Fully-Succinct Multi-Key Homomorphic Signatures from Standard Assumptions

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19th August 2024

CRYPTO 2024

Computing on Authenticated Data

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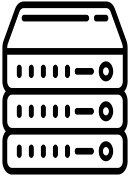

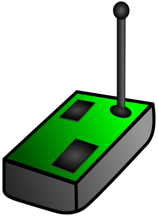

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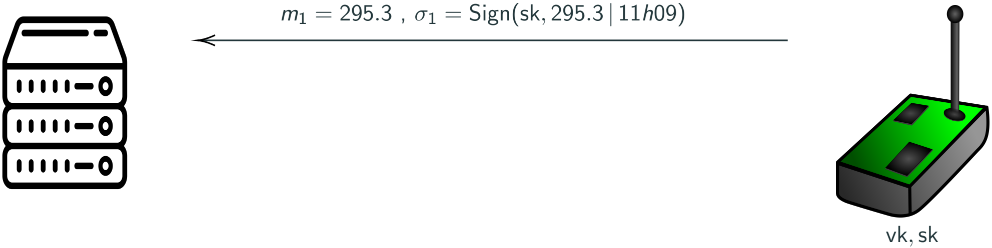

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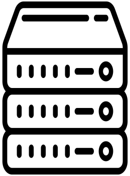

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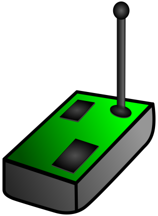

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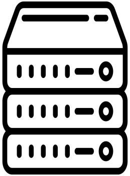

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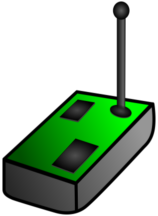

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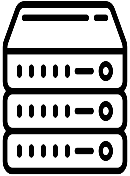

Compute MAX temp
from 0h00 to 23h59

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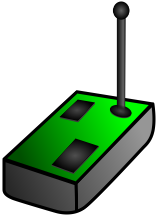

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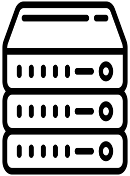

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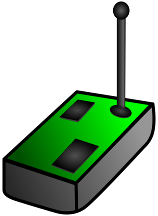

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Can we have a short, publicly verifiable proof that the MAX temperature is computed correctly on **today***'s* **authentic** *temperatures?*

Homomorphic Signatures [JMSW02]

vk, sk

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$\mathsf{Eval}(f, \boldsymbol{\ell}, \boldsymbol{m}, \mathsf{vk}, \boldsymbol{\sigma}) \longrightarrow \boldsymbol{y}, \sigma_{f,y}$

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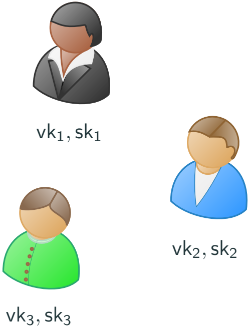
- $\sigma_{f,y}$ is *succinct*: does not grow with n or $|f|$.

Multi-Key Homomorphic Signatures [FMNP16]



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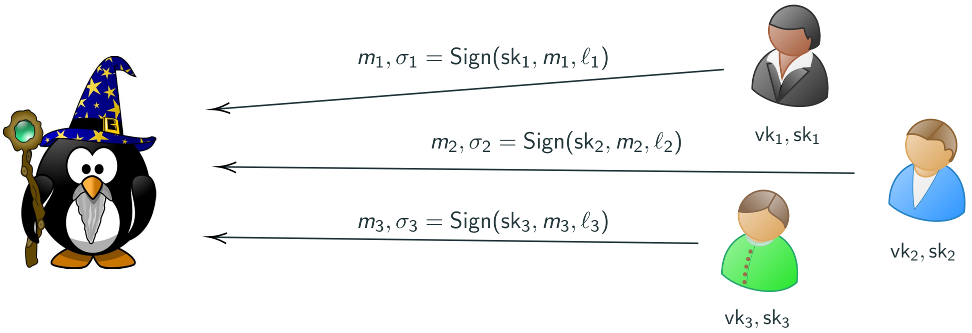

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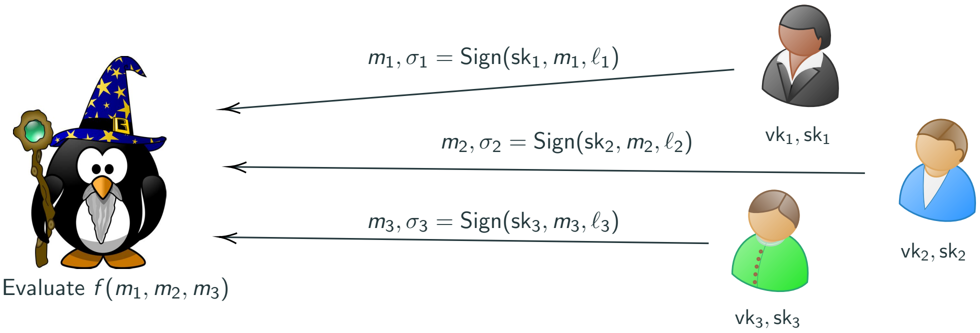

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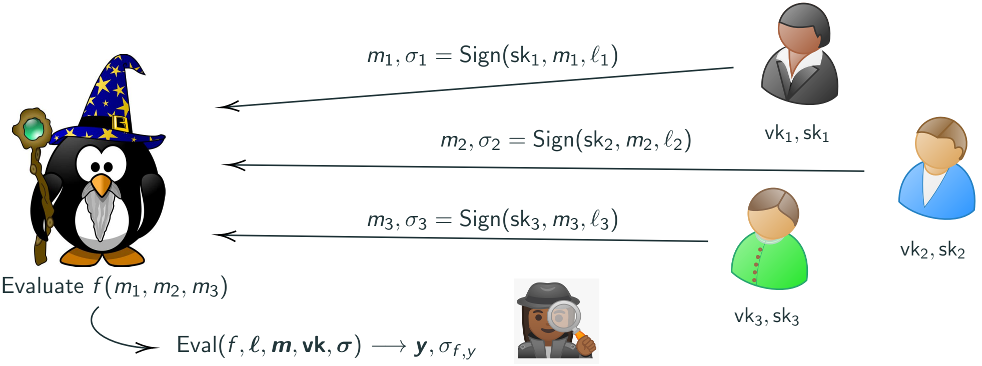

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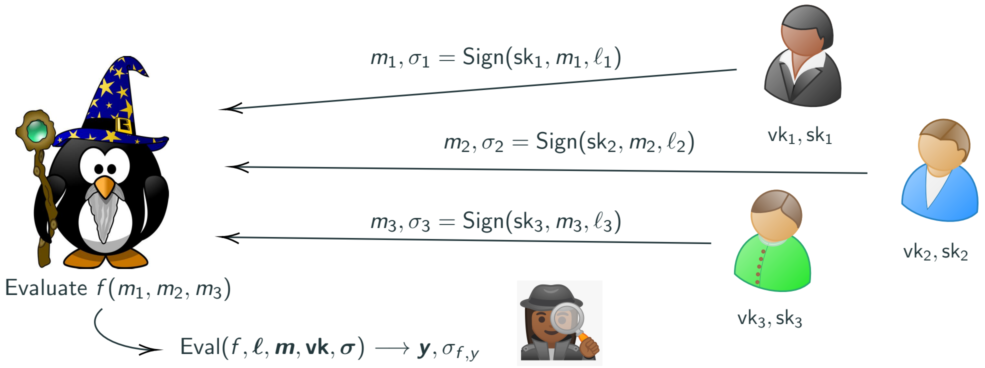

- $\mathsf{Setup}(1^\lambda)$, $\mathsf{KeyGen}(\mathsf{pp})$
- $\mathsf{Sign}(\mathsf{sk}, m, \ell) \to \sigma$
- $\mathsf{Eval}(\mathsf{pp}, (f, \boldsymbol{\ell}), \boldsymbol{m}, \mathbf{vk}, \boldsymbol{\sigma}) \to \sigma_{f,y}$
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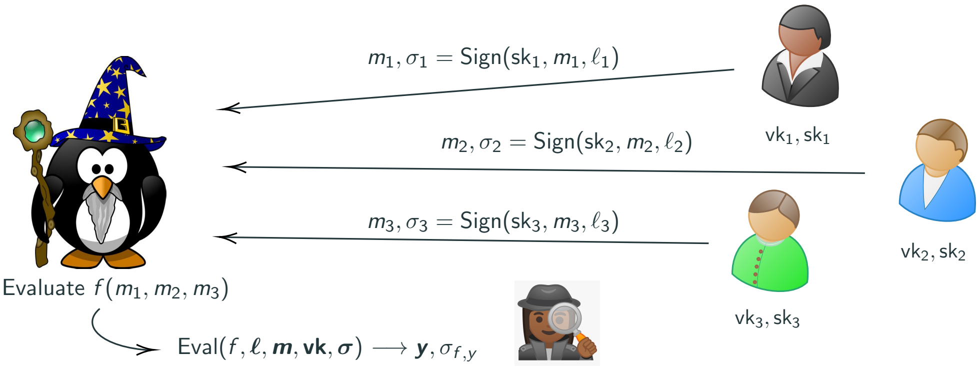

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- **Succinctness:** $|\sigma_{f,y}| \leq p(\lambda)$. Succinct in:
  - nº of inputs n,
  - function size $|f|$,
  - *nº of parties t.*

Our Contribution

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✓ Adaptive security, (sequential) multi-hop evaluation, pre-processing.

✓ Instantiations from e.g. k-Lin or LWE.

✗ Non black-box use of cryptographic primitives.

Roadmap

Batch arguments for NP: aggregating signatures

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Batch arguments for NP: aggregating signatures $\Longrightarrow$ **Homomorphic evaluation: functional commitments**

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$\Rightarrow$

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$\Downarrow$

Achieving security: Linking messages and commitments

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Instantiations

Batch Arguments for NP [KPY19, CJJ21]

Let $(\mathtt{x}_1, \mathtt{w}_1), \ldots, (\mathtt{x}_n, \mathtt{w}_n)$ be statement-witness pairs from an NP relation $\mathcal{C}(\mathtt{x}_i, \mathtt{w}_i) = 1$.

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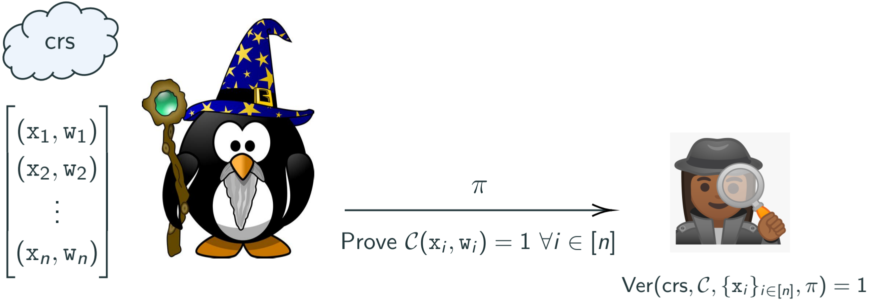

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crs′

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td, i^*

π

Prove $\mathcal{C}(\mathtt{x}_i, \mathtt{w}_i) = 1 \ \forall i \in [n]$

$\mathsf{Ver}(\mathsf{crs}, \mathcal{C}, \{\mathtt{x}_i\}_{i \in [n]}, \pi) = 1$

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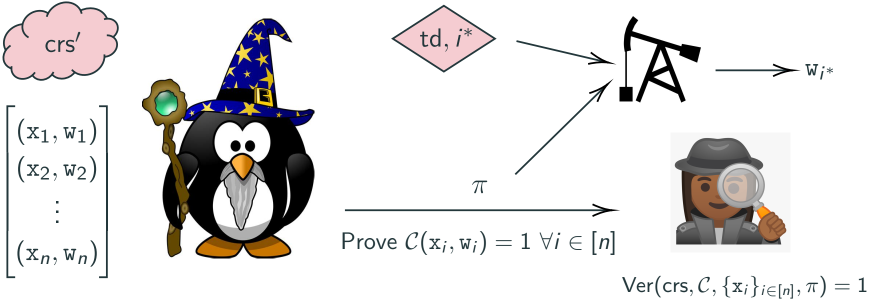

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- **Somewhere extractability:** td extracts a valid $\mathtt{w}_{i^*}$.

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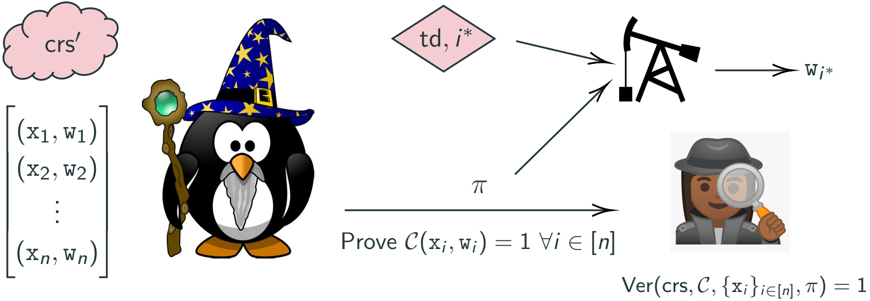

Aggregate *n* signatures [WW22, DGKV22]: Let $\mathtt{x}_i = (\mathsf{vk}_i, m_i)$, $\mathtt{w}_i = \sigma_i$. Prove

$$\underline{\mathcal{C}(\mathtt{x}_i, \mathtt{w}_i) :} \ \Sigma.\mathsf{Ver}(\mathsf{vk}_i, m_i, \sigma_i) = 1$$

Homomorphic Evaluation of f

- **Naive attempt:** Let $\mathtt{x}_i = (\mathsf{vk}_i, \ell_i)$, $\mathtt{w}_i = (m_i, \sigma_i)$ and prove:

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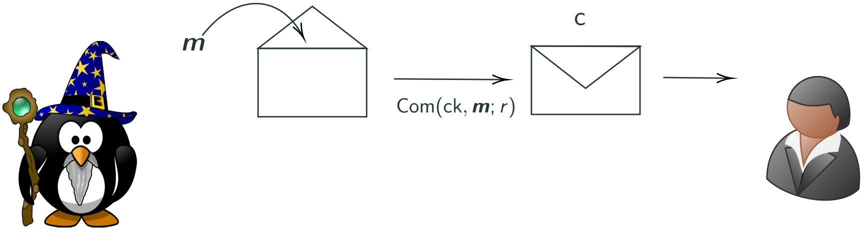

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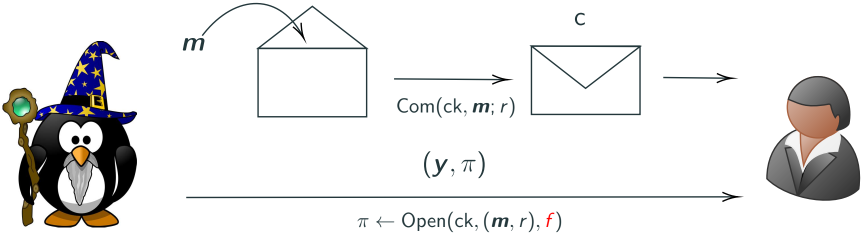

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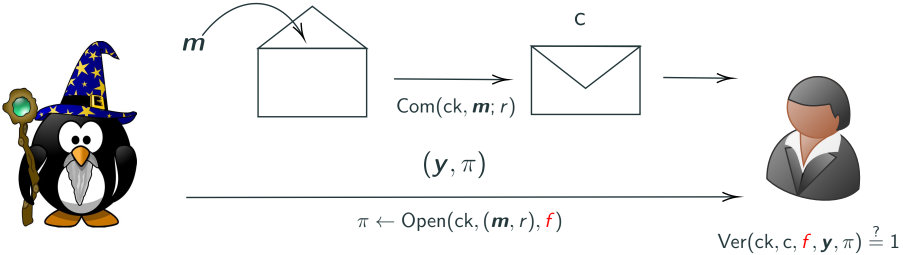

Putting Everything Together

Candidate MKHS:

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Solution: iteratively compute c inside $\mathcal{C}$.

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- Let $\mathtt{x}_i = (\mathsf{vk}_i, \ell_i)$ and $\mathtt{w}_i = (m_i, \sigma_i)$.
- Do a *BARG proof* for $\underline{\mathcal{C}(\mathtt{x}_i, \mathtt{w}_i) :}$ $\Sigma.\mathsf{Ver}(\mathsf{vk}_i, m_i | \ell_i, \sigma_i) = 1$.
- Obtain $\mathsf{c} \leftarrow \mathsf{FC.Com}(\mathsf{ck}, (m_1, \ldots, m_n))$ and *open* c to $f(m_1, \ldots, m_n)$.

Correct, but insecure... the committed m_i may differ from $\mathtt{w}_i$!

Solution: iteratively compute c inside $\mathcal{C}$.

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- Define *partial* $\mathsf{c}_0, \ldots, \mathsf{c}_n$, where c_i commits to $(m_1, \ldots, m_i, 0, \ldots, 0)$.
- $\mathcal{C}(\mathtt{x}_i, \mathtt{w}_i)$ checks that *c_i and c_{i-1} differ on m_i* at position i.

Our MKHS Construction

Description of $\mathcal{C}(\mathtt{x}_i, \mathtt{w}_i)$ (simplified):

Statement: $\mathtt{x}_i = (\mathsf{vk}_i, \ell_i, \mathsf{ck}_i, i)$

Witness: $\mathtt{w}_i = (m_i, \sigma_i, \pi_i, \mathsf{c}_{i-1}, \mathsf{c}_i)$

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For the security proof to work, we also need a somewhere extractable commitment (SEC).

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From HiKer and k-Lin for $k \geq 2$, there exists a MKHS for arithmetic circuits of width w where:

- $|\mathsf{pp}| = \mathcal{O}(w^5)$
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- $|\sigma_{f,y}| = \mathsf{poly}(\lambda)$.

Open Questions

Make a **fully algebraic** and practical construction?

Support arbitrary **multi-hop evaluation**? [ACG24]

Apply similar techniques to **other advanced primitives**?

Takeaways

- Multi-key homomorphic signatures:
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Thank you!

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david.balbas@imdea.org

Security: Unforgeability

- Security is game-based [FMNP16]. Adversary $\mathcal{A}$ and challenger interact via oracles:
  - $\mathcal{O}^{\mathsf{KeyGen}}(\mathsf{id}) \to \mathsf{pk}_{\mathsf{id}}$
  - $\mathcal{O}^{\mathsf{Sign}}(\mathsf{id}, m, \ell) \to \sigma$. Only *one query per label* is allowed!
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- $\mathcal{A}$ wins if $\sigma_{f,y}^*$ *verifies* and either:
  1. Exists i such that $\mathcal{O}^{\mathsf{Sign}}(\ell_i^*, \cdot)$ was never queried.
  2. For all i, (ℓ_i^*, m_i) honest but $\boldsymbol{y}^* \neq f^*(m_1, \ldots, m_n)$.

Proving Security

- The proof proceeds by partitioning the winning condition in multiple events.
- **Interesting event:** when $\boldsymbol{y} \neq f(m_1, \ldots, m_n)$ and the (deterministic) commitment to the messages c^* *is dishonest*, $\mathsf{c}^* \neq \mathsf{FC.Com}(\mathsf{ck}, (m_1, \ldots, m_n))$.
- Strategy is to *gradually* show that *each partial c_i must be honest*. Multiple hybrids for each $i \in [n]$, where:
  1. Program the BARG crs and extract at i,
  2. Compare the extracted c_i to the honest one,
  3. Extract m_i and σ_i (a potential forgery) and certify the validity of the commitment update from c_{i-1} to c_i.
  4. "Reboot" the extraction to step $i + 1$.
- Add a *somewhere extractable commitment* to follow a sliding window approach.