On Central Primitives for Quantum Cryptography with Classical Communication

Kai-Min Chung, **Eli Goldin**, Matthew Gray

(Classical) Cryptography Topography

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Key Exchange can be used to build one way functions

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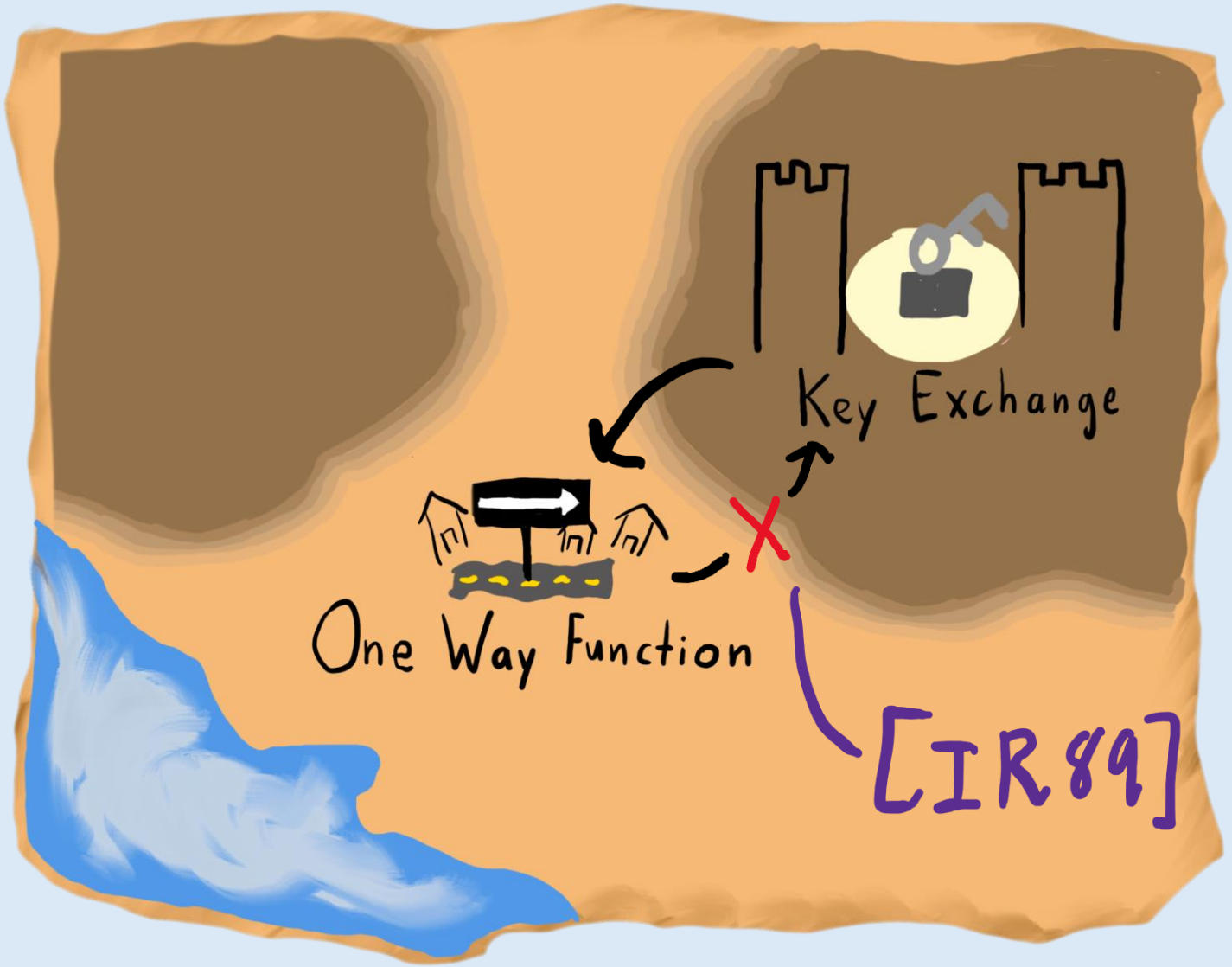

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Hard to build KE from OWF

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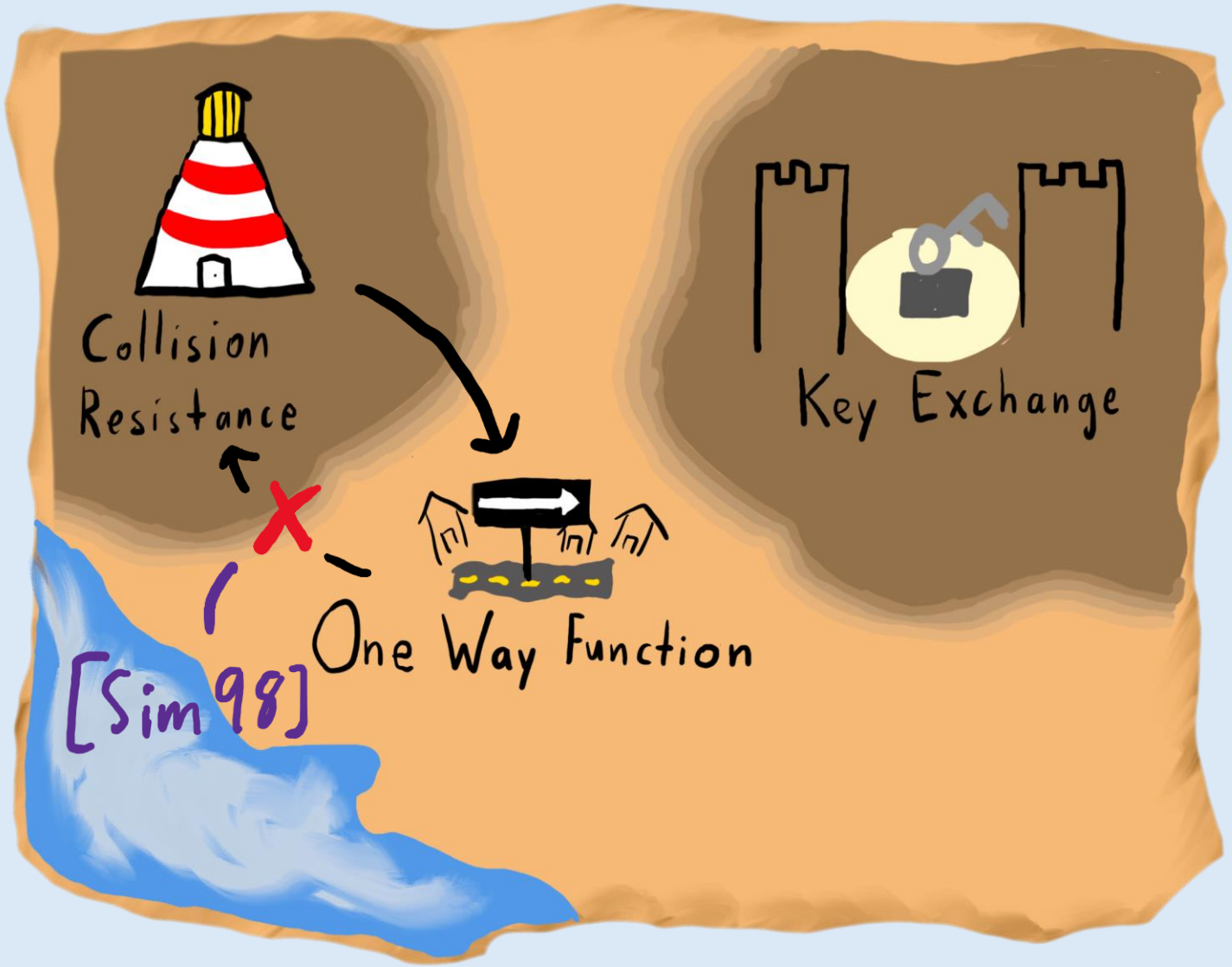

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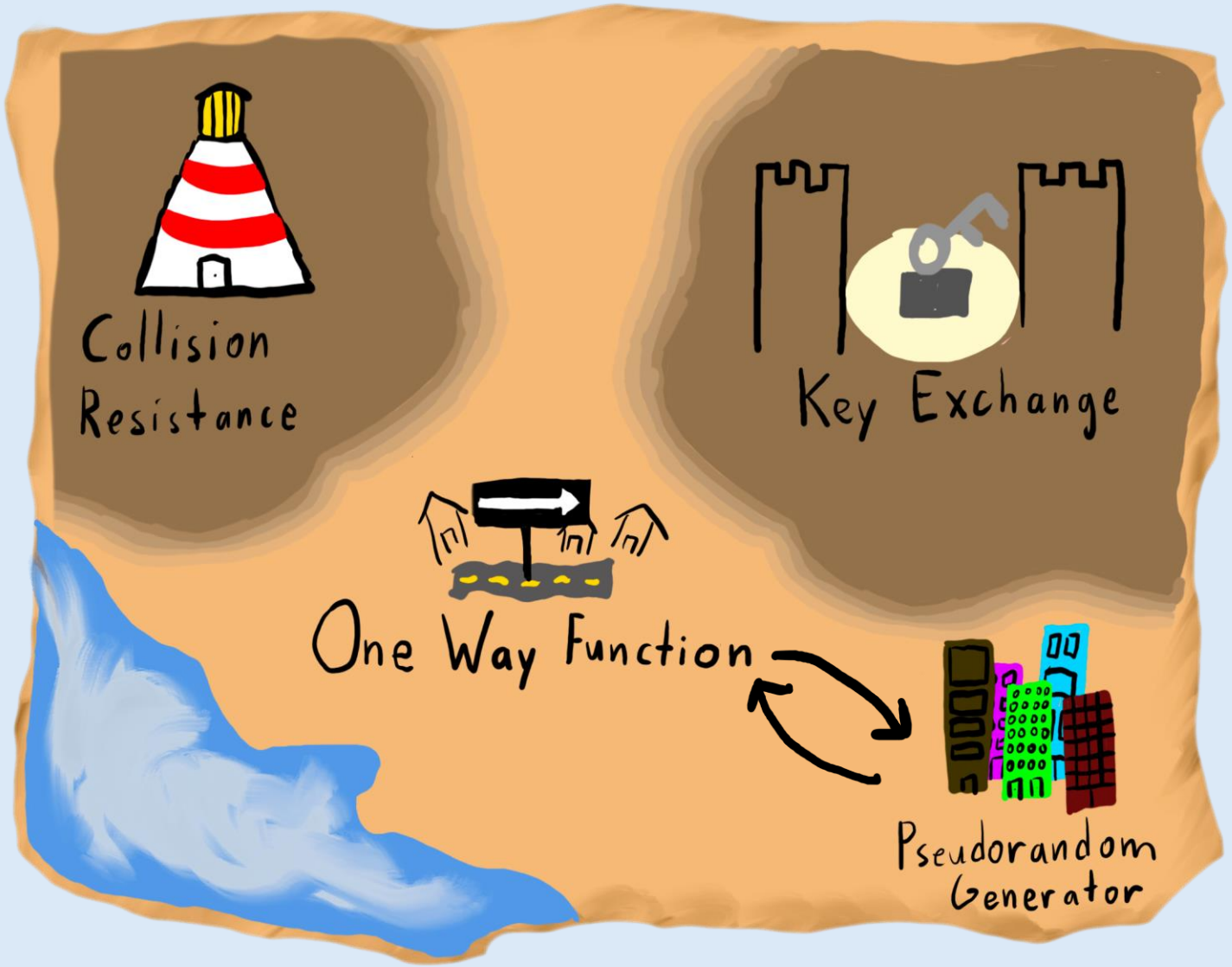

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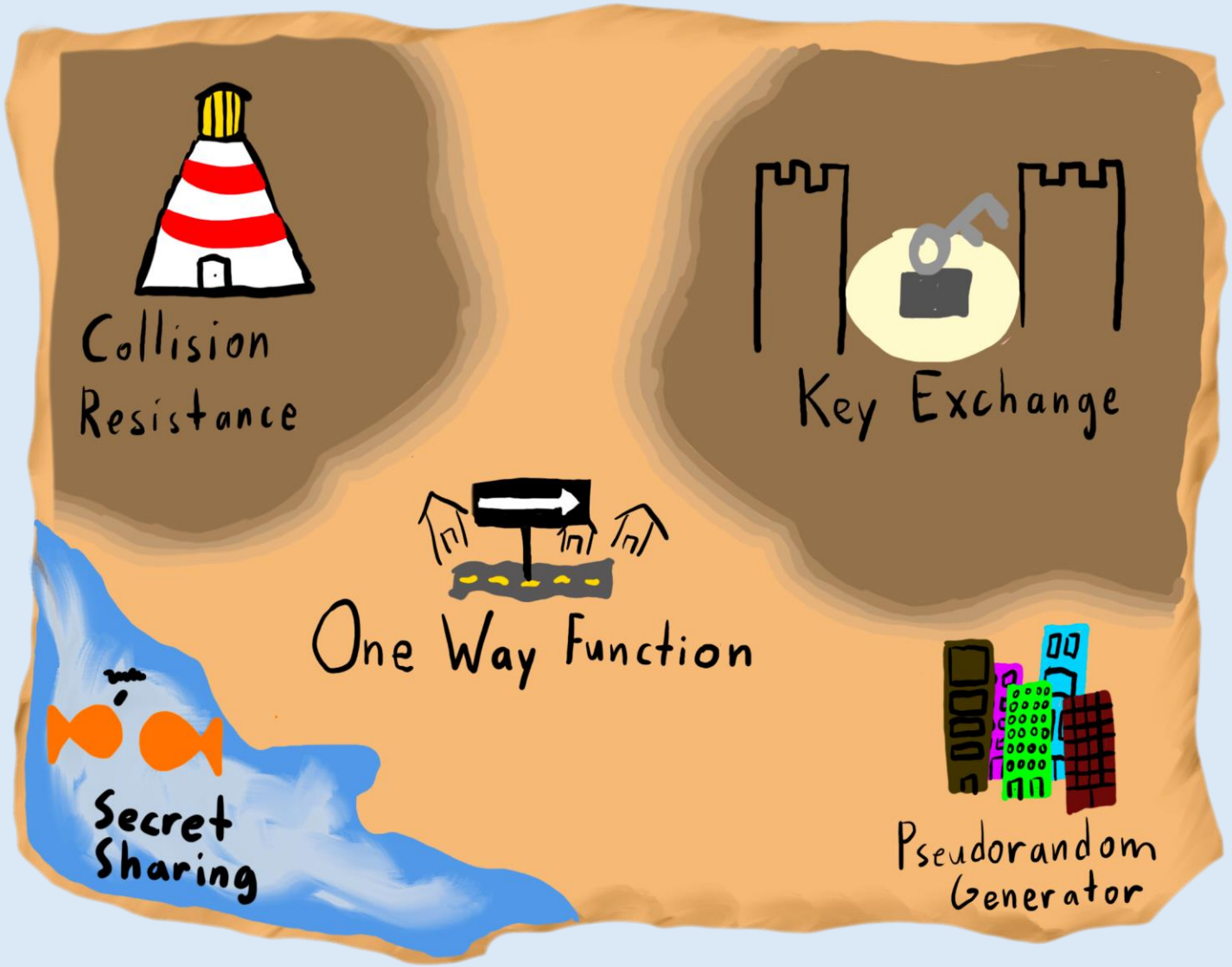
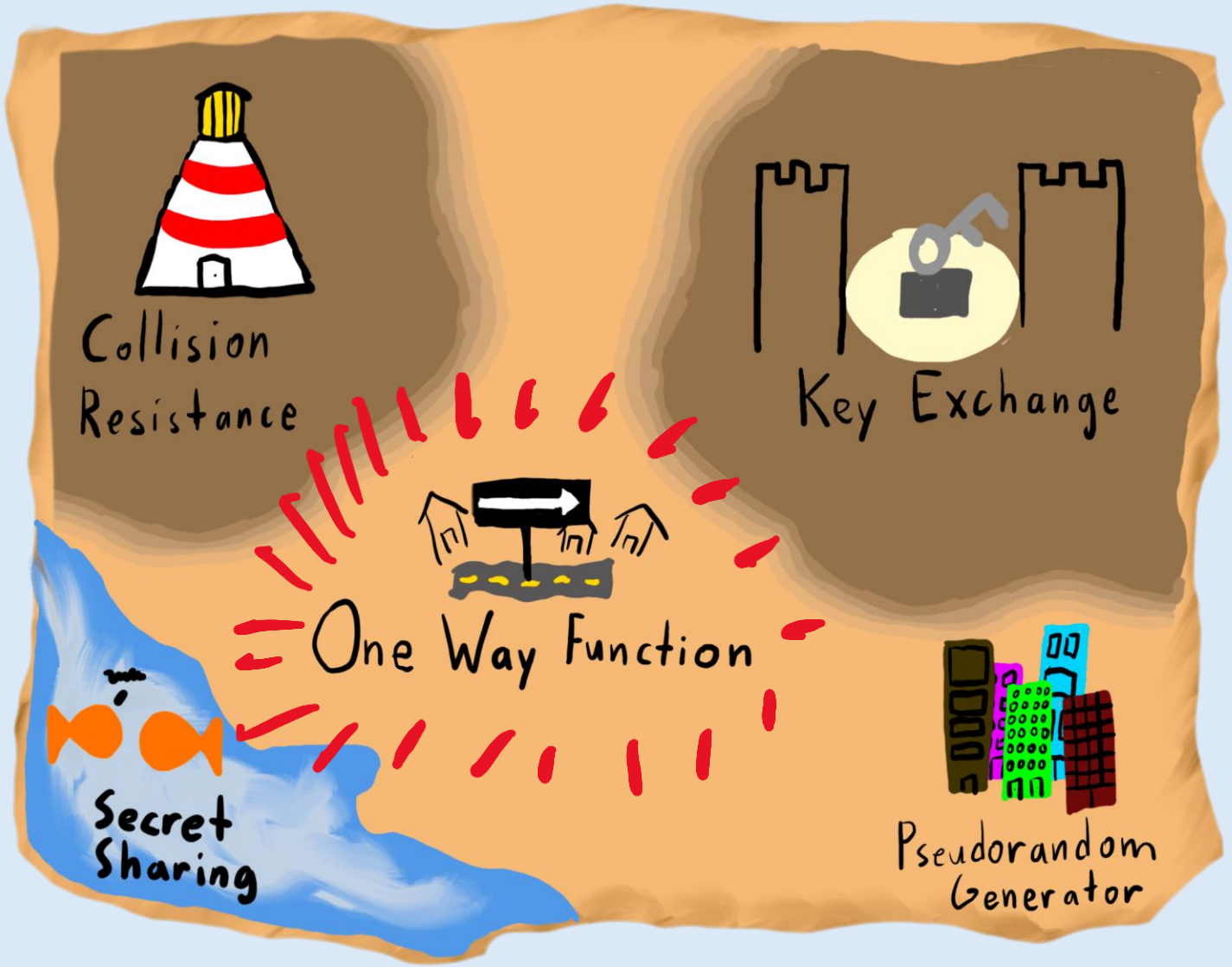


Secret sharing exists unconditionally (and has information theoretic security)

The Center of the World: OWFs



OWFs are special. Very useful starting place for understanding the map of Cryptography.

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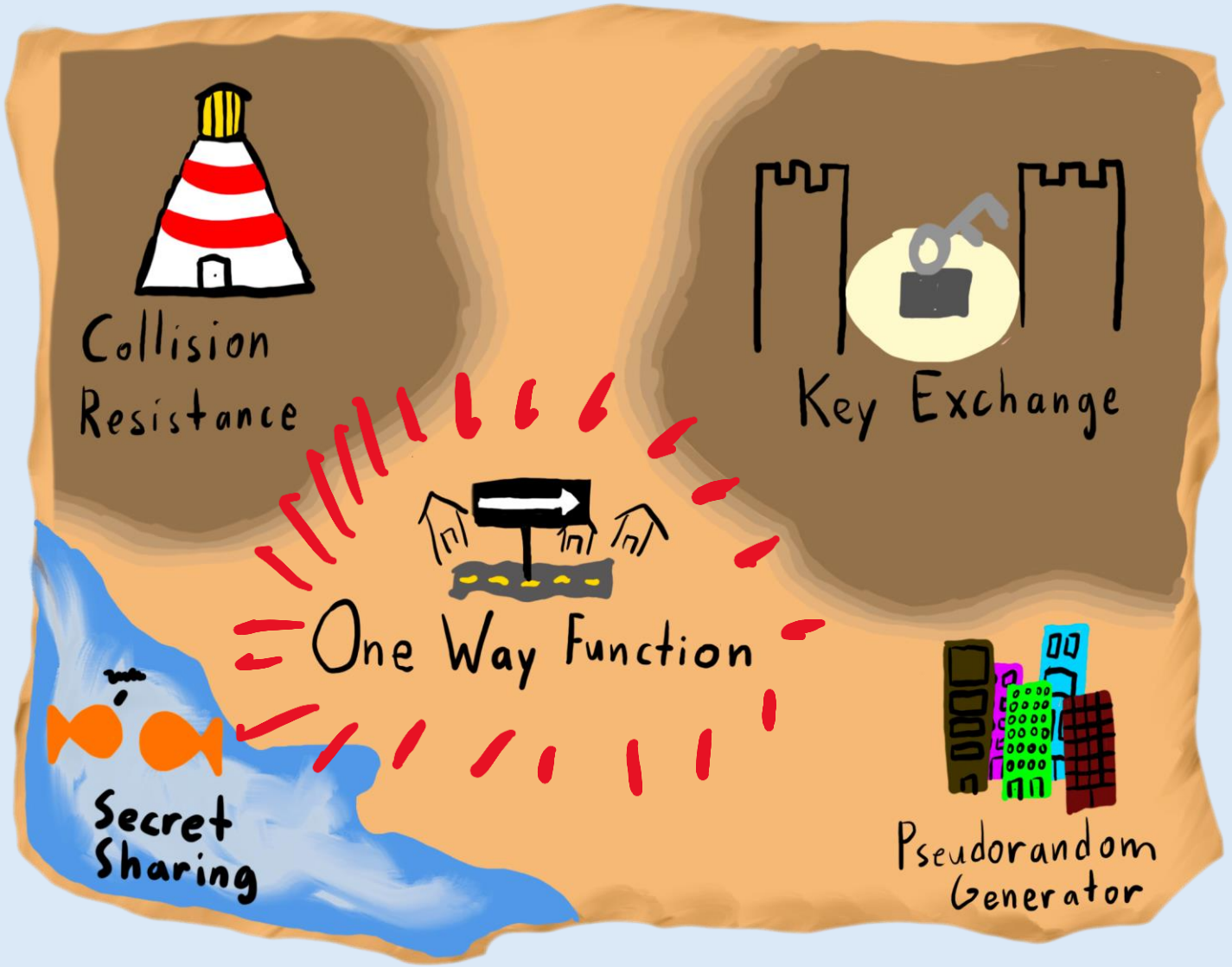

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Why?

The Center of the World: OWFs

Why are one way functions so important?

1. Simple
2. Minimal
3. Useful
4. Flexible

The Center of the World: OWFs

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1. **Simple**: "Easy to compute, hard to invert"
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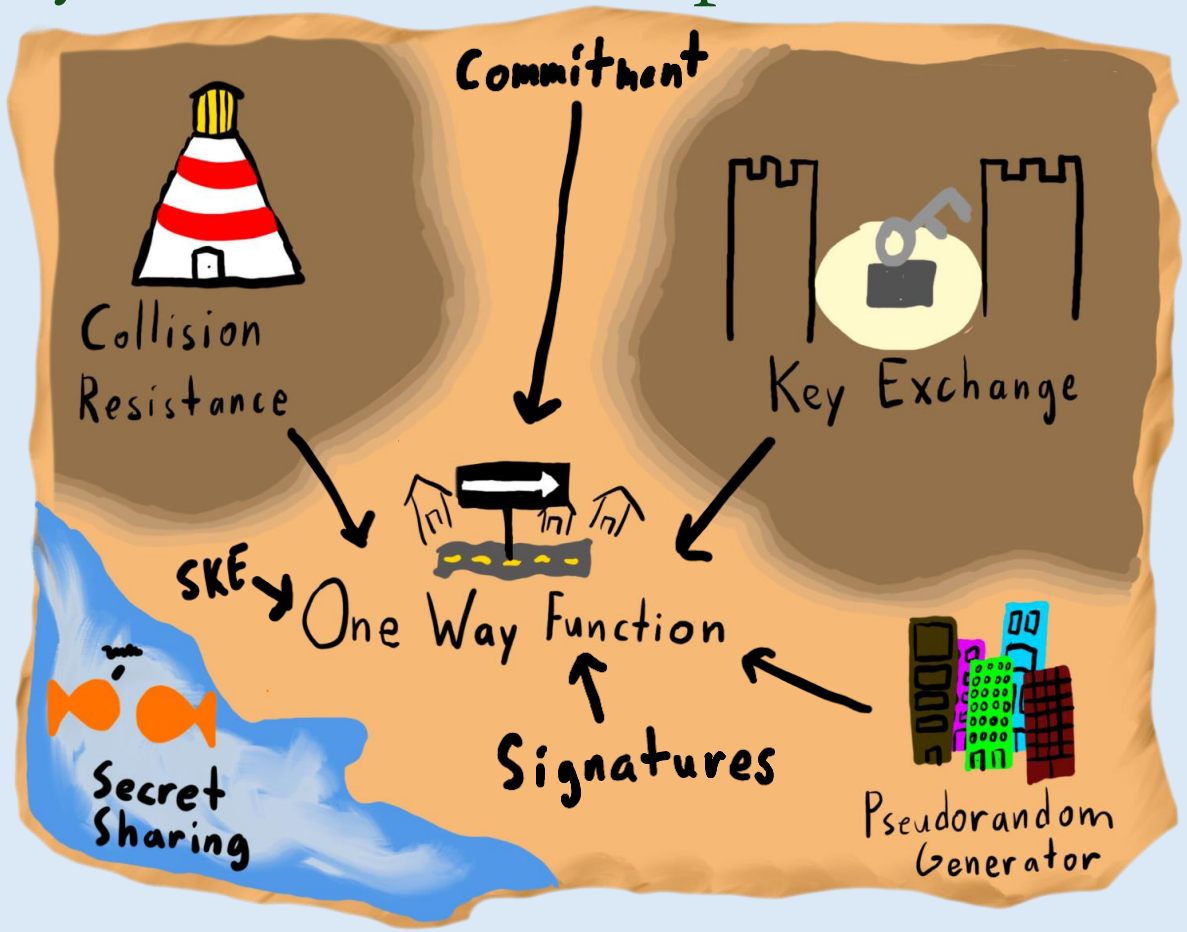

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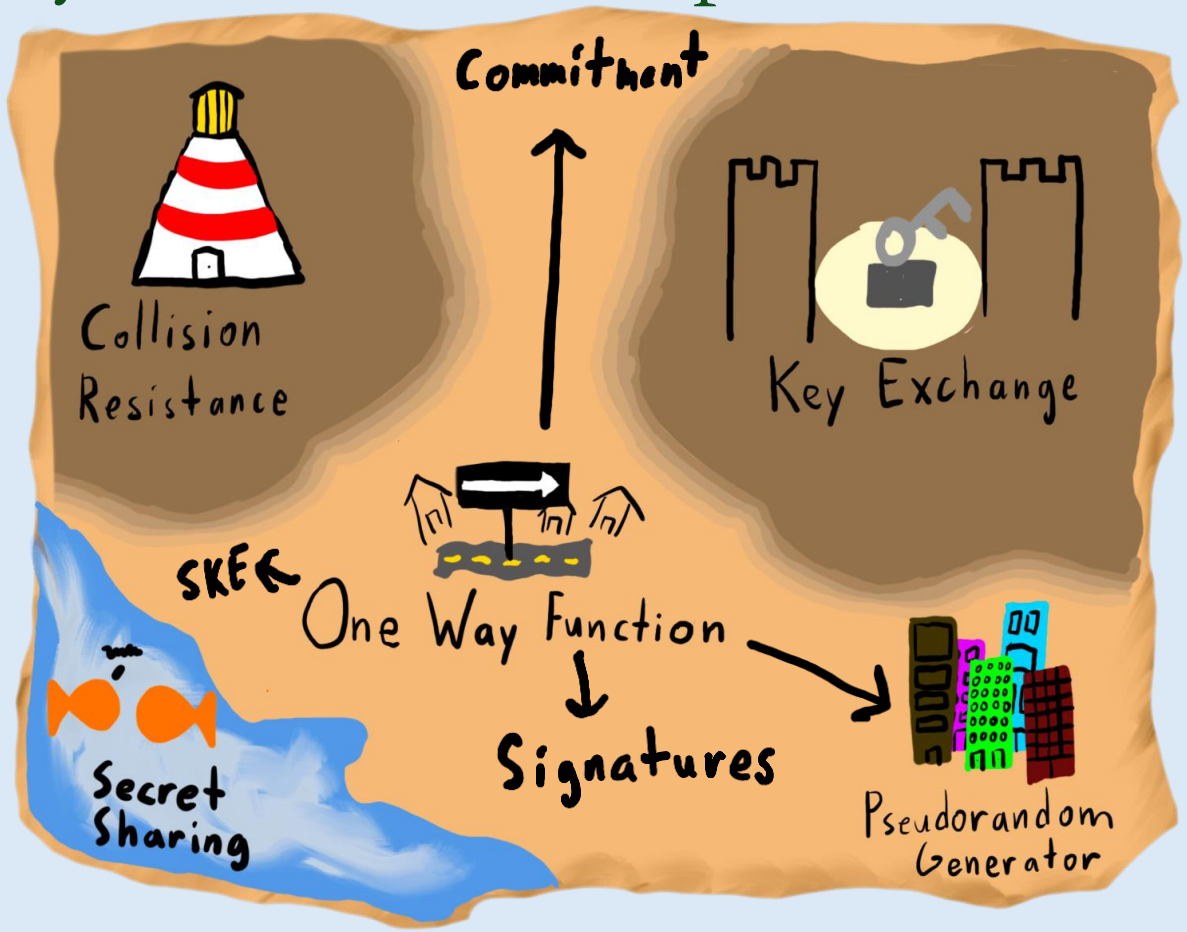

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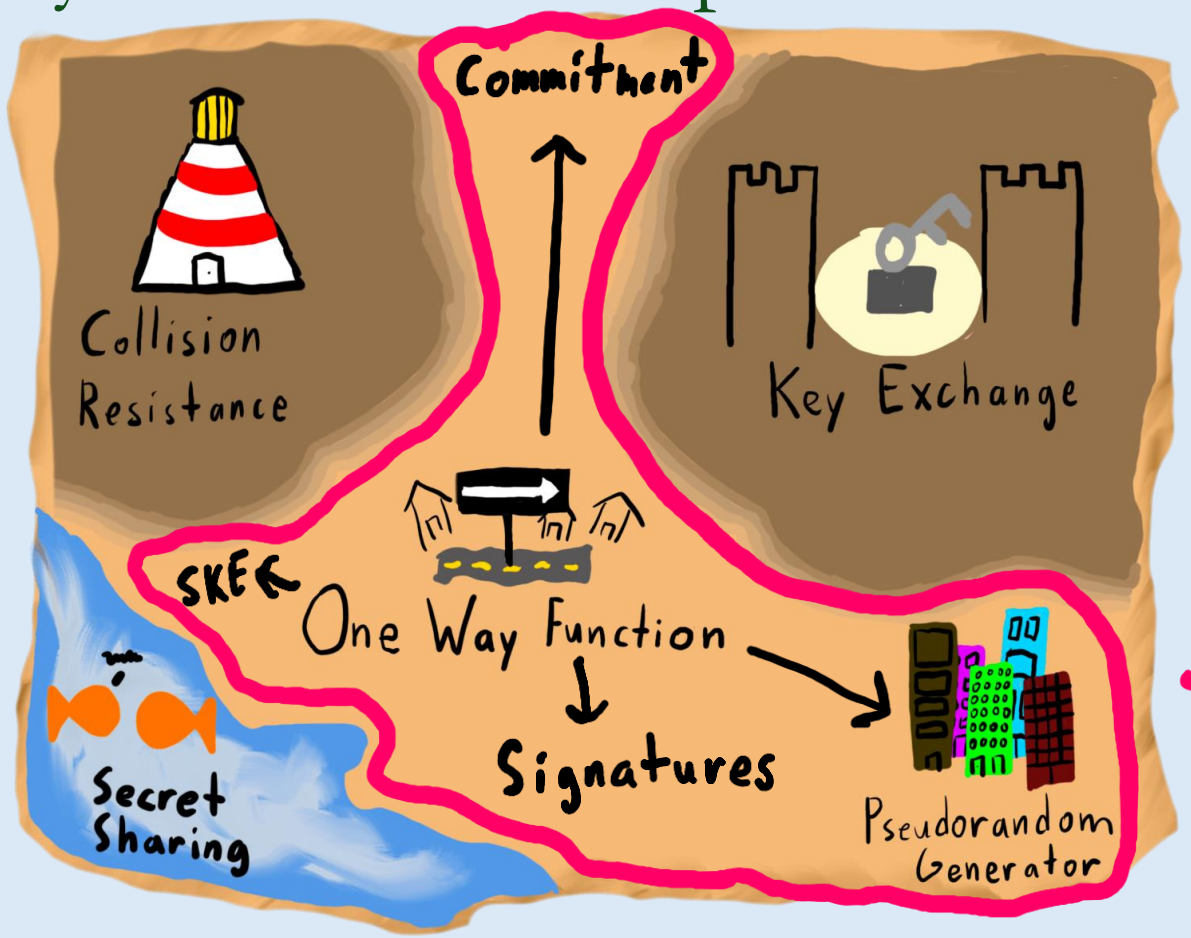

MiniCrypt

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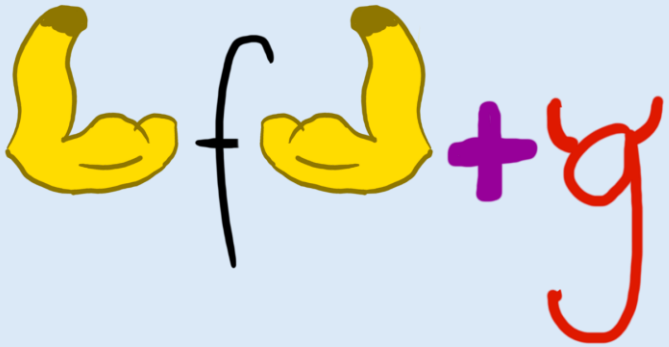

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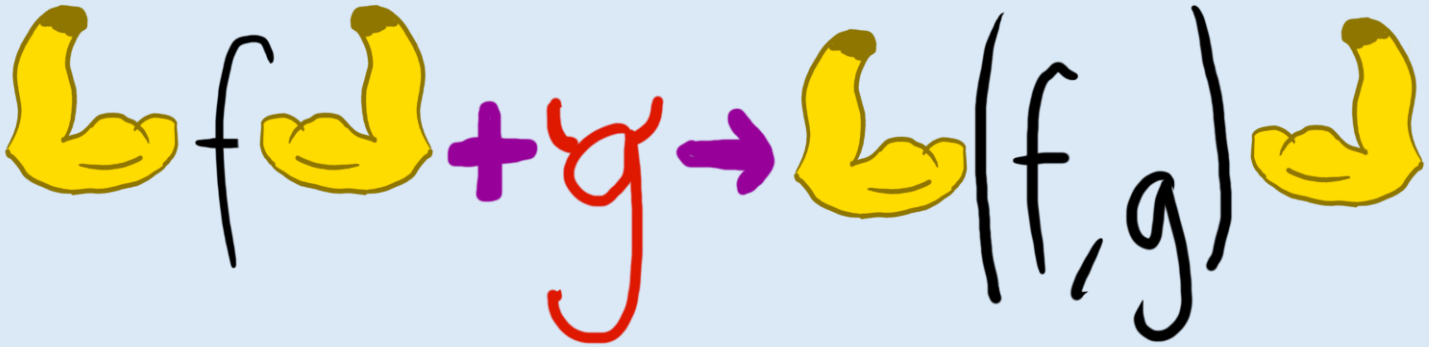

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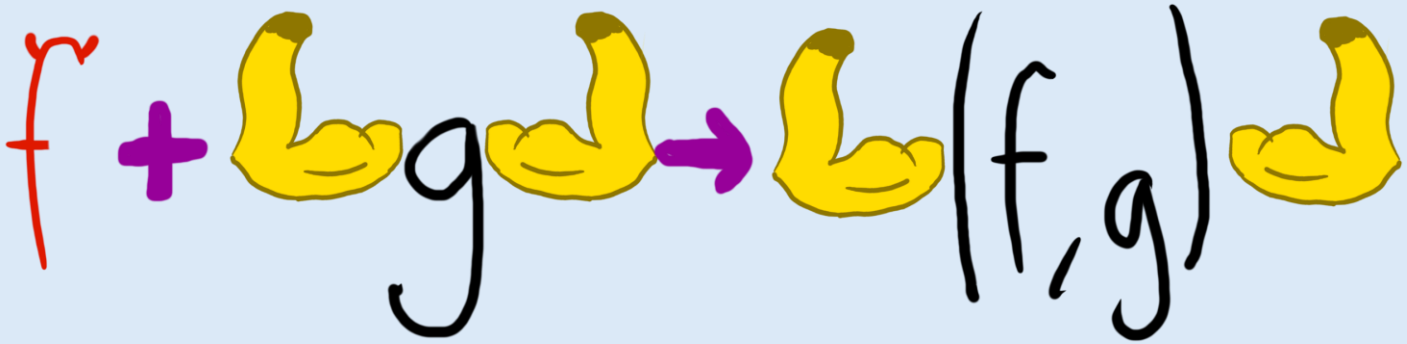

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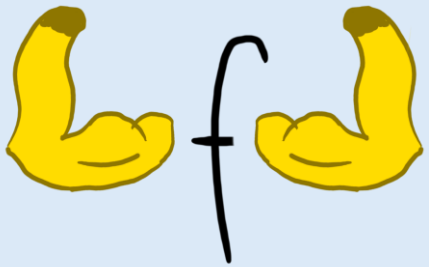

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"strong" OWF

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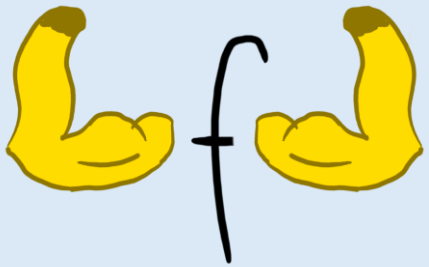

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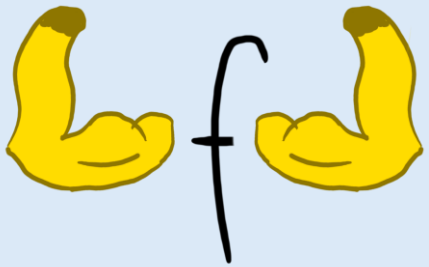

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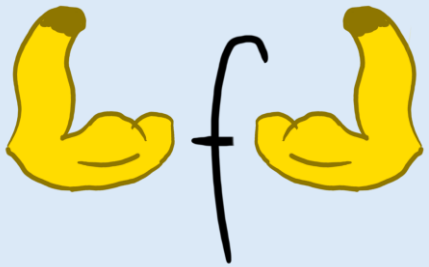

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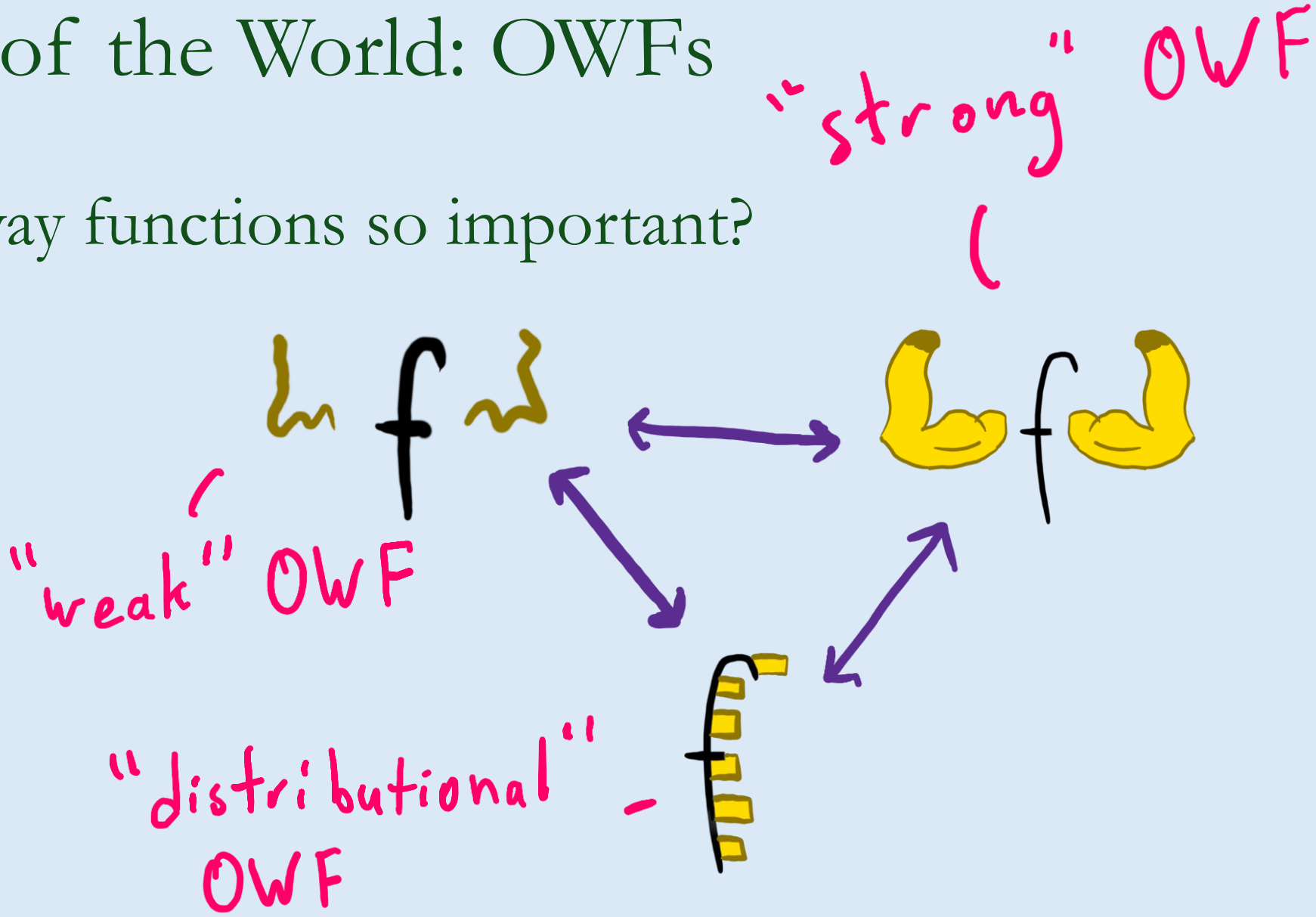

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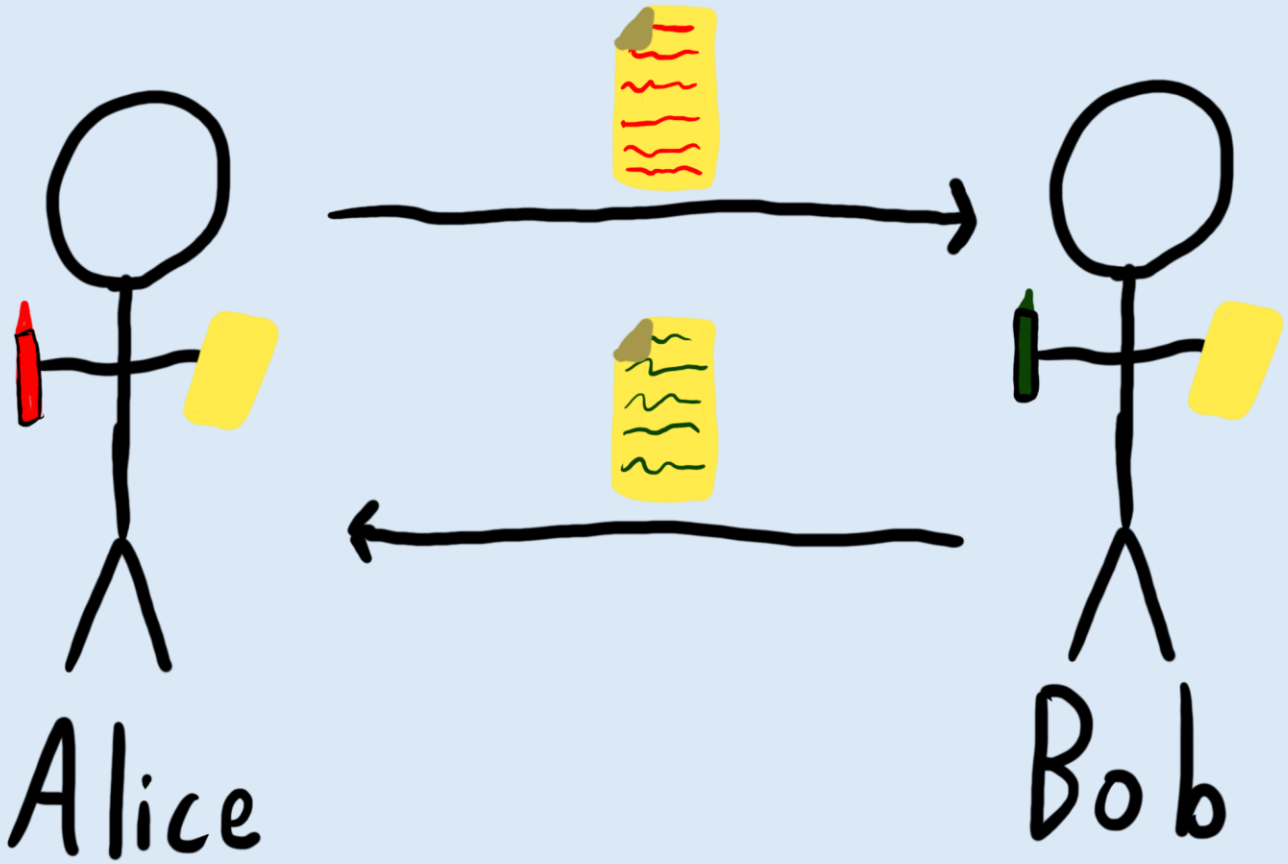

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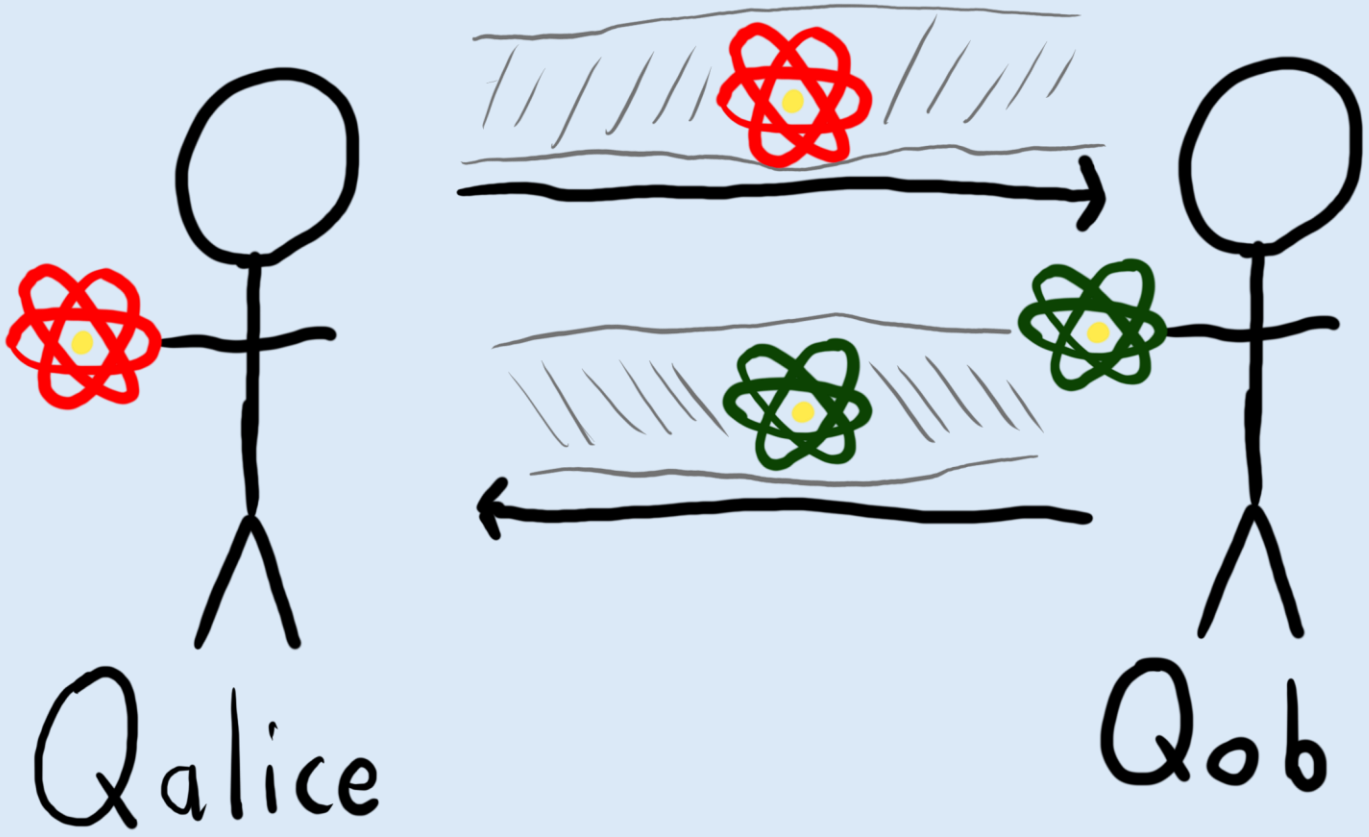
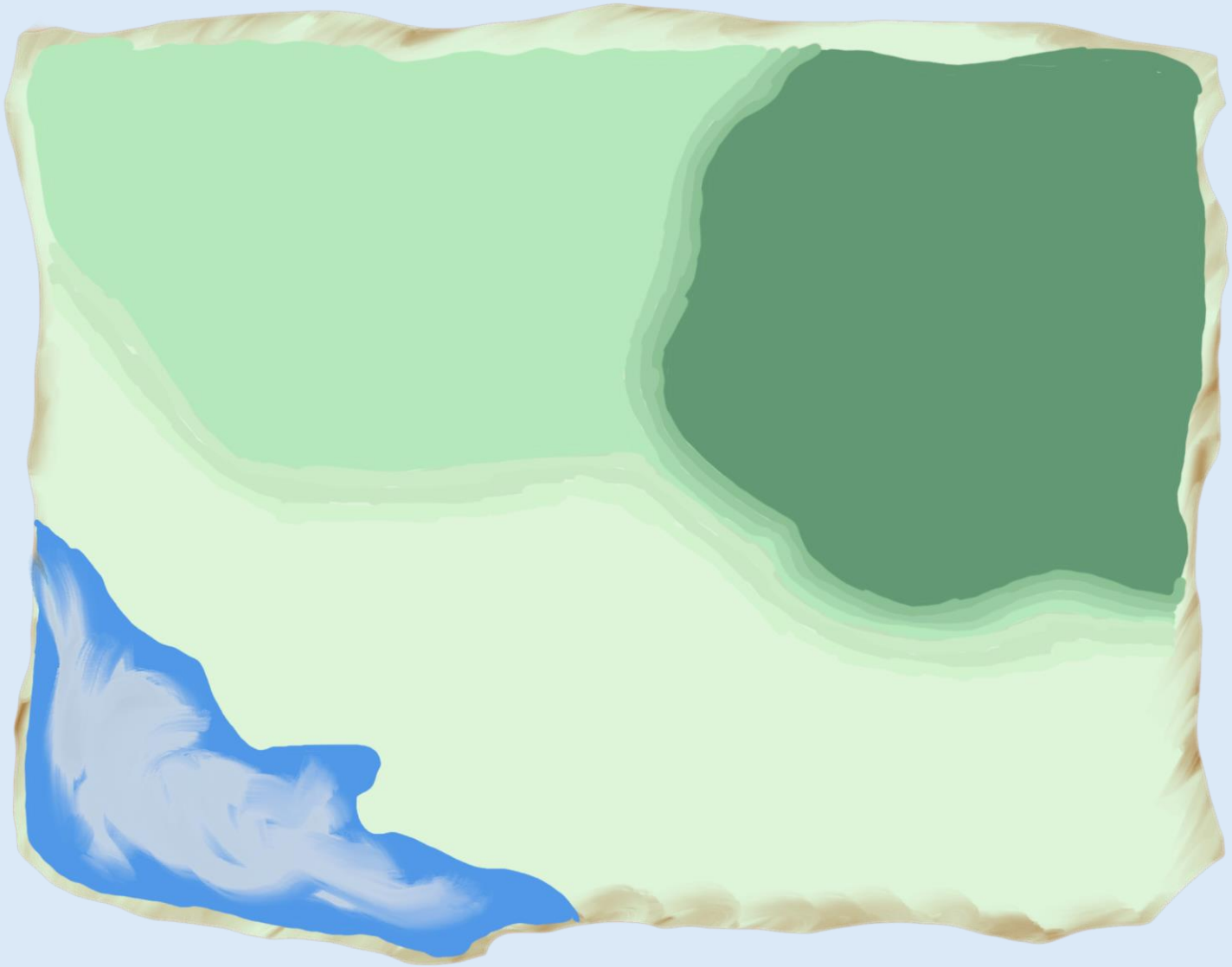


Quantum Cryptography Topography



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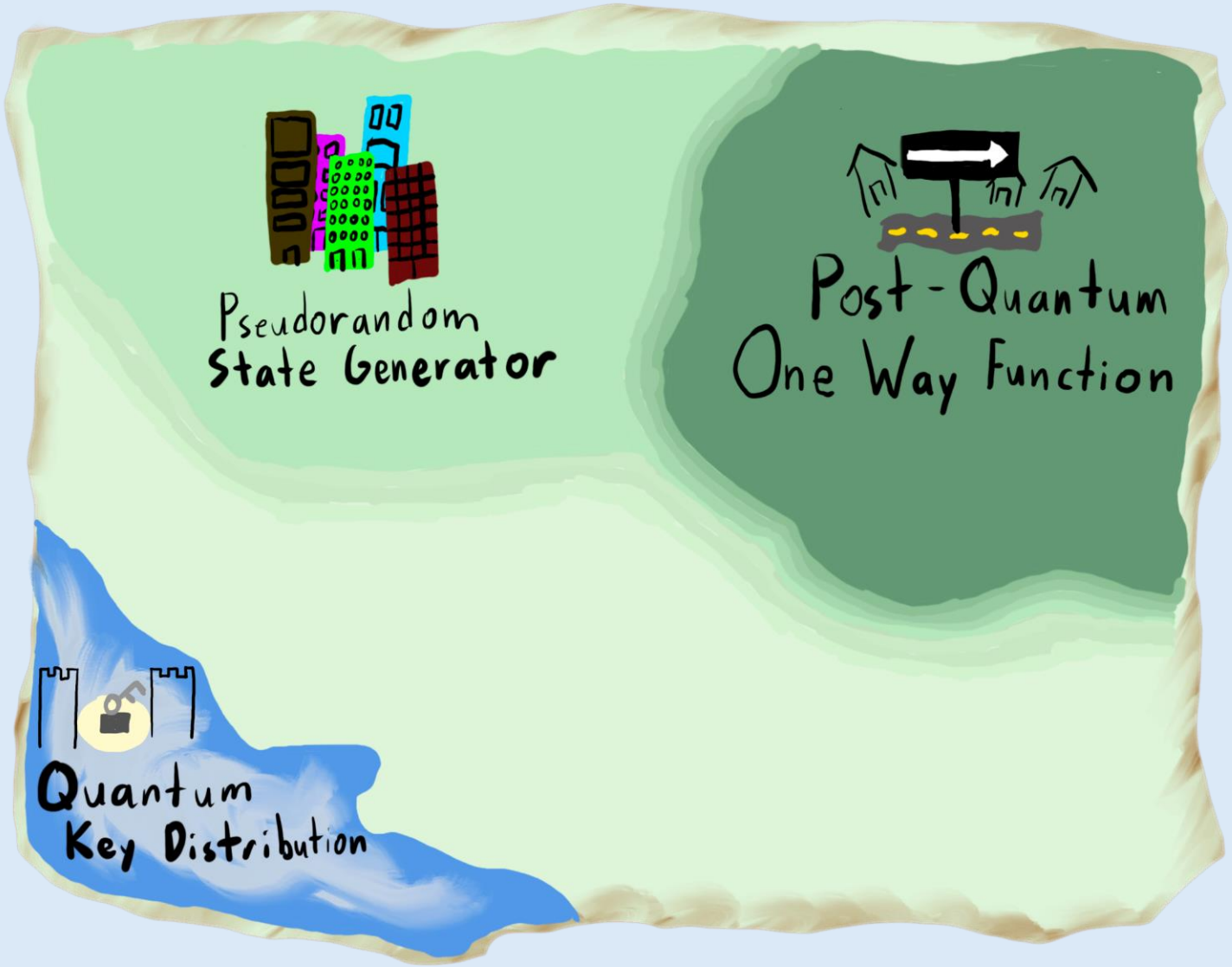

Map different!

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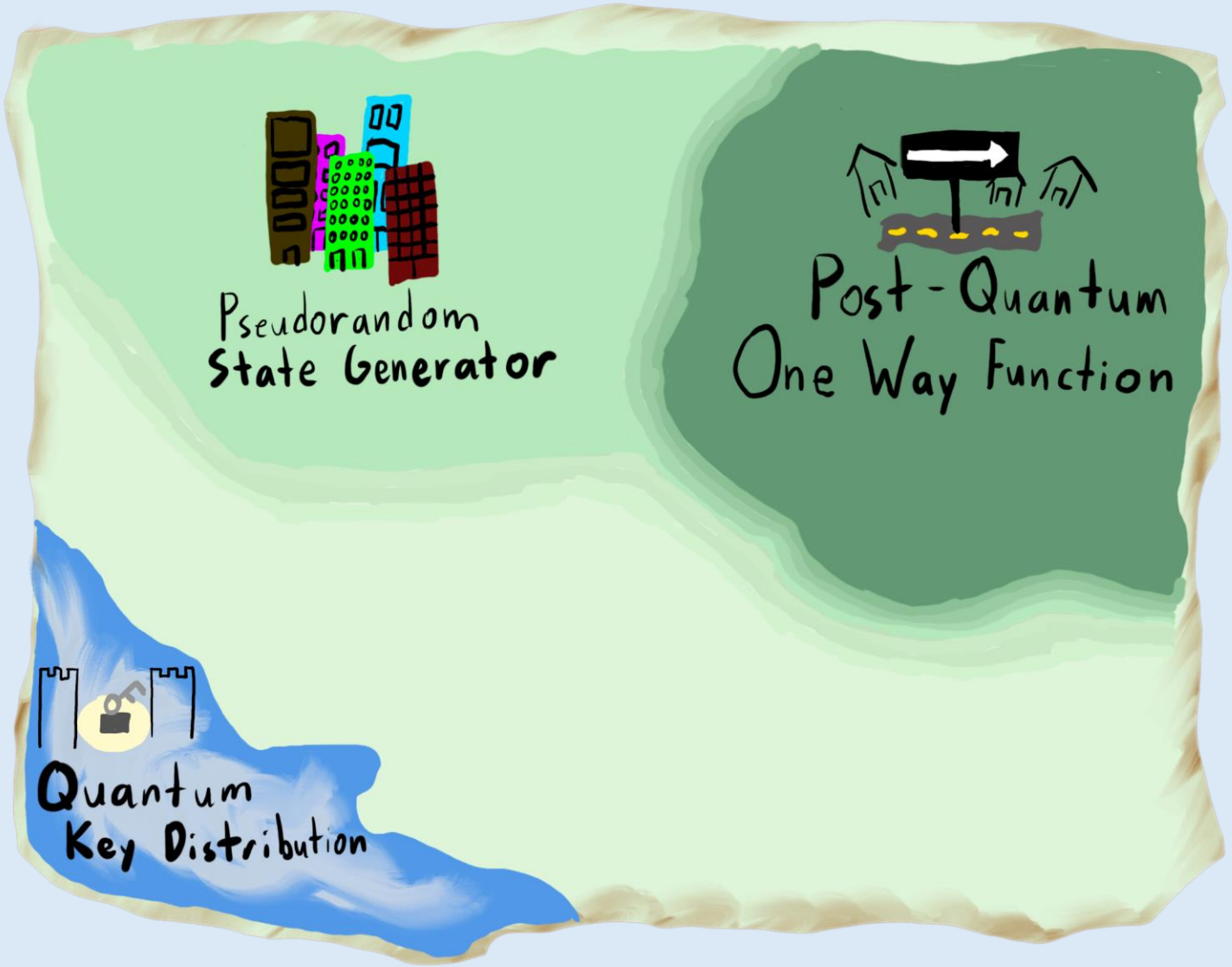

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Is there a central primitive?

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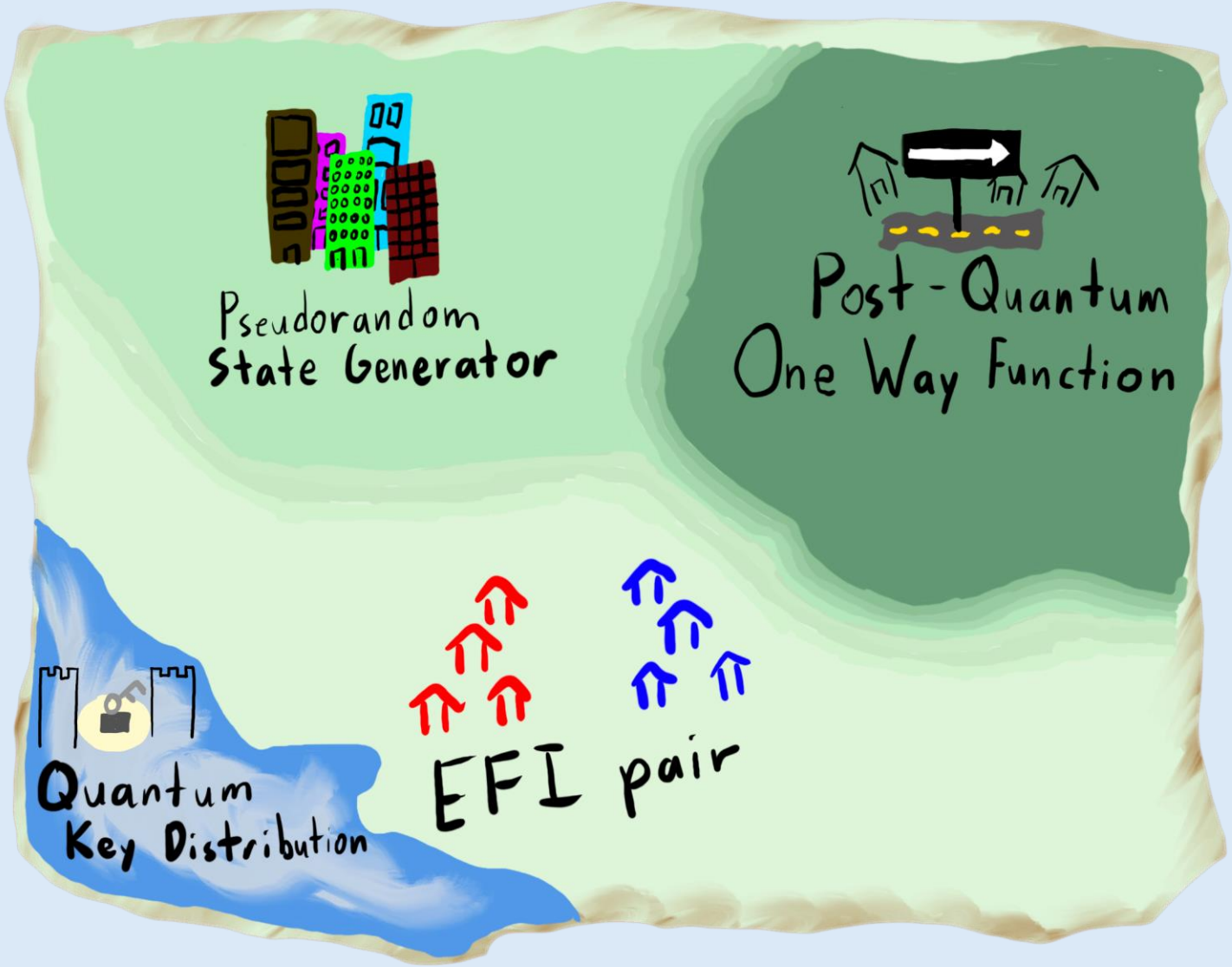

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Pretty good candidate: EFI pairs

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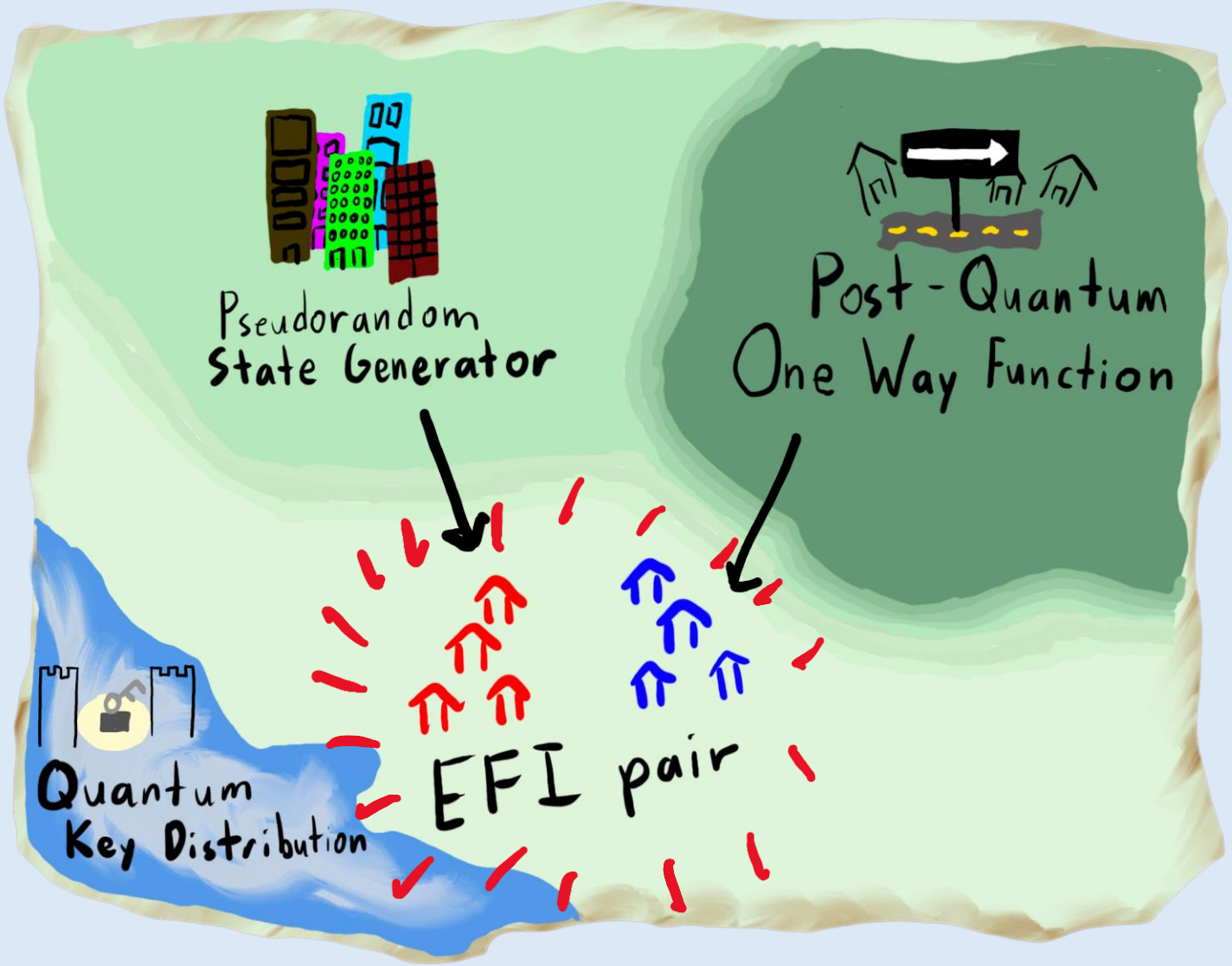

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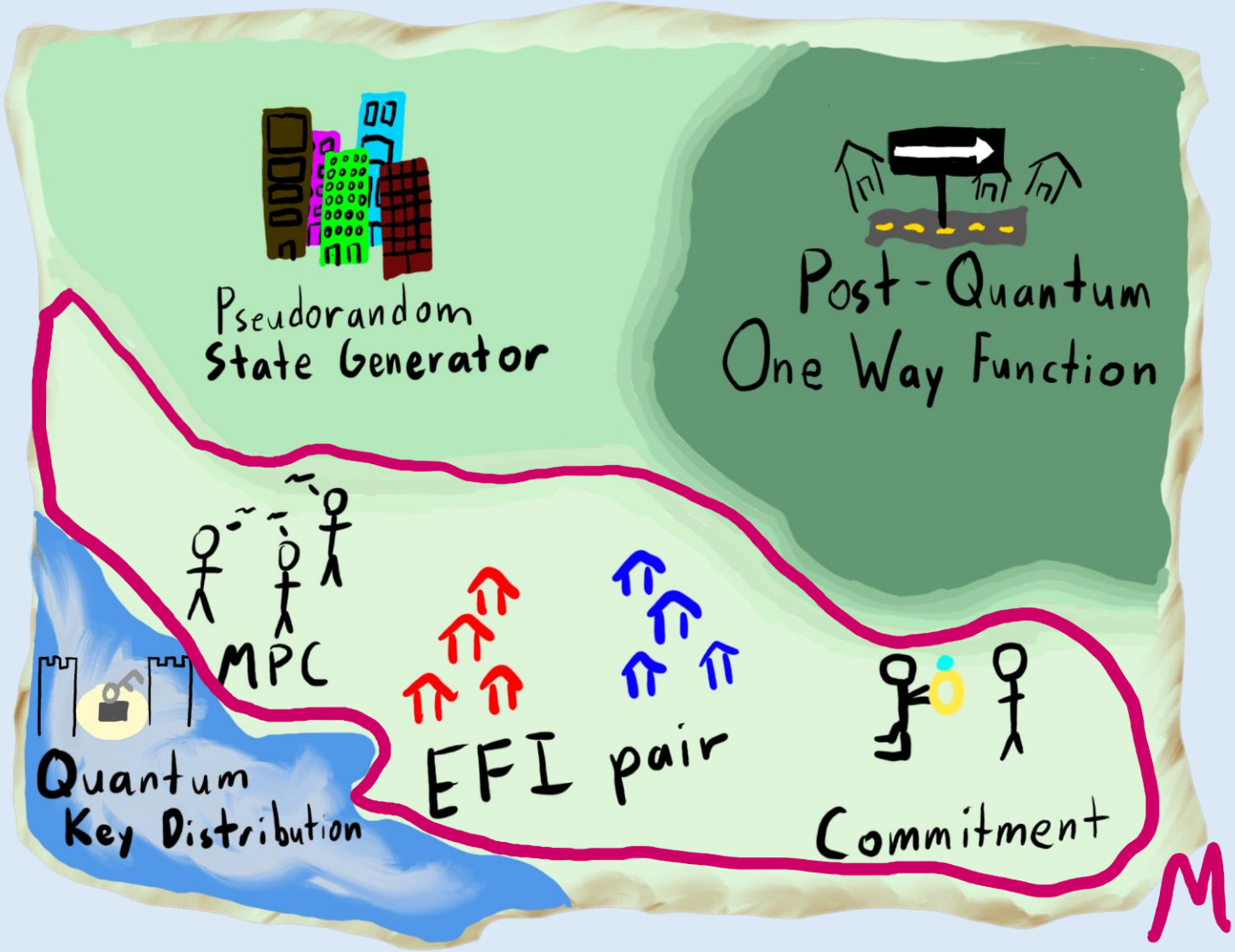

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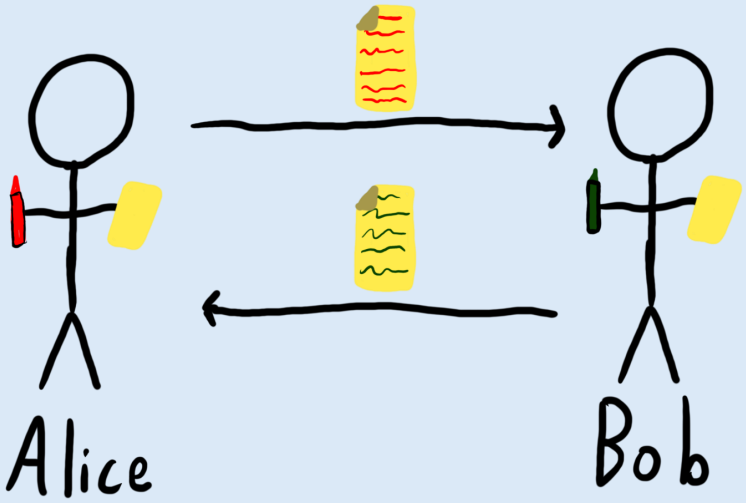
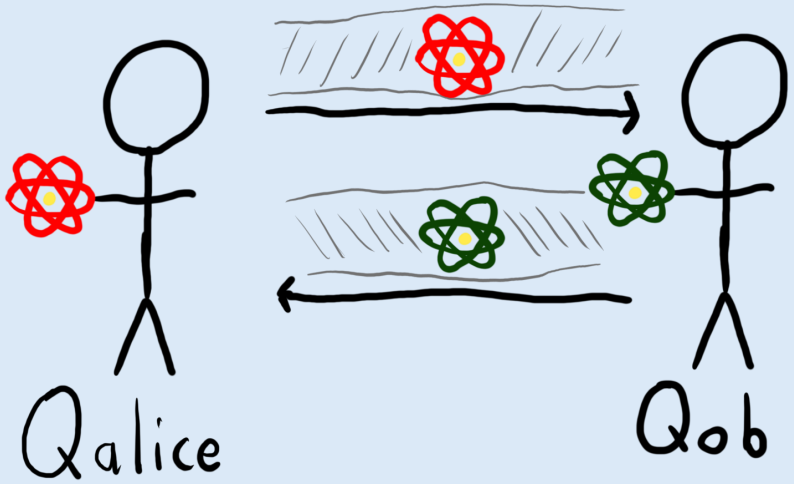
MicroQrypt

Our focus: The QCCC (QC) setting



OWF



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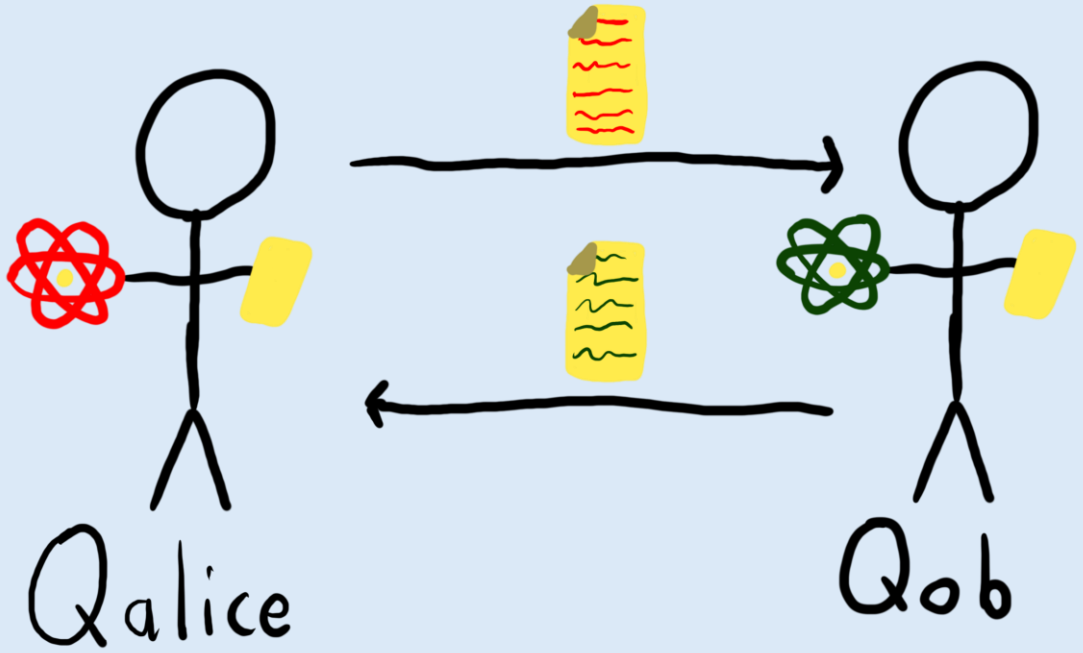
"Quantum Computation with Classical Communication"



Our Goals

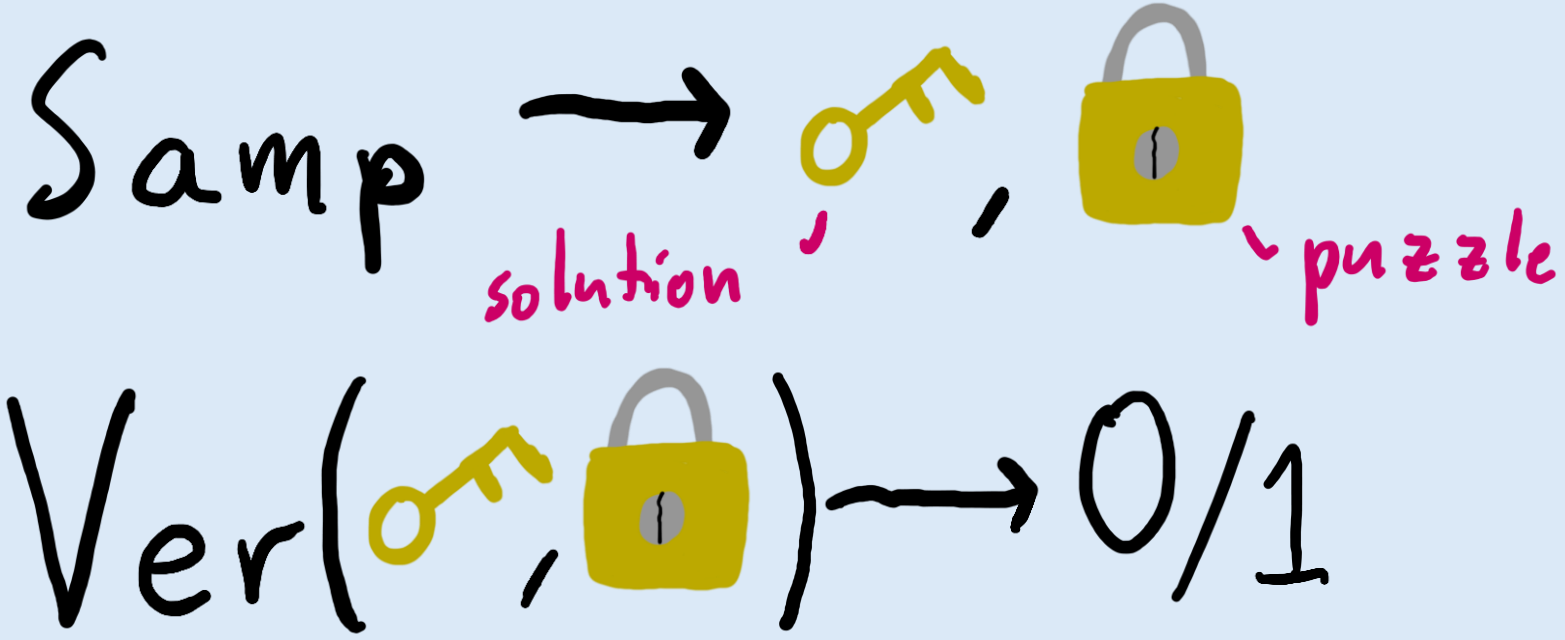
What does the “map” of QC cryptography look like?

Is there a central primitive in the QC setting?

Prior Work

Candidate: One Way Puzzles [KT24]

Puzzle/solution pairs easy to generate, hard to solve.



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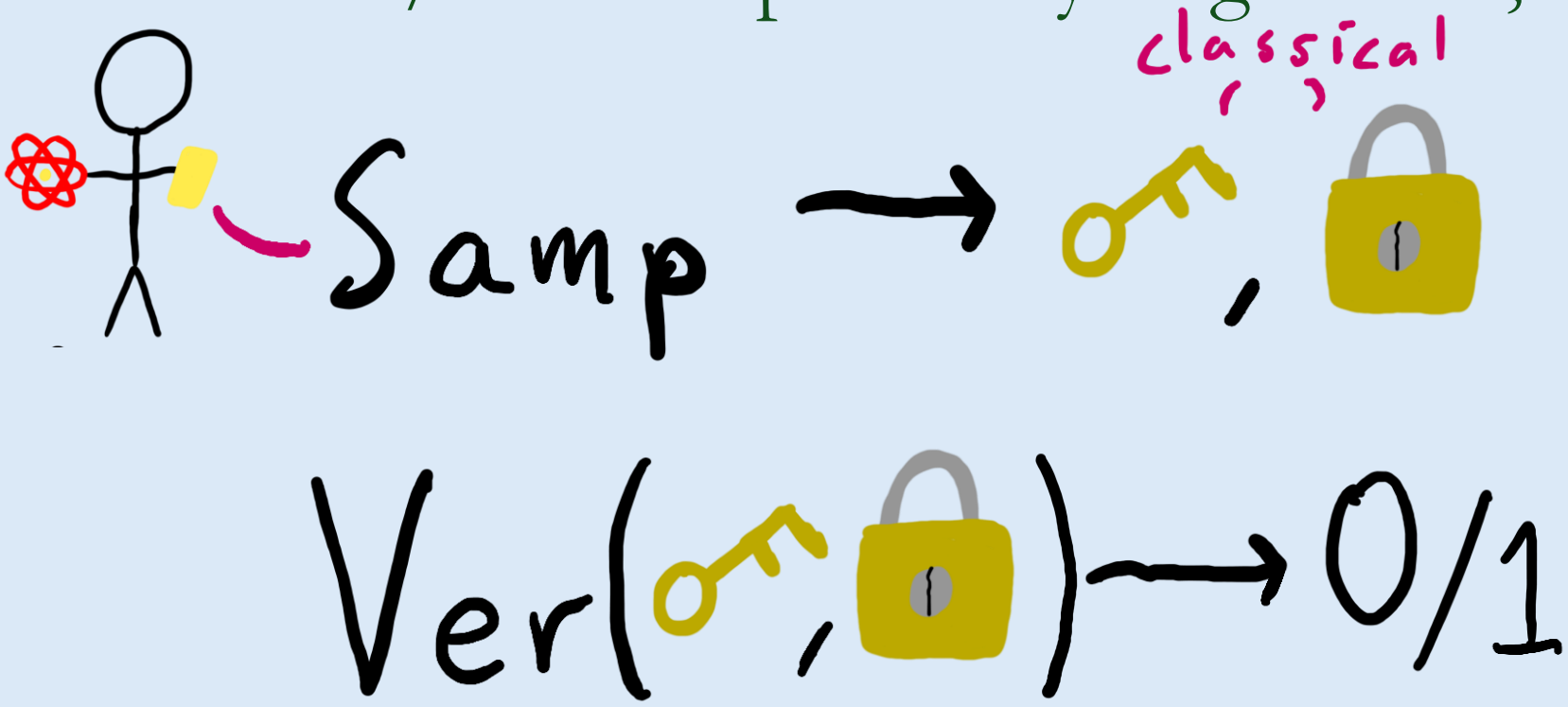

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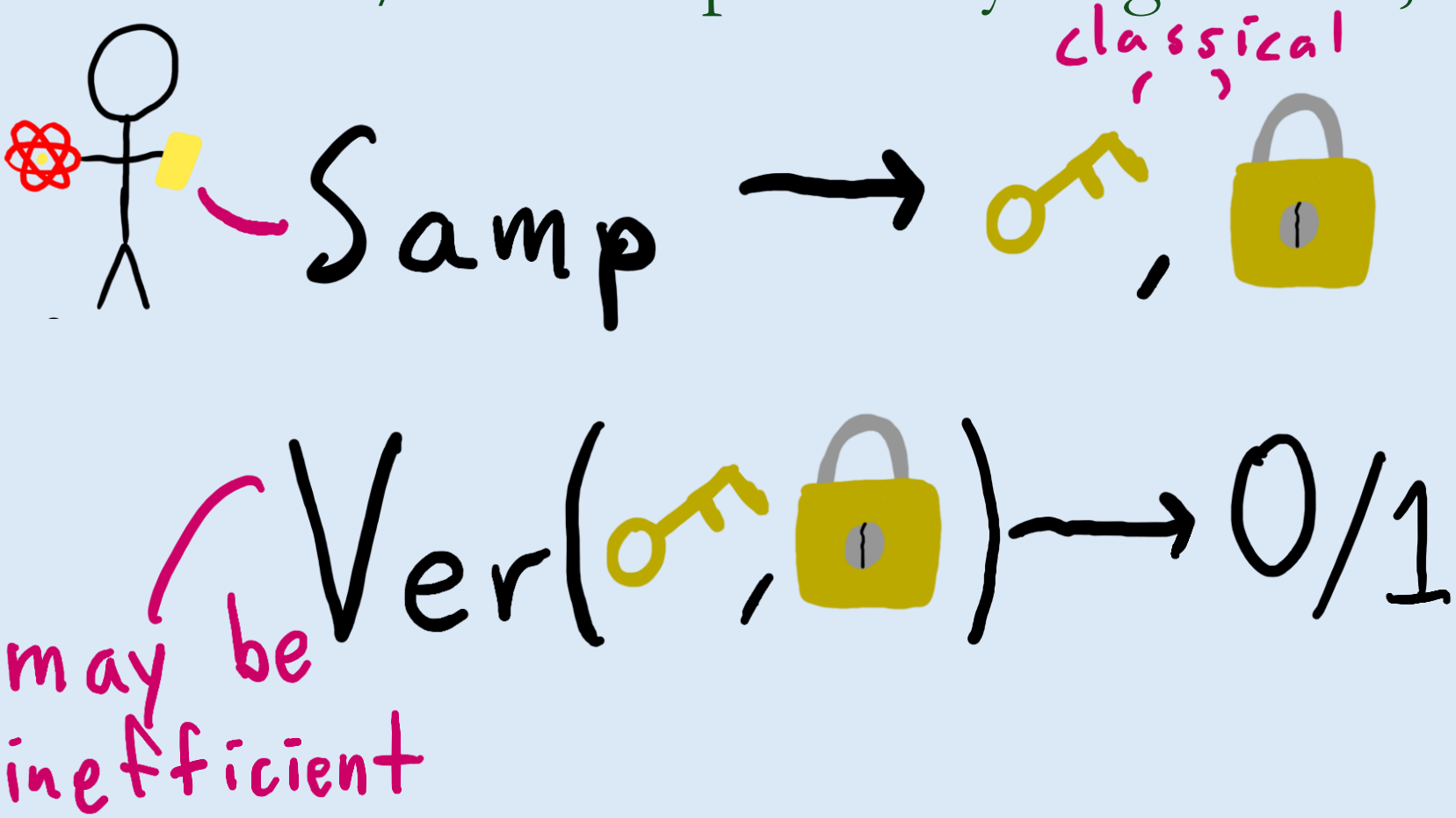

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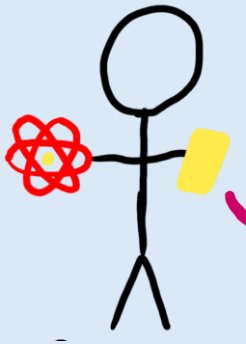

Samp $\longrightarrow$ 

classical

Ver() $\longrightarrow$ 0/1

may be inefficient

Correctness

$\Pr[\mathrm{Ver}(\text{🔑}, \text{🔒}) \to 1] \geq 1 - \mathrm{negl}$

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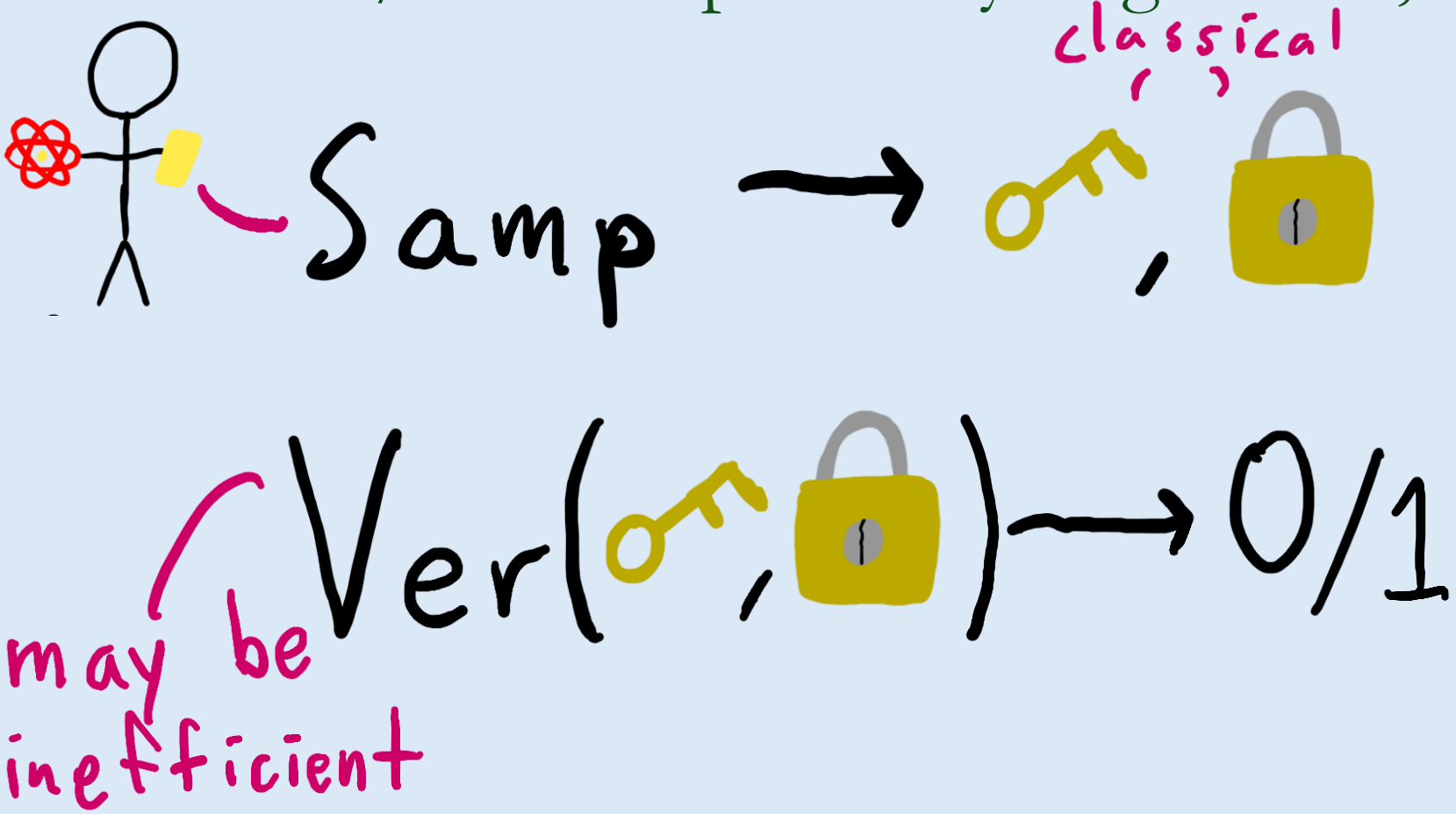

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classically $\approx$ OWF!

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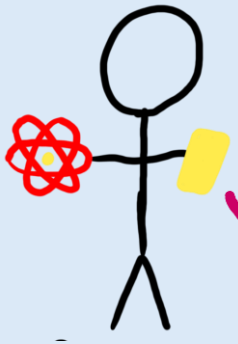

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Are one way puzzles a good central primitive?

Important properties of central primitives

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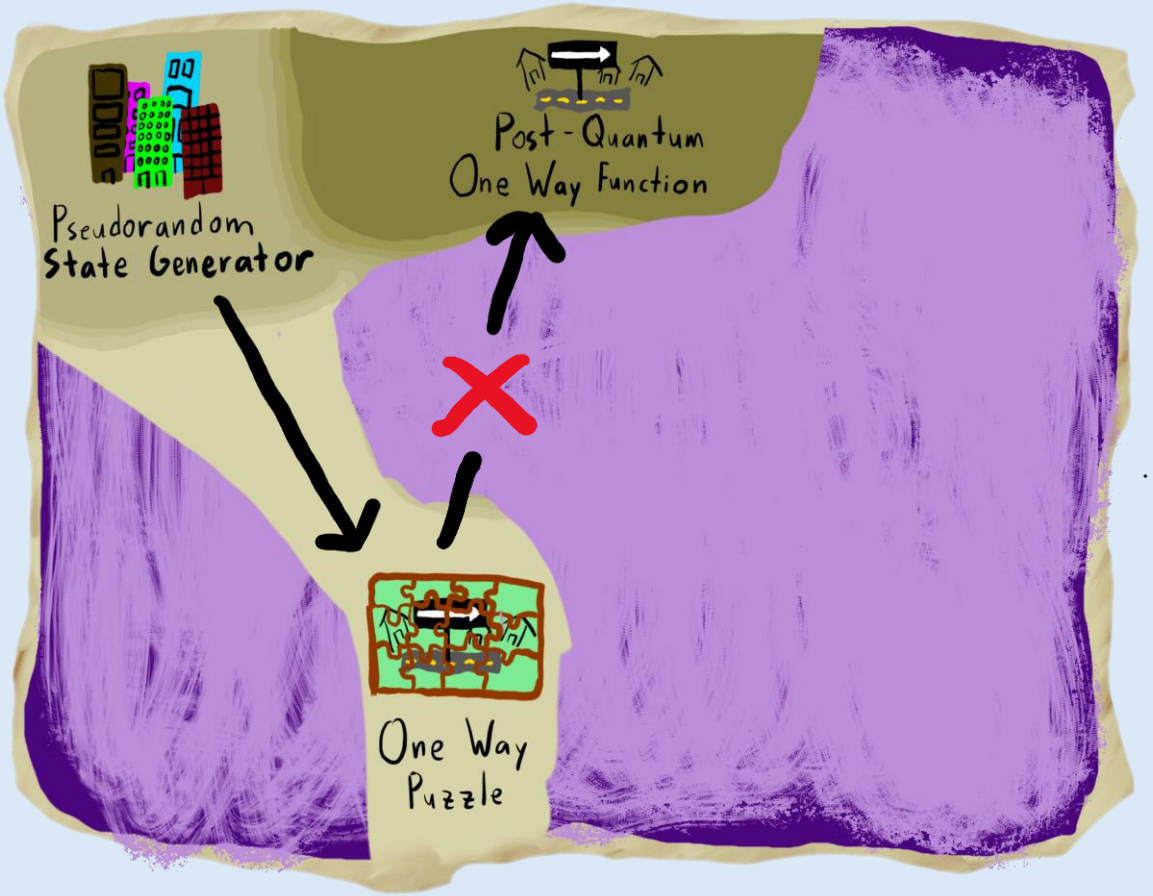

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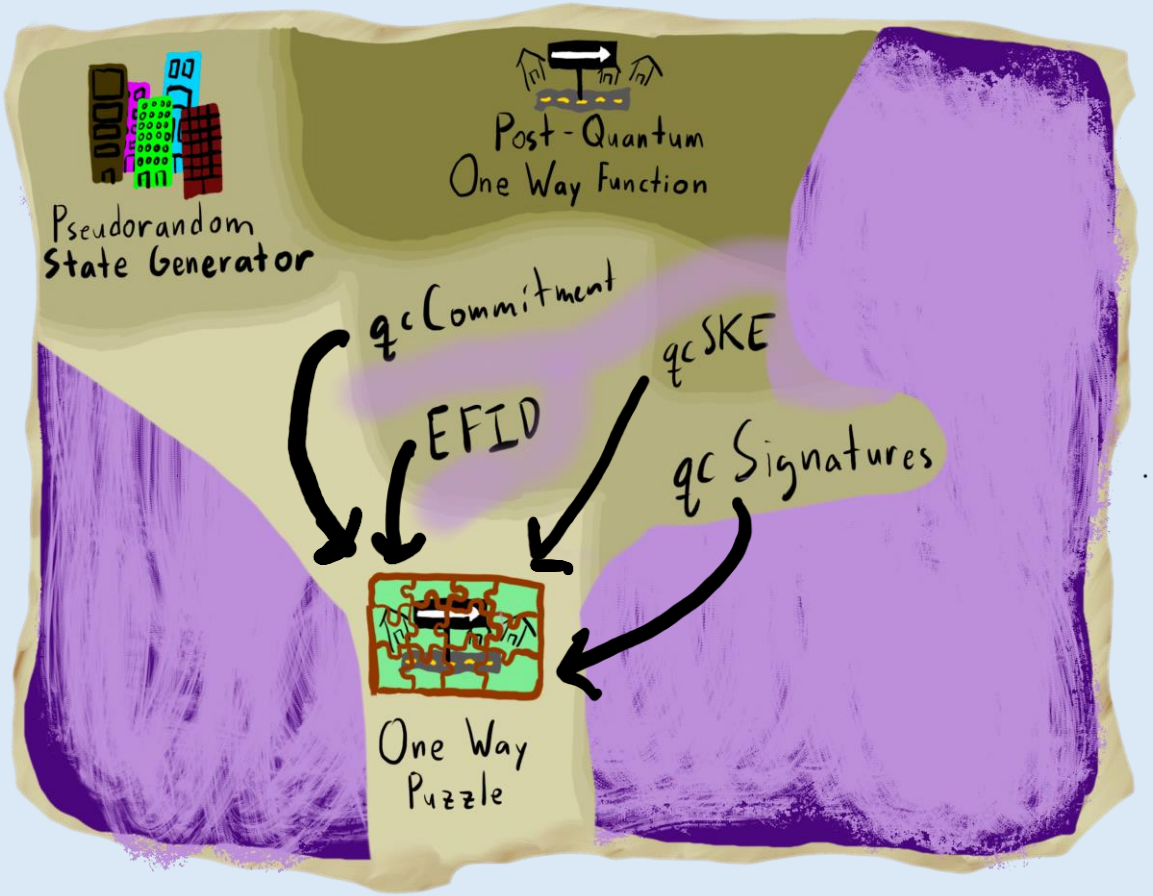

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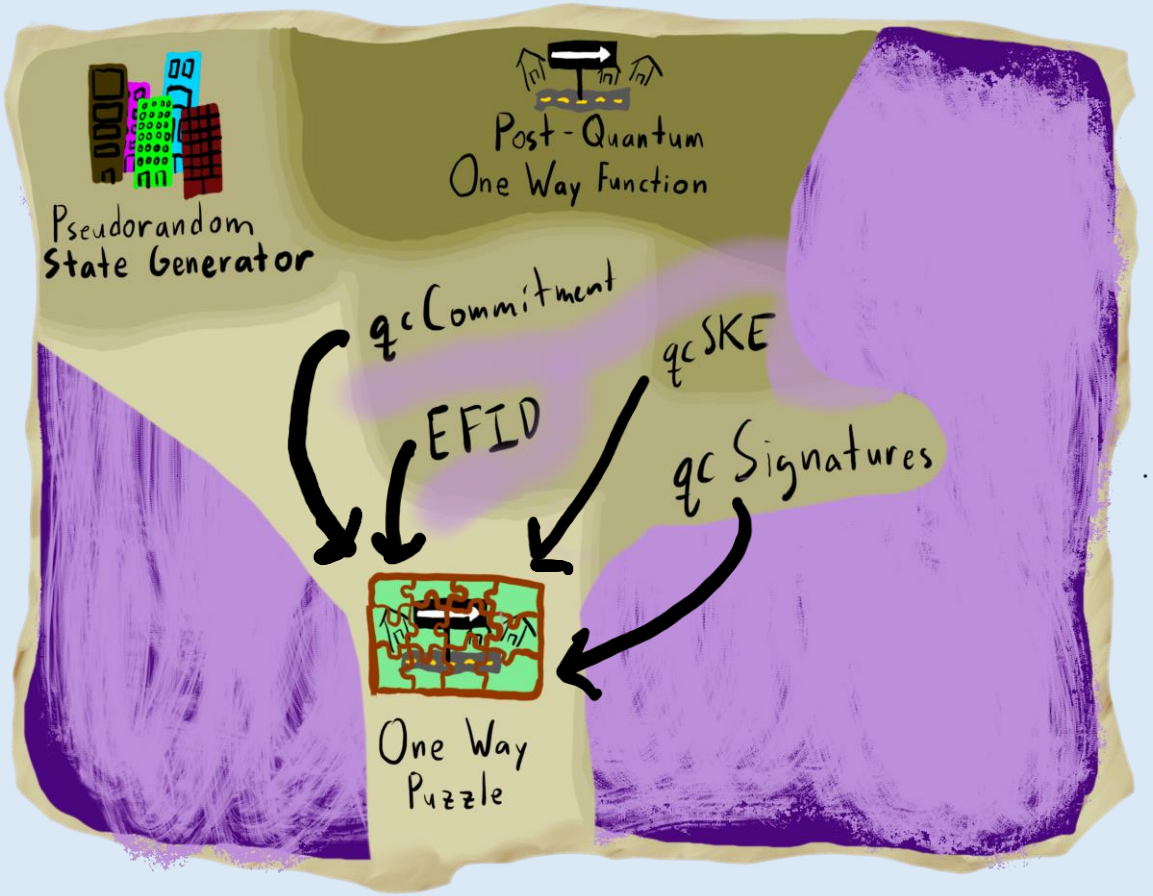

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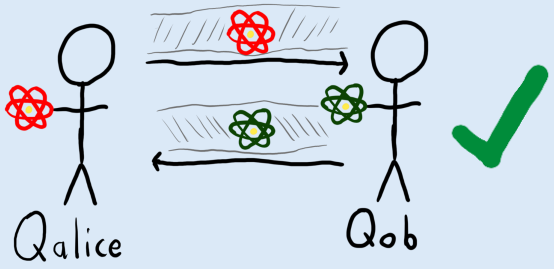

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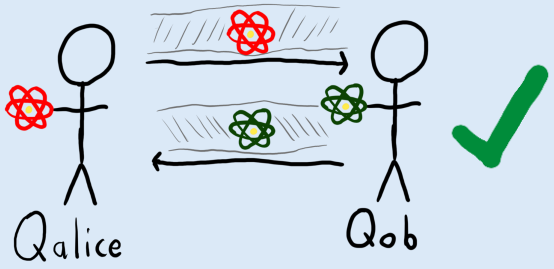

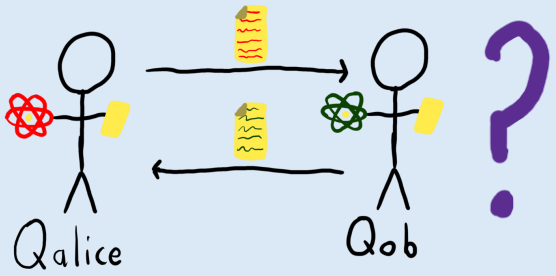

Our Questions

1. Are there any QC primitives implied by one way puzzles?
2. Are one way puzzles well-behaved?

Our Results

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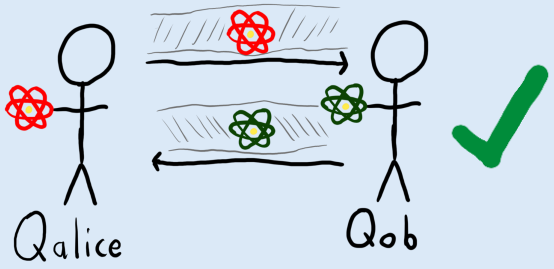

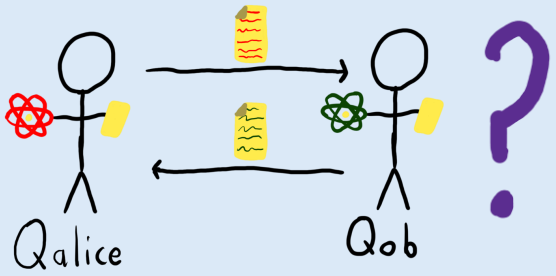

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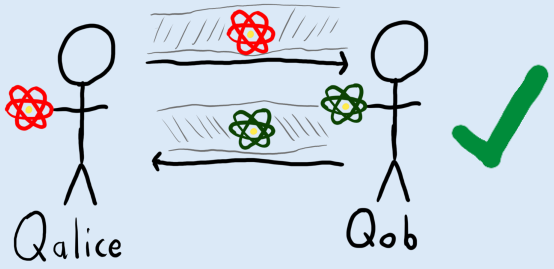

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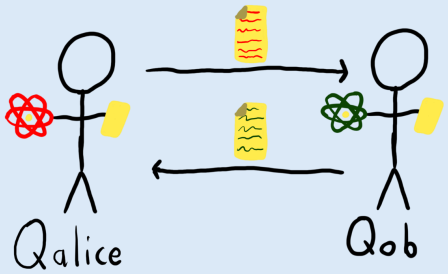

sorta

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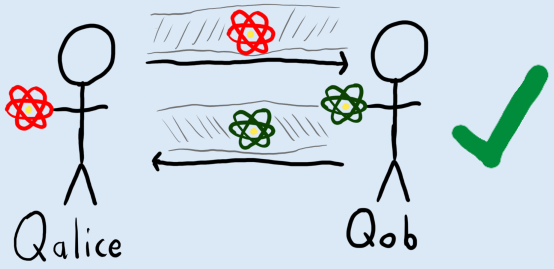

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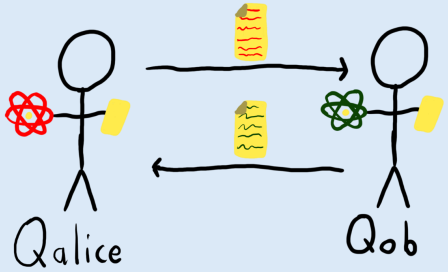

sorta

One Way Puzzles are Minimal [KT24]

Signature → One Way Puzzle

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Signature $\longrightarrow$ One Way Puzzle

$\mathsf{Gen} \rightarrow sk, vk$

$\mathsf{Sign}(sk, m) \rightarrow \sigma$

$\mathsf{VerSig}(vk, m, \sigma) \rightarrow 0/1$

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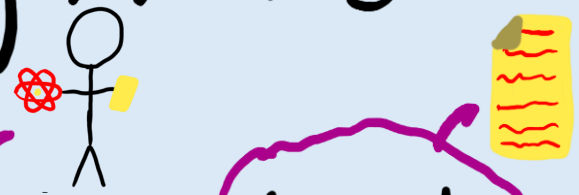

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Signature → One Way Puzzle

Gen → sk, vk

Sign(sk, m) → σ

VerSig(vk, m, σ) → 0/1

Samp →  = σ,  = vk

Sign(sk, 0)

k

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$\mathrm{Ver}(\sigma, vk) = \mathrm{VerSig}(vk, 0, \sigma)$

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Ver(σ, vk) = VerSig(vk, 0, σ)

can be inefficient

Are One Way Puzzles Useful?

Signature → One Way Puzzle

Gen → sk, vk

Sign(sk, m) → σ

VerSig(vk, m, σ) → 0/1

Samp →  = σ,  = vk

k s

Sign(sk, 0)

Ver(σ, vk) = VerSig(vk, 0, σ)

efficient!

Are One Way Puzzles Useful?

Signature $\xrightarrow{\text{EV}}$ One Way Puzzle

$\mathrm{Gen} \to sk, vk$

$\mathrm{Sign}(sk, m) \to \sigma$

$\mathrm{VerSig}(vk, m, \sigma) \to 0/1$

$\mathrm{Samp} \to$  $= \sigma$,  $= vk$

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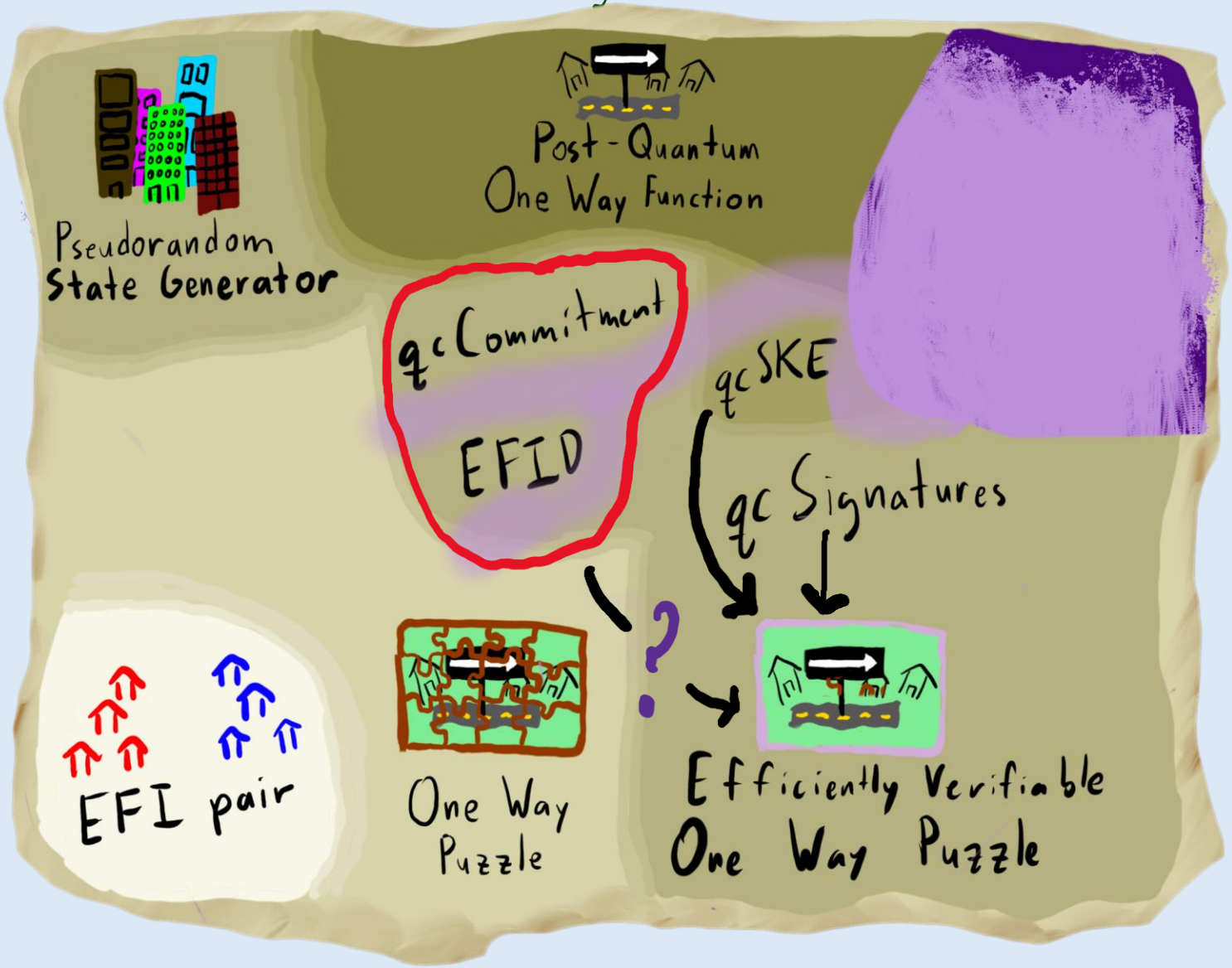
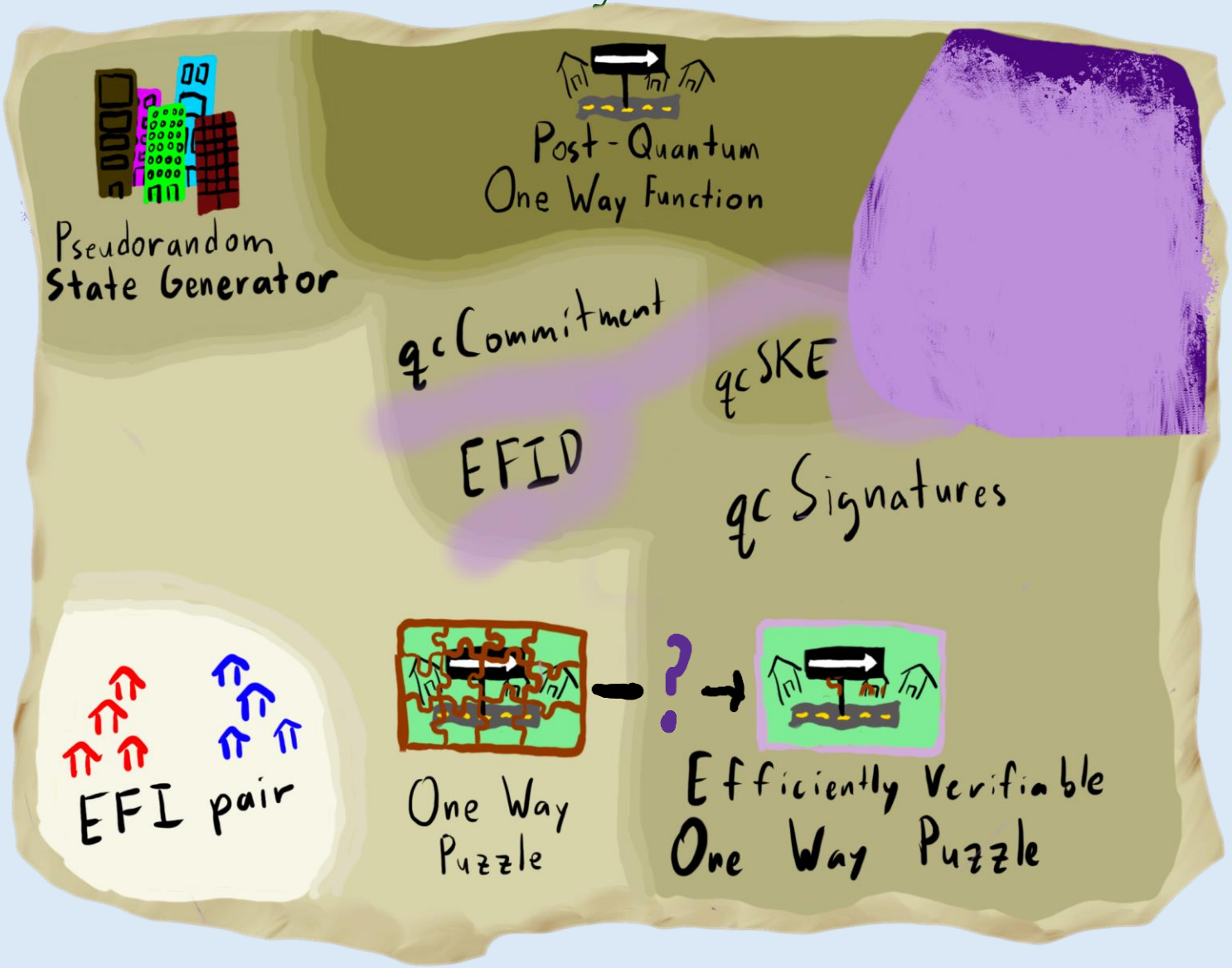


Notable exception: commitments

Are One Way Puzzles Useful?

EV-OWPuzz are (= 1-time sigs)

Are One Way Puzzles Useful?



Necessary and sufficient condition for usefulness of OWPuzz

Are One Way Puzzles Useful?

- Can we build other QC primitives from OWPuzz?

NO

exception: maybe commitments

Are One Way Puzzles Useful?

- Can we build other QC primitives from OWPuzz?

Are One Way Puzzles Useful?

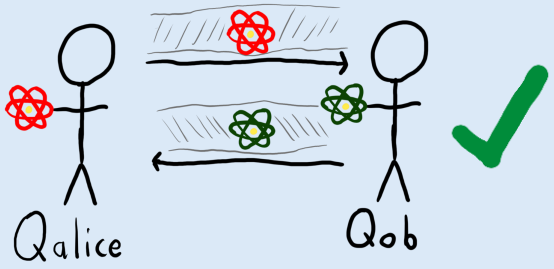
- Can we build other QC primitives from OWPuzz?

- OWPuzz can exist when BQP = QCMA [Kreschmer21]
- EV-OWPuzz broken in QCMA [INNRY22][ABOBS22]

Are one way puzzles a good central primitive?

Important properties of central primitives

1. Simple ✓
2. Minimal ✓
3. **Useful**
4. Flexible ?



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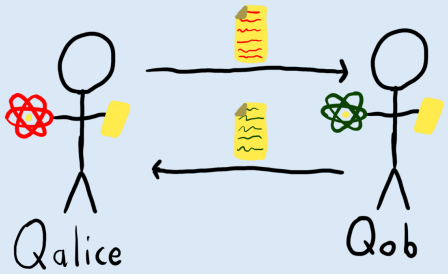

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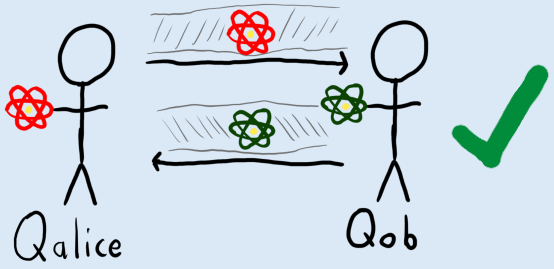

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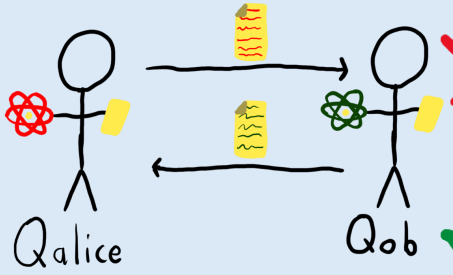

✗ OWPuzz

✓ EV-OWPuzz

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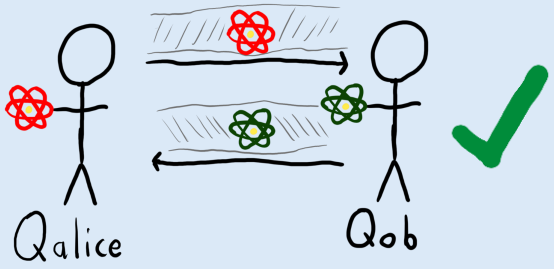

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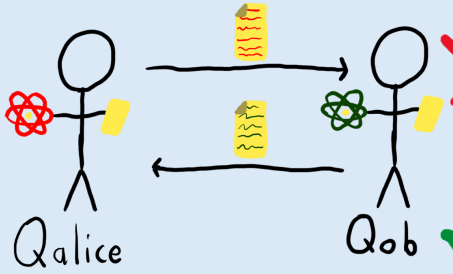

✗ OWPuzz

✓ EV-OWPuzz

$$(\mathsf{Samp}, \mathsf{Ver}) + (\mathsf{Samp}', \mathsf{Ver}') \to (\widetilde{\mathsf{Samp}}, \widetilde{\mathsf{Ver}})$$

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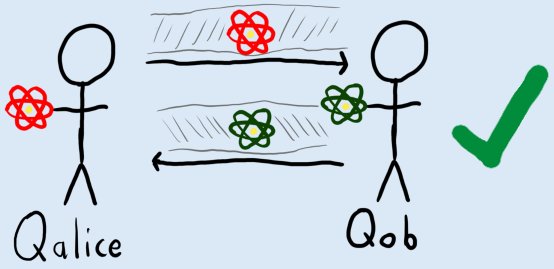

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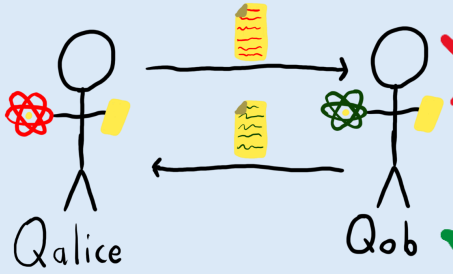

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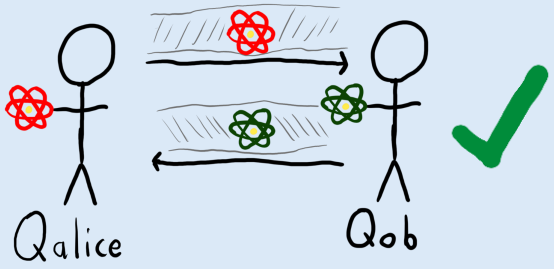

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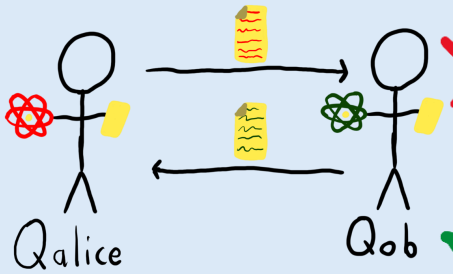

✗ OWPuzz

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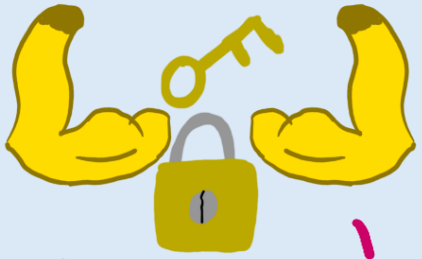
"Dist OWPuzz"

"Weak OWPuzz"



"Strong OWPuzz"

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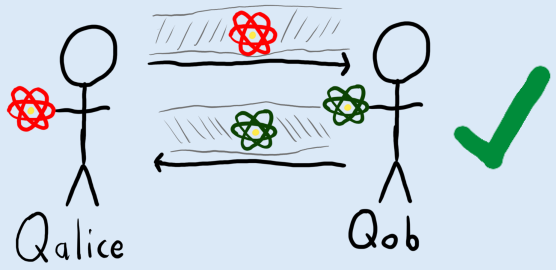

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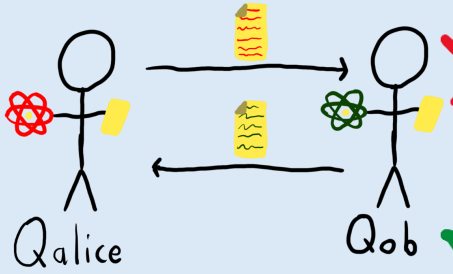

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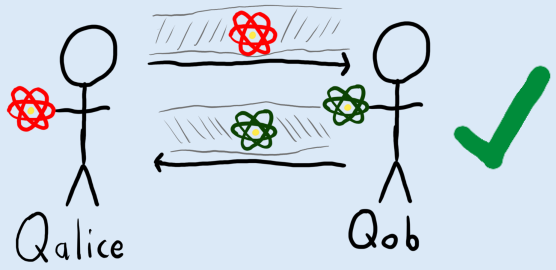

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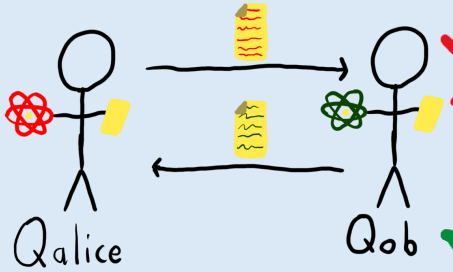

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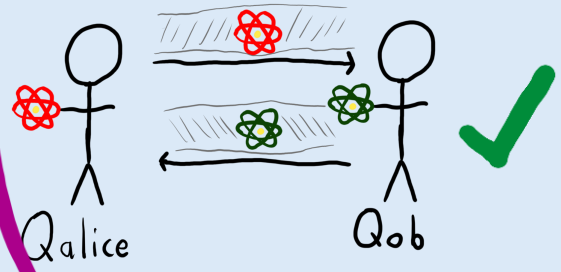

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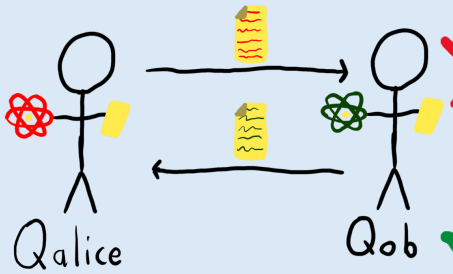

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For both OWPuzz & EV-OWPuzz

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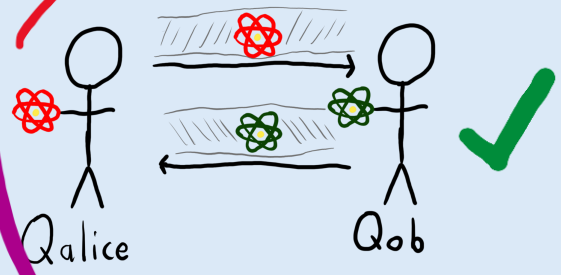
exception: commitments



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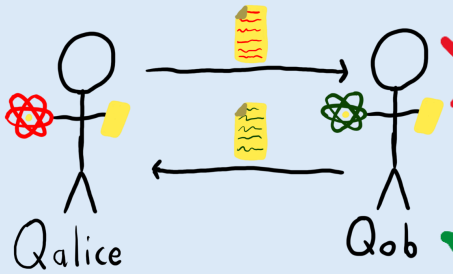

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QC Topography

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One Way Puzzle Amplification

Why is this hard?

We *can* amplify soundness for efficient 3 round quantum interactive protocols via parallel repetition [BQSY23].

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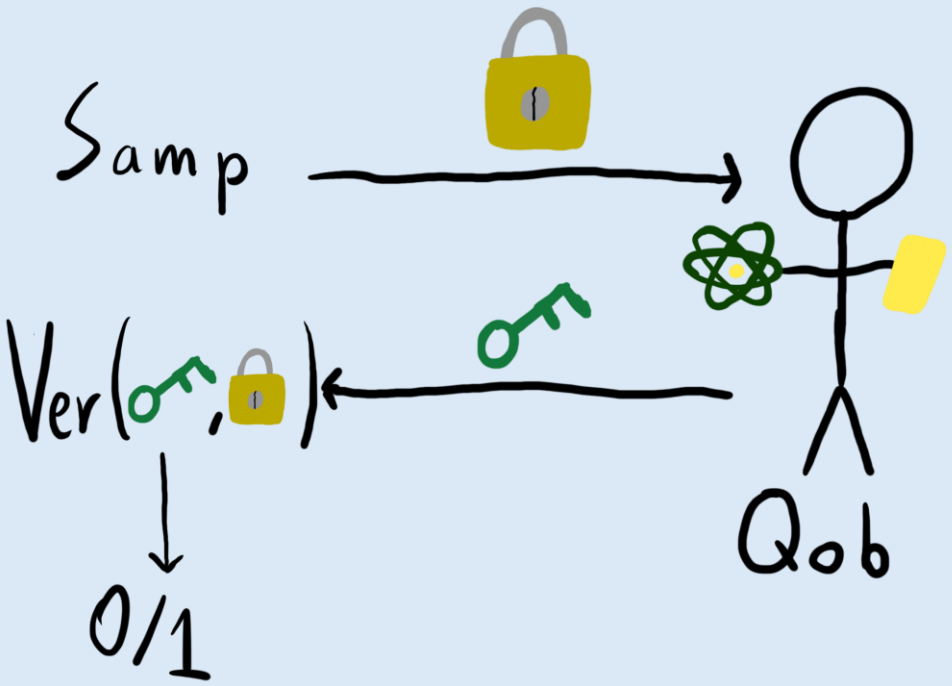
Security Game



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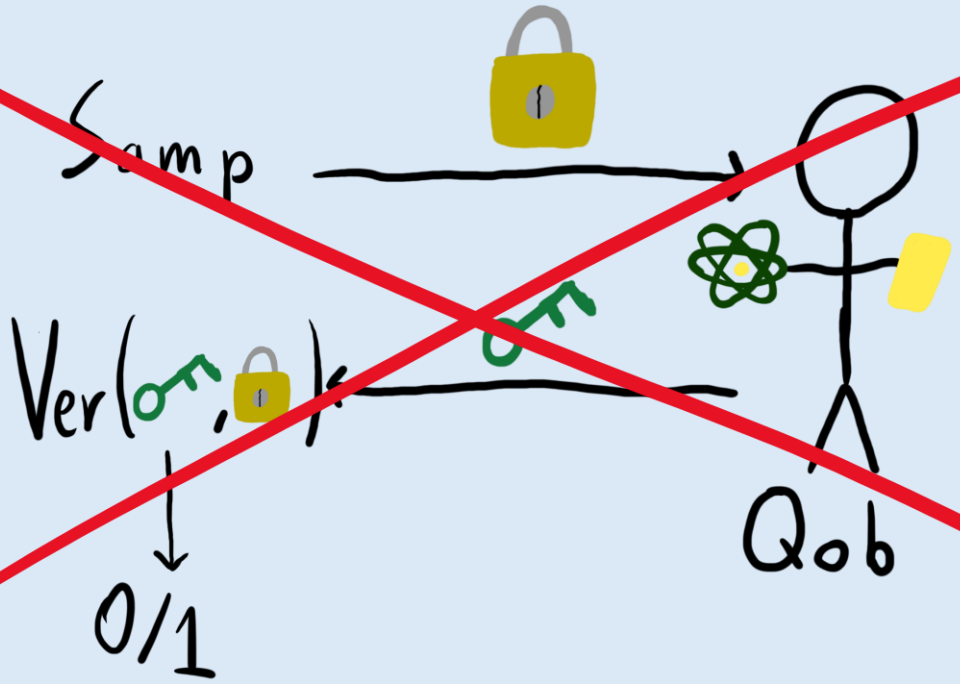

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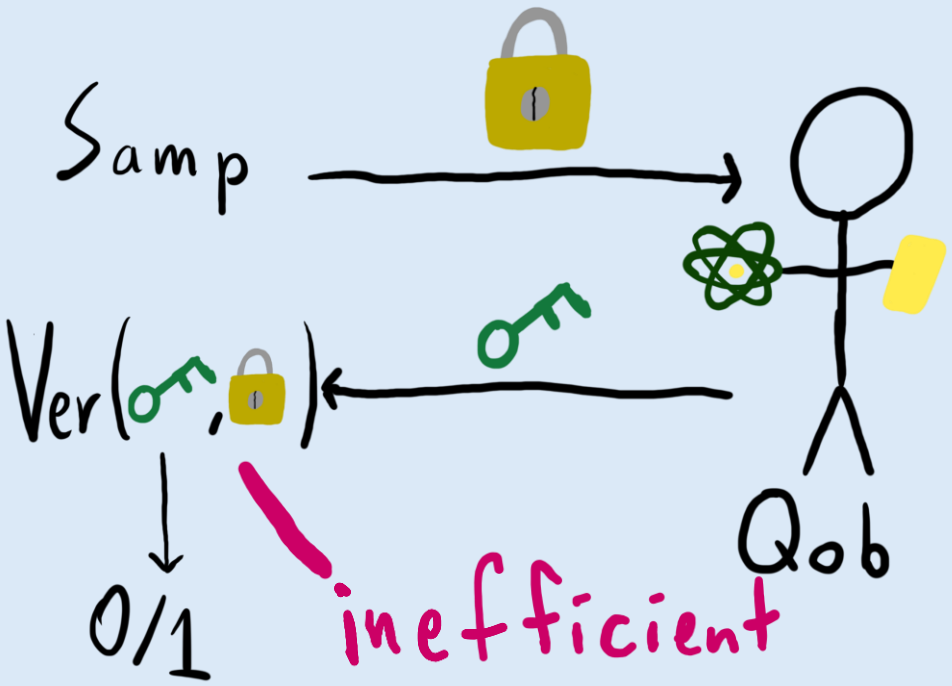

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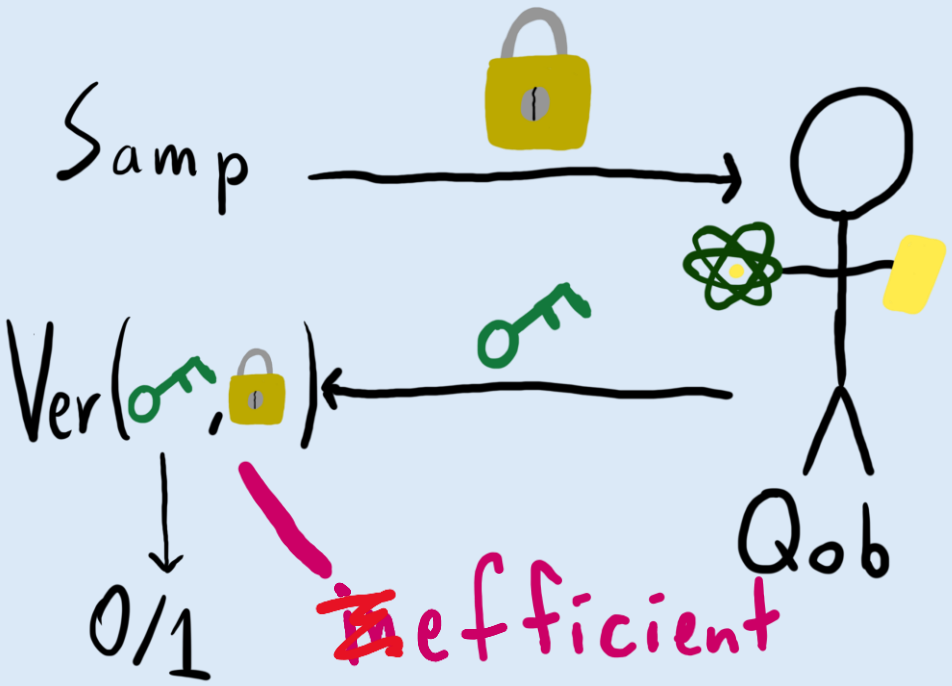

Works for EV-OWPuzz

Our approach

- [KT24] builds "kinda PRGs" (EFID) from OWPuzz using techniques of [HILL99]

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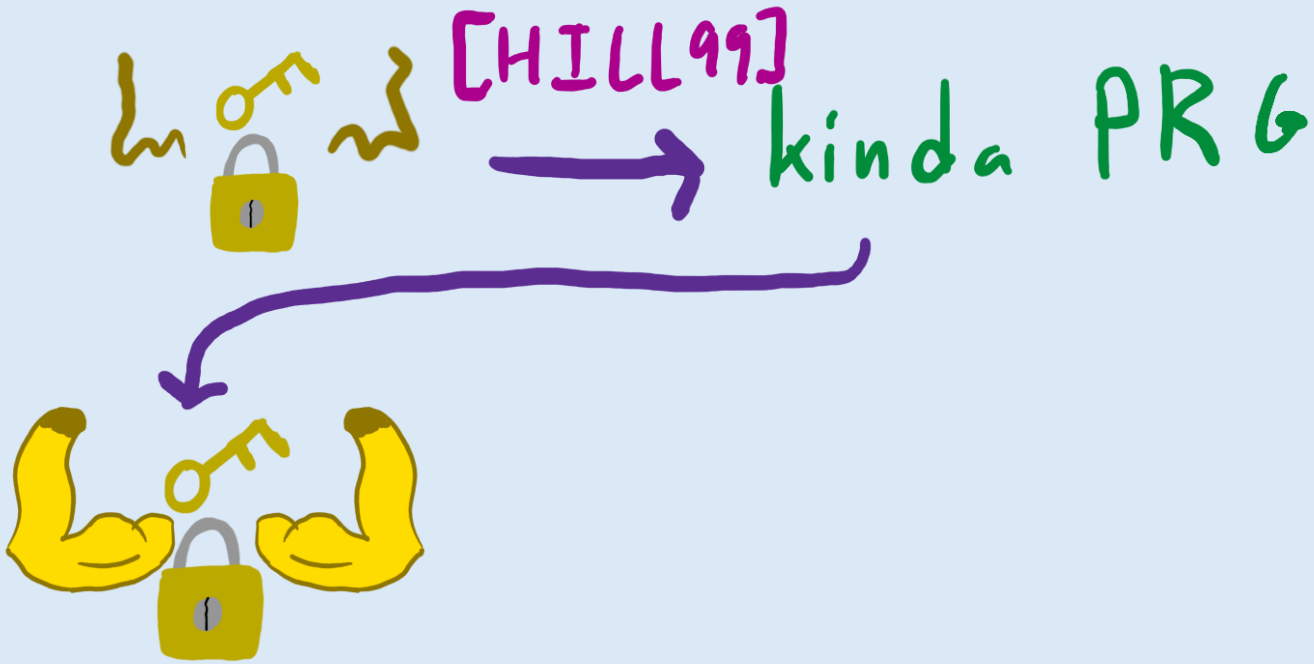

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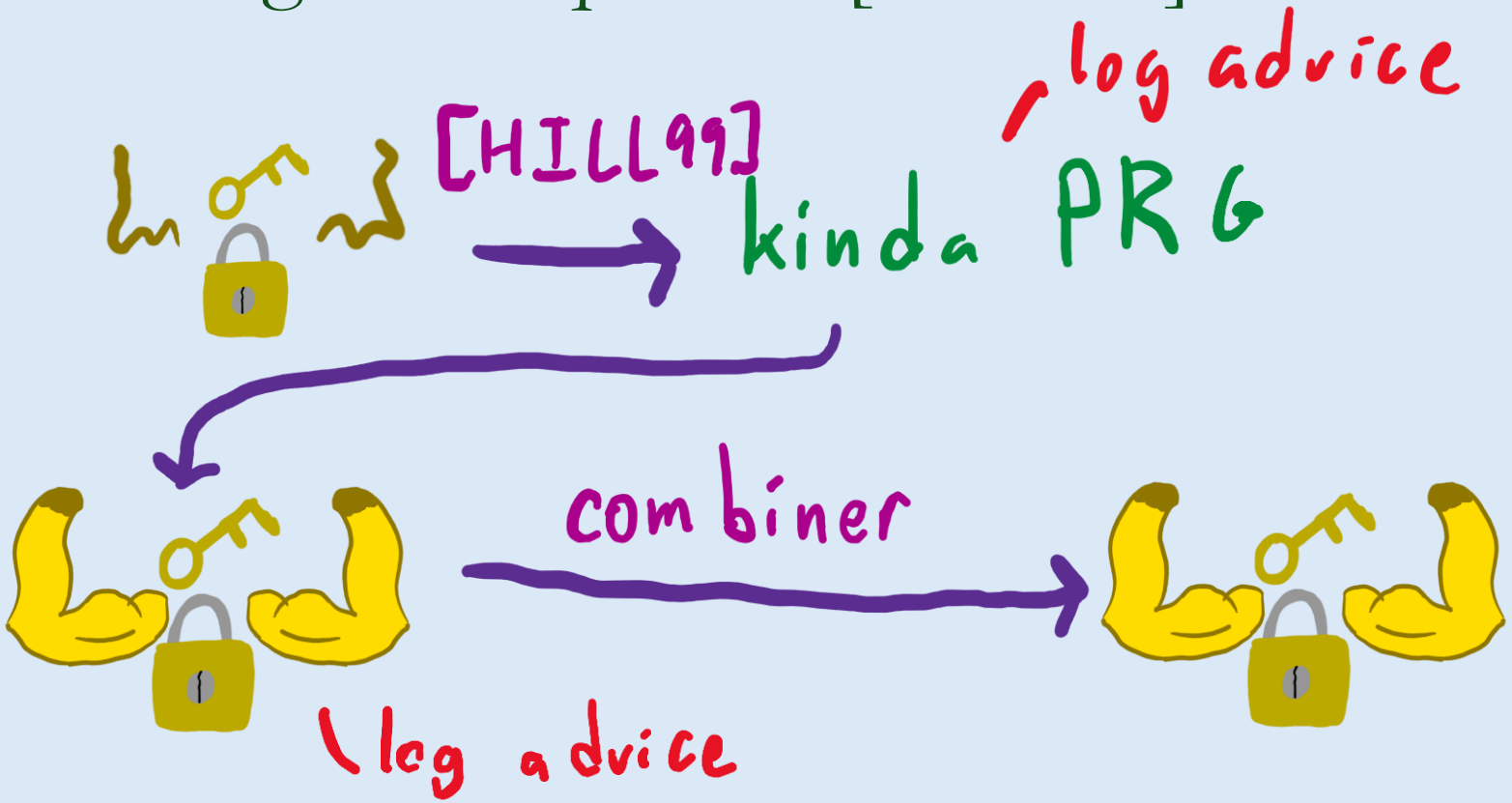

Our approach

GL can only extract 1 bit of randomness

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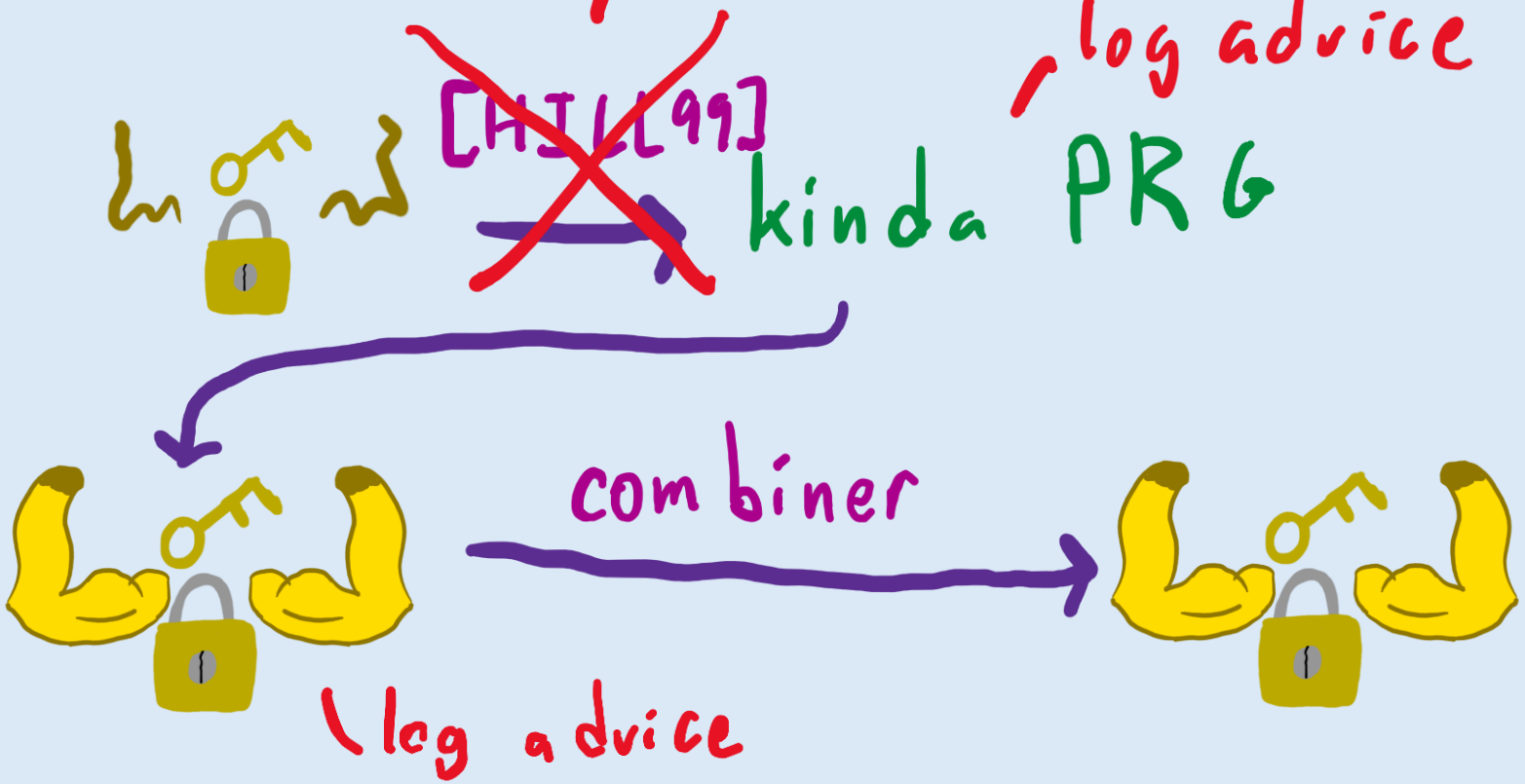

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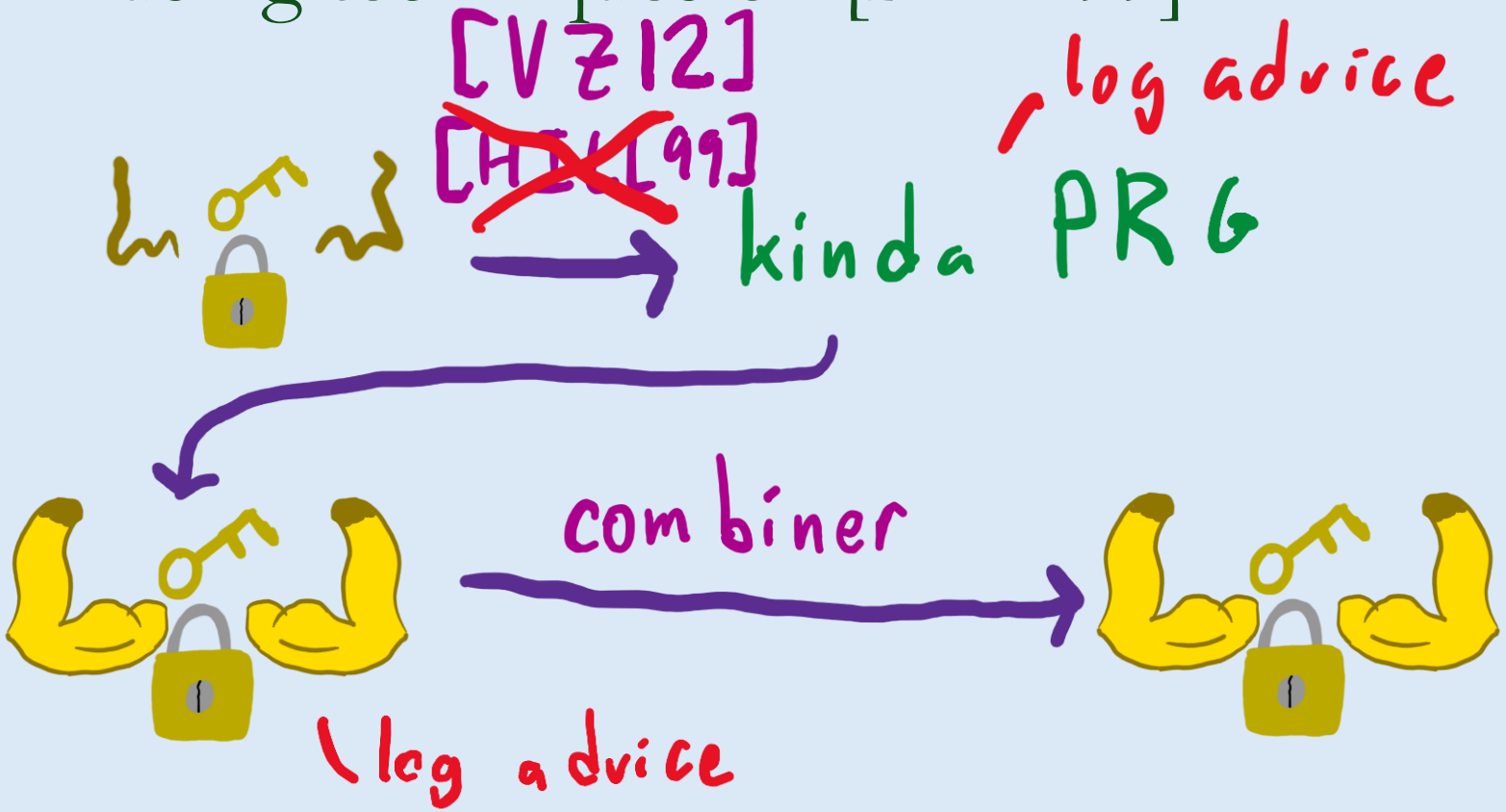

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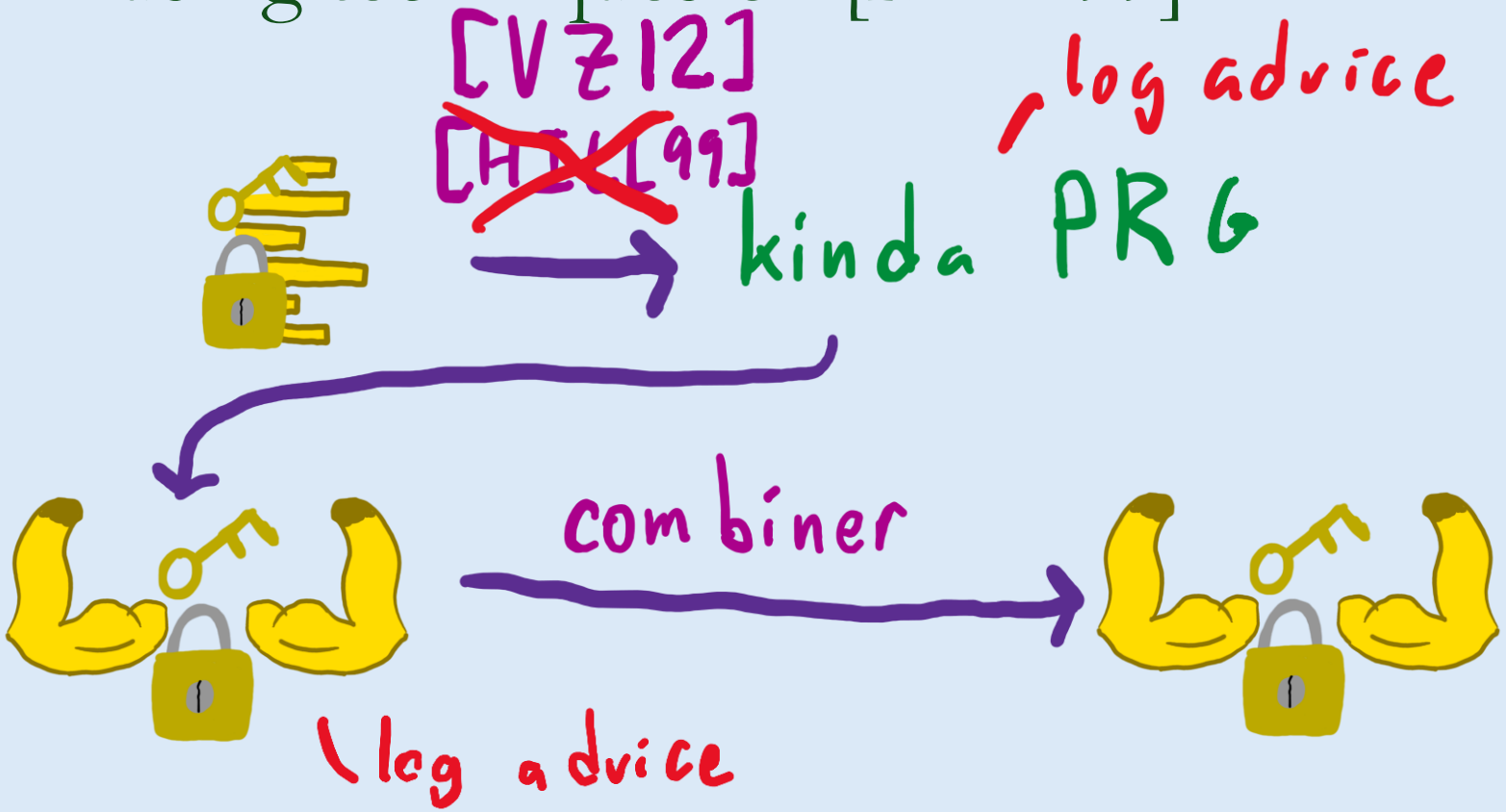

Additional Results

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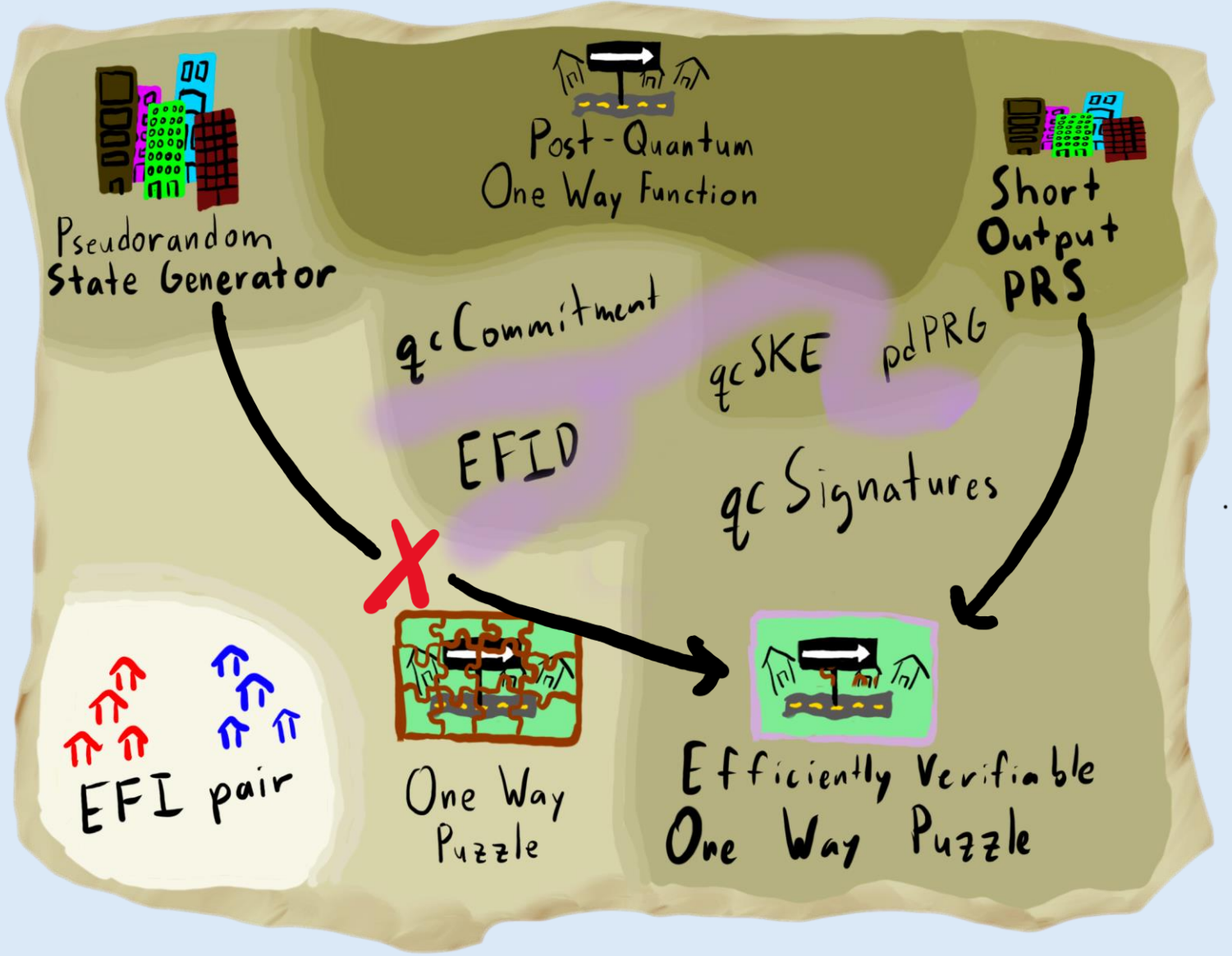

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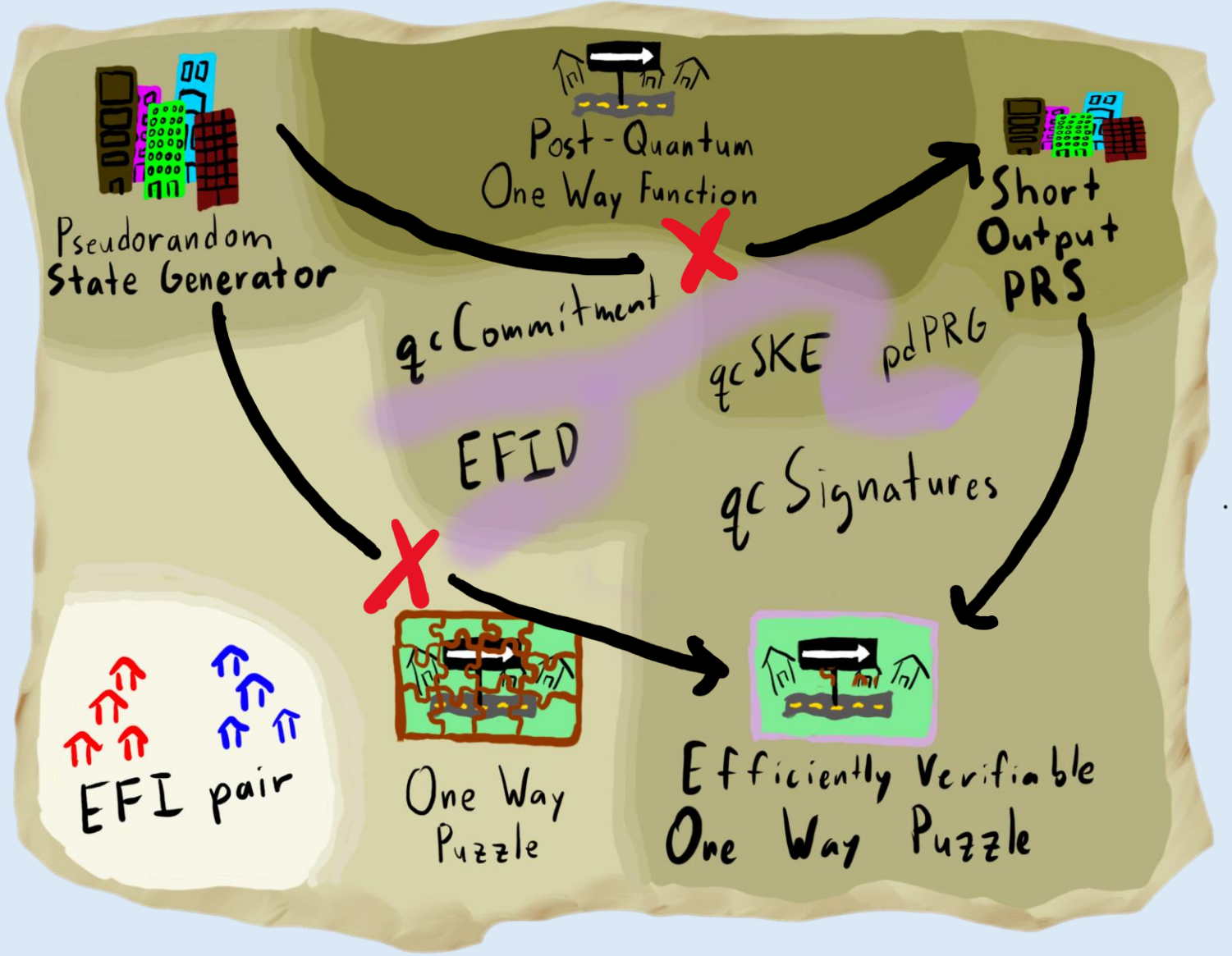

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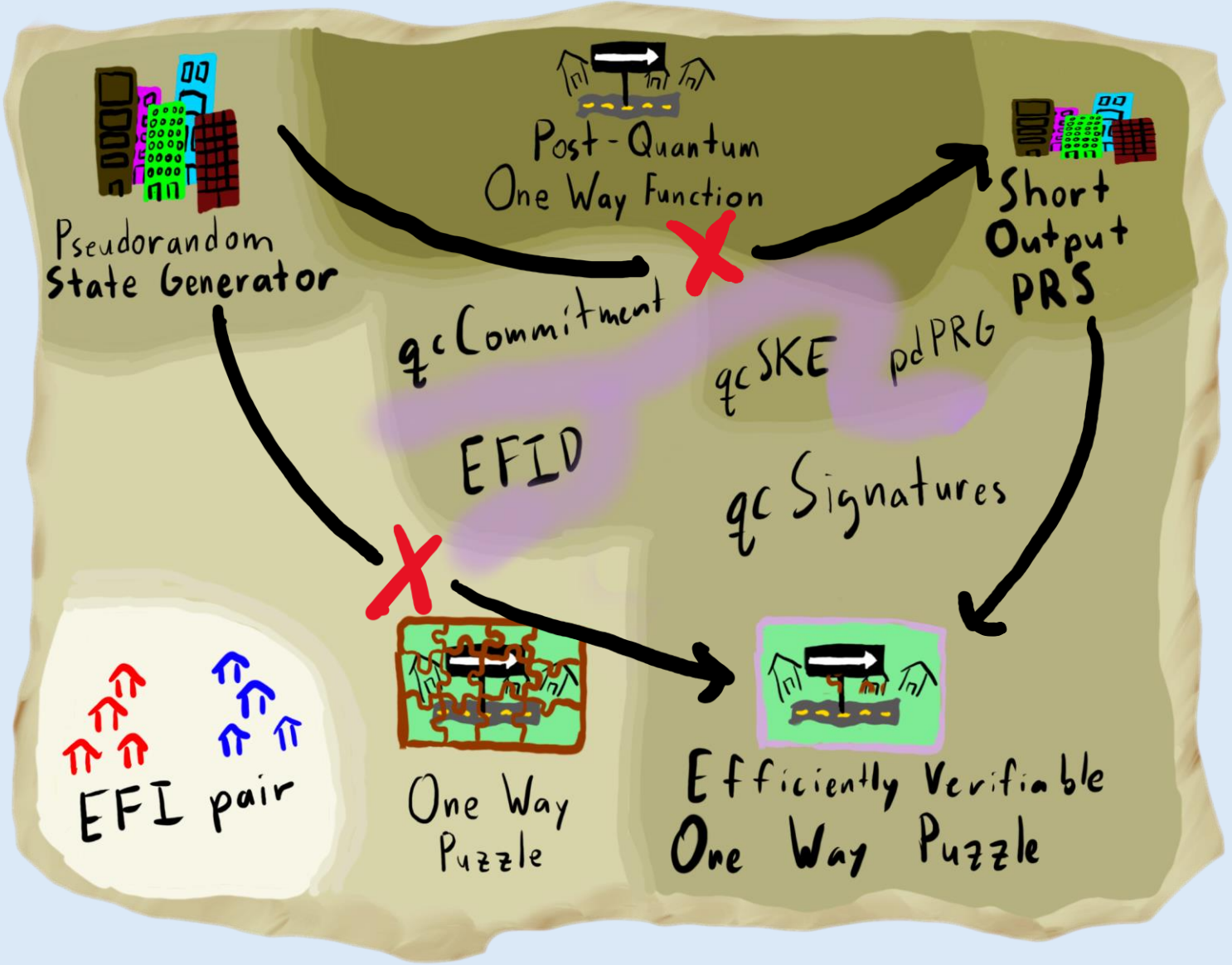
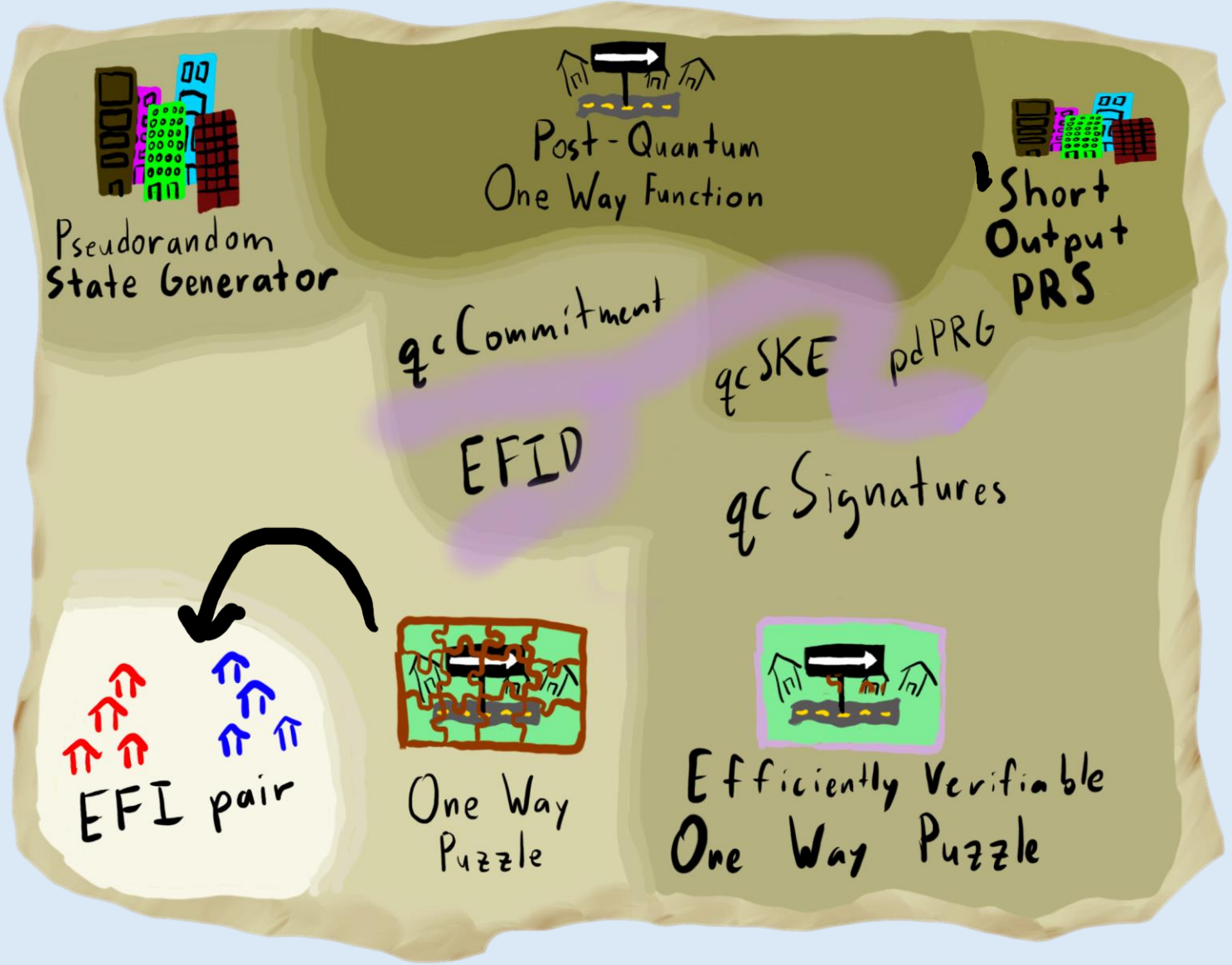


Separation between pseudorandom states with long output and short output.

Concurrently, [CM24][BM24]

Additional Results



Cleaner construction of EFI from OWPuzz using techniques of [VZ12]

Open Questions

$$\text{OWPuzz} \xrightarrow{?} \text{qc Commitments}$$

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Open Questions

OWPuzz $\xrightarrow{?}$ qc Commitments

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EV-OWPuzz $\xrightarrow{?}$ anything qc besides signatures