Field-Agnostic SNARKs from Expand-Accumulate Codes

CRYPTO 2024

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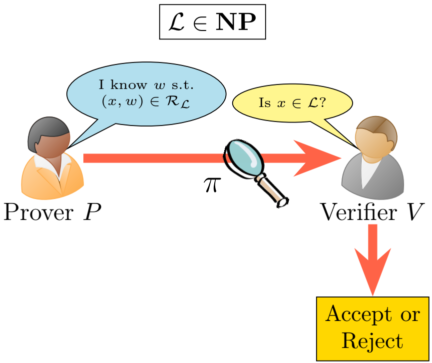
[5]University of Maryland
[6]University of Illinois Urbana-Champaign

Succinct **N**on-interactive **AR**guments of **K**nowledge



Completeness: $\forall (x, w) \in \mathcal{R}_\mathcal{L}$:

$$\Pr[V(x, \pi) = 1 \mid \pi \leftarrow P(x, w)] = 1$$

ε**-Soundness:** $\forall x \notin \mathcal{L}$, $\forall$ PPT P^*:

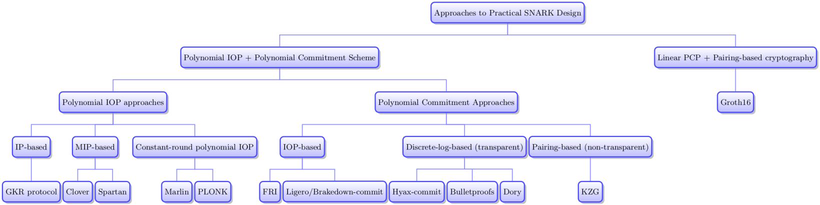
$$\Pr[V(x, \pi^*) = 1 \mid \pi^* \xleftarrow{\$} P^*(x)] \leqslant \varepsilon(x, \lambda)$$

ε**-Knowledge Soundness:** $\exists$ PPT extractor $\mathcal{E}$ such that $\forall x$ and $\forall$ PPT P^*:

$$\Pr[(x, \mathcal{E}^{P^*}(x)) \in \mathcal{R}_\mathcal{L}] + \varepsilon(x, \lambda) \geqslant \Pr[V(x, \pi^*) = 1 \mid \pi^* \xleftarrow{\$} P^*(x)]$$

Succinctness: $|\pi| = o_\lambda(|w|)$; ideally $O_\lambda(\text{polylog}(|w|))$

Building Concretely Efficient SNARKs [Tha22]



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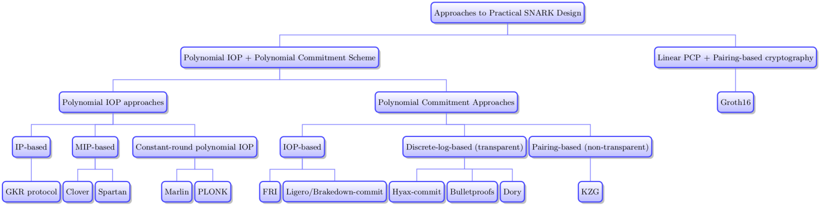

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Build SNARKs from Error-correcting Codes

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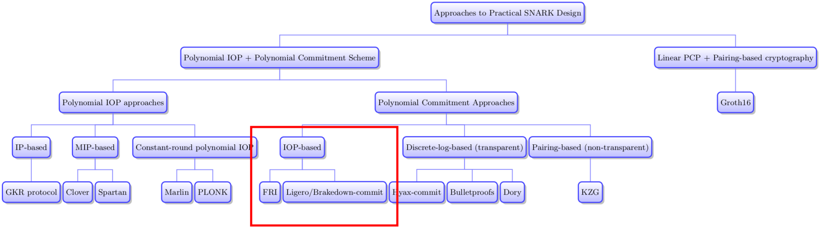

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Code(-based) SNARKs

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- SNARKs from Codes are built upon the PIOP + PCS paradigm

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$$\boxed{\text{Code SNARK}} = \boxed{\text{PIOP}} + \boxed{\text{Code-based PCS}}$$

Goal: design Code-based *Polynomial Commitment Scheme*

PCS from any Linear Code

Ligero/Brakedown-based PCS

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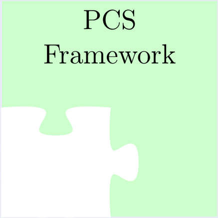

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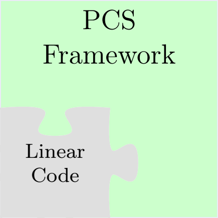

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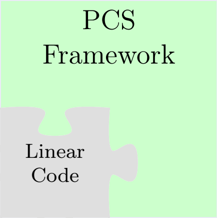

PCS from any linear code (derived from [BCG20])

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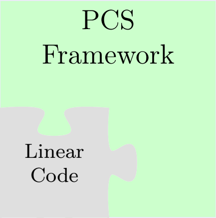

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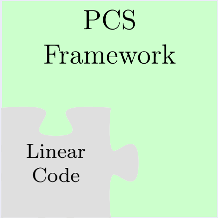

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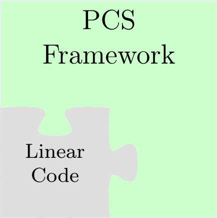

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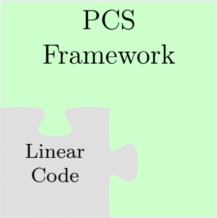

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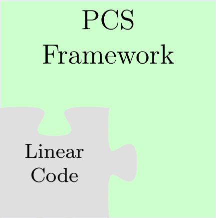

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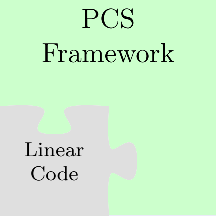

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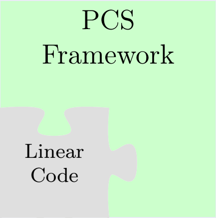

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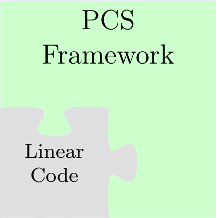

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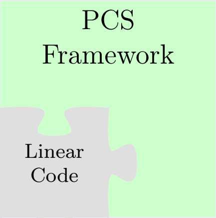

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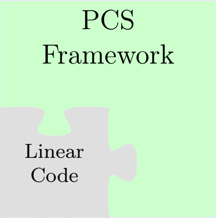

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Why field-agnostic?

Prover can experience $\approx 25\times$ slow down if the SNARK doesn't support field of the underlying computation!

Our Results: Bird's Eye View

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New Code-PCS from Expand-Accumulate Codes
via the Brakedown PCS Framework

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Improved distance analysis of binary EA codes

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Provide an alternate distance analysis + proof via classic coding techniques

Better concrete parameters than [BCG+22]

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Extend binary analysis to FFs

Answers open problem of [BCG+22]

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Concretely efficient SNARKs from EA codes

- Field-agnostic
- Proof size + verifier time comparable with [Brakedown], only $\approx 1.2\times$ prover overhead

Our Results: Comparison

Table 1: Performance of field-agnostic SNARKs based on linear codes for a statement modeled as an arithmetic circuit of size M and depth d.

	Prover Time	Proof Size	Verifier Time
[Brakedown]	$O(M)$	$O(\sqrt{M})$	$O(\sqrt{M})$
[BaseFold]	$O(M \log M)$	$O(\log^2 M)$	$O(\log^2 M)$
This Work	$O(M \log M)$	$O(\sqrt{M})$	$O(\sqrt{M})$
Proof of ECDSA verification			
[Brakedown]	0.17s	2.2MB	0.062s
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Remainder of the Talk

- Error-correcting codes overview
- EA Codes overview
- IOWE technique for distance analysis
- EA Code over any finite field analysis
- Experimental results

Definition 1 (Linear Codes)

Linear Error-Correcting Codes

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Let $\mathbb{F}$ be a finite field. A $[N, n, d]$ linear error correcting code C for $n \leqslant N$ is a n-dimensional subspace of $\mathbb{F}^N$ such that the minimum distance of C, denoted as $\Delta(C)$, is d, where $\Delta(C) := \min_{y \in C \setminus \{0^N\}} \{\mathsf{wt}(y)\}$.

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Equivalently

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- **Relative Distance:** $\delta(C) := \Delta(C)/N$

Expand-Accumulate Codes [BCG+22]

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$$
\begin{array}{cc}
 & N \\
n & \begin{bmatrix}
\mathbb{F} & & & & & \mathbb{F} & & \\
 & \mathbb{F} & \mathbb{F} & & & \mathbb{F} & & \\
 & & & \mathbb{F} & & & & \mathbb{F} \\
 & & & & \mathbb{F} & & &
\end{bmatrix}
\end{array}
$$

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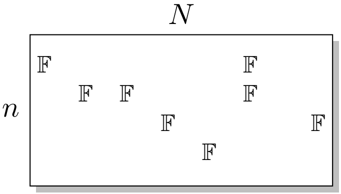

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\end{array}
\qquad
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N & \\
\begin{bmatrix}
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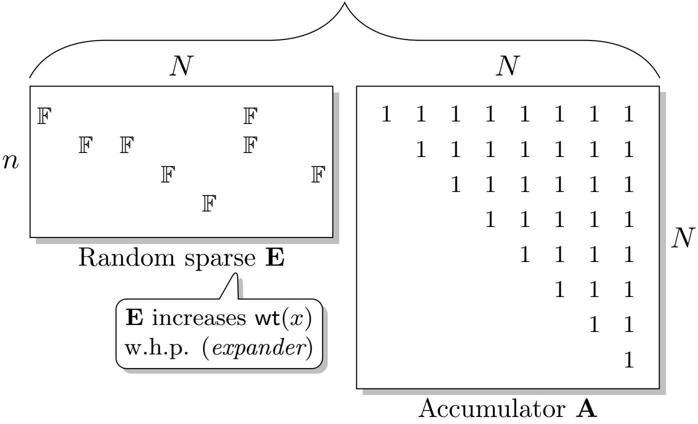

How to sample $\mathbf{E}$?

Sampling Expander Matrix

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(Generalized) Bernoulli

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$$\mathsf{Ber}_p(\mathbb{F}) := \begin{cases} x \xleftarrow{\$} \mathbb{F} \setminus \{0\} & \text{w.p. } p \\ 0 & \text{w.p. } 1-p \end{cases}$$

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Fixed Row Weights

$$\mathbf{E}_i \leftarrow \mathsf{Fixed}(N, t, \mathbb{F}), \forall i \in [n]$$

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EA Codes: Our Approach

Juxtaposed EA Code

$$C[\mathbf{E}_1, \mathbf{E}_2](x) := (x\mathbf{E}_1\mathbf{A}) \| (x\mathbf{E}_2\mathbf{A})$$

$$\mathbf{E}_1 \leftarrow \mathsf{Fixed}(n, N, t, \mathbb{F}) \text{ and } \mathbf{E}_2 \leftarrow \mathsf{Ber}_p^{n\times N}(\mathbb{F})$$

$$R = n/N = \Theta(1),\ t = \Theta(\log(N)),\ p = t/N$$

Juxtaposed EA Codes: Our Results

Theorem 1

Over any $\mathbb{F}$*, for* $R = n/N$ *constant, there exist constants* $\delta \in (0, 1/2)$ *and* $c^* > 5$ *such that for* $t = \Theta(\log(N))$ *and* $p = t/N$*, the juxtaposed EA code* $C[\mathbf{E}_1, \mathbf{E}_2]$ *over* $\mathbb{F}$ *has constant relative distance* δ *with at least* $1 - 1/\text{poly}(N^{5-c^*})$ *probability.*

If $\mathbb{F} = \mathbb{F}_2$*, then the above holds for* $c^* > 4$ *with probability at least* $1 - 1/\text{poly}(N^{4-c^*})$

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- We consider juxtaposed EA codes due to limitations in our analysis of $\mathbf{E}_1$.

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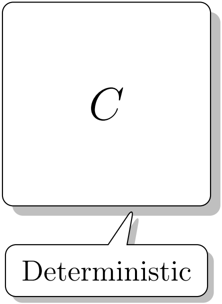
- We consider juxtaposed EA codes due to limitations in our analysis of $\mathbf{E}_1$.
- We conjecture both $\mathbf{E}_1\mathbf{A}$ and $\mathbf{E}_2\mathbf{A}$ are good codes.
- Parameters in above theorem are nowhere near tight; can be tightened up with better Stirling approximations.

IOWE Technique

Input-Output Weight Enumerator

IOWE Technique

Input-Output Weight Enumerator



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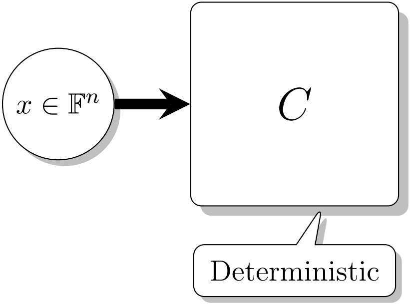

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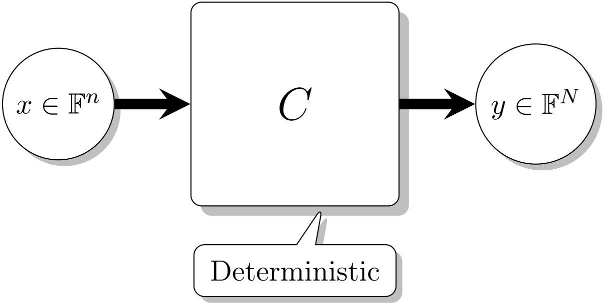

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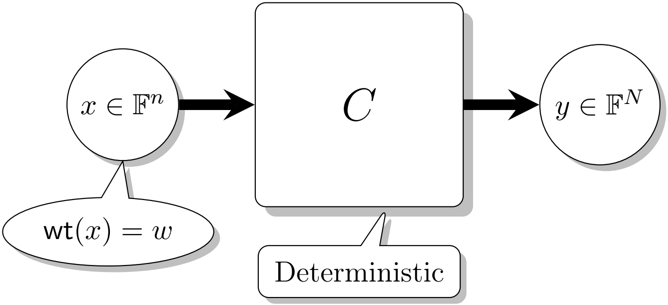

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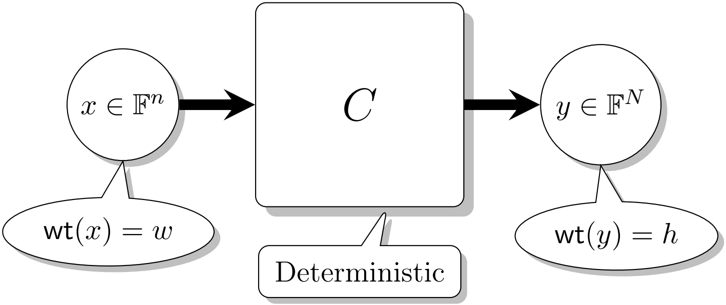

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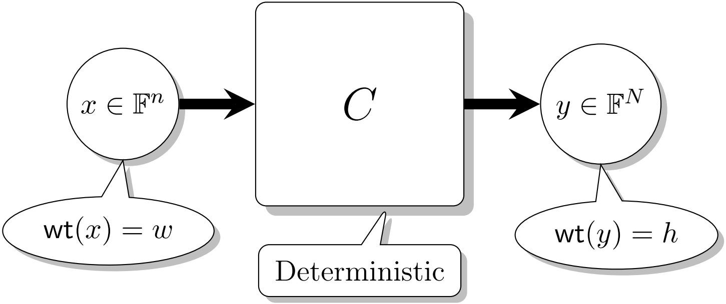
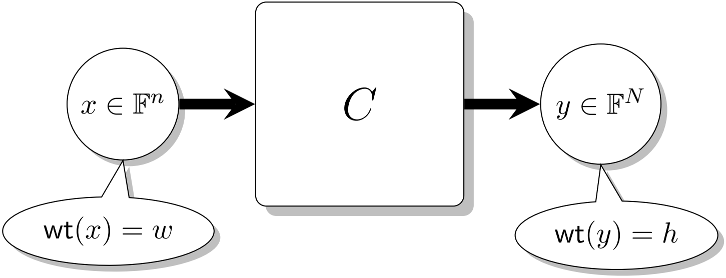
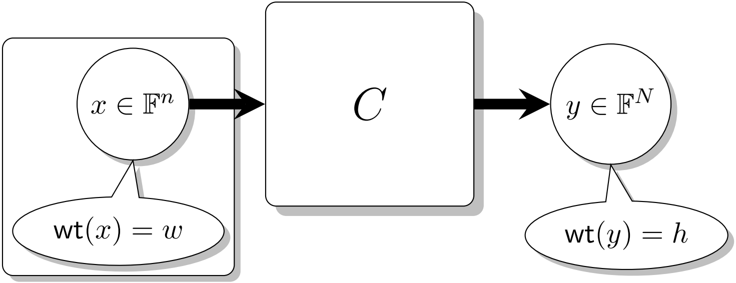


Definition 2 (IOWE)

$$C^N_{w,h} := \left|\{x \in \mathbb{F}^n : \mathsf{wt}(x) = w \ \wedge \ \mathsf{wt}(C(x)) = h\}\right|$$

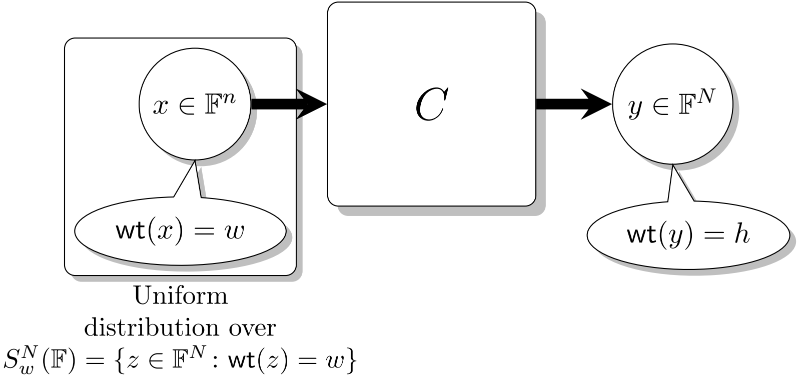
IOWE Technique





Uniform
distribution over
$S_w^N(\mathbb{F}) = \{z \in \mathbb{F}^N : \mathsf{wt}(z) = w\}$

IOWE Technique



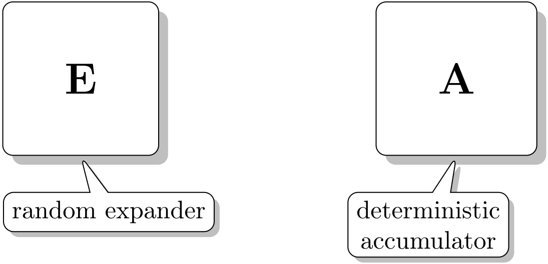
$$\Pr[\mathsf{wt}(C(x)) = h \colon x \xleftarrow{\$} S_w^N(\mathbb{F})] = \frac{C_{w,h}^N}{\binom{n}{w}}$$

IOWE Technique for EA Codes

E

A

IOWE Technique for EA Codes



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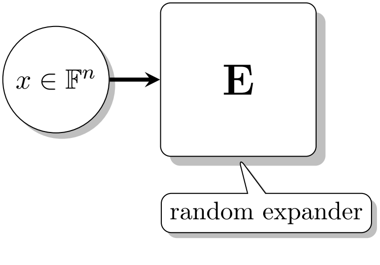

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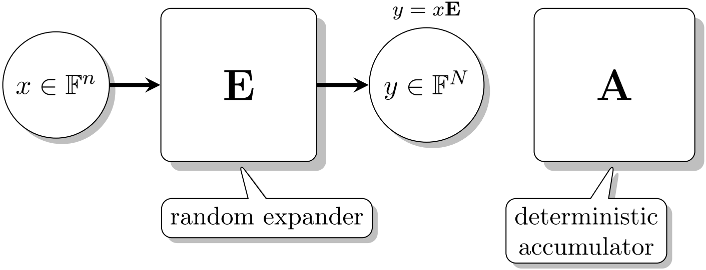

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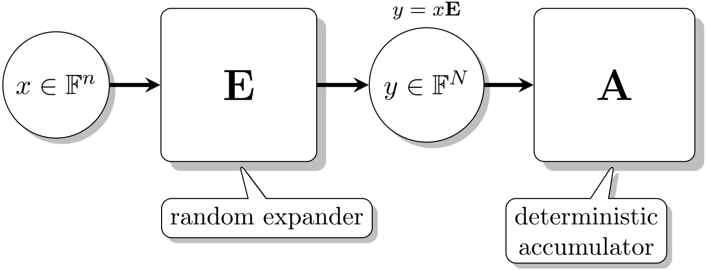

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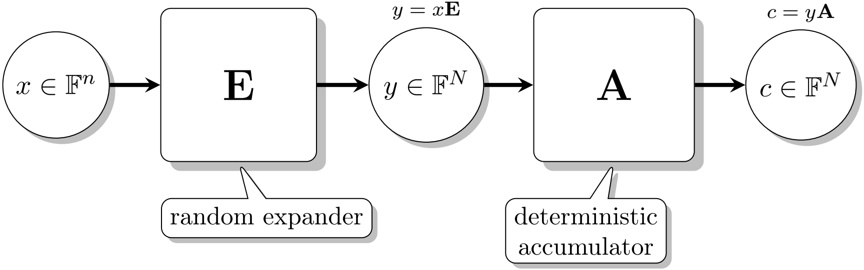

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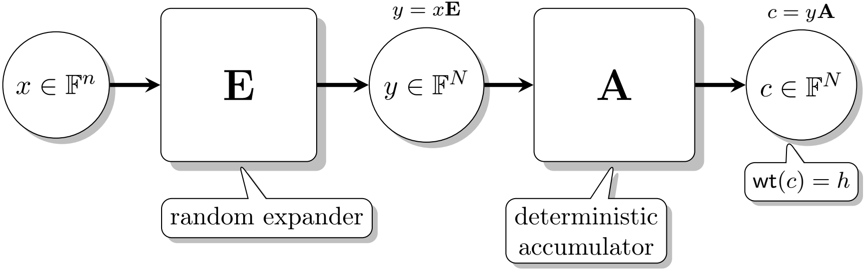

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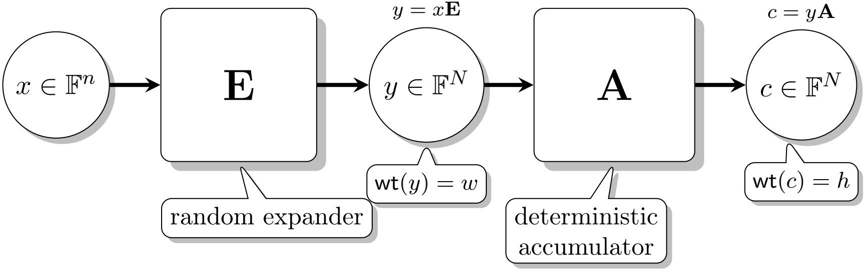

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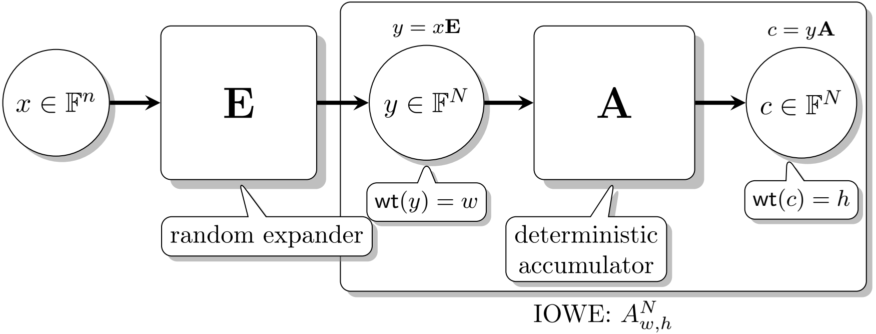

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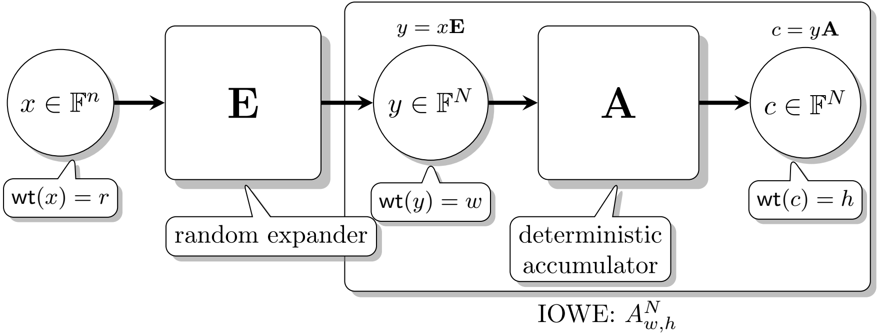

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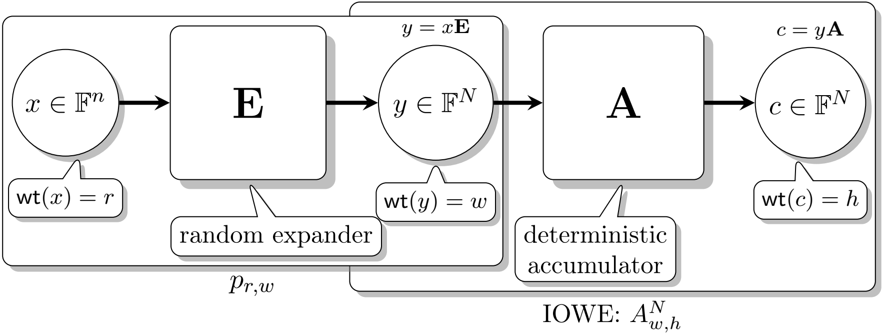

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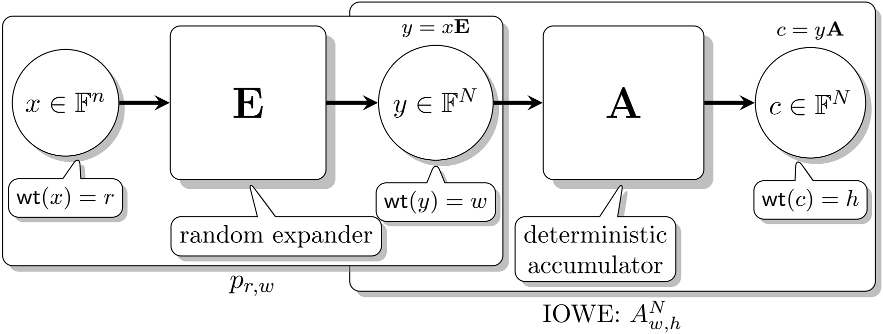
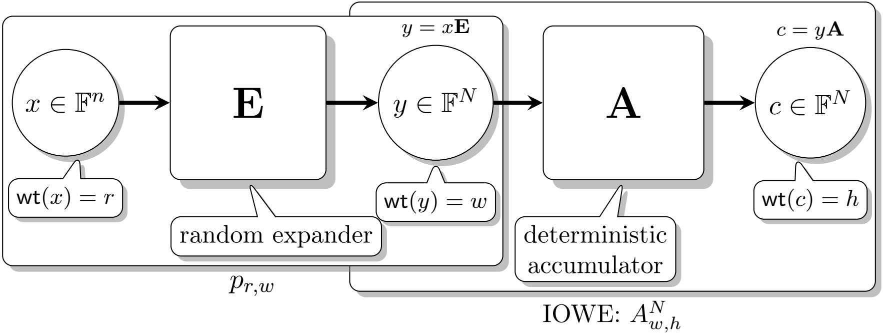


Properties of $\mathbf{E}$

IOWE Technique for EA Codes



Properties of $\mathbf{E}$

- $p_{r,w} = \Pr_{\mathbf{E}}[\mathsf{wt}(x\mathbf{E}) = w \mid \mathsf{wt}(x) = r]$

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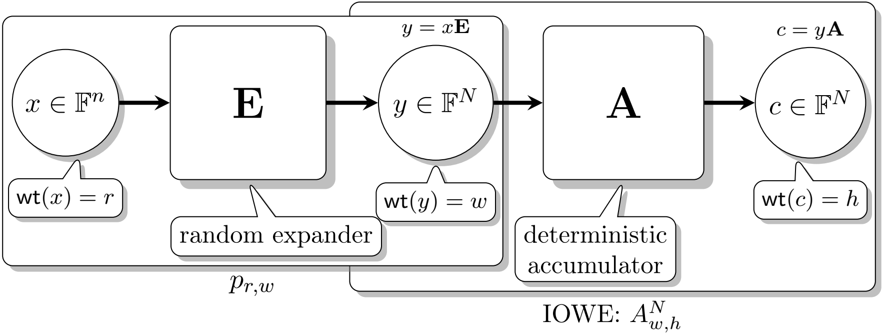

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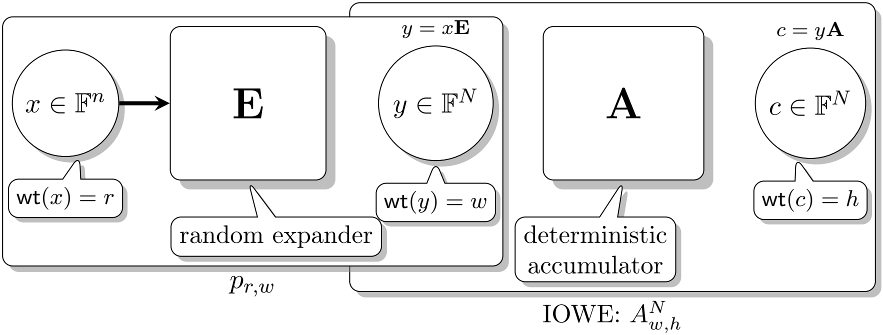

Distance Analysis (Binary Case)

$$\Pr_{\mathbf{E}}\left[\exists x \in \mathbb{F}_2^n \setminus \{0^n\} \colon \mathsf{wt}(x\mathbf{EA}) \leqslant \delta N\right] \leqslant$$

$$\sum_{r=1}^{n} \binom{n}{r} \cdot \sum_{w=1}^{N} p_{r,w} \cdot \sum_{h=1}^{\delta N} A^N_{w,h} \Big/ \binom{N}{w}$$

Accumulator IOWE

Binary IOWE Accumulator [DJM98]

$$A^{N,2}_{w,h} = \binom{h-1}{\lceil w/2 \rceil - 1}\binom{N-h}{\lfloor w/2 \rfloor}$$

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Theorem 2

For finite field $\mathbb{F}_q$, $N \in \mathbb{N}$, and $w, h \in [N]$, the IOWE of the $N \times N$ accumulator matrix over $\mathbb{F}_q$ is

$$A_{w,h}^{N,q} = \sum_{i=0}^{w-1} \binom{h-1}{\left\lceil \frac{w-i}{2} \right\rceil - 1}\binom{N-h}{\left\lfloor \frac{w-i}{2} \right\rfloor}\binom{h - \left\lceil \frac{w-i}{2} \right\rceil}{i}(q-1)^{\left\lceil \frac{w-i}{2} \right\rceil}(q-2)^i.$$

Distance of EA Codes over Finite Fields

- Given IOWE $A_{w,h}^{N,q}$, we can directly bound the distance of the EA code over any $\mathbb{F}$

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Distance Analysis ($\mathbb{F}_q$ Case)

$$\sum_{r=1}^{n} \binom{n}{r} (q-1)^r \sum_{w=1}^{N} p_{r,w} \sum_{h=1}^{\delta N} A_{w,h}^{N,q} / \binom{N}{w} (q-1)^w =$$

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We were unable to bound this for $q > 4$!

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Looks like binary case; able to bound this!

Experiments

- Implementation of PCS + SNARK in Rust
- SNARK relies on Spartan PIOP [Set20]
- Artifact available: https://artifacts.iacr.org/crypto/2024/a10/

Parameters

- Distance $\delta = 1/10$ with probability 2^{-100}, calculated numerically
- Rate $R = 1/2$, $n \in 2^{\{10,11,12\}}$, $N = n/R$
- Sparsity $t \geqslant 18 \log(N)$
- $\mathbb{F}$ is the scalar field of the BN254 curve unless otherwise stated.
- "Brakedown-improved" refers to using the improved Brakedown parameters due to [Hab23]

Experiments

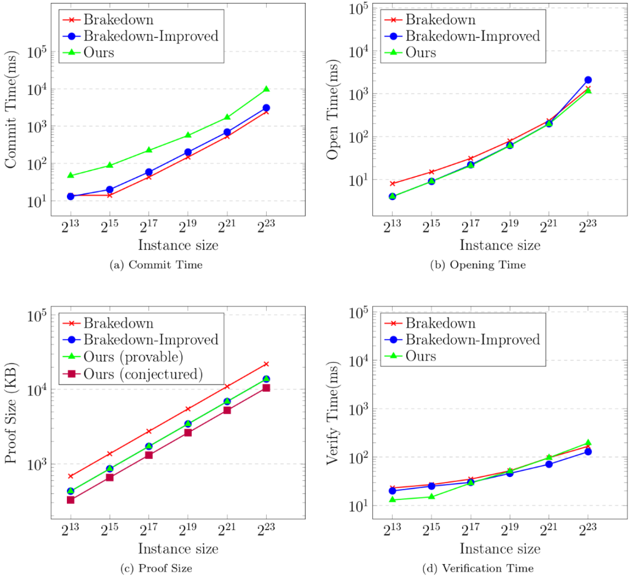

Figure 1: Performance of polynomial commitment schemes.

EXPERIMENTS

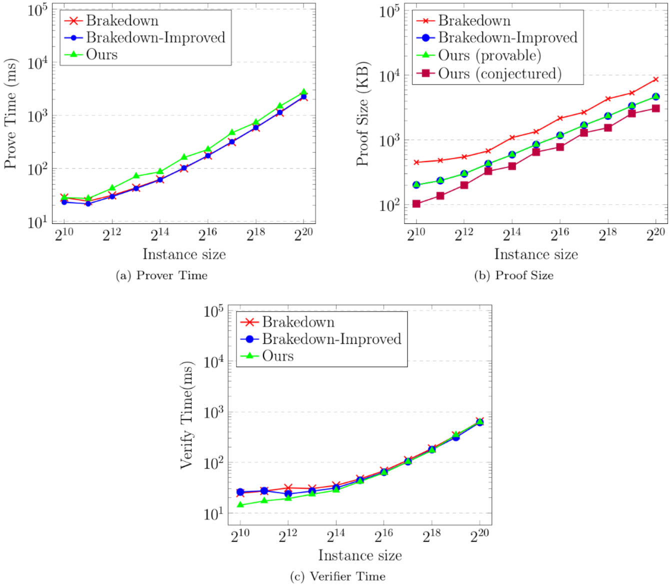

Figure 2: Performance of SNARKs on random R1CS instances.

Table 2: Performance of different SNARKs for ECDSA verification. Native field is the scalar field of the secp256k1 curve.

R1CS Size	Scheme	Prover time	Proof size	Verifier time
2^{21} (non-native)	Ligero	103s	20 MB	57 s
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2^{21} (non-native)	Groth16	149s	128 B	2 ms
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Concretely small proofs

Comparable to Brakedown

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2^{21} (non-native)	Aurora	534s	148 KB	15.2 s
2^{21} (non-native)	Groth16	149s	128 B	2 ms
2^{16} (native)	Brakedown	0.17s	2.2 MB	62 ms
2^{16} (native)	Brakedown-Improved	0.17s	1.1 MB	64 ms
2^{16} (native)	Ours (provable)	0.23s	1.1 MB	68 ms
2^{16} (native)	Ours (conjectured)	0.23s	778 KB	67 ms

Slightly slower than Brakedown

Concretely small proofs

Comparable to Brakedown

Summary

New Code-PCS from Expand-Accumulate Codes via the Brakedown PCS Framework

1. Improved distance analysis of binary EA codes

- Provide an alternate distance analysis + proof via classic coding techniques
- Better concrete parameters than [BCG+22]

2. Generalize EA codes to arbitrary finite fields

- Extend binary analysis to FFs
- Answers open problem of [BCG+22]
- Quasi-linear encoding, concretly large min distance

3. Concretely efficient SNARKs from EA codes

- Field-agnostic
- Proof size + verifier time comparable with [GLS+23], only $\approx 1.2\times$ prover overhead

Thank you!

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