Improved Alternating-Moduli PRFs and Post-Quantum Signatures

Navid Alamati, **Guru Vamsi Policharla**, Srinivasan Raghuraman, and Peter Rindal

Designing Crypto Primitives

Provable security

Heuristic constructions

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- Security from hardness of DDH, SXDH, LPN, LWE, etc.

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Can we have fast, simple to describe, crypto primitives that are MPC/ZK friendly?

The Alternating-Moduli Paradigm

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Why do they resist cryptanalysis?

High algebraic degree when represented in a single field

What do they look like?

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$$y = B \cdot_3 (K \cdot_2 x)$$

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Map $\mathbb{F}_2 \to \mathbb{F}_3$

What do they look like?

[BIP+18]: weak PRF

$$y = \underbrace{B}_{\mathbb{F}_3} \; \underbrace{K}_{\mathbb{F}_2} \; \underbrace{x}_{\mathbb{F}_2}$$

- $K \sim 384 \times 384$
- Can be circulant for faster evaluation
- Cryptanalysis + fixes in [CCKK21]. More variants in [BIP+18, DGI+21].

What do they look like?

[DGH+21]: OWF

Input

$$y = B \cdot_3 (A \cdot_2 x)$$

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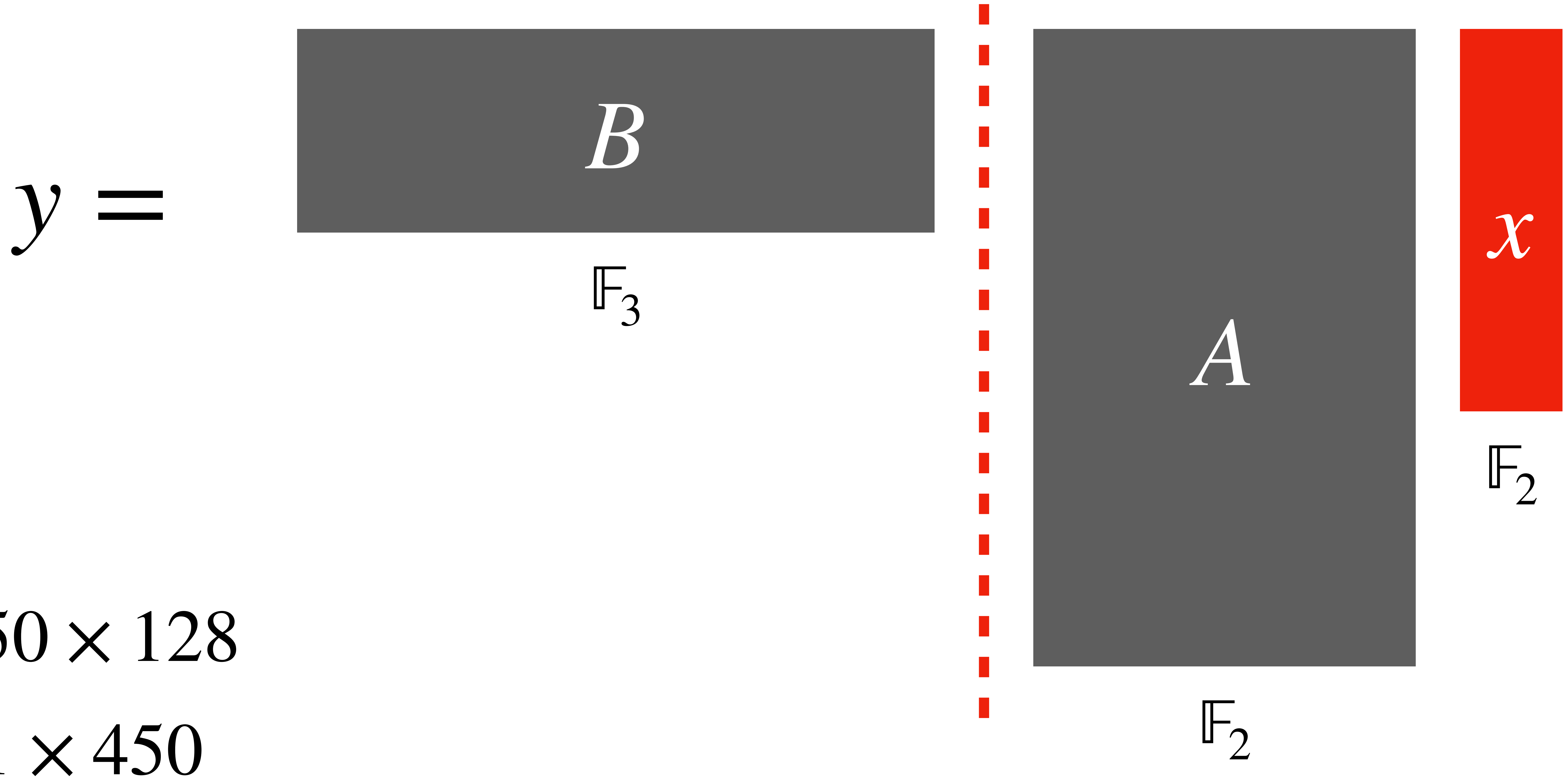
[DGH+21]: OWF



- $A \sim 450 \times 128$
- $B \sim 81 \times 450$

Efficient PQ Signatures

MPC-in-the-Head [IKOS07]

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MPCitH: Convert an MPC protocol into a zero-knowledge proof

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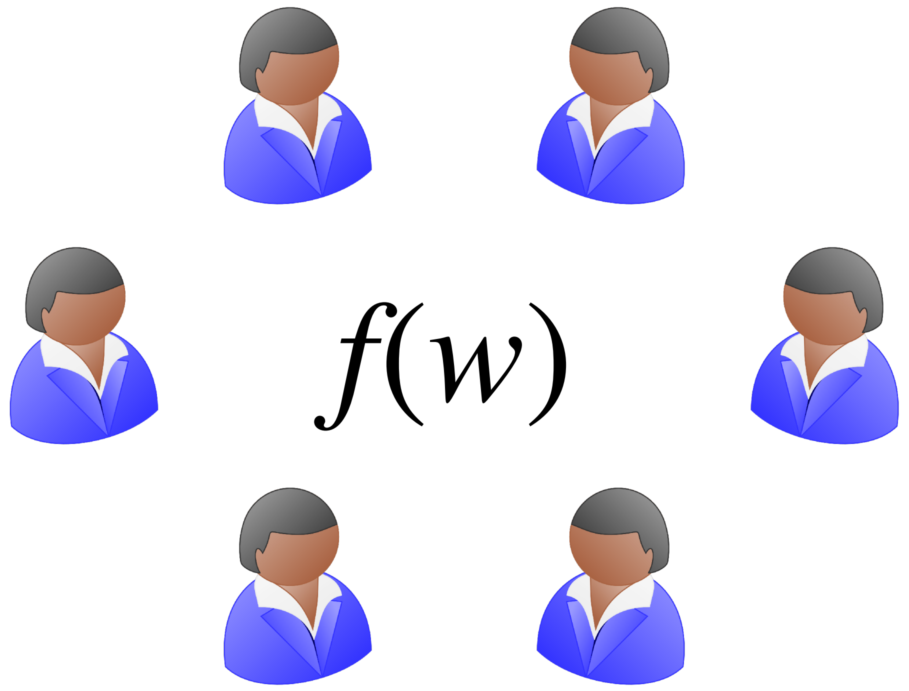
Each party has a "view" containing inputs/randomness/messages



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Prover **Verifier**

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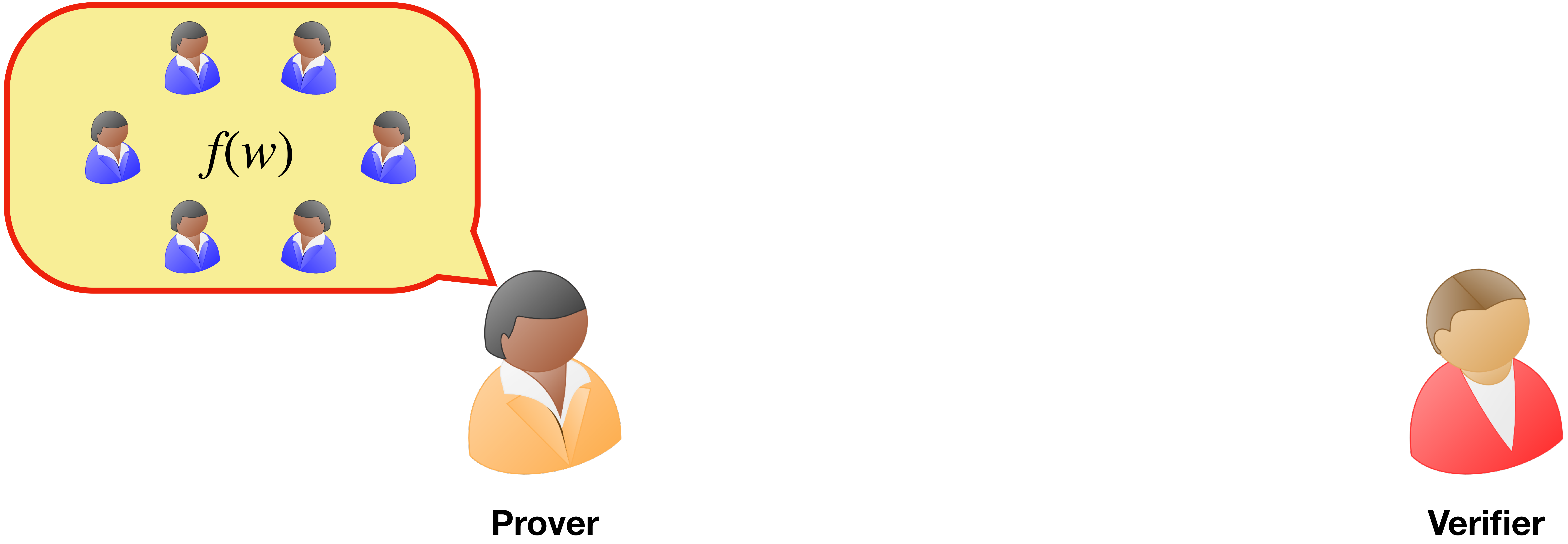

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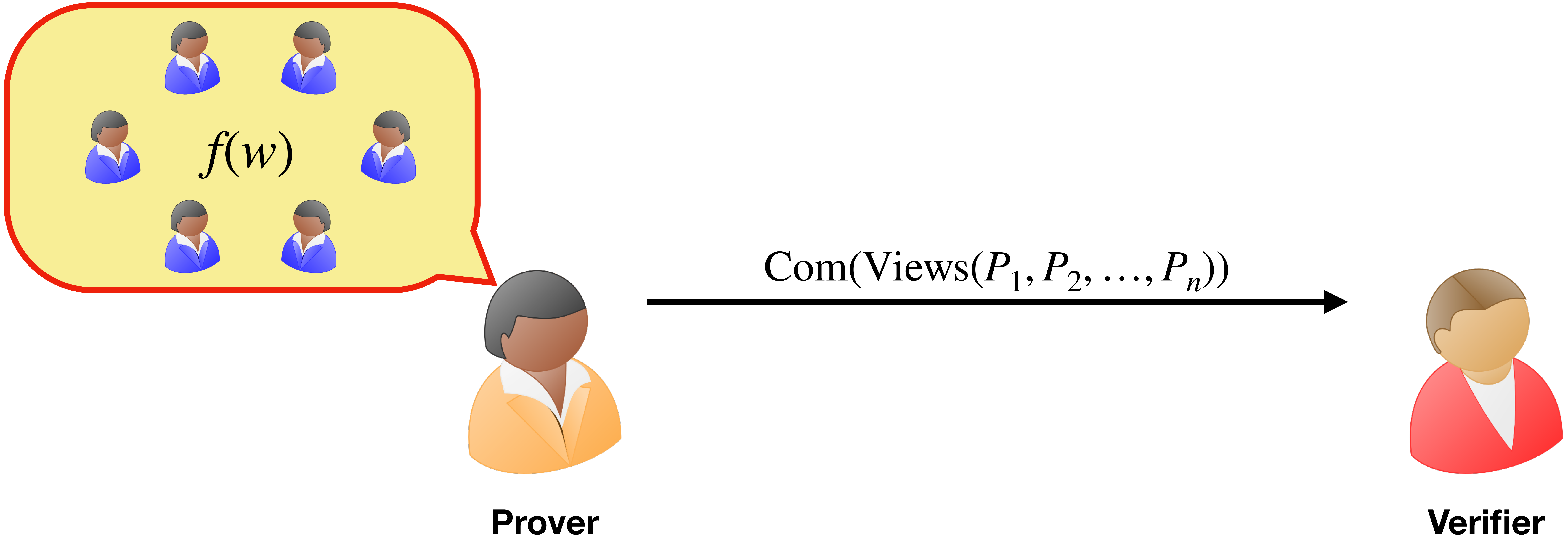

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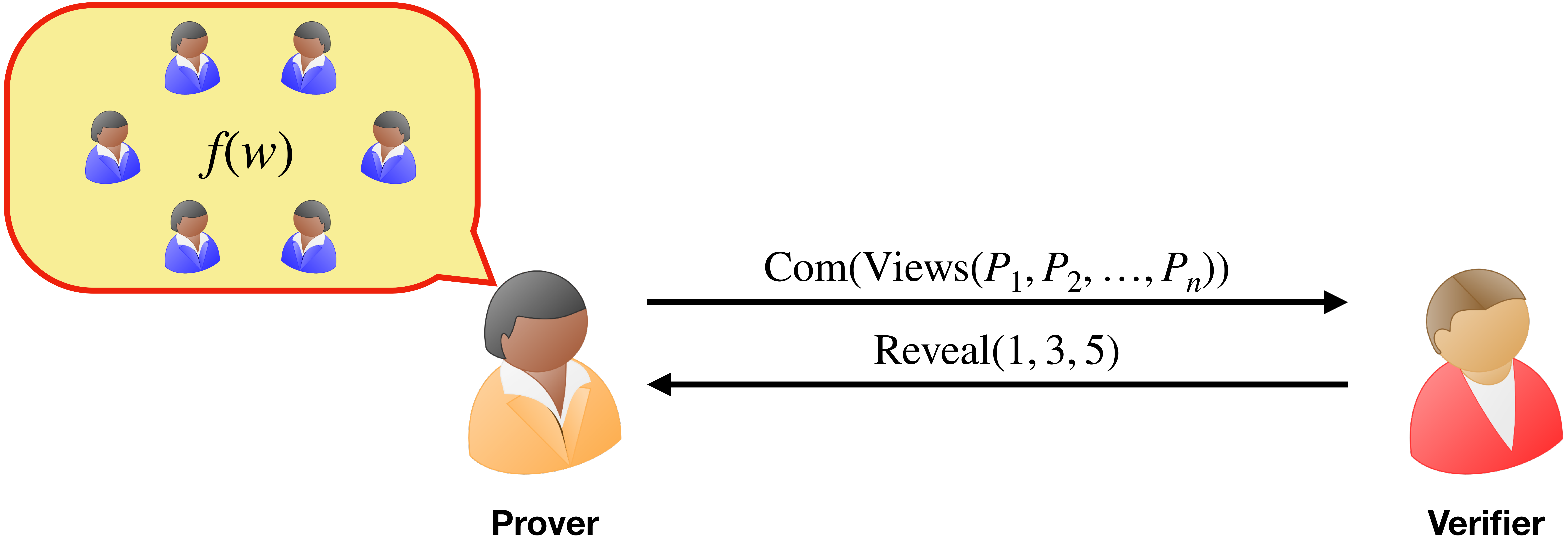

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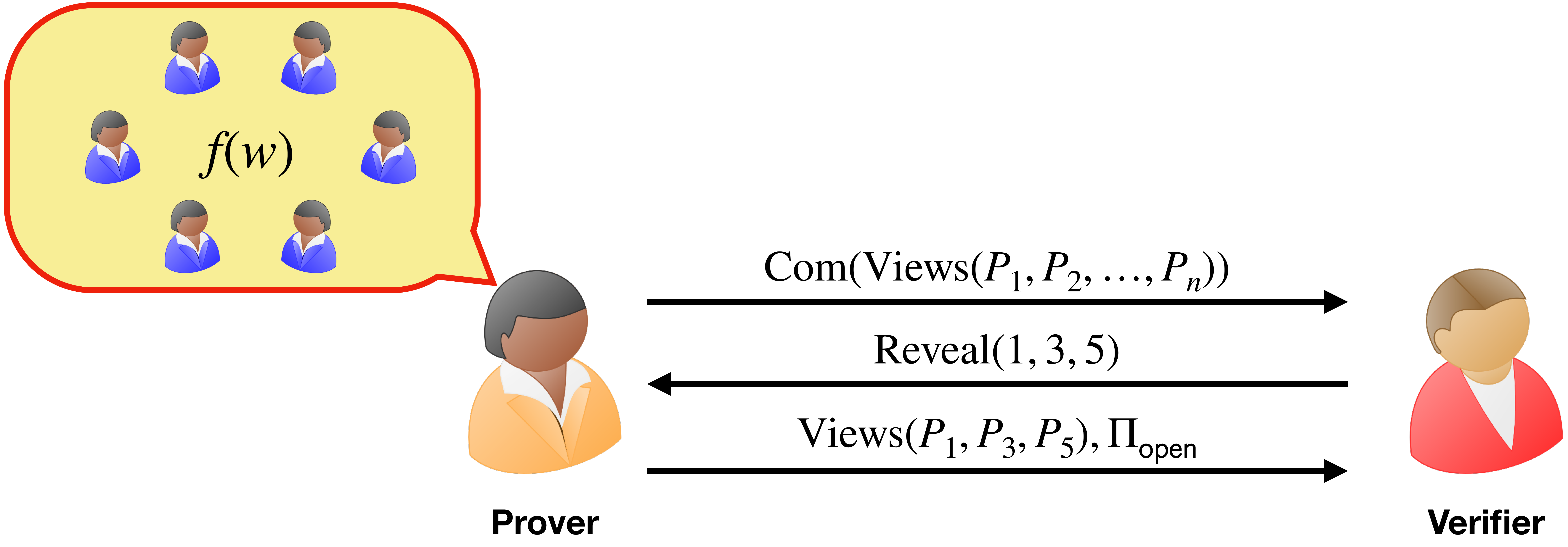

OWF → Signature

Can build a signature scheme given a OWF and a NIZK:

$$\Pi = \{\textsf{sk} \mid \textsf{pk} = f(\textsf{sk}) \wedge m\}$$

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Can we do better with a custom proof for the AM-OWF? 🤔

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2. $[m]_3 = z + (1 - 2z) \odot [r]_3$

MPCitH for AM-OWF [DHG+21]

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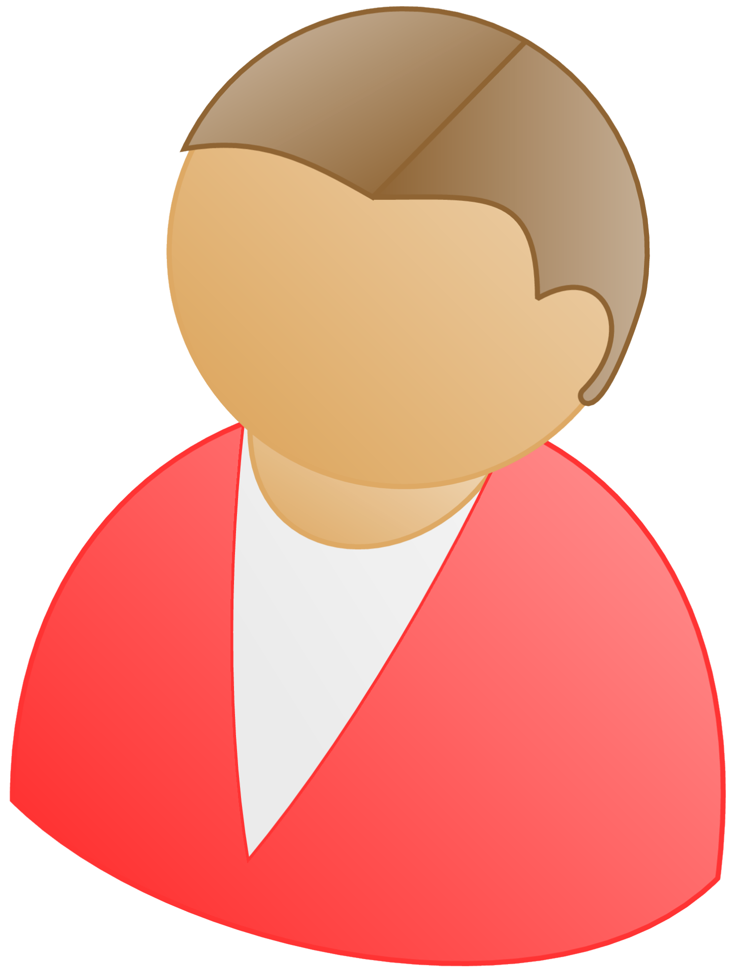
Prover



Verifier

MPCitH for AM-OWF [DHG+21]

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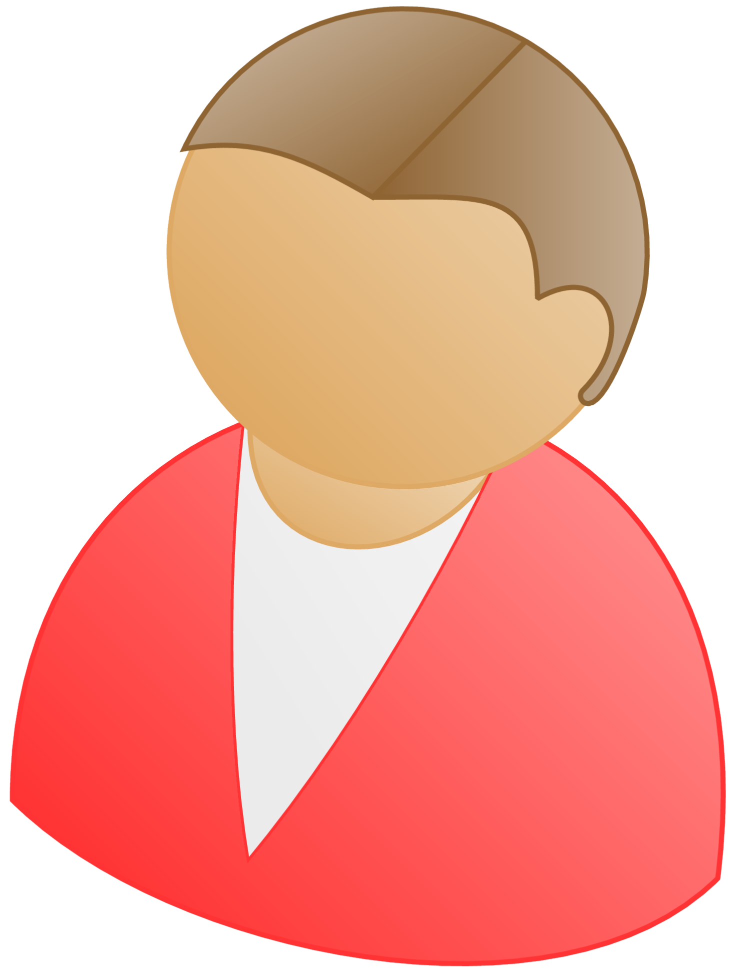
1 $\xrightarrow[]{[x^{(i)}] \quad ([r^{(i)}]_2, [r^{(i)}]_3)}$

Prover



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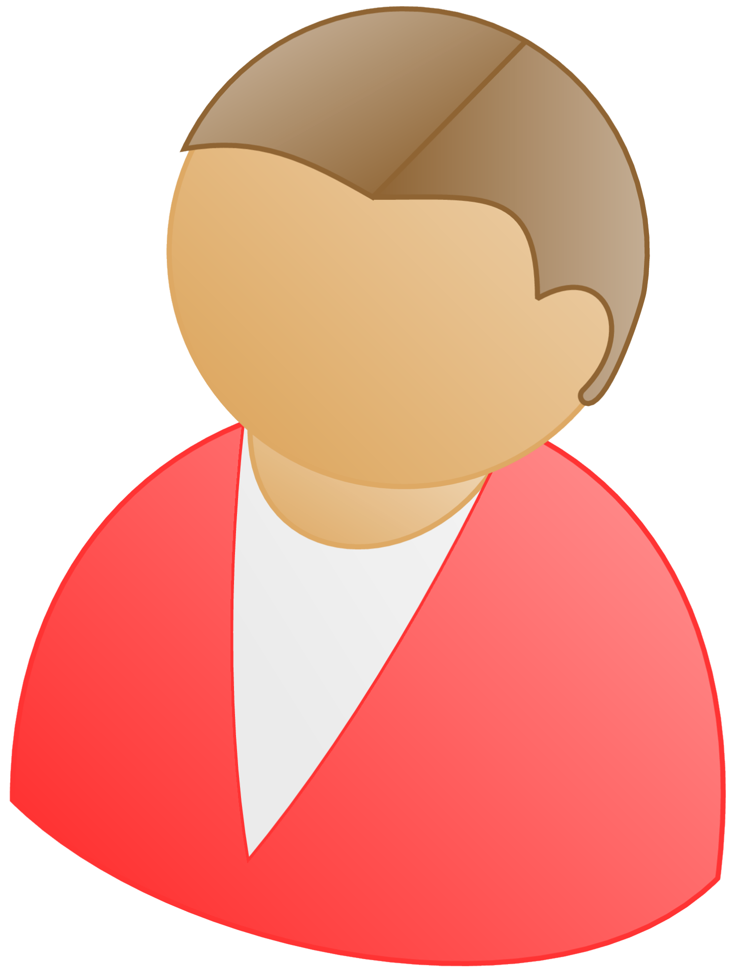

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via Cut-and-Choose

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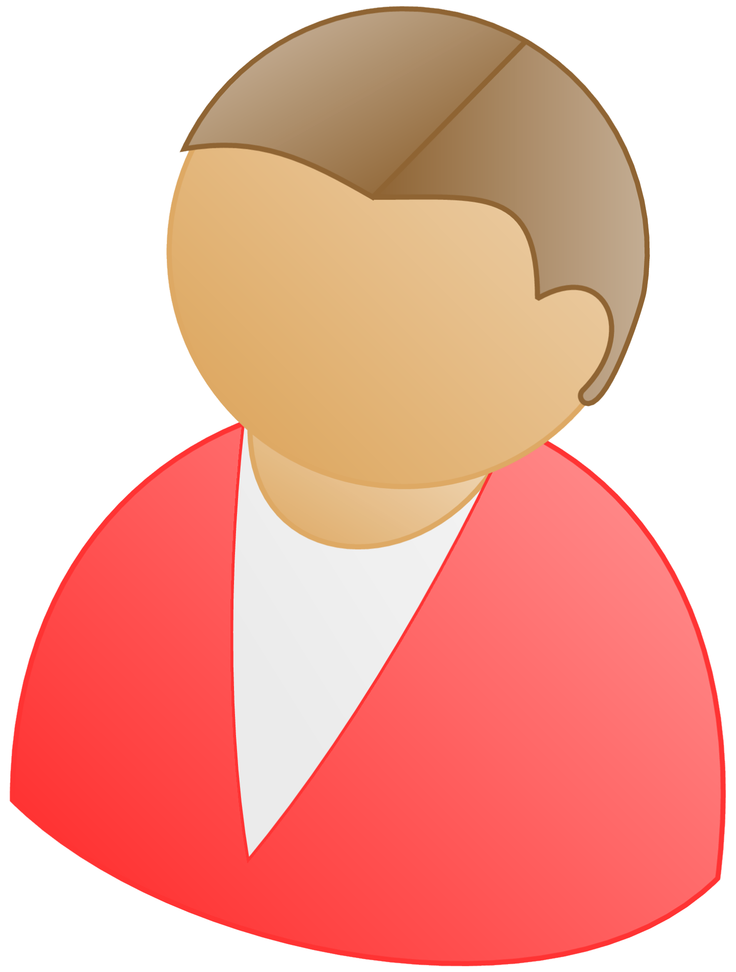

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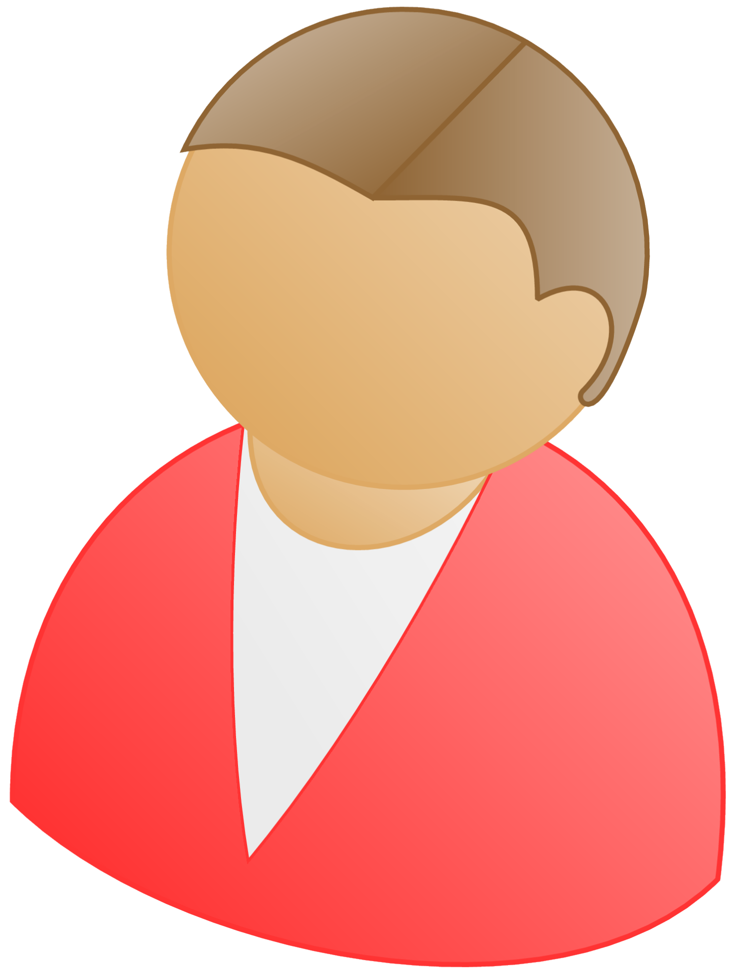

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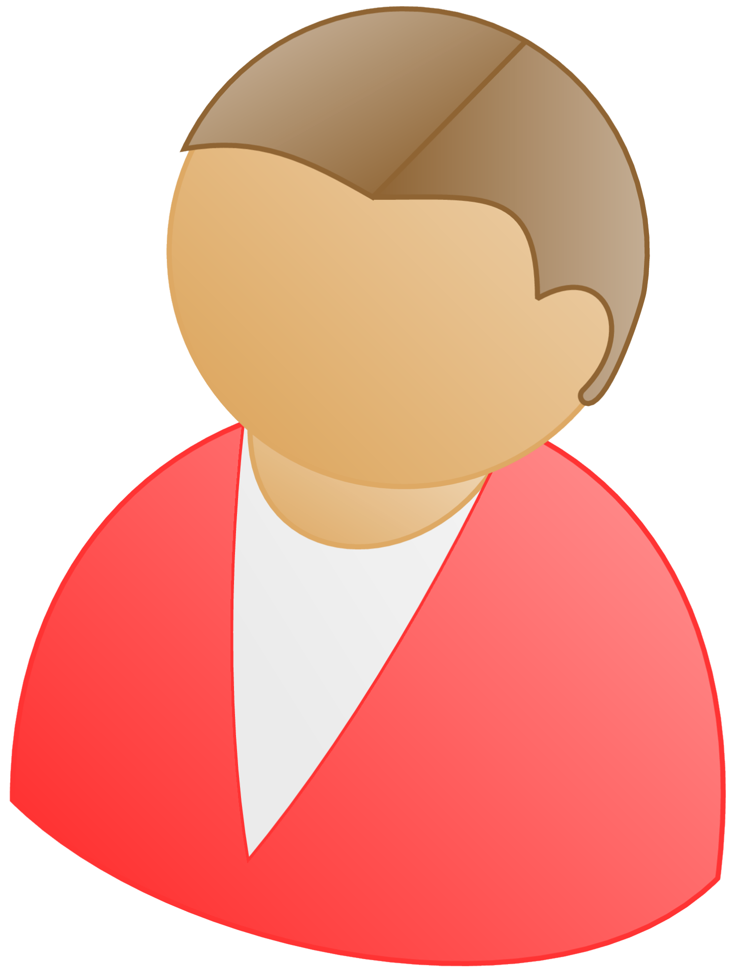

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Costs 5-7 KB! 😞

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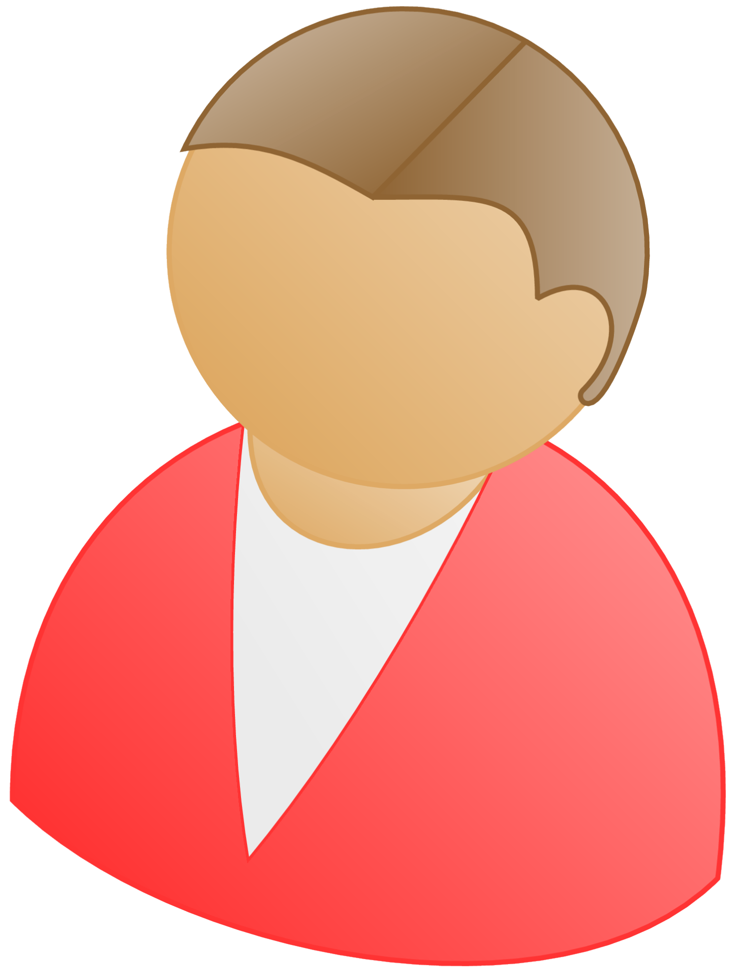

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$[x^{(i)}]$

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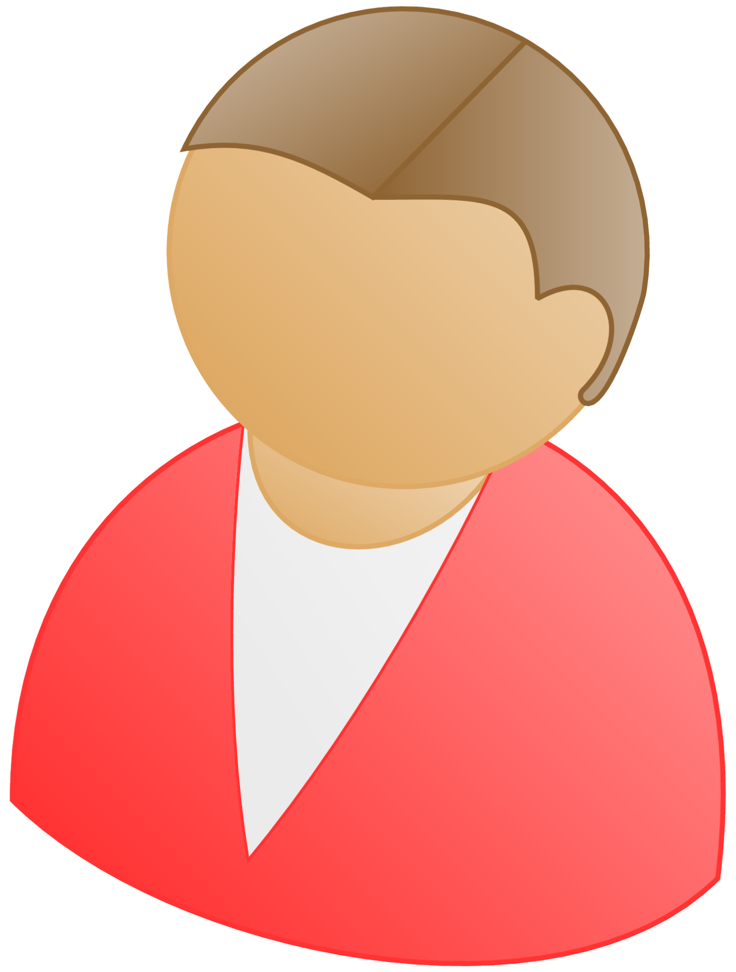
③ $\longrightarrow$



Verifier

Can we avoid Cut-and-Choose?

MPCitH for AM-OWF [This Work]

$$\Pi = \{x \mid y = B \cdot_3 (A \cdot_2 x)\}$$

$[x^{(i)}]$

Prover

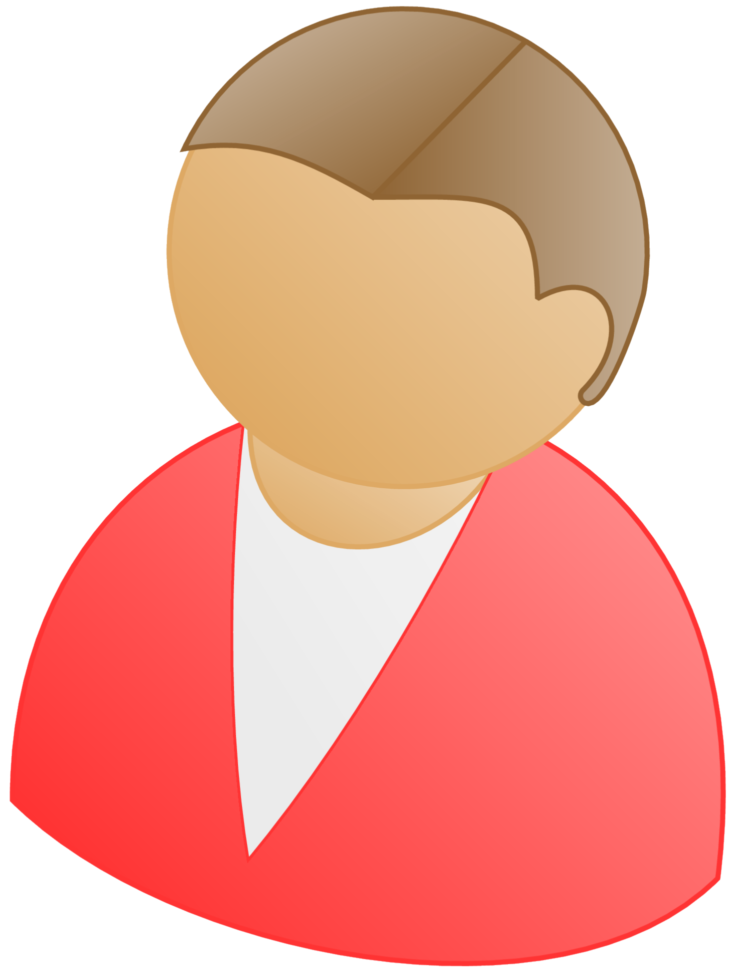
1 $([r^{(i)}]_2, [r^{(i)}]_3)$

Permute $([r^{(i)}]_2, [r^{(i)}]_3)$

2

Send transcript of “online phase”

3



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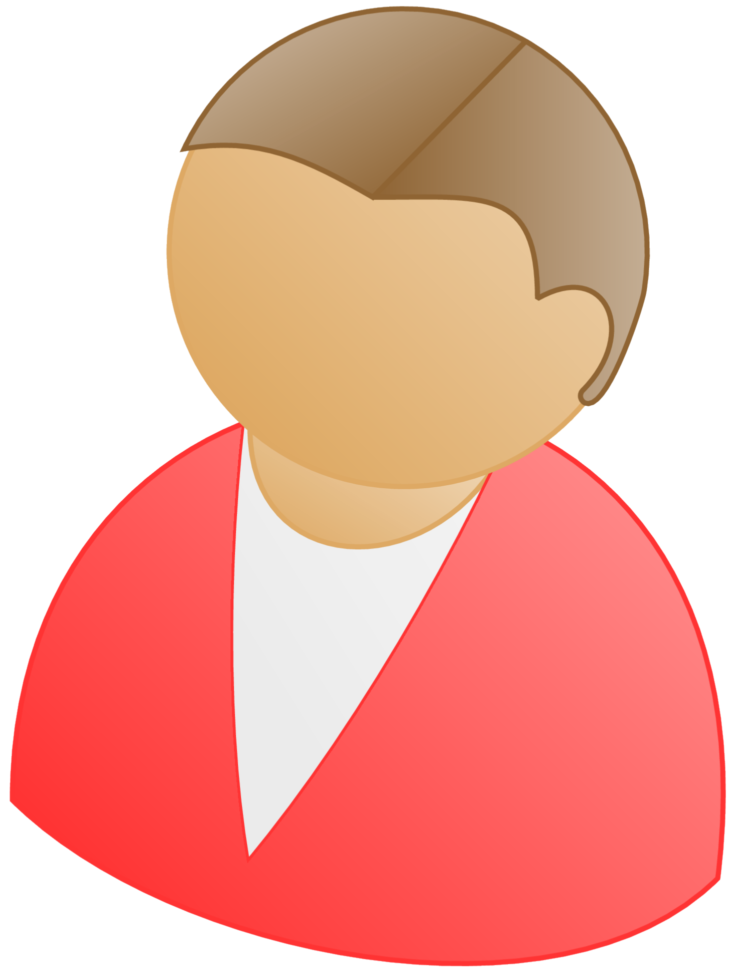

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Soundness via a careful analysis.
Similar techniques in [CCJ23]

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2 Free! 😃

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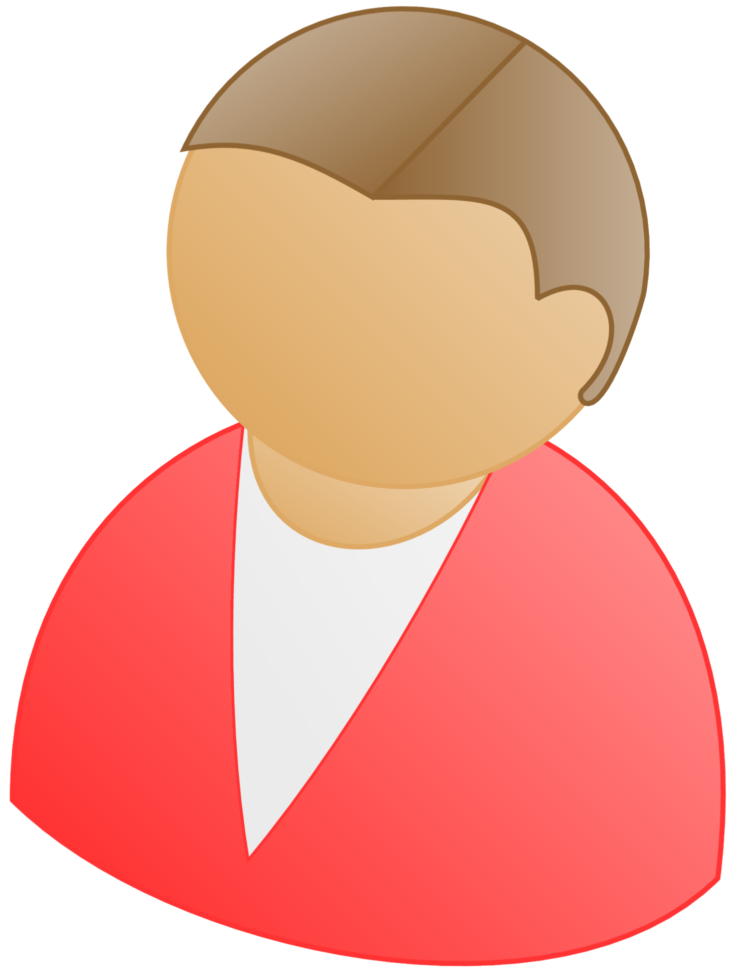
3

Prover



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Comparison

	Fast (KB)	Short (KB)	Assumption
SPHINCS+	16.7	7.7	SHA256
CCJ23	11.3	7.8	f-almost RSD
AGH+23	9.7	4.5	SD over GF(256)
BBdSG+22	5.6	4.5	EM-AES
KZ22	5.8	4.4	Rain
ARZV23	7.7	4.4	MinRank
KHS23	5.8	3.8	AIM
Our Work	**5.5**	**4.0**	**AM-OWF**

Many more works! Dropped due to lack of space ☹️

New wPRF

Why do we want a new wPRF?

Secret Key Matrix

[BIP+18]: $$y = B \cdot_3 (\underbrace{K \cdot_2 x}_{})$$

Requires $O(\lambda^2)$ multiplications in MPC

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Requires $O(\lambda^2)$ multiplications in MPC

Can reduce communication by using circulant matrices

But work is still $\omega(\lambda)$ ☹️

New wPRF

Input

[BIP+18]: $y = B \cdot_3 (K \cdot_2 x)$

Secret Key Matrix

Input

Ours: $y = B \cdot_3 (A \cdot_2 (k \odot_2 x))$

Secret Key Vector

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[BIP+18]: $y = B \cdot_3 (\mathbf{K} \cdot_2 x)$

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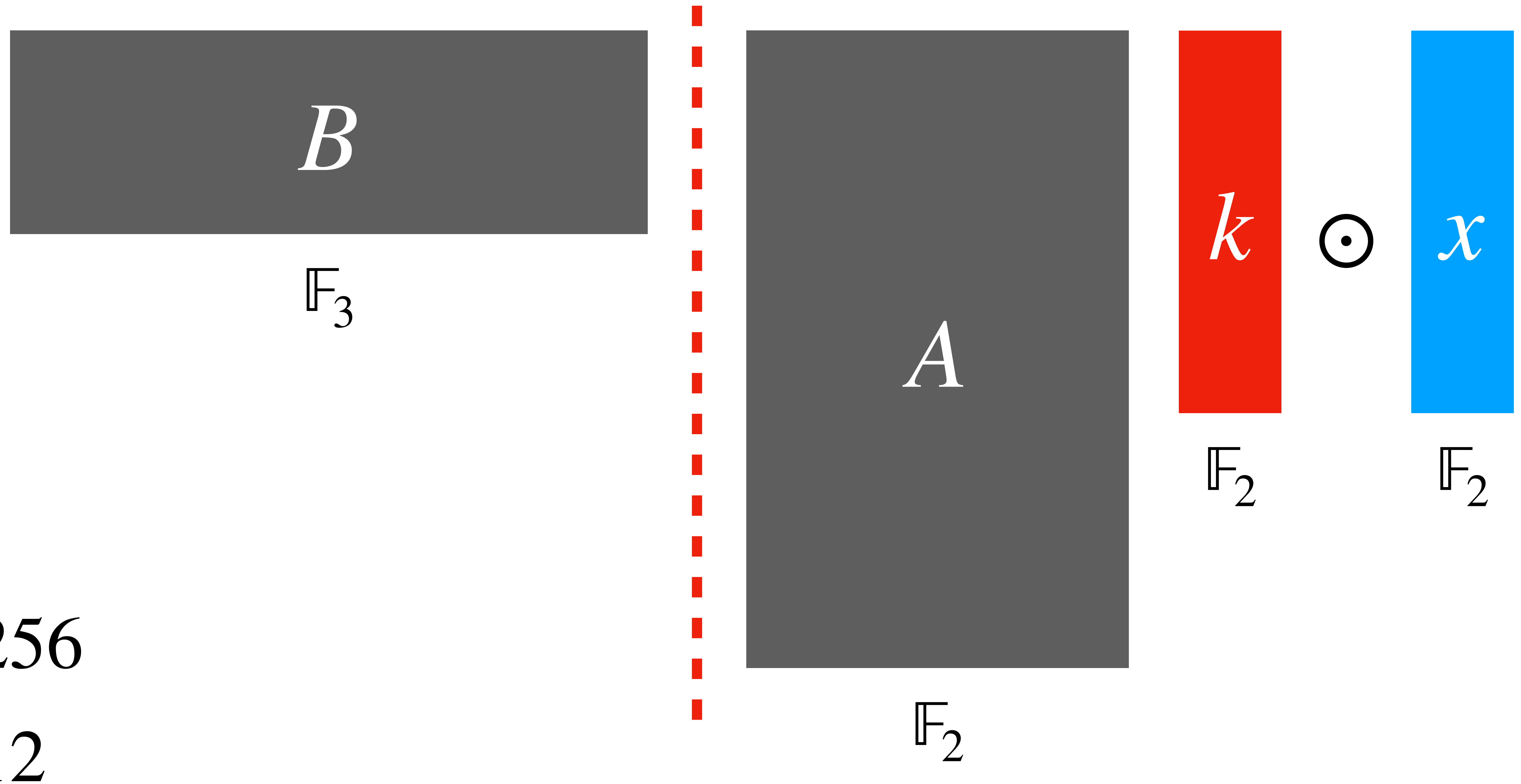
Ours: $y = B \cdot_3 (A \cdot_2 (k \odot_2 x))$

Linear time

Secret Key Vector

New wPRF

$y =$



- $A \sim 512 \times 256$
- $B \sim 81 \times 512$

OPRF Protocol

OPRF

OPRF

$$\mathsf{PRF}(k, x) \to y$$

OPRF

$$\mathsf{PRF}(k, x) \rightarrow y$$

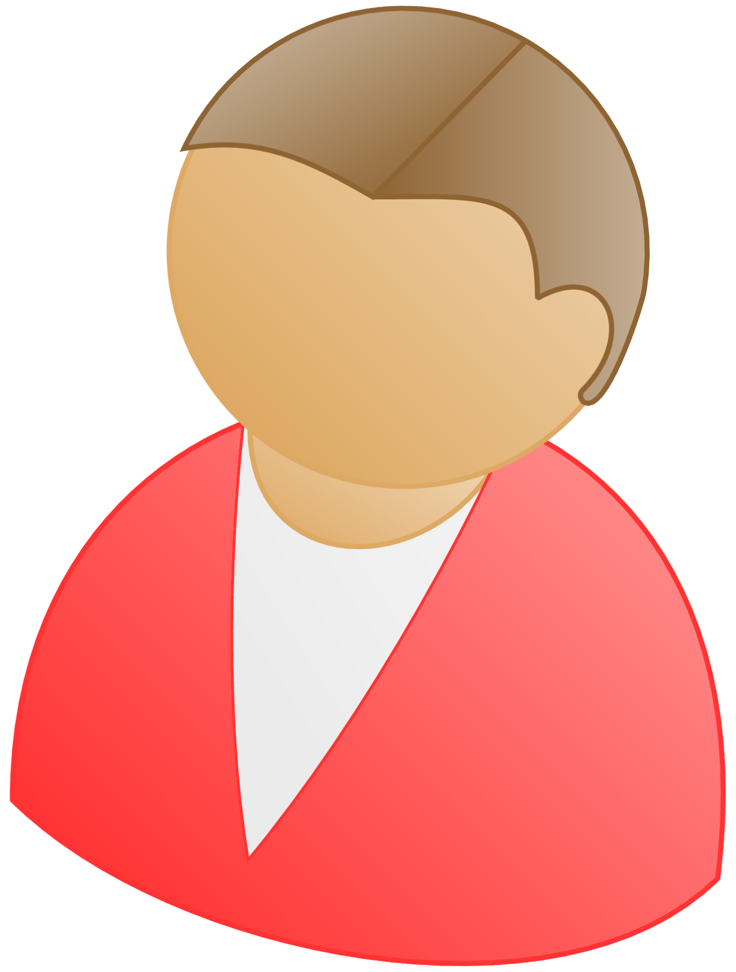
x

OPRF

$$\mathrm{PRF}(k, x) \rightarrow y$$

x

k



OPRF

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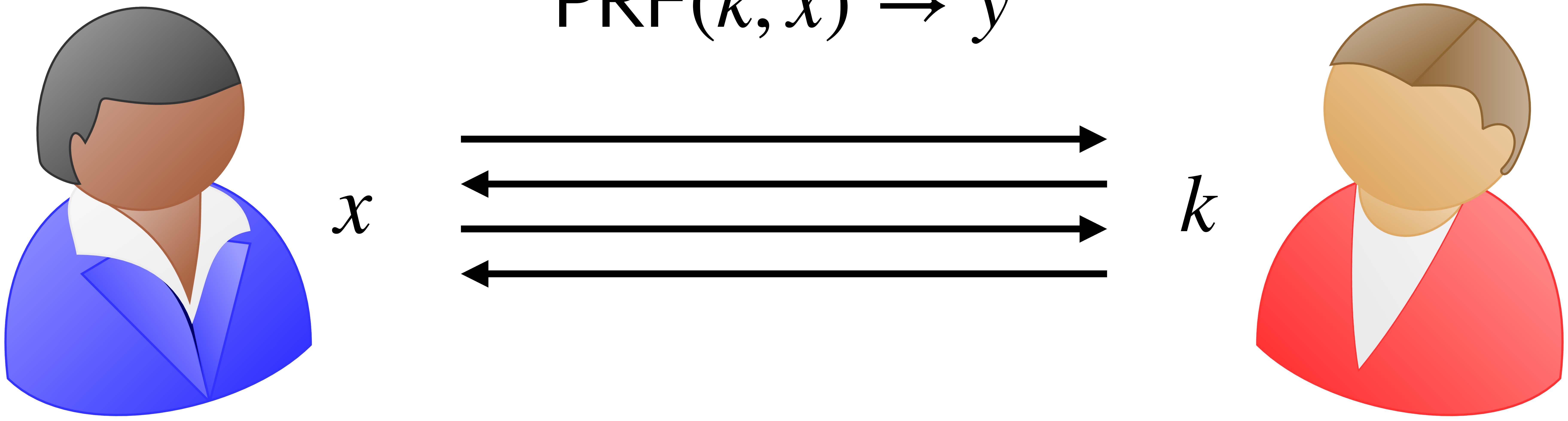

OPRF

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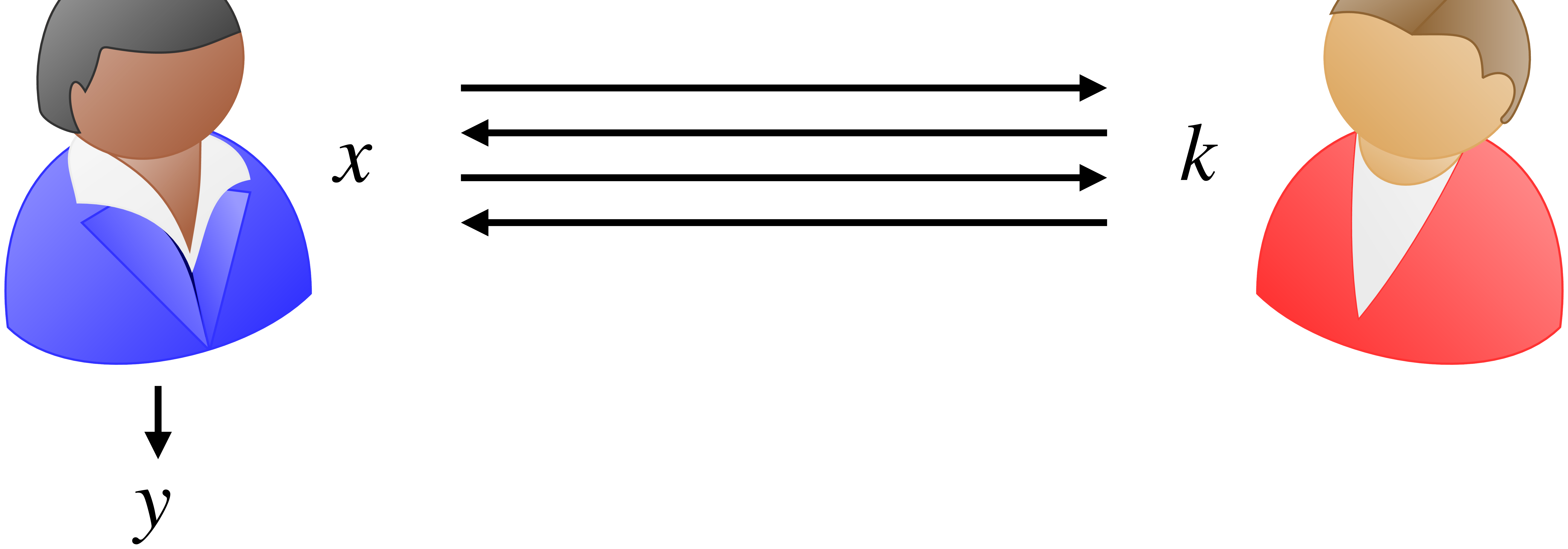

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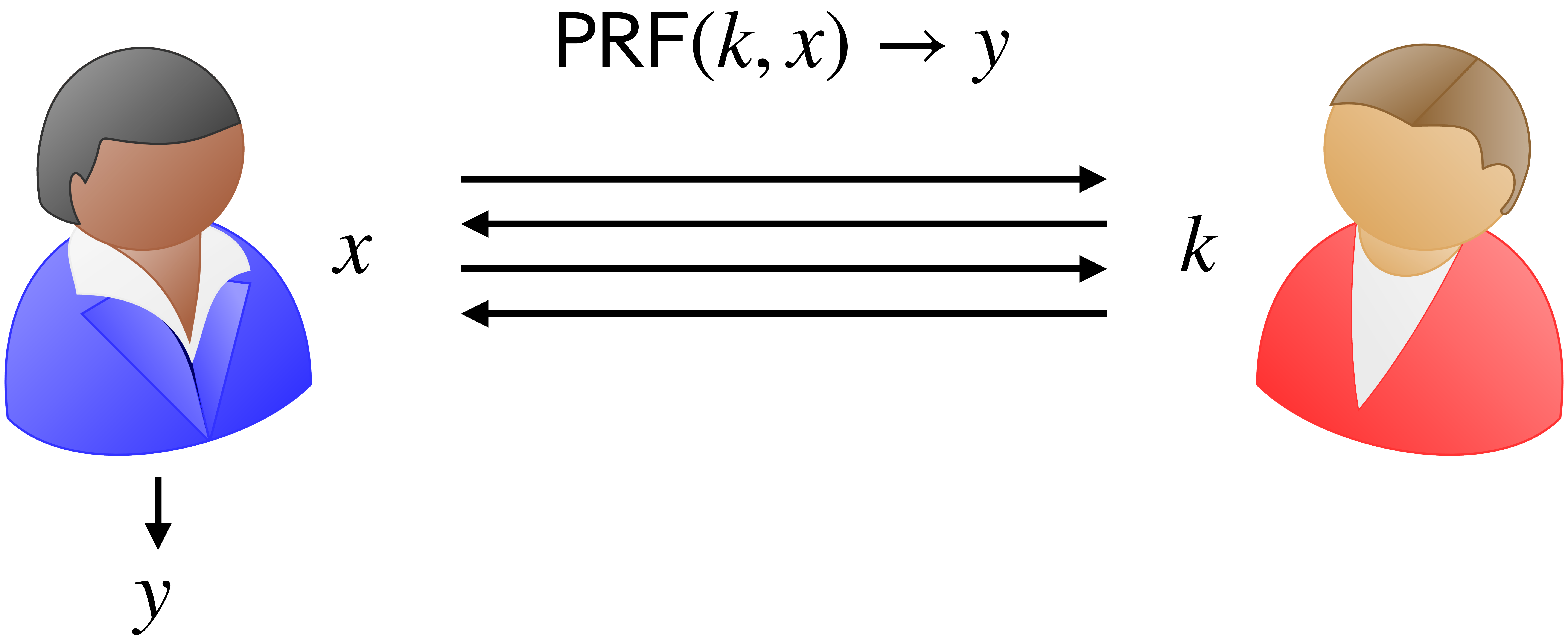

Learns nothing about k

Learns nothing about x

Distributed wPRF evaluation

$$y = B \cdot_3 (A \cdot_2 (k \odot_2 x))$$

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Implementing $k \odot x$:

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Implementing $k \odot x$:

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$\Longrightarrow$ Can **reuse** OT correlations across evaluations!

Distributed wPRF evaluation

$$y = B \cdot_3 (A \cdot_2 (k \odot_2 x))$$

Non-Linear

Two approaches to implement $\mathbb{F}_2 \to \mathbb{F}_3$:

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Non-Linear

Two approaches to implement $\mathbb{F}_2 \rightarrow \mathbb{F}_3$:

- OT - Less communication but “consumes” OTs

Distributed wPRF evaluation

$$y = B \cdot_3 (A \cdot_2 (\textcolor{red}{k} \odot_2 \textcolor{blue}{x}))$$

Non-Linear

Two approaches to implement $\mathbb{F}_2 \rightarrow \mathbb{F}_3$:

- OT - Less communication but "consumes" OTs
- Garbling - More communication but no correlations needed

Evaluation

	Rounds	Comm. (bits)	Time (μs)
DDH [Mea86]	2	512	121
[DHG+21]	2*	65 + 1252	$25.4^{\dagger}$ + 6.1
Our Work (OT)	2*	38 + 916	7.0 + 0.4
Our Work (Garble)	2*	2^{15}	0.0 + $4.0^{\dagger}$
Our Work (Shared output)	1*	2^{15}	0.0 + $4.0^{\dagger}$

** excludes preprocessing rounds*
† denotes estimates

Wish List 🧞

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   - Post-Quantum cryptanalysis
2. **New primitives?**
   - We have OWFs and (w)PRFs. Can we have CRH? … or maybe even a Random Oracle?
3. **Better protocols!**
   - Only scratched the surface in terms of optimizing protocols
   - Also need better implementations!

Thank you!