Hintless Single-Server PIR

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Private Information Retrieval

Client$(i \in [m])$		**Server**$(\mathrm{db} \in \Omega^m)$
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- **Secure**: $\forall i, j \in [m]$, $\{\mathsf{Query}(i)\} \approx_c \{\mathsf{Query}(j)\}$

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- TiptoePIR [HDCZ SOSP23] (Concurrent to our work)
  - Pros: Removes the hint from SimplePIR!
  - Cons: $+\Omega(n^2 \log q)$ ($\approx$ 23MB) per-query bandwidth cost

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  - Speed up with novel precomputation argument
  - Also a PIR with $O(\sqrt[3]{|\mathrm{db}|})$ bandwidth scaling (TensorPIR)

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- Con: Requires n RLWE ciphertexts, $\Theta(n^2 \log q)$ bandwidth

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- Use standard RLWE (and gadget RLWE) encryption with plaintext packing

$\diamond$-Product: Standard Domain

$\diamond$-Product

Let $\vec{\mathrm{ct}} \in (R_{n,q} \times R_{n,q})^\ell$ be a collection of ℓ RLWE ciphertexts. For a polynomial $a \in R_{n,q}$, define

$$a \diamond \vec{\mathrm{ct}} = \sum_{i \in [\ell]} \mathrm{GadgetDecompose}(a)_i * \vec{\mathrm{ct}}_i$$

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    - And GSW/FHEW/TFHE/etc.
  - Implies hom. rotation $\vec{s} \mapsto \mathrm{rot}(\vec{s})$
- As written, is $O(n^2 \log q)$ time due to $*$

$\diamond$-Product: NTT Domain

- If we assume $a, \vec{\mathsf{ct}}$ start in NTT domain $(\widehat{a}, \widehat{\vec{\mathsf{ct}}})$, and want result in NTT domain, instead must compute

$$\widehat{a \diamond \vec{\mathsf{ct}}} = \sum_{i \in [\ell]} \mathsf{NTT}(\mathsf{GadgetDecompose}(\mathsf{NTT}^{-1}(\widehat{a}))_i) \circ \widehat{\vec{\mathsf{ct}}_i}$$

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- Precompute?

Composable Preprocessing

- For RLWE ctxt $\mathsf{ct} = [a, b]$, define

$$\alpha(\mathsf{ct}) = a, \qquad \beta(\mathsf{ct}) = b$$

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  - Precompute all NTTs ($O(\log n)\times$ speedup)

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- Apply composable preprocessing to preprocess all NTTs that occur
- Net Result: Can hom. compute $(A, \mathsf{Enc}_{\vec{s}}(\vec{x})) \mapsto \mathsf{Enc}_{\vec{s}}(A \cdot \vec{x})$ within a multiplicative constant of the cost in plaintext

Composable Preprocessing Limitations

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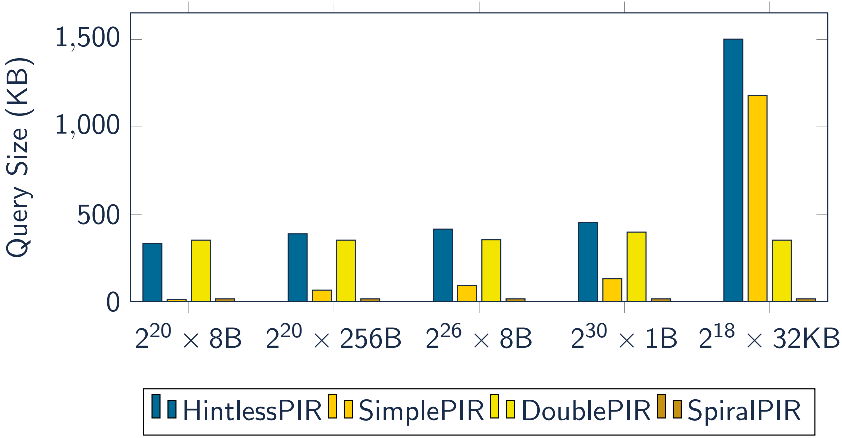
Contents

Evaluation

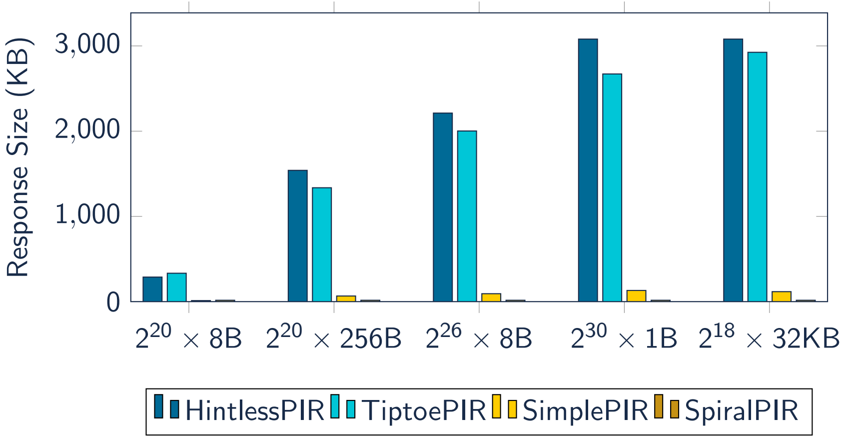
- Implemented our protocols
  - `https://github.com/google/hintless_pir`
- Compared HintlessPIR to other HE-based PIR
  - SimplePIR, DoublePIR, TiptoePIR, SpiralPIR

Bandwidth: Query Size



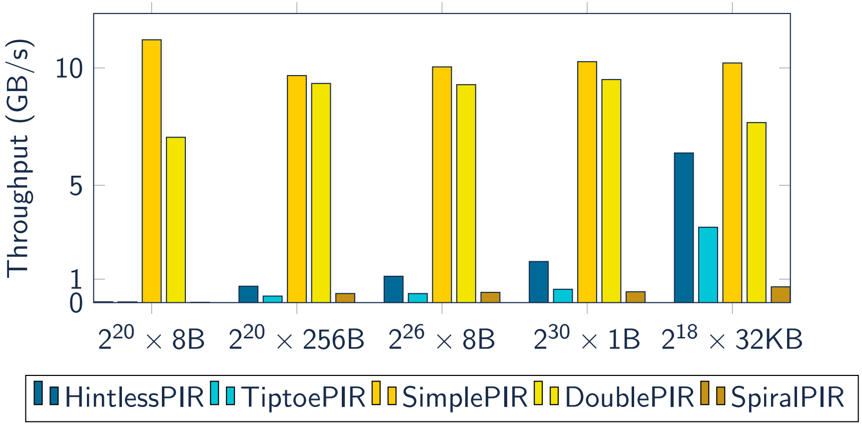
- TiptoePIR: ≈ 23MB for all database sizes

Bandwidth: Response Size



- DoublePIR's response depends only on record size. Ranges from 44KB (1B records) to 1273MB (32KB records)

Throughput


Throughput (GB/s)
10
5
1
0
$2^{20} \times$ 8B
$2^{20} \times$ 256B
$2^{26} \times$ 8B
$2^{30} \times$ 1B
$2^{18} \times$ 32KB
HintlessPIR
TiptoePIR
SimplePIR
DoublePIR
SpiralPIR

Hint Size

Database	SimplePIR	DoublePIR
$2^{20} \times$ 8B(8MB)	200% (16MB)	3025% (242MB)
$2^{20} \times$ 256B(286MB)	34% (92MB)	2573% (6897MB)
$2^{26} \times$ 8B(537MB)	24% (131MB)	45% (242MB)
$2^{30} \times$ 1B(1074MB)	17% (185MB)	2.8% (31MB)
$2^{18} \times$ 32KB(8389MB)	2% (185MB)	10418% (874000MB)