Crypto 2024

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FuLeakage:

Breaking FuLeeca by Learning Attacks

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joint work with Wessel van Woerden[3]

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DLR

Motivation and Overview

FuLeeca

ia.cr/2023/377

FuLeeca

- is a code-based signature scheme,
- uses quasi-cyclic codes in the Lee metric, and
- was presented at CBCrypto 2023 and submitted to NIST's additional call for signatures [Ritterhoff et al., 2023].

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Outline

1. FuLeeca, Codes, and Lattices
2. Leaked-Sublattice Attack
3. Learning Attack

Lee-Metric Codes and Euclidean Lattices

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$\boldsymbol{x} = (x_1, \dots, x_n) \in \mathbb{F}_p^n$ has Lee weight $\mathrm{wt}_L(\boldsymbol{x}) = \sum_i \lvert x_i \rvert$.	$\boldsymbol{x} = (x_1, \dots, x_n) \in \mathbb{R}^n$ has Euclidean norm $\lVert \boldsymbol{x} \rVert_2 = \sqrt{\sum_i x_i^2}$.

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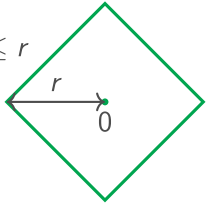
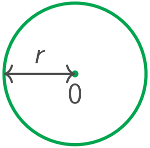
$\mathrm{wt}_L(\boldsymbol{x}) \leq r$ 	 $\lVert \boldsymbol{x} \rVert_2 \leq r$

Ball of radius r

Hash-and-Sign Signatures

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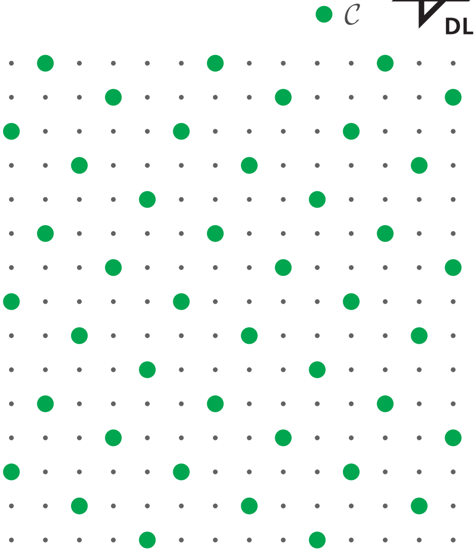

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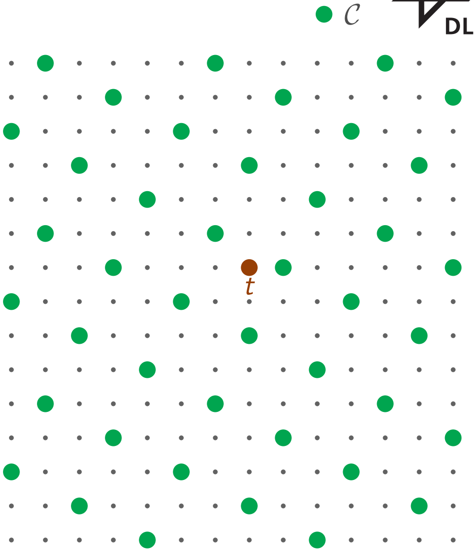

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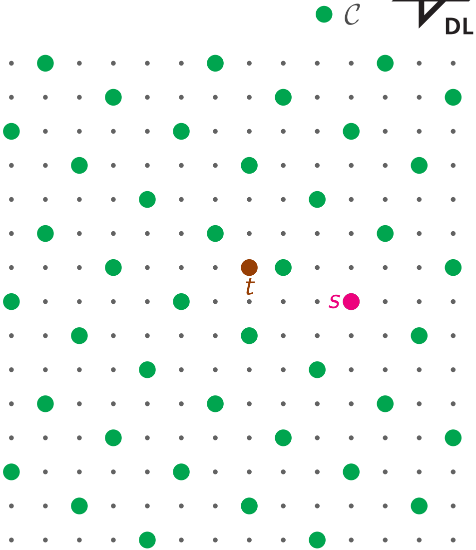

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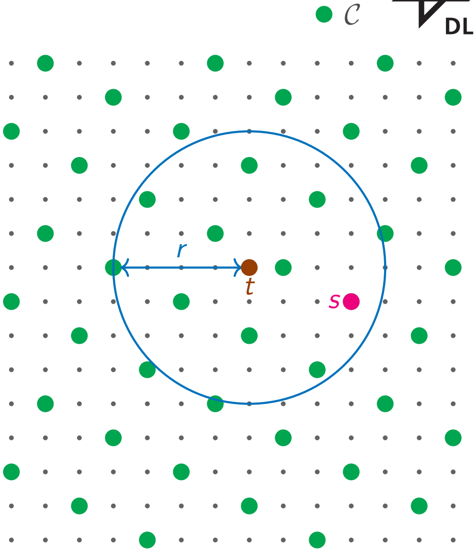

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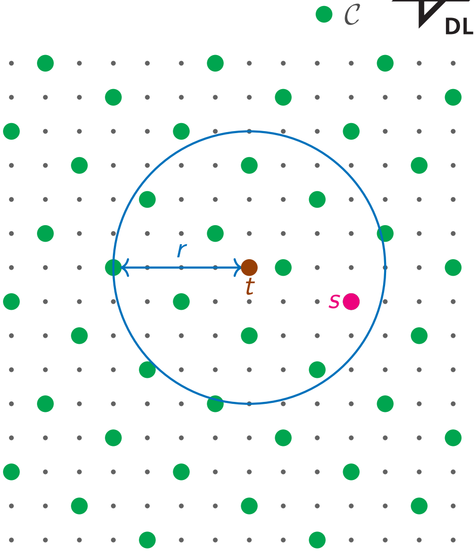
The signer needs a good description of the code but any description works for the verifier.



FuLeeca Key Generation [Ritterhoff et al., 2023]

Generate a secret vector $\boldsymbol{g} = (\boldsymbol{a} \mid \boldsymbol{b}) \in \mathbb{F}_p^n$ with $n = 2k$ by drawing $\boldsymbol{a}$ and $\boldsymbol{b}$ uniformly at random from $\{\boldsymbol{x} \in \mathbb{F}_p^k : \mathrm{wt}_L(\boldsymbol{x}) = w_{\mathrm{key}}\}$.

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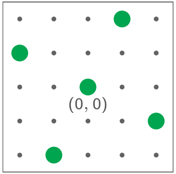
Goal: Recover the secret vector $\boldsymbol{g}$ (or any of its quasi-circular shifts $\boldsymbol{g}_1, \dots, \boldsymbol{g}_k$).

Lattices from Codes: Construction A

$\mathbb{F}_p^2$

● $\mathcal{C}$



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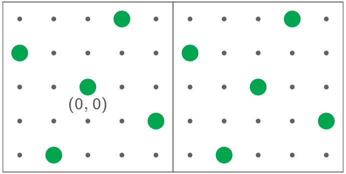

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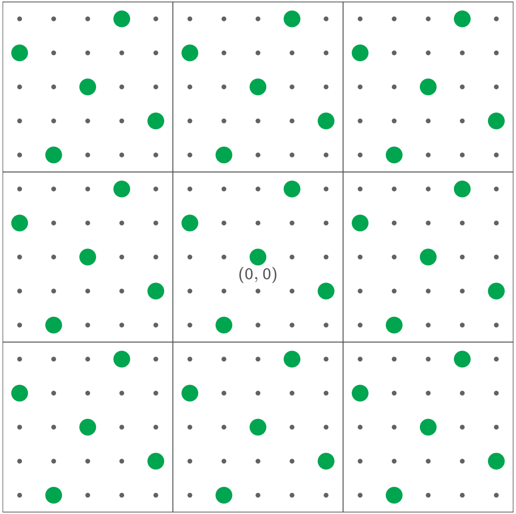

$$\mathcal{L}_{\mathrm{A}}(\mathcal{C}) := \{\boldsymbol{v} \in \mathbb{Z}^n : \boldsymbol{v} \pmod{p} \in \mathcal{C}\}$$

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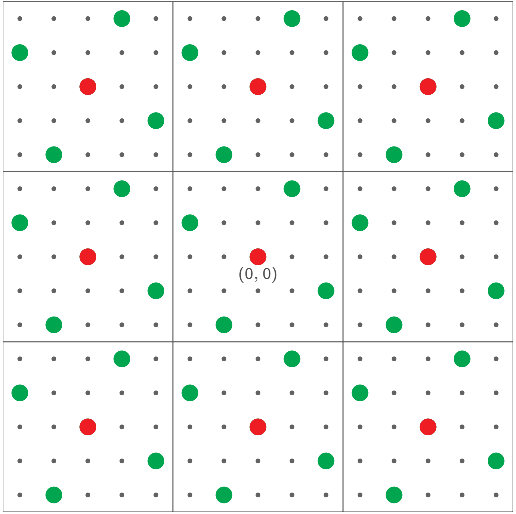

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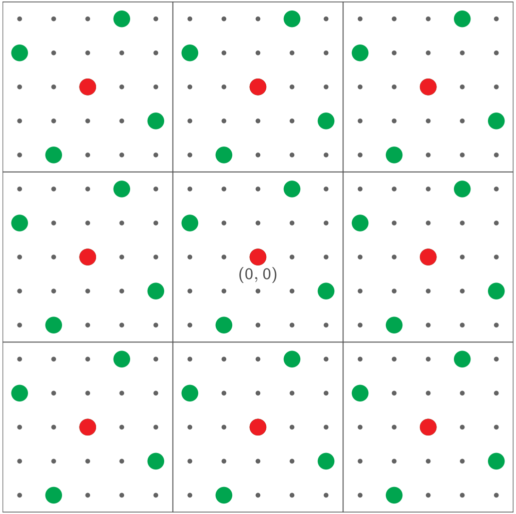

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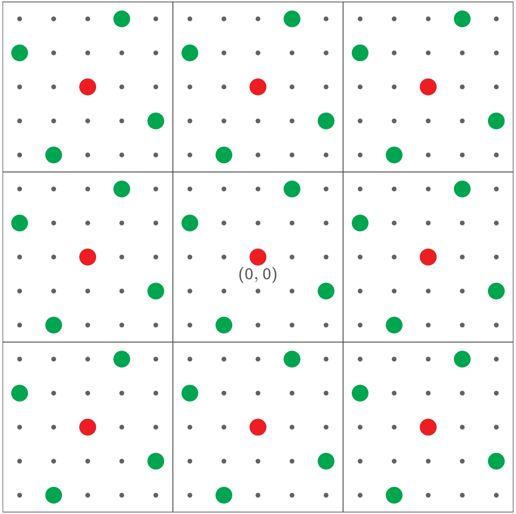

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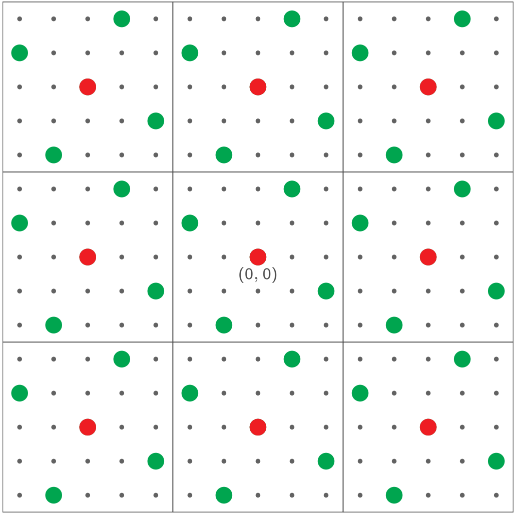

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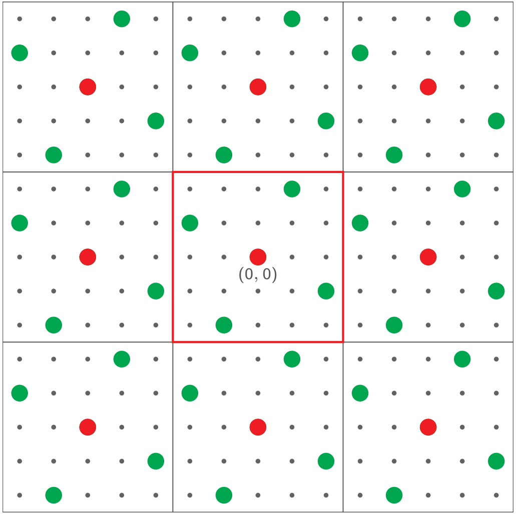

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Observe: In the central square, the Lee metric corresponds to the ℓ_1-norm.

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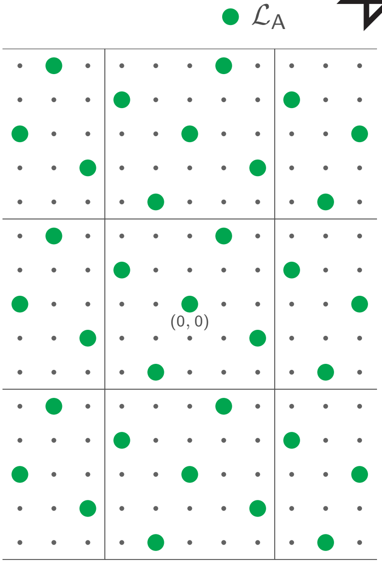
A Lower-Rank Sublattice

$$\mathcal{L}_{\mathrm{A}} = \mathbb{Z}^n \cdot \begin{pmatrix} \mathbf{I}_k & \boldsymbol{A}^{-1}\boldsymbol{B} \\ \mathbf{0} & p\,\mathbf{I}_{n-k} \end{pmatrix} \subset \mathbb{R}^n$$



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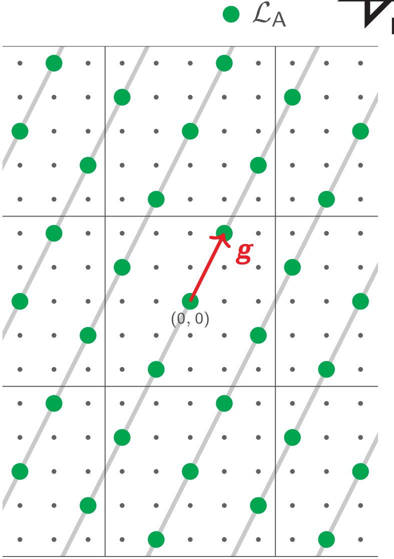

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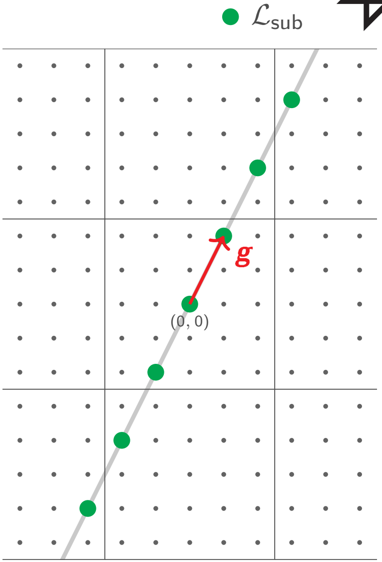

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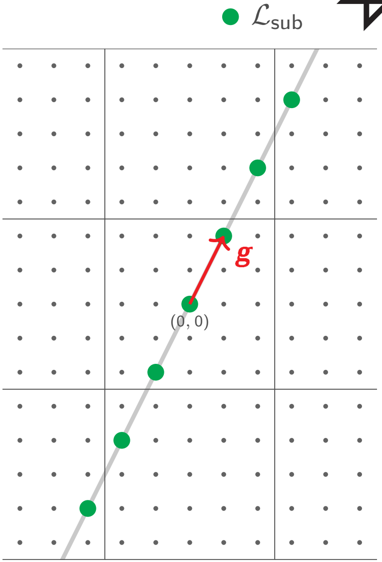
$\mathcal{L}_{\text{sub}}$ has rank $k = n/2$ and lattice techniques can recover short vectors with complexity $O\left(2^{\frac{0.292n}{2}}\right)$.



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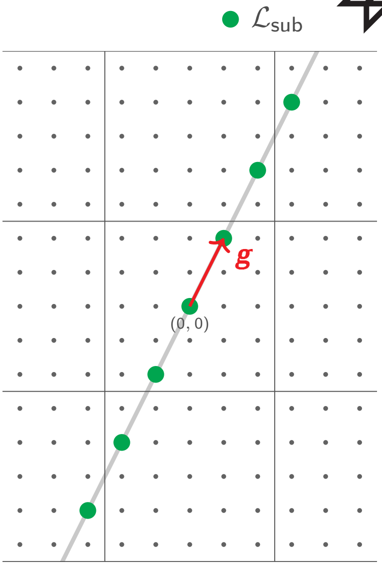

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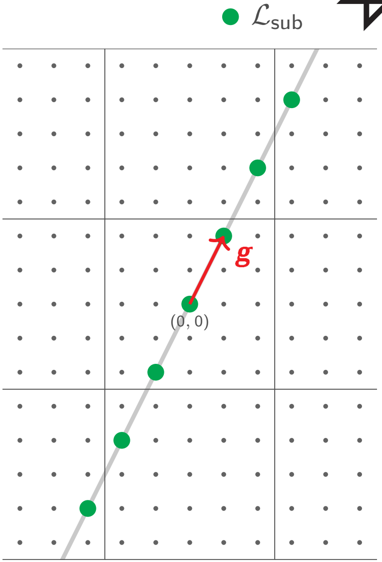
Remark: The quasicyclic structure of $\mathcal{L}_{\text{sub}}$ enables a polynomial-time quantum attack.



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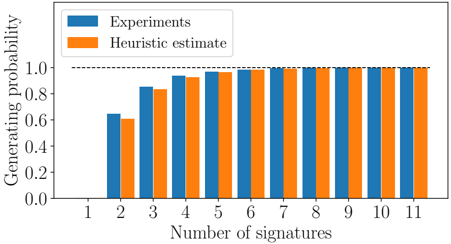

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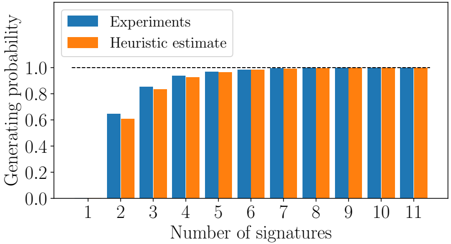

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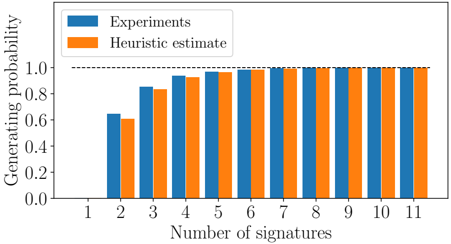

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$\Longrightarrow$ The NIST standards are not met. ✖

Outline

1. FuLeeca, Codes, and Lattices
2. Leaked-Sublattice Attack
3. Learning Attack

Observed Bias in FuLeeca

Recall: The method $\boldsymbol{v} = \texttt{concentrate}(\boldsymbol{c}, \boldsymbol{v}, \boldsymbol{G}_{\text{sec}})$ in the signature generation tries to improve $\boldsymbol{v}$ by successively adding $\pm\boldsymbol{g}_i$ for $i = 1, \ldots, k$.

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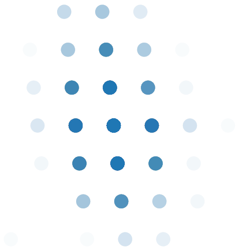

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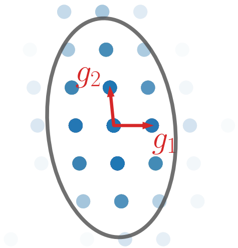

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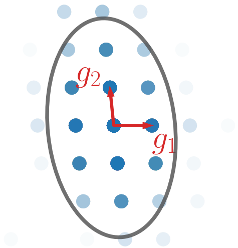

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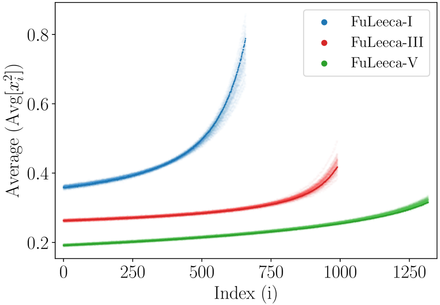

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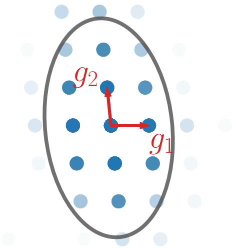

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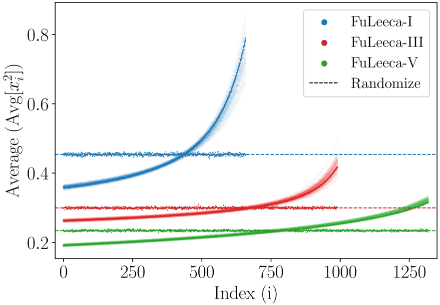

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Exploiting the Bias

Collect FuLeeca signatures $\boldsymbol{v}_1, \ldots, \boldsymbol{v}_N$ with $\boldsymbol{v}_i = \boldsymbol{x}_i \boldsymbol{G}_{\text{sec}} = (\underbrace{\boldsymbol{x}_i \boldsymbol{A}}_{=:\boldsymbol{w}_i} \mid \boldsymbol{x}_i \boldsymbol{B})$ for $i = 1, \ldots, N$.

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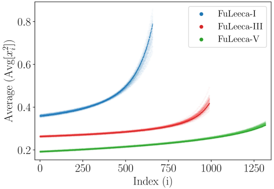
- $\boldsymbol{A} = \mathrm{Shift}(\boldsymbol{a})$ is circulant and
- $\boldsymbol{D}$ has an increasing diagonal.



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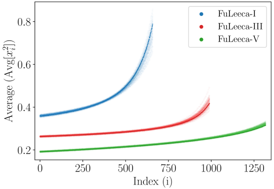
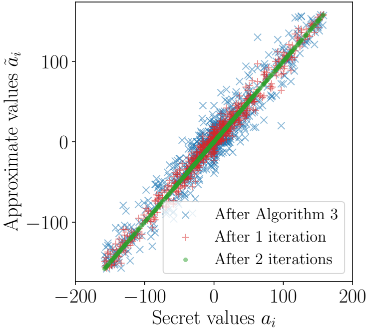


We recover $\boldsymbol{a}$ with high probability:

1. Get an approximation of $\boldsymbol{a}$.
2. Turn the guess into an exact solution iteratively.

Success of the Learning Attack

From approximation to exact solution.



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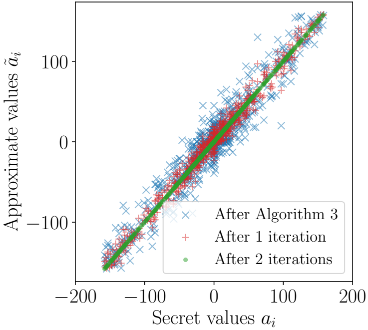
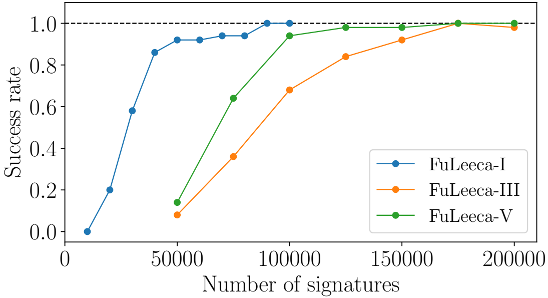


Success rate of the learning attack.

Averaged over 50 keys for each parameter set.



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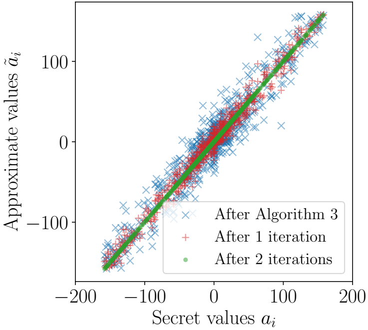

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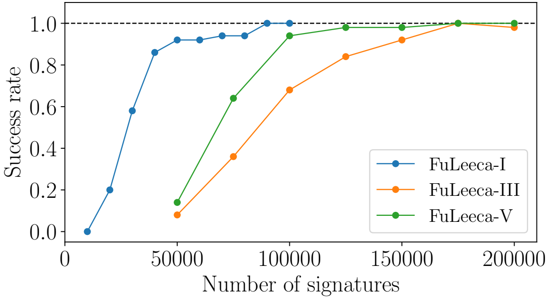

$\Longrightarrow$ 175,000 signatures are enough to fully break FuLeeca!

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Find FuLeakage here:

- `ia.cr/2024/353`
- </> `artifacts.iacr.org/crypto/2024/a12`

References

[Ritterhoff et al., 2023] Ritterhoff, S., Maringer, G., Bitzer, S., Weger, V., Karl, P., Schamberger, T., Schupp, J., and Wachter-Zeh, A. (2023).
FuLeeca: A Lee-based signature scheme.
In *Code-Based Cryptography - 11th International Workshop, CBCrypto 2023*, volume 14311 of *Lecture Notes in Computer Science*, pages 56–83. Springer.
See `https://csrc.nist.gov/csrc/media/Projects/pqc-dig-sig/documents/round-1/spec-files/FuLeeca-spec-web.pdf` for the submission to NIST's call for additional digital signature schemes.