Sometimes You Can't Distribute Random-Oracle-Based Proofs

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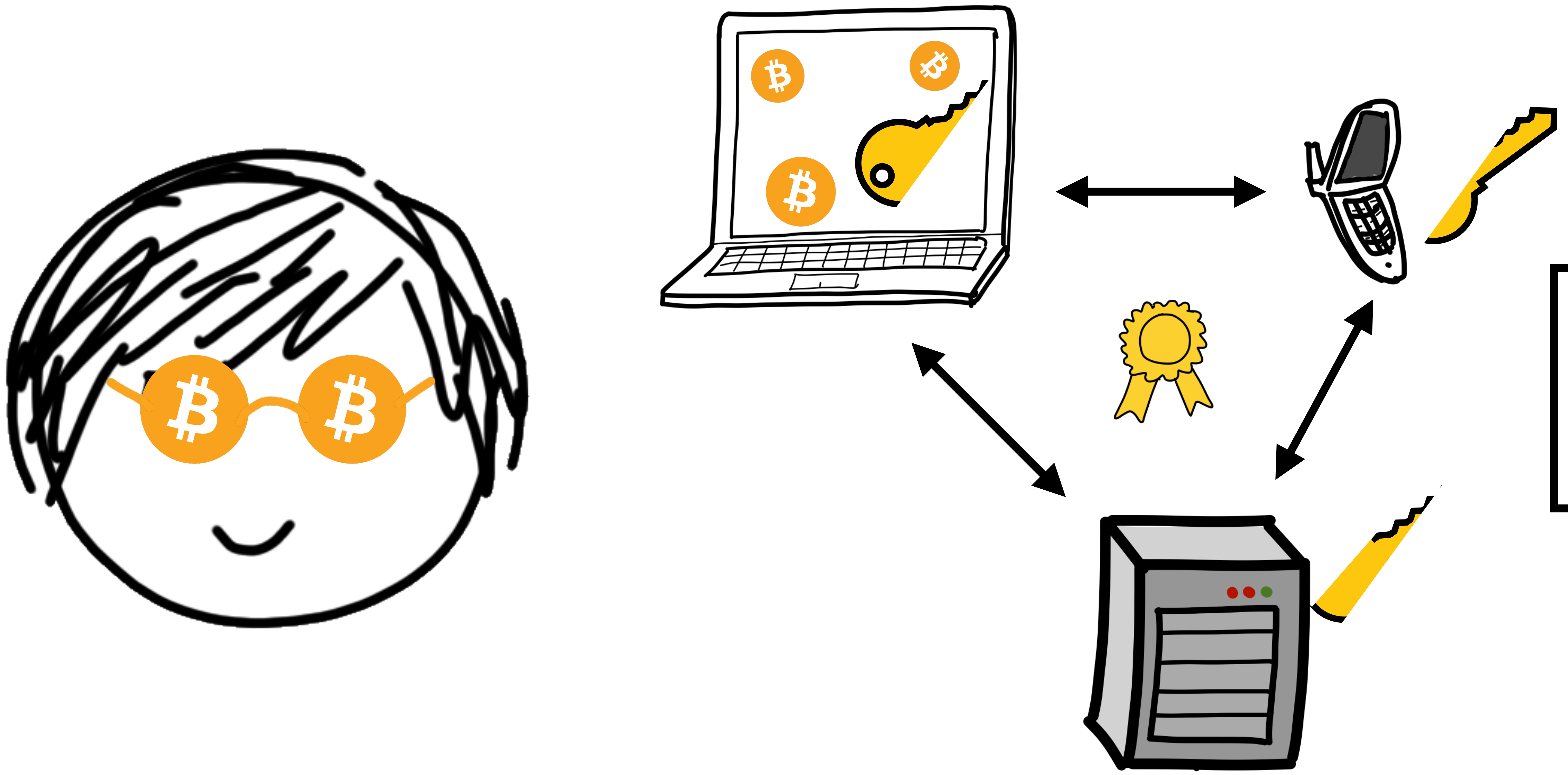
v1.2

Threshold / Distributed Signing

- Specialized Multiparty Computation (MPC) protocols to securely compute $\mathsf{Sign}(\mathsf{sk}, m)$ from secret shares of sk

- Commonly applied to decentralize key management



Mitigates risk of key theft

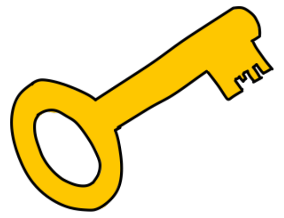
- **Compatibility**:
  Verifies w.r.t. original algorithm

- **Corruption Resilience**:
  Compromising some devices does not leak the signing key

- This talk: Signatures ⇔ Non-interactive Zero-knowledge

 ⇔ π

(

, ) ⇔ (x, w)

Distributed Signing ⇔ Distributed Proving

How to Distribute Signing

- Any signing scheme can be distributed via general MPC
- "Practical" efficiency usually requires more fine-grained notions than just feasibility
- As one proxy, practical distributed signing protocols make **blackbox use** of complex components of the signing algorithm:
  - Integer arithmetic in $\mathbb{Z}_q$ or $\mathbb{Z}_N^*$
  - Elliptic curve group operations
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RSA, Schnorr/EdDSA, ECDSA, BLS, BBS+, custom constructions using lattices, isogenies, etc.

What about Purely Hash Based?

- Proof size, verifier time linear in #provers
  [Ozdemir Boneh 22]: distributed version of Fractal
  [Cui Zhang Chen Liu Yu 21]: distributed MPC-in-the-head

- Prove statements about circuit representation of hash function
  [Khaburzaniya Chalkias Lewi Malvai 21]: aggregate Lamport signatures with STARKs

- Hash-based proofs that are designed to be hard to distribute
  [Dziembowski Faust Lizurej 23]: Individual Cryptography
  [Kelkar Babel Daian Austgen Buterin Juels 23]: Complete Knowledge

This Work: Limitations

- For some hash based NIZKs[1], there is an inherent barrier[2] to designing practical protocols[3] to distribute their computation.

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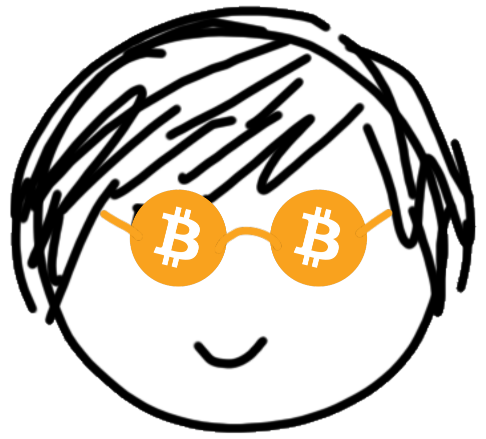
1. NIZKs that have straight-line extractors in the Random-Oracle Model
2. Attack that completely recovers the witness by corrupting all-but-one distributed provers
3. Protocol that is blackbox in the same hash function (i.e. Random Oracle) as the NIZK

Implications for distributing…

- Signing for standard schemes based on MPC-in-the-head
- NIZKs/signatures obtained by compiling Sigma protocols via:
  - Pass' or Fischlin's transformations (tight/concurrent security)
  - Unruh's transformation (post-quantum)
- PCPs/IOPs compiled via hash functions

Proofs of Knowledge

- What does it mean for a proof to certify "knowledge" of a witness?
- "Proof of Knowledge" is formalized by an "extractor" Ext



$P(x, w)$:

(NI)Zero-knowledge Proof:
"I know w such that $(x, w) \in L$"

$V(x)$

Accept/Reject

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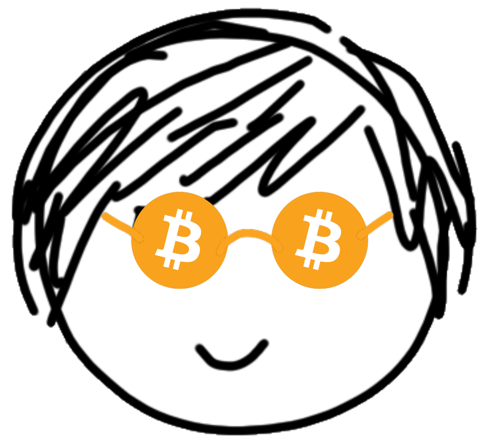

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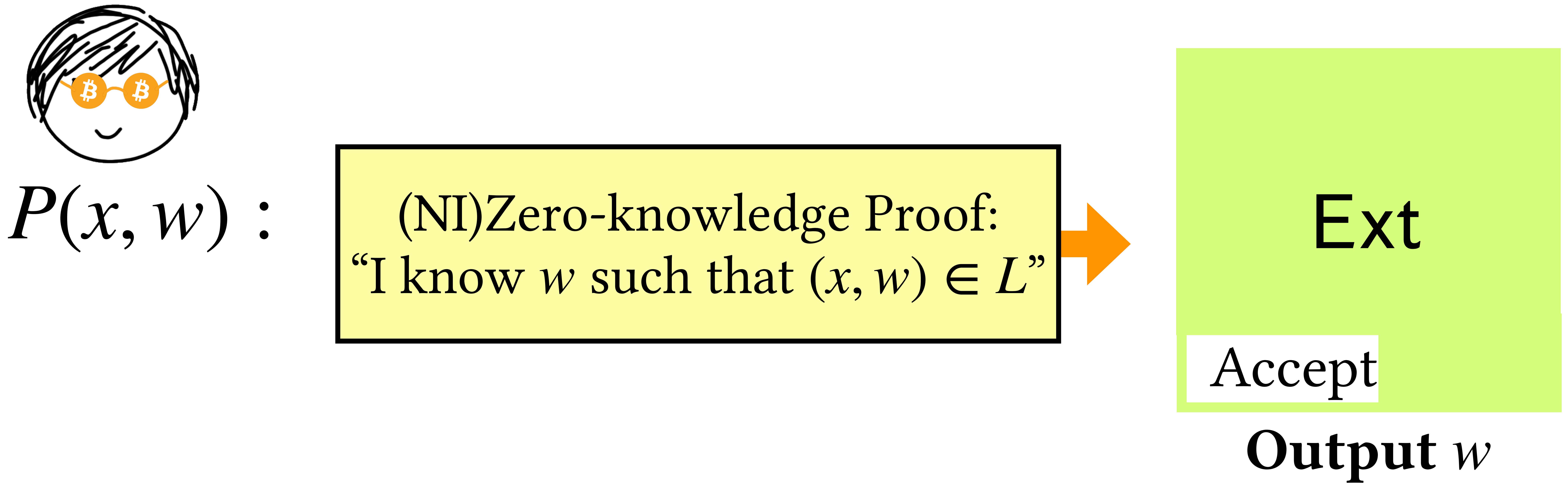

Why is Ext special?

- Clearly, Ext must not be an algorithm that just anybody can run

- Ext has carefully chosen special privileges:
  - Powerful enough to accomplish extraction
  - Still meaningful as a security claim

- “Straight-line” Extraction (SLE): no rewinding. Instead, use other trapdoor like CRS, RO, etc.

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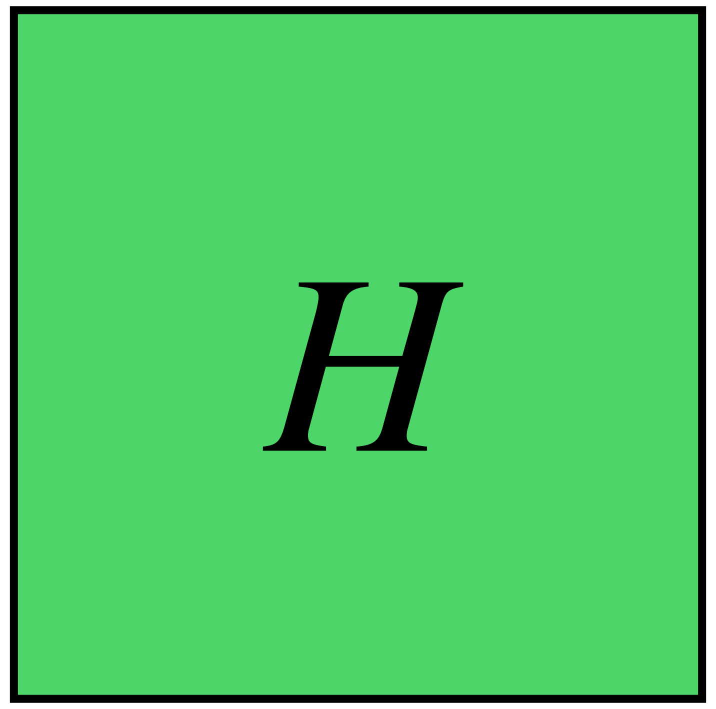
<u>Bad for</u>:

- Quantum
- Concurrency
- Tightness

Random Oracle Model



$$H : \{0,1\}^* \mapsto \{0,1\}^\ell$$

Random Oracles as Ext Privilege

[Pass 03]

H

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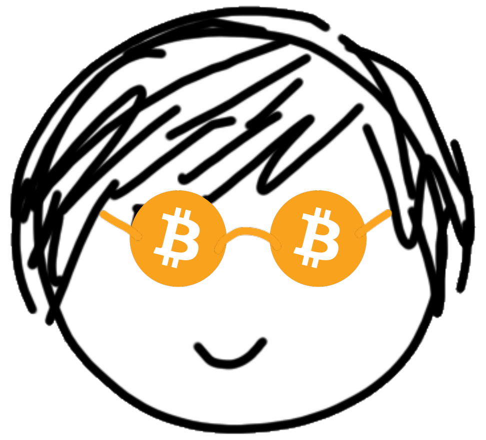

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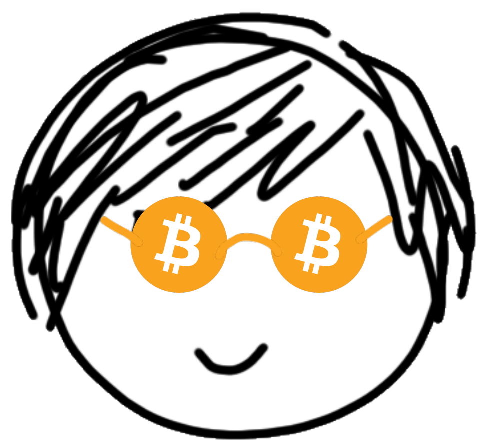

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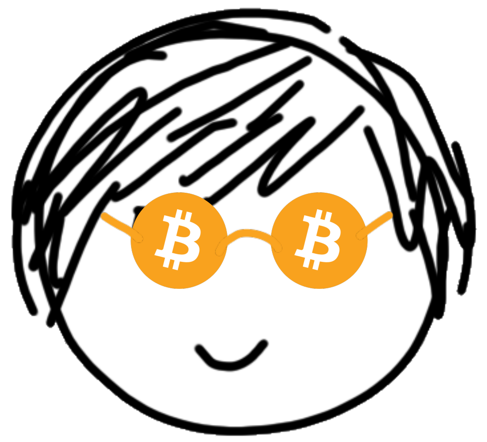

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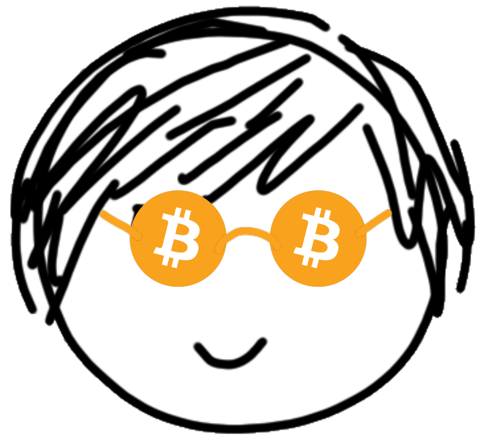

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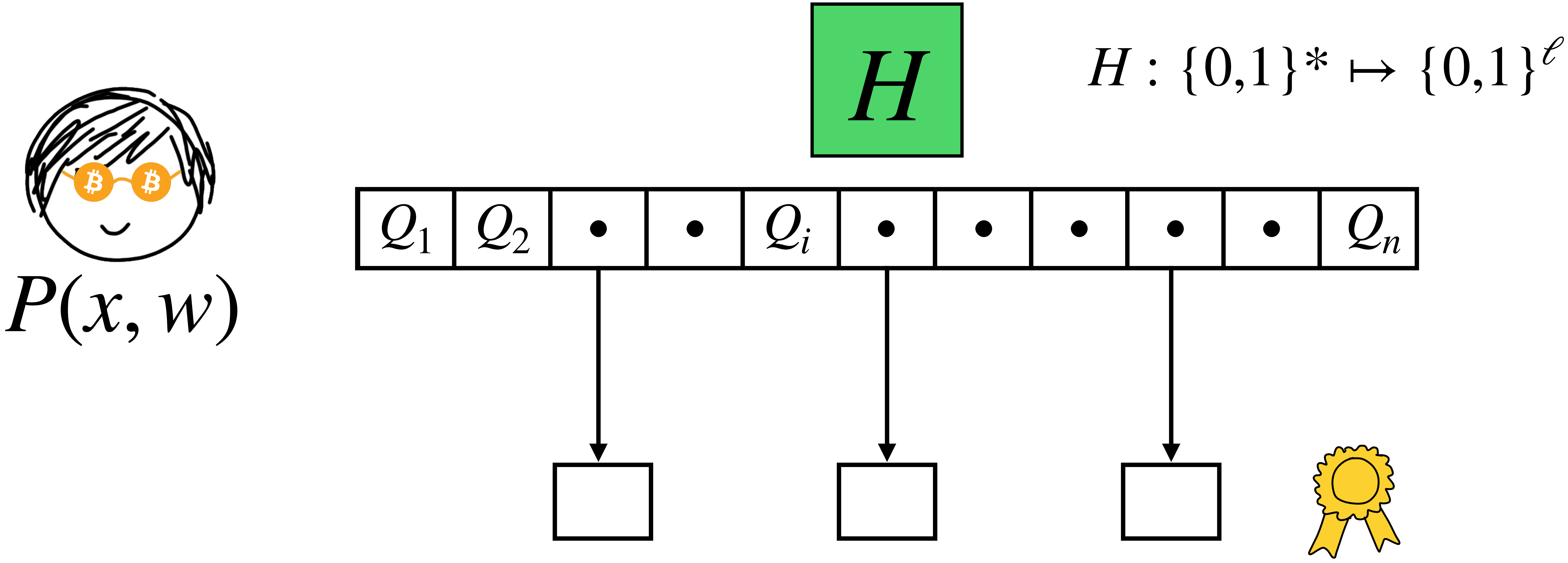

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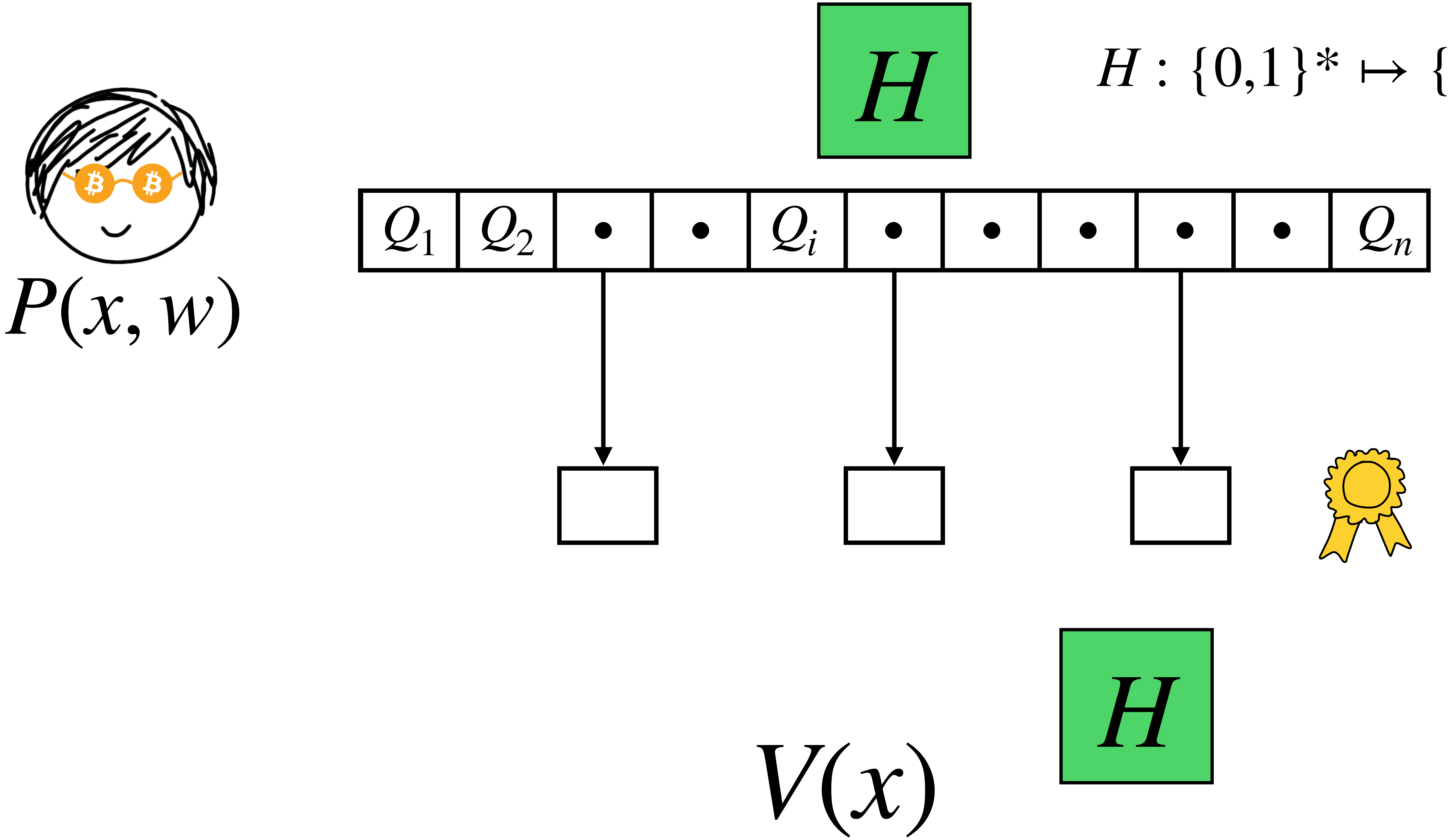

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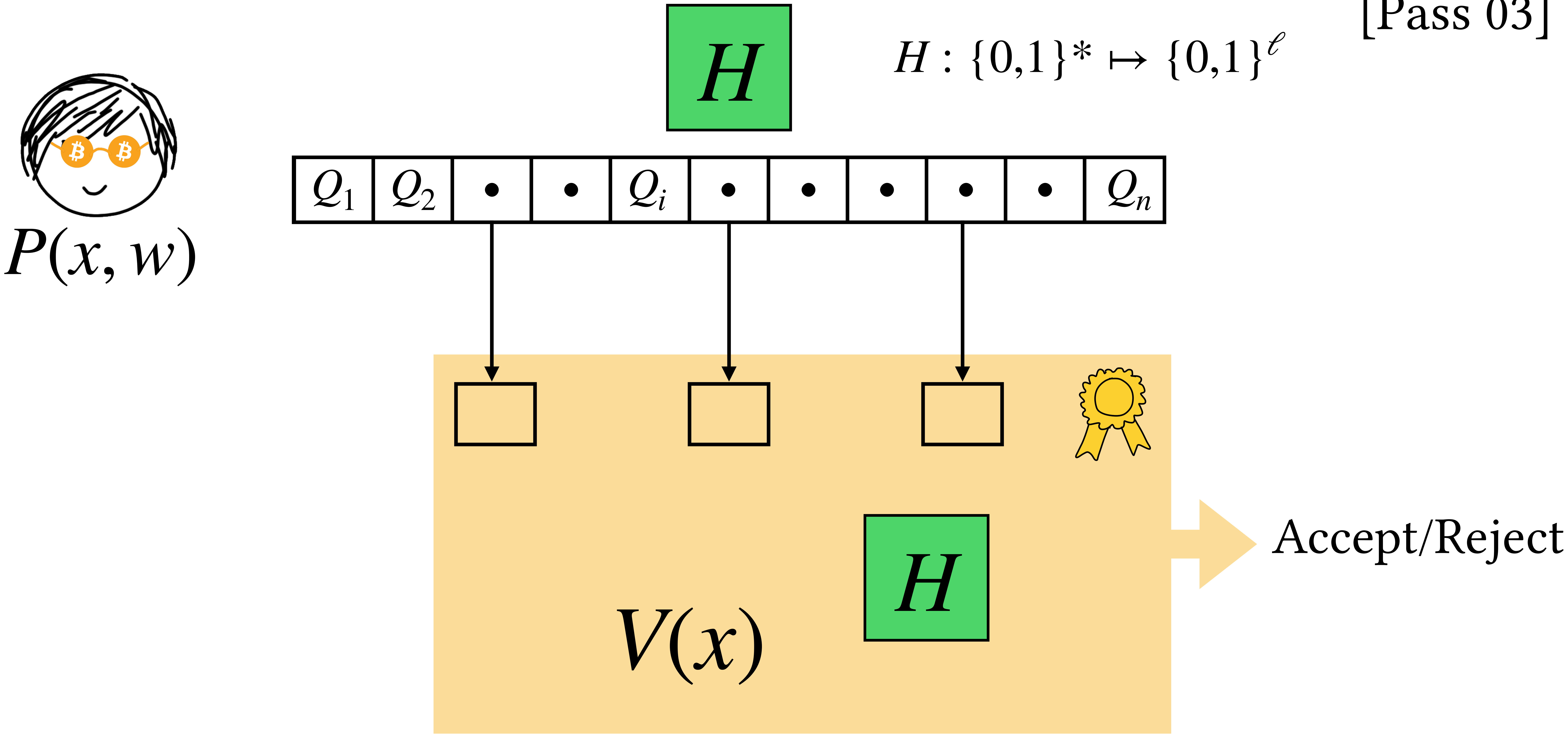

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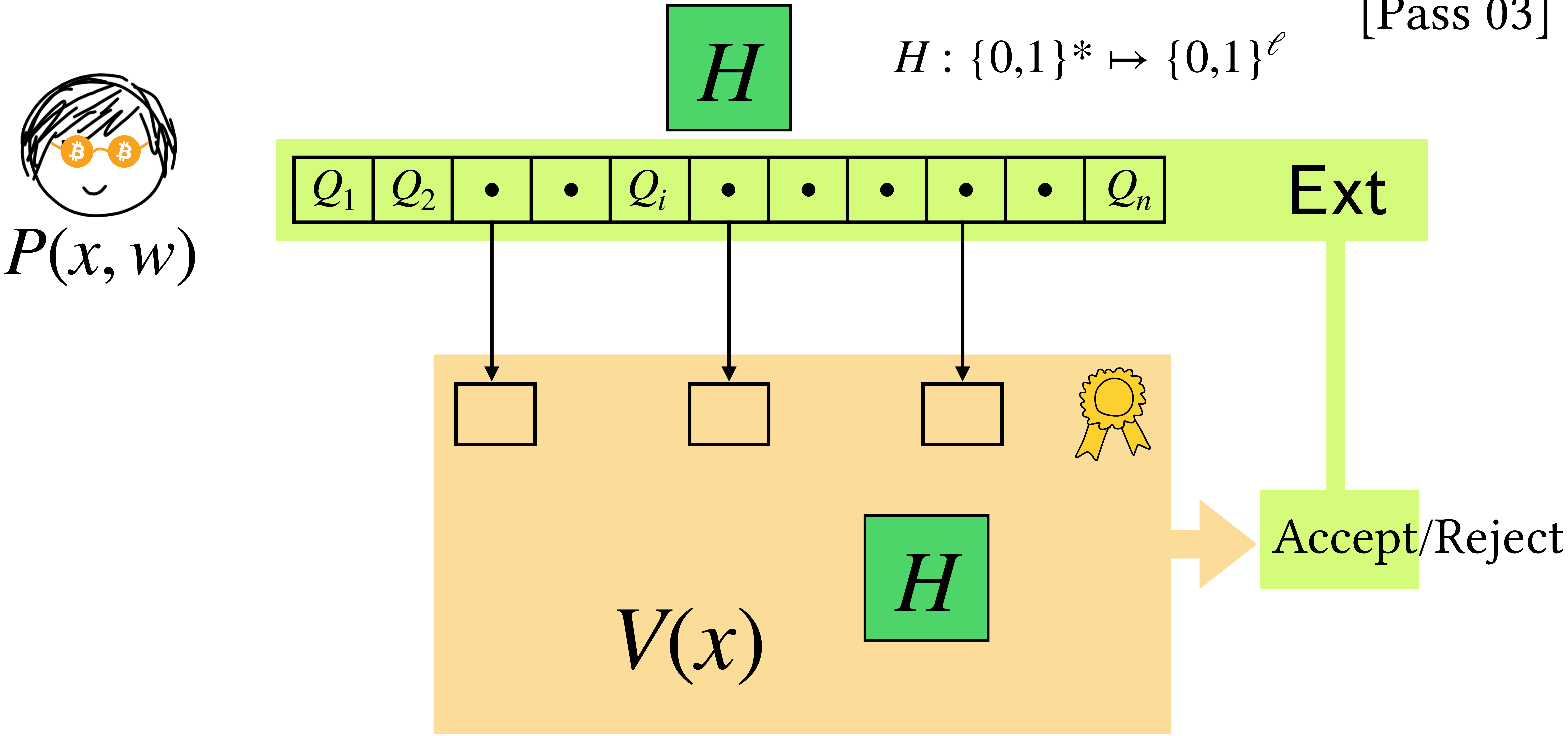

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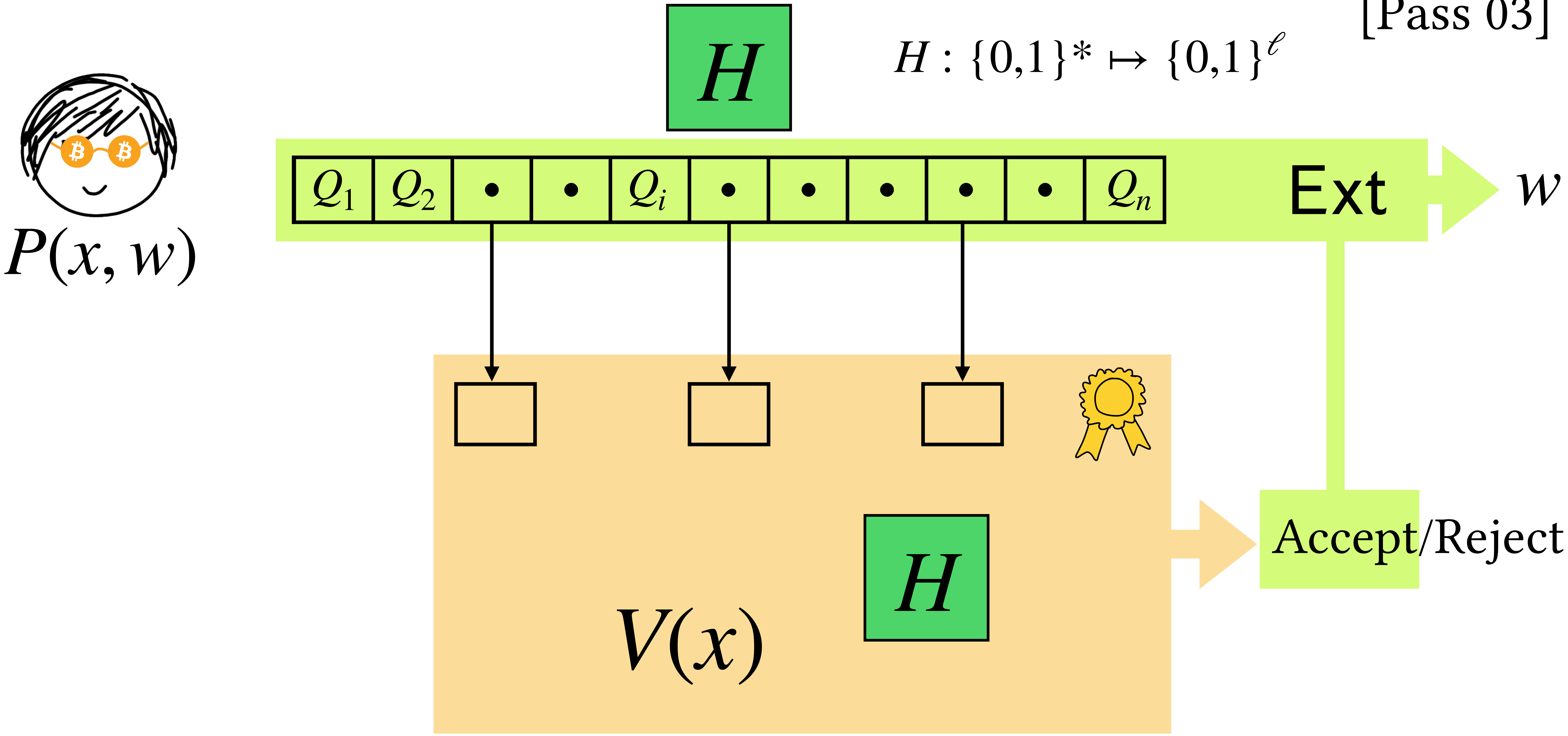


Random Oracles as Ext Privilege

[Pass 03]

- Why is it a meaningful trapdoor?
  - Hash functions are complex and highly unstructured
  - Prover must "query" each Q_i to H to obtain $H(Q_i)$
- Practical usage:
  - No "trusted setup", each query is very cheap
  - Many NIZKs **happen to achieve** SLE in the ROM

Distributing NIZKs in the ROM

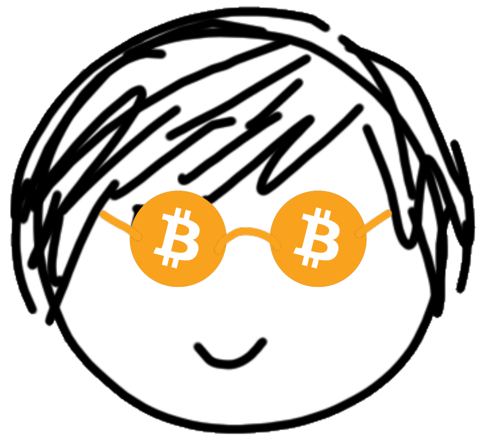
- Multiparty protocols to securely compute RO-based NIZKs should *ideally* make blackbox use of H
  - Conceptually: H should not have a circuit description
  - Practically: hash functions have large circuits
- We call them "Oracle Respecting Distributed" (ORD) protocols

Oracle Respecting Distribution is Leaky

- Consider a proof system (P^H, V^H) for some language
- Assumption: $n \in \mathsf{poly}(\kappa)$ is a strict upper bound on queries made by V to the random oracle H
  - Holds for most 'natural' schemes
- We will show: any $n+1$-party protocol that ORD-computes P^H will leak the witness to n parties

Trimming Resilience

H



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π

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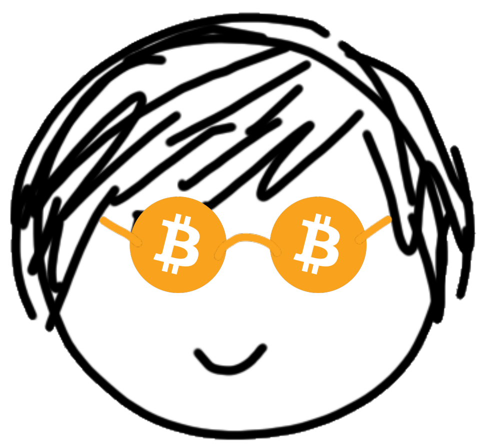

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V checks at most $n = 2$ queries

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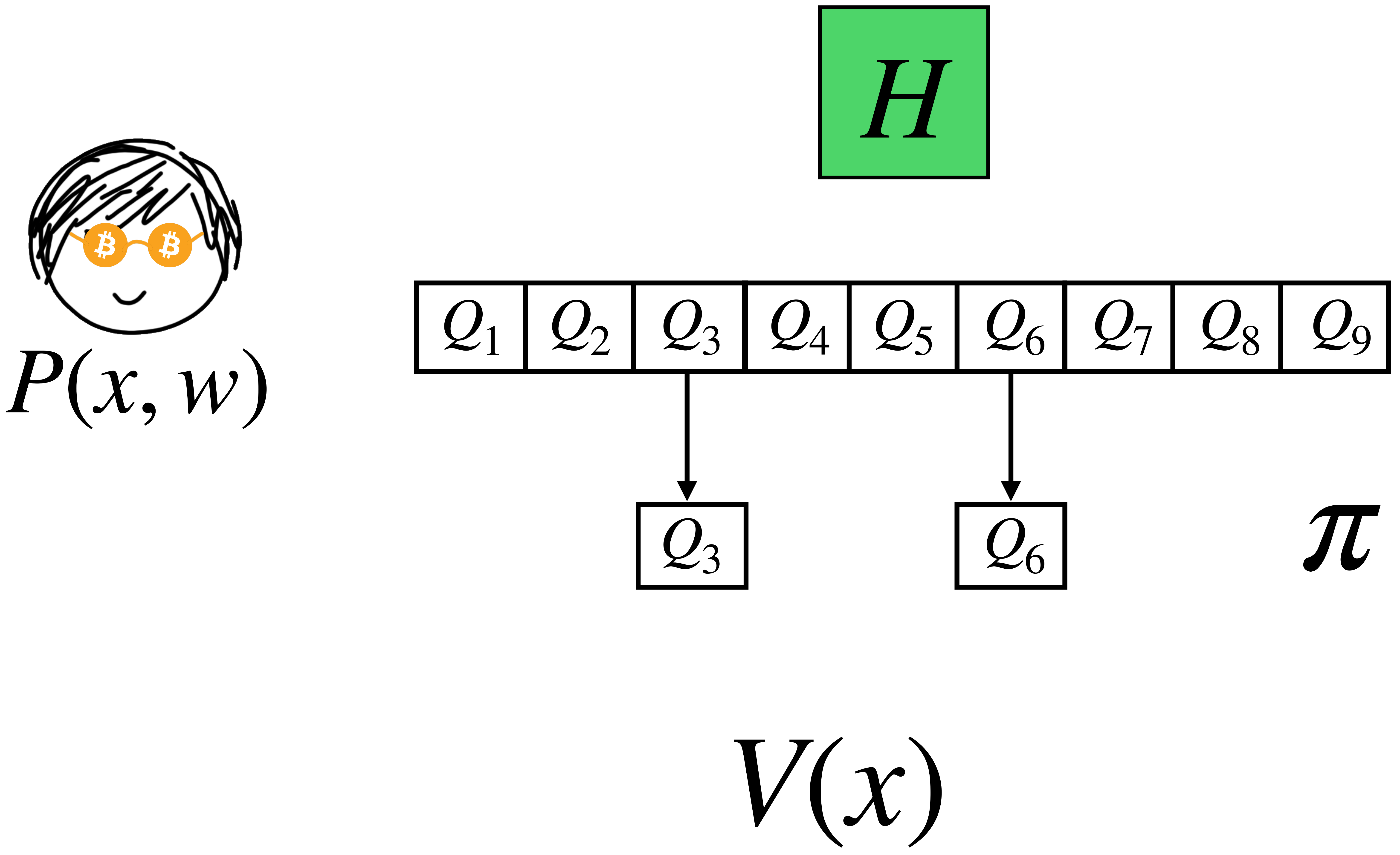

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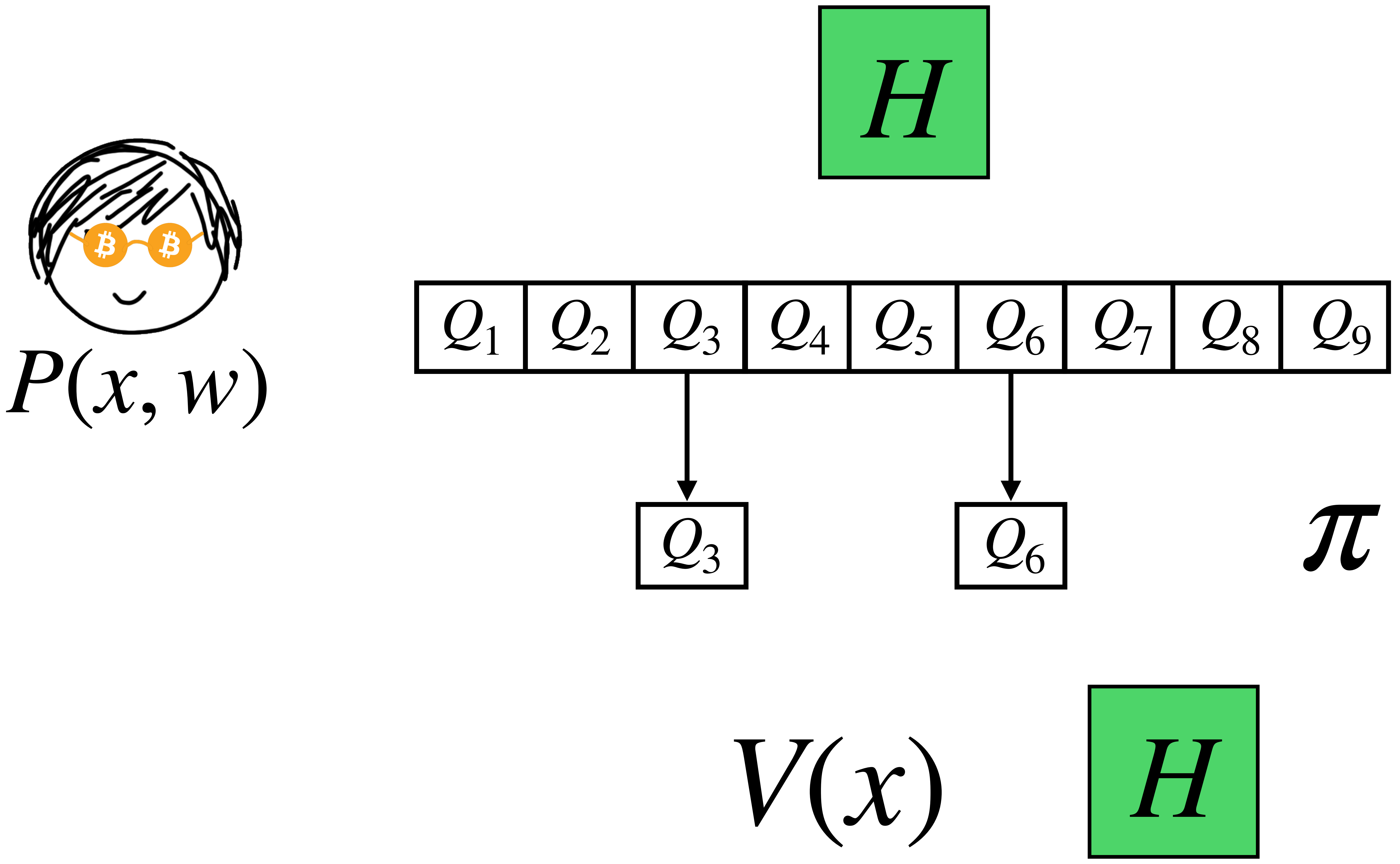

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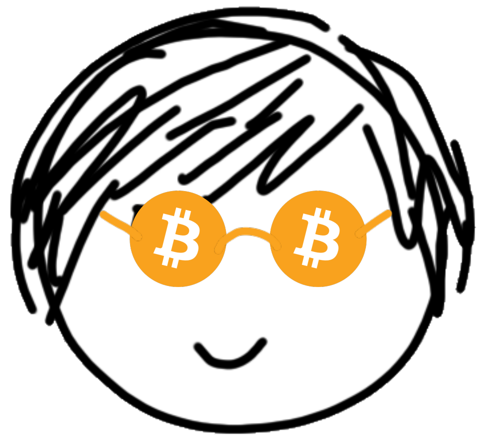

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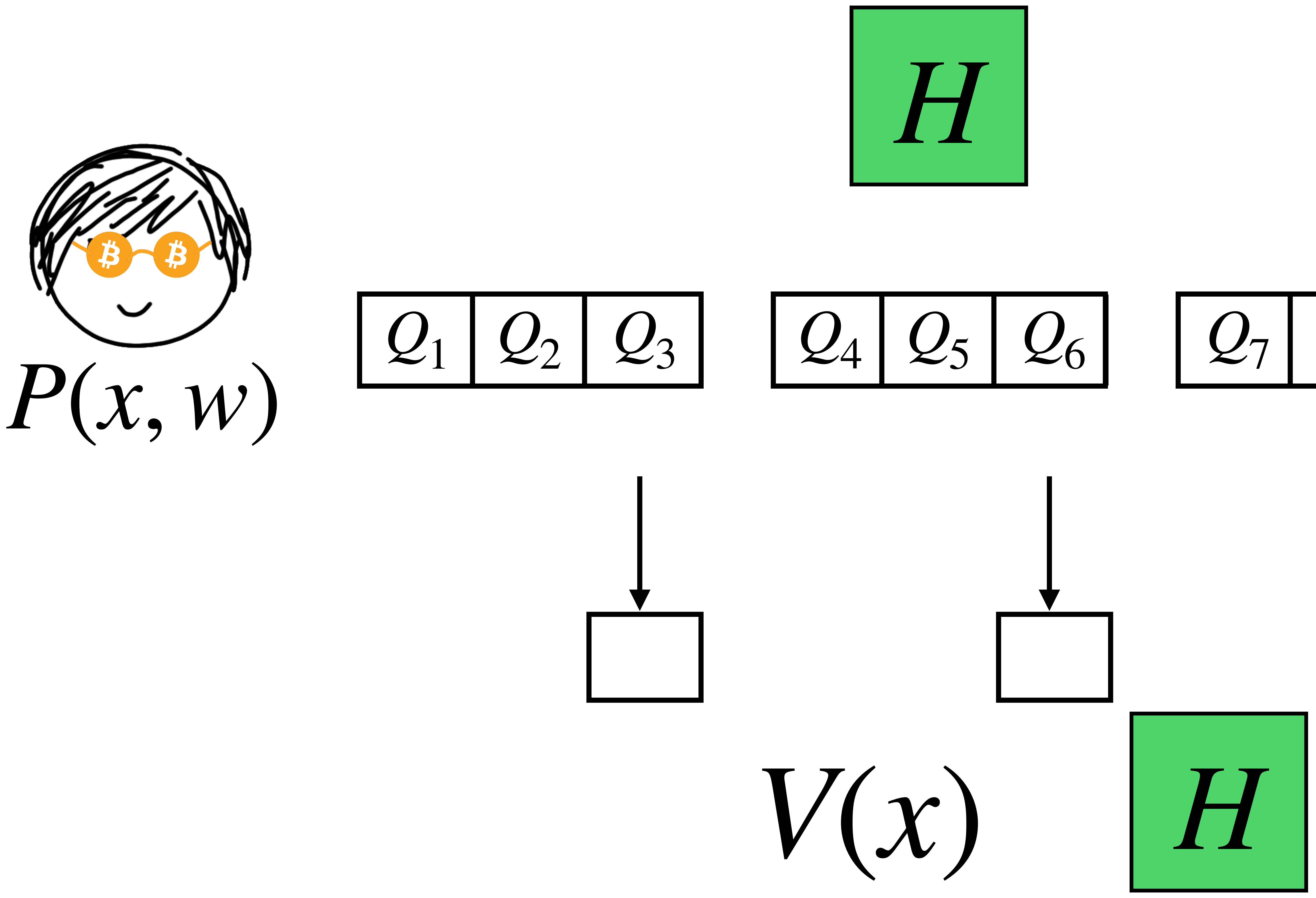

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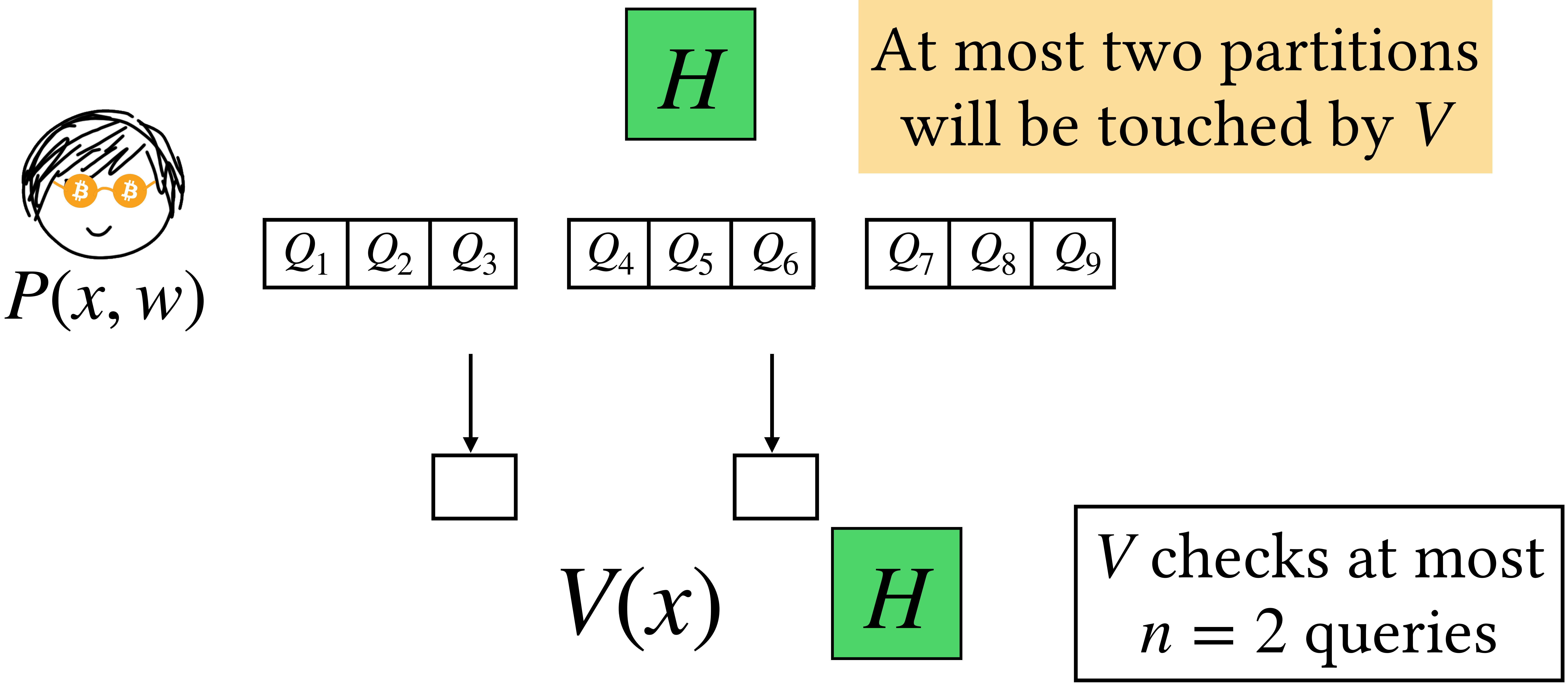

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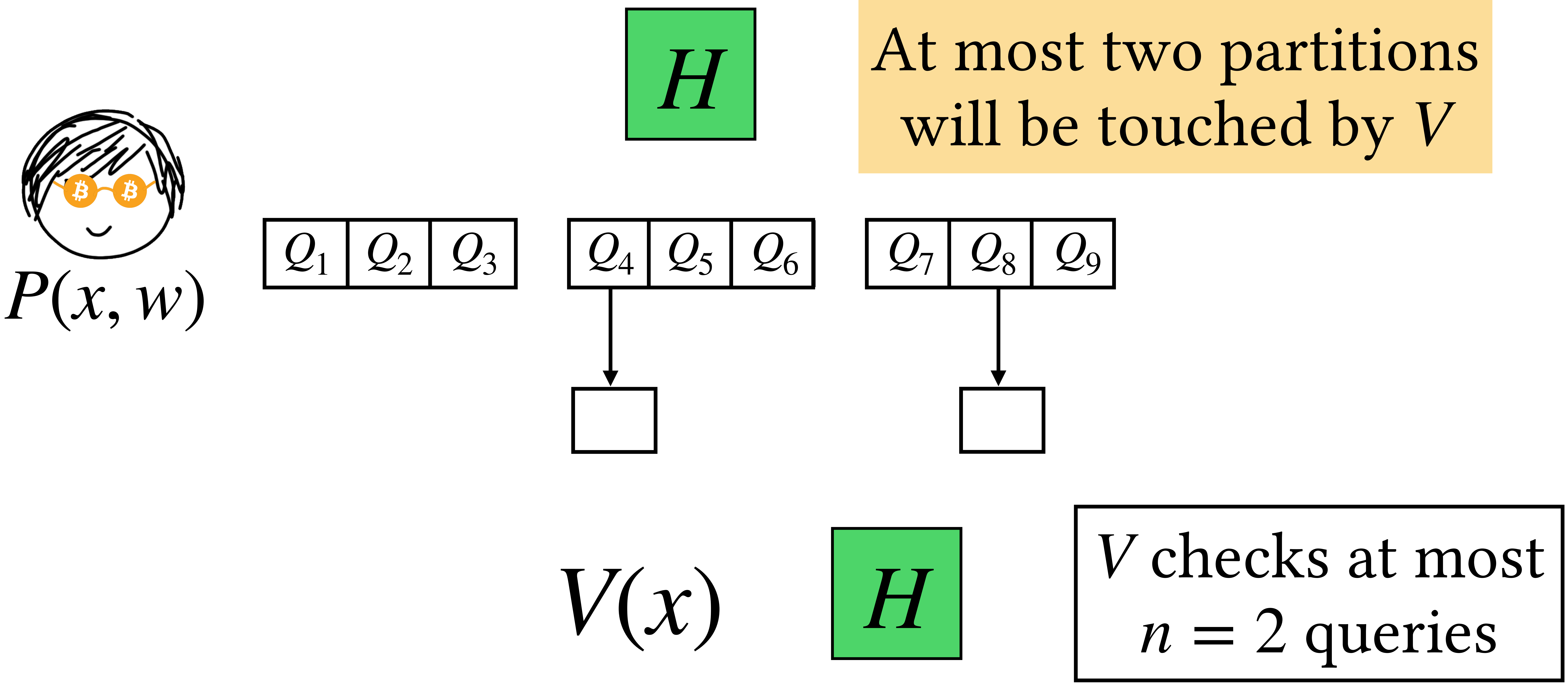

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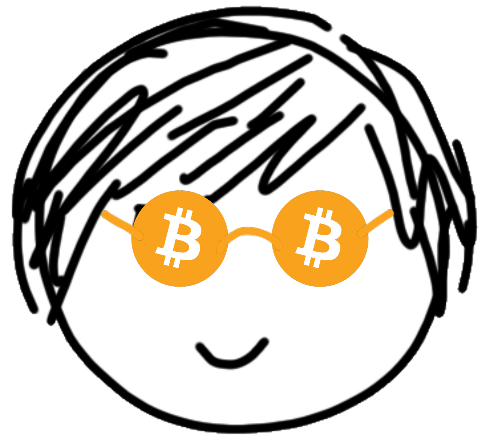
At most two partitions will be touched by V



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Randomly selected partition:
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$V(x)$

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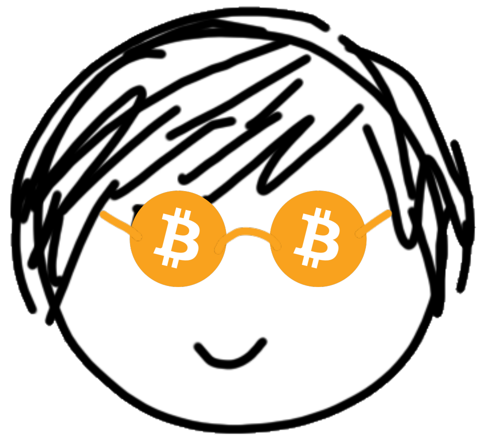

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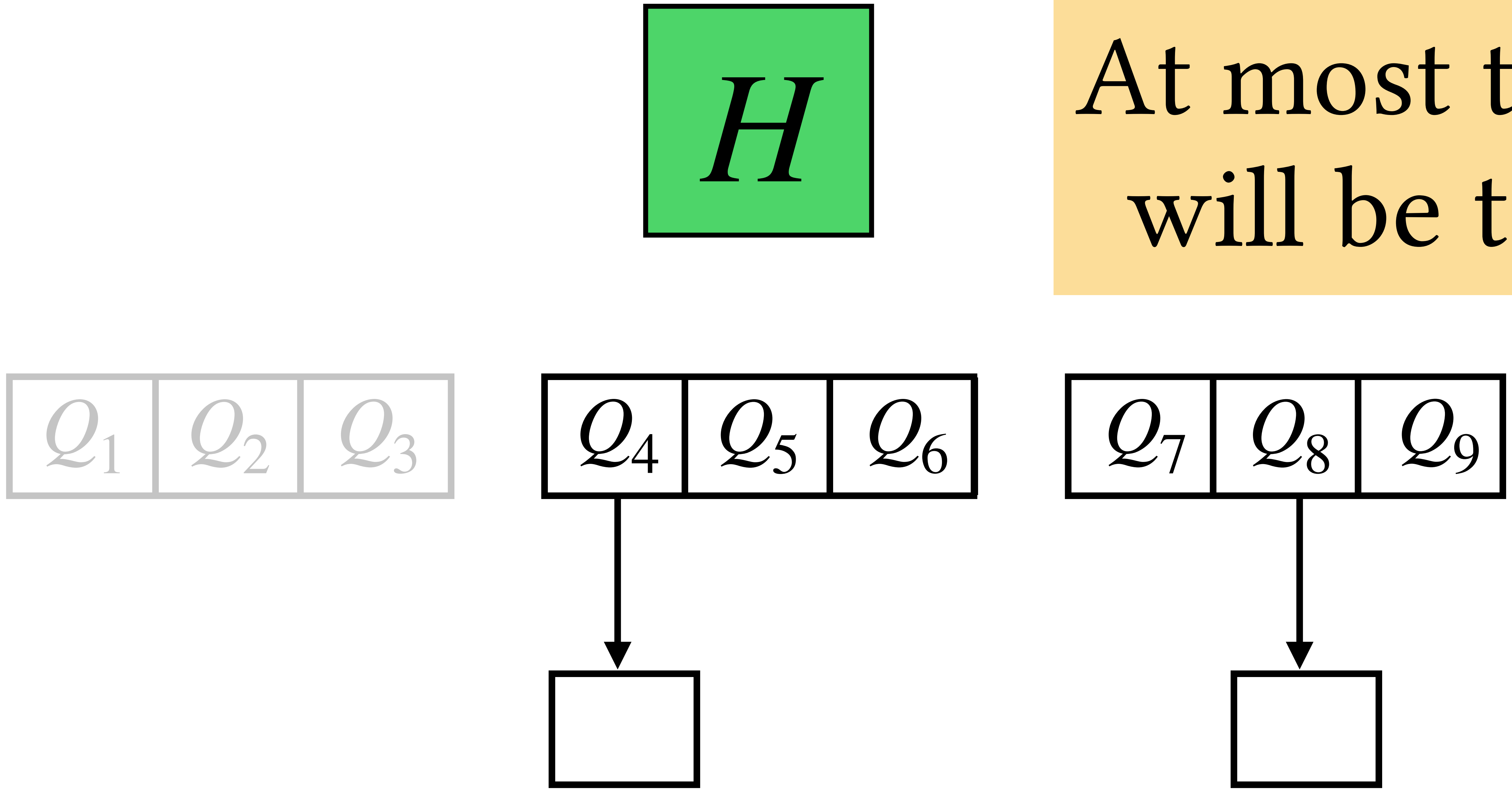

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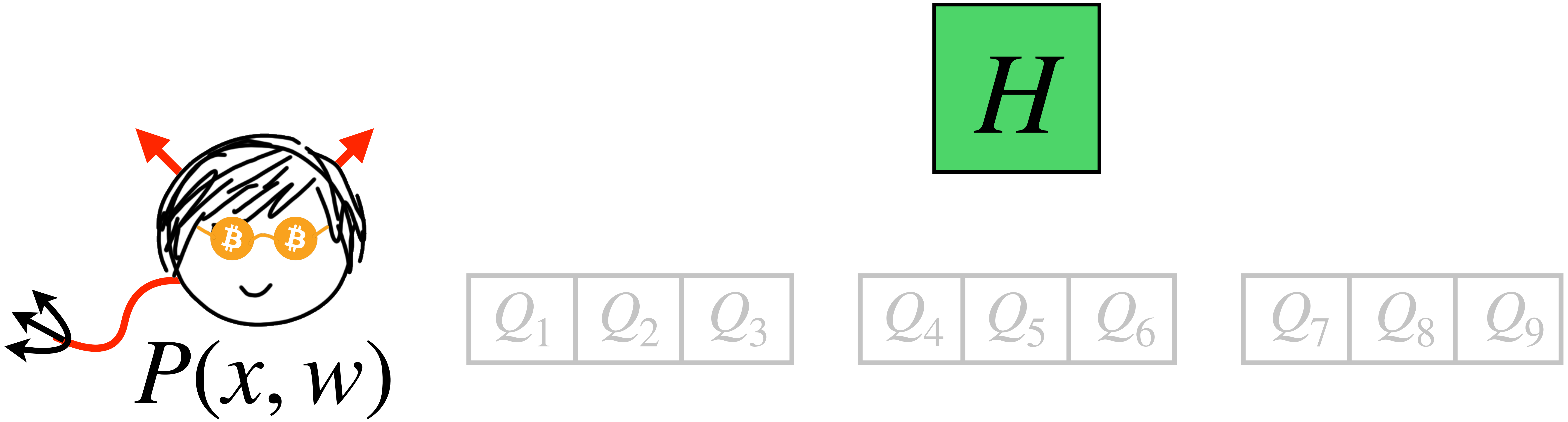

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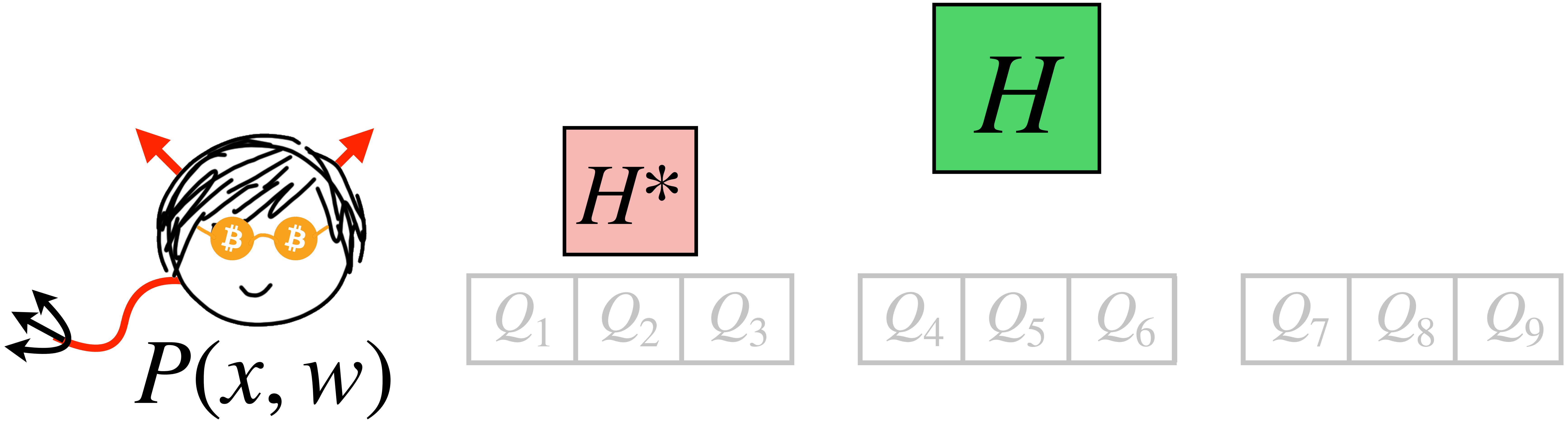

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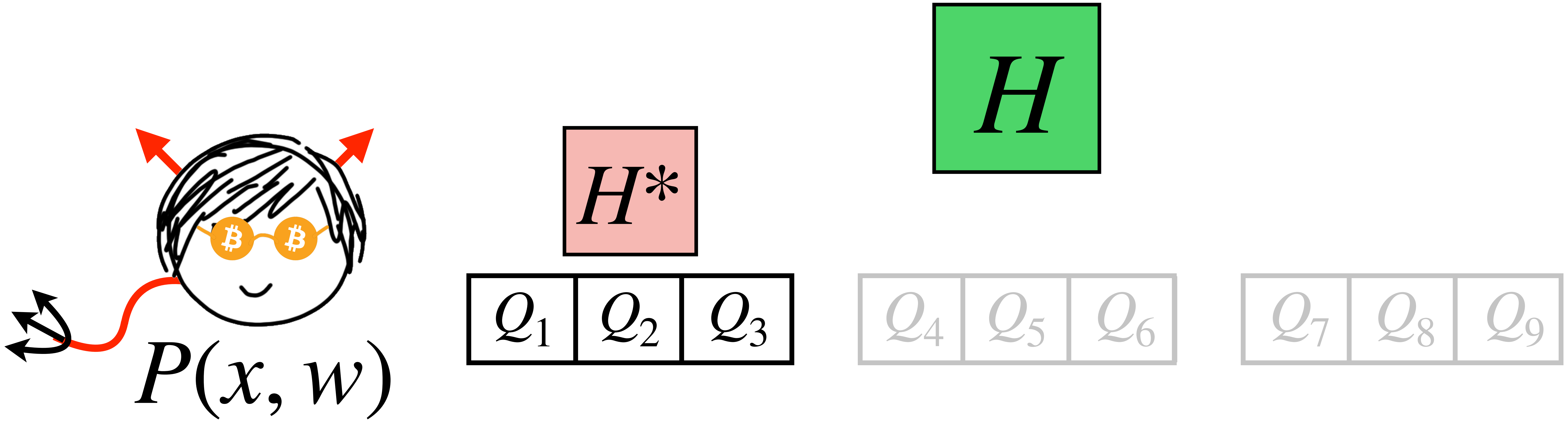

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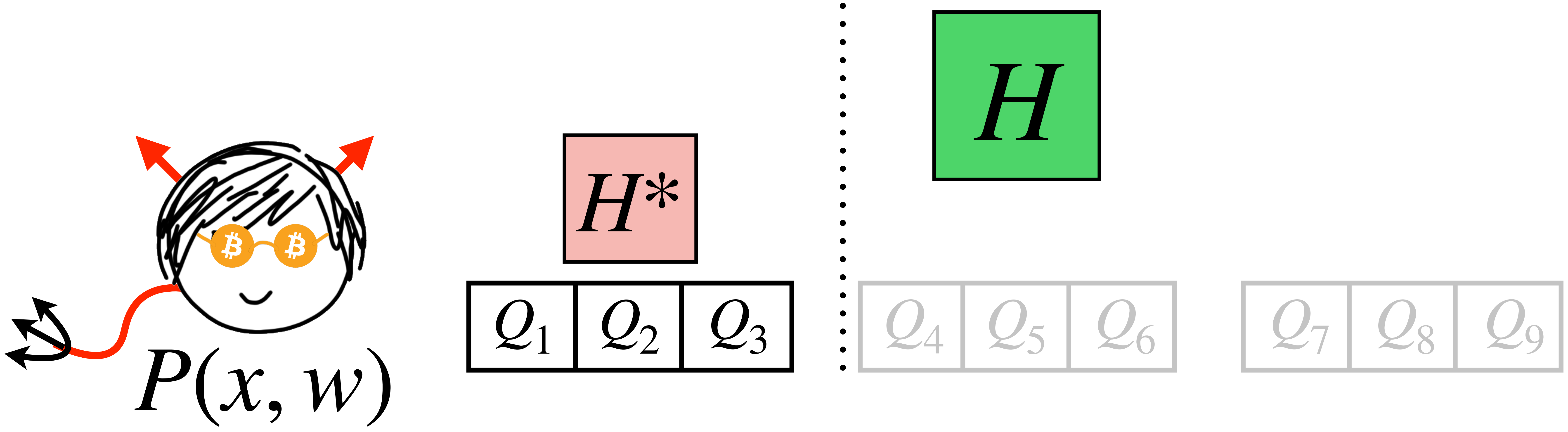

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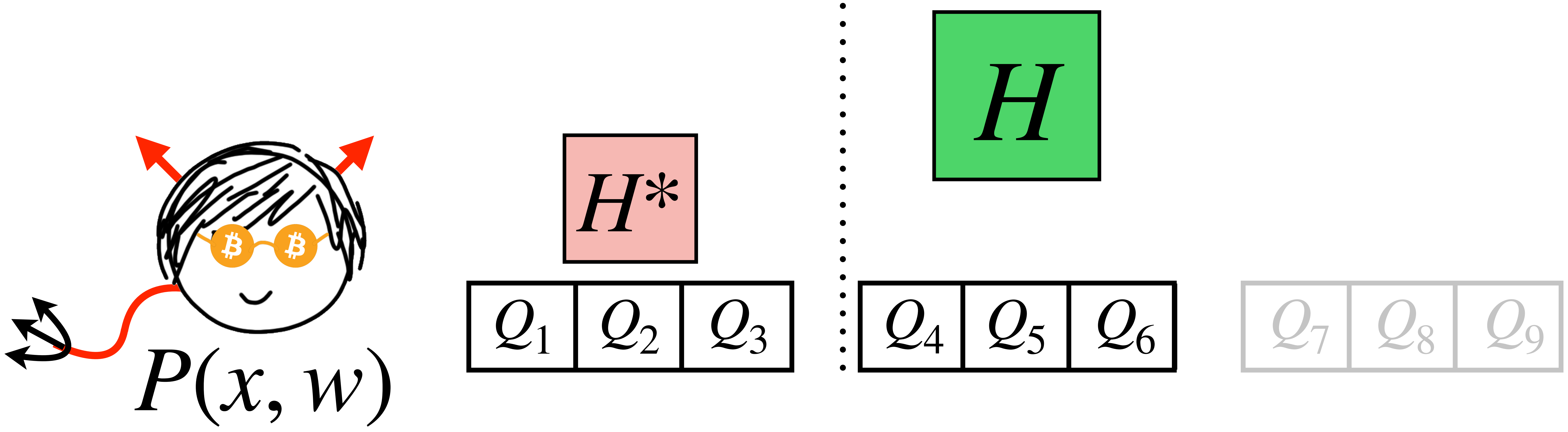

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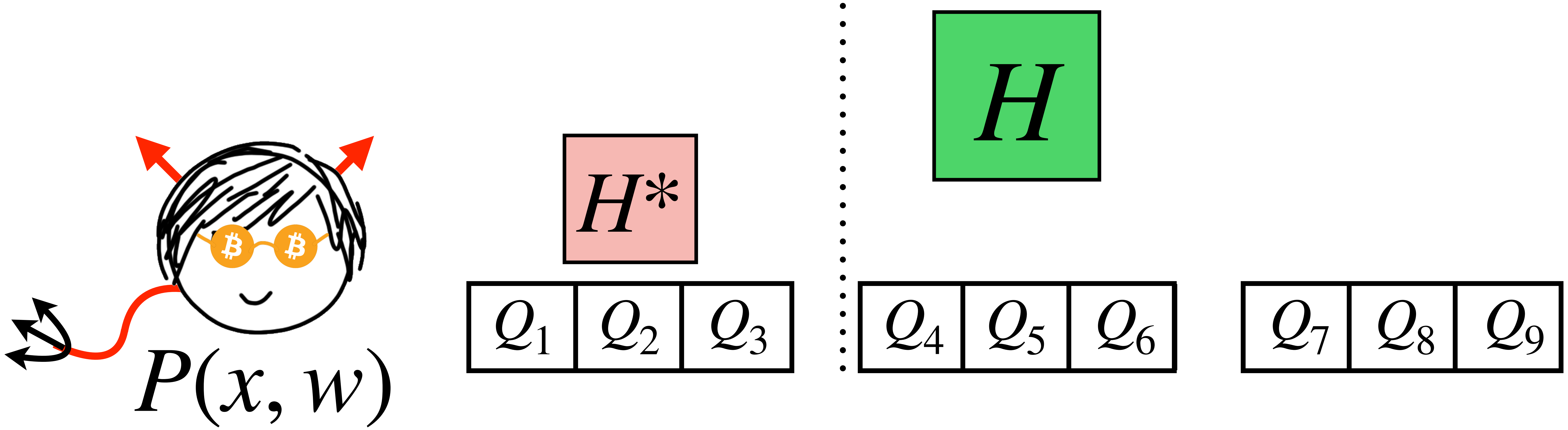

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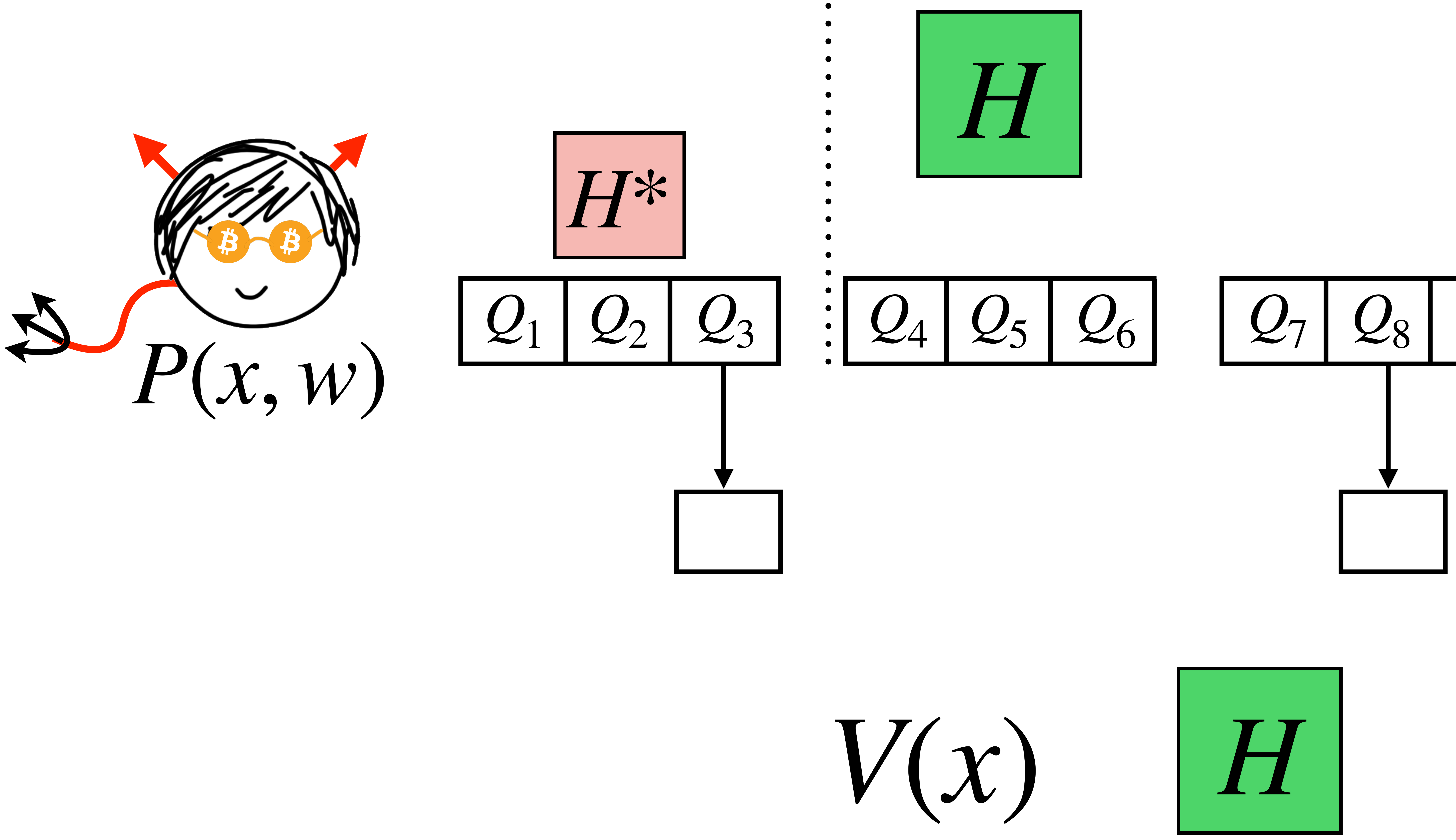

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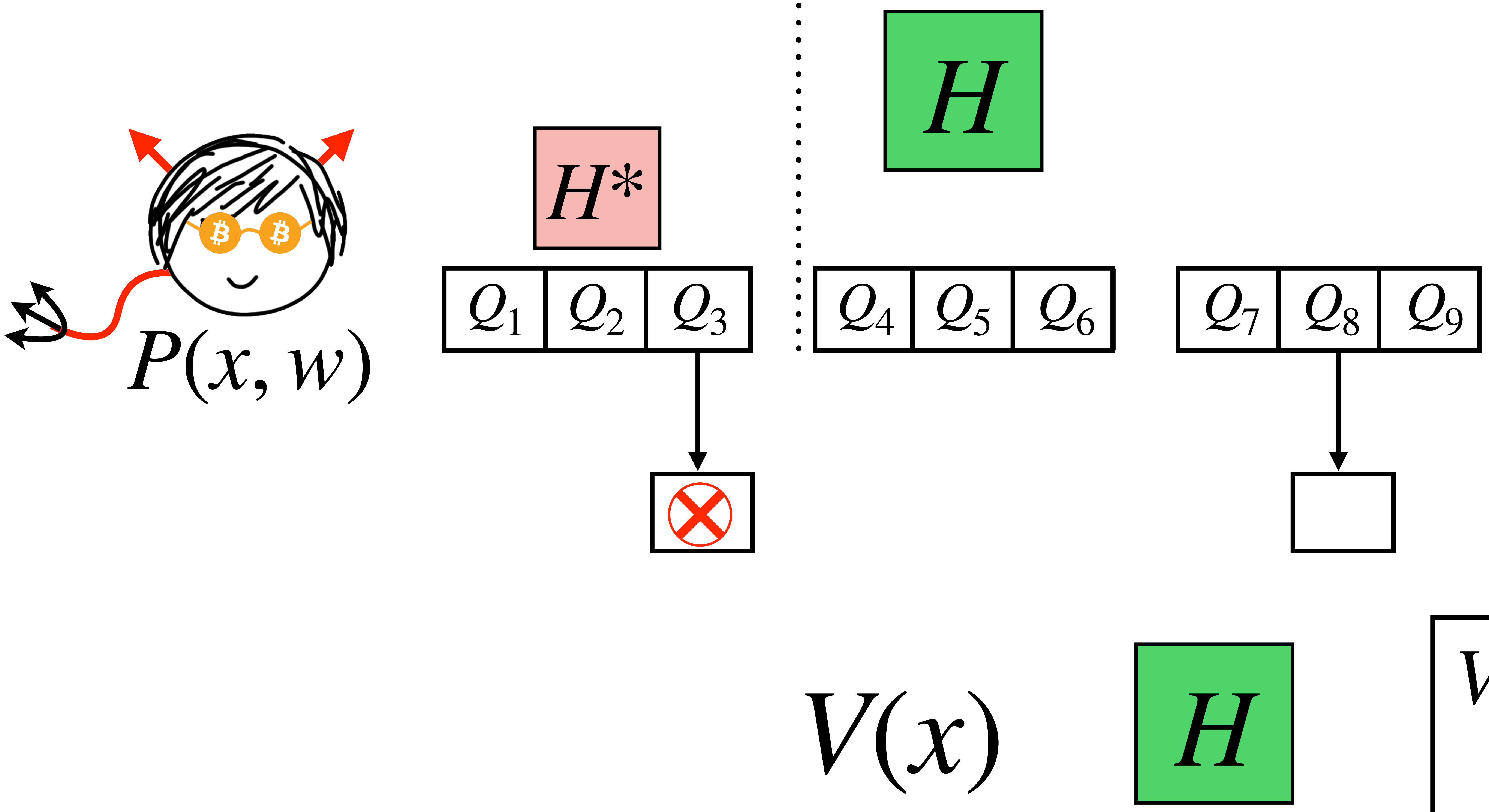

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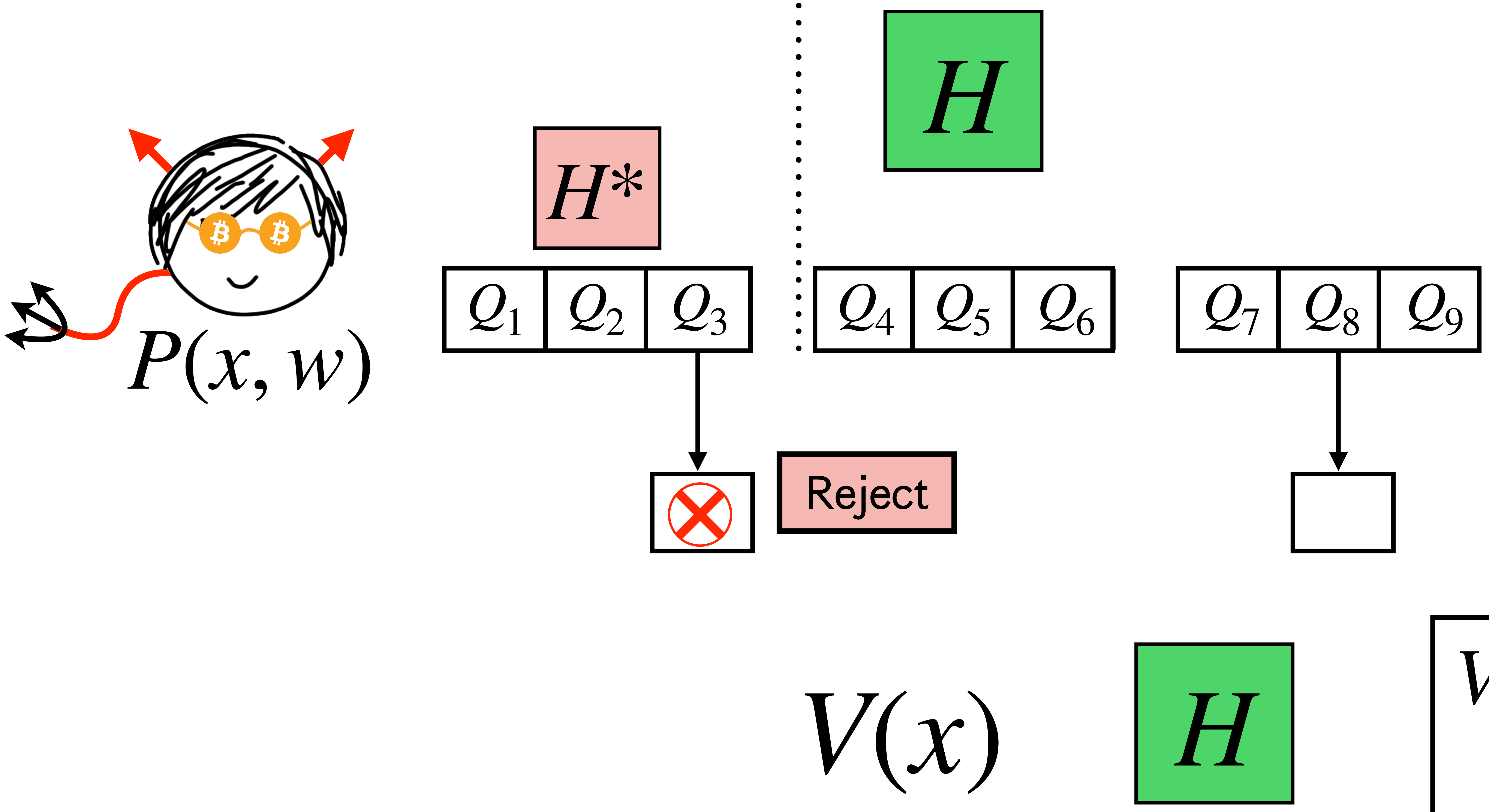

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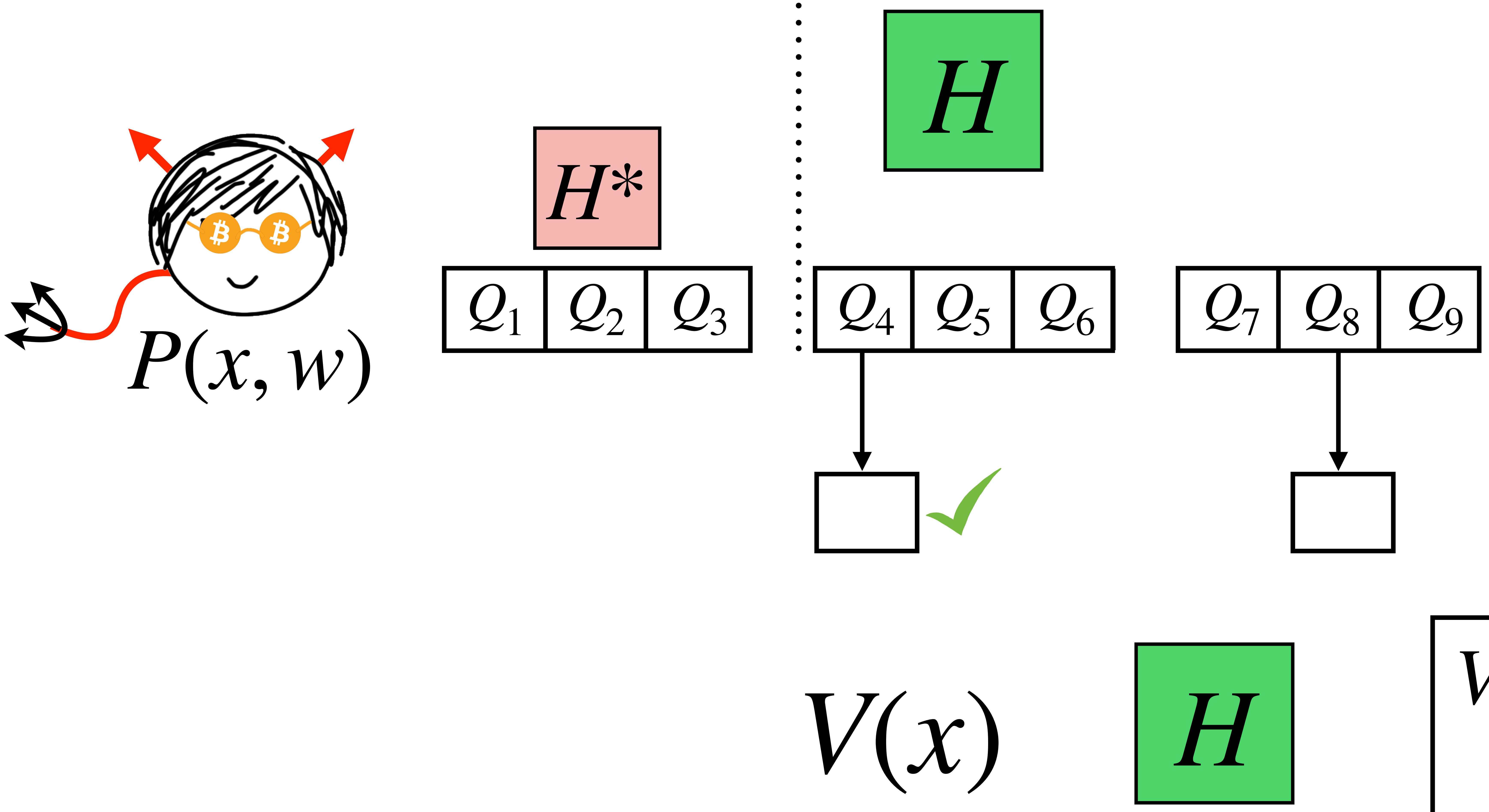

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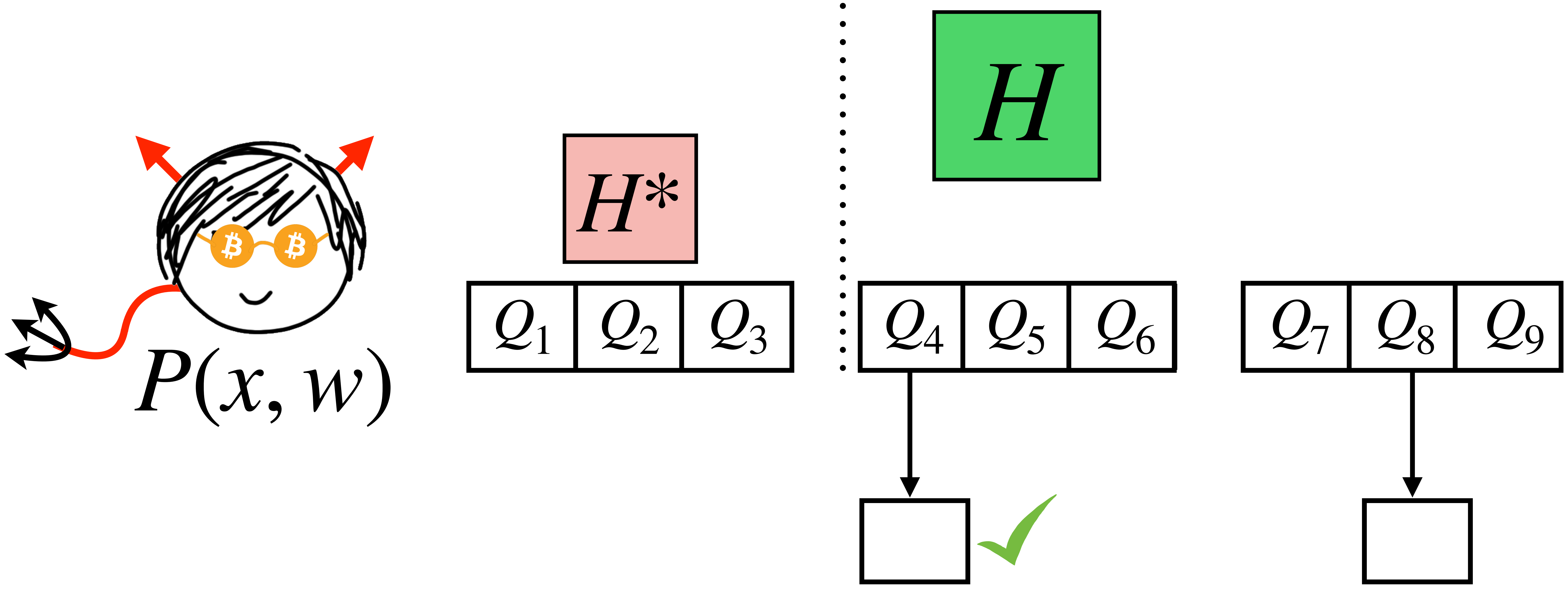

Pr[V accepts] $\geq 1/3$

$V(x)$

H

V checks at most $n = 2$ queries

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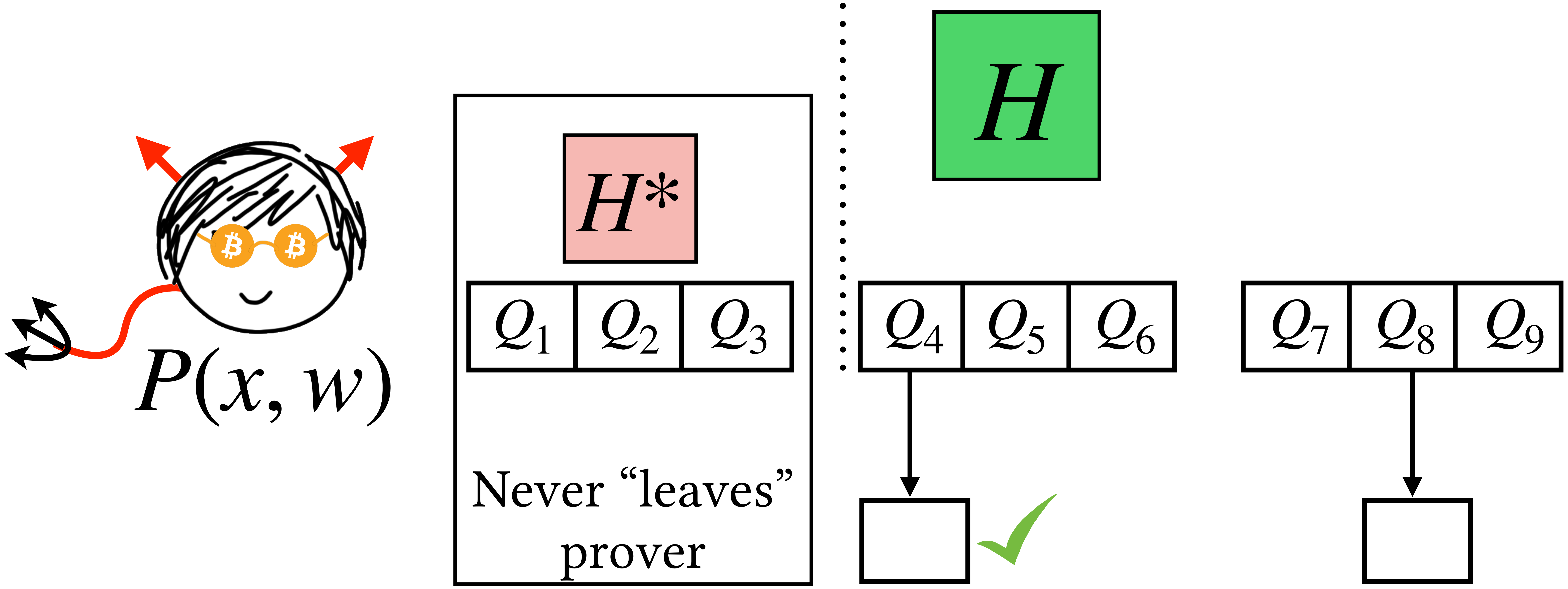

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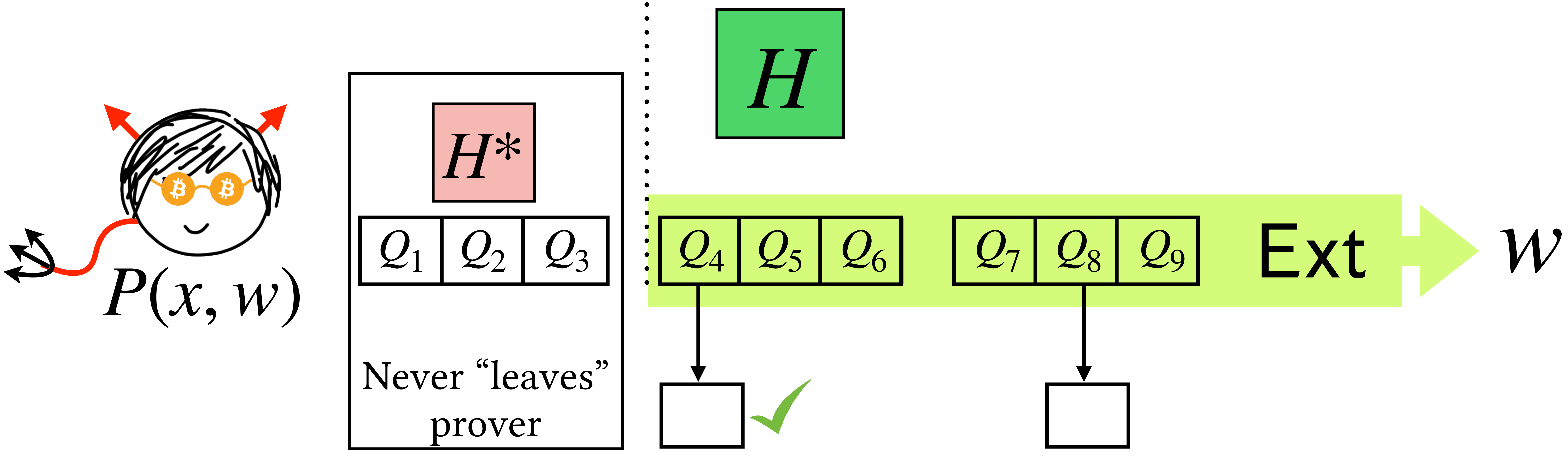

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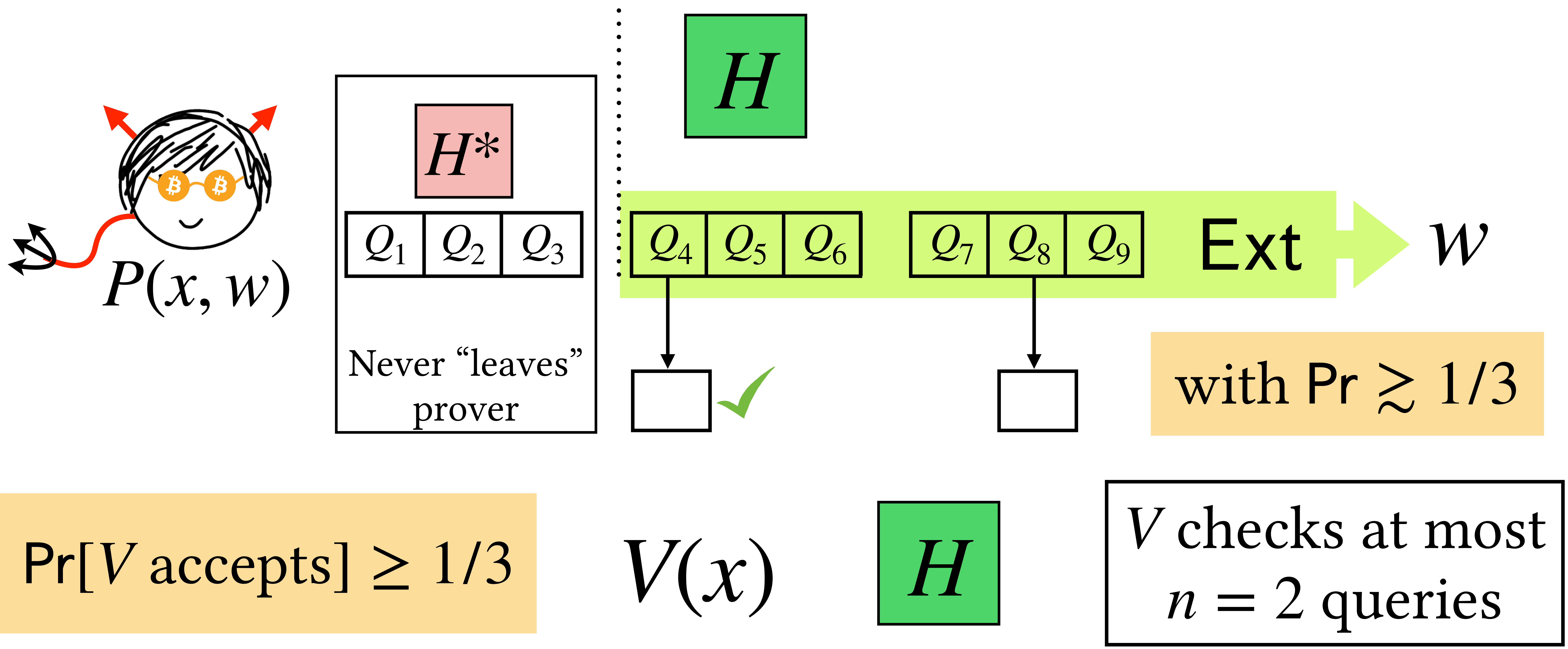

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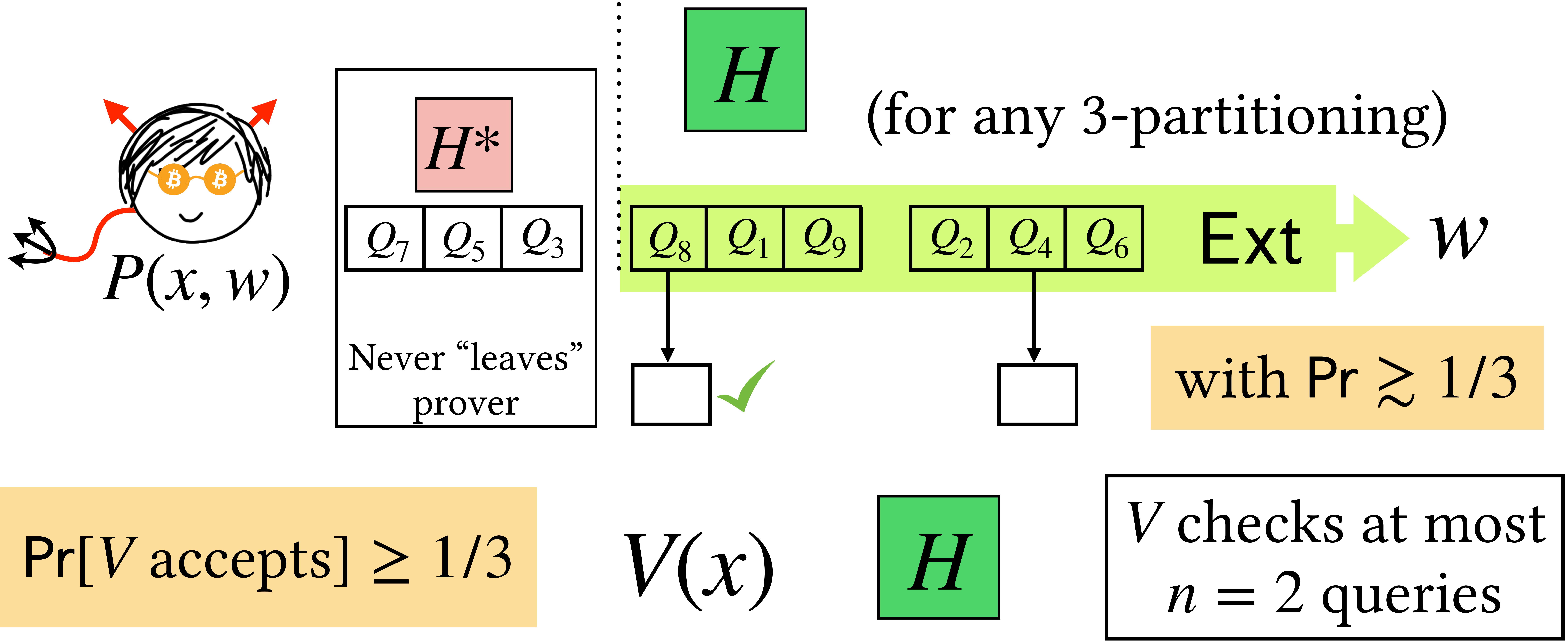
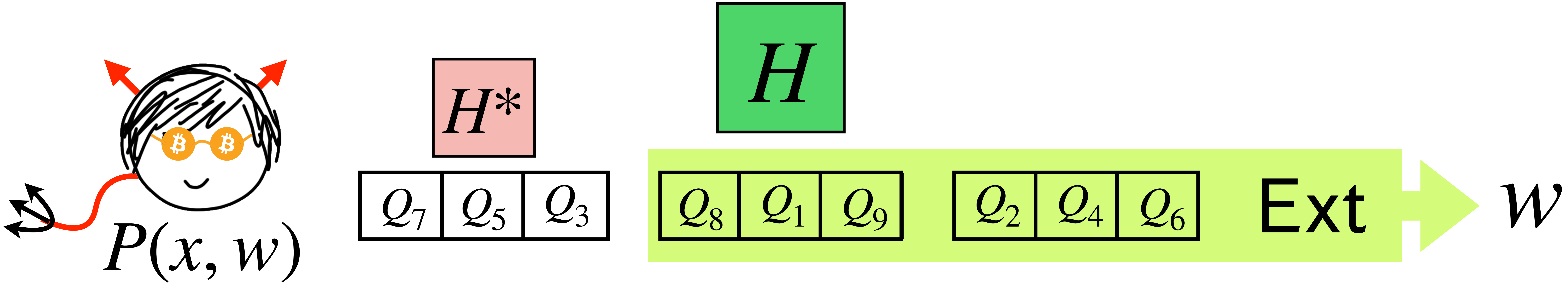


Trimming Resilience

Lemma: For any $n + 1$-partitioning of RO queries, omitting *one* partition still allows extraction



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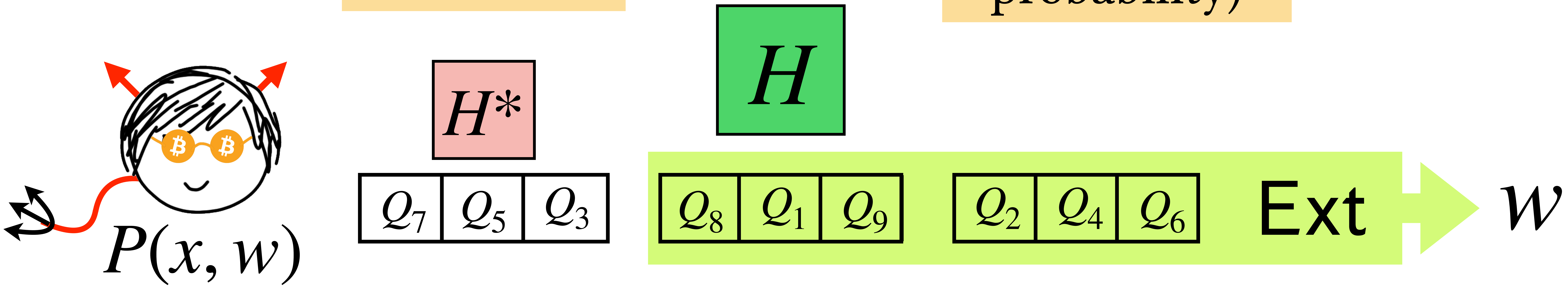
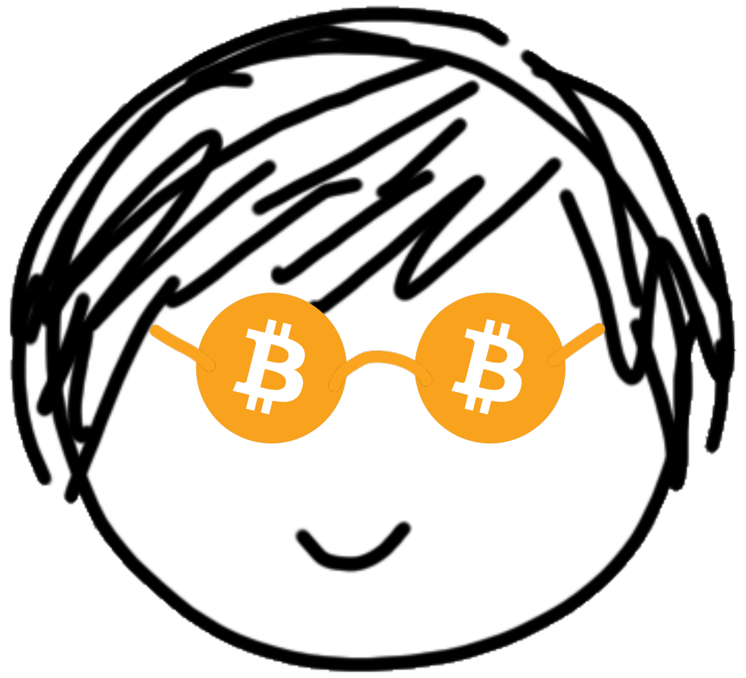
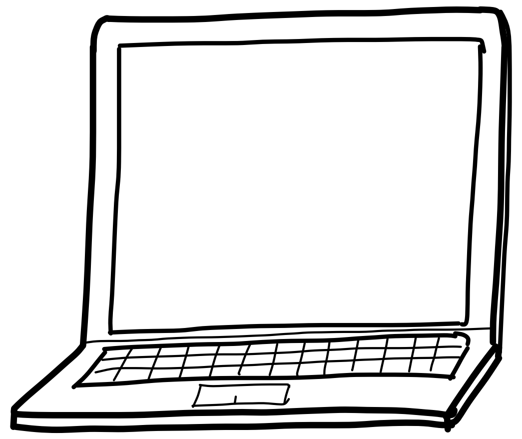
(random)

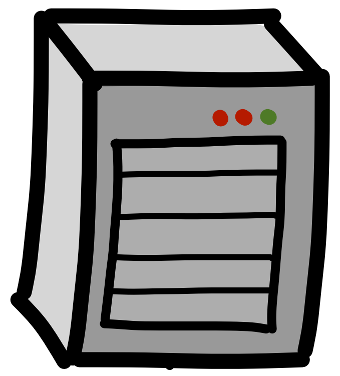
(w. noticeable probability)



Oracle Respecting Distribution







(x, w)

$w_0 , w_1 , w_2 \leftarrow \mathsf{Share}(w)$

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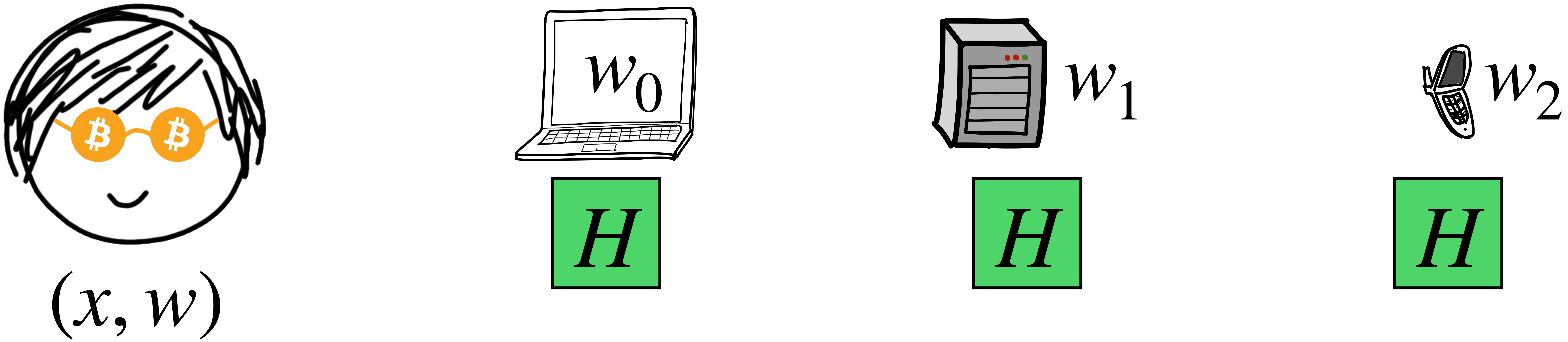

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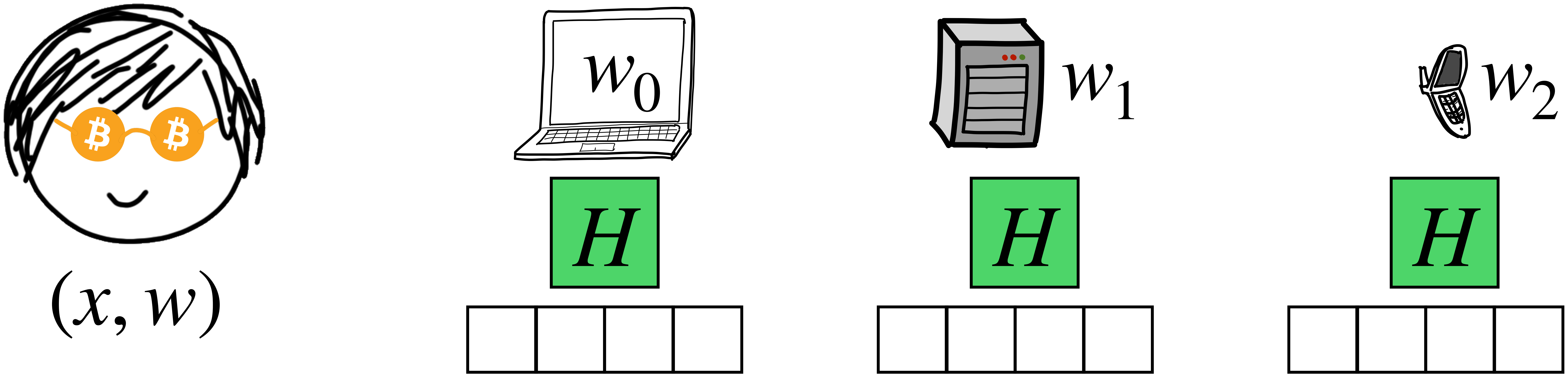
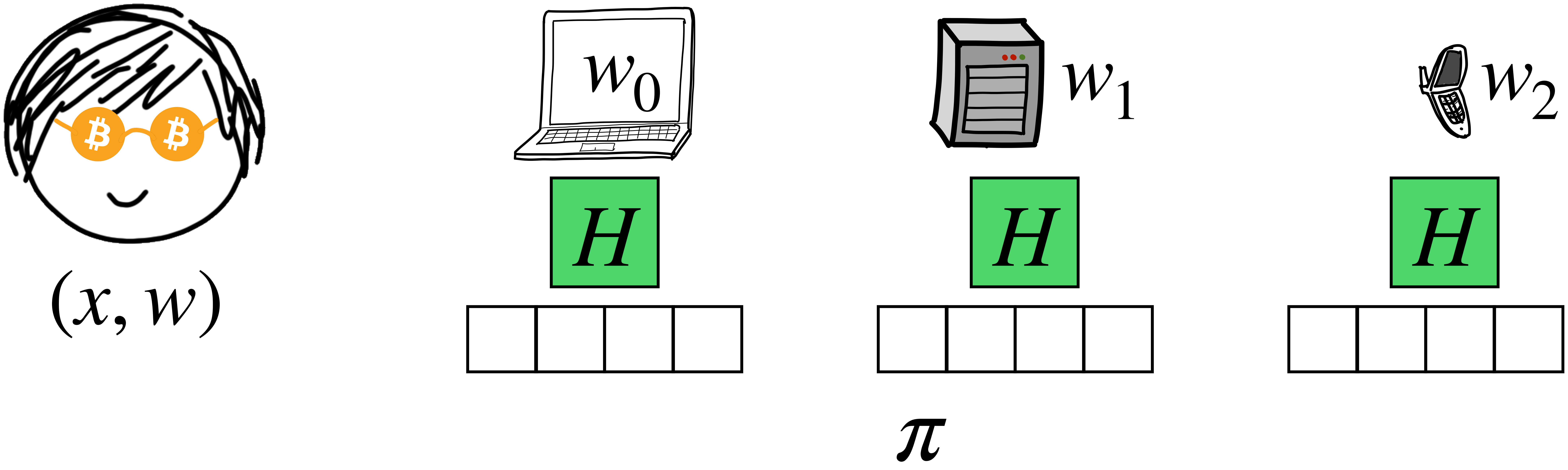


Oracle Respecting Distribution



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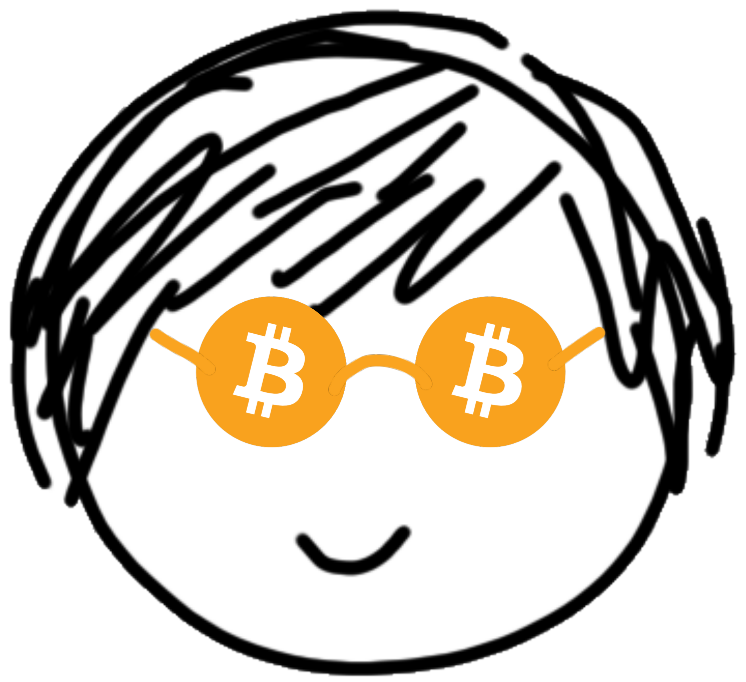
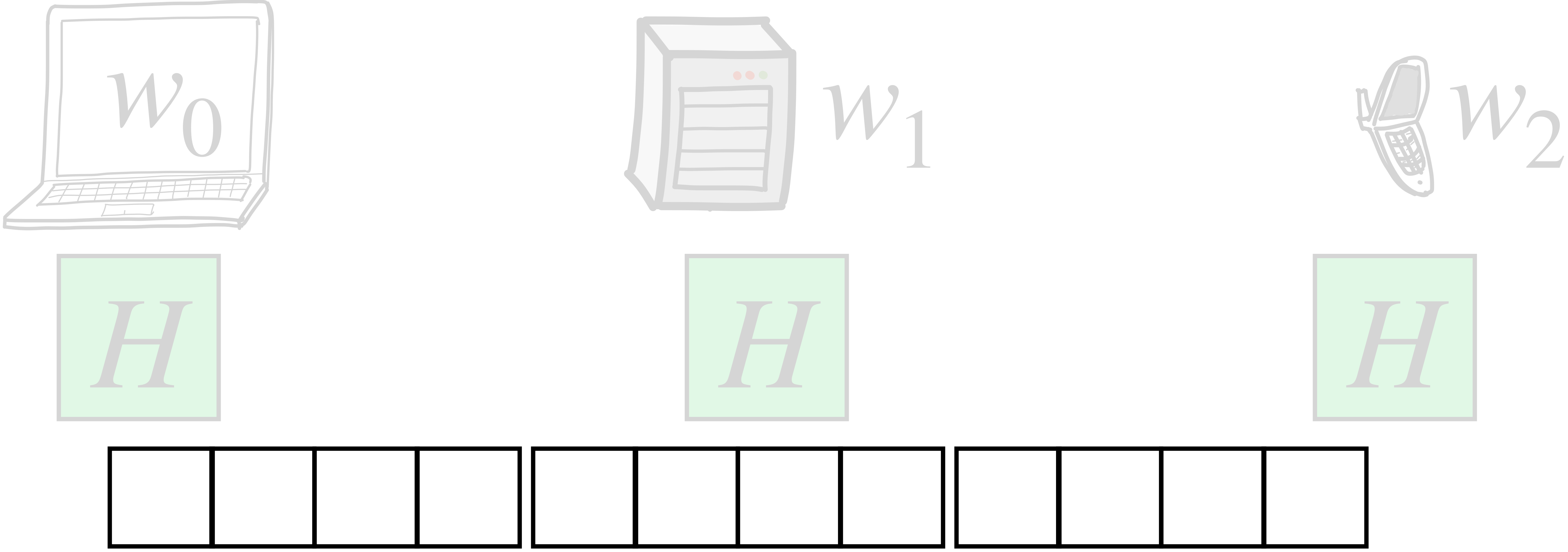


(x, w)



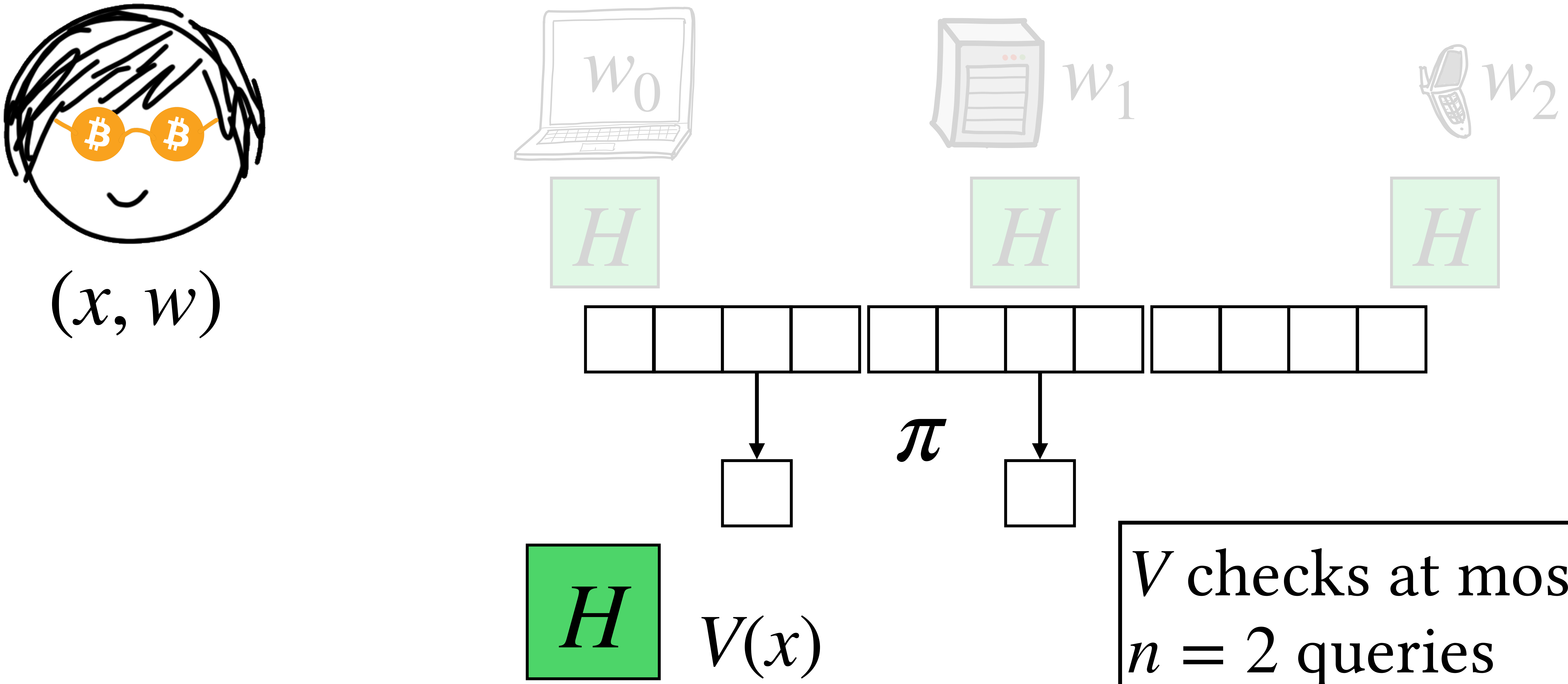
π

H

$V(x)$

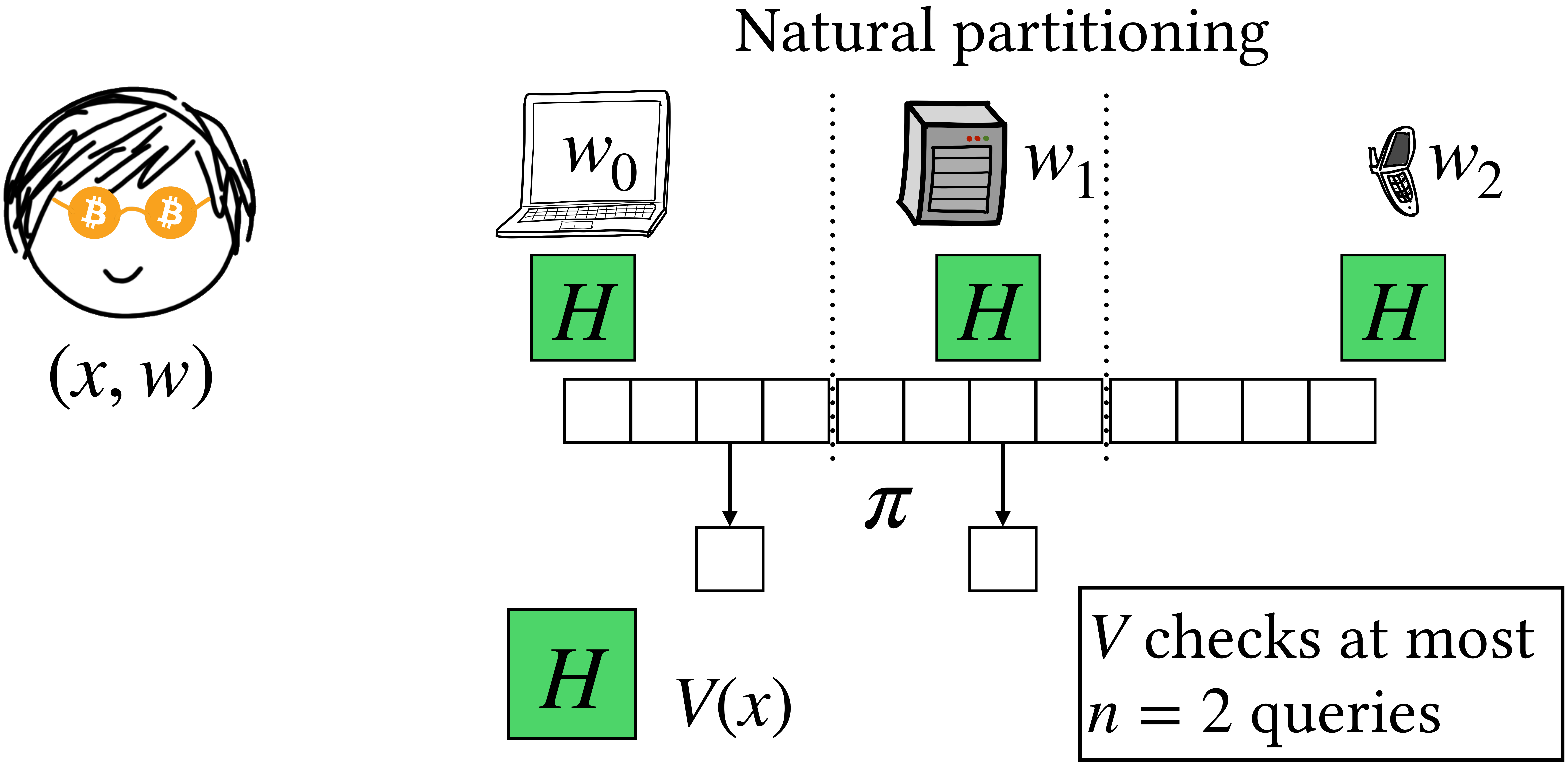
V checks at most $n = 2$ queries

Oracle Respecting Distribution



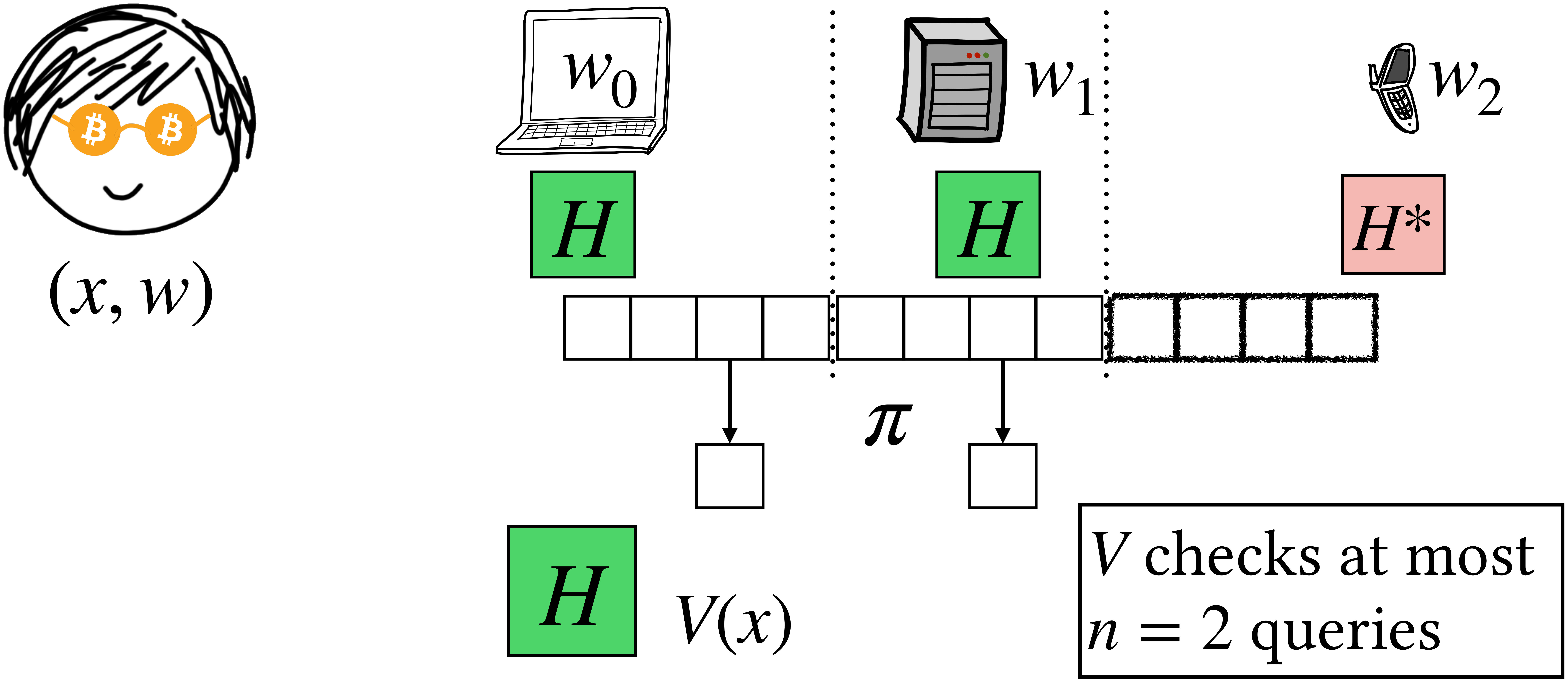
V checks at most $n = 2$ queries

Oracle Respecting Distribution



Oracle Respecting Distribution

Natural partitioning



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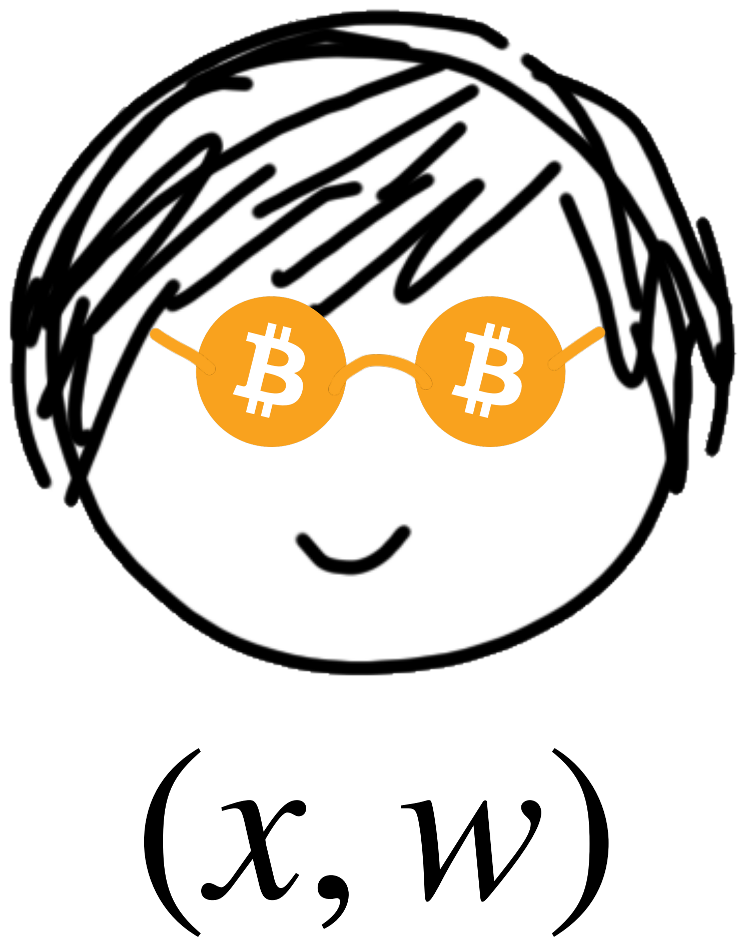

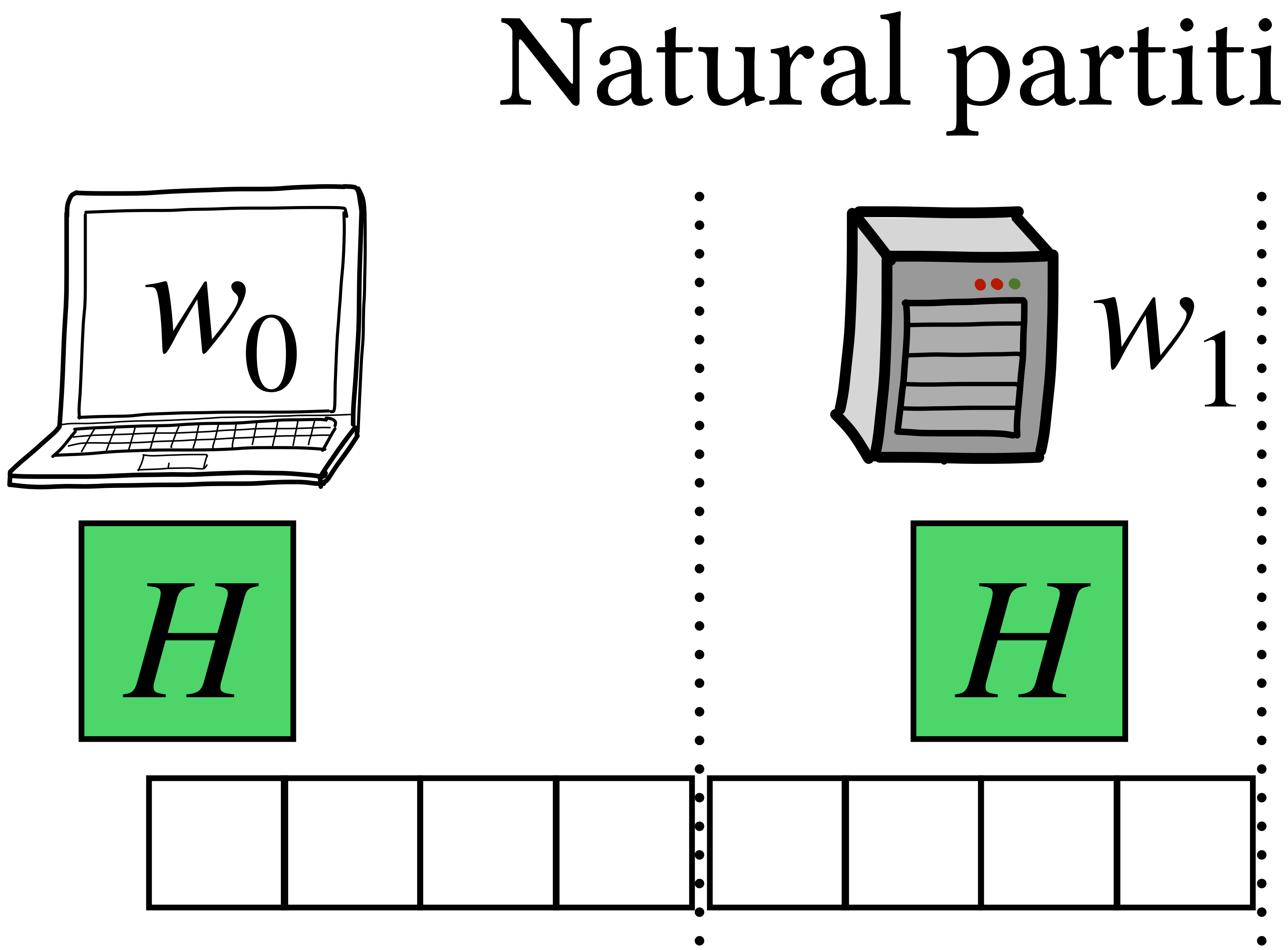

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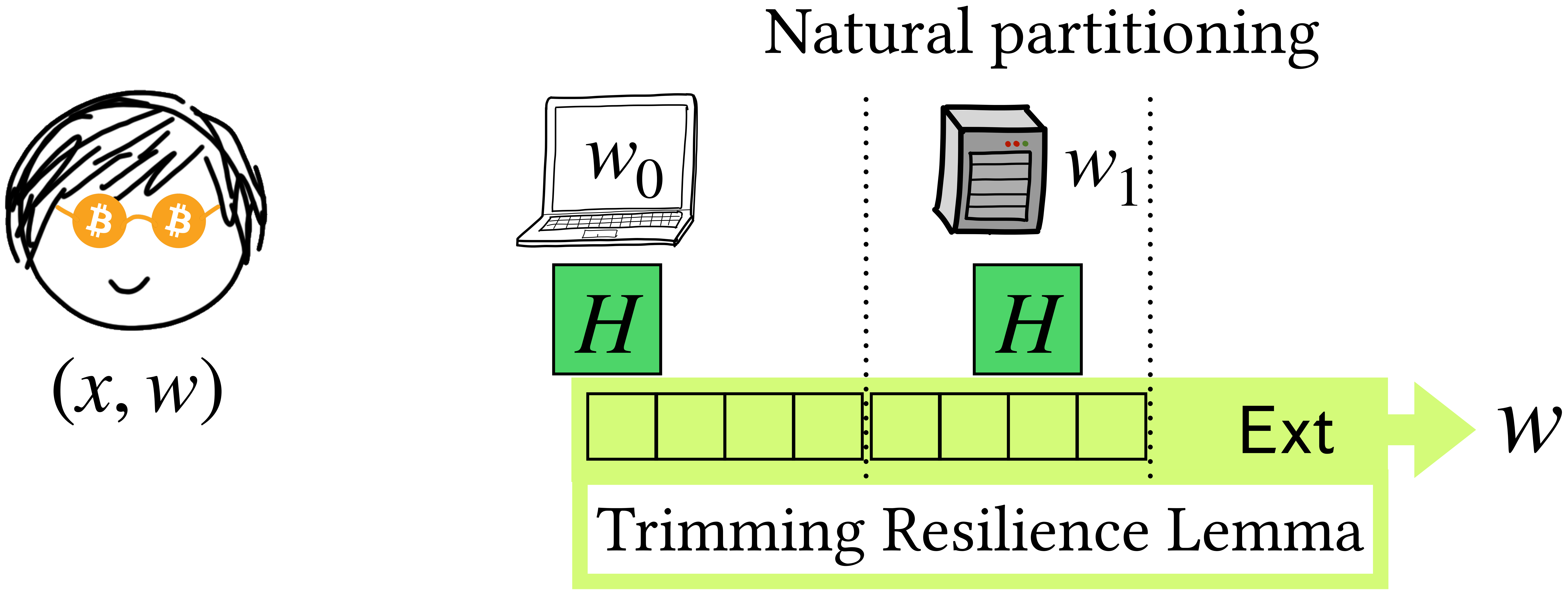

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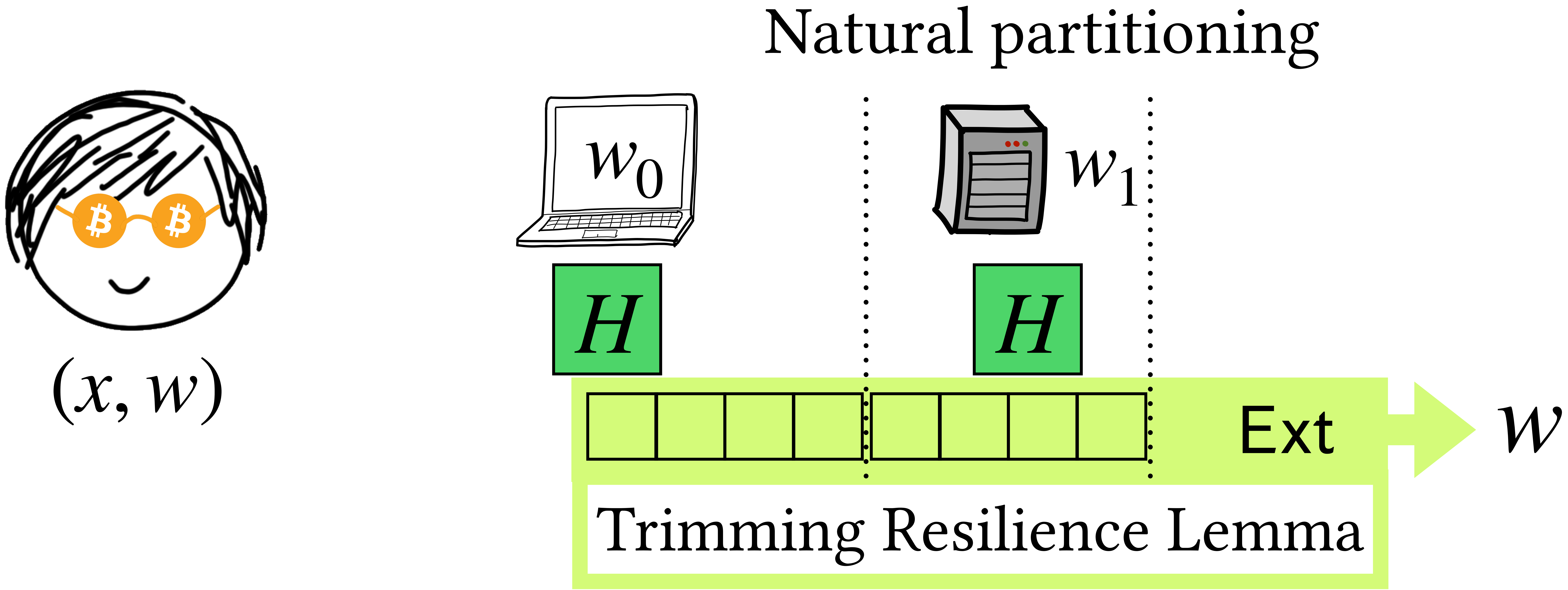
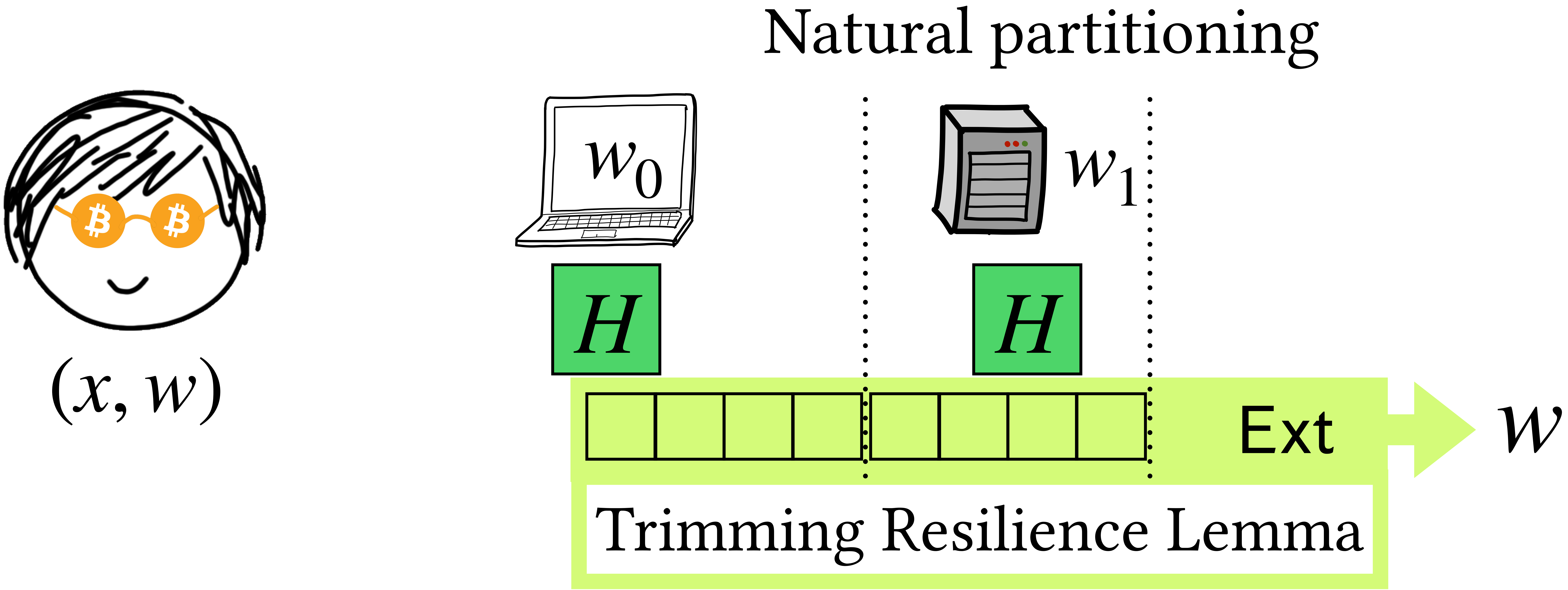


Two views are sufficient to reconstruct the witness

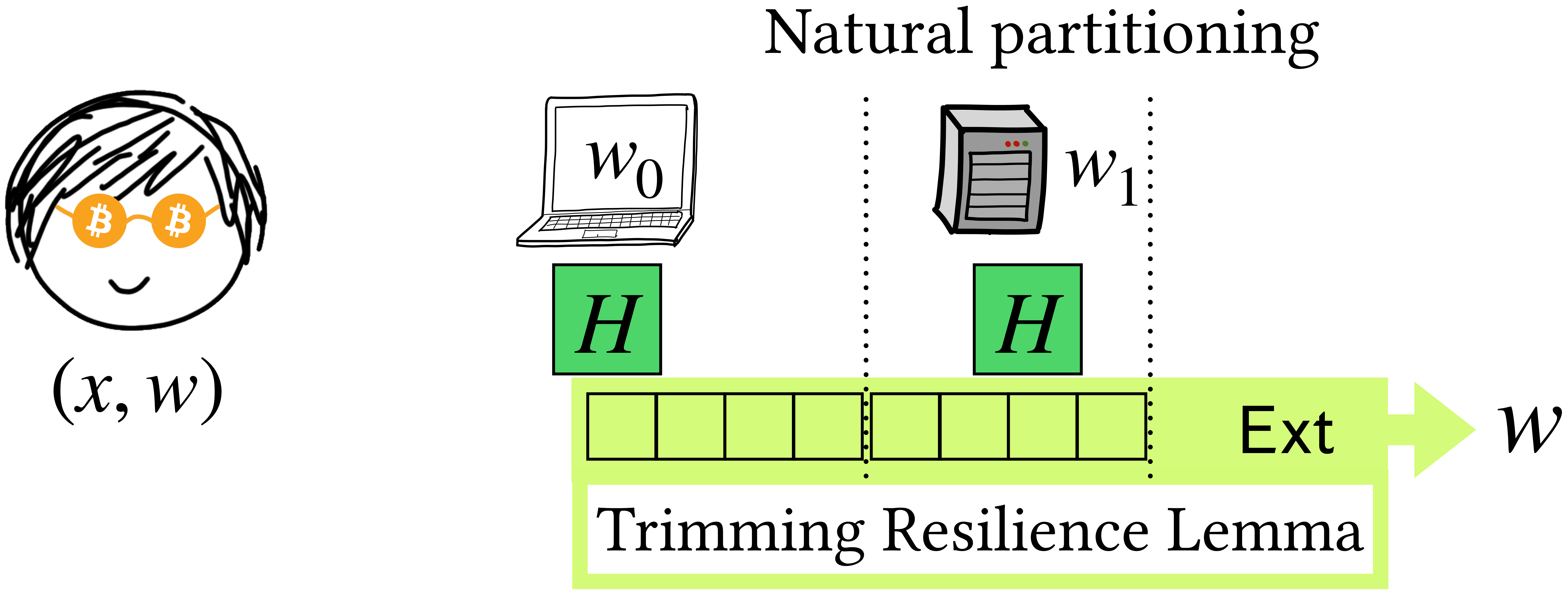
Oracle Respecting Distribution



Two views are sufficient to reconstruct the witness

3 party ORD protocol can not withstand 2 passive corruptions

Oracle Respecting Distribution



Two views are sufficient to reconstruct the witness

n party ORD protocol can not withstand n-1 passive corruptions

Caveats

- The n-party protocol must be mapped to a single party algorithm to apply the trimming lemma

- This mapping induces one of two artefacts:
  - Protocol property: Each RO query in the protocol must "traceable" to the party that first made it

    OR

  - NIZK property: $\mathsf{Ext}(\overrightarrow{Q}, \pi)$ does not actually need $H(\overrightarrow{Q})$

Fewer than $n - 1$ Corrupt?

- In the paper:
  - Extend impossibility for $n - O(1)$ corruptions
  - Notes on further barriers for many natural NIZKPoKs

- Impossibility itself does not generalize to $O(1)$ *fraction* of corruptions: $\exists$ NIZK that permits n-party ORD protocol with $\mathsf{const} \cdot n$ corruptions

Conclusion

- We showed that n-party protocols to securely compute certain hash-based signatures/NIZKs can not make blackbox use of the same hash function
  - Includes MPC-in-the-head, Fischlin/Unruh/Pass/Ks22 transform, PCPs/IOPs

- Dist. NIZK Verifier must depend on #parties—could it indicate that thresh. signature must grow with #signers?

Thanks!

eprint: 2023/1381

Thanks **Eysa Lee** for