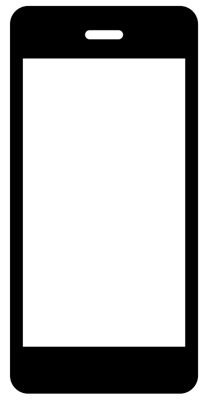
How to construct Quantum FHE, Generically

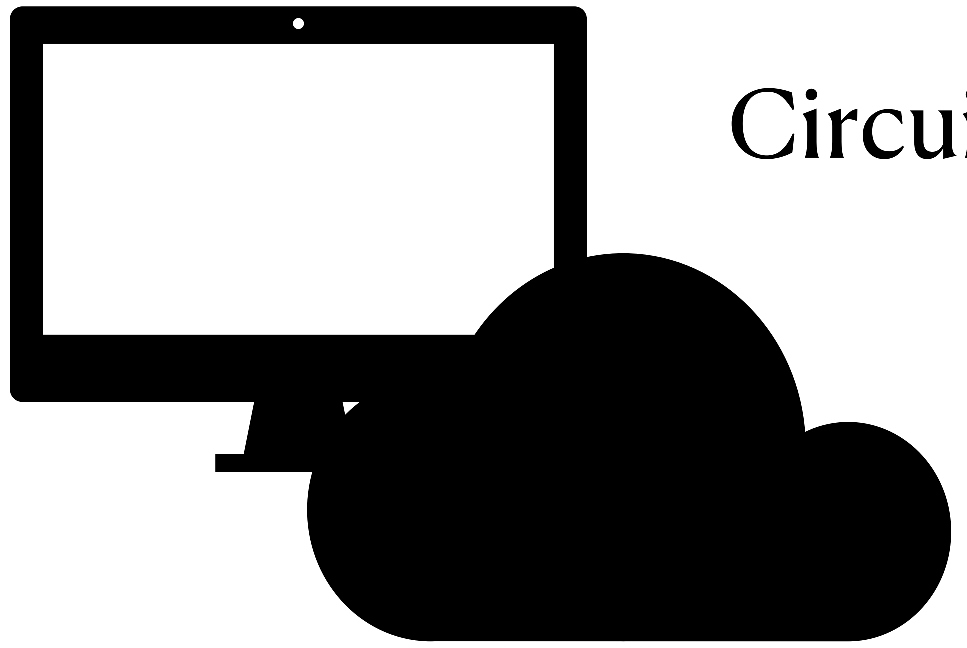
Aparna Gupte
MIT

Vinod Vaikuntanathan
MIT

Classical Fully Homomorphic Encryption (FHE)

Input x





Circuit C

Server should not learn anything about x

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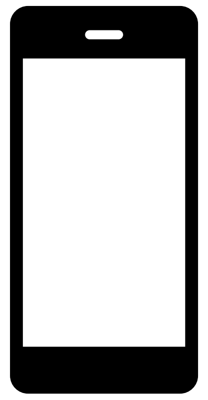
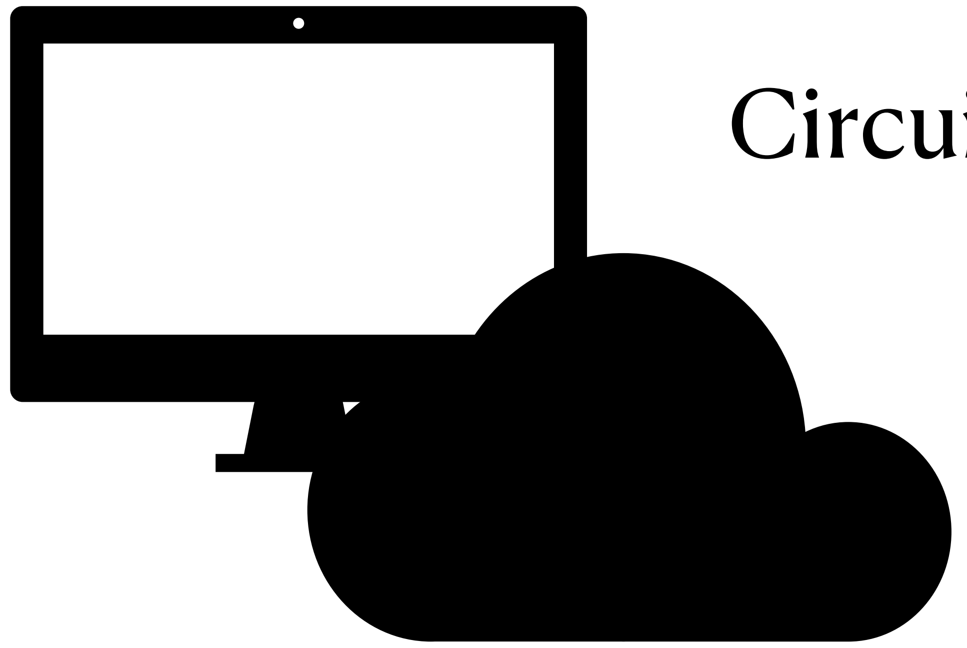


c, evk

$\longrightarrow$



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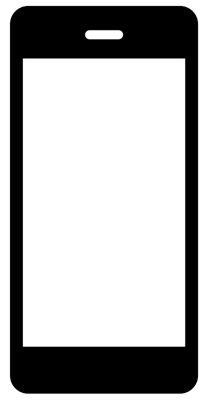

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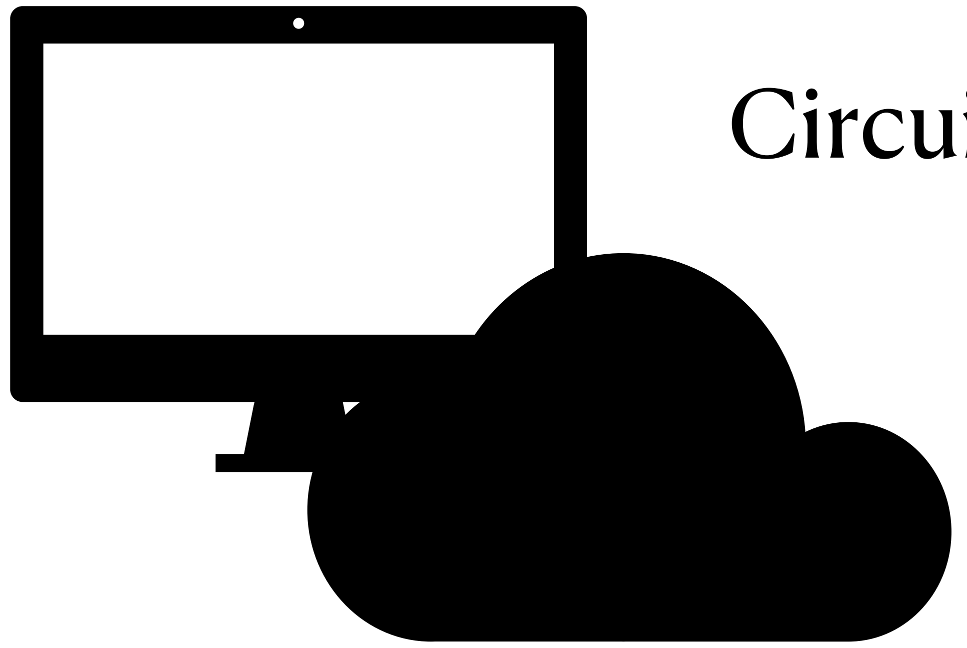

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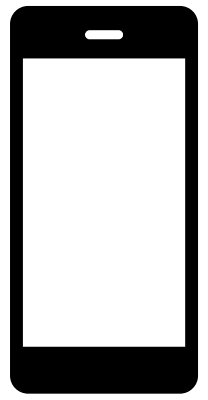

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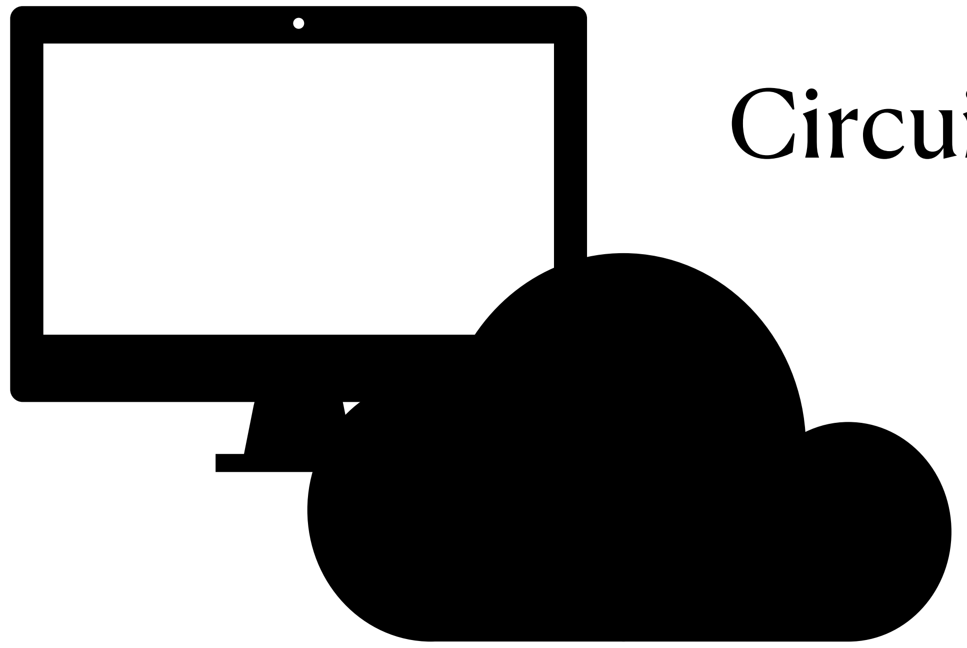

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Quantum Fully Homomorphic Encryption (QFHE)

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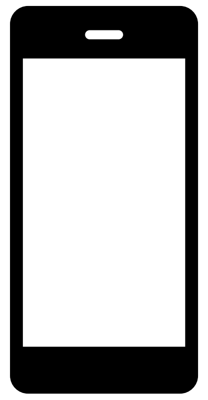

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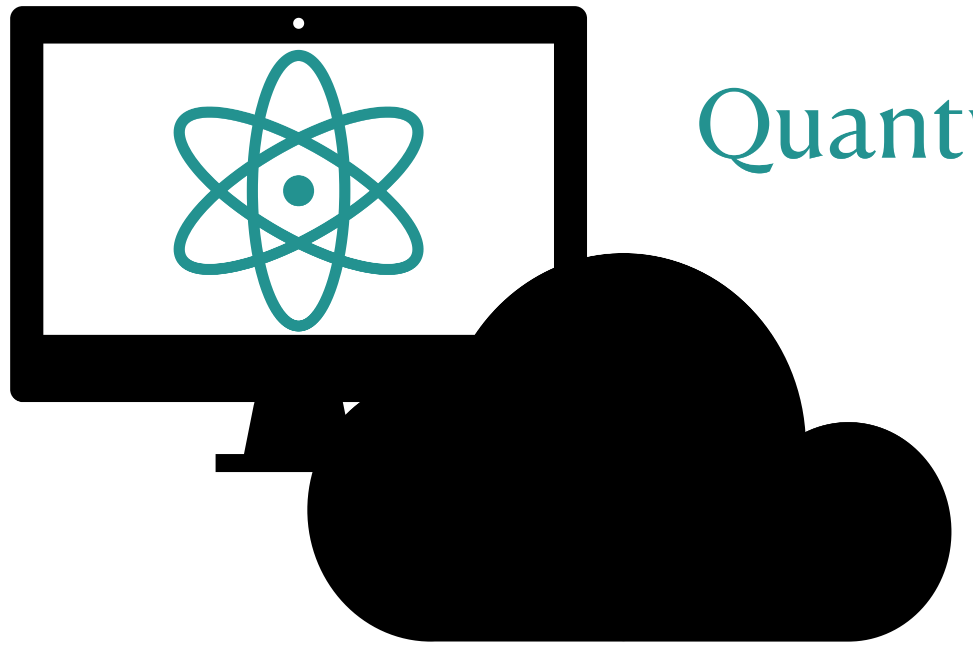

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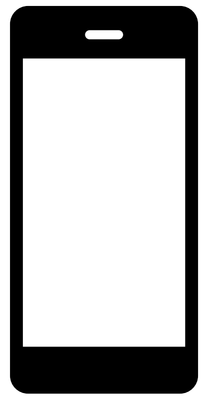

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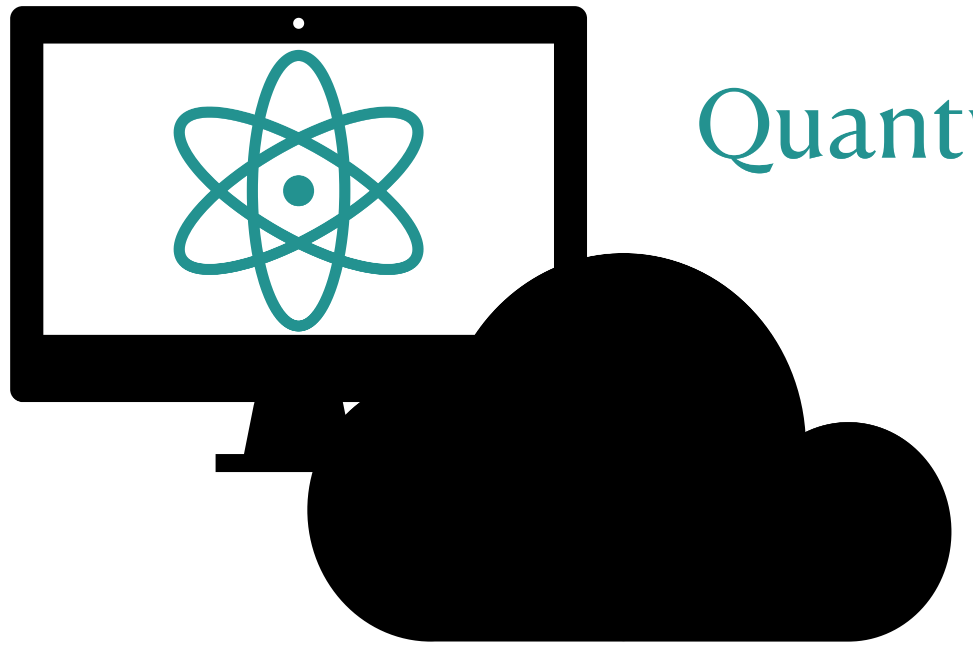

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Quantum Algorithms for Lattice Problems

Yilei Chen*

April 18, 2024

Why should we diversify assumptions?

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Dream Theorem. Any post-quantum classical FHE $\Longrightarrow$ QFHE.

Our Results (QFHE)

Theorem 1 [G-Vaikuntanathan]

pq. Classical FHE* + pq. Dual-mode Trapdoor Functions $\implies$ QFHE

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$\Uparrow$

- LWE [BV11, BV14]
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Non-compact QFHE

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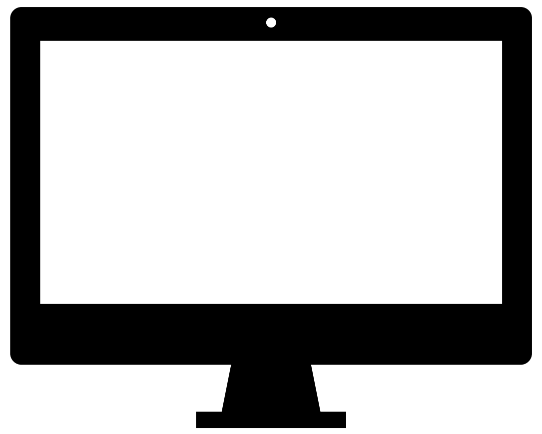

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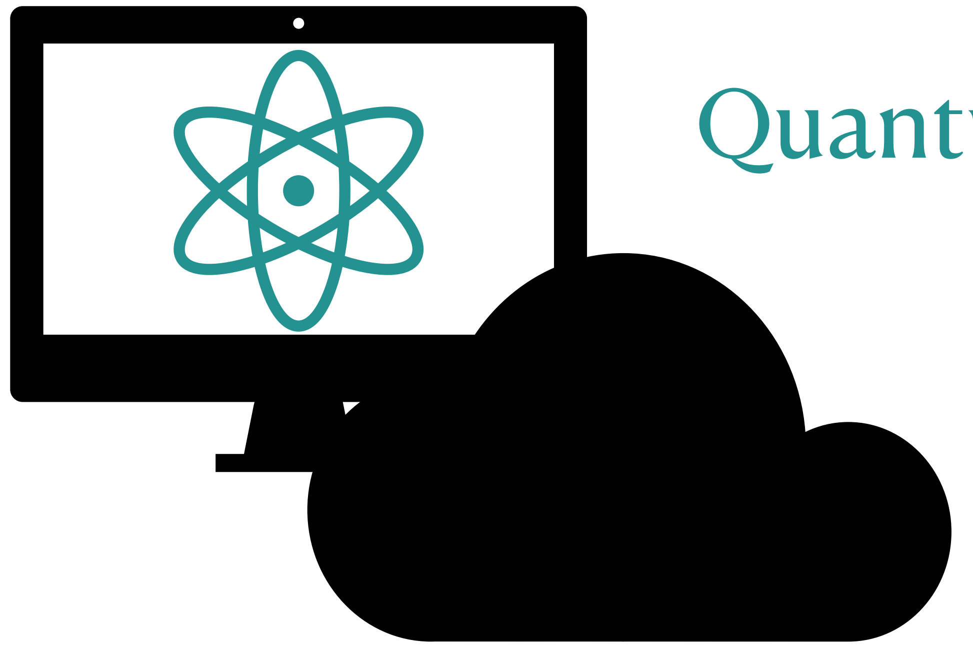

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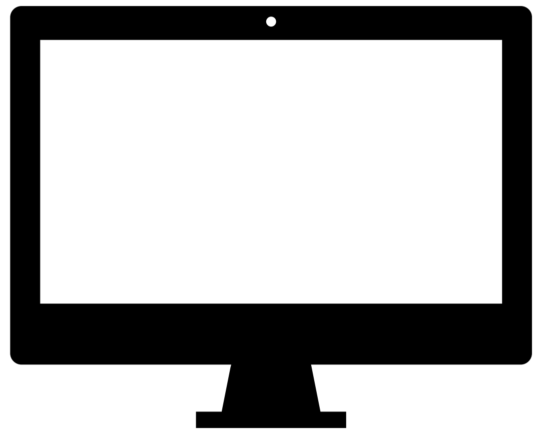

c, evk $\longrightarrow$

$\longrightarrow$ $\longleftarrow$ $\longrightarrow$ … $\longrightarrow$

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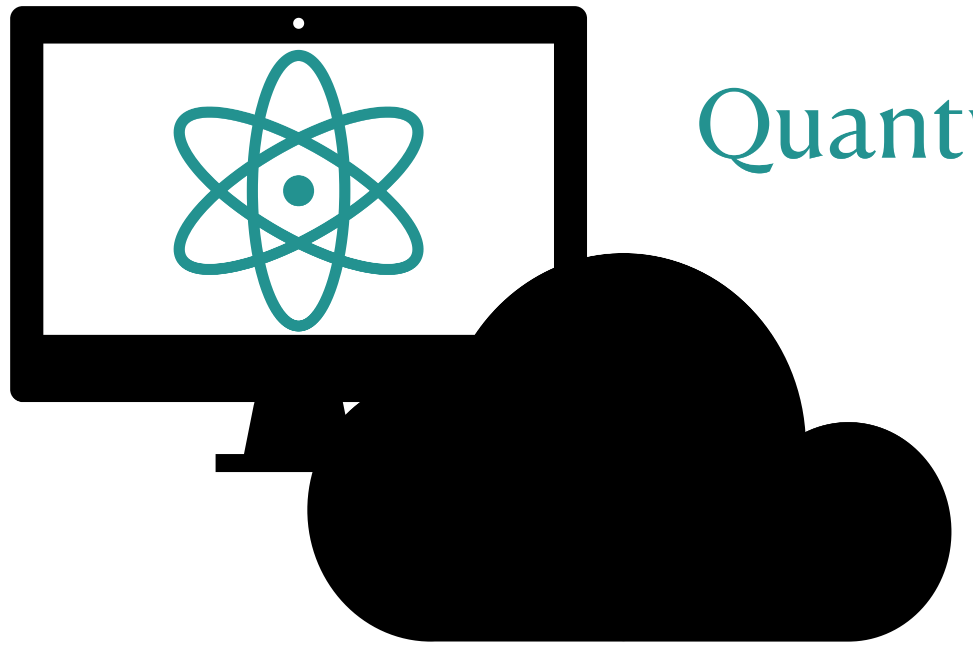

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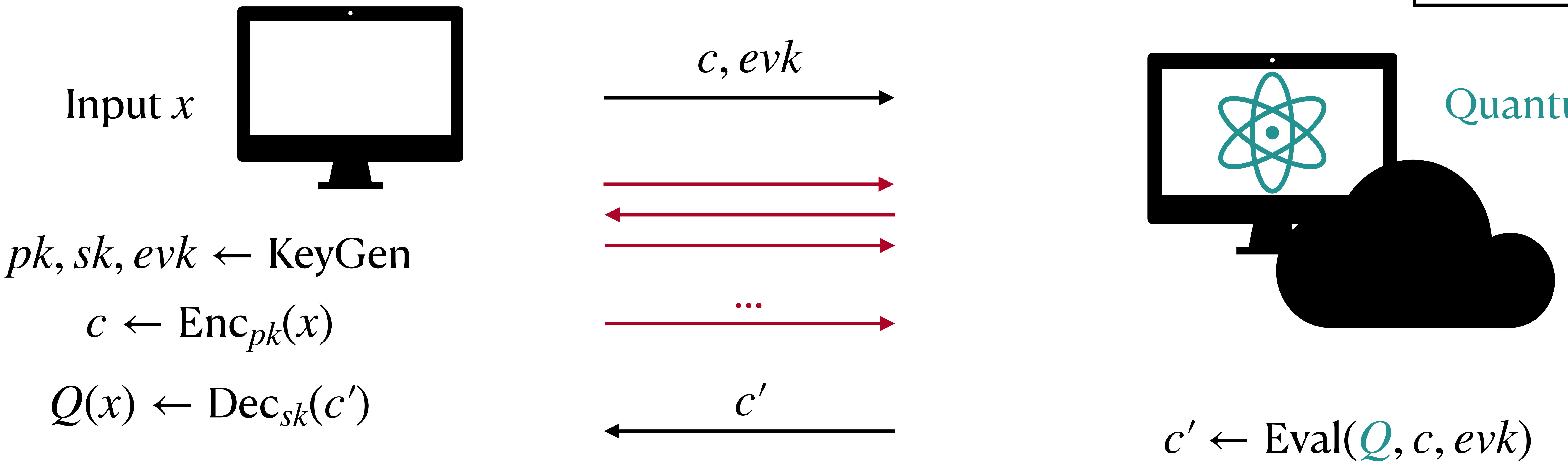
Non-compact QFHE

- Called Blind Delegated Quantum Computation
- Previously known only with LWE



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Corollary [GV24]. Group actions $\Longrightarrow$ Non-compact QFHE

Not known to imply classical (compact) FHE

Our scheme for (compact) QFHE

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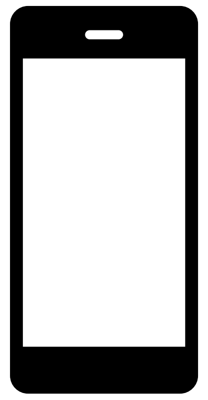

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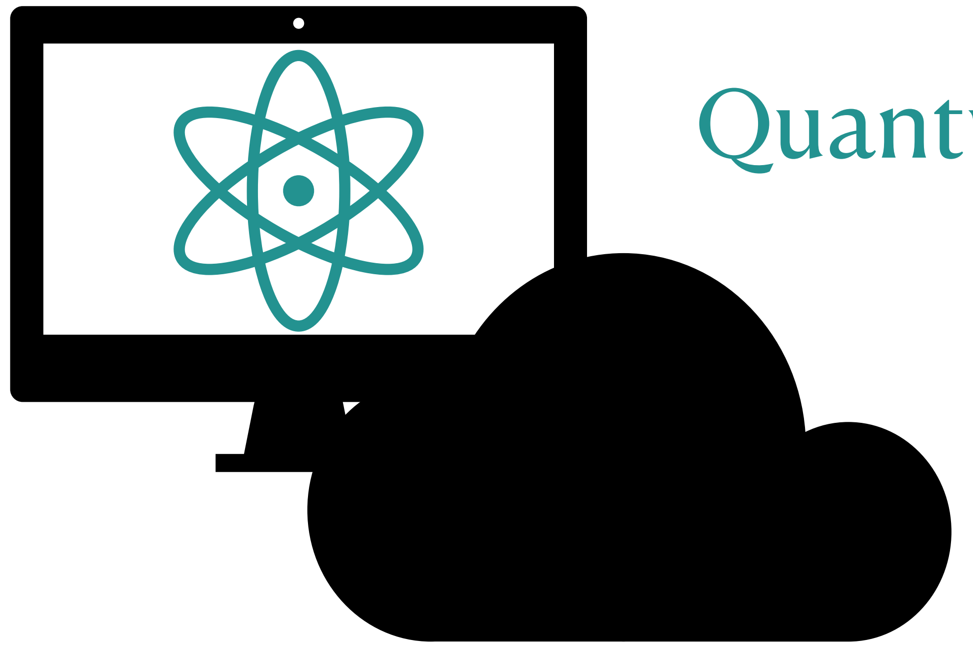

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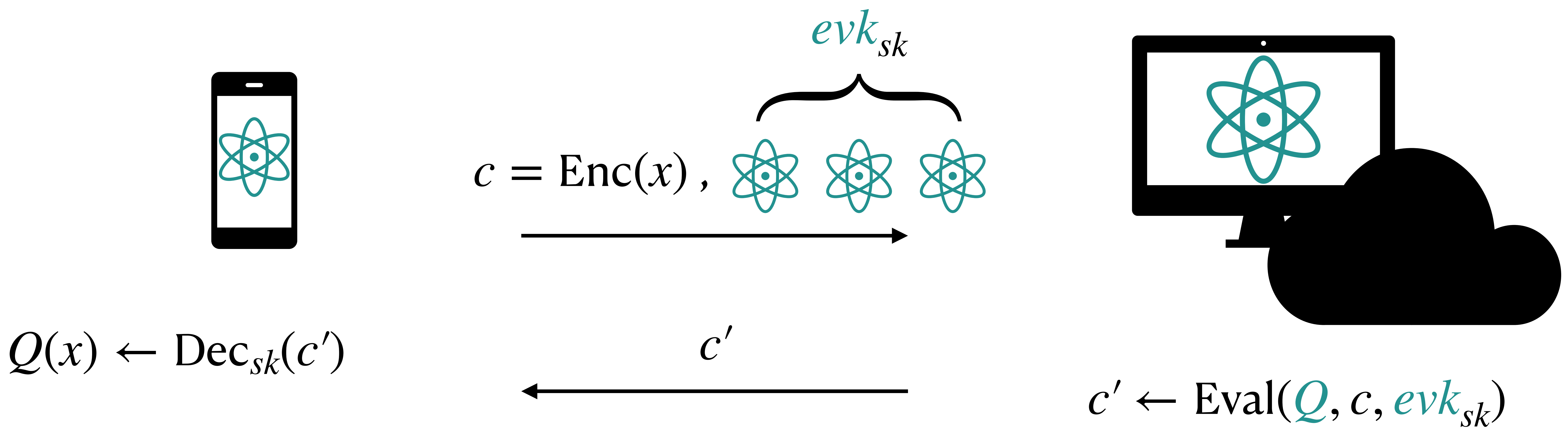
Starting point: Dulek-Schaffner-Speelman'16



- Client needs to be quantum 🙁
- Needs to prepare and send quantum evaluation keys evk (which is a function of sk)

Our Idea: Use RSP

Remote State Preparation (RSP)

[Djunko-Kashefi'16, Gheorghiu-Vidick'19,
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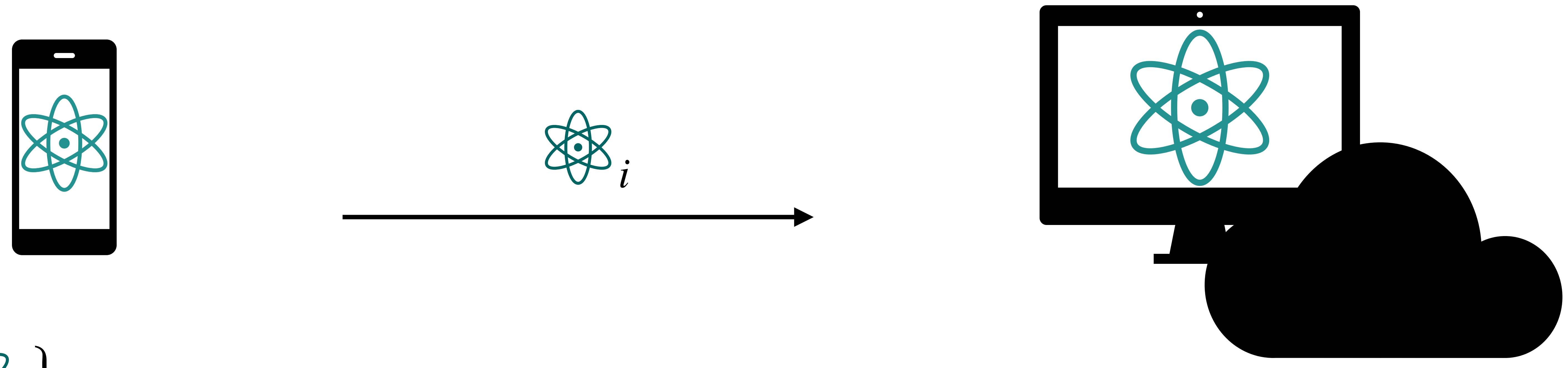

Family of states $\{\ {}_i\}_{i\in I}$

- Goal: Replace quantum communication in some protocols.

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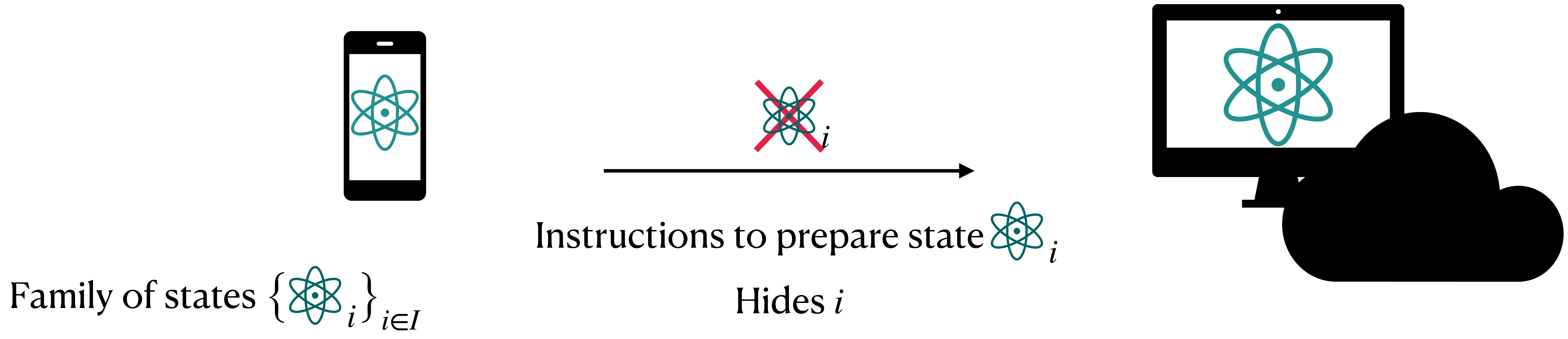

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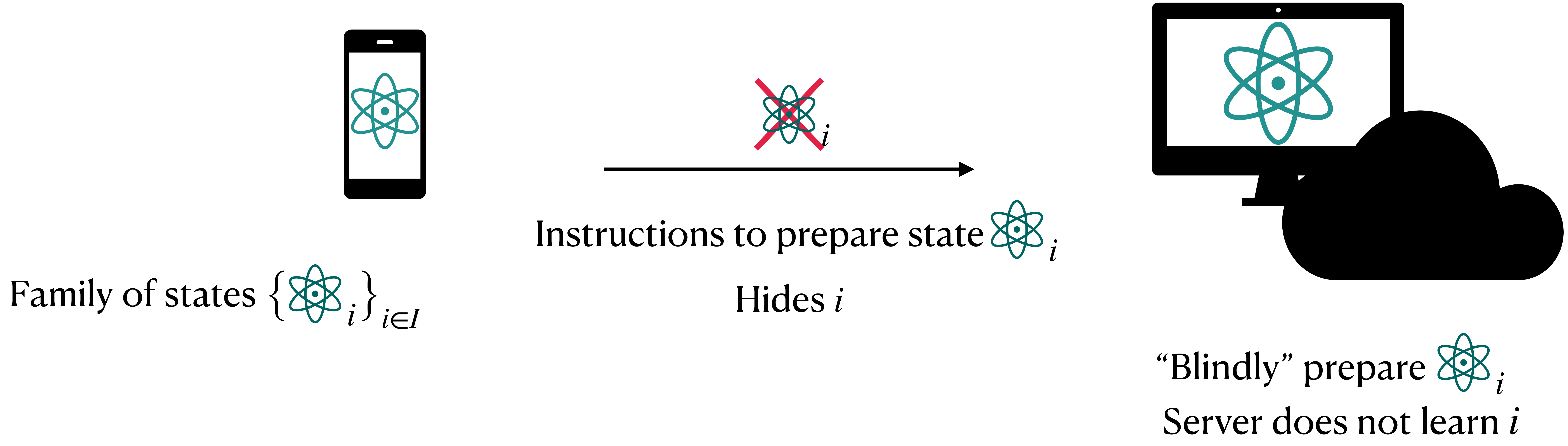

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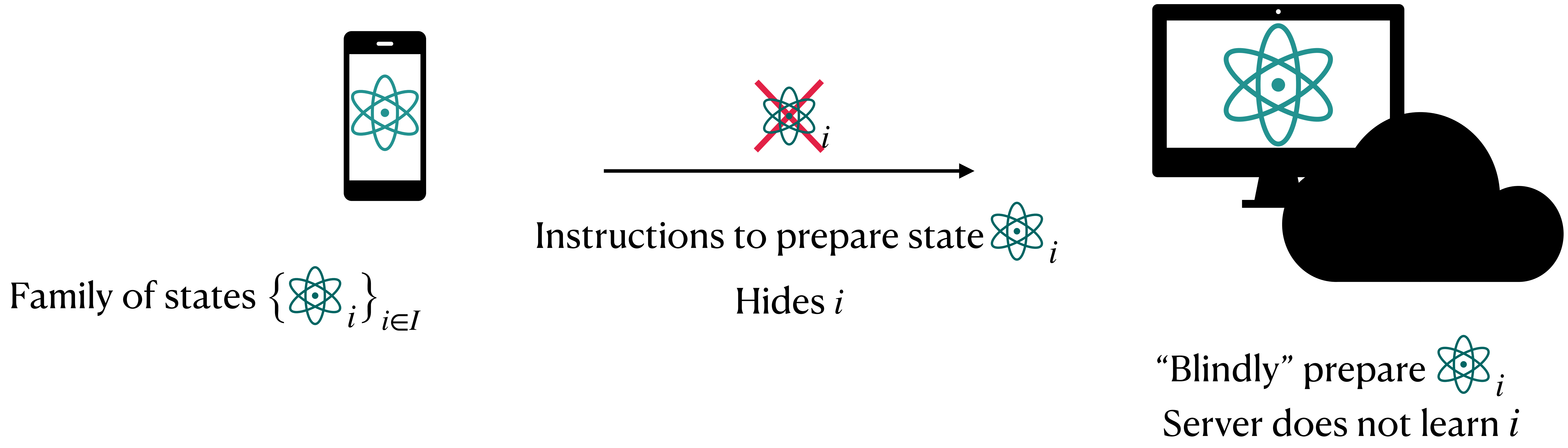

- Goal: Replace quantum communication in some protocols.
- What is known? Can RSP BB84 states assuming dual-mode trapdoor functions [GV19, GMP22].

Our Main Technical Contribution

RSP the DSS evaluation keys

- Can we do remote state preparation for the DSS quantum *evk*?

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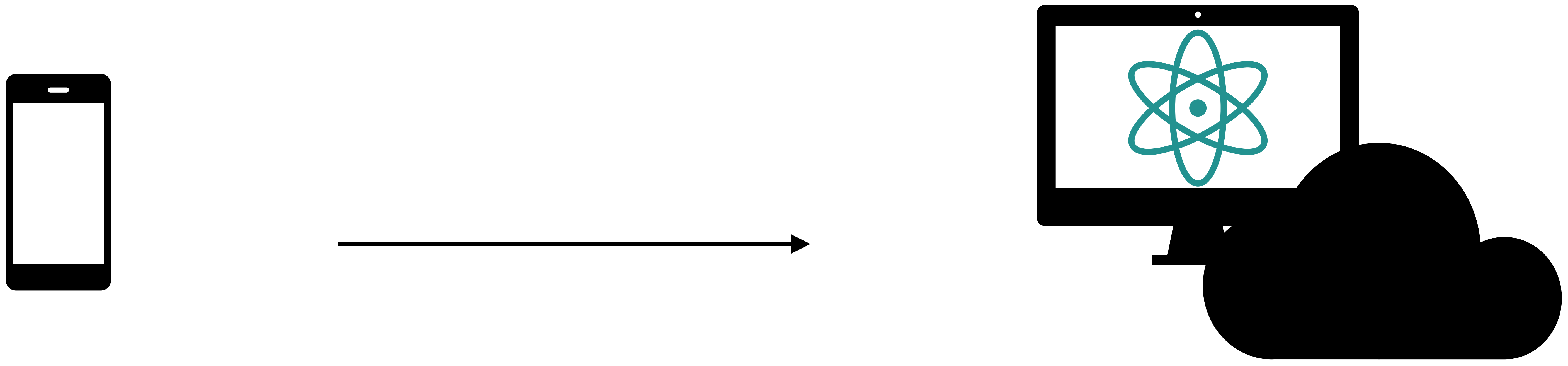

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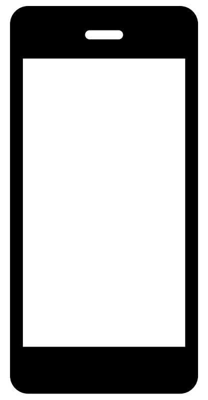

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Instructions to prepare
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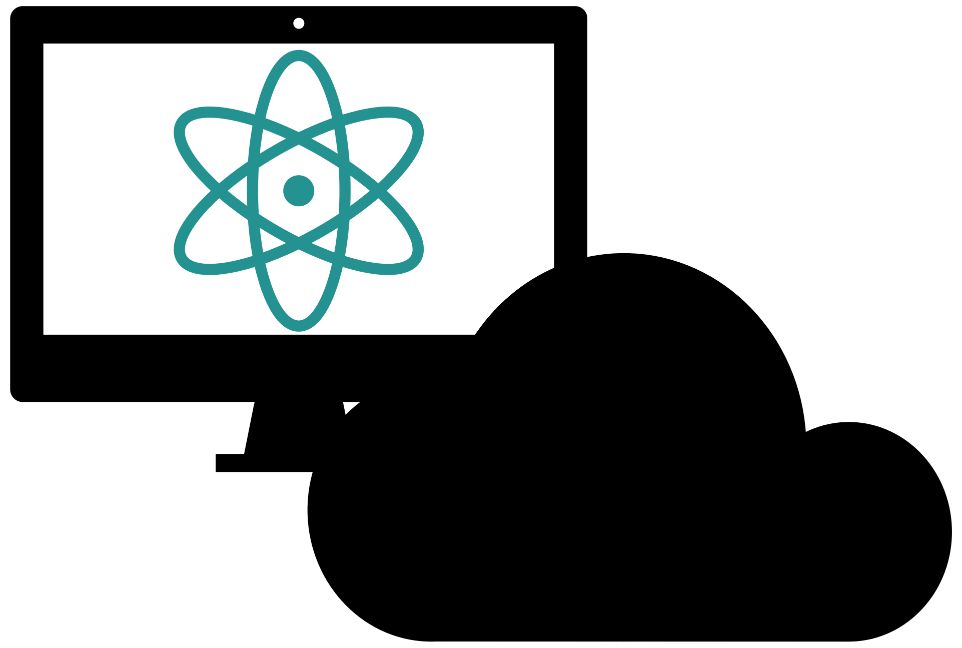

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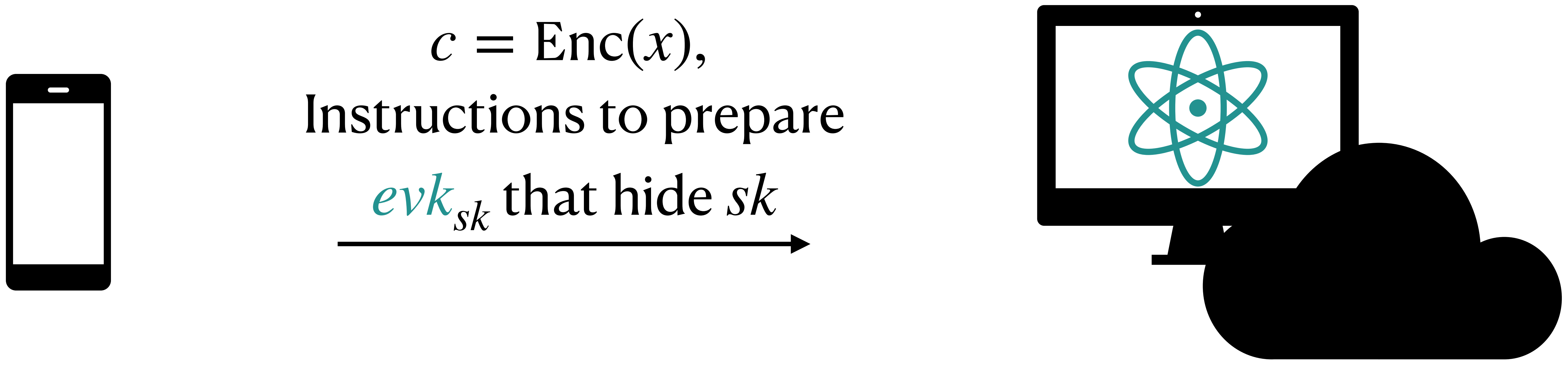

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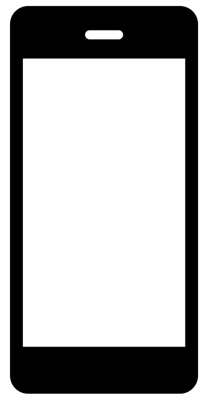

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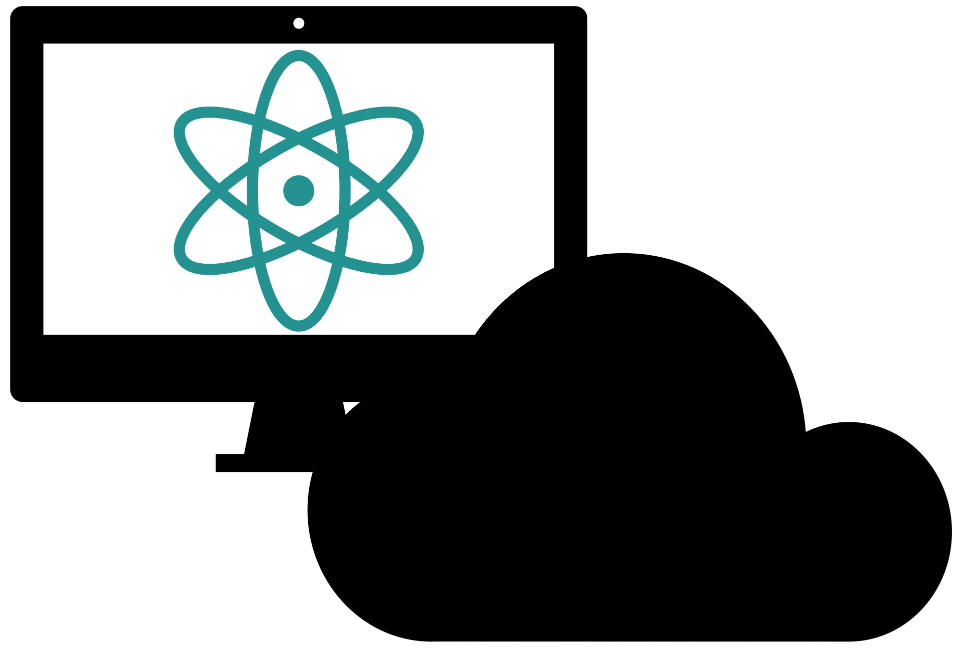

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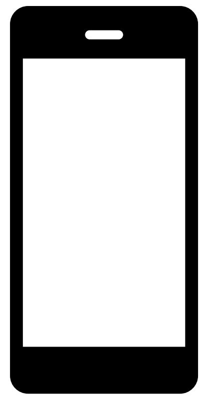

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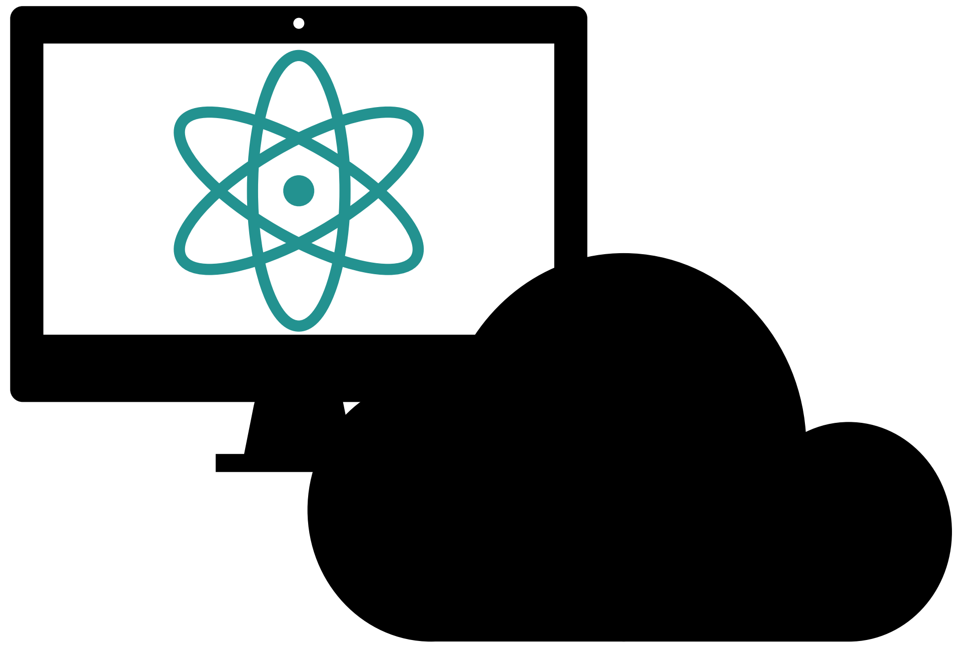

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Client is classical!

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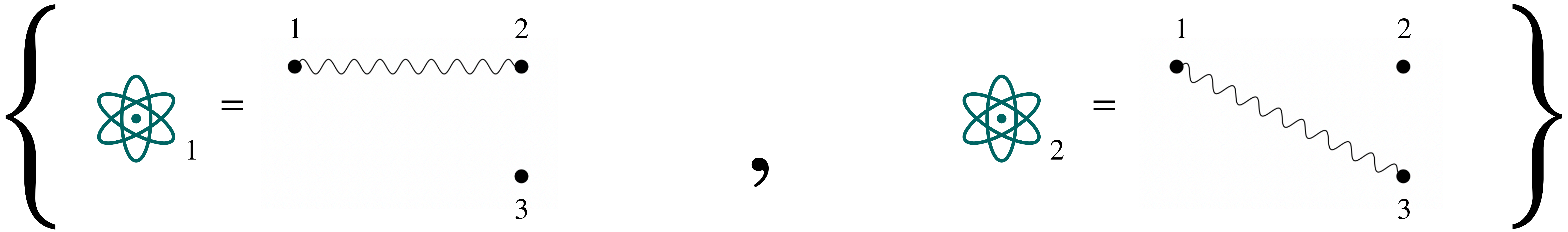
Step (1) It (essentially) suffices to RSP one of these two states:



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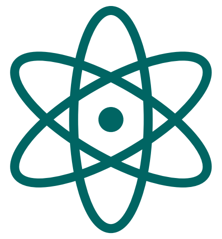
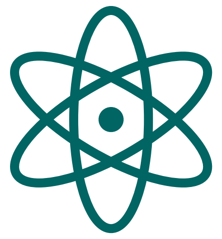
Step (2) Dual-mode trapdoor functions $\Longrightarrow$ RSP of  $_1$ or  $_2$.

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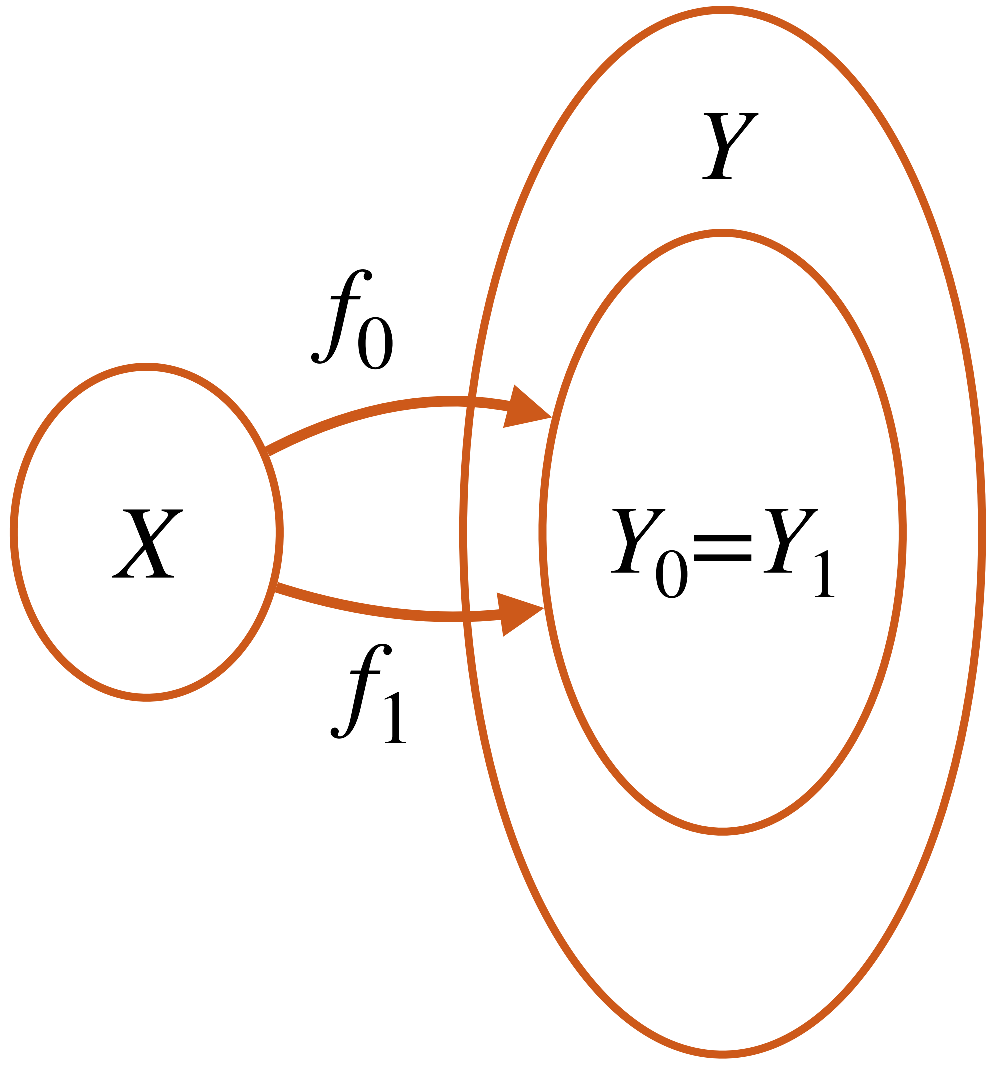

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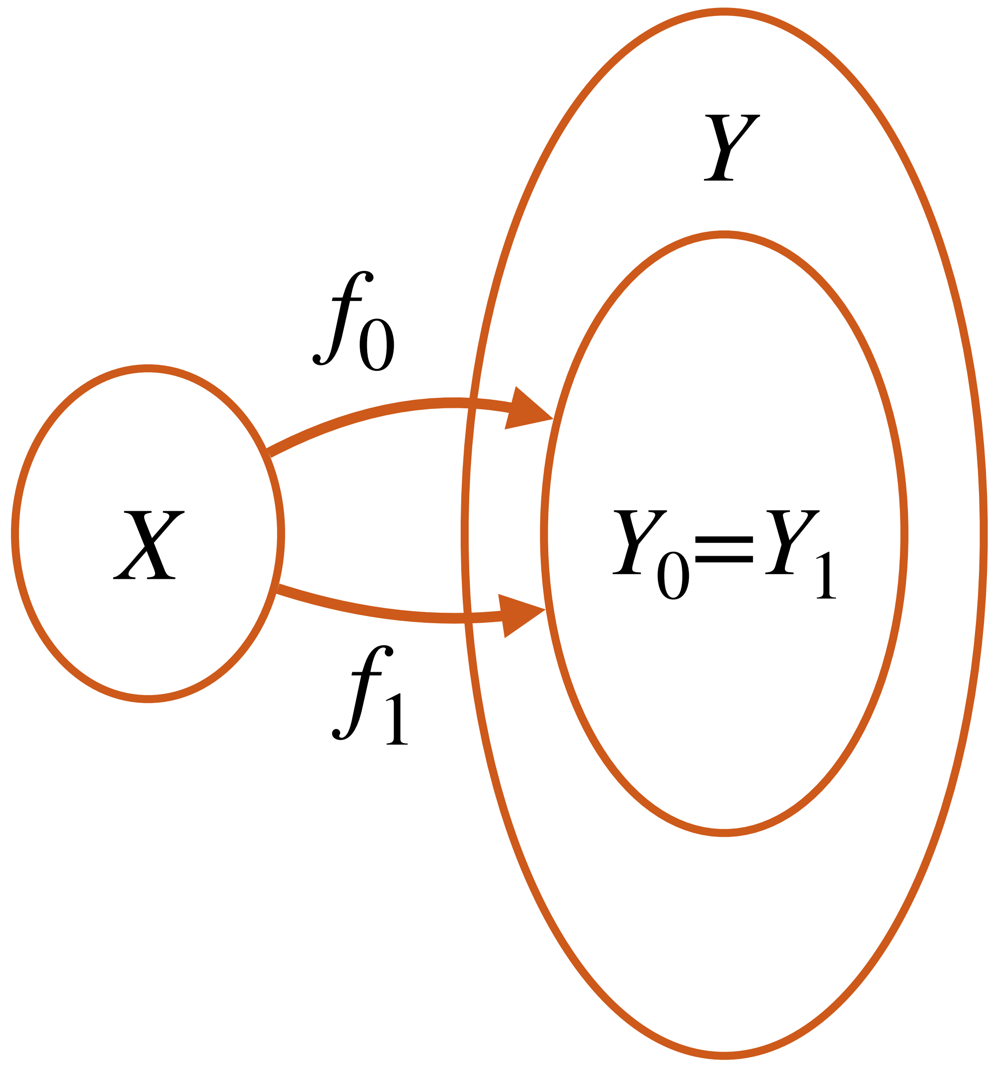

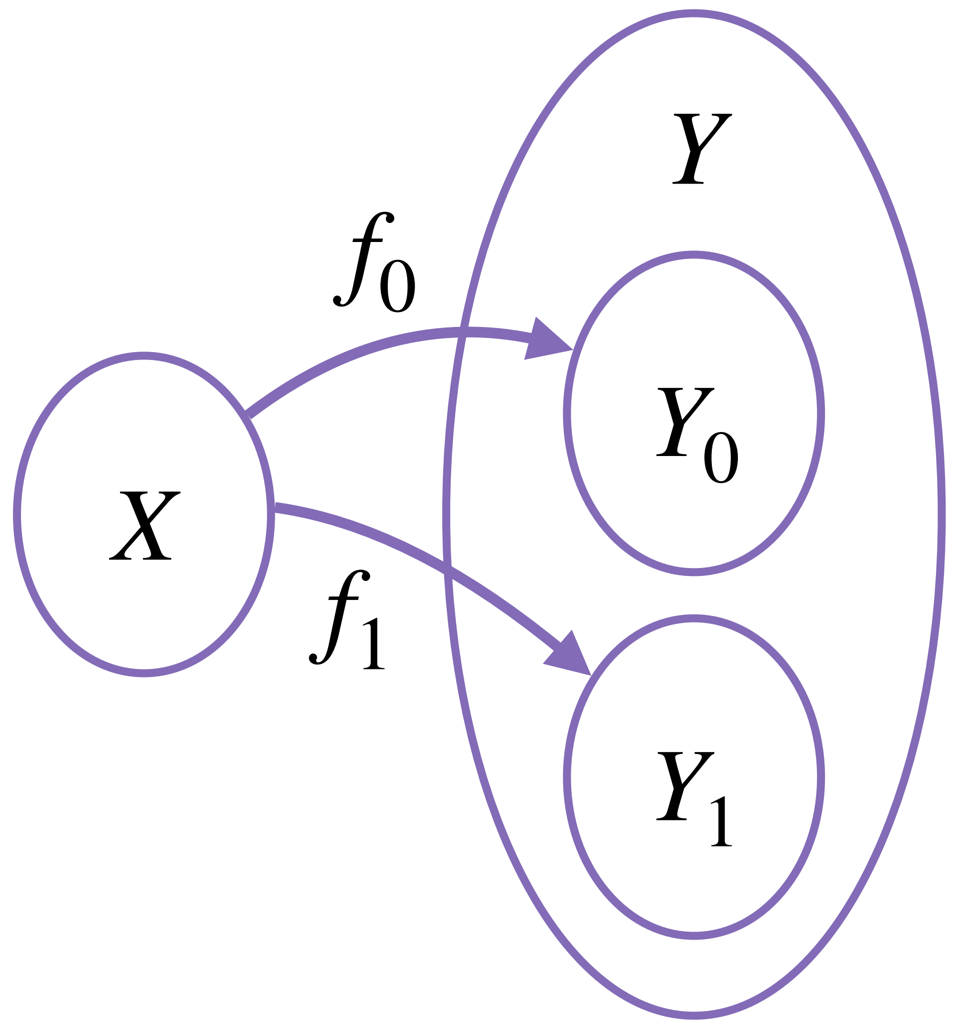

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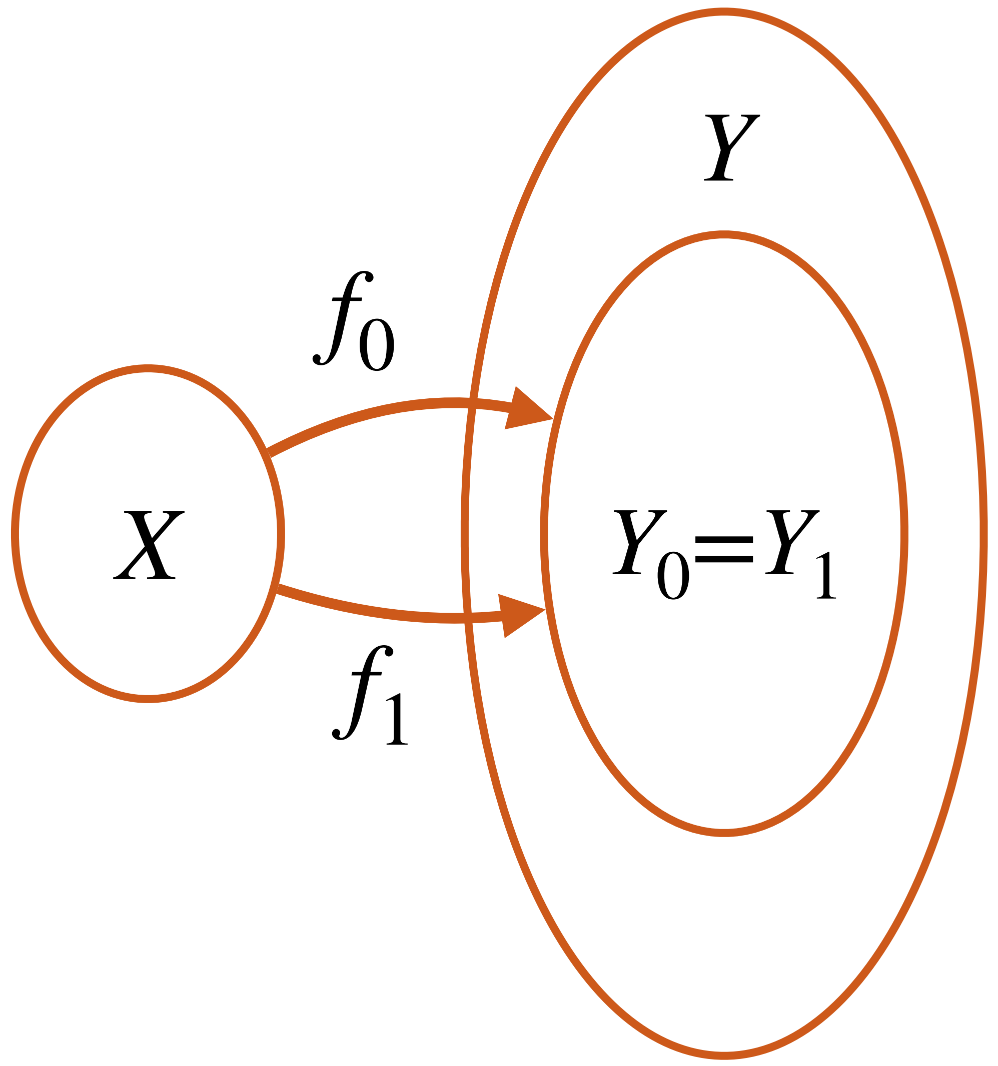
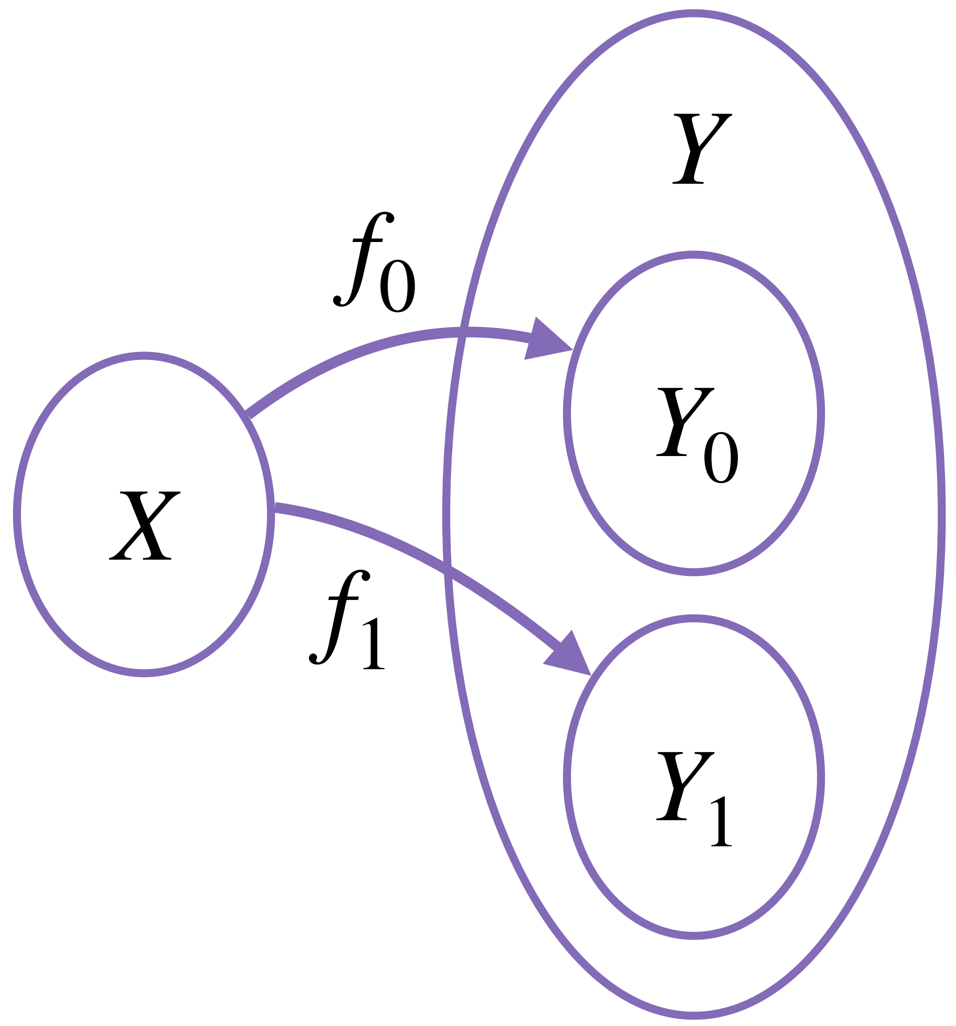


$\approx_c$

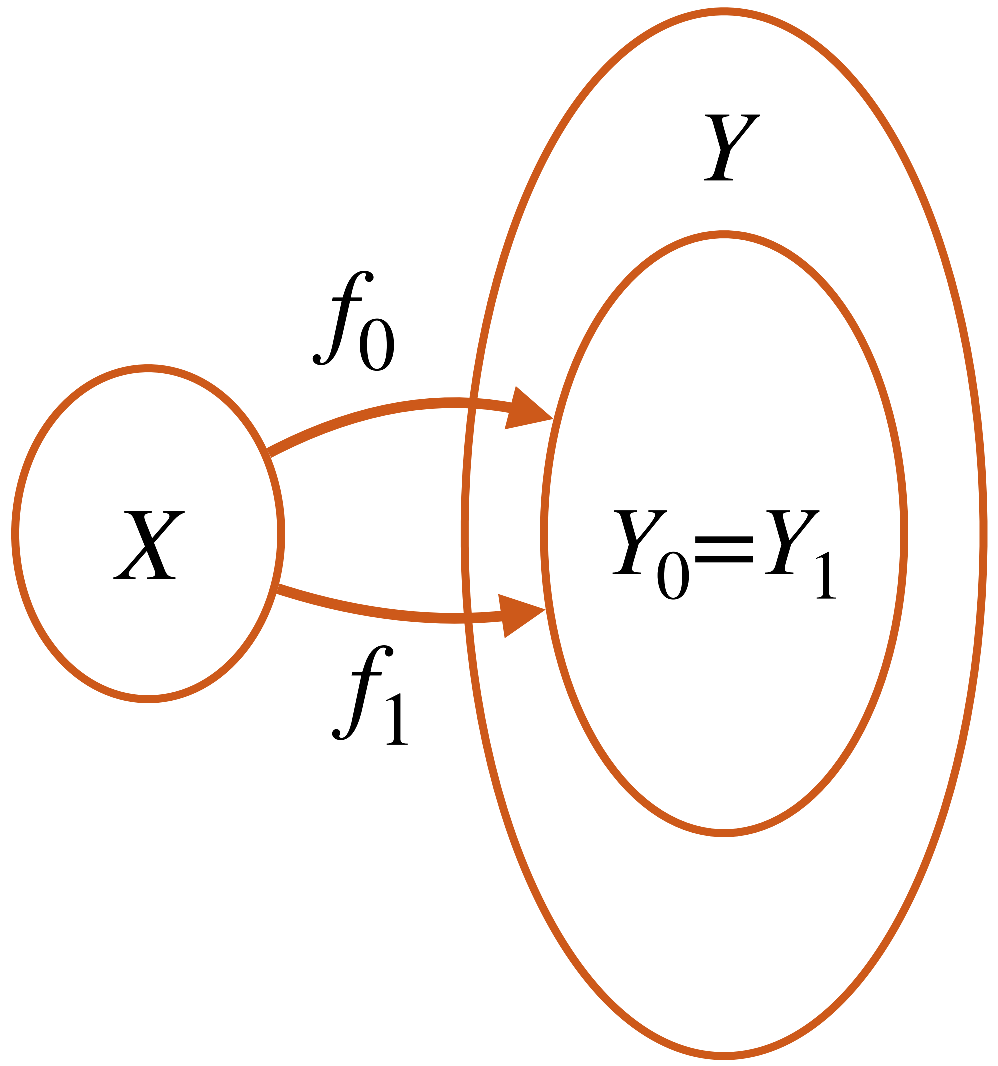


- **Dual-mode:** Given keys for f_0, f_1, it is hard to tell whether they are in mode (1) or (2).

Dual-mode trapdoor functions

Definition. Family of pairs of injective functions $f_0, f_1 : X \to Y$ such that either

(1) $\mathrm{Im}(f_0) = \mathrm{Im}(f_1)$ or (2) $\mathrm{Im}(f_0) \cap \mathrm{Im}(f_1) = \varnothing$.



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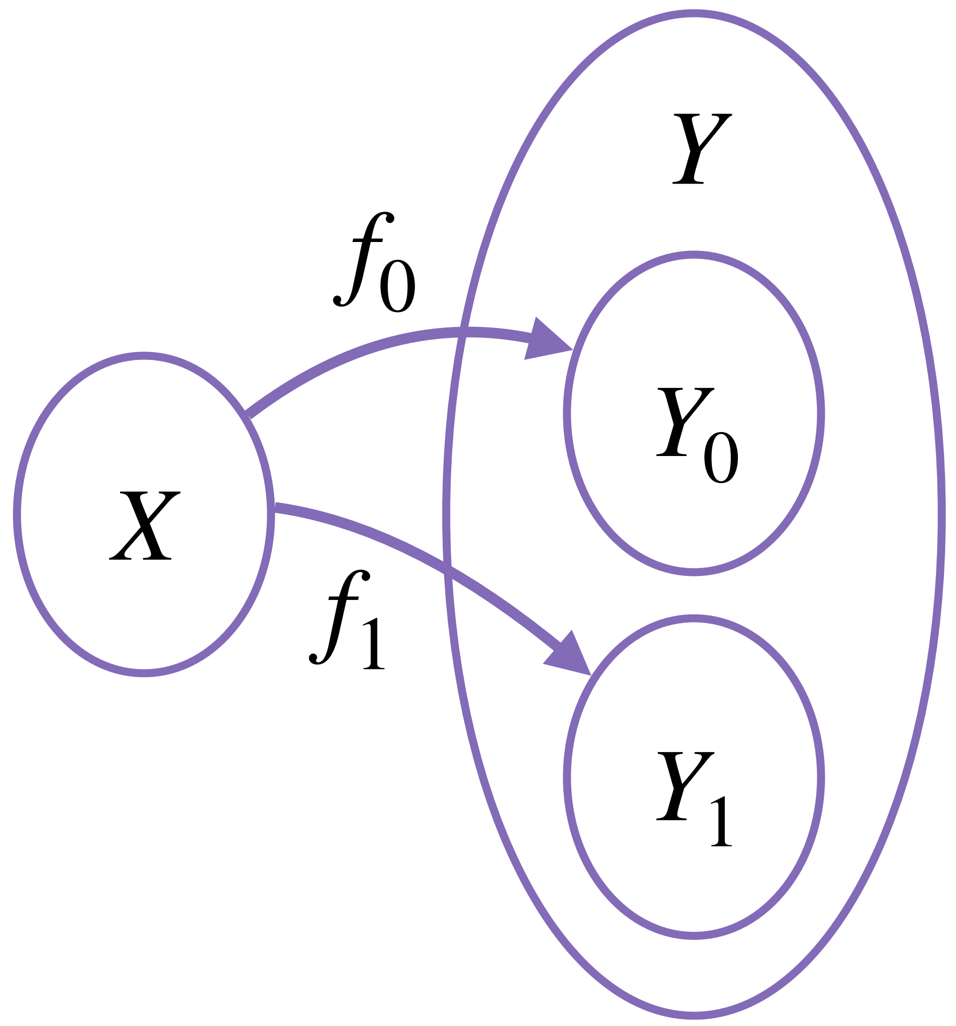

- **Dual-mode:** Given keys for f_0, f_1, it is hard to tell whether they are in mode (1) or (2).
- **Trapdoor:** allows efficient inversion in both modes.

Warmup: Remote State Preparation of BB84 states

[GV19, GMP22]

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BB84 states: $\left\{ |0\rangle, \quad |1\rangle, \quad \frac{1}{\sqrt{2}}\big(|0\rangle + |1\rangle\big), \quad \frac{1}{\sqrt{2}}\big(|0\rangle - |1\rangle\big) \right\}$

$_1 \qquad _2 \qquad _3 \qquad _4$

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1. Prepare state $\sum_{b\in\{0,1\}} \sum_{x\in X} |b\rangle |x\rangle$

Warmup: Remote State Preparation of BB84 states

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$_1 \qquad _2 \qquad _3 \qquad _4$

1. Prepare state $\sum_{b\in\{0,1\}} \sum_{x\in X} |b\rangle\,|x\rangle\,|f_b(x)\rangle$

2. Measure register #3 to get $y \in Y$

- In mode (1) $\mathrm{Im}(f_0) = \mathrm{Im}(f_1)$

$$|0\rangle\,|x_0\rangle + |1\rangle\,|x_1\rangle$$

- In mode (2) $\mathrm{Im}(f_0) \cap \mathrm{Im}(f_1) = \varnothing$

$$|b\rangle\,|x_b\rangle$$

Warmup: Remote State Preparation of BB84 states

[GV19, GMP22]

BB84 states: $\left\{ \underset{{}_1}{|0\rangle}, \quad \underset{{}_2}{|1\rangle}, \quad \frac{1}{\sqrt{2}}\underset{{}_3}{(|0\rangle + |1\rangle)}, \quad \frac{1}{\sqrt{2}}\underset{{}_4}{(|0\rangle - |1\rangle)} \right\}$

1. Prepare state $\sum_{b\in\{0,1\}} \sum_{x\in X} |b\rangle |x\rangle |f_b(x)\rangle$

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     $|b\rangle|x_b\rangle$

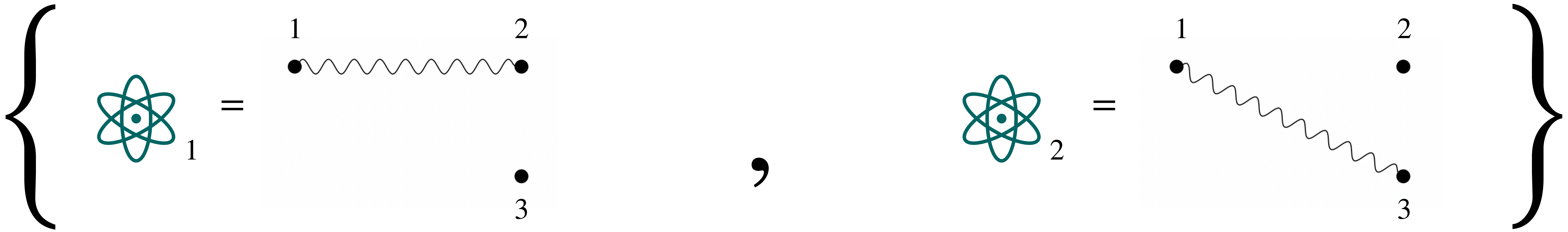
3. Measure register #2 in Hadamard basis to get rid of x_b

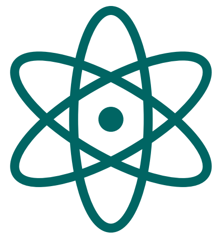
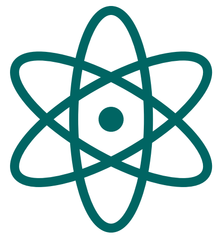
   - In mode (1) $\mathrm{Im}(f_0) = \mathrm{Im}(f_1)$ $\quad {}_3 \quad {}_4$
   - In mode (2) $\mathrm{Im}(f_0) \cap \mathrm{Im}(f_1) = \varnothing$ $\quad {}_1 \quad {}_2$

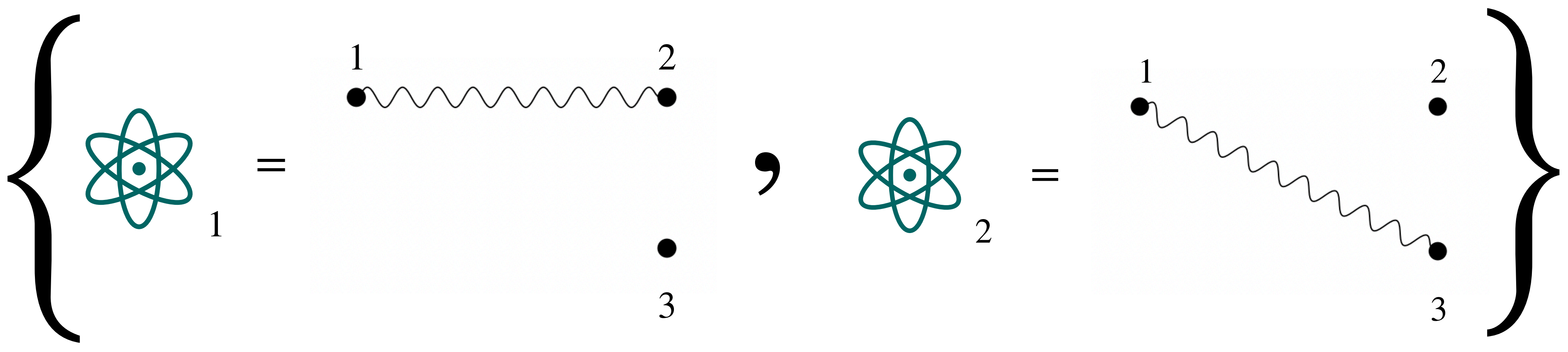
Our Main Technical Contribution

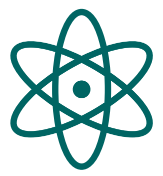
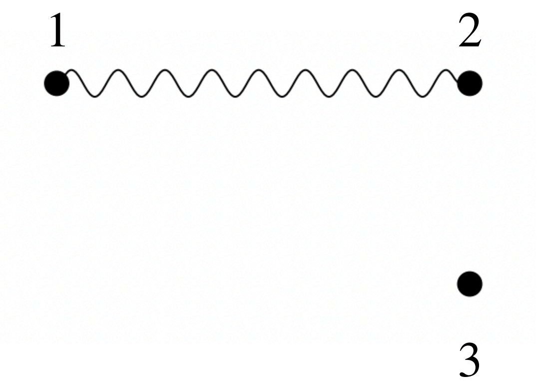
RSP the DSS evaluation keys

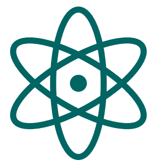
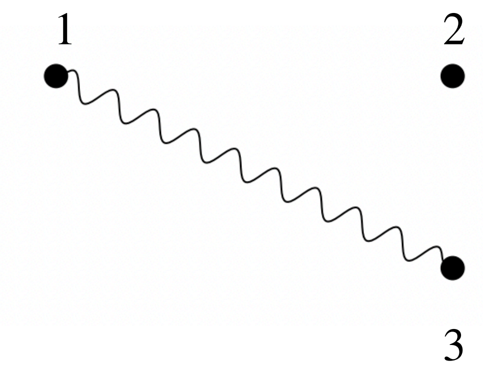
Step (1) It (essentially) suffices to RSP one of these two states:



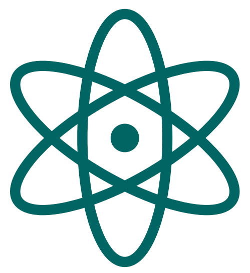
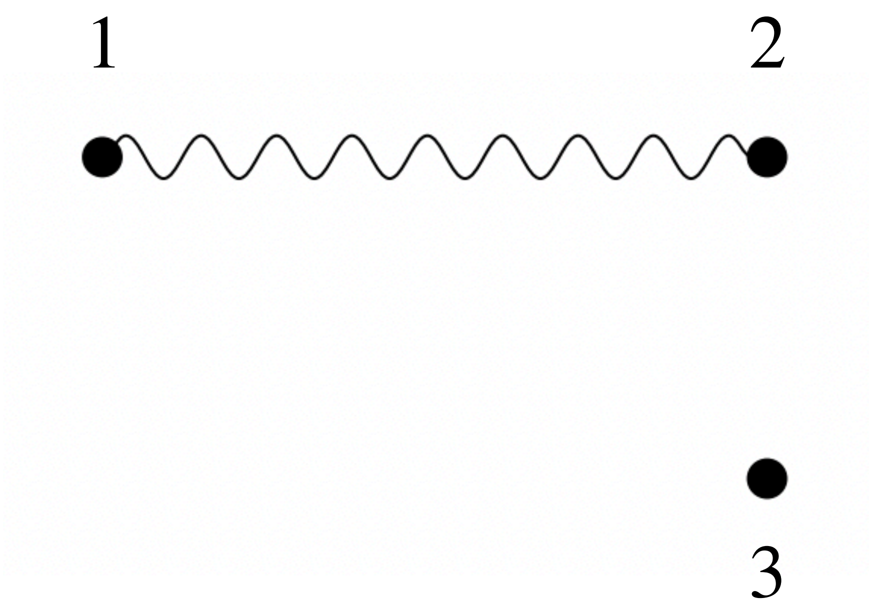
Step (2) Dual-mode trapdoor functions $\Longrightarrow$ RSP of  $_1$ or  $_2$.

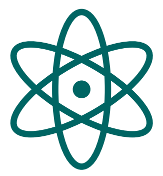
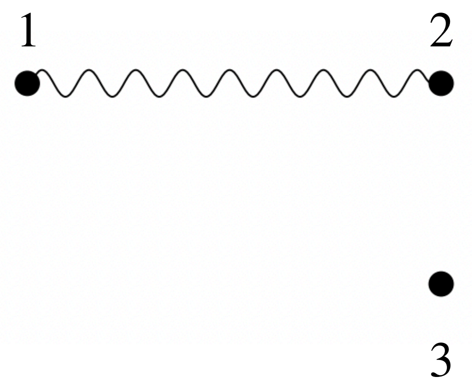
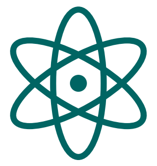
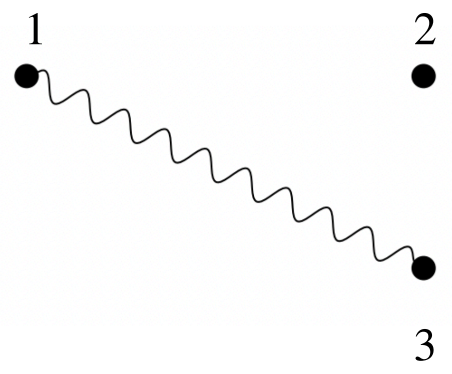
Lemma [GV24]. Dual-mode trapdoor functions $\Longrightarrow$ RSP for 

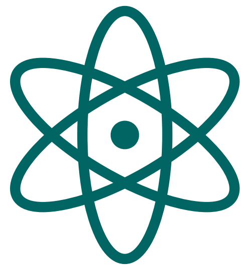
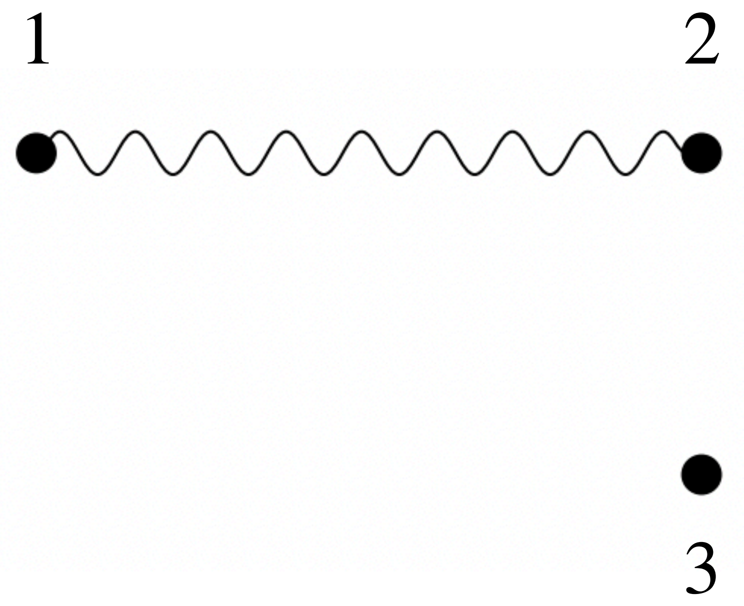
Lemma [GV24]. Dual-mode trapdoor functions $\Longrightarrow$ RSP for $\Big\{$  $_1$ = 

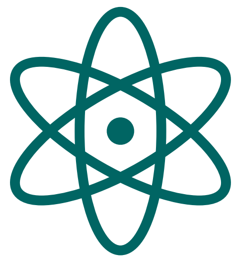
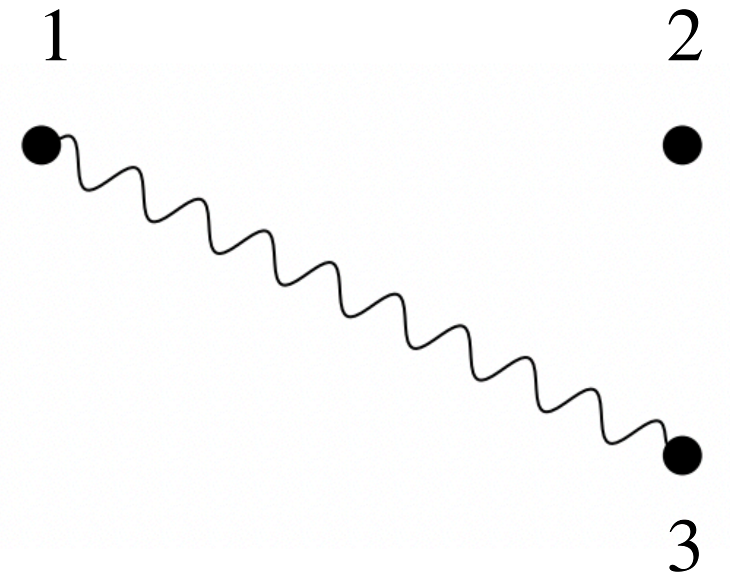
,  $_2$ = 

$\Big\}$

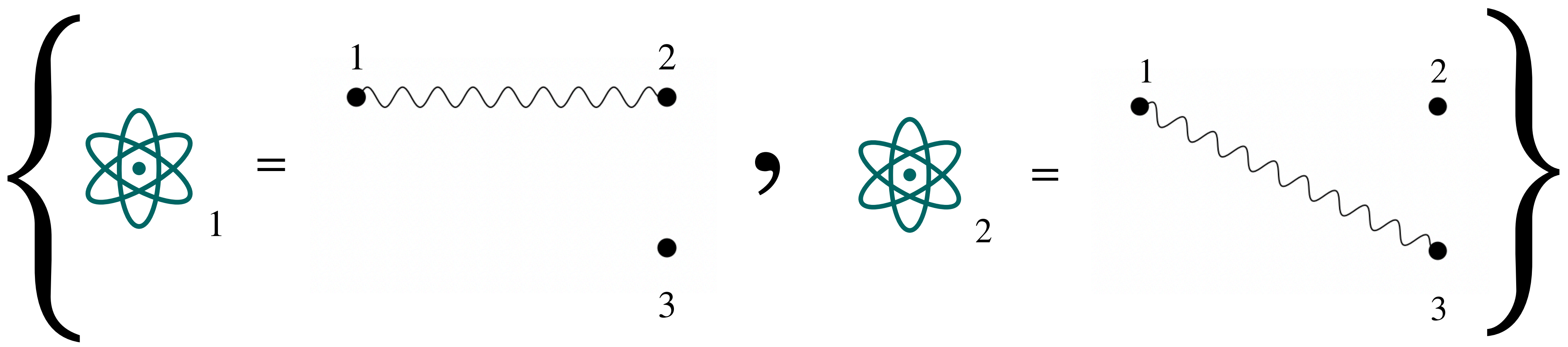
 $_1 = \frac{1}{2}\Big(|000\rangle + |001\rangle + |110\rangle + |111\rangle \Big) = \sum_{u=v, w\in\{0,1\}} |uvw\rangle$ = 

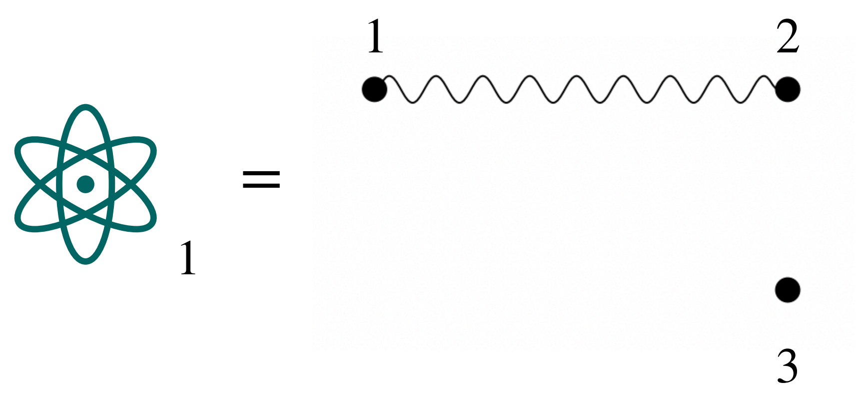
Lemma [GV24]. Dual-mode trapdoor functions $\Longrightarrow$ RSP for $\left\{ \right.$  $_1 =$  ,  $_2 =$  $\left. \right\}$

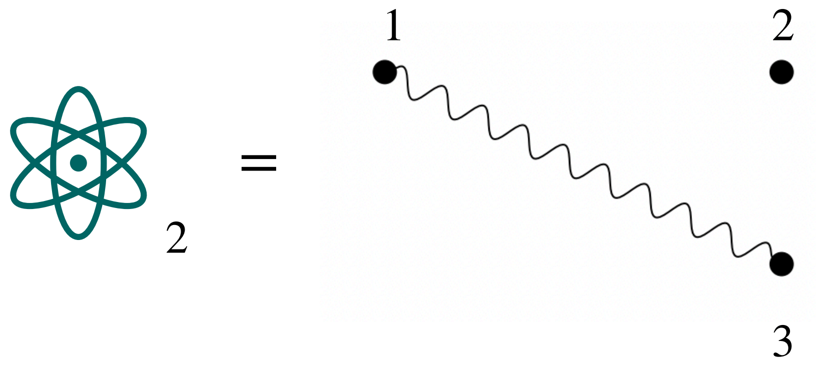
 $_1$ $= \frac{1}{2}\Big(|000\rangle + |001\rangle + |110\rangle + |111\rangle \Big) = \sum_{u=v, w\in\{0,1\}} |uvw\rangle$ $=$ 

 $_2$ $= \frac{1}{2}\Big(|000\rangle + |010\rangle + |101\rangle + |111\rangle \Big) = \sum_{u=w, v\in\{0,1\}} |uvw\rangle$ $=$ 

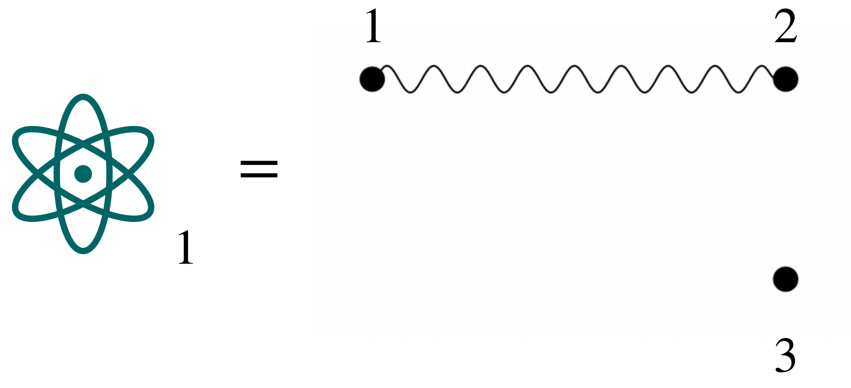
Lemma [GV24]. Dual-mode trapdoor functions $\Longrightarrow$ RSP for

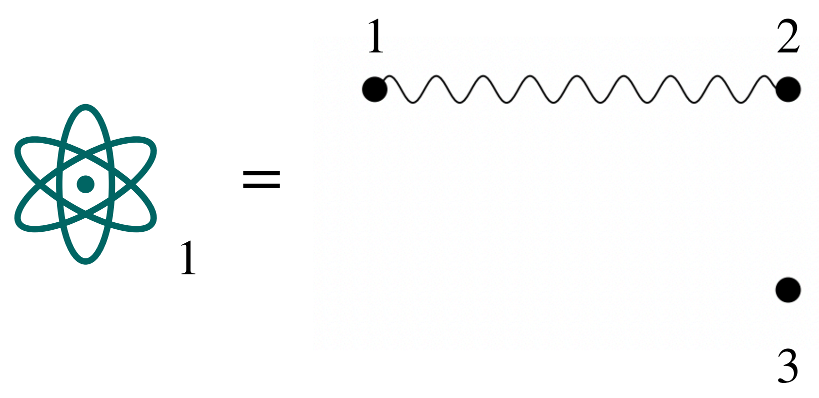


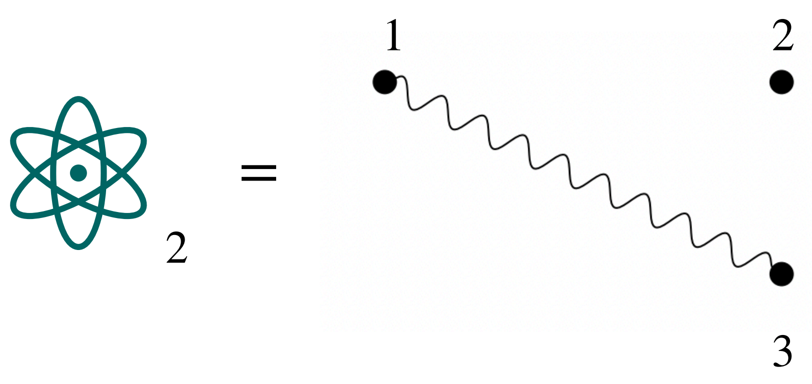
Lemma [GV24]. Dual-mode trapdoor functions $\Longrightarrow$ RSP for $\Big\{$ 

, 

$\Big\}$

If client wants to prepare 

Lemma [GV24]. Dual-mode trapdoor functions $\Longrightarrow$ RSP for $\Big\{$ 

, 

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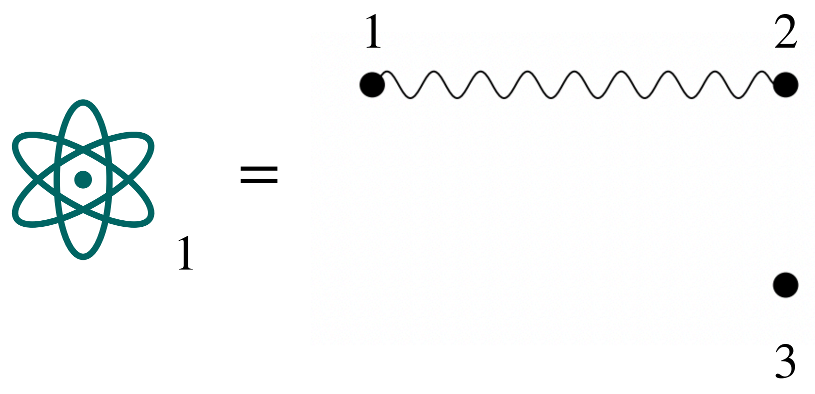
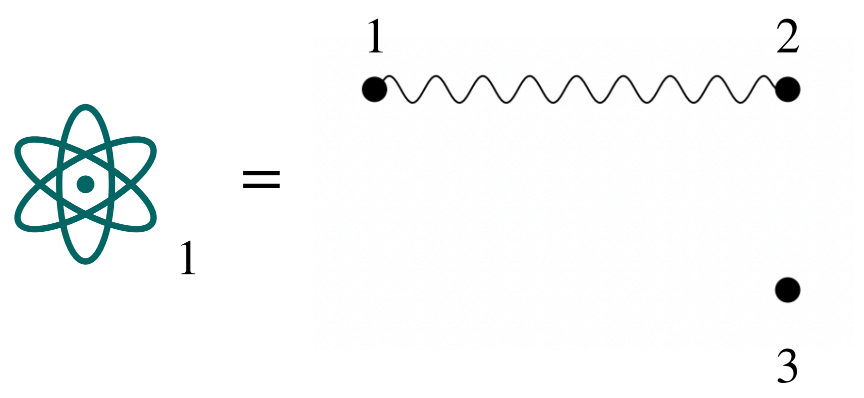
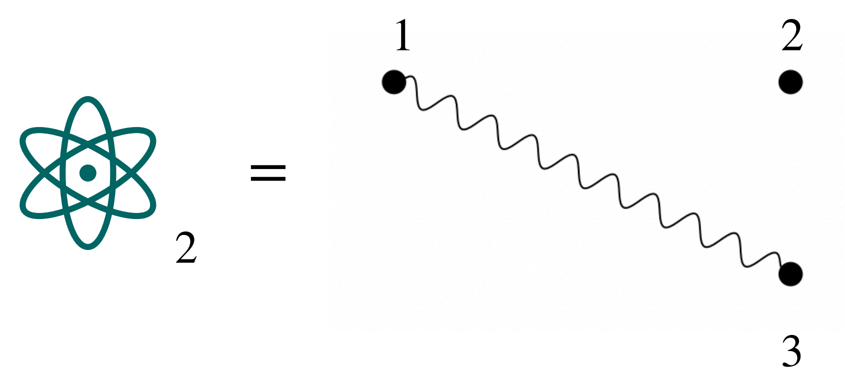
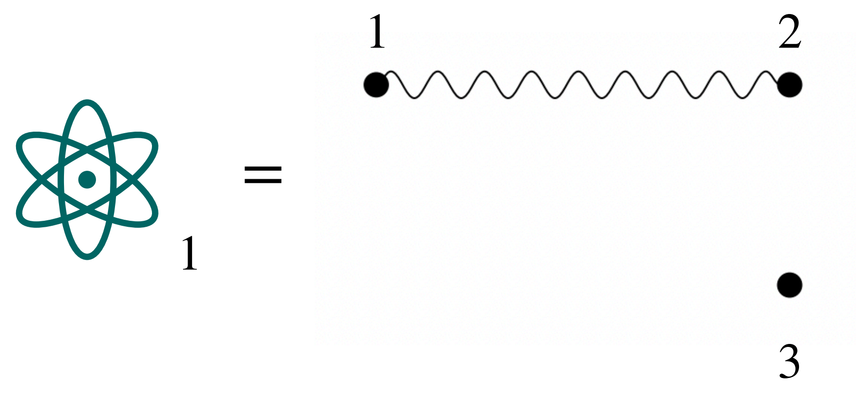
If client wants to prepare 

want to project to the blue basis vectors.

$$\frac{1}{2\sqrt{2}}\Big(|000\rangle + |001\rangle + |010\rangle + |011\rangle + |100\rangle + |101\rangle + |110\rangle + |111\rangle \Big)$$

Lemma [GV24]. Dual-mode trapdoor functions $\Longrightarrow$ RSP for $\Big\{$  ,  $\Big\}$

If client wants to prepare  want to project to the blue basis vectors.

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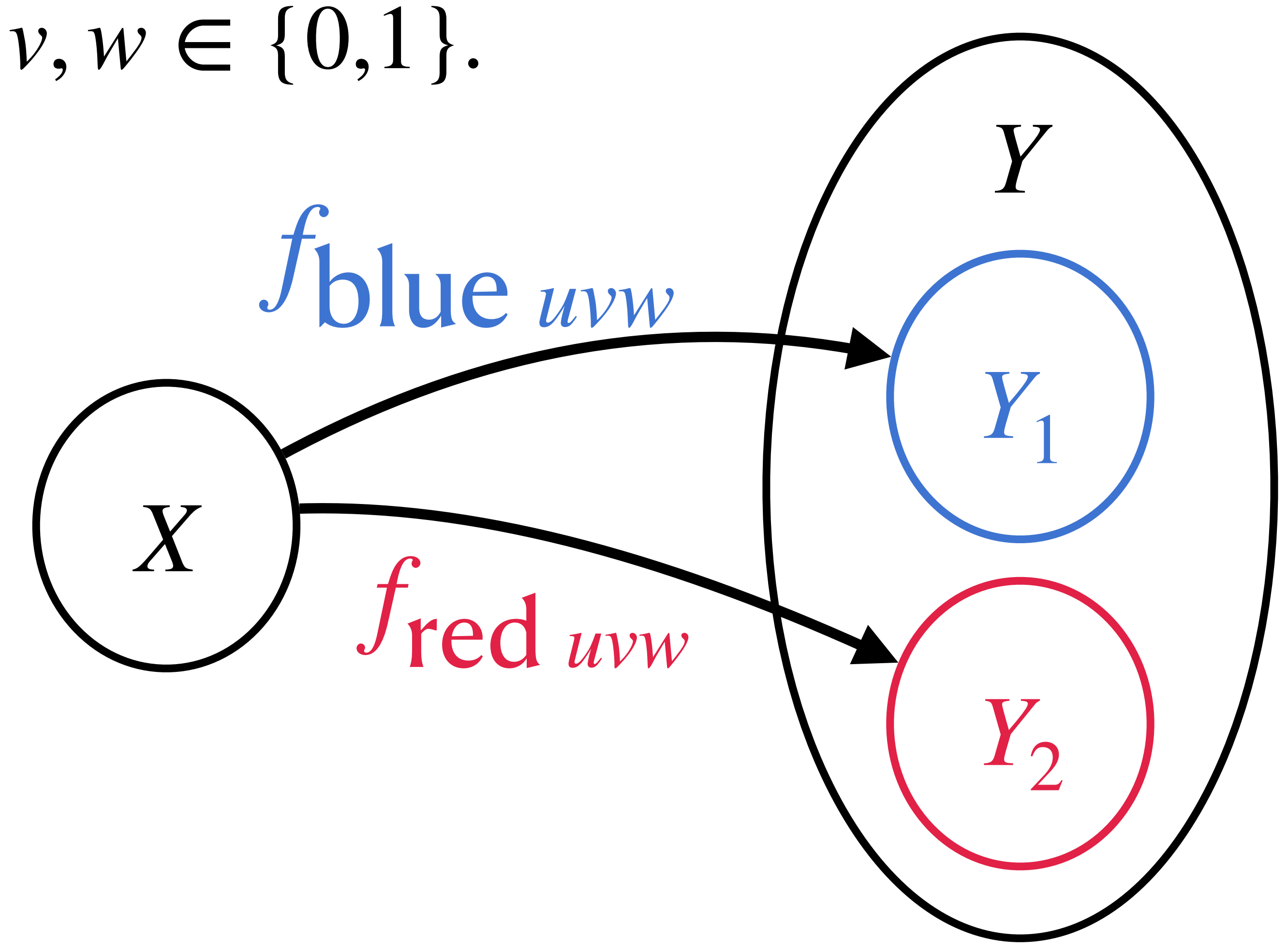
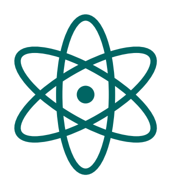
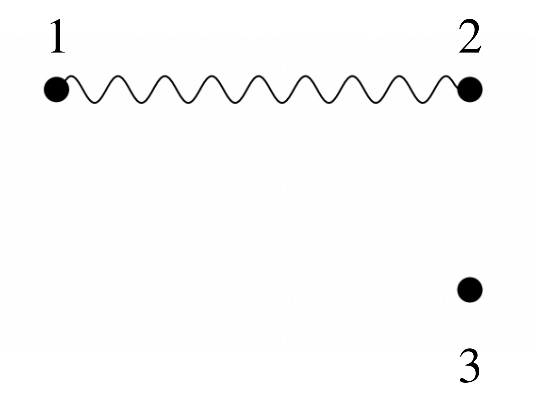
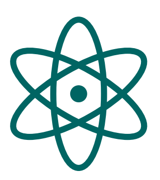
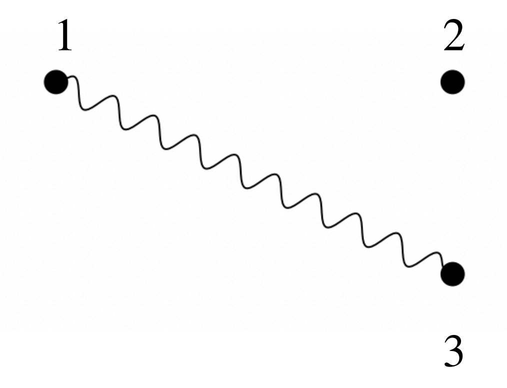
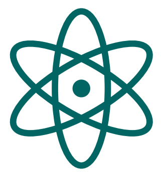
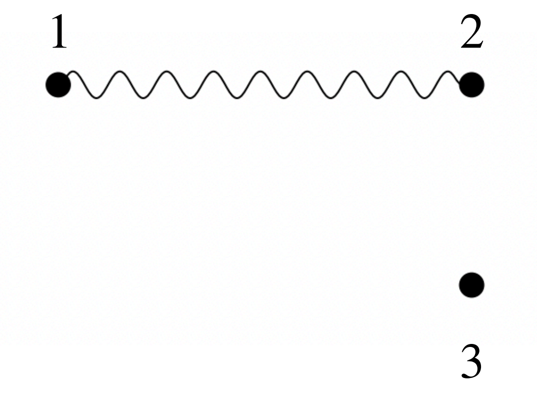


Lemma [GV24]. Dual-mode trapdoor functions $\Longrightarrow$ RSP for $\Big\{$  $_1$ = 

,  $_2$ = 

$\Big\}$

If client wants to prepare  $_1$ = 

want to project to the blue basis vectors.

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1. Prepare $\sum_{u,v,w\in\{0,1\}} \sum_{x\in X} |uvw\rangle|x\rangle|f_{uvw}(x)\rangle$ for 8 functions $f_{uvw} : X \to Y, u, v, w \in \{0,1\}$.

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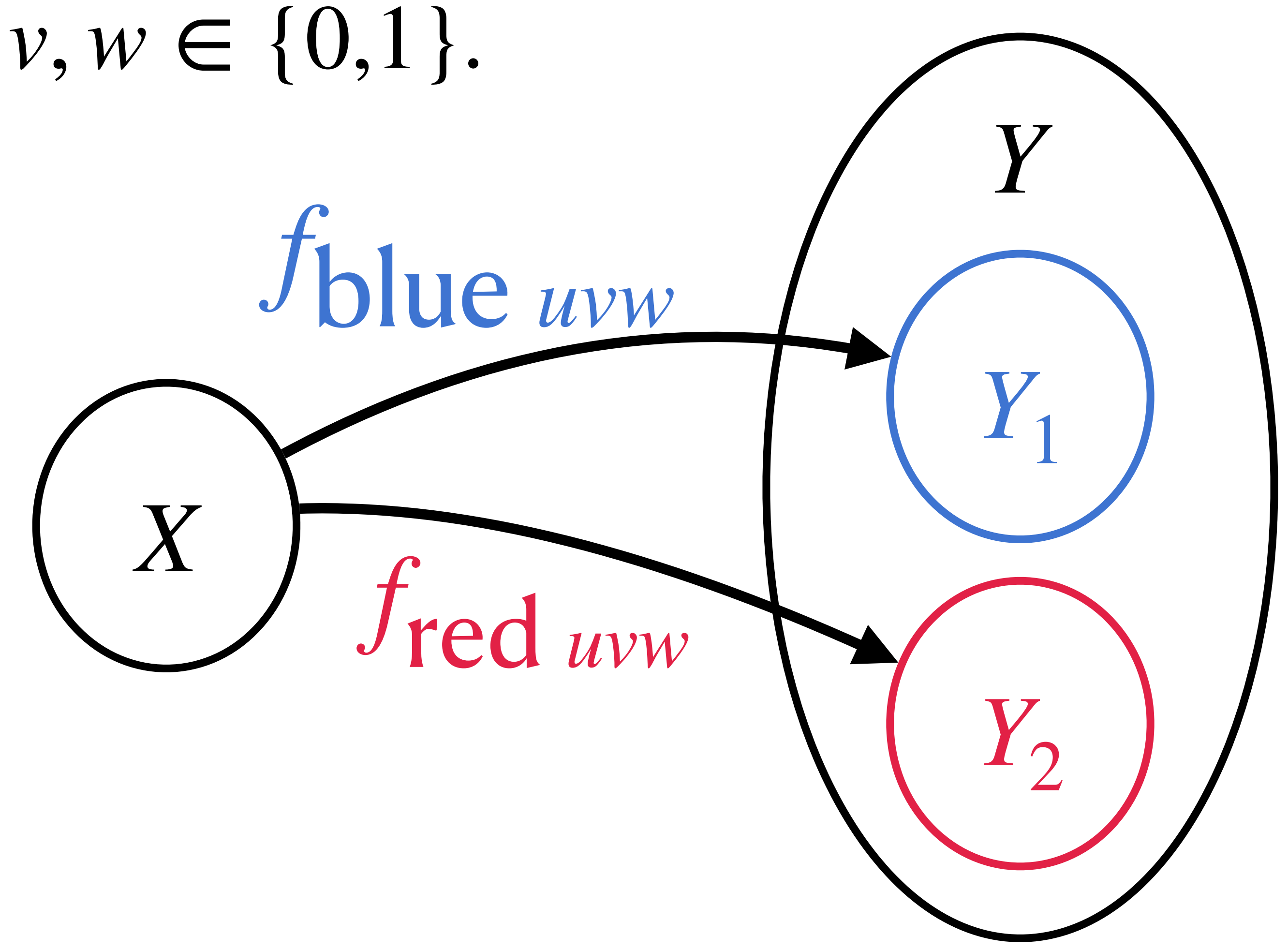
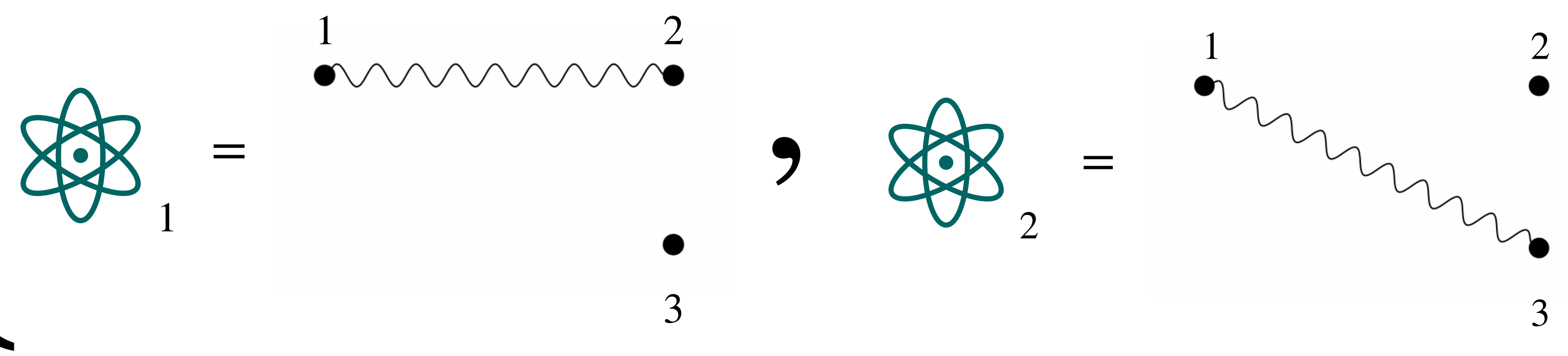
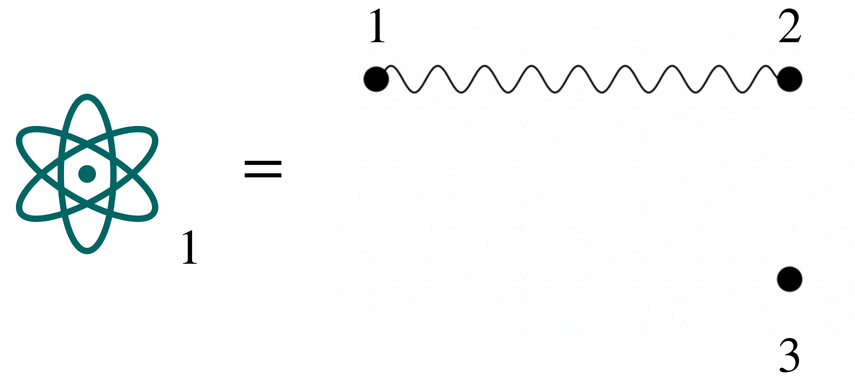


Lemma [GV24]. Dual-mode trapdoor functions $\Longrightarrow$ RSP for $\{$  $\}$

If client wants to prepare  want to project to the blue basis vectors.

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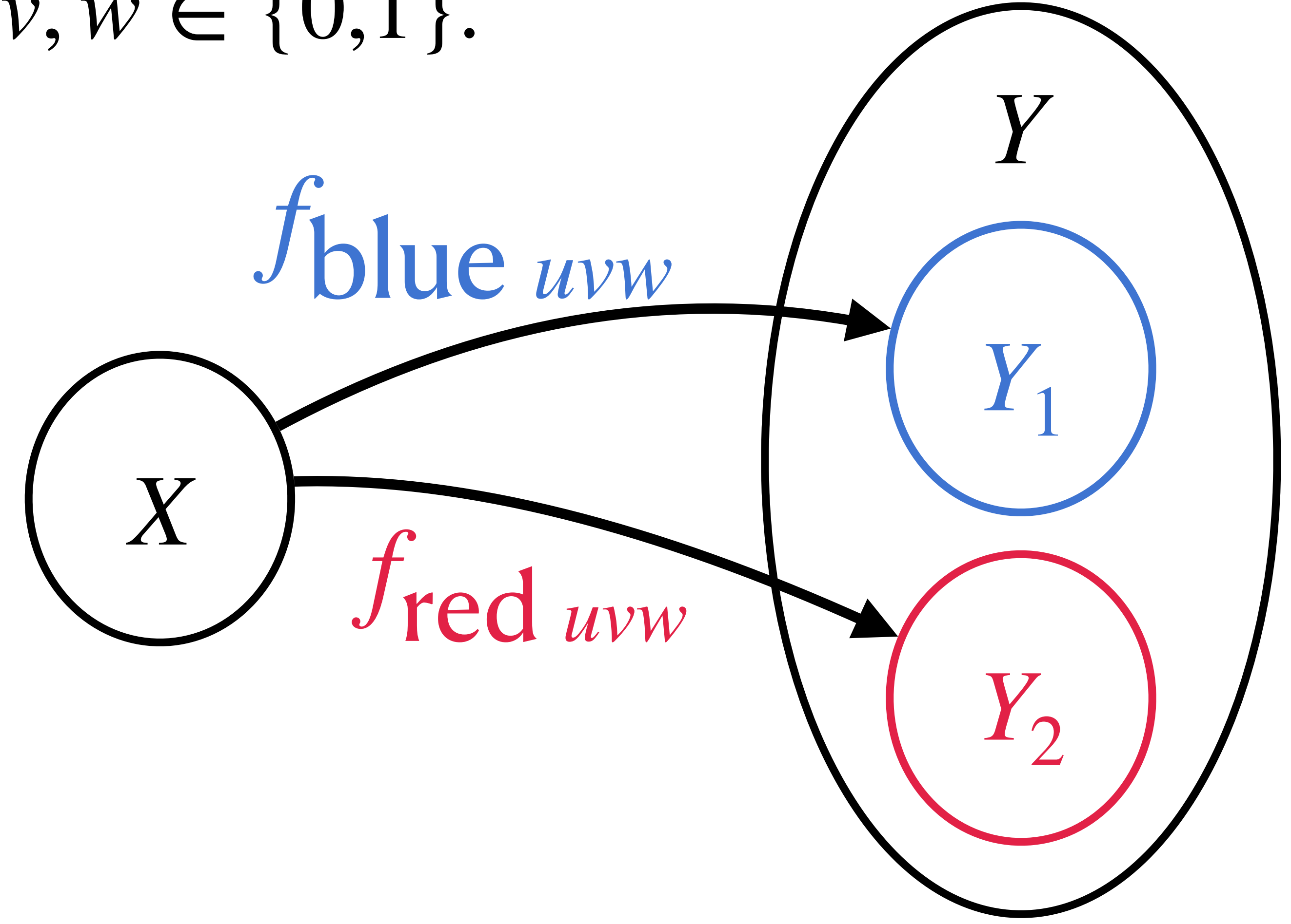
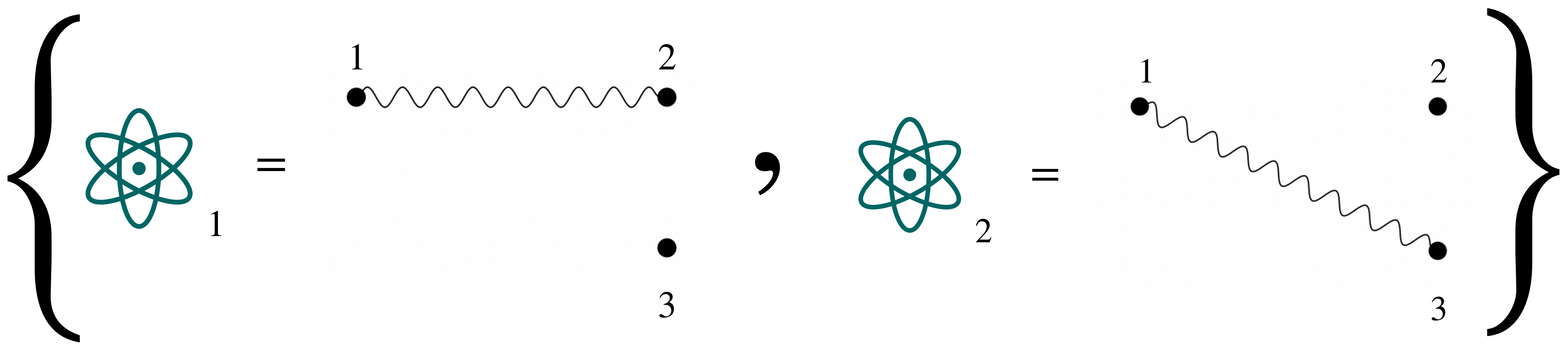
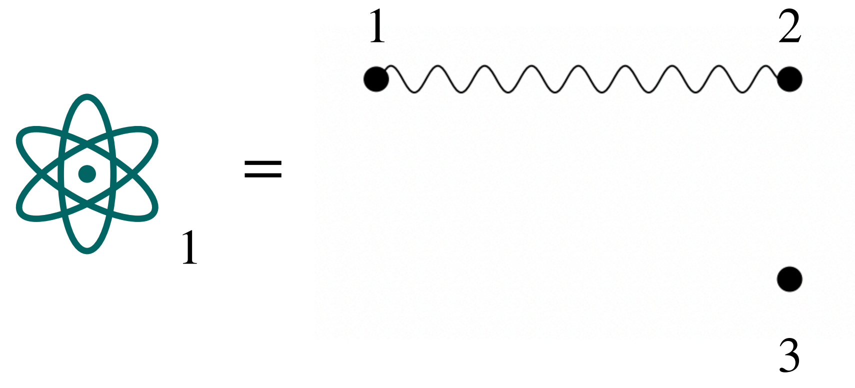


Lemma [GV24]. Dual-mode trapdoor functions $\Longrightarrow$ RSP for 

If client wants to prepare 

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Dual-mode trapdoor functions

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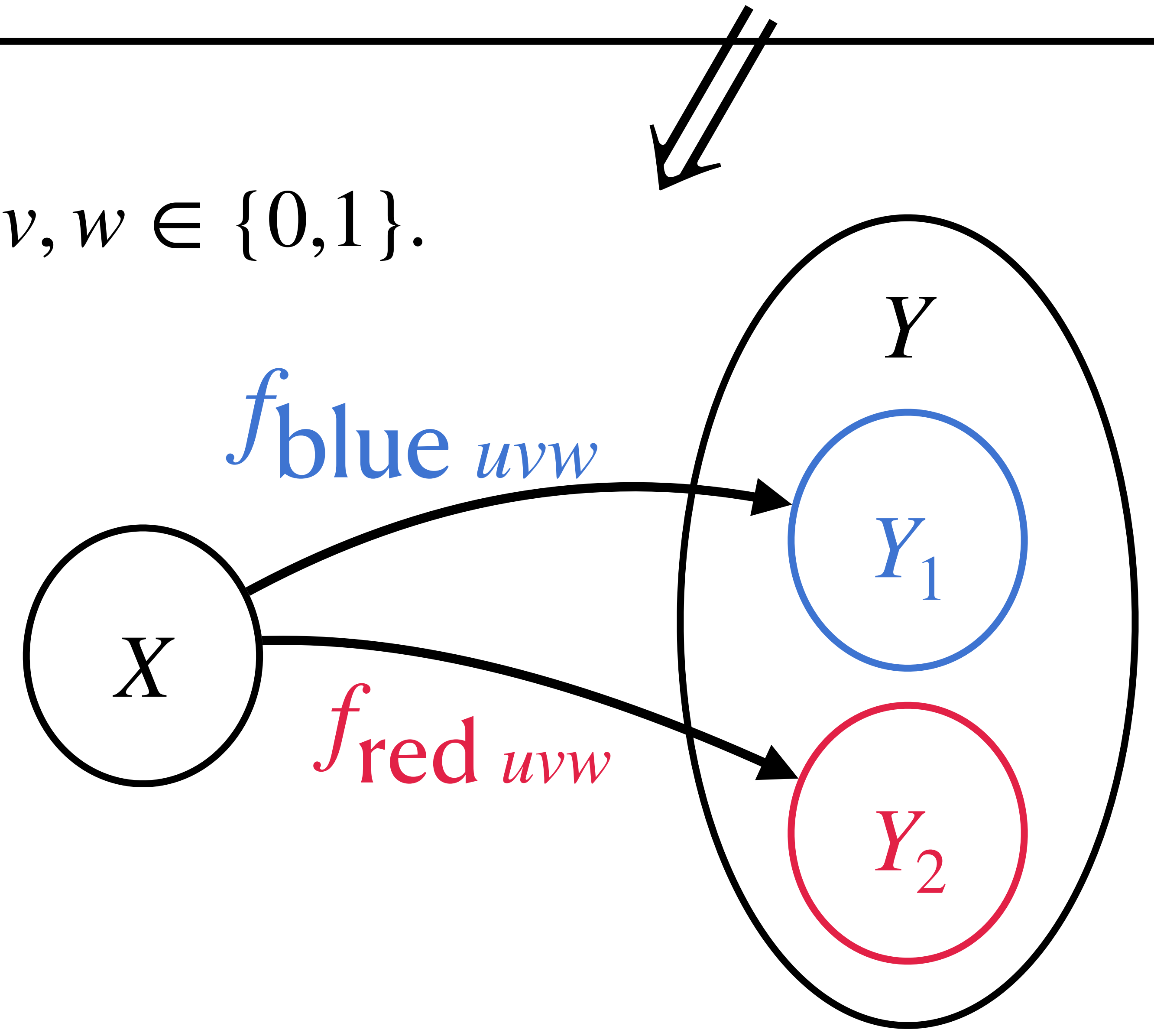
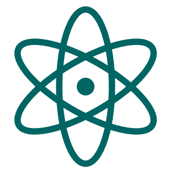
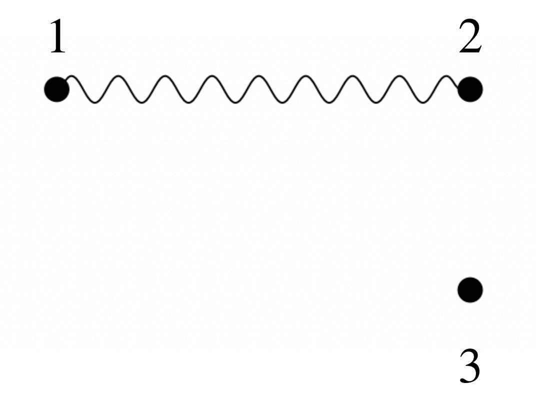
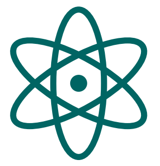
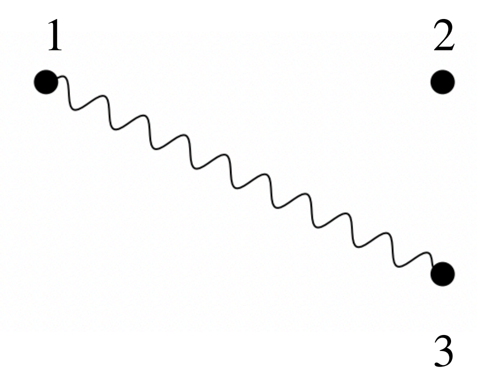
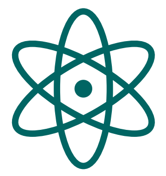
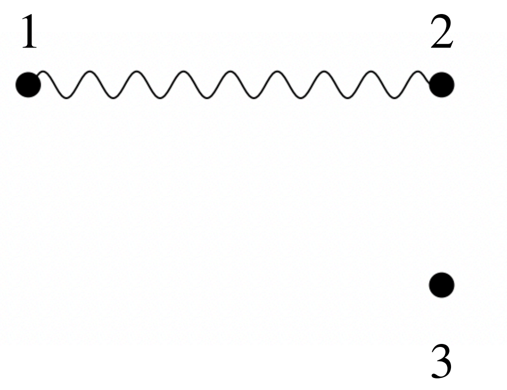


Lemma [GV24]. Dual-mode trapdoor functions $\Longrightarrow$ RSP for $\Big\{$  $_1$ =  ,  $_2$ =  $\Big\}$

If client wants to prepare  $_1$ =  want to project to the blue basis vectors.

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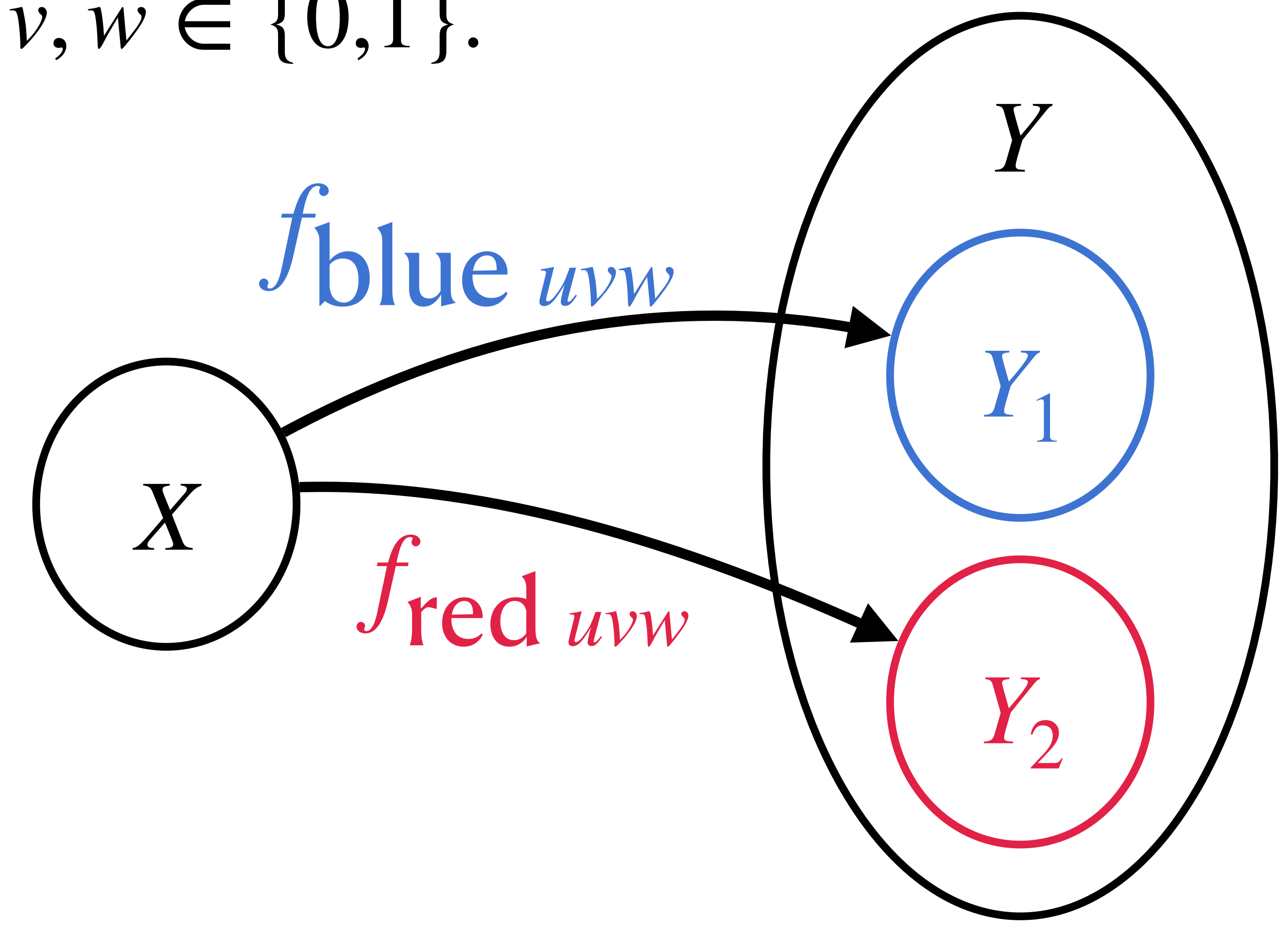
Dual-mode trapdoor functions

1. Prepare $\sum_{u,v,w\in\{0,1\}} \sum_{x\in X} |uvw\rangle|x\rangle|f_{uvw}(x)\rangle$ for 8 functions $f_{uvw}: X \to Y, u, v, w \in \{0,1\}$.
2. Measure 3rd register, get $y \in Y_1$, $\sum_{\text{blue } u,v,w} |uvw\rangle|x_{uvw}\rangle$.
3. Measure 2nd register in Hadamard basis get rid of x_{uvw}.



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Open Question #1

- Is the dream theorem true?

Dream Theorem. Any post-quantum classical FHE $\Longrightarrow$ QFHE.

Open Question #2

Non-compact QFHE from minimal assumptions

- We don’t need classical FHE
- *Our work*: Dual-mode trapdoor functions suffice
- Information-theoretic security unlikely [Morimae-Nishimura-Takeuchi-Tani’18, Aaronson-Cojocaru-Gheorghiu-Kashefi’19]
- Can you construct non-compact QFHE from one-way functions?