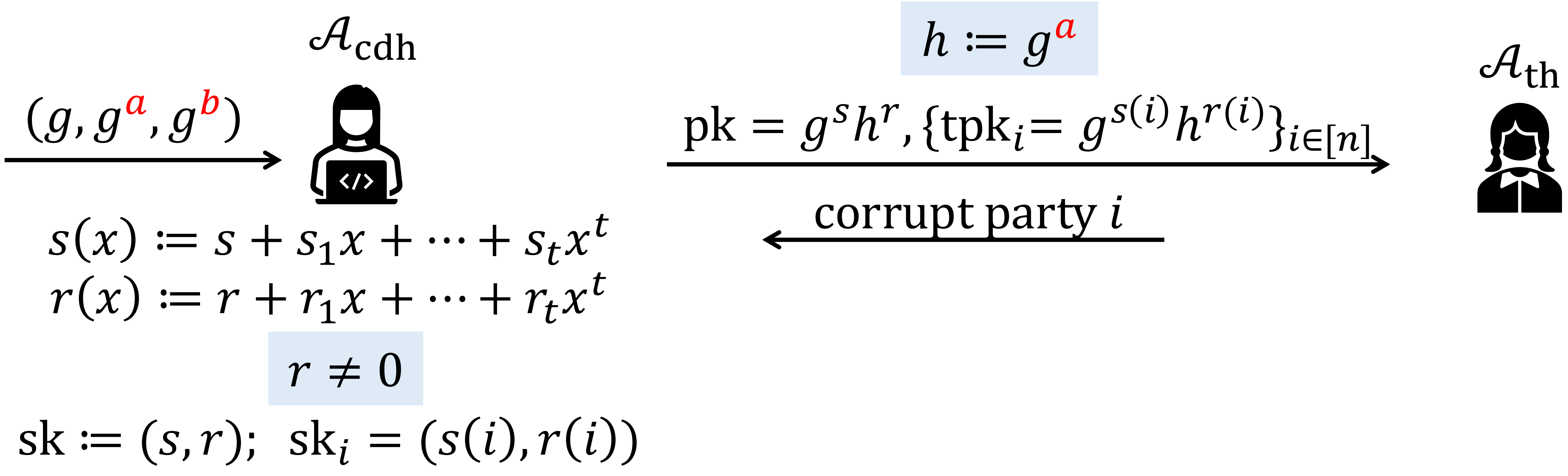
$$s(x) \coloneqq s + s_1 x + \cdots + s_t x^t$$

$$r(x) \coloneqq r + r_1 x + \cdots + r_t x^t$$

$$r \neq 0$$

$$\text{sk} \coloneqq (s, r);\ \ \text{sk}_i = (s(i), r(i))$$

Our Proof: Rigged Public Key



$\mathcal{A}_{\mathrm{cdh}}$

(g, g^{a}, g^{b})

$h := g^{a}$

$\mathrm{pk} = g^{s} h^{r}, \{\mathrm{tpk}_i = g^{s(i)} h^{r(i)}\}_{i \in [n]}$

corrupt party i

$\mathcal{A}_{\mathrm{th}}$

$$s(x) := s + s_1 x + \cdots + s_t x^t$$
$$r(x) := r + r_1 x + \cdots + r_t x^t$$

$$r \neq 0$$

$$\mathrm{sk} := (s, r);\;\; \mathrm{sk}_i = (s(i), r(i))$$

Our Proof: Rigged Public Key

$\mathcal{A}_{\text{cdh}}$

$\xrightarrow{(g, g^{a}, g^{b})}$

$$s(x) := s + s_1 x + \cdots + s_t x^t$$

$$r(x) := r + r_1 x + \cdots + r_t x^t$$

$$\boxed{r \neq 0}$$

$$\text{sk} := (s, r);\ \ \text{sk}_i = (s(i), r(i))$$

$$\boxed{h := g^{a}}$$

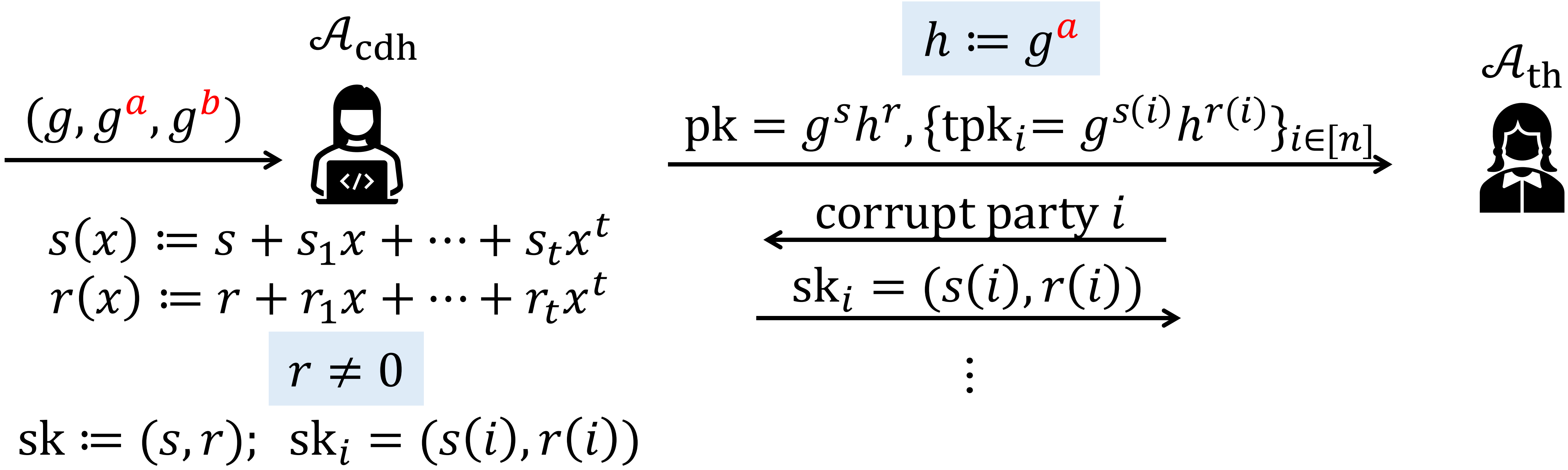
$\mathcal{A}_{\text{th}}$

$$\xrightarrow{\text{pk} = g^s h^r, \{\text{tpk}_i = g^{s(i)} h^{r(i)}\}_{i \in [n]}}$$

$$\xleftarrow{\text{corrupt party } i}$$

$$\xrightarrow{\text{sk}_i = (s(i), r(i))}$$

Our Proof: Rigged Public Key



$\mathcal{A}_{\text{cdh}}$

$\xrightarrow{(g, g^a, g^b)}$

$s(x) \coloneqq s + s_1 x + \cdots + s_t x^t$

$r(x) \coloneqq r + r_1 x + \cdots + r_t x^t$

$r \neq 0$

$\text{sk} \coloneqq (s, r);\ \ \text{sk}_i = (s(i), r(i))$

$h \coloneqq g^a$

$\xrightarrow{\text{pk} = g^s h^r, \{\text{tpk}_i = g^{s(i)} h^{r(i)}\}_{i \in [n]}}$

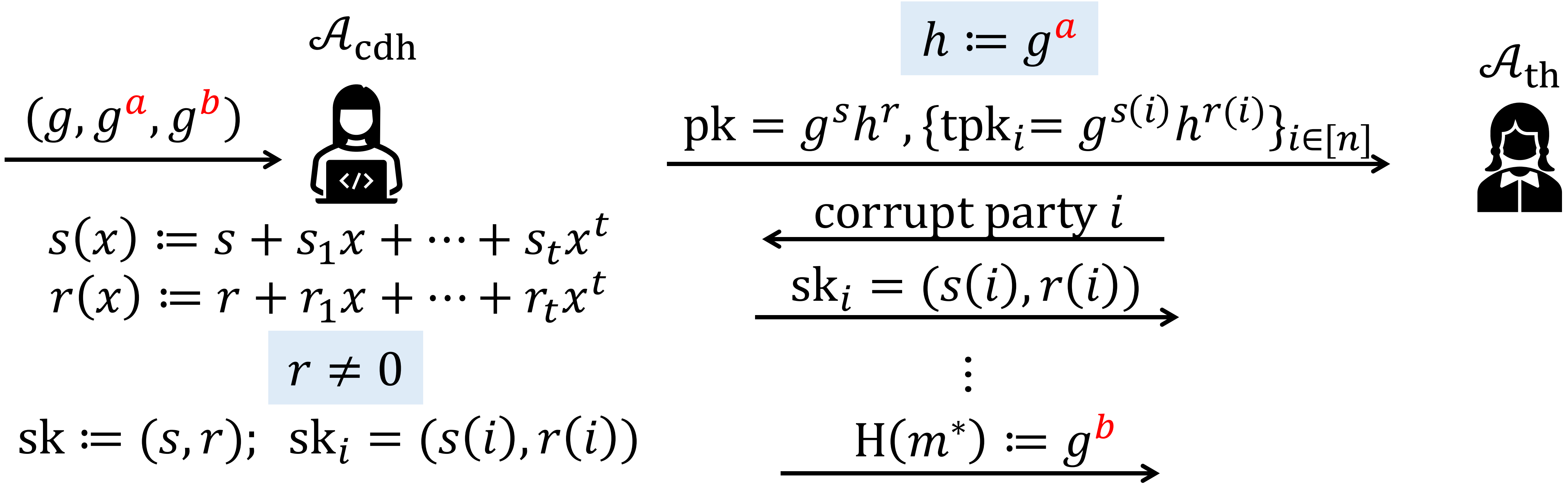
$\mathcal{A}_{\text{th}}$

$\xleftarrow{\text{corrupt party } i}$

$\xrightarrow{\text{sk}_i = (s(i), r(i))}$

$\vdots$

Our Proof: Rigged Public Key



$\mathcal{A}_{\text{cdh}}$

$(g, g^{a}, g^{b}) \rightarrow$

$$s(x) \coloneqq s + s_1 x + \cdots + s_t x^t$$

$$r(x) \coloneqq r + r_1 x + \cdots + r_t x^t$$

$$\boxed{r \neq 0}$$

$$\text{sk} \coloneqq (s, r);\ \ \text{sk}_i = (s(i), r(i))$$

$$\boxed{h \coloneqq g^{a}}$$

$\mathcal{A}_{\text{th}}$

$\xrightarrow{\text{pk} = g^s h^r, \{\text{tpk}_i = g^{s(i)} h^{r(i)}\}_{i \in [n]}}$

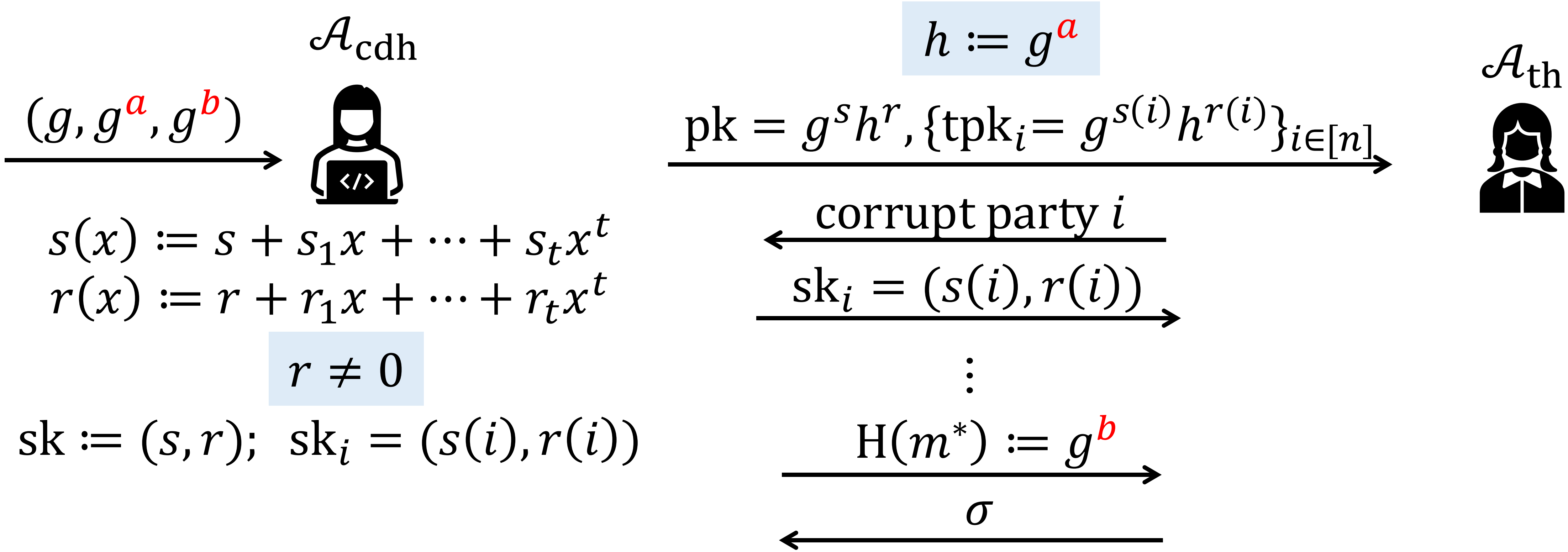
$\xleftarrow{\text{corrupt party } i}$

$\xrightarrow{\text{sk}_i = (s(i), r(i))}$

$\vdots$

$\xrightarrow{\text{H}(m^*) \coloneqq g^{b}}$

Our Proof: Rigged Public Key



$\mathcal{A}_{\text{cdh}}$

$\xrightarrow{(g, g^{a}, g^{b})}$

$$s(x) \coloneqq s + s_1 x + \cdots + s_t x^t$$
$$r(x) \coloneqq r + r_1 x + \cdots + r_t x^t$$

$\boxed{r \neq 0}$

$$\text{sk} \coloneqq (s, r); \;\; \text{sk}_i = (s(i), r(i))$$

$\boxed{h \coloneqq g^{a}}$

$\mathcal{A}_{\text{th}}$

$\xrightarrow{\text{pk} = g^s h^r, \{\text{tpk}_i = g^{s(i)} h^{r(i)}\}_{i \in [n]}}$

$\xleftarrow{\text{corrupt party } i}$

$\xrightarrow{\text{sk}_i = (s(i), r(i))}$

$\vdots$

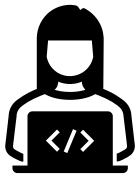
$\xrightarrow{\text{H}(m^*) \coloneqq g^{b}}$

$\xleftarrow{\sigma}$

Our Proof: Rigged Public Key

$\mathcal{A}_{\text{cdh}}$

$$\xrightarrow{(g, g^{a}, g^{b})}$$



$$s(x) \coloneqq s + s_1 x + \cdots + s_t x^t$$

$$r(x) \coloneqq r + r_1 x + \cdots + r_t x^t$$

$$r \neq 0$$

$$\text{sk} \coloneqq (s, r);\ \ \text{sk}_i = (s(i), r(i))$$

$$h \coloneqq g^{a}$$

$\mathcal{A}_{\text{th}}$

$$\xrightarrow{\text{pk} = g^{s} h^{r}, \{\text{tpk}_i = g^{s(i)} h^{r(i)}\}_{i \in [n]}}$$

$$\xleftarrow{\text{corrupt party } i}$$

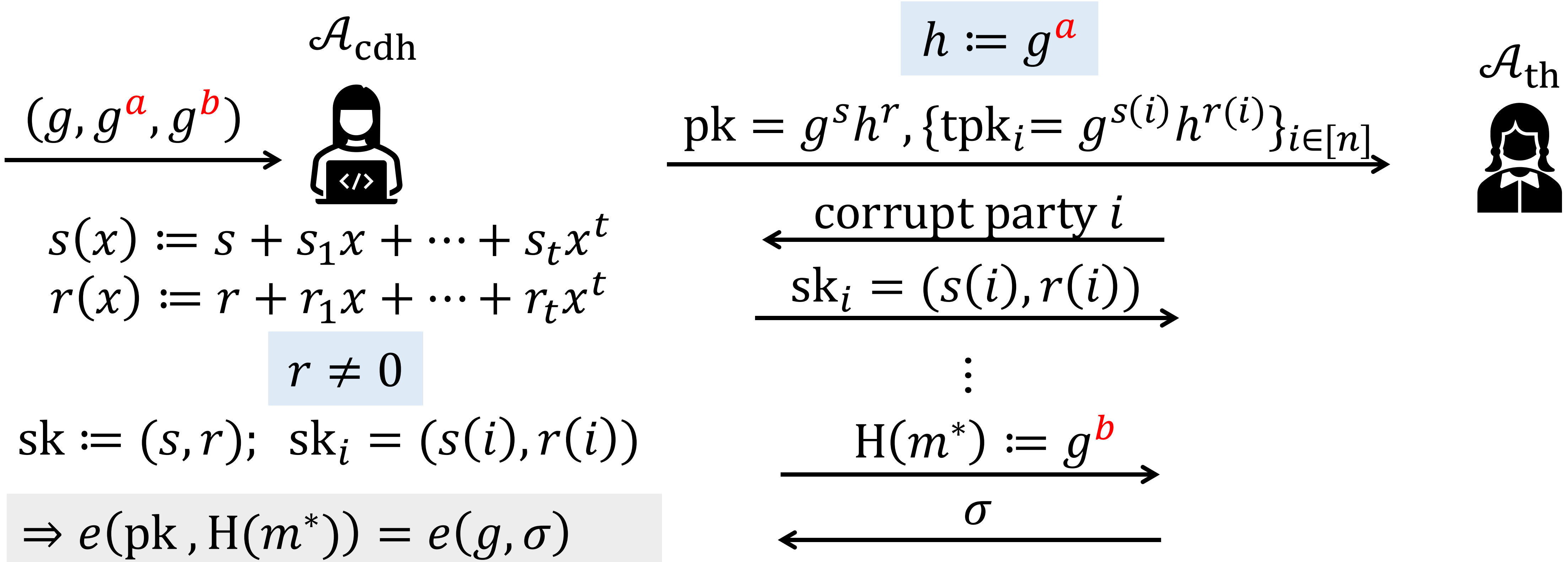
$$\xrightarrow{\text{sk}_i = (s(i), r(i))}$$

$$\vdots$$

$$\xrightarrow{\text{H}(m^*) \coloneqq g^{b}}$$

$$\xleftarrow{\sigma}$$

Our Proof: Rigged Public Key



$\mathcal{A}_{\mathrm{cdh}}$

$(g, g^{a}, g^{b}) \longrightarrow$

$h \coloneqq g^{a}$

$\mathcal{A}_{\mathrm{th}}$

$\xrightarrow{\mathrm{pk} = g^{s}h^{r}, \{\mathrm{tpk}_i = g^{s(i)}h^{r(i)}\}_{i\in[n]}}$

$\xleftarrow{\text{corrupt party } i}$

$\xrightarrow{\mathrm{sk}_i = (s(i), r(i))}$

$\vdots$

$\xrightarrow{\mathrm{H}(m^*) \coloneqq g^{b}}$

$\xleftarrow{\sigma}$

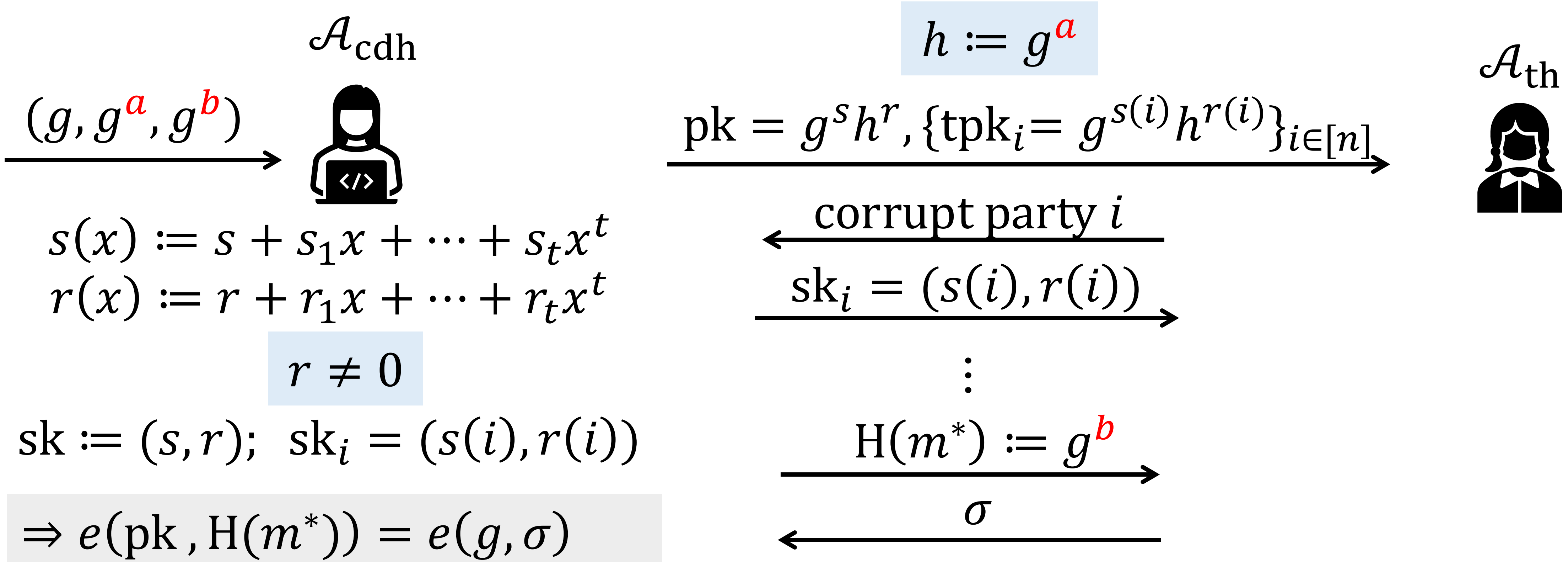
$$s(x) \coloneqq s + s_1 x + \cdots + s_t x^t$$
$$r(x) \coloneqq r + r_1 x + \cdots + r_t x^t$$

$$r \neq 0$$

$$\mathrm{sk} \coloneqq (s, r);\ \ \mathrm{sk}_i = (s(i), r(i))$$

$$\Rightarrow e(\mathrm{pk}, \mathrm{H}(m^*)) = e(g, \sigma)$$

Our Proof: Rigged Public Key



$\mathcal{A}_{\mathrm{cdh}}$

$(g, g^{a}, g^{b}) \rightarrow$

$$s(x) \coloneqq s + s_1 x + \cdots + s_t x^t$$

$$r(x) \coloneqq r + r_1 x + \cdots + r_t x^t$$

$$\boxed{r \neq 0}$$

$$\mathrm{sk} \coloneqq (s, r);\ \ \mathrm{sk}_i = (s(i), r(i))$$

$$\boxed{h \coloneqq g^{a}}$$

$\mathcal{A}_{\mathrm{th}}$

$\mathrm{pk} = g^{s} h^{r}, \{\mathrm{tpk}_i = g^{s(i)} h^{r(i)}\}_{i \in [n]} \rightarrow$

$\leftarrow$ corrupt party i

$\mathrm{sk}_i = (s(i), r(i)) \rightarrow$

$\vdots$

$\mathrm{H}(m^*) \coloneqq g^{b} \rightarrow$

$\leftarrow \sigma$

$$\Rightarrow e(\mathrm{pk}, \mathrm{H}(m^*)) = e(g, \sigma)$$

$$\Rightarrow e\left(g^{s} h^{r}, g^{b}\right) = e(g, \sigma)$$

Our Proof: Rigged Public Key

$\mathcal{A}_{\text{cdh}}$

$$\xrightarrow{(g, g^{a}, g^{b})}$$

$$s(x) \coloneqq s + s_1 x + \cdots + s_t x^t$$
$$r(x) \coloneqq r + r_1 x + \cdots + r_t x^t$$

$$r \neq 0$$

$$\text{sk} \coloneqq (s, r);\ \ \text{sk}_i = (s(i), r(i))$$

$$\Rightarrow e(\text{pk}, \text{H}(m^*)) = e(g, \sigma)$$
$$\Rightarrow e\left(g^s h^r, g^{b}\right) = e(g, \sigma)$$
$$\Rightarrow e\left(g^s g^{ar}, g^{b}\right) = e(g, \sigma)$$

$$h \coloneqq g^{a}$$

$\mathcal{A}_{\text{th}}$

$$\xrightarrow{\text{pk} = g^s h^r, \{\text{tpk}_i = g^{s(i)} h^{r(i)}\}_{i \in [n]}}$$

$$\xleftarrow{\text{corrupt party } i}$$

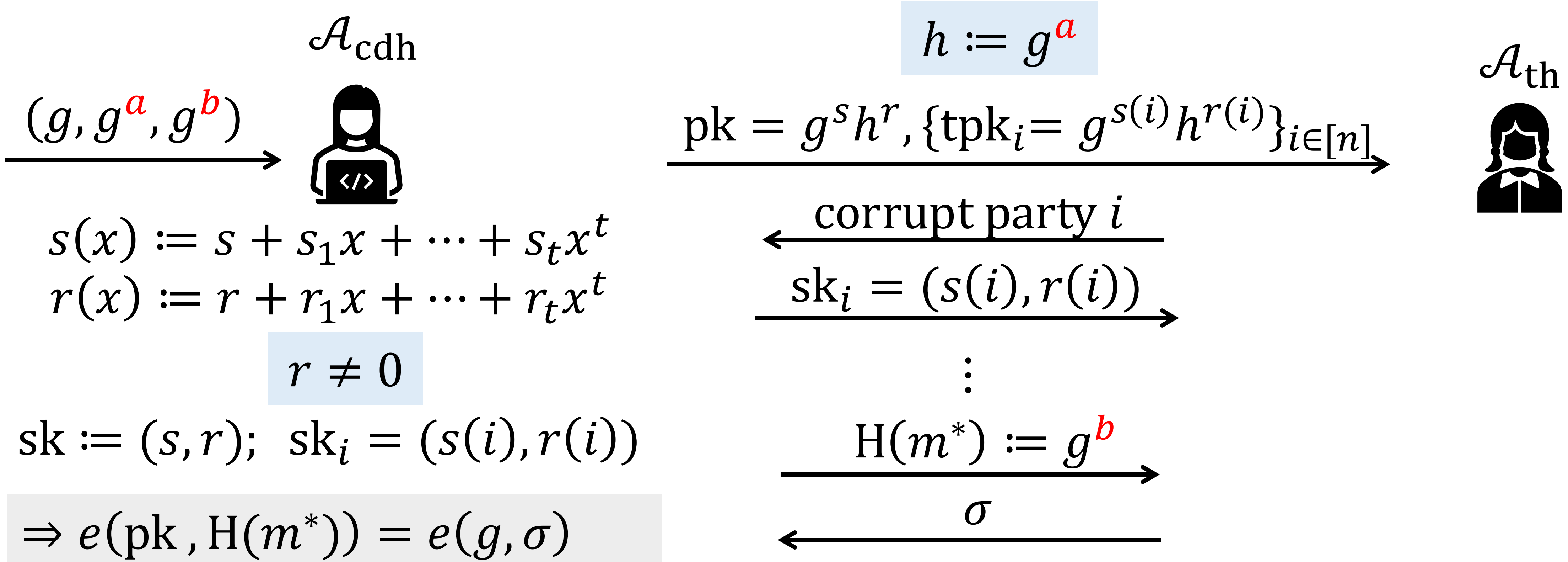
$$\xrightarrow{\text{sk}_i = (s(i), r(i))}$$

$$\vdots$$

$$\xrightarrow{\text{H}(m^*) \coloneqq g^{b}}$$

$$\xleftarrow{\sigma}$$

Our Proof: Rigged Public Key



$\mathcal{A}_{\text{cdh}}$

$$(g, g^{a}, g^{b}) \longrightarrow$$

$$s(x) \coloneqq s + s_1 x + \cdots + s_t x^t$$
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$$r \neq 0$$

$$\text{sk} \coloneqq (s, r);\ \ \text{sk}_i = (s(i), r(i))$$

$$h \coloneqq g^{a}$$

$$\xrightarrow{\text{pk} = g^s h^r, \{\text{tpk}_i = g^{s(i)} h^{r(i)}\}_{i \in [n]}}$$

$$\xleftarrow{\text{corrupt party } i}$$

$$\xrightarrow{\text{sk}_i = (s(i), r(i))}$$

$$\vdots$$

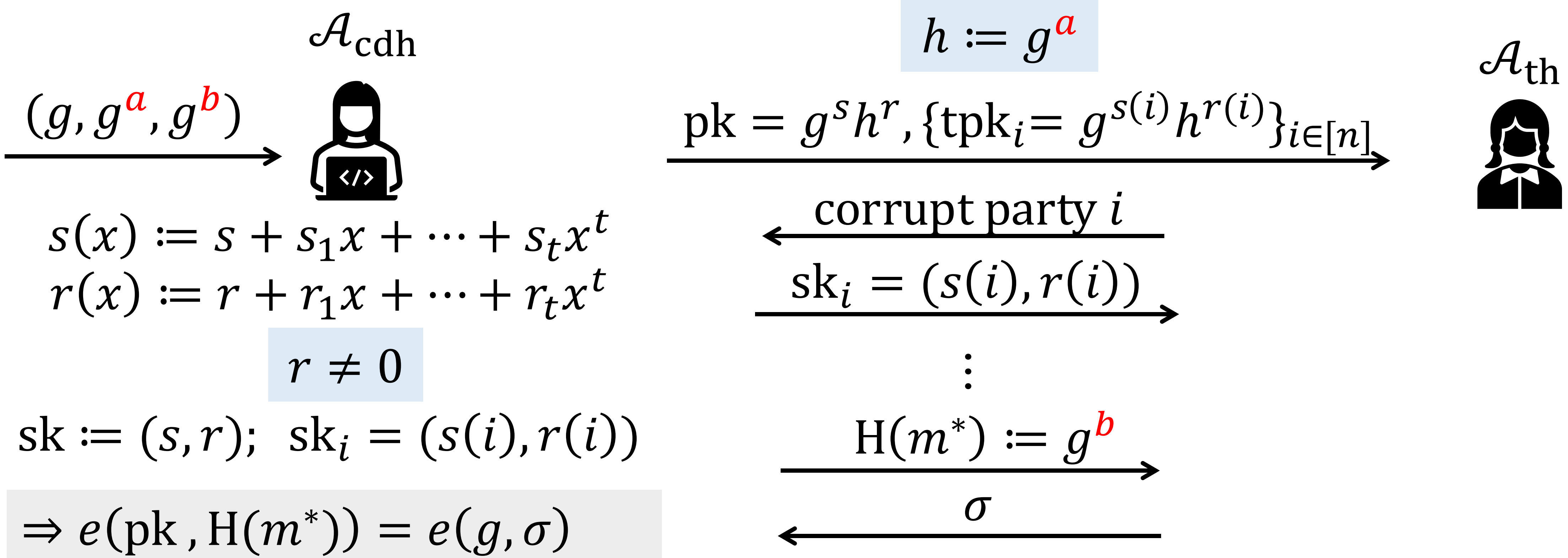
$$\xrightarrow{\text{H}(m^*) \coloneqq g^{b}}$$

$$\xleftarrow{\sigma}$$

$\mathcal{A}_{\text{th}}$

$$\Rightarrow e(\text{pk}, \text{H}(m^*)) = e(g, \sigma)$$
$$\Rightarrow e\left(g^s h^r, g^{b}\right) = e(g, \sigma)$$
$$\Rightarrow e\left(g^s g^{ar}, g^{b}\right) = e(g, \sigma)$$
$$\Rightarrow g^{ab} = \sigma^{r^{-1}} \cdot g^{-sbr^{-1}}$$

Our Proof: Rigged Public Key



$\mathcal{A}_{\mathrm{cdh}}$

(g, g^{a}, g^{b})

$s(x) \coloneqq s + s_1 x + \cdots + s_t x^t$

$r(x) \coloneqq r + r_1 x + \cdots + r_t x^t$

$r \neq 0$

$\mathrm{sk} \coloneqq (s, r);\ \ \mathrm{sk}_i = (s(i), r(i))$

$h \coloneqq g^{a}$

$\mathcal{A}_{\mathrm{th}}$

$\mathrm{pk} = g^s h^r, \{\mathrm{tpk}_i = g^{s(i)} h^{r(i)}\}_{i \in [n]}$

corrupt party i

$\mathrm{sk}_i = (s(i), r(i))$

$\vdots$

$\mathrm{H}(m^*) \coloneqq g^{b}$

σ

$\Rightarrow e(\mathrm{pk}, \mathrm{H}(m^*)) = e(g, \sigma)$

$\Rightarrow e\left(g^s h^r, g^{b}\right) = e(g, \sigma)$

$\Rightarrow e\left(g^s g^{ar}, g^{b}\right) = e(g, \sigma)$

$\Rightarrow g^{ab} = \sigma^{r^{-1}} \cdot g^{-sbr^{-1}}$

$\mathcal{A}_{\mathrm{cdh}}$ knows both $s(x)$ and $r(x)$, hence s and r

Simulating Signing Queries

Simulating Signing Queries

$\mathcal{A}_{\text{cdh}}$

$\xrightarrow{(g, g^{a}, g^{b})}$

$h \coloneqq g^{a}$

$\mathcal{A}_{\text{th}}$

Simulating Signing Queries

$\mathcal{A}_{\mathrm{cdh}}$

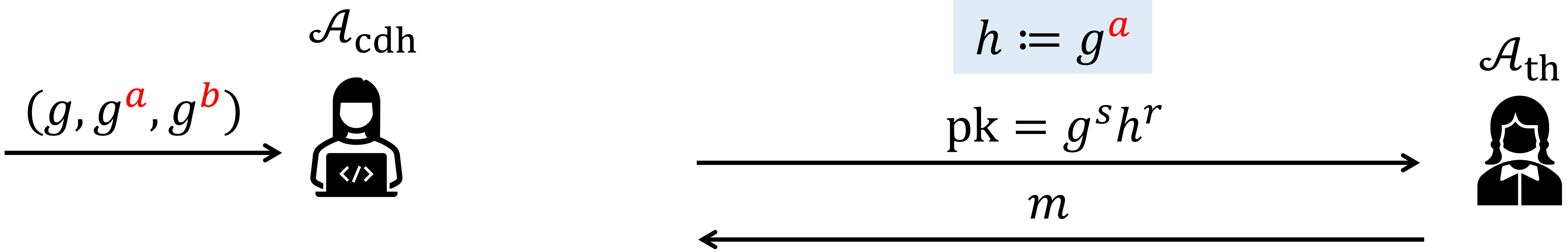
(g, g^{a}, g^{b})

$h := g^{a}$

$\mathrm{pk} = g^{s} h^{r}$

$\mathcal{A}_{\mathrm{th}}$

Simulating Signing Queries



$\mathcal{A}_{\text{cdh}}$

(g, g^{a}, g^{b})

$h \coloneqq g^{a}$

$\mathrm{pk} = g^{s}h^{r}$

m

$\mathcal{A}_{\text{th}}$

Simulating Signing Queries

$\xrightarrow{(g, g^{a}, g^{b})}$ $\mathcal{A}_{\mathrm{cdh}}$

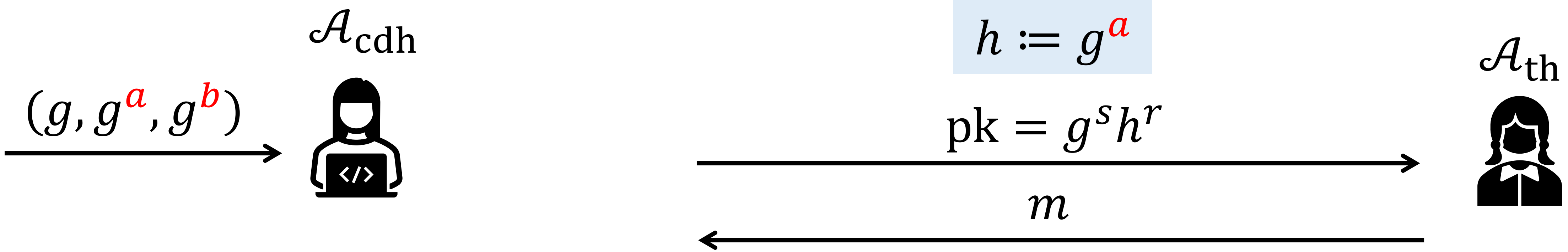
$h := g^{a}$

$\xrightarrow{\mathrm{pk} = g^{s} h^{r}}$ $\mathcal{A}_{\mathrm{th}}$

$\xleftarrow{m}$

1. Program $\mathrm{H}(m) = g^{x}$

Simulating Signing Queries



$\mathcal{A}_{\text{cdh}}$

(g, g^{a}, g^{b})

$h \coloneqq g^{a}$

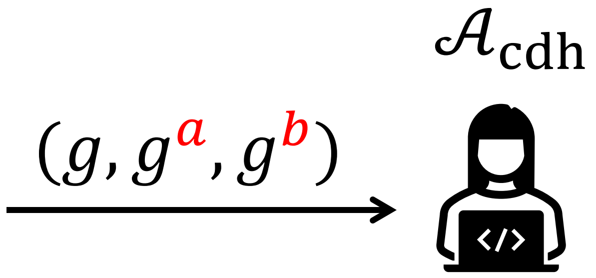
$\mathcal{A}_{\text{th}}$

$\text{pk} = g^{s}h^{r}$

m

1. Program $\text{H}(m) = g^{x}$
2. Let $\sigma = \text{H}(m)^{s}$

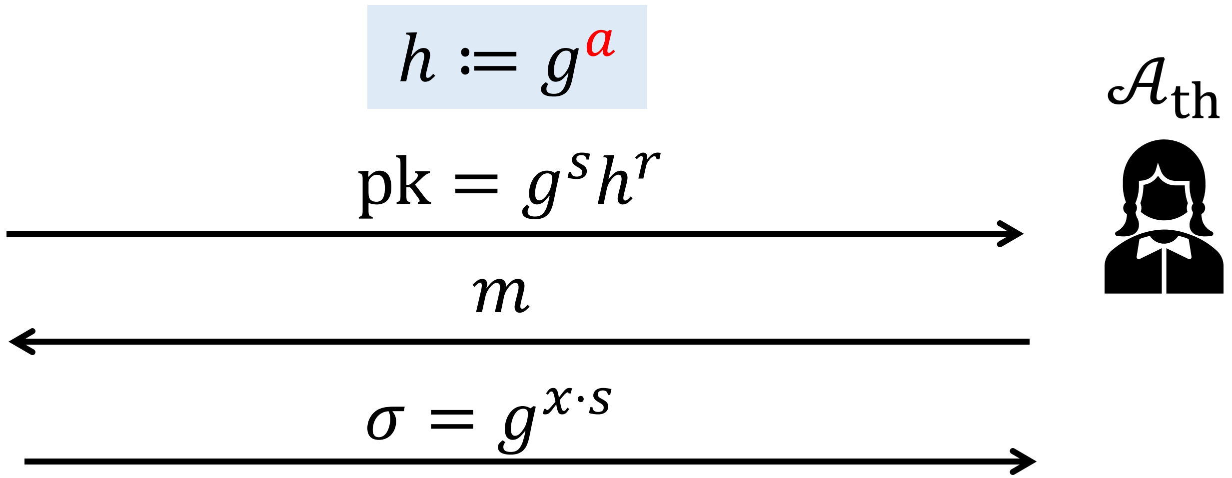
Simulating Signing Queries



$\mathcal{A}_{\text{cdh}}$

(g, g^{a}, g^{b})

1. Program $\text{H}(m) = g^{x}$
2. Let $\sigma = \text{H}(m)^{s}$



$h \coloneqq g^{a}$

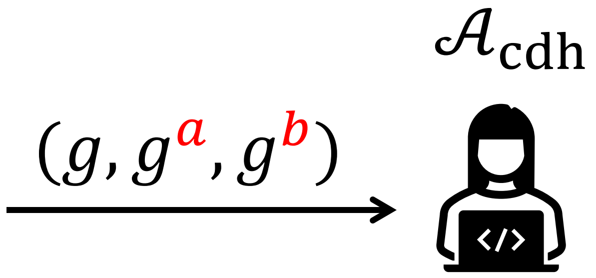
$\mathcal{A}_{\text{th}}$

$\text{pk} = g^{s}h^{r}$

m

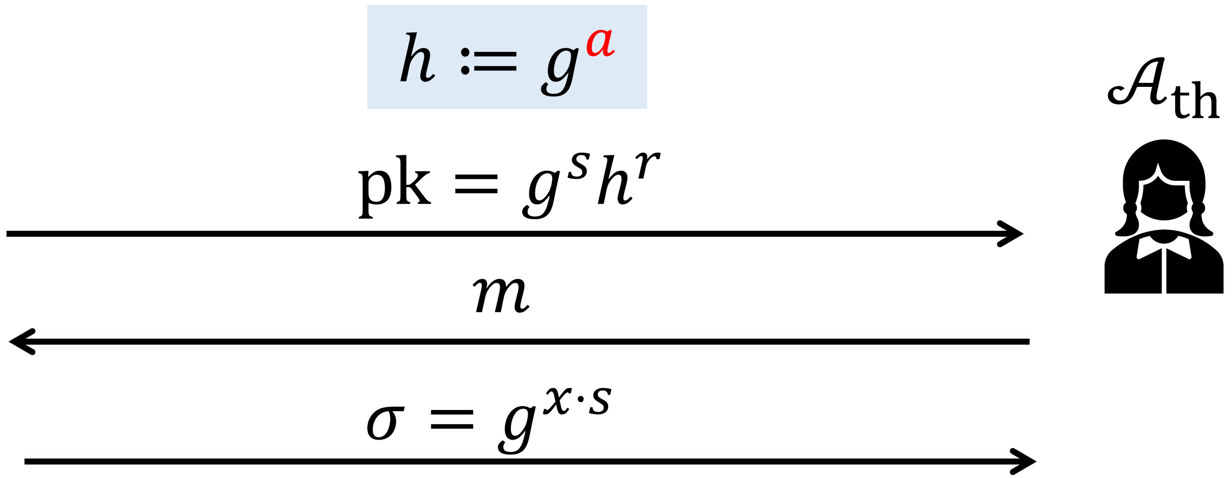
$\sigma = g^{x \cdot s}$

Simulating Signing Queries



$\mathcal{A}_{\mathrm{cdh}}$

(g, g^{a}, g^{b})



$h := g^{a}$

$\mathcal{A}_{\mathrm{th}}$

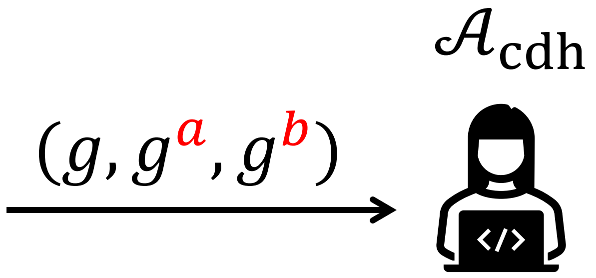
$\mathrm{pk} = g^{s}h^{r}$

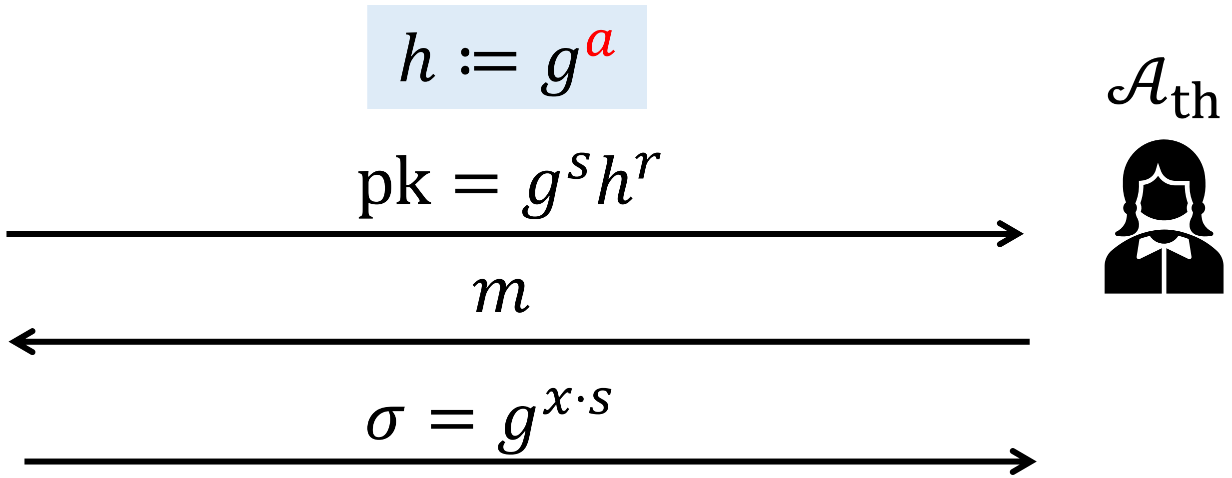
m

$\sigma = g^{x \cdot s}$

1. Program $\mathrm{H}(m) = g^{x}$
2. Let $\sigma = \mathrm{H}(m)^{s}$

Simulating Signing Queries





1. Program $\mathrm{H}(m) = g^{x}$
2. Let $\sigma = \mathrm{H}(m)^{s}$

$\Rightarrow e(\text{pk}, \mathrm{H}(m)) = e(g, \sigma)$

Simulating Signing Queries

$\mathcal{A}_{\mathrm{cdh}}$

$\xrightarrow{(g, g^{a}, g^{b})}$

1. Program $\mathrm{H}(m) = g^{x}$
2. Let $\sigma = \mathrm{H}(m)^{s}$

$h := g^{a}$

$\mathcal{A}_{\mathrm{th}}$

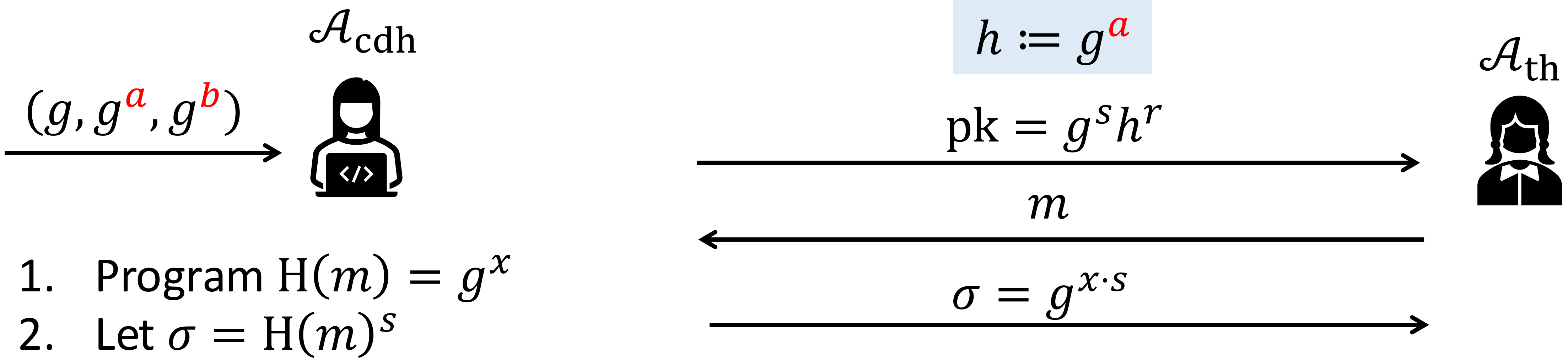
$\xrightarrow{\mathrm{pk} = g^{s} h^{r}}$

$\xleftarrow{m}$

$\xrightarrow{\sigma = g^{x \cdot s}}$

$$\Rightarrow e(\mathrm{pk}, \mathrm{H}(m)) = e(g, \sigma)$$
$$\Rightarrow e(g^{s+ar}, g^{x}) = e(g, g^{x \cdot s})$$

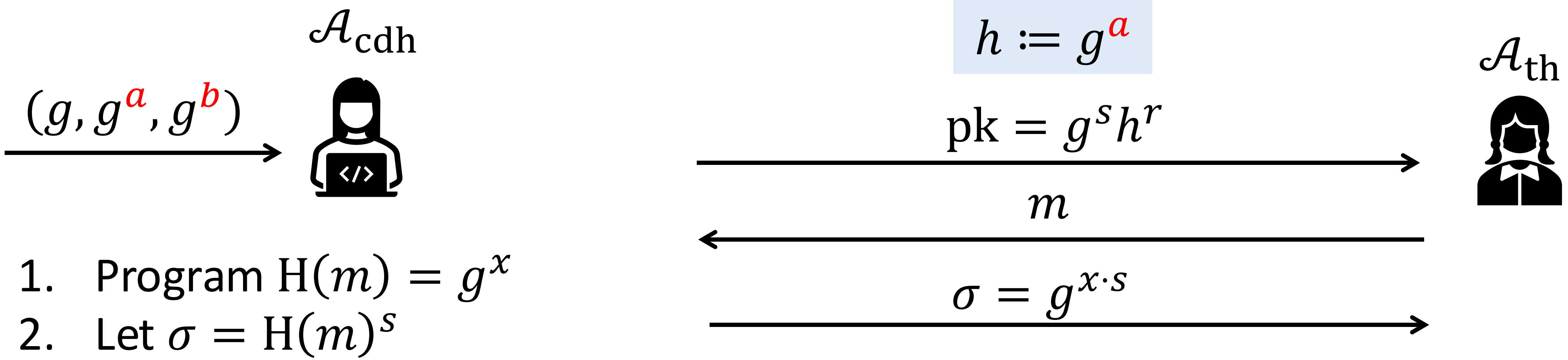
Simulating Signing Queries



1. Program $\text{H}(m) = g^{x}$
2. Let $\sigma = \text{H}(m)^{s}$

$$\Rightarrow e(\text{pk}, \text{H}(m)) = e(g, \sigma)$$
$$\Rightarrow e(g^{s+ar}, g^{x}) = e(g, g^{x \cdot s})$$
$$\Rightarrow r = 0$$

Simulating Signing Queries



$\mathcal{A}_{\text{cdh}}$

(g, g^{a}, g^{b})

$h \coloneqq g^{a}$

$\mathcal{A}_{\text{th}}$

$\text{pk} = g^{s}h^{r}$

m

$\sigma = g^{x \cdot s}$

1. Program $\text{H}(m) = g^{x}$
2. Let $\sigma = \text{H}(m)^{s}$

$\Rightarrow e(\text{pk}, \text{H}(m)) = e(g, \sigma)$

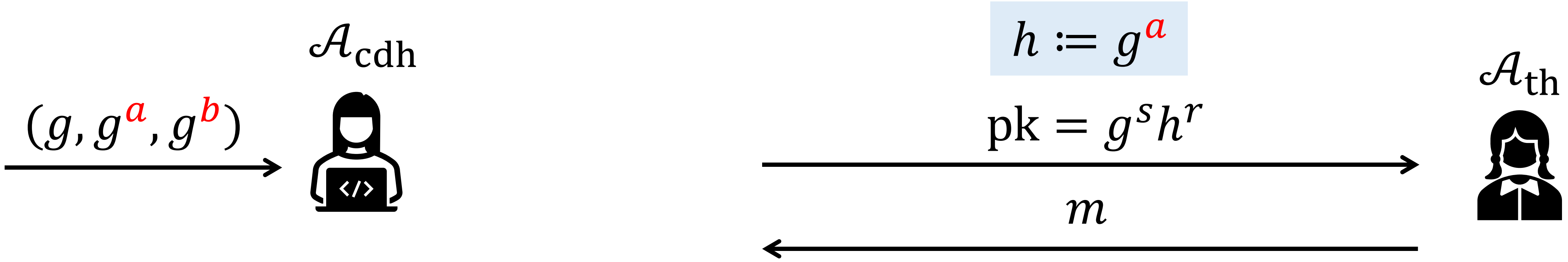
$\Rightarrow e(g^{s+ar}, g^{x}) = e(g, g^{x \cdot s})$

$\Rightarrow r = 0$

We will need new ideas

Simulating Signing Queries

Simulating Signing Queries



$\mathcal{A}_{\mathrm{cdh}}$

(g, g^{a}, g^{b})

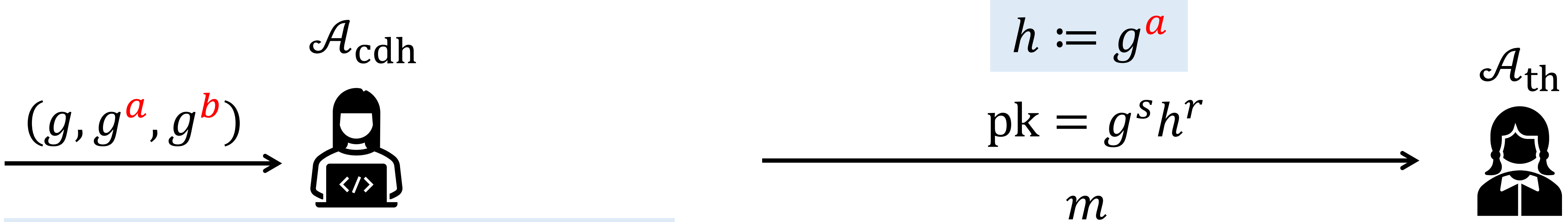
$h := g^{a}$

$\mathrm{pk} = g^{s} h^{r}$

m

$\mathcal{A}_{\mathrm{th}}$

Simulating Signing Queries


$\mathcal{A}_{\text{cdh}}$

(g, g^{a}, g^{b})

$h := g^{a}$

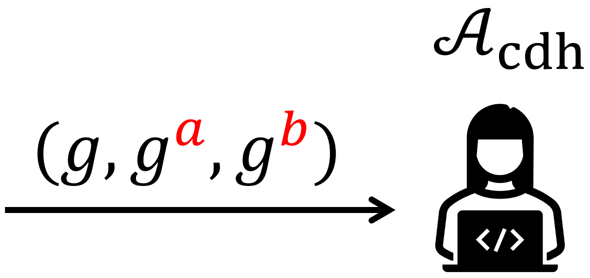
$\mathcal{A}_{\text{th}}$

$\text{pk} = g^{s}h^{r}$

m

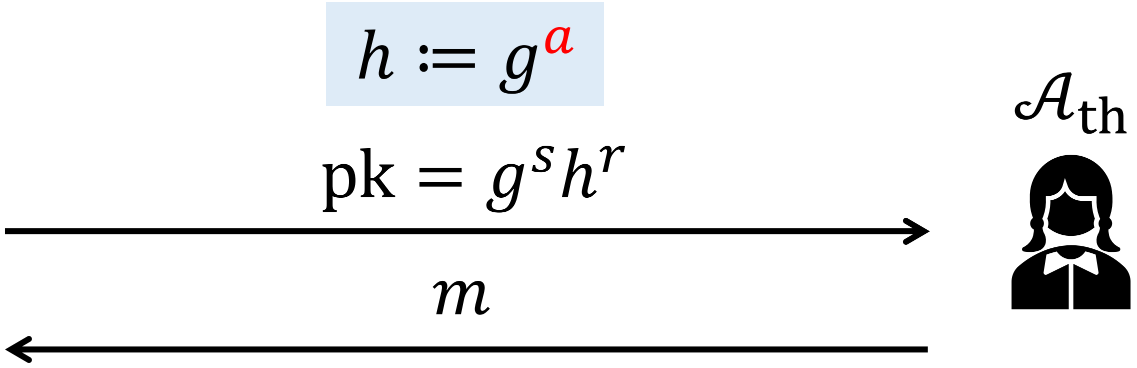
Idea: Use **two** random oracles!

Simulating Signing Queries



$\mathcal{A}_{\mathrm{cdh}}$

(g, g^{a}, g^{b})



$h := g^{a}$

$\mathcal{A}_{\mathrm{th}}$

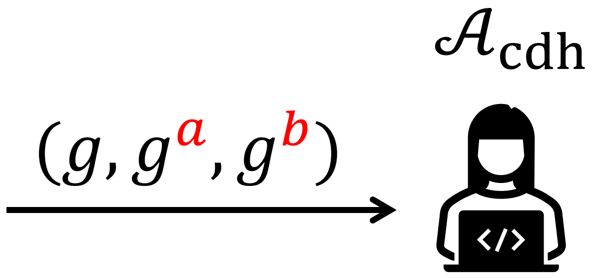
$\mathrm{pk} = g^{s}h^{r}$

m

Idea: Use **two** random oracles!

$\mathrm{H}(m) = g^{x}$ $\qquad \widehat{\mathrm{H}}(m) = g^{y}$

Simulating Signing Queries

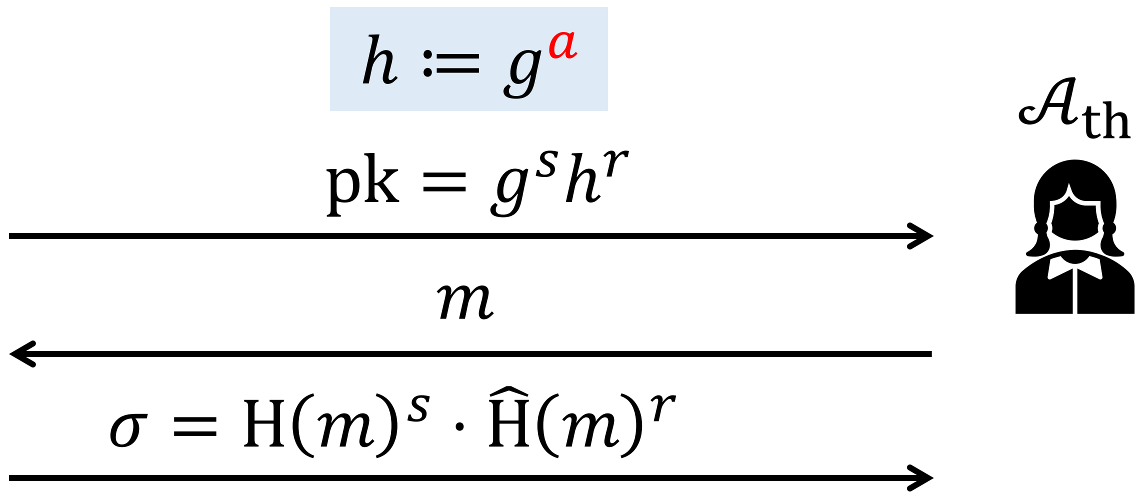


$\mathcal{A}_{\text{cdh}}$

(g, g^{a}, g^{b})

Idea: Use **two** random oracles!

$\mathrm{H}(m) = g^{x}$ $\qquad$ $\widehat{\mathrm{H}}(m) = g^{y}$



$h \coloneqq g^{a}$

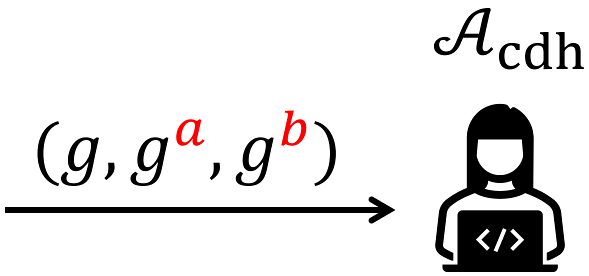
$\mathcal{A}_{\text{th}}$

$\mathrm{pk} = g^{s} h^{r}$

m

$\sigma = \mathrm{H}(m)^{s} \cdot \widehat{\mathrm{H}}(m)^{r}$

Simulating Signing Queries

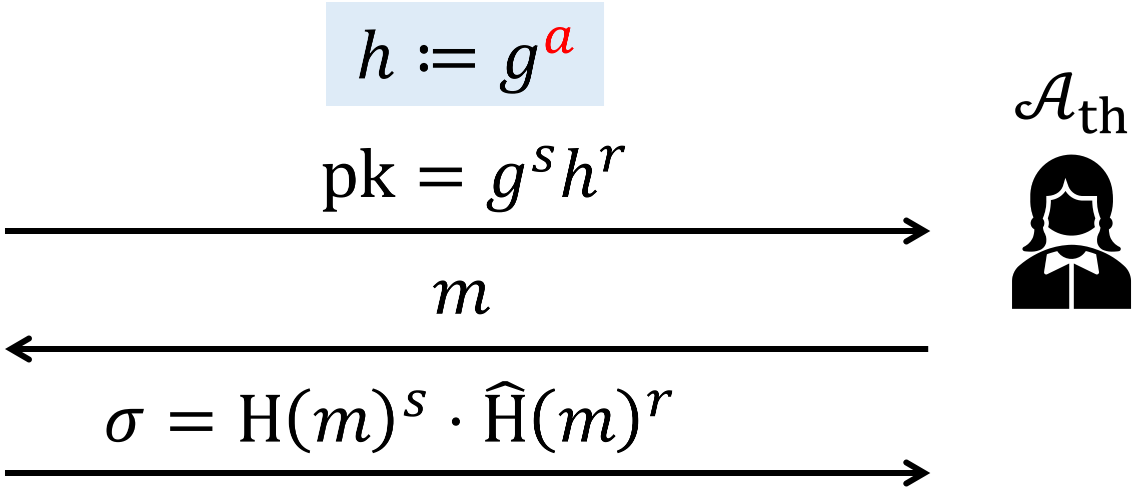


$\mathcal{A}_{\text{cdh}}$

(g, g^{a}, g^{b})

Idea: Use **two** random oracles!

$\mathrm{H}(m) = g^{x}$ $\widehat{\mathrm{H}}(m) = g^{y}$



$h := g^{a}$

$\mathcal{A}_{\text{th}}$

$\text{pk} = g^{s}h^{r}$

m

$\sigma = \mathrm{H}(m)^{s} \cdot \widehat{\mathrm{H}}(m)^{r}$

Simulating Signing Queries

$\mathcal{A}_{\text{cdh}}$

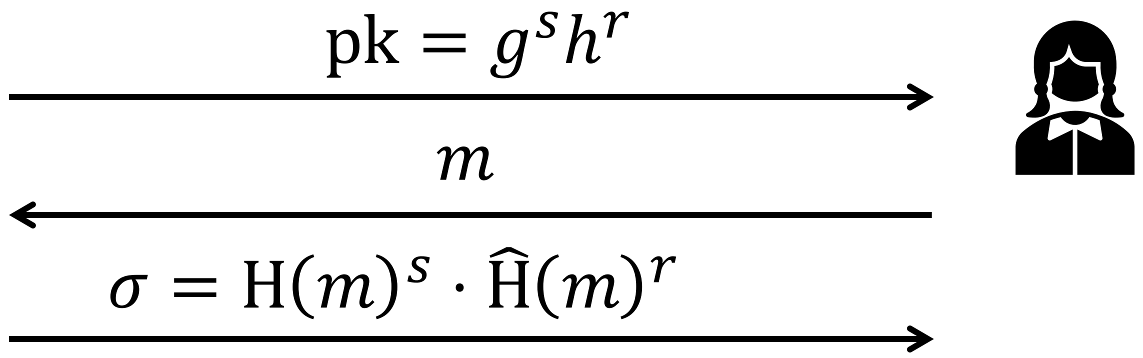
$\xrightarrow{(g, g^{a}, g^{b})}$

Idea: Use **two** random oracles!

$\mathrm{H}(m) = g^{x} \qquad \widehat{\mathrm{H}}(m) = g^{y}$

$h \coloneqq g^{a}$

$\mathcal{A}_{\text{th}}$



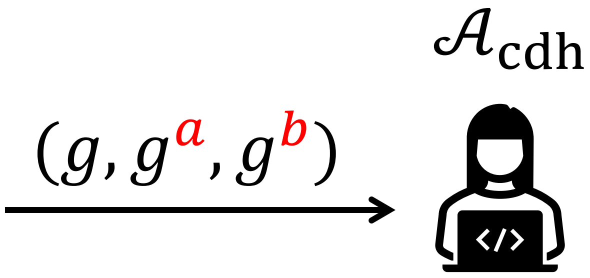
$\xrightarrow{\mathrm{pk} = g^{s} h^{r}}$

$\xleftarrow{m}$

$\xrightarrow{\sigma = \mathrm{H}(m)^{s} \cdot \widehat{\mathrm{H}}(m)^{r}}$

$\Rightarrow e(\mathrm{pk}, \mathrm{H}(m)) = e(g, \sigma)$

Simulating Signing Queries

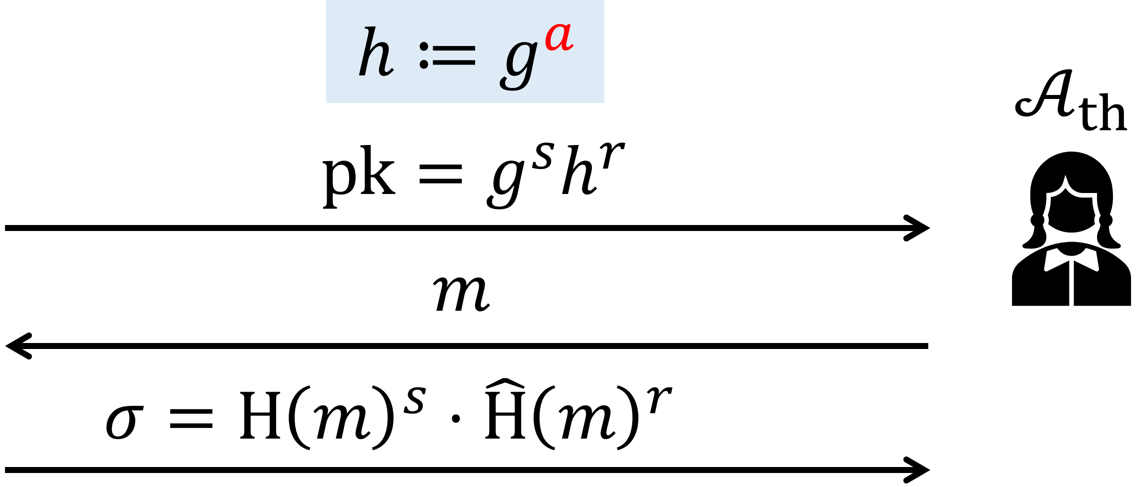


$\mathcal{A}_{\text{cdh}}$

(g, g^{a}, g^{b})

Idea: Use **two** random oracles!

$\text{H}(m) = g^{x} \qquad \widehat{\text{H}}(m) = g^{y}$



$h := g^{a}$

$\mathcal{A}_{\text{th}}$

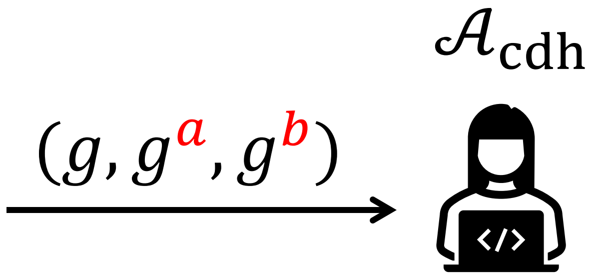
$\xrightarrow{\text{pk} = g^{s}h^{r}}$

$\xleftarrow{m}$

$\xrightarrow{\sigma = \text{H}(m)^{s} \cdot \widehat{\text{H}}(m)^{r}}$

$$\Rightarrow e(\text{pk}, \text{H}(m)) = e(g, \sigma)$$
$$\Rightarrow e(g^{s+ar}, g^{x}) = e\left(g, \text{H}(m)^{s} \cdot \widehat{\text{H}}(m)^{r}\right)$$

Simulating Signing Queries

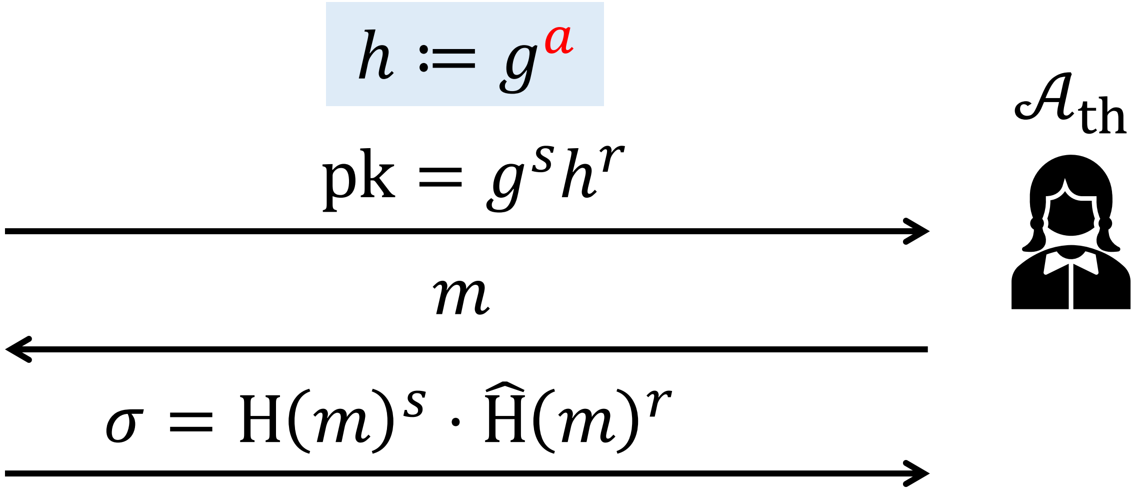


$\mathcal{A}_{\mathrm{cdh}}$

(g, g^{a}, g^{b})

Idea: Use **two** random oracles!

$$\mathrm{H}(m) = g^{x} \qquad \widehat{\mathrm{H}}(m) = g^{y}$$



$h := g^{a}$

$\mathcal{A}_{\mathrm{th}}$

$\mathrm{pk} = g^{s} h^{r}$

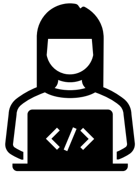
m

$\sigma = \mathrm{H}(m)^{s} \cdot \widehat{\mathrm{H}}(m)^{r}$

$$\Rightarrow e(\mathrm{pk}, \mathrm{H}(m)) = e(g, \sigma)$$
$$\Rightarrow e(g^{s+ar}, g^{x}) = e\left(g, \mathrm{H}(m)^{s} \cdot \widehat{\mathrm{H}}(m)^{r}\right)$$
$$\Rightarrow ax = y$$

Simulating Signing Queries

$\mathcal{A}_{\text{cdh}}$



$(g, g^{a}, g^{b}) \longrightarrow$

Idea: Use **two** random oracles!

$\mathrm{H}(m) = g^{x} \qquad \widehat{\mathrm{H}}(m) = g^{y}$

$h \coloneqq g^{a}$

$\mathcal{A}_{\text{th}}$

$\xrightarrow{\mathrm{pk} = g^{s} h^{r}}$

$\xleftarrow{m}$

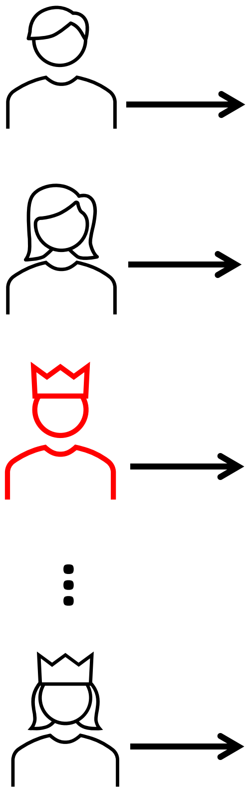
$\xrightarrow{\sigma = \mathrm{H}(m)^{s} \cdot \widehat{\mathrm{H}}(m)^{r}}$

$$\Rightarrow e(\mathrm{pk}\,, \mathrm{H}(m)) = e(g, \sigma)$$
$$\Rightarrow e(g^{s+ar}, g^{x}) = e\left(g, \mathrm{H}(m)^{s} \cdot \widehat{\mathrm{H}}(m)^{r}\right)$$
$$\Rightarrow ax = y$$

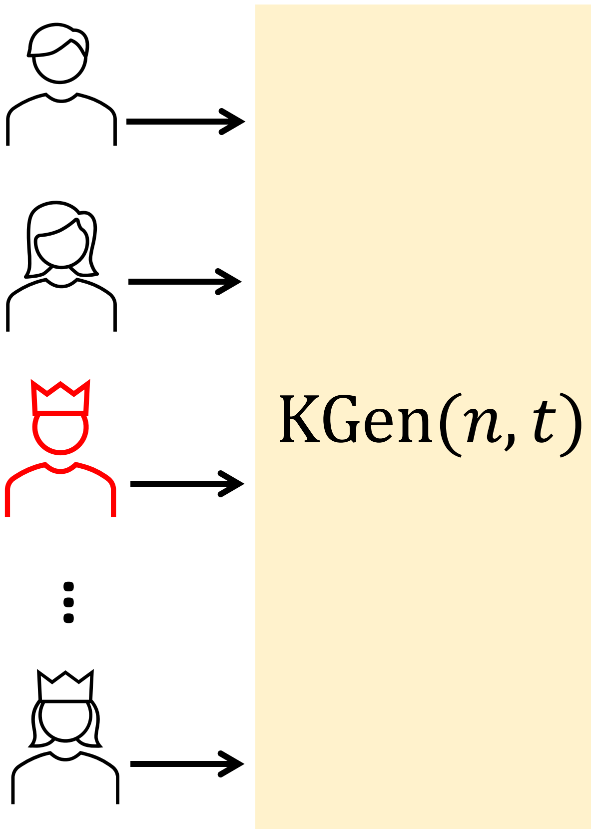
No security proof for this construction :(

Our Final Protocol: Key Generation

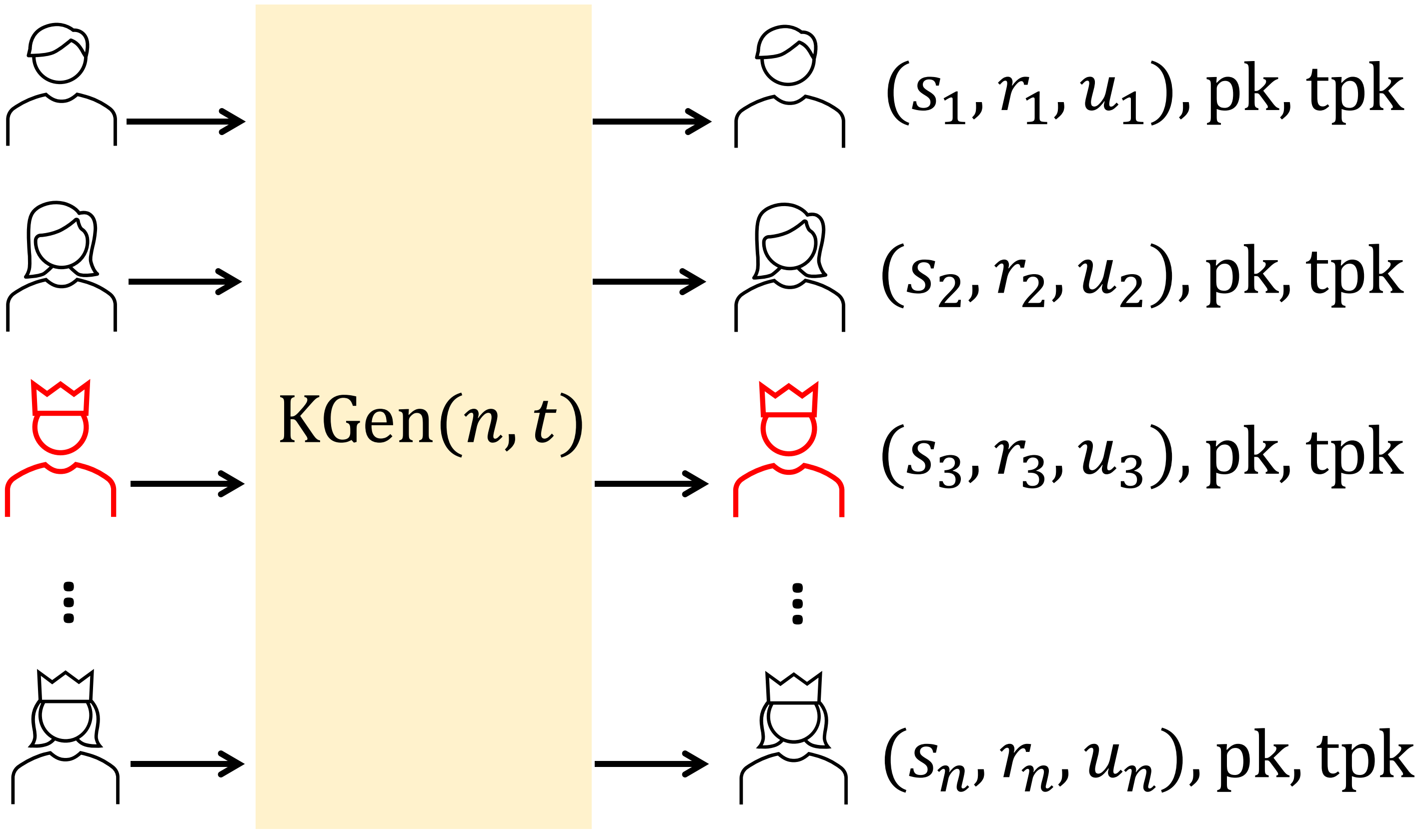
Our Final Protocol: Key Generation



Our Final Protocol: Key Generation



Our Final Protocol: Key Generation



$\mathrm{KGen}(n, t)$

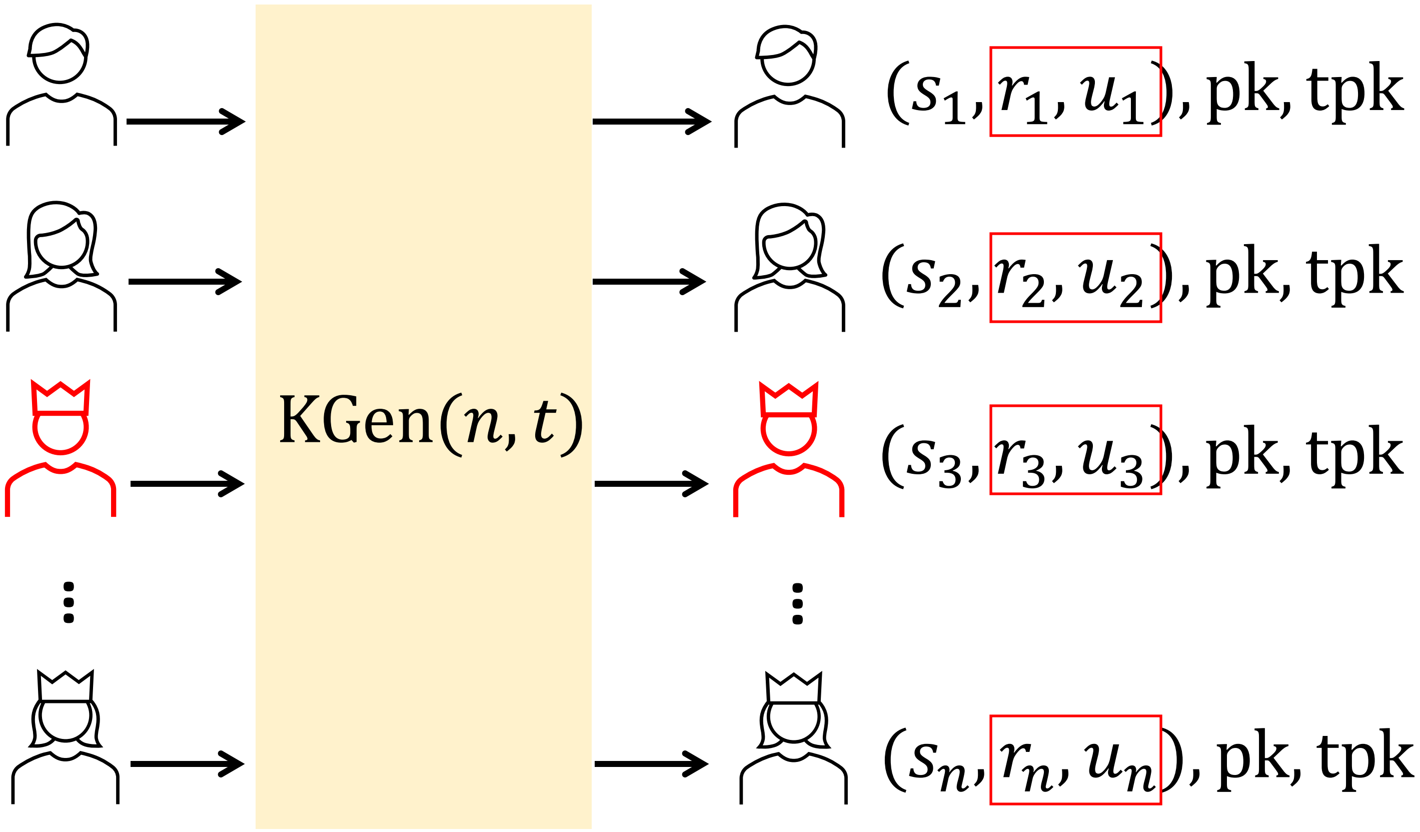
$(s_1, r_1, u_1), \mathrm{pk}, \mathrm{tpk}$

$(s_2, r_2, u_2), \mathrm{pk}, \mathrm{tpk}$

$(s_3, r_3, u_3), \mathrm{pk}, \mathrm{tpk}$

$(s_n, r_n, u_n), \mathrm{pk}, \mathrm{tpk}$

Our Final Protocol: Key Generation



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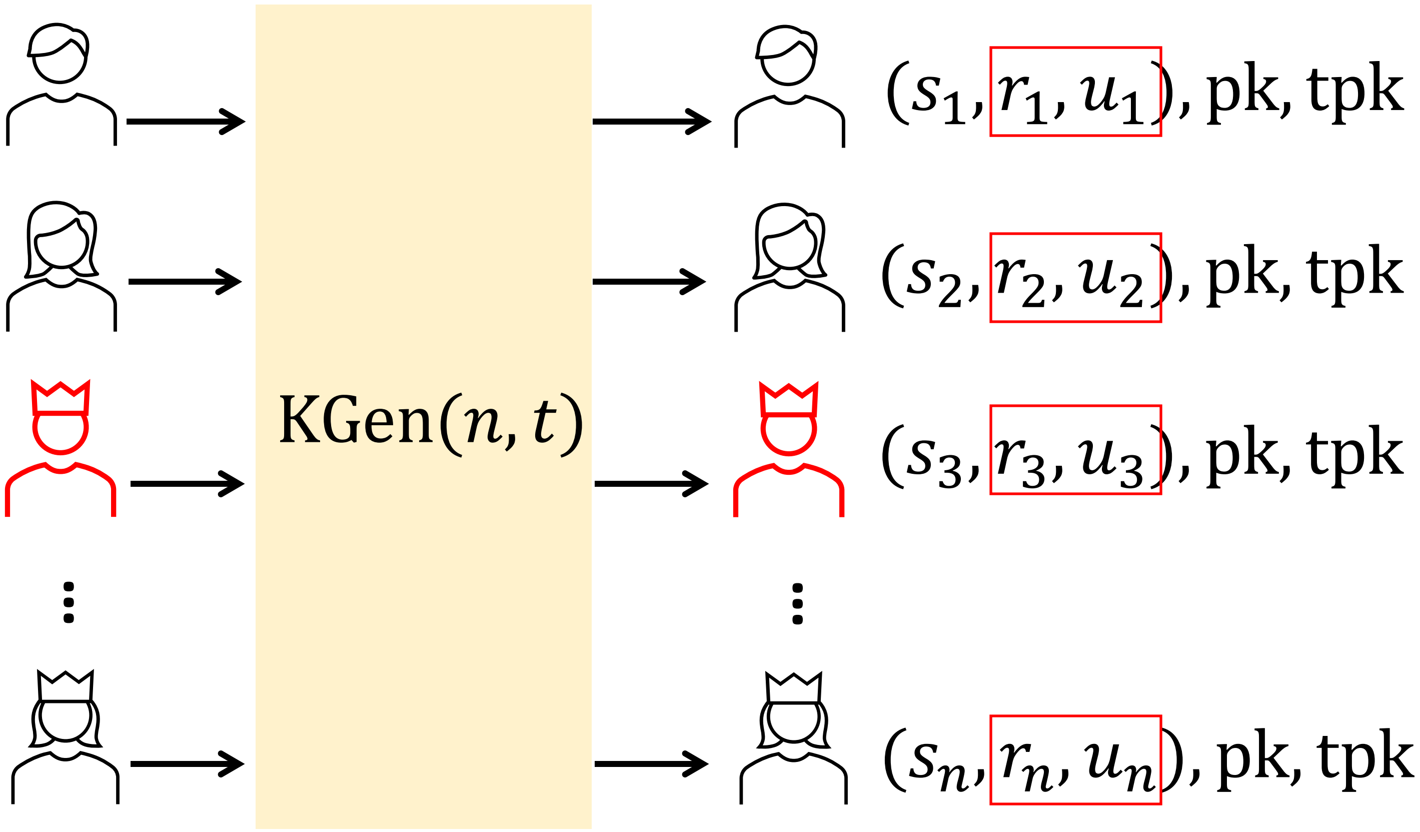

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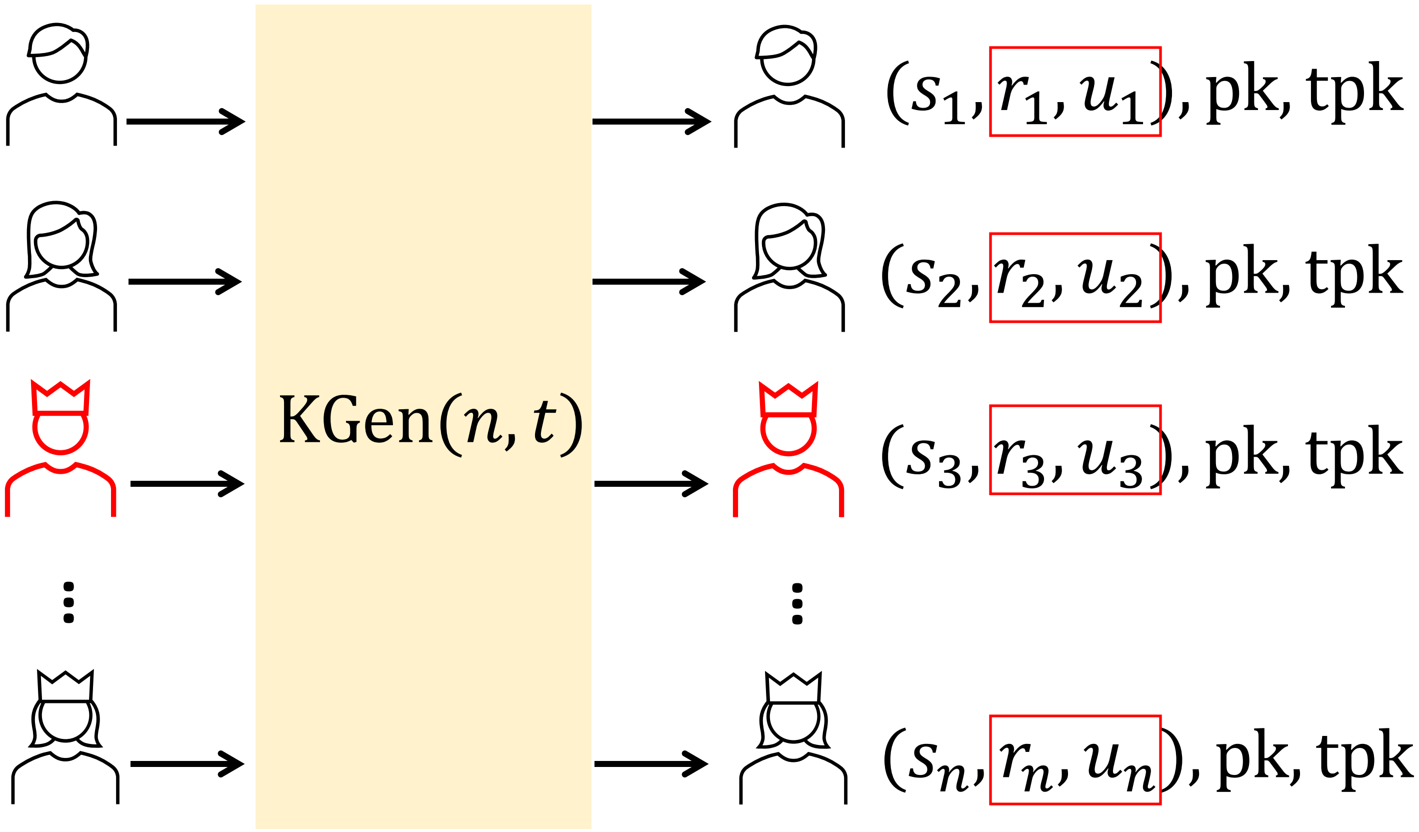

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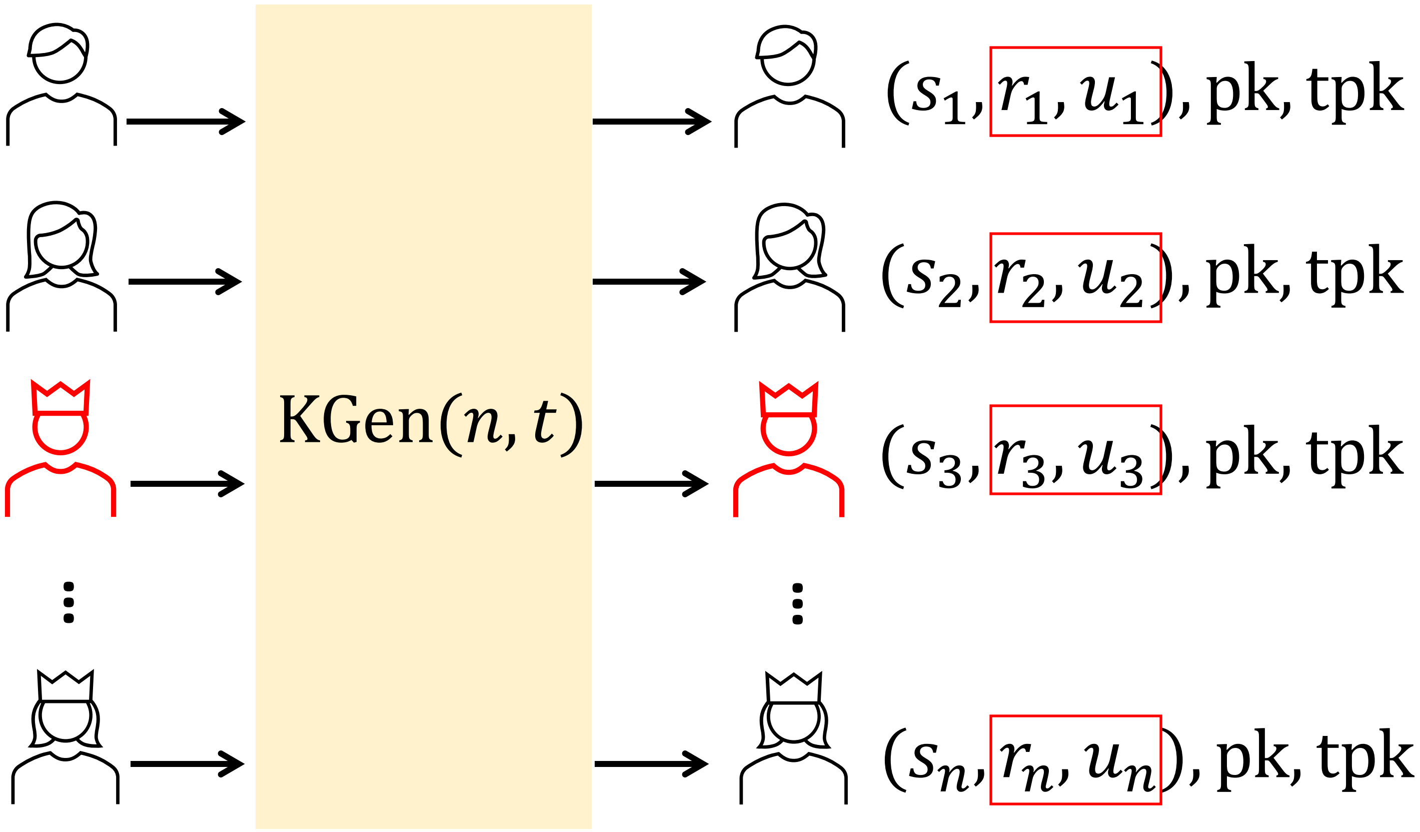

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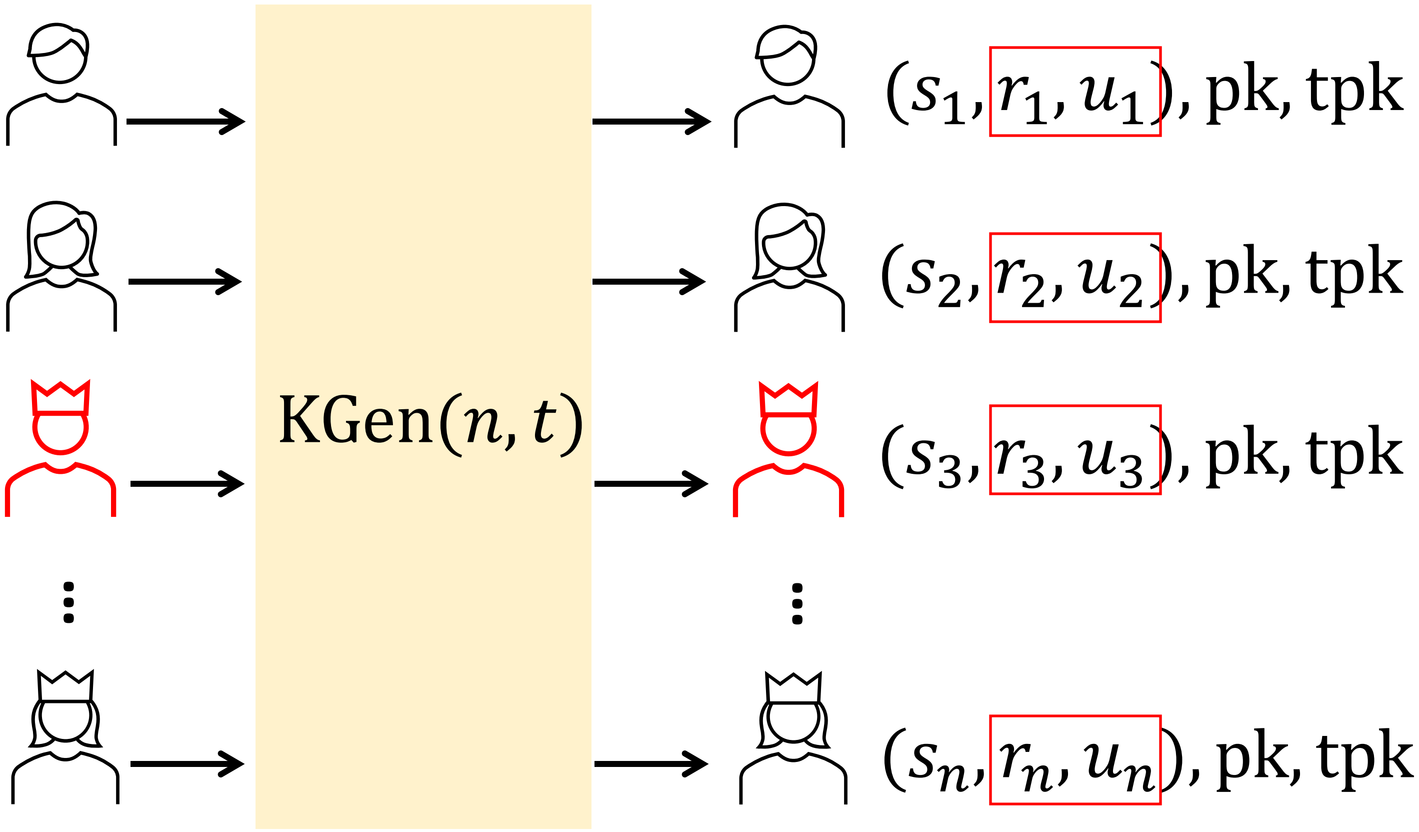

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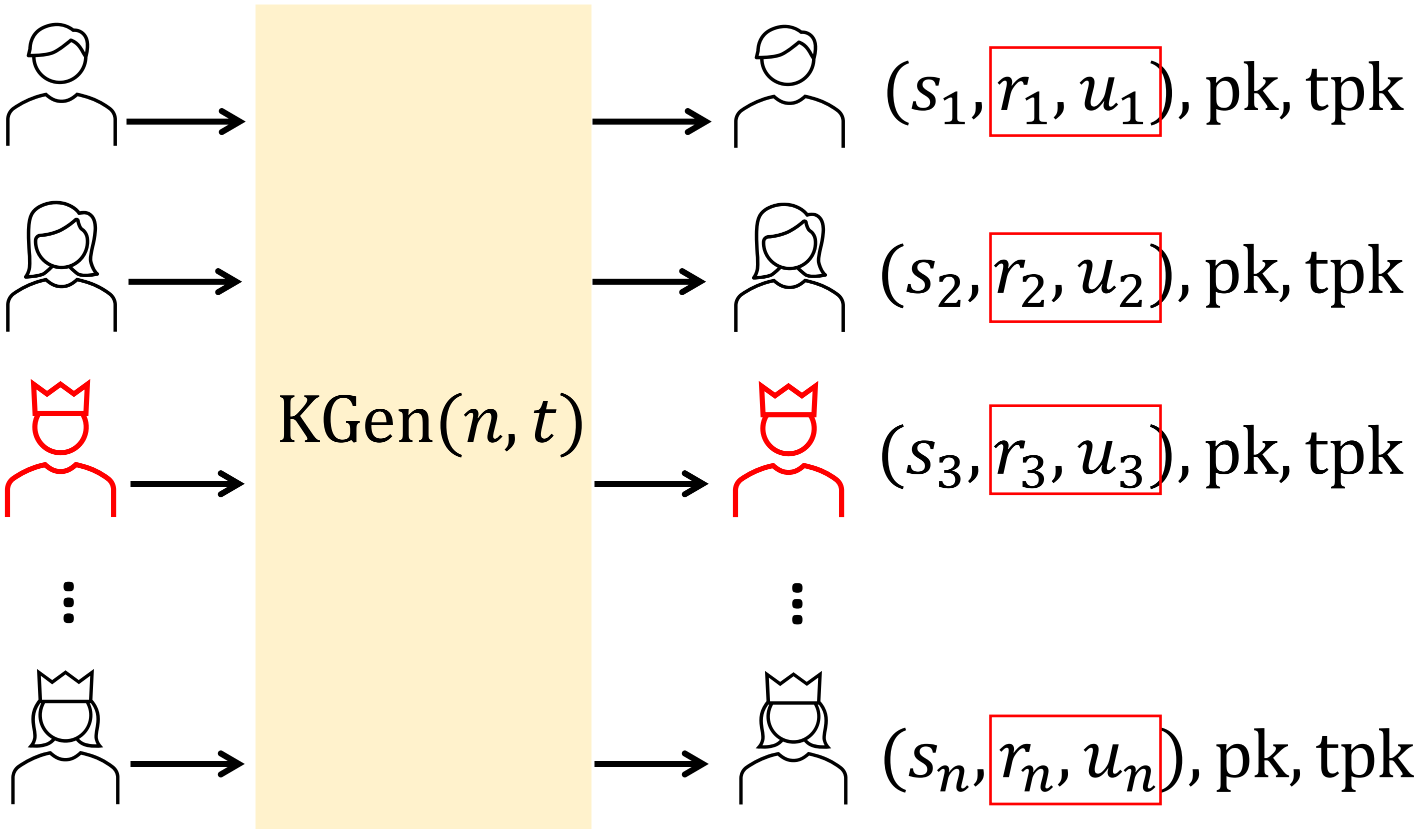

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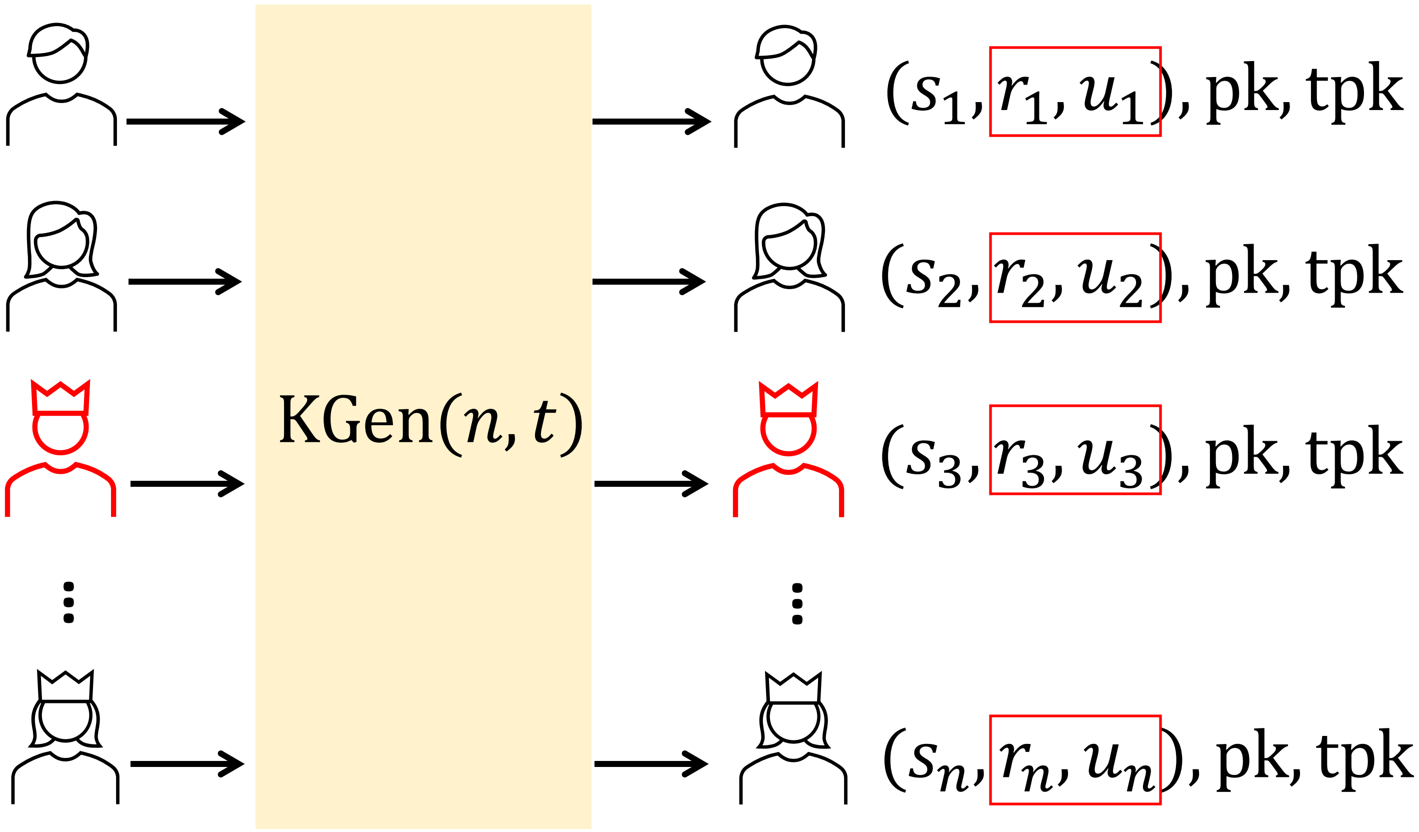

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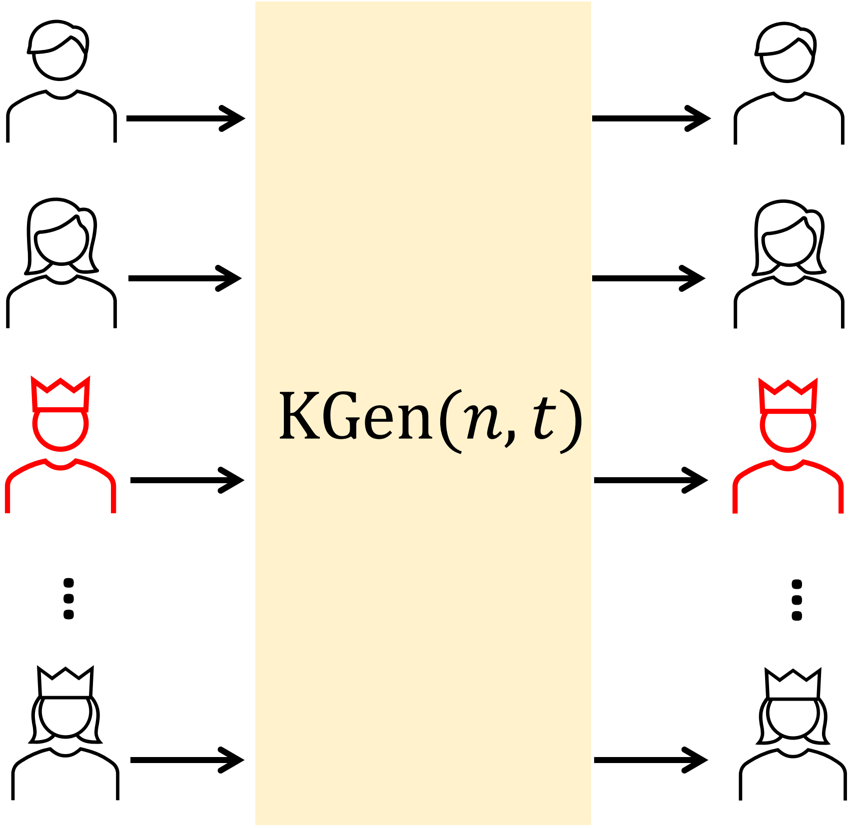

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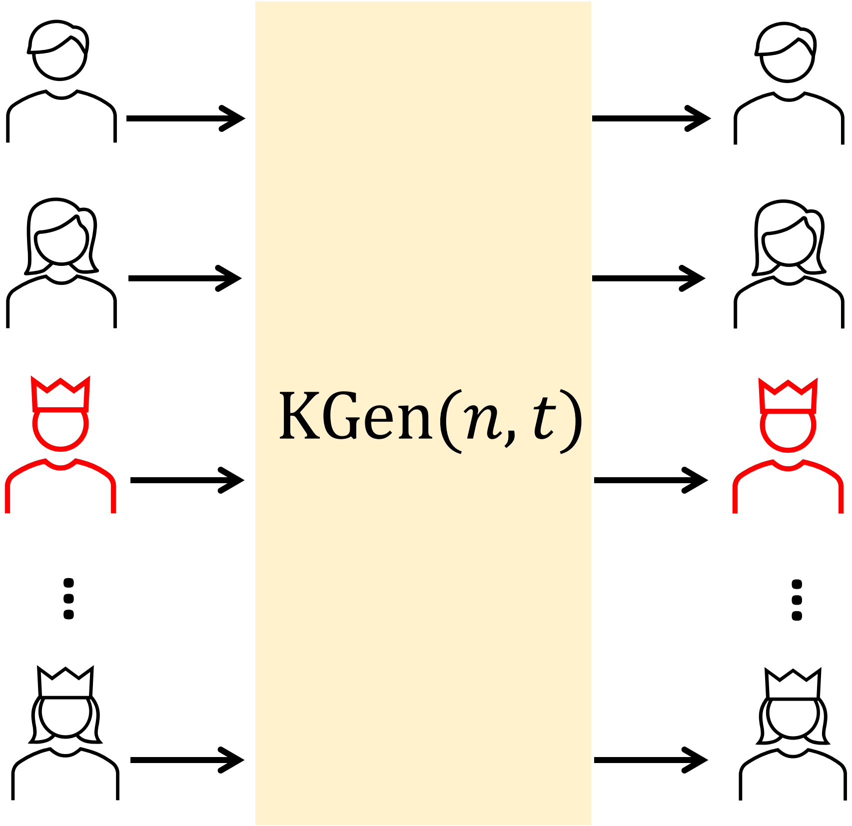

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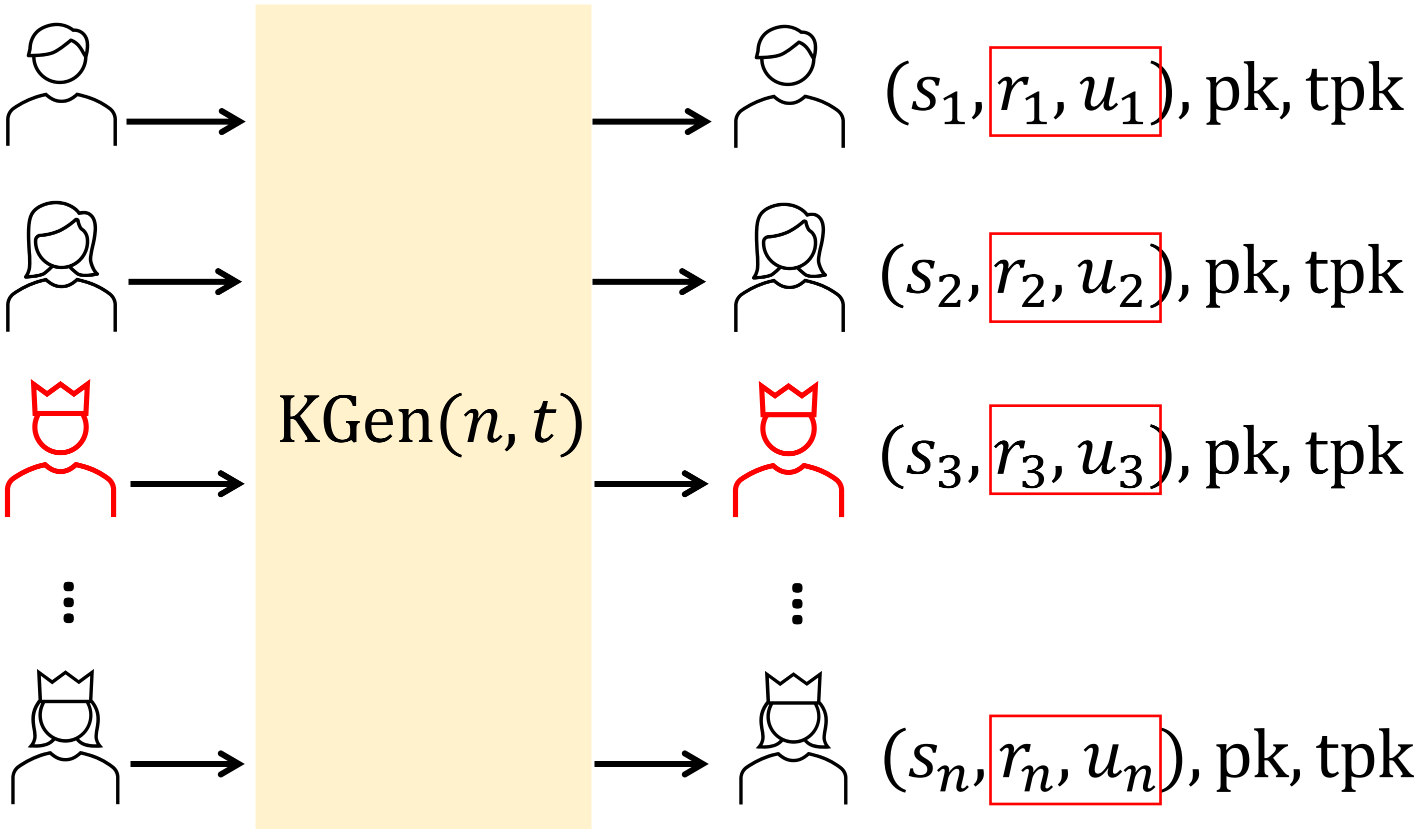

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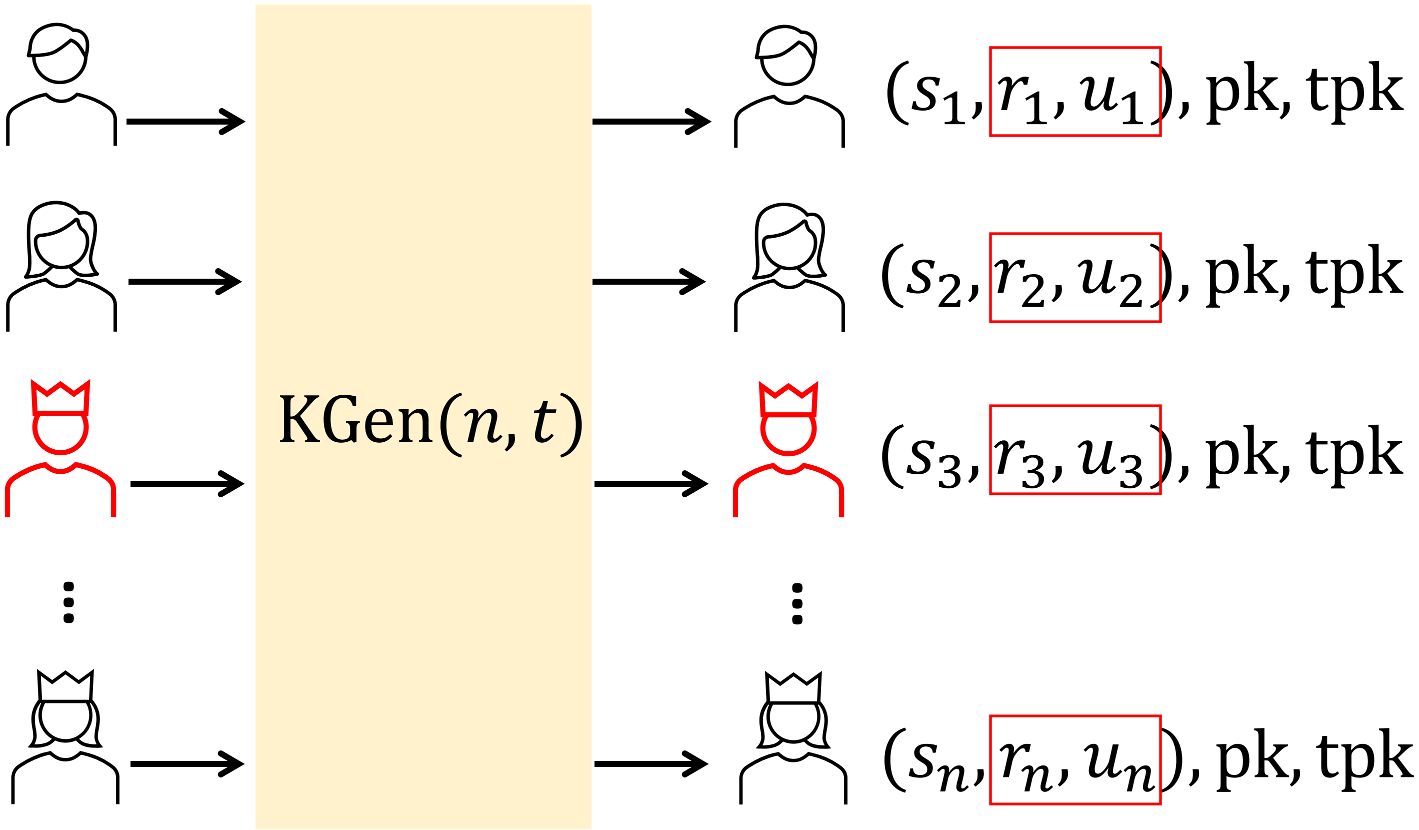

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See paper for the proof!

Implementation and Evaluation

Evaluation

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- gnark-crypto library, bls12-381 curve

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- **Baselines:**

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Common case aggregation time (for t=64) is 7.7 ms for all three schemes!

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Implementation

My website

Thank you (souravd2@Illinois.edu)