Stochastic Secret Sharing with 1-Bit Shares and Applications to MPC

Benny Applebaum **Eliran Kachlon**

TEL AVIV UNIVERSITY

Sharing a Bit by Bits

astic Secret Sharing
it Shares
and Ap MPC

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Combiners

Sharing a Bit by Bits

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Combiners

PIR

Sharing a Bit by Bits

Combiners

Secure computation of **Boolean** circuits

PIR

Dream Version: "Shamir over $\mathbb{F}_2$"

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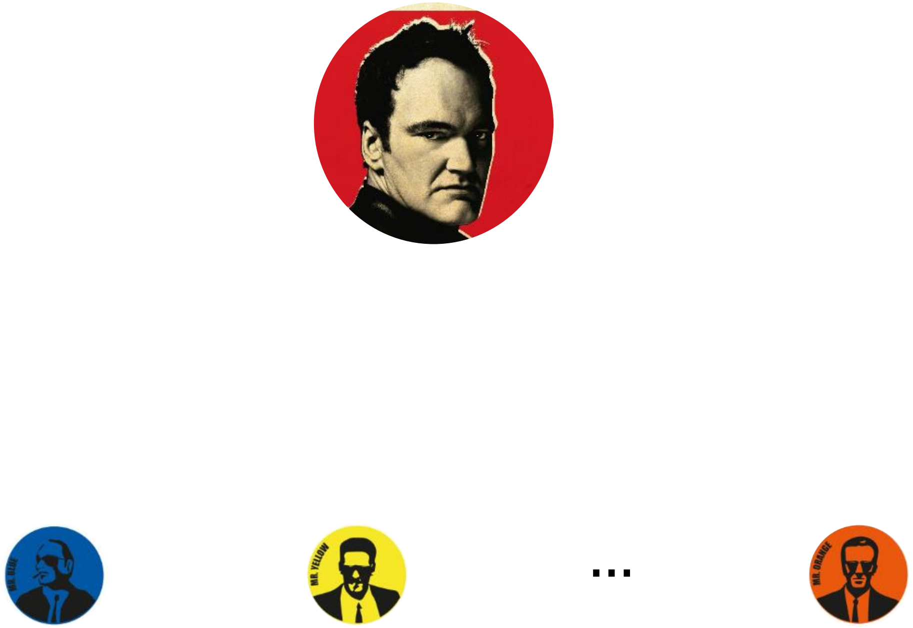

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$s \in \{0,1\}$

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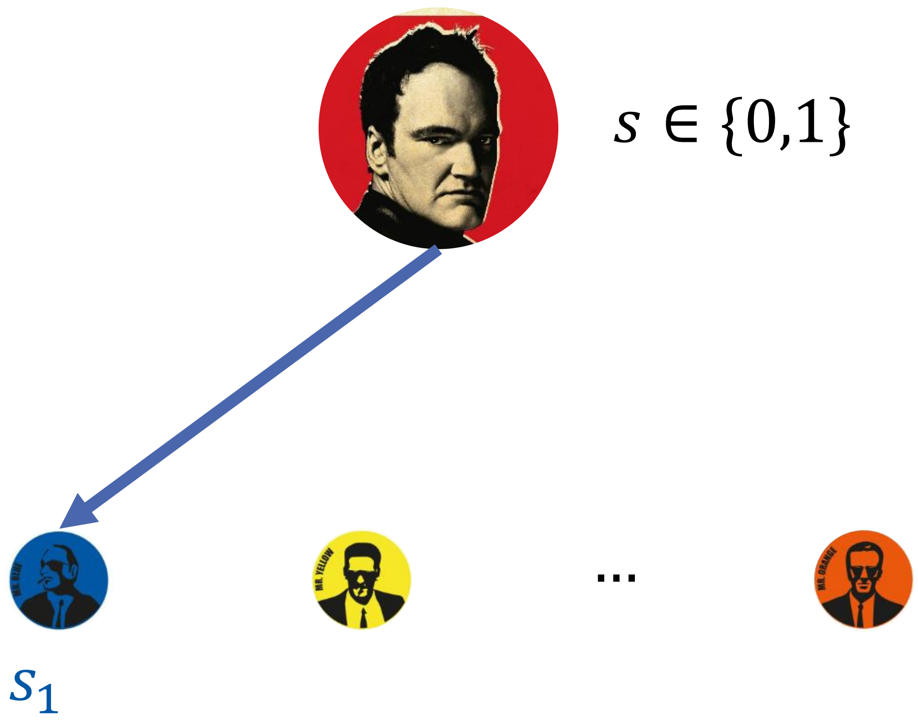

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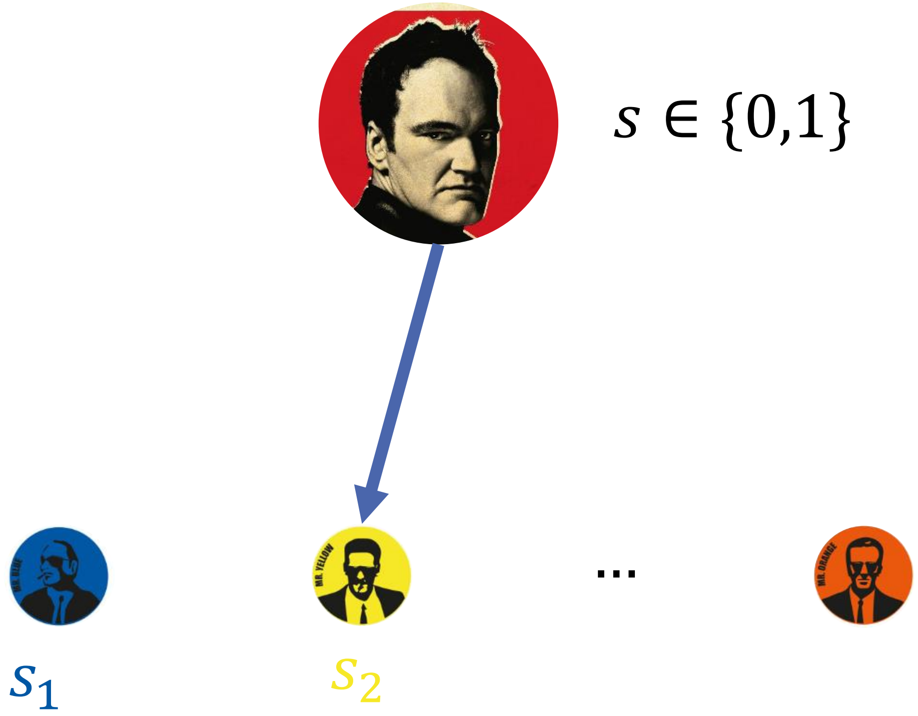

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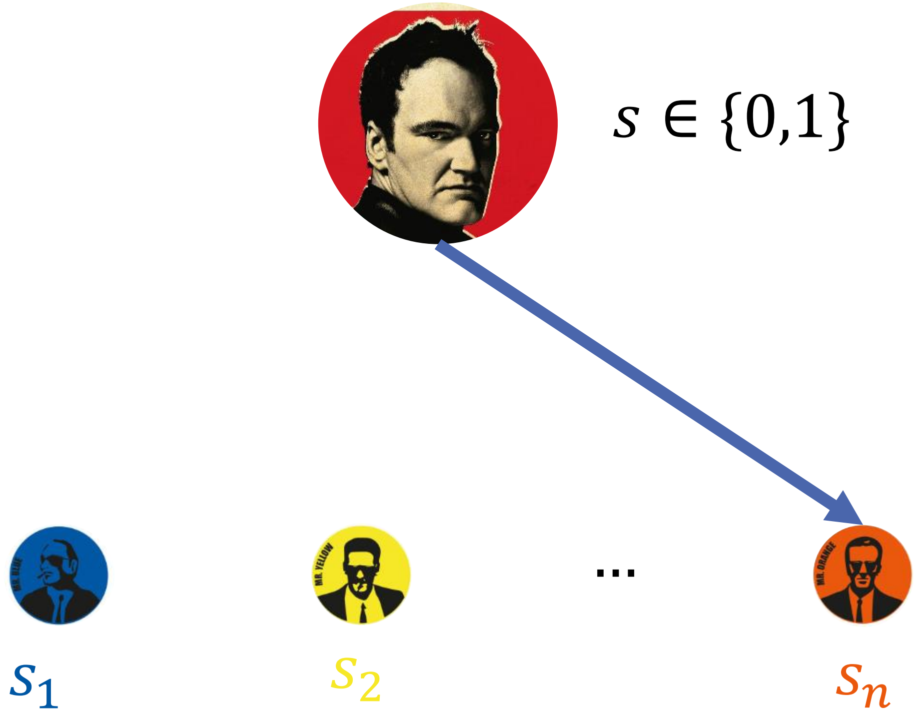

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1-bit shares!

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$s + s' \Rightarrow$	$s_1 + s_1'$	$s_2 + s_2'$		$s_n + s_n'$

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$s \cdot s' \Leftarrow$	$s_1 \cdot s'_1$	$s_2 \cdot s'_2$		$s_n \cdot s'_n$

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$\Omega(\log(n))$ is necessary for threshold secret sharing [CDN15]

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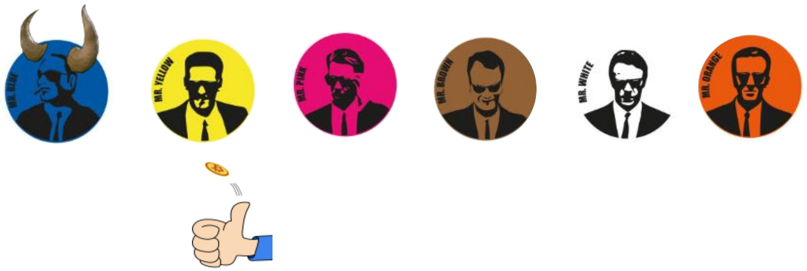

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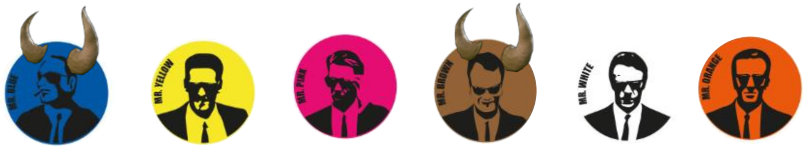

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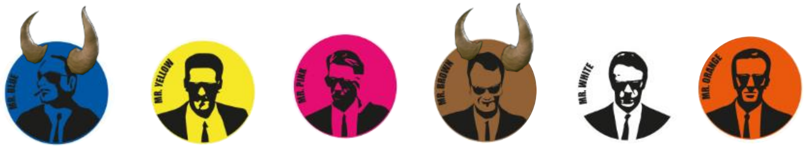

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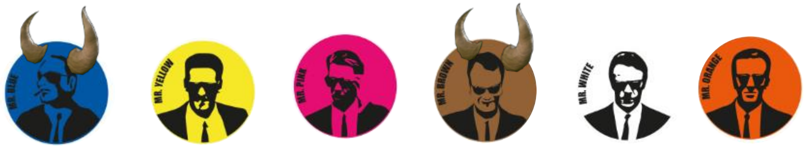

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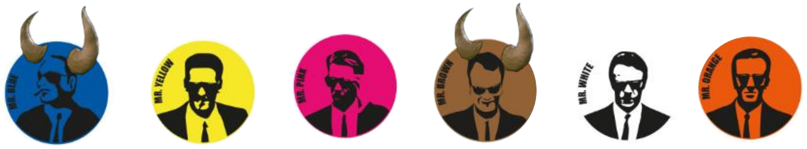

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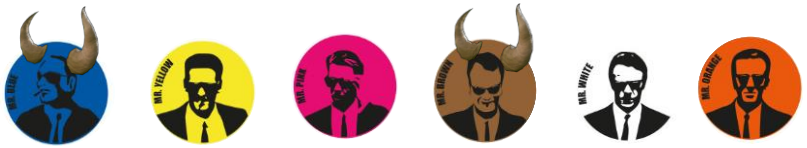

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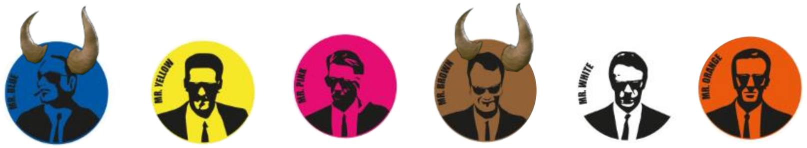

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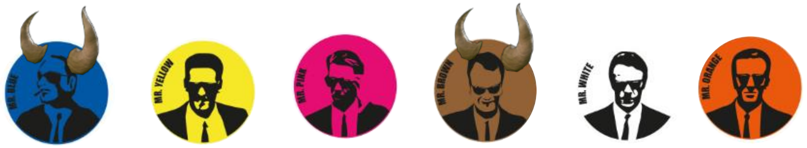

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- $\Rightarrow p < 1/2$

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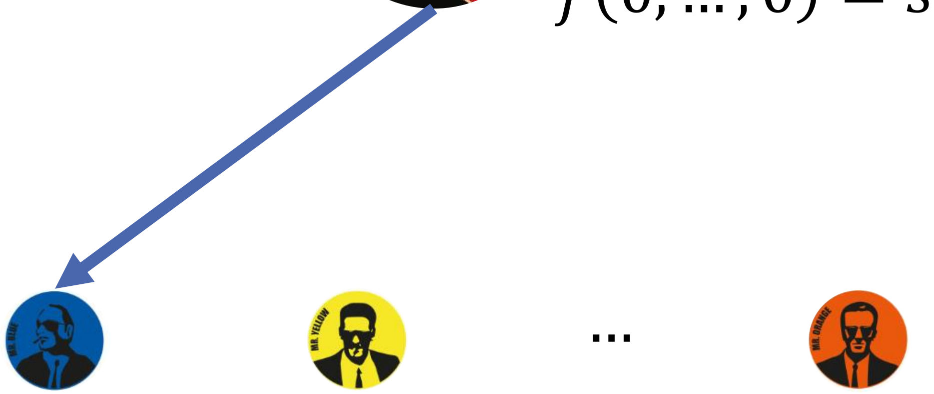

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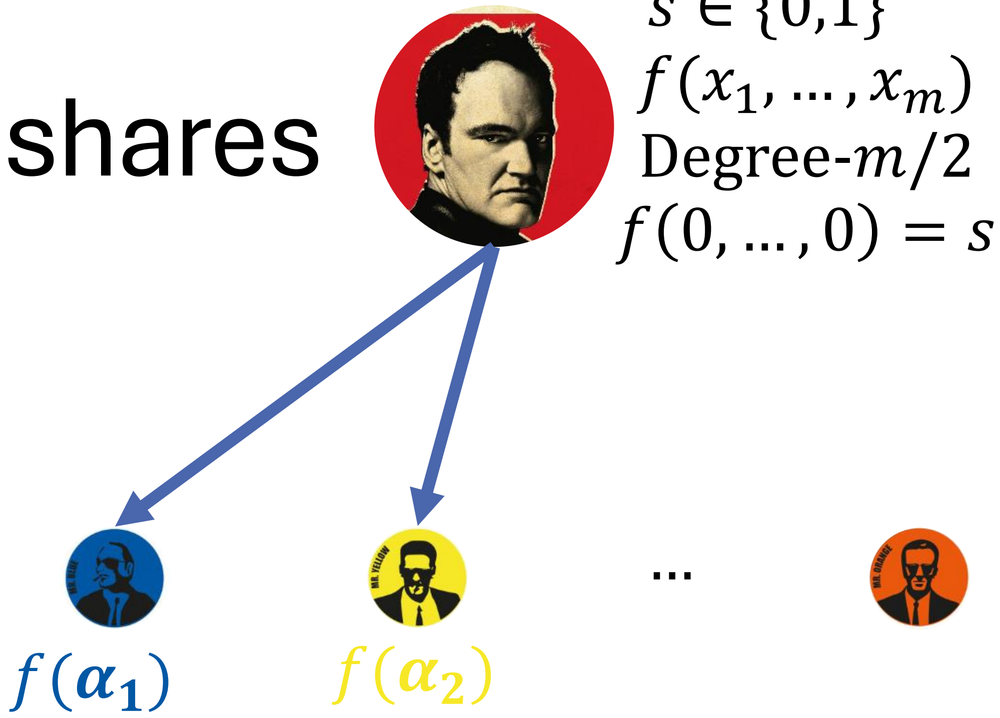

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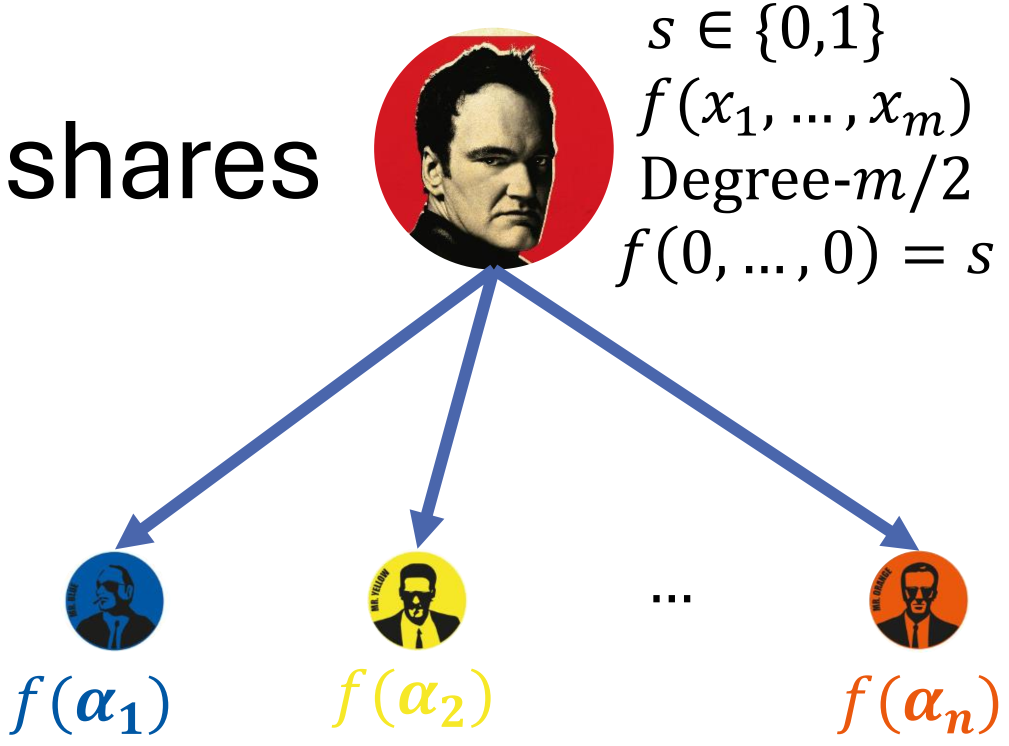

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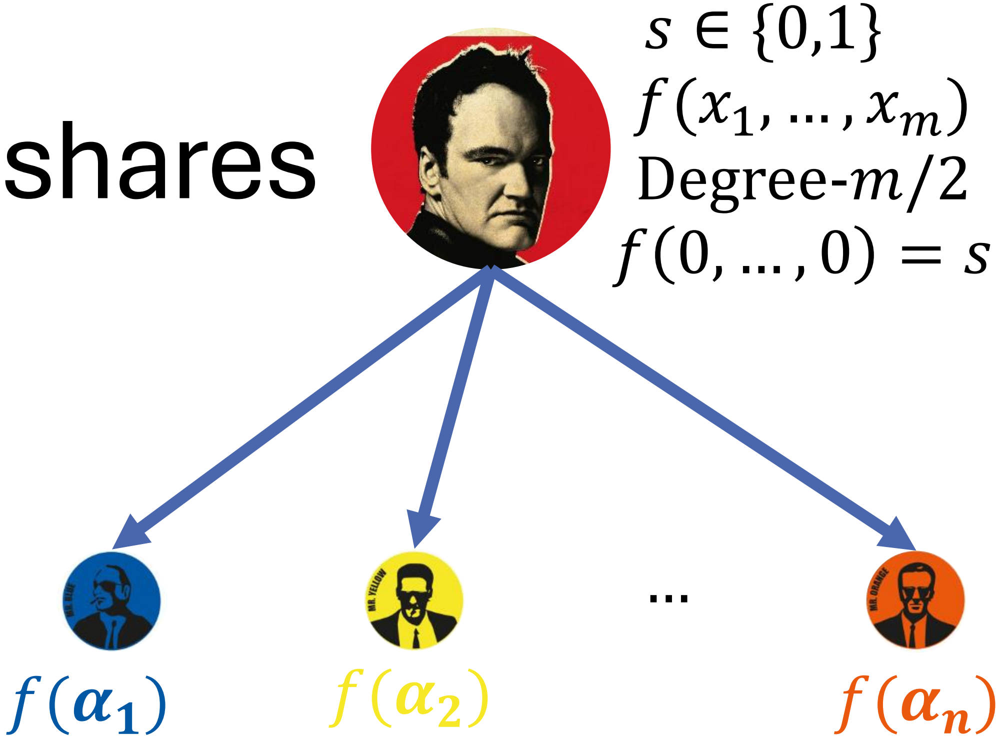

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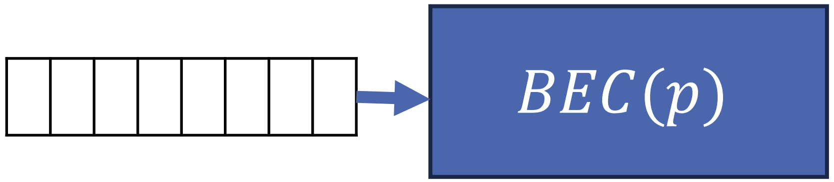

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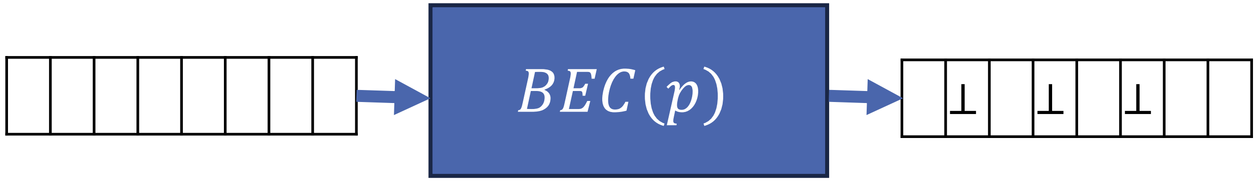

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Reed-Muller achieves capacity over $BEC(p)$ [KKMPSU17]

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Our Results: SSS with 1-bit shares

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Linear-size circuit!

Standard corruptions?

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- Static secret sharing

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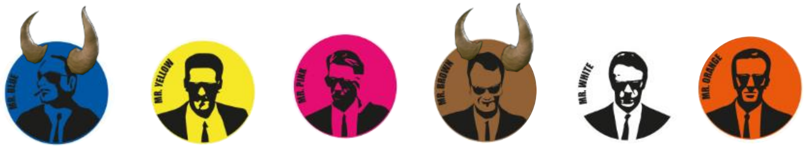

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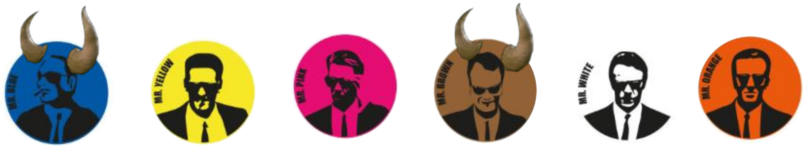

s_1 s_2 s_3 s_4 s_5 s_6

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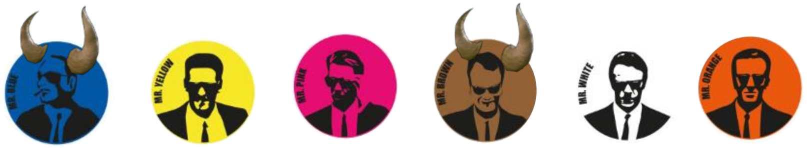

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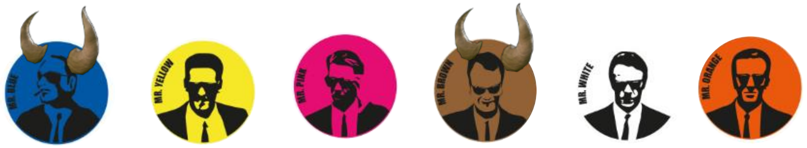
  - Public randomness chosen once and for all
    - Can be reused by the dealer and other dealers



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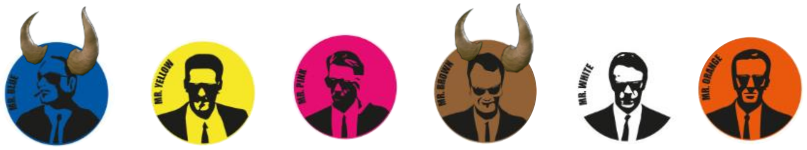
From Stochastic to Static

R



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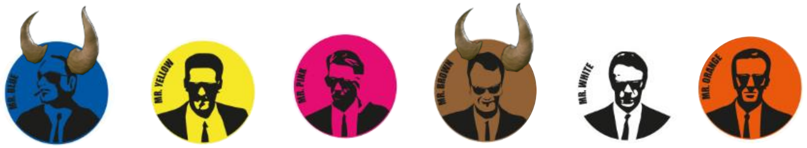


p-stochastic secret sharing with error δ

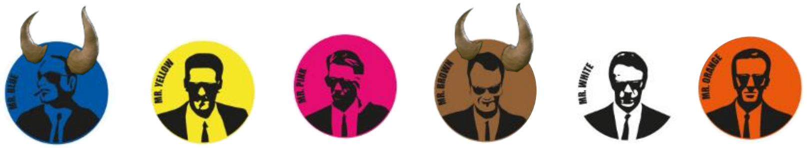
From Stochastic to Static

R

p-stochastic secret sharing with error δ

↓

Static secret sharing
threshold $t = \lfloor pn \rfloor$
error 2δ



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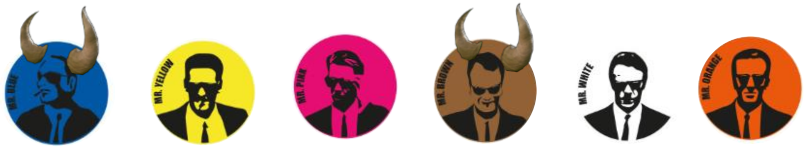


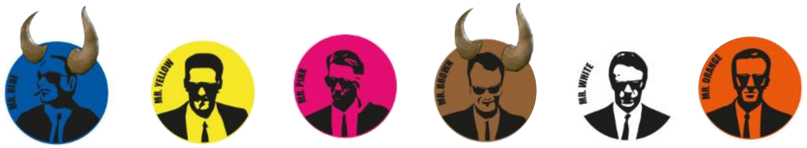
p-stochastic secret sharing with error δ

Permute the parties!

Static secret sharing threshold $t = \lfloor pn \rfloor$ error 2δ

From Stochastic to Static

π



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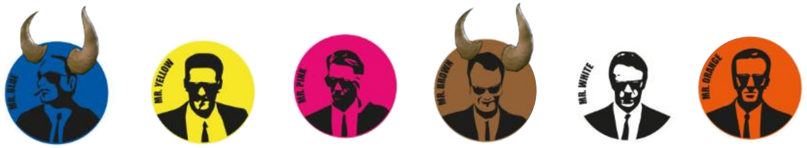

$s_{\pi(1)}$

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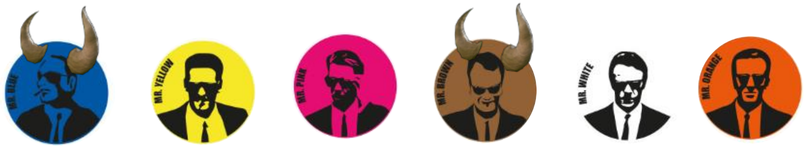

$s_{\pi(1)}$ $s_{\pi(2)}$

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Optimal computation! Amortized linear-size circuits!

Applications: Communication Efficient MPC

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Statistical MPC for **Boolean circuits**

Applications: Communication Efficient MPC

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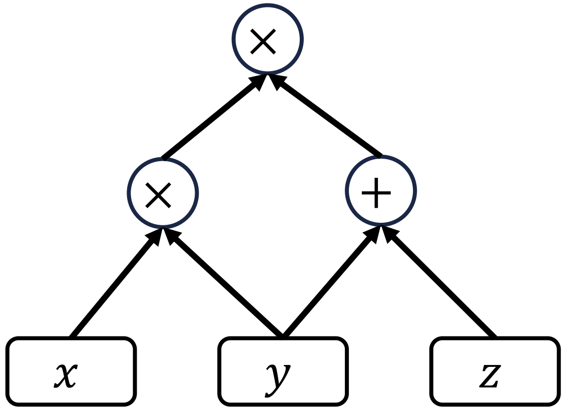

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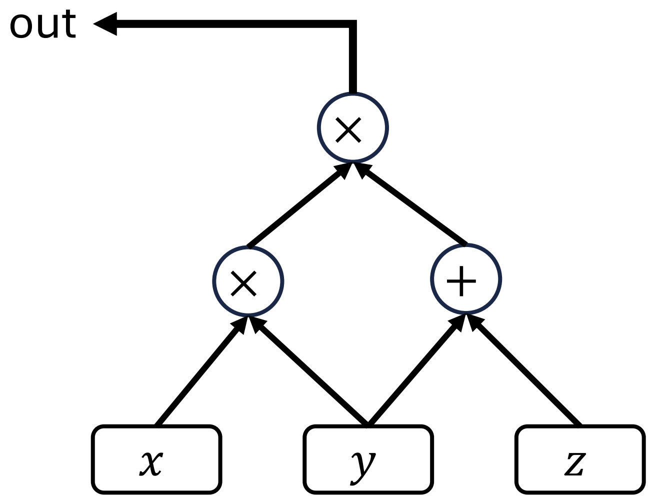

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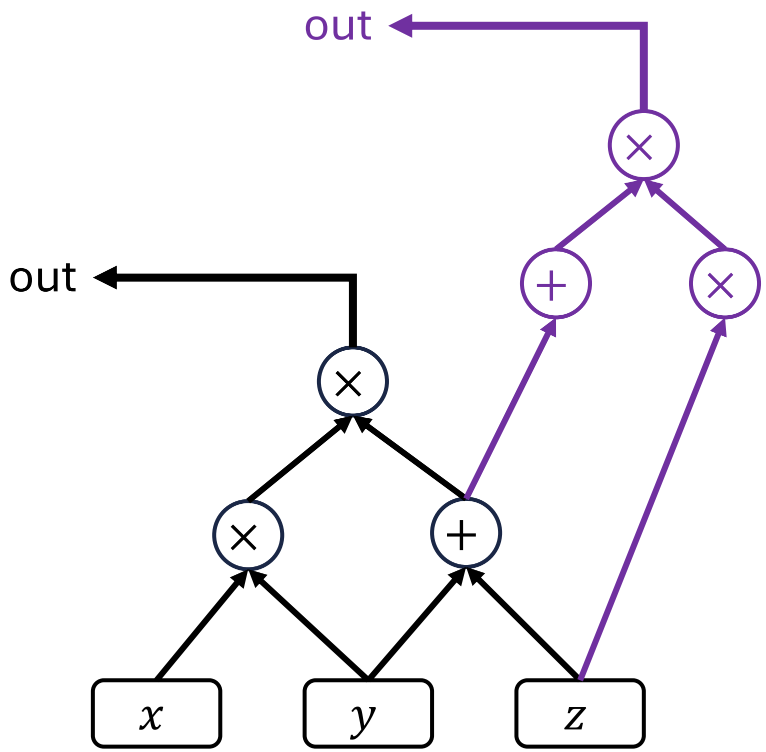

Conclusion

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Thank You!