HyperNova

Recursive Arguments for Customizable Constraint Systems

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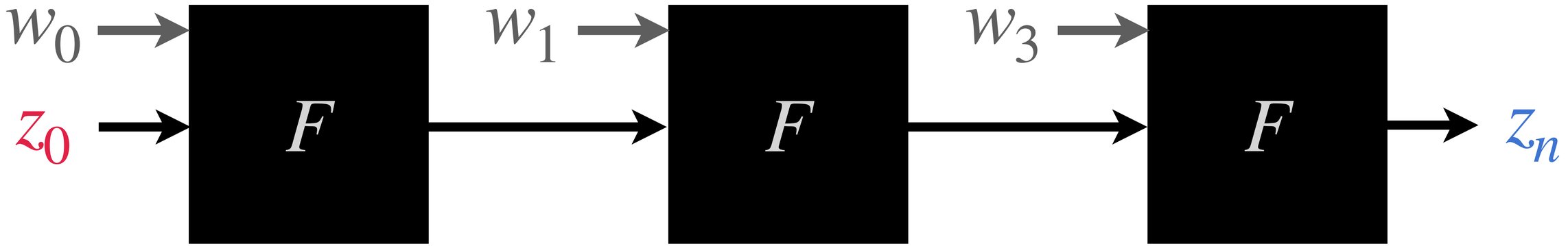
ia.cr/2023/573

Crypto 2024, UCSB

Goal: Practical zkSNARKs for Recursive Computation

Prove that (non-deterministic) function F applied n times to initial input z_0 results in z_n



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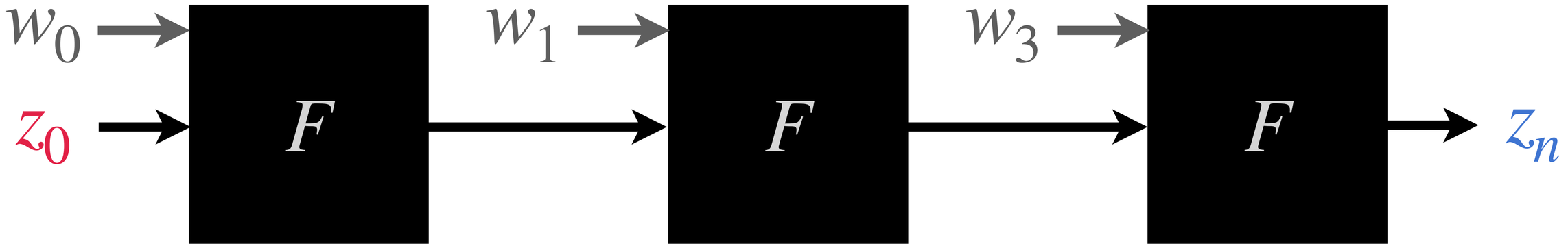
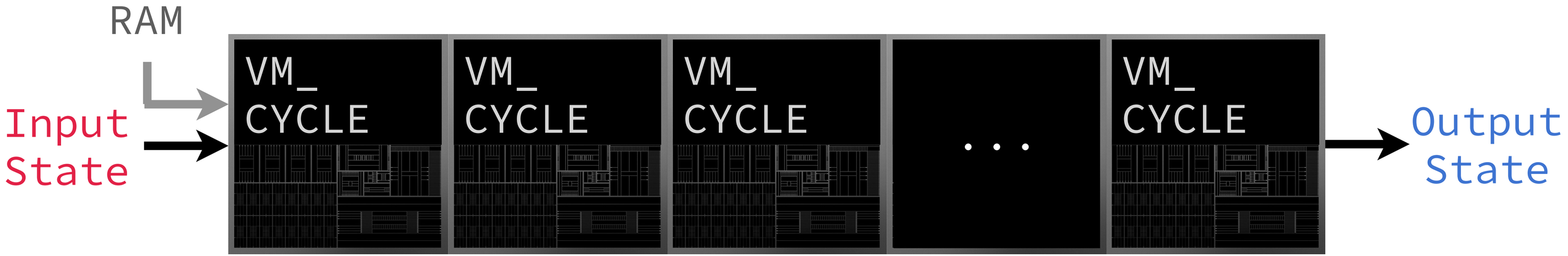


Example Applications

- Verifiable Delay Function: Let F be a delay function [BBBF19]
- ZK-rollups: Let F validate new blockchain transactions

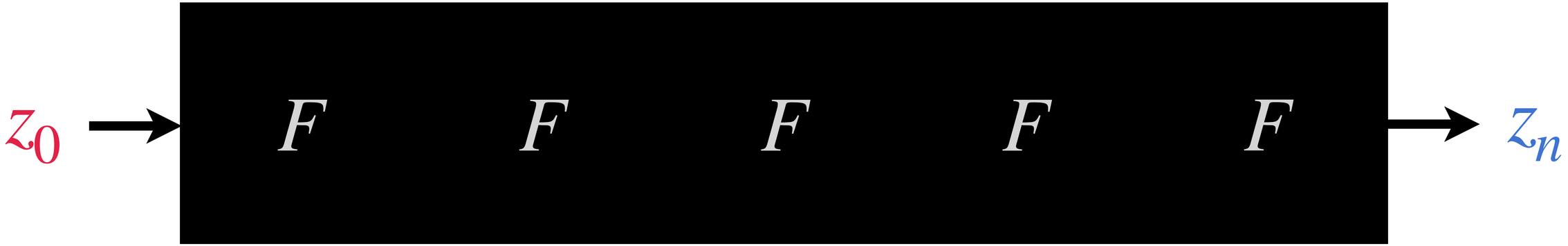
Converging Application: Zero-Knowledge Virtual Machines

Proofs for recursive computations imply proofs for VM execution.



Direct Approach: Memory-Bound [BCTV13,WSHRBW15,AST23]

Use a SNARK to check the entire unrolled computation trace.



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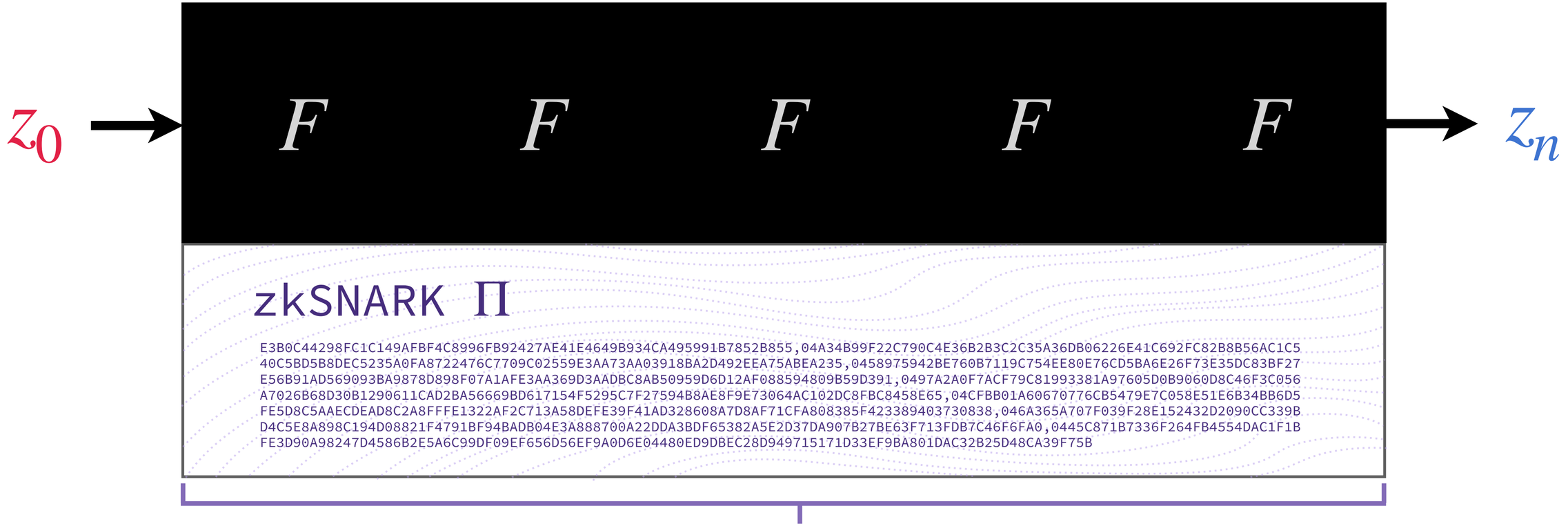

Jolt [AST23] proves 17M RISC-V cycles in 125 seconds using 512GB.

Limitation: Memory scales linearly with iterations.

Memory-Efficient Approach: IVC [Val08,BGH19,BCLMS21,KST21]

Solution: Locally process each chunk of execution and incrementally update the proof without increasing its size.

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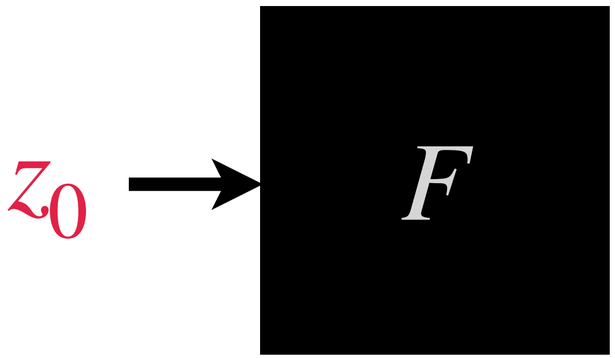

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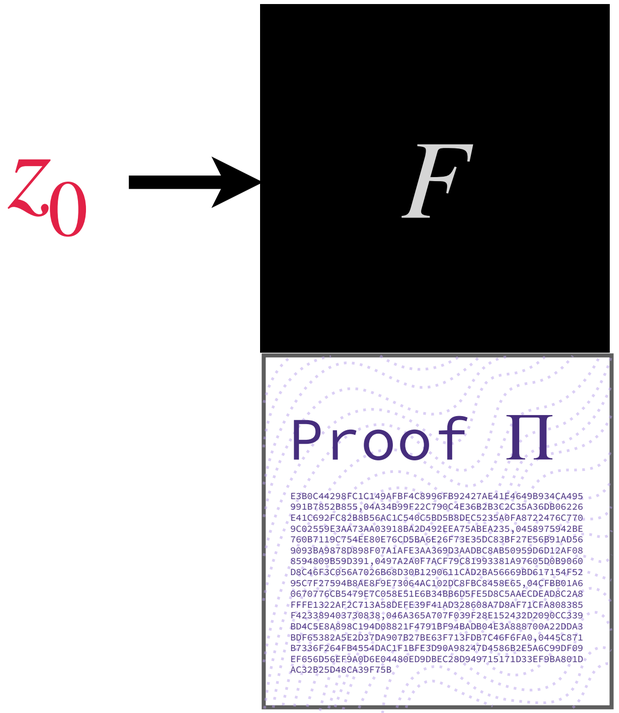

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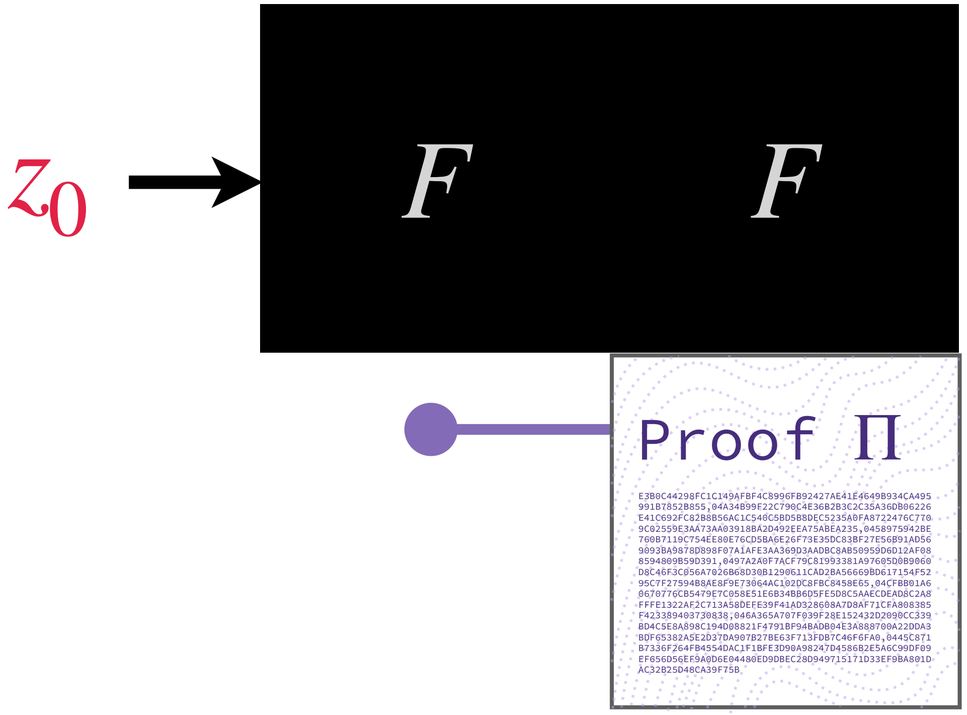

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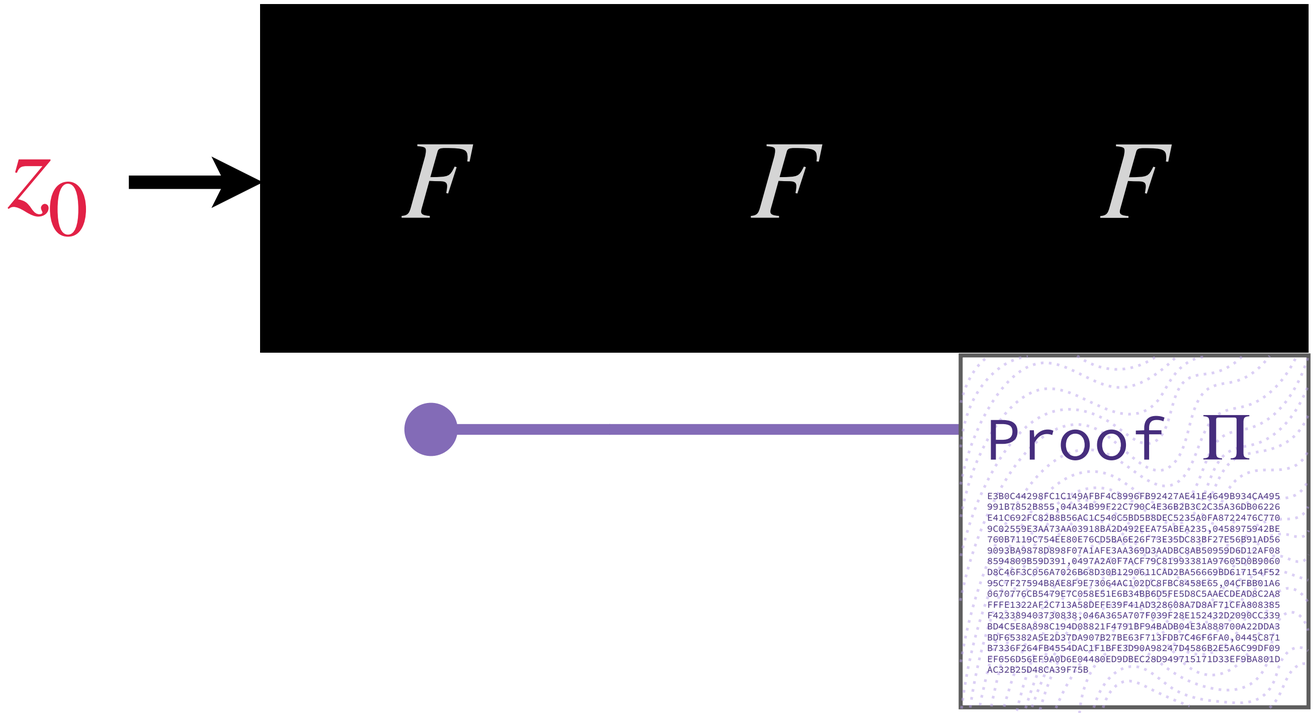

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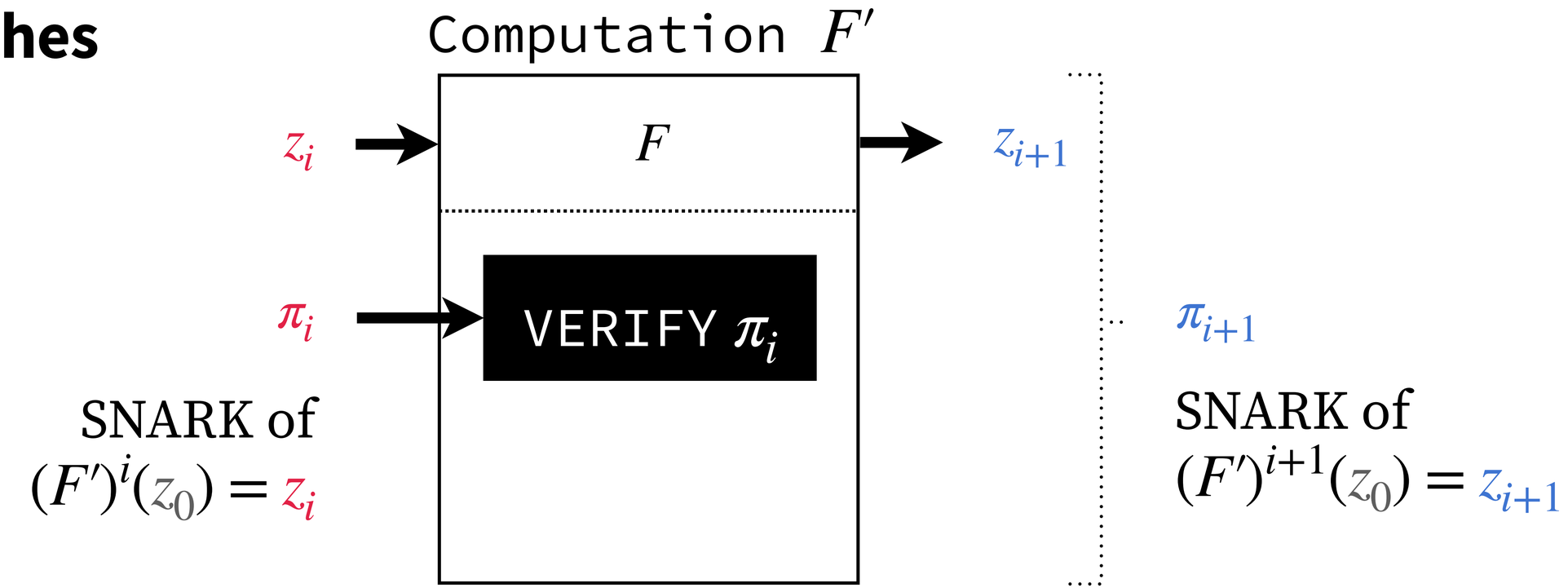
IVC Approaches



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Prover	Produces SNARK [10MSMs on pairing cycle]	Produces SNARK [10MSMs on standard cycle]	Only commits to witness and auxiliary materials [2MSMs]
Verifier	Verifies entire SNARK	Partially verifies SNARK	Constant grp ops and hashes

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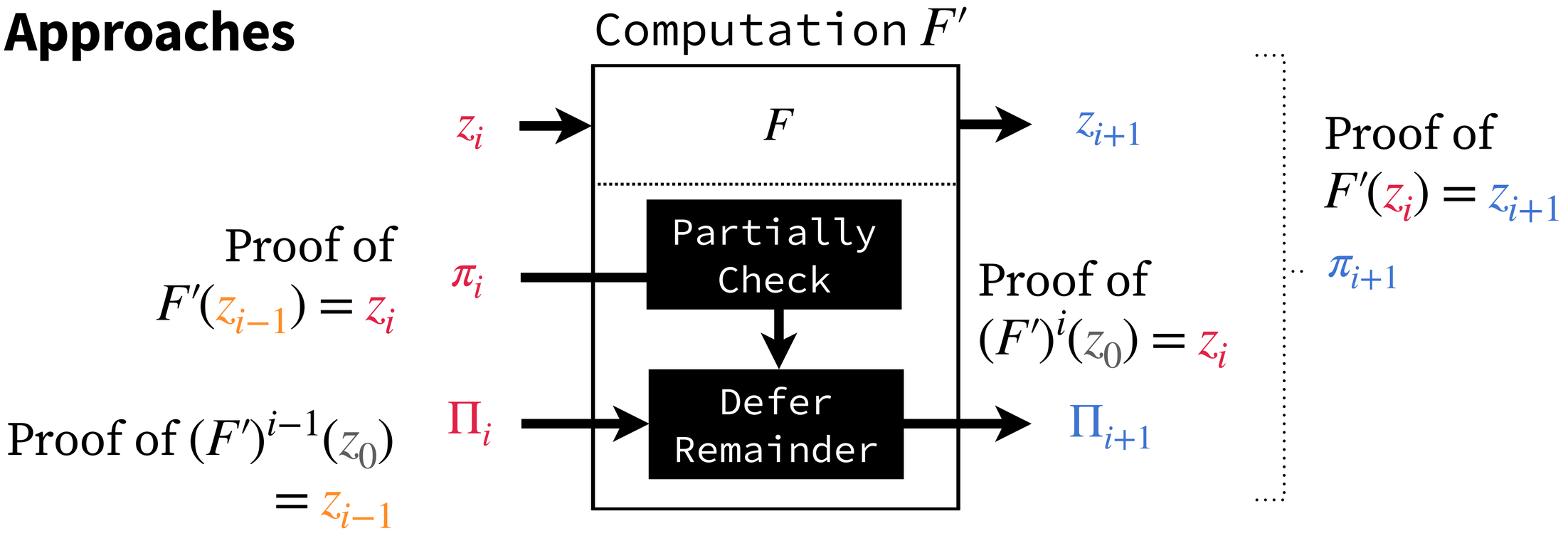

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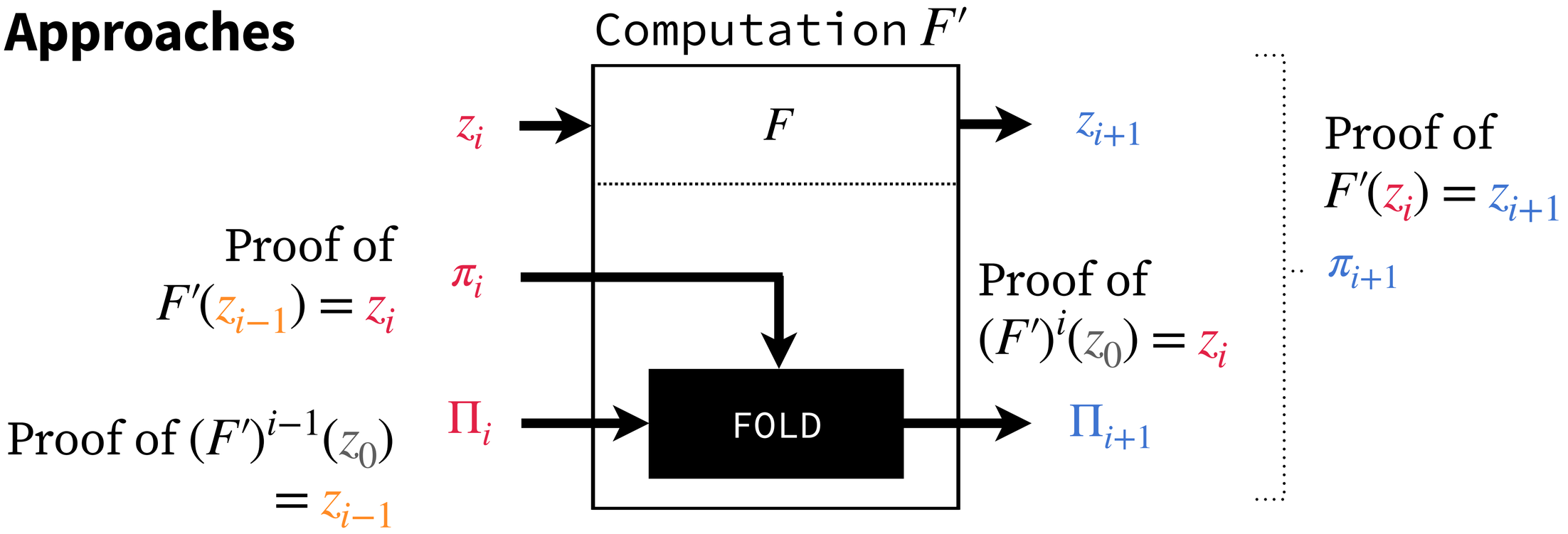

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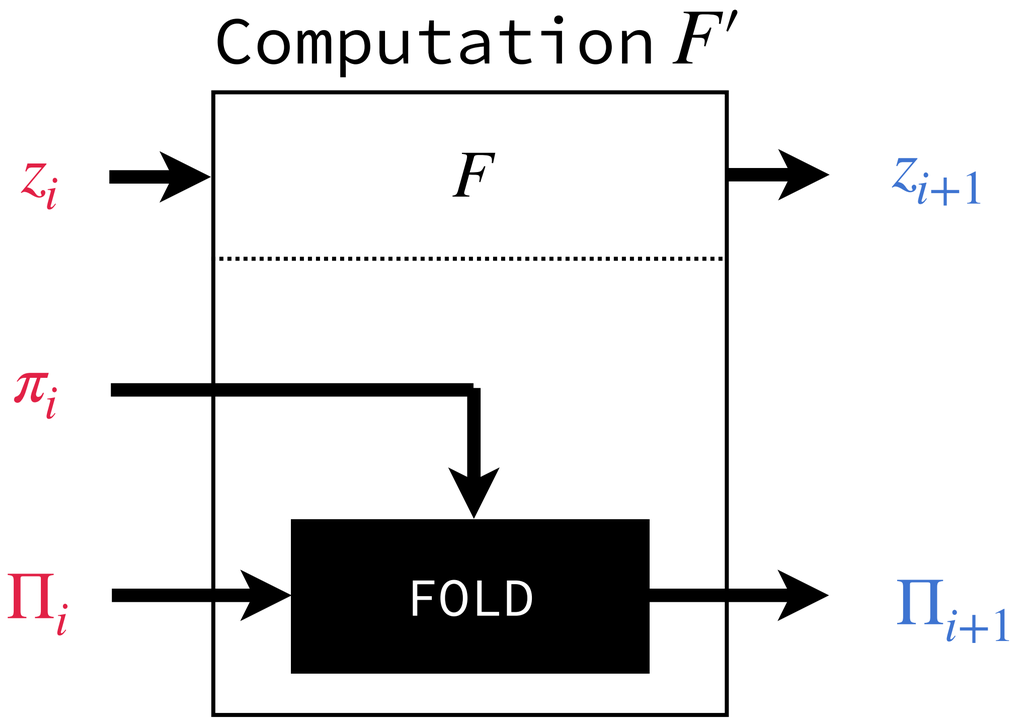
Limitations with Prior Folding-Based IVC Schemes



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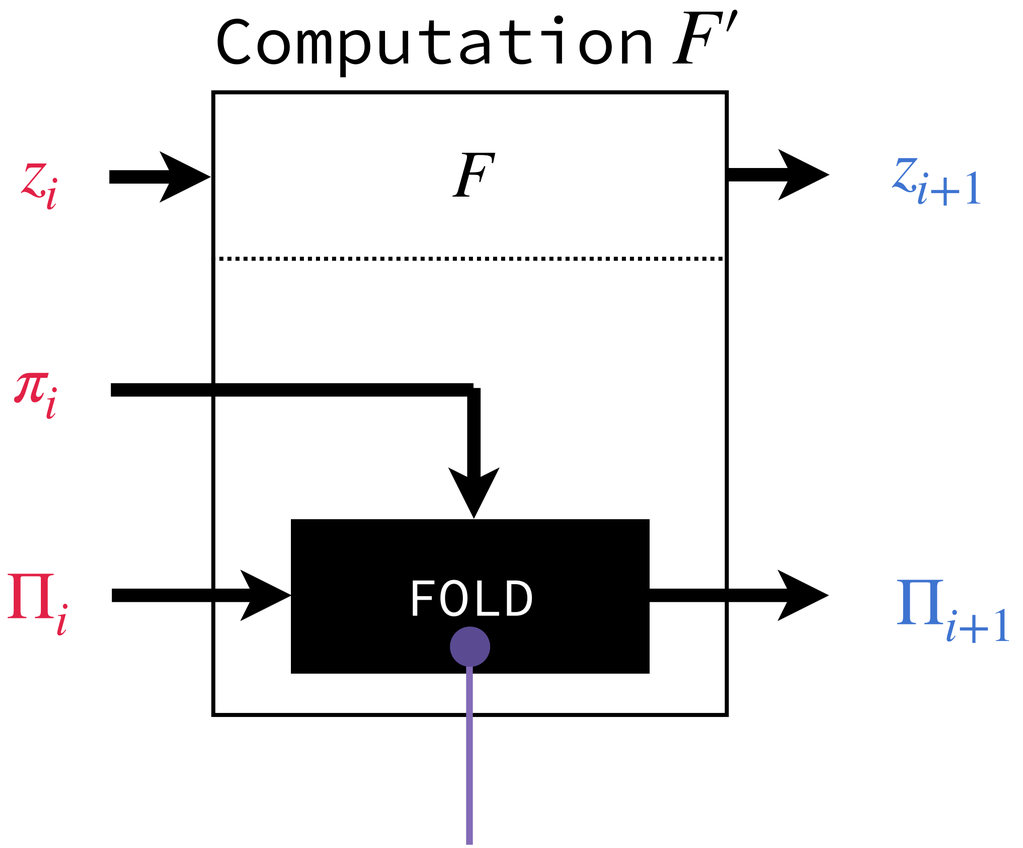

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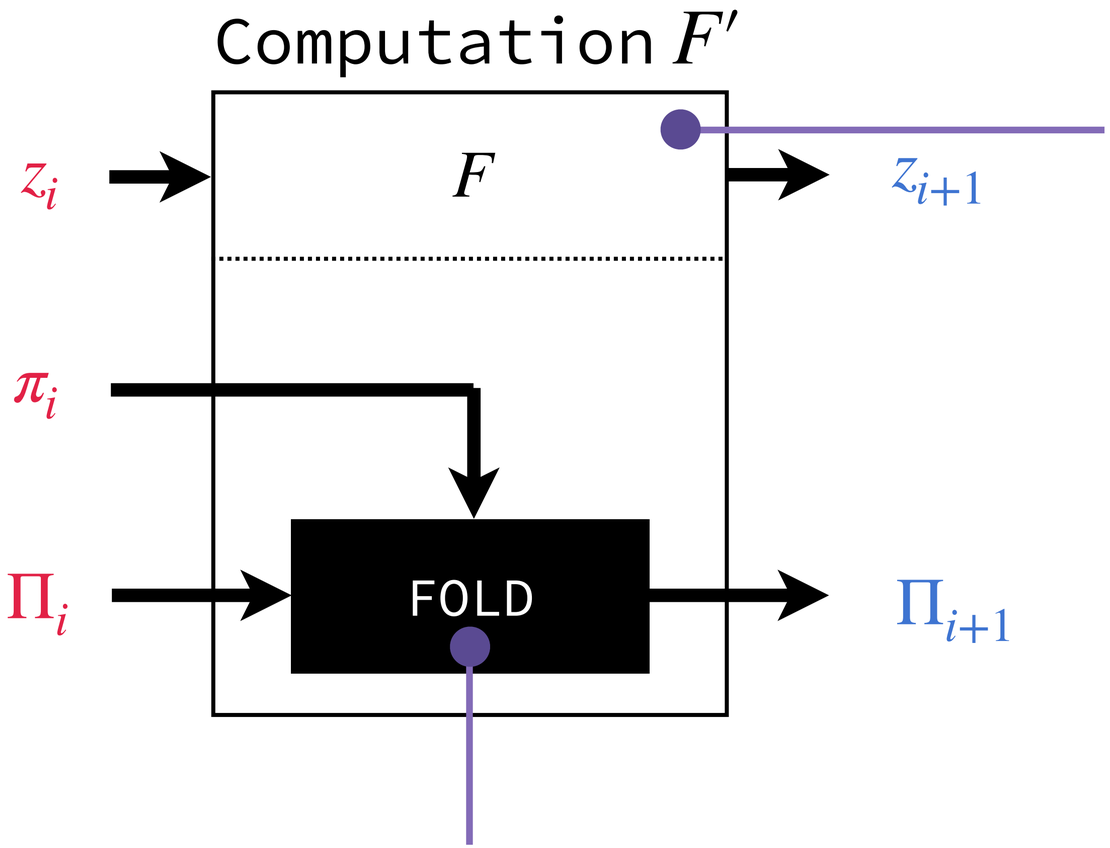

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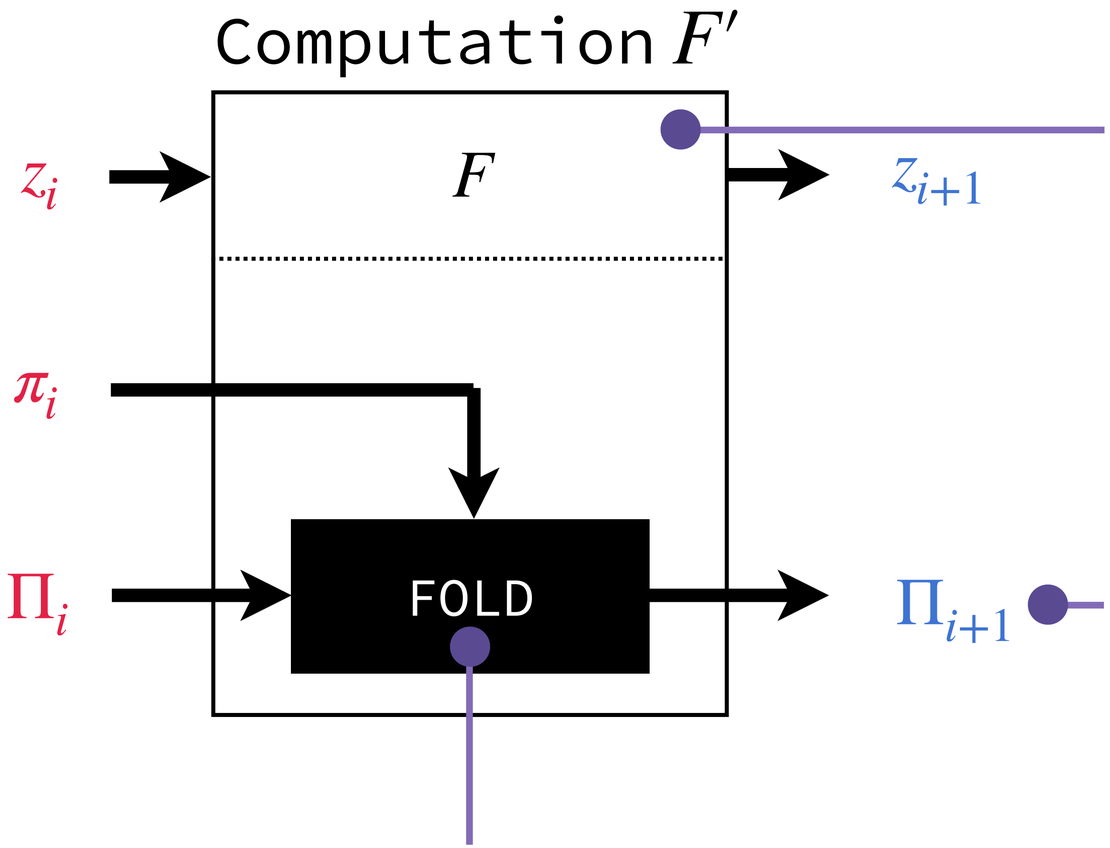

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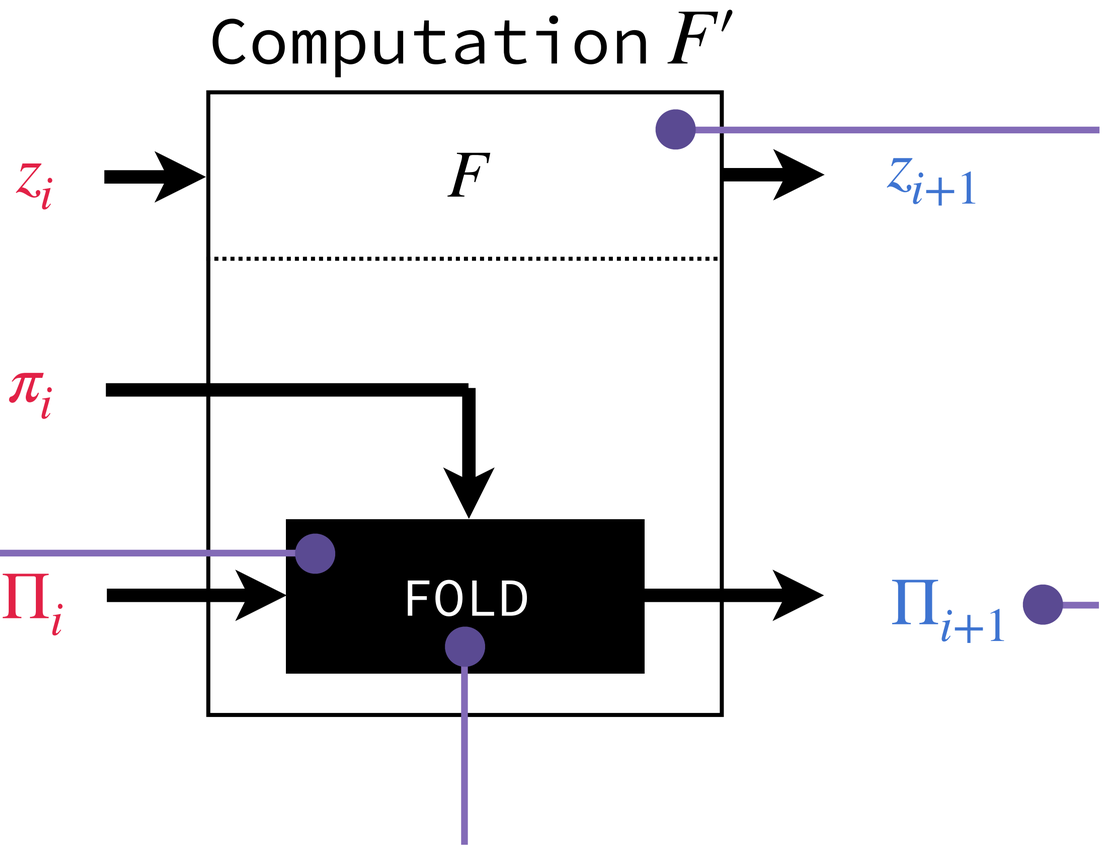

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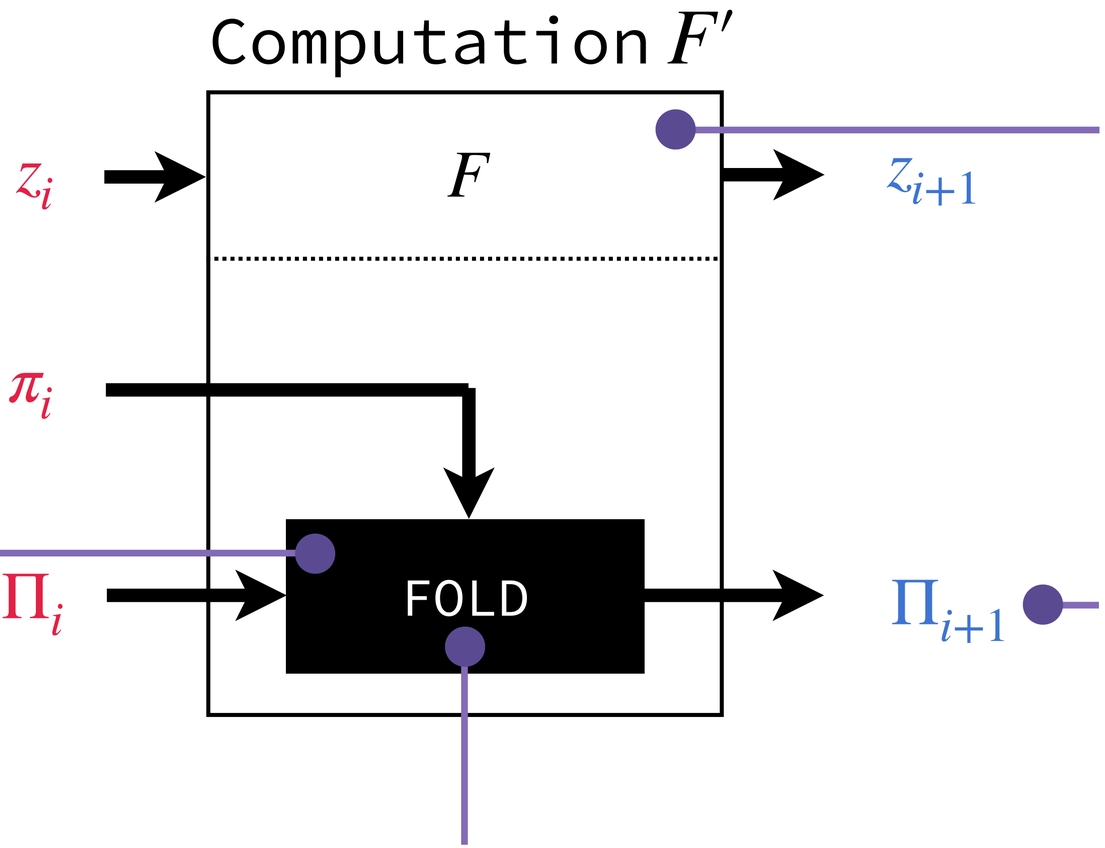
HyperNova Contributions



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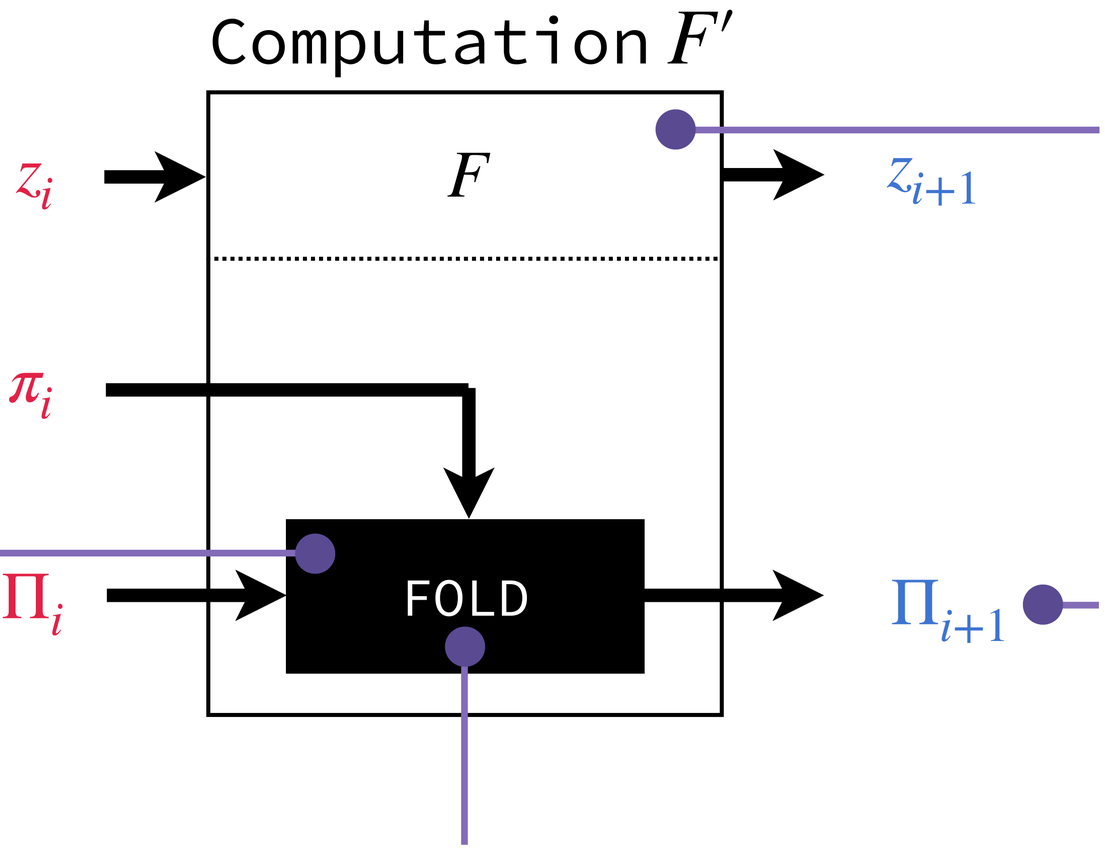

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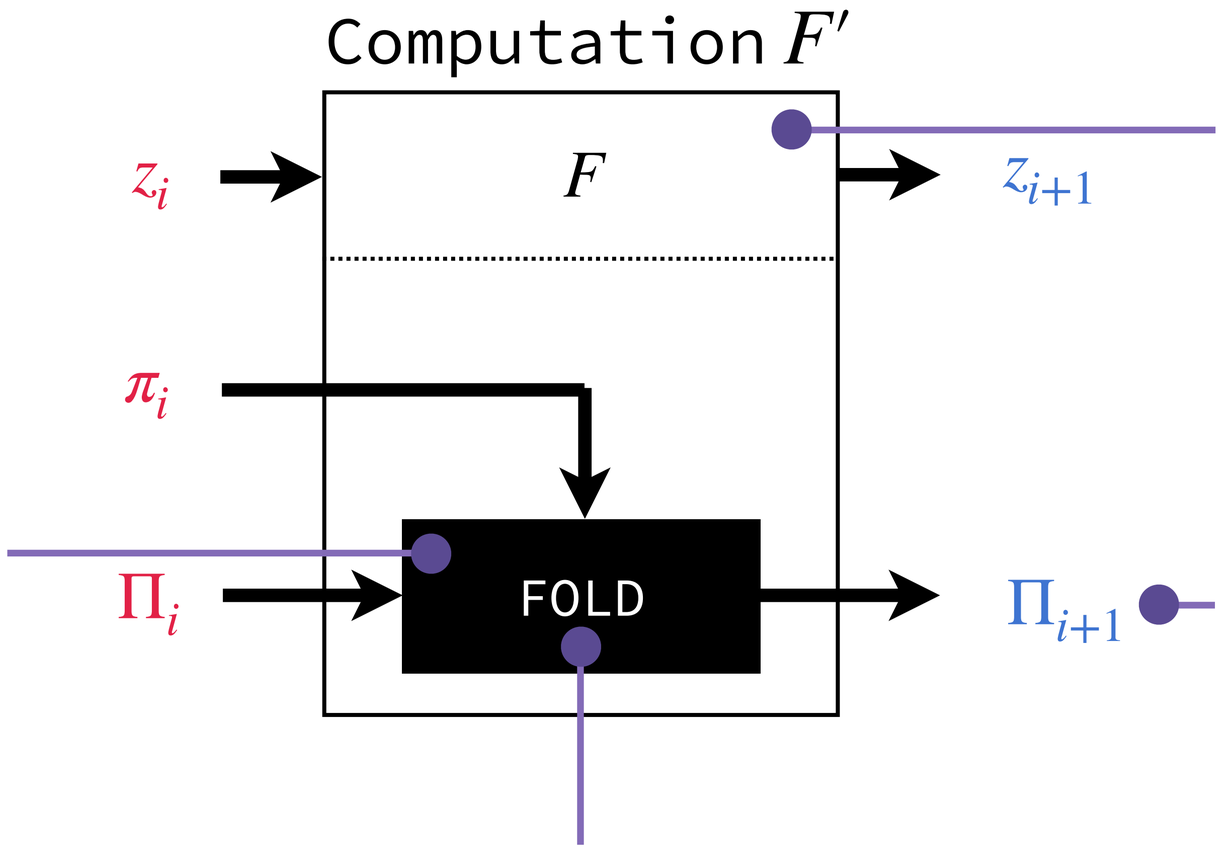

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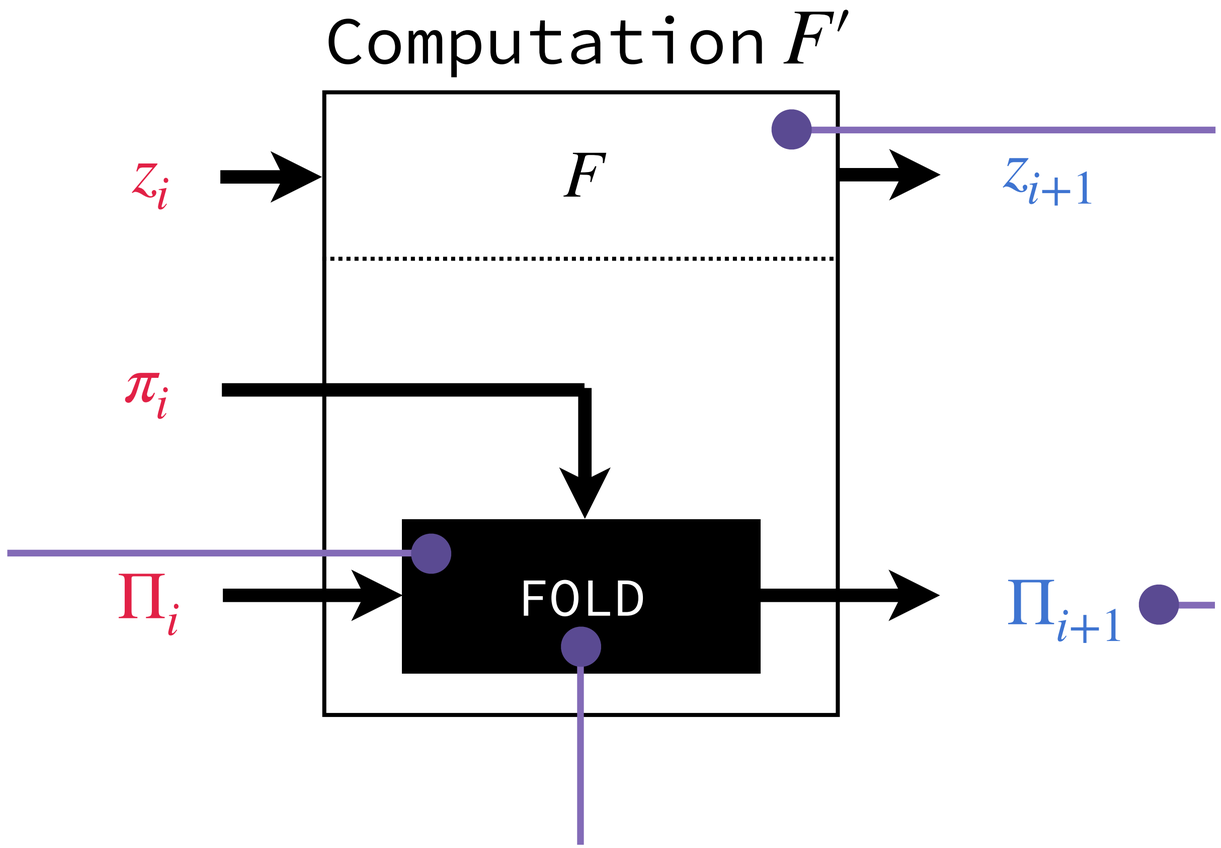

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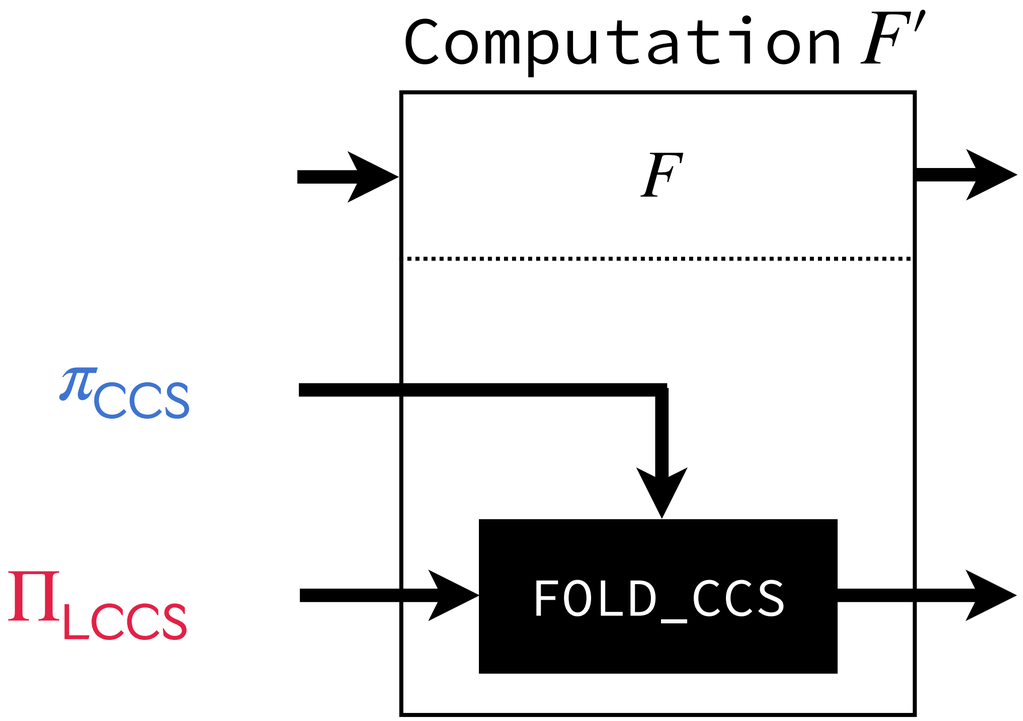
Folding High-Degree CCS Constraints



The core idea is to use the sumcheck protocol to reduce non-linear constraints into linear constraints.

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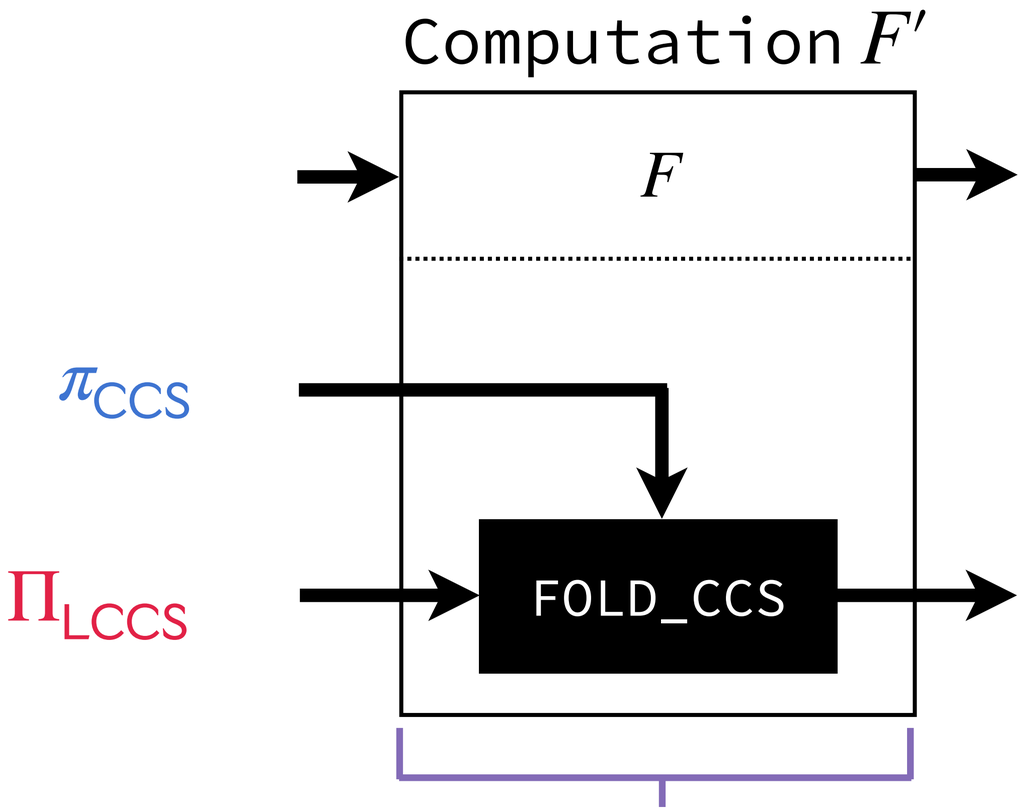

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We achieve a recursion overhead dominated by 1 GSM and $d \log |F'|$ CHRFs.

Folding CCS

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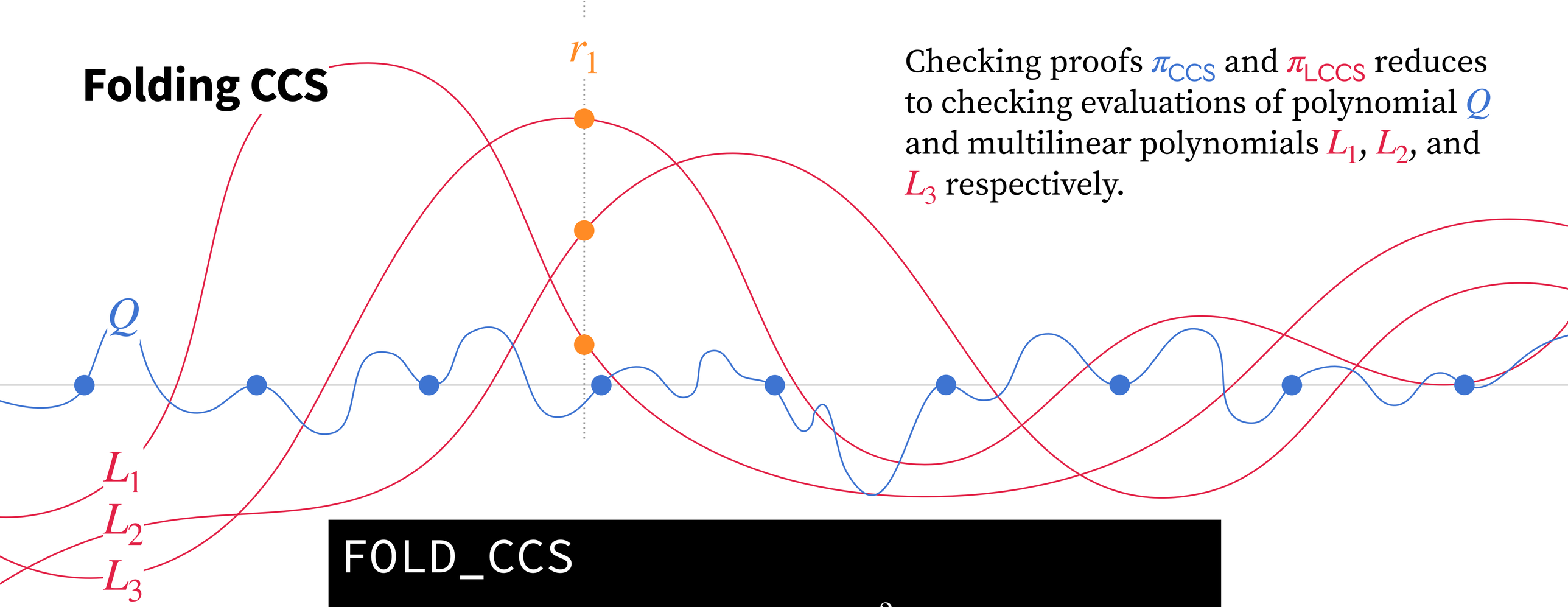


Checking proofs π_{CCS} and π_{LCCS} reduces to checking evaluations of polynomial Q and multilinear polynomials L_1, L_2, and L_3 respectively.

$\pi_{\mathsf{CCS}} \rightarrow$

$\Pi_{\mathsf{LCCS}} \rightarrow$

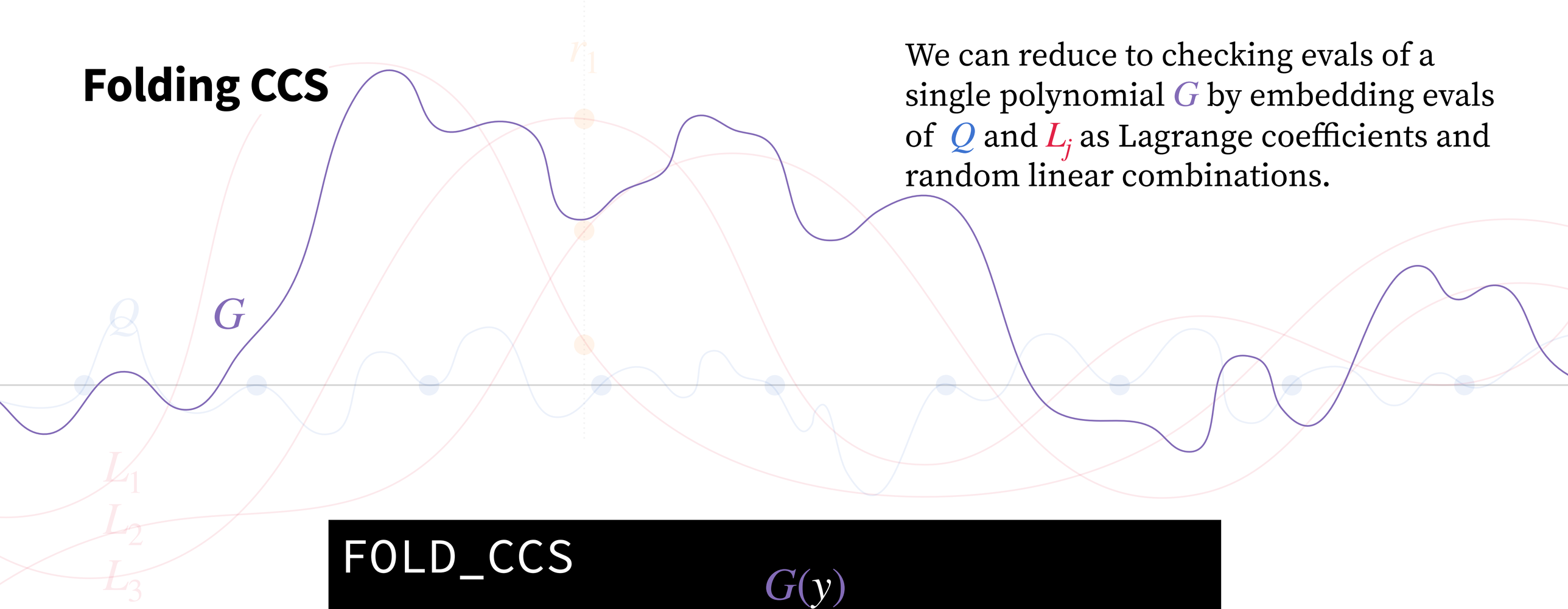
FOLD_CCS

$$\forall x \in [n] \,.\, Q(x) \stackrel{?}{=} 0$$

$$\forall j \in [t] \,.\, L_j(r_1) \stackrel{?}{=} v_j$$

Folding CCS

We can reduce to checking evals of a single polynomial G by embedding evals of Q and L_j as Lagrange coefficients and random linear combinations.



FOLD_CCS

$G(y)$

π_{CCS} →

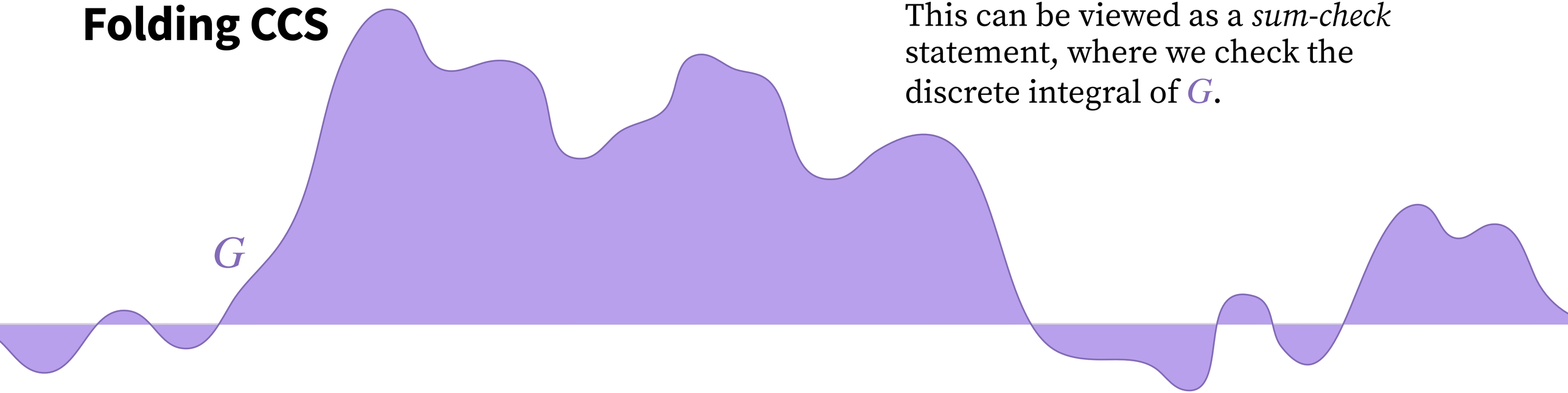
$$\sum_{y} \gamma^{t+1} \cdot \mathrm{eq}(r_2, y) \cdot Q(y) + \mathrm{eq}(r_1, y) \sum_{j \in [t]} \gamma^{j} \cdot L_j(y)$$

Π_{LCCS} →

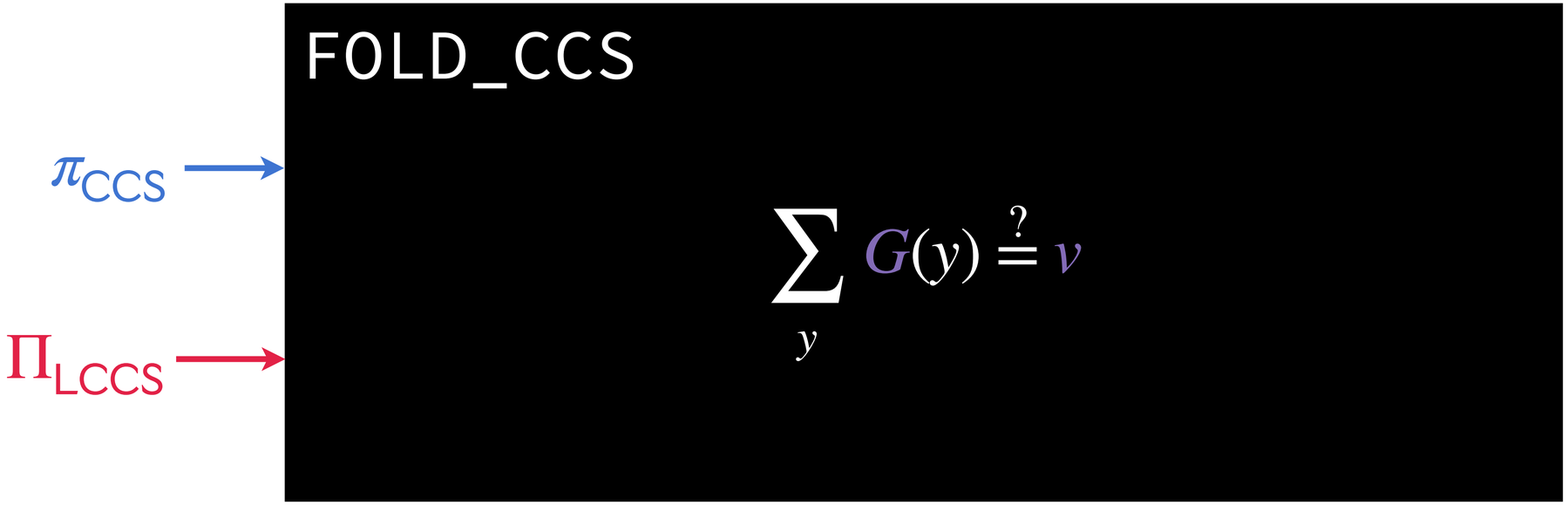
$$\stackrel{?}{=} \sum_{j \in [t]} \gamma^{j} \cdot v_j \quad v$$

For random γ, r_2

Folding CCS

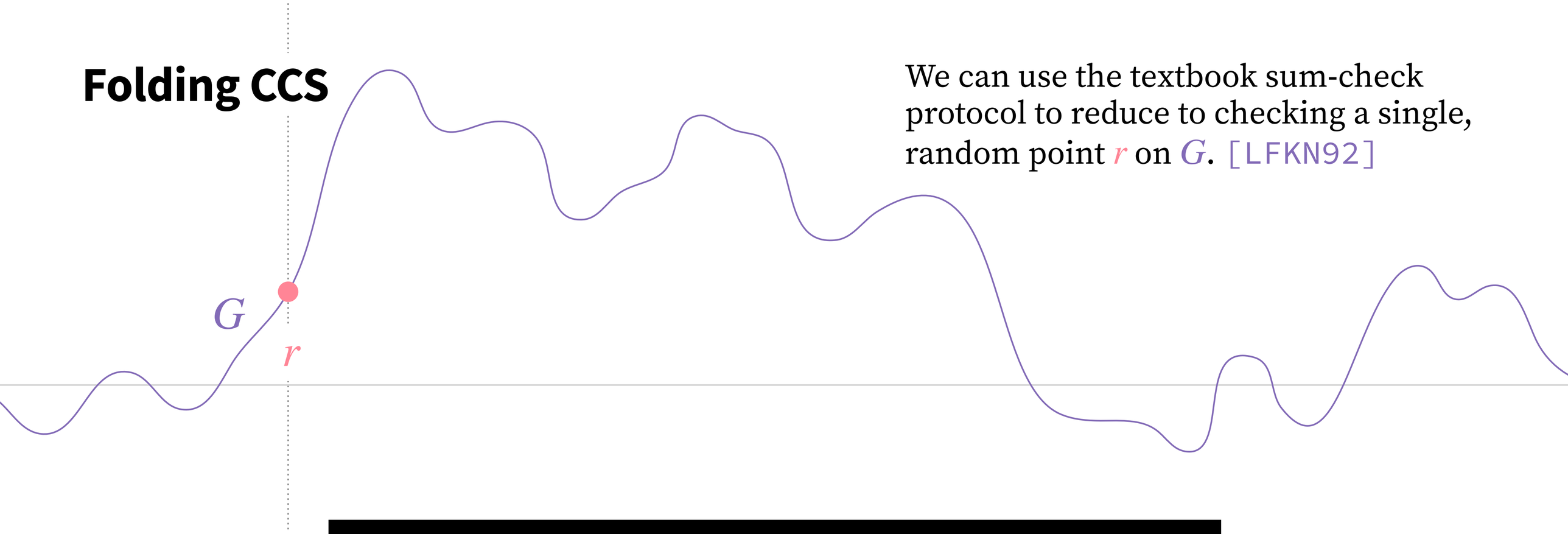


This can be viewed as a *sum-check* statement, where we check the discrete integral of G.



Folding CCS

We can use the textbook sum-check protocol to reduce to checking a single, random point r on G. [LFKN92]



π_{CCS} →

Π_{LCCS} →

FOLD_CCS

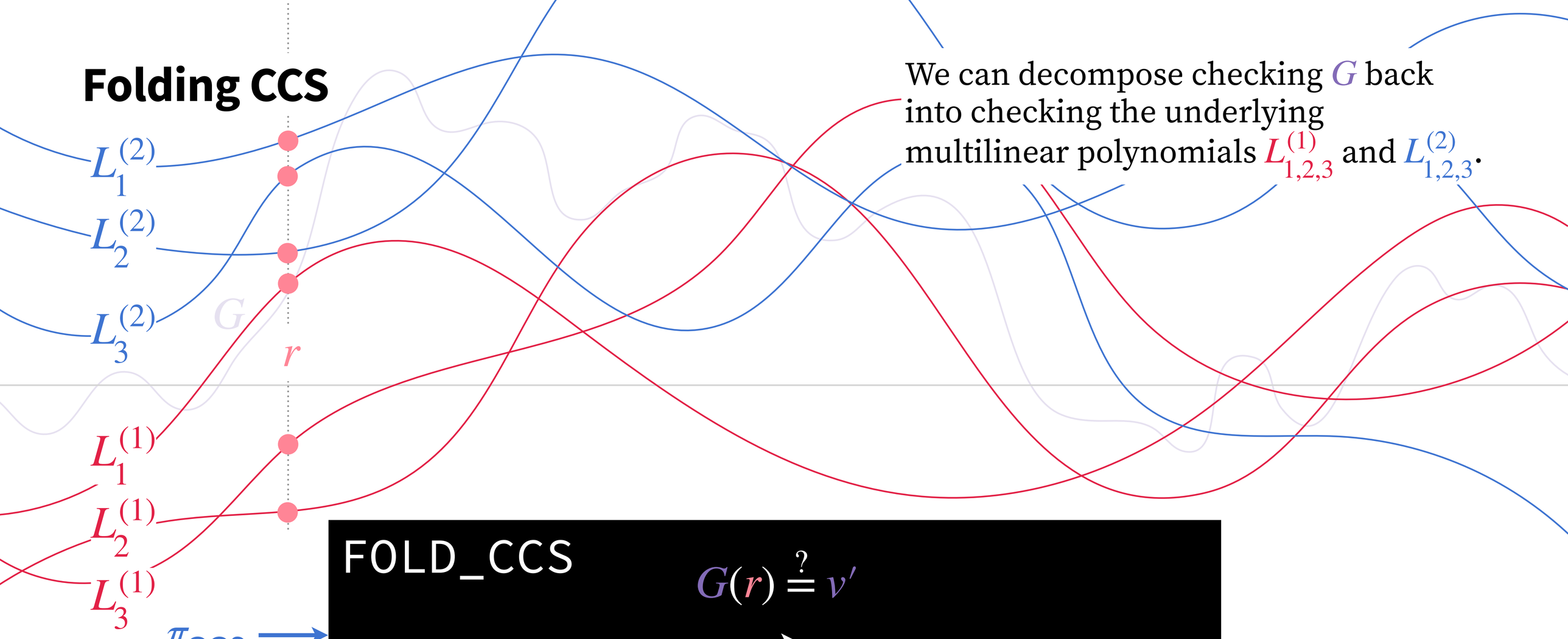
$$\sum_y G(y) \stackrel{?}{=} v$$

→

$$G(r) \stackrel{?}{=} v'$$

Folding CCS

We can decompose checking G back into checking the underlying multilinear polynomials $L^{(1)}_{1,2,3}$ and $L^{(2)}_{1,2,3}$.



π_{CCS} →

Π_{LCCS} →

FOLD_CCS

$$G(r) \stackrel{?}{=} v'$$

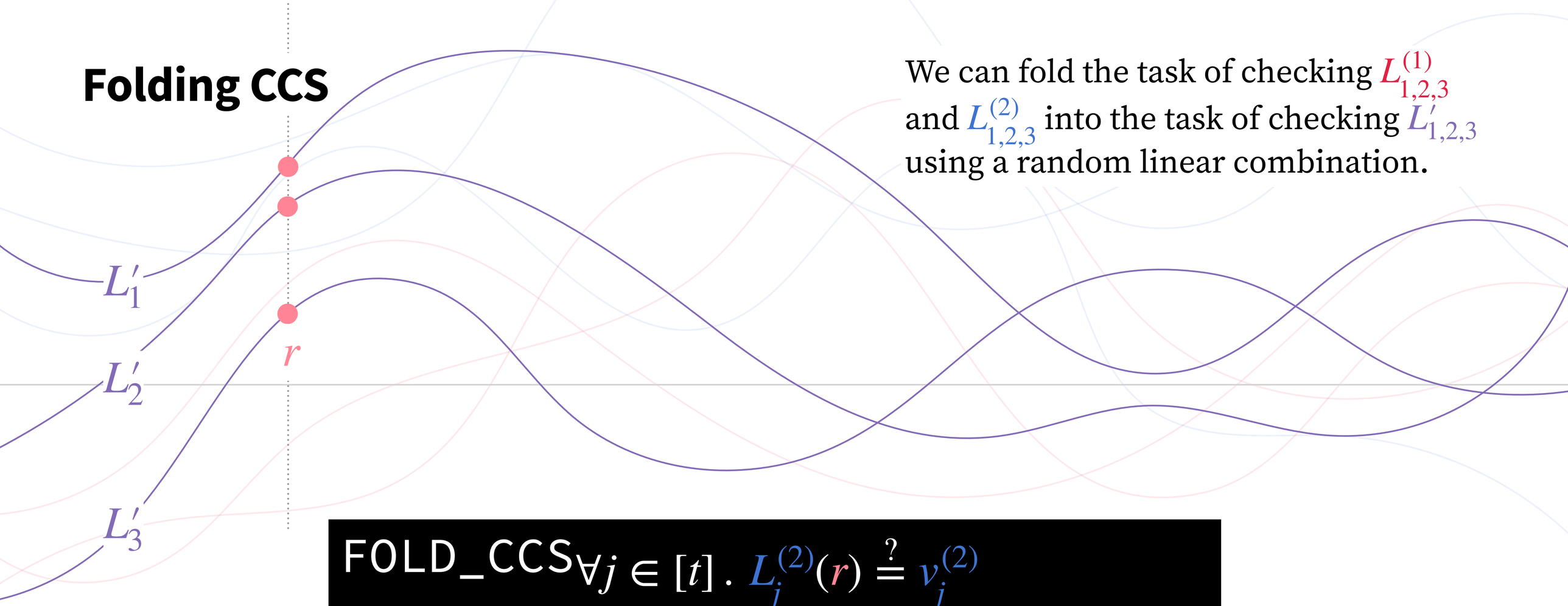
→

$$\forall j \in [t] \,.\, L_j^{(2)}(r) \stackrel{?}{=} v_j^{(2)}$$

$$\forall j \in [t] \,.\, L_j^{(1)}(r) \stackrel{?}{=} v_j^{(1)}$$

Folding CCS

We can fold the task of checking $L^{(1)}_{1,2,3}$ and $L^{(2)}_{1,2,3}$ into the task of checking $L'_{1,2,3}$ using a random linear combination.



π_{CCS} →

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FOLD_CCS

$\forall j \in [t] \,.\, L_j^{(2)}(r) \stackrel{?}{=} v_j^{(2)}$

$\forall j \in [t] \,.\, L_j^{(1)}(r) \stackrel{?}{=} v_j^{(1)}$

→

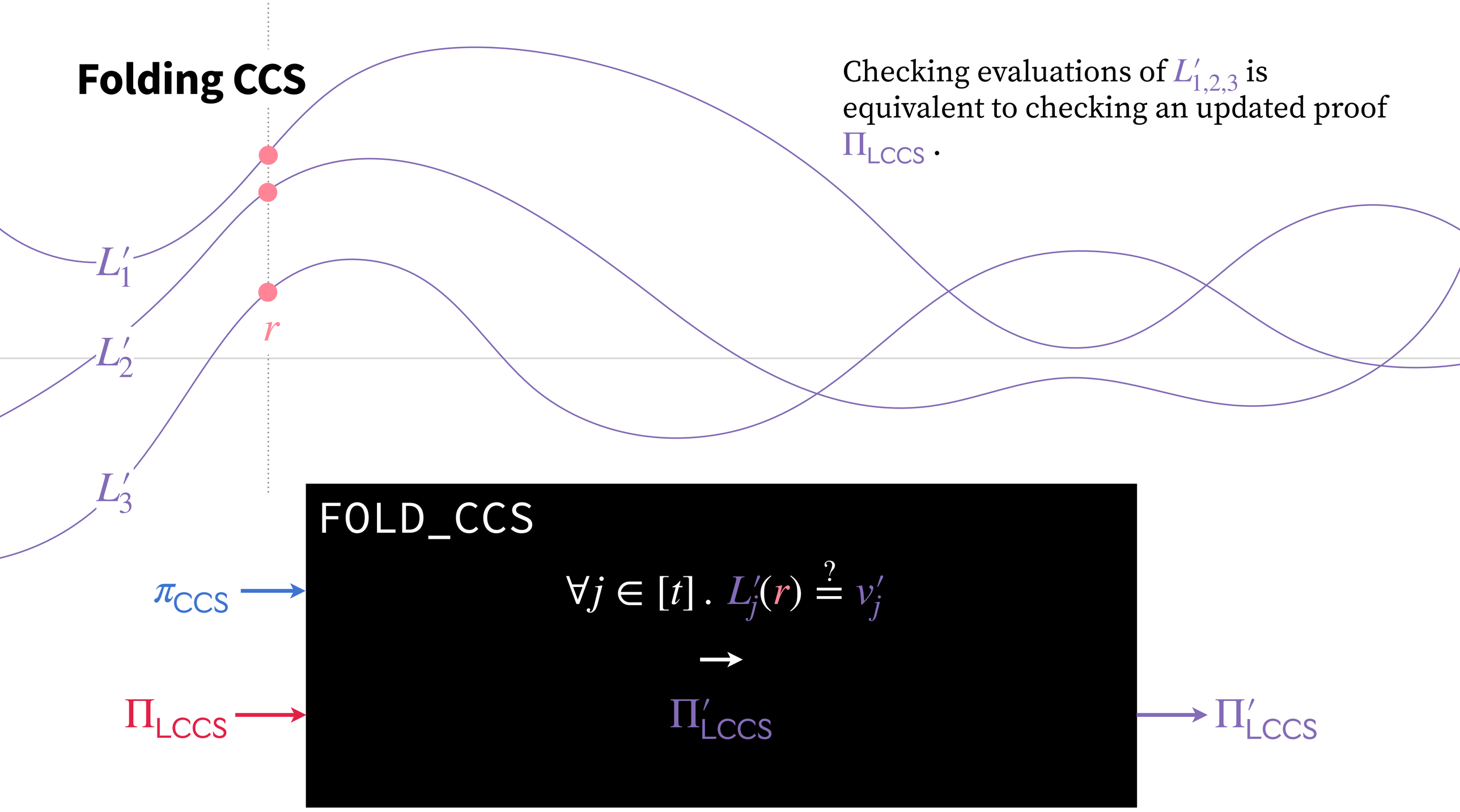
$\forall j \in [t] \,.\, L'_j(r) \stackrel{?}{=} v'_j$

For $L'_j = L_j^{(1)} + \rho \cdot L_j^{(2)}$

For random ρ

Folding CCS

Checking evaluations of $L'_{1,2,3}$ is equivalent to checking an updated proof Π_{LCCS}.



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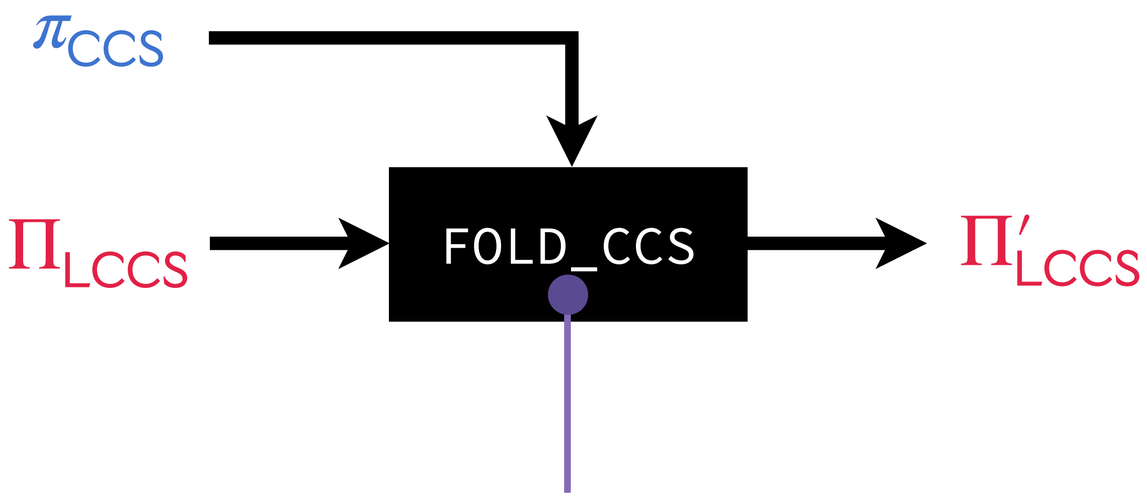

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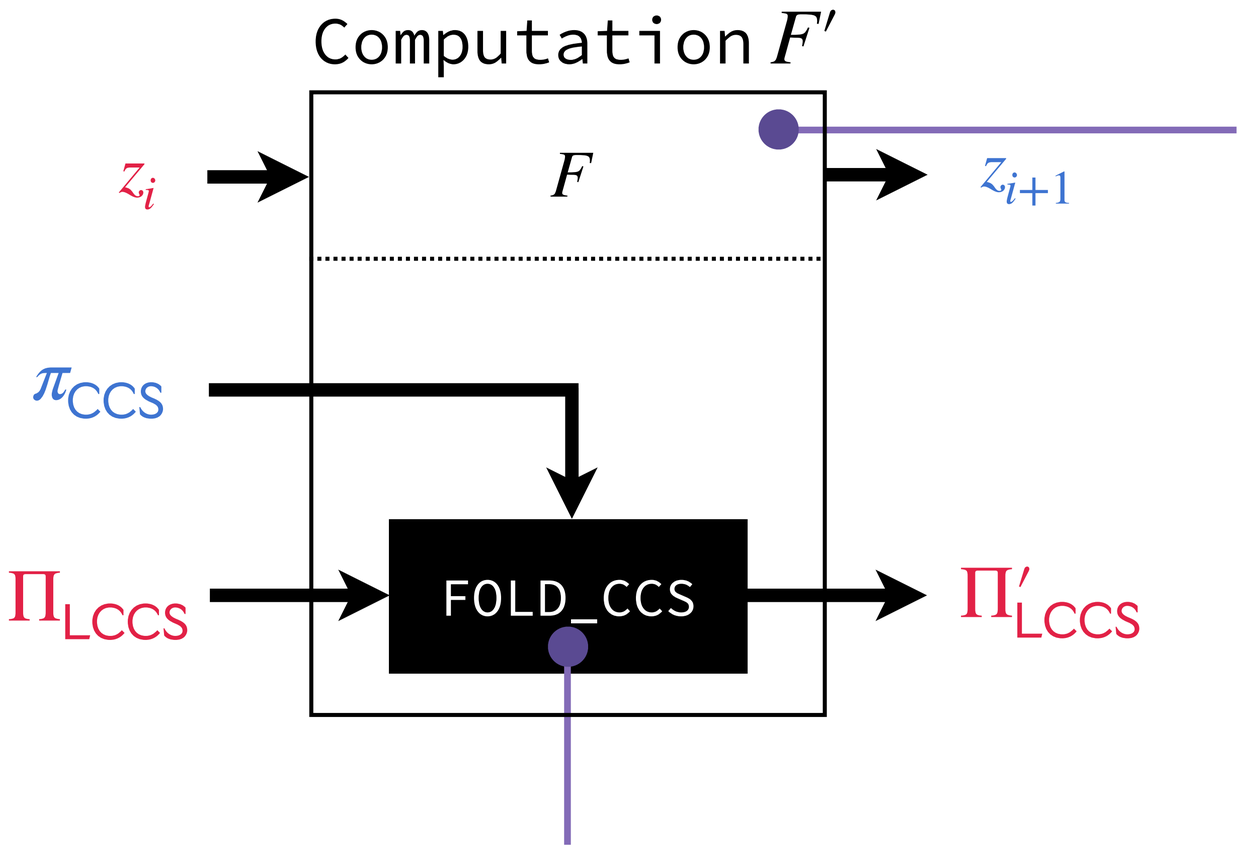

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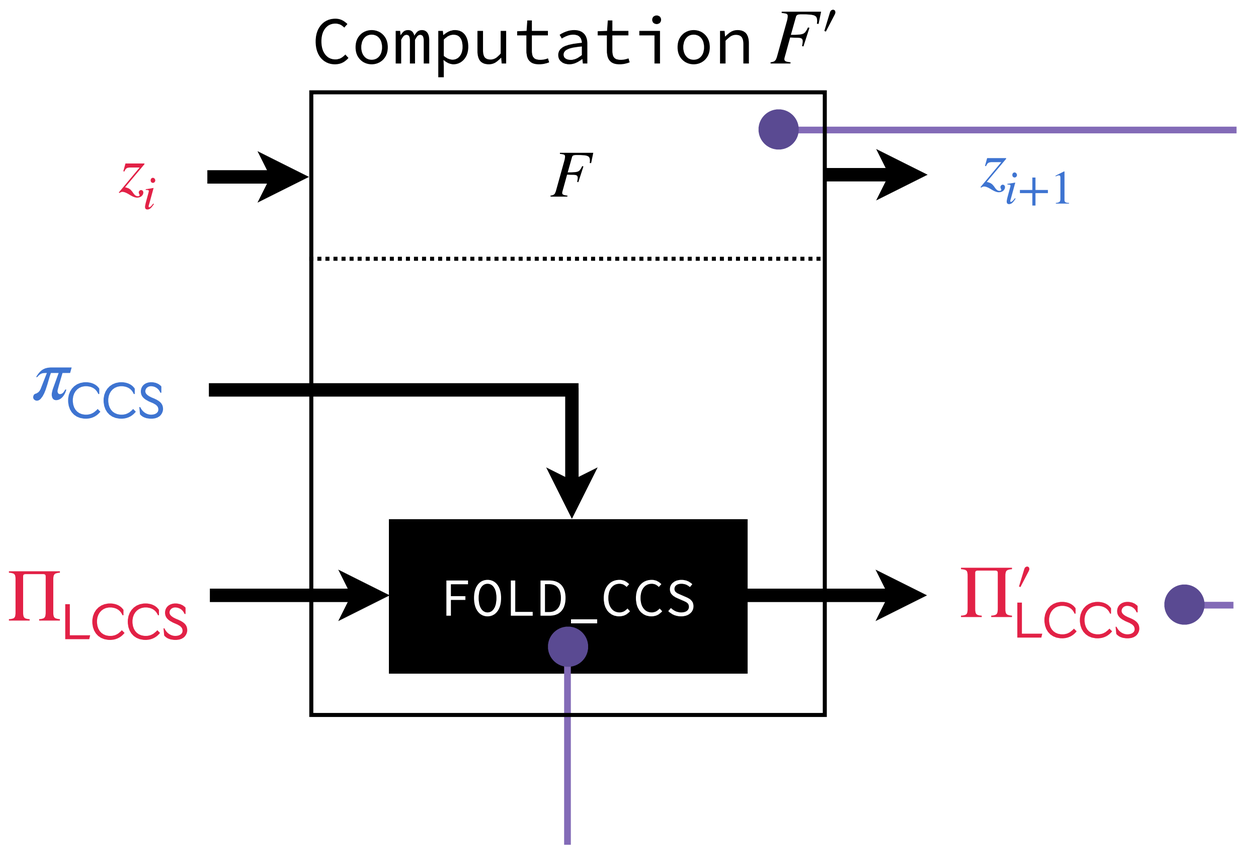

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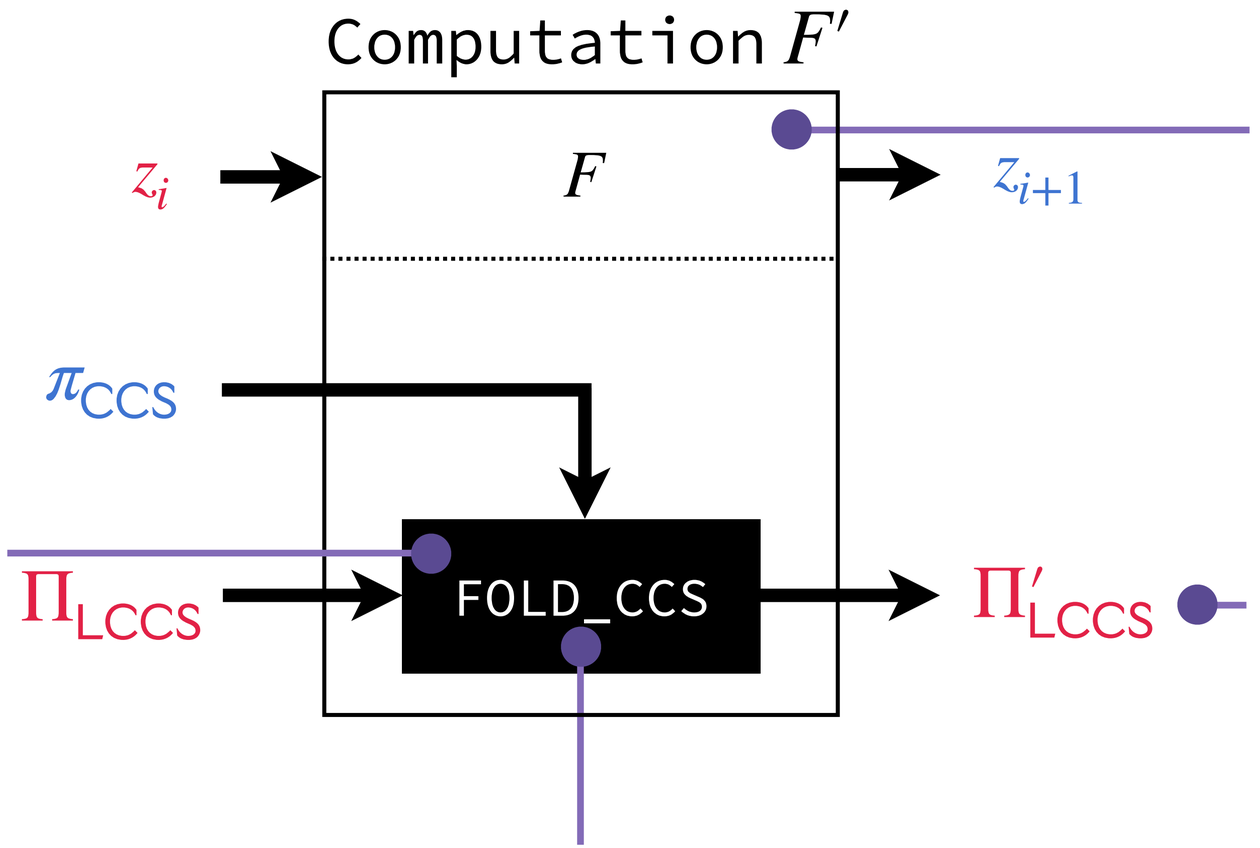
(4) We introduce CycleFold, a general methodology to efficiently instantiate folding schemes over cycles of curves.



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