

# How to Prove Statements Obliviously?

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## Succinct Non-interactive Arguments (SNARG)



$st, w$



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- Completeness, Soundness.
- Succinctness: Verification time and proof size  $|\pi|$  are  $\ll |w|$
- A large body of works — [Kil92, Mic94, Gro16, BCG+17, BCG+18, XZZ+19, GWC19, Set20, BCG20, CHM+20, Lee21, KMP20, ZLW+21, BCL22 .....]

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## Example I: Verifiable Delegation of Computation



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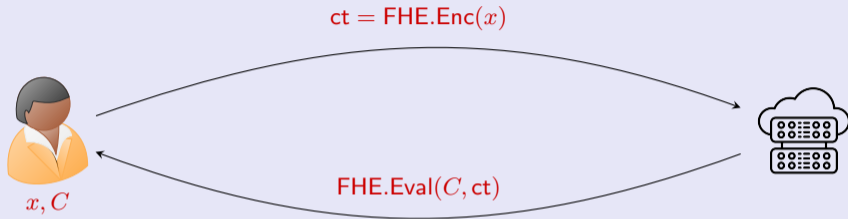
$$ct = \text{FHE.Enc}(x)$$



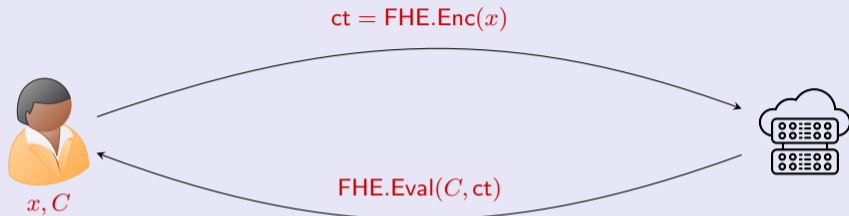
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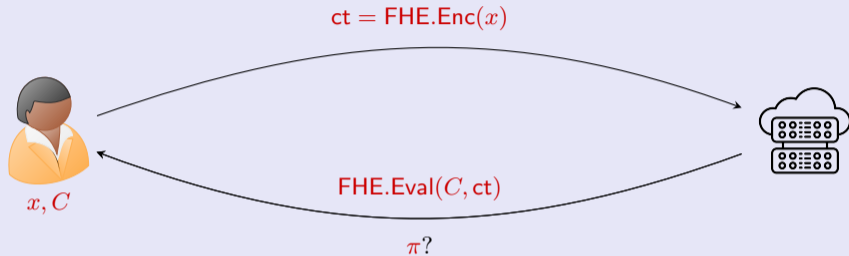


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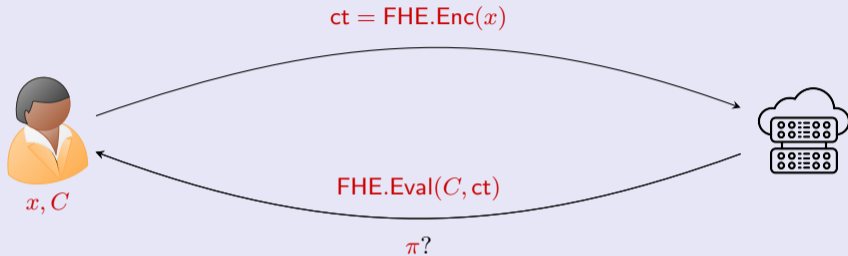
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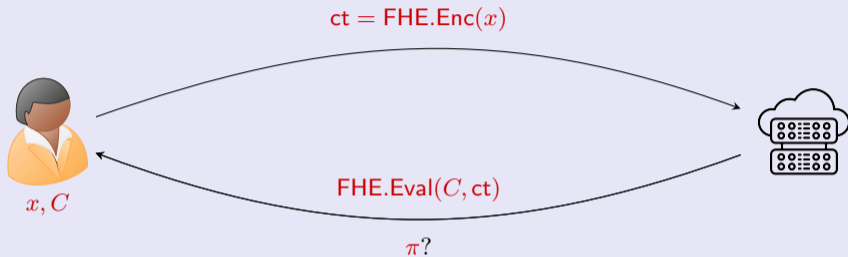
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- **Private** verifiability is acceptable



## Example II: Verifiable Public Keys Aggregation



$pk_1, \dots, pk_n$

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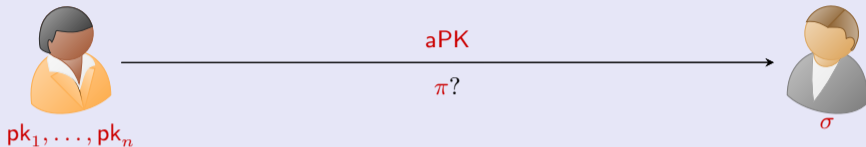
aPK



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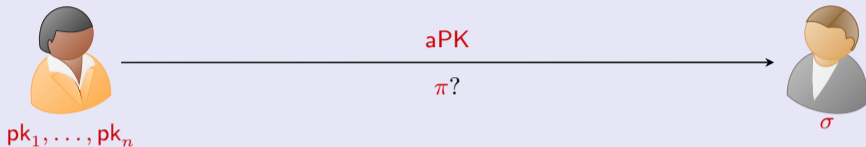
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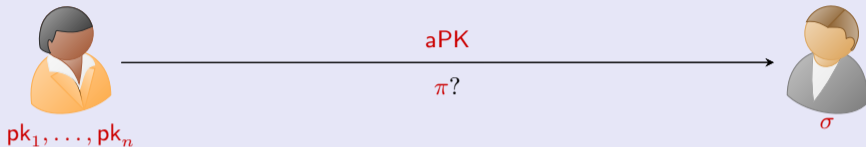
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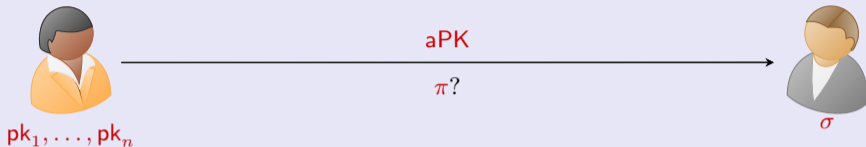


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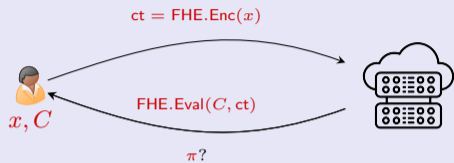
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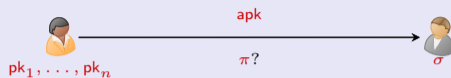
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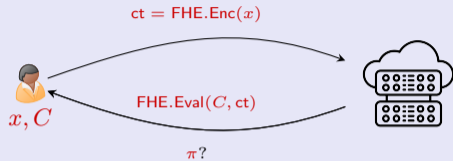
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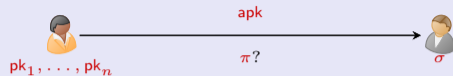
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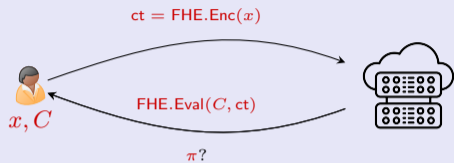
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- Translate homomorphic operations (**FHE.Eval**, group operation) as circuits and generically apply SNARKs

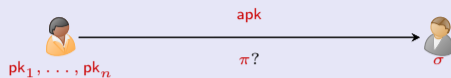


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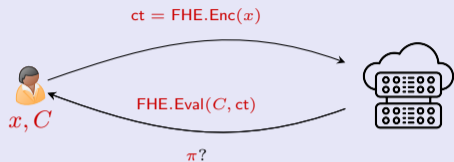


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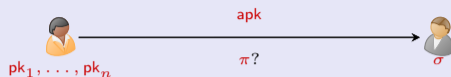
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*How can the prover generate a proof obliviously with  
only **black-box use** of the encapsulation scheme?*

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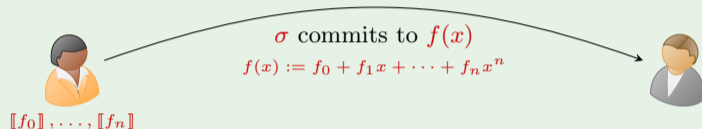


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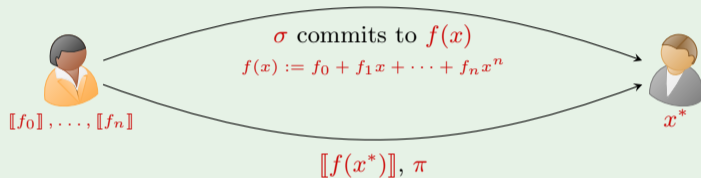


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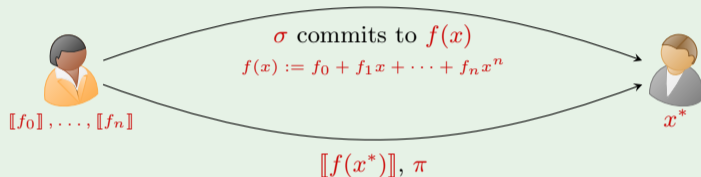
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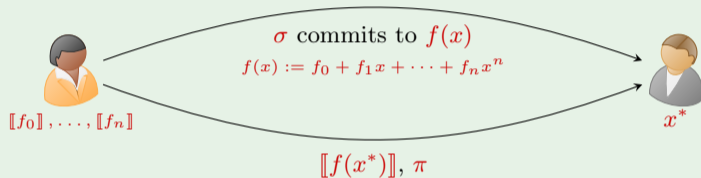


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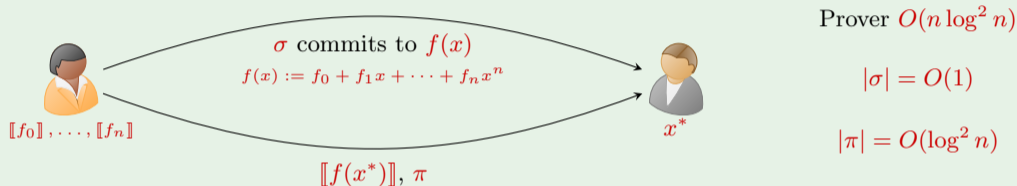
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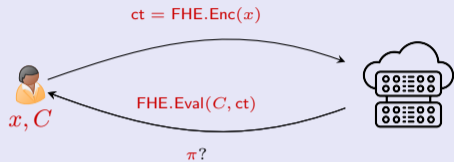
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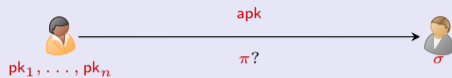
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  - E.g., group exponentiation, ElGamal, ...
- An adaptation of the celebrated FRI proof system [Ben-Sasson-Bentov-Horesh-Riabzev'18]  
**Black-box** in  $[\cdot]$  and achieve the same efficiency as FRI

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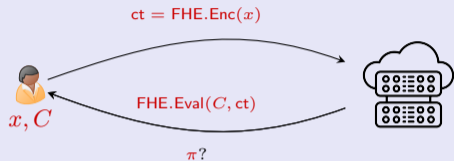
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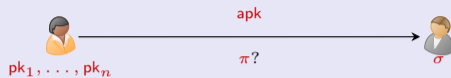
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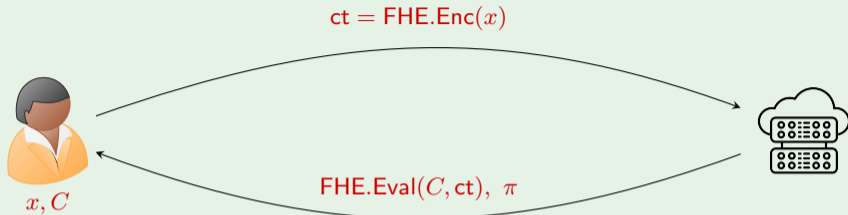


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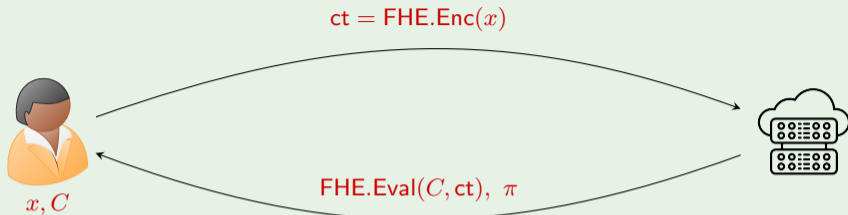
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- Existing techniques: polynomial commitment  $\implies$  SNARKs
- We show: **FRI on hidden values**  $\implies$  oblivious (black-box) proof generation

## Application: Verifiable Delegation of Computation

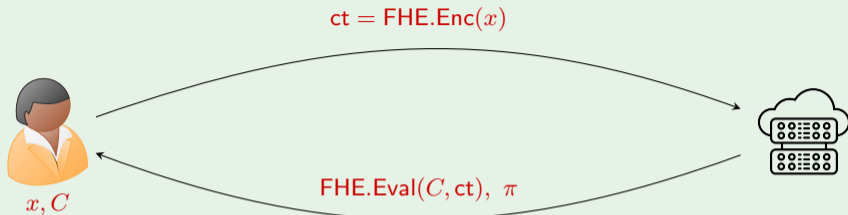


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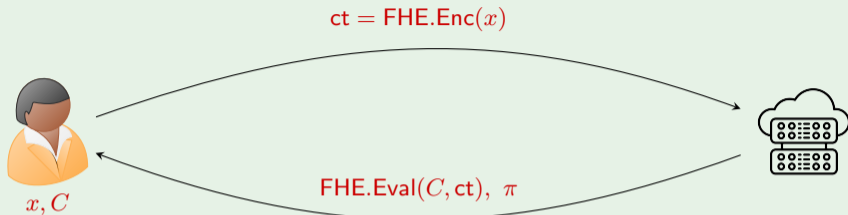
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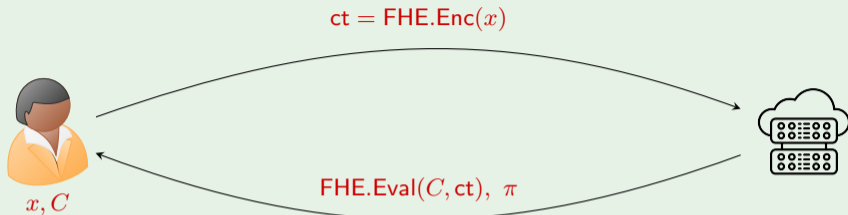


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- Require the client to perform  $|C|$  FHE operations [Gennaro-Gentry-Parno'10, Applebaum-Ishai-Kushilevitz'10, Chung-Kalai-Vadhan'10, Benabbas-Gennaro-Vahlis'11, ...]

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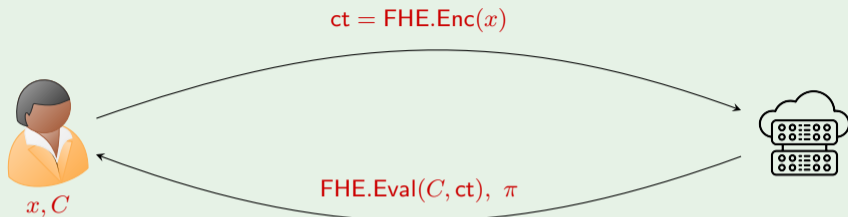
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- Makes **non-black-box** use of FHE [Fiore-Gennaro-Pastr'14, Fiore-Nitulescu-Pointcheval'20, Bois-Cascudo-Fiore-Kim'21, ...]

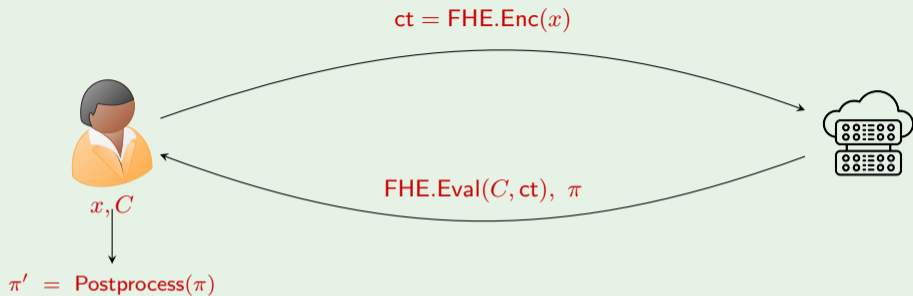


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## Public Verifiability

- Client Postprocessing:  $O(\log^2 |C|)$
- Application to [the delegation of zkSNARKs](#) to **untrusted** server (see paper!)

## Application: Weighted Threshold Signature without DKG



$pk_1$



$pk_2$

...



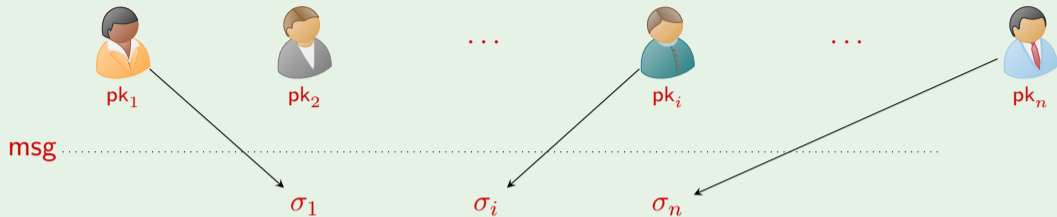
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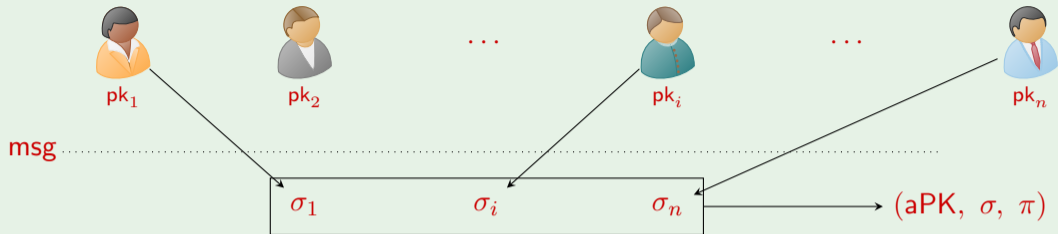


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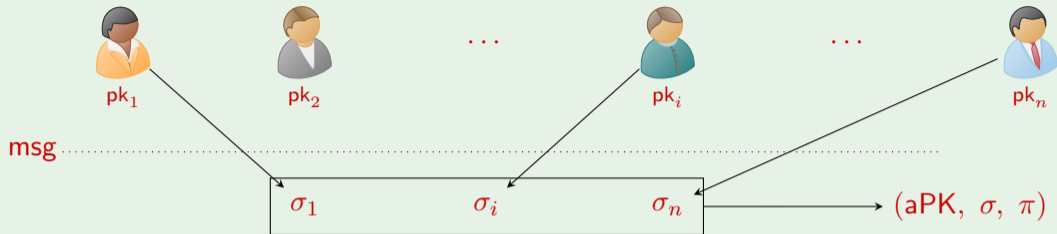


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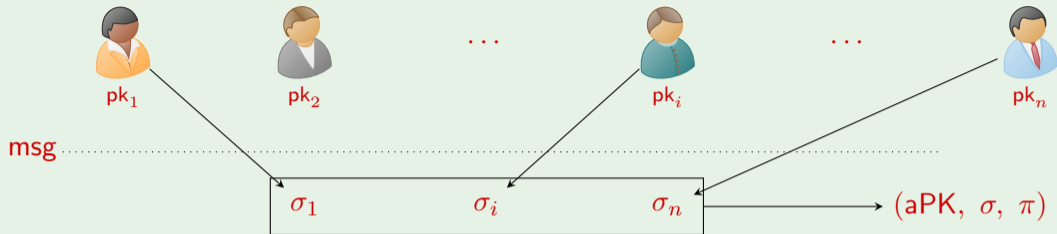
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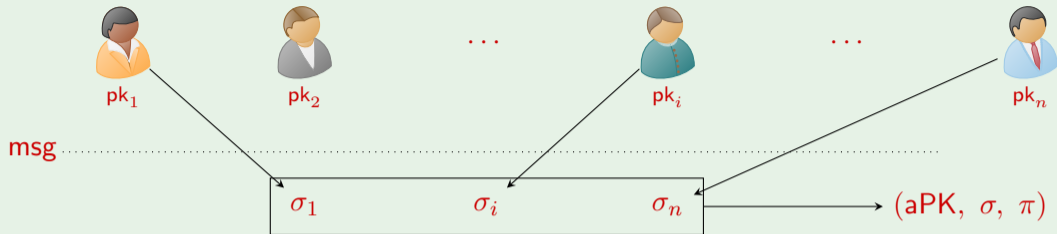
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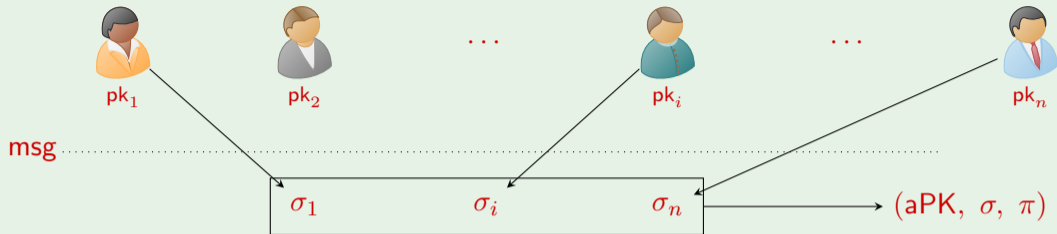
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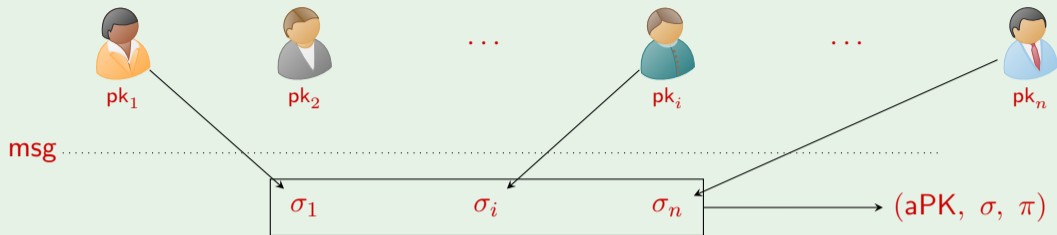


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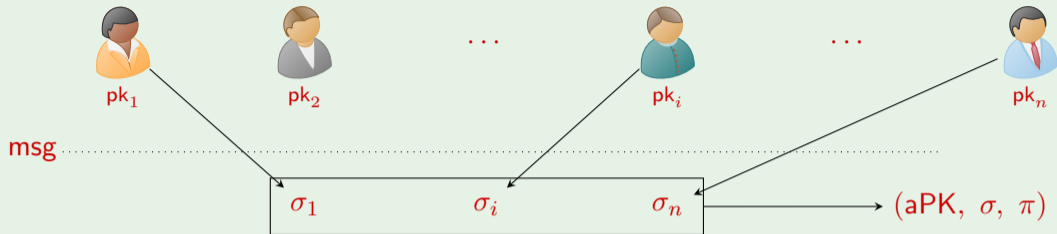


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## Prior Works

- Pairing-based SNARKs:  $O(n)$ -size SRS,  $O(n)$ -size  $pk_i$   
[Garg-Jain-Mukherjee-Sinha-W-Zhang'24, Das-Camacho-Xiang-Nieto-Bunz-Ren'23]

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- Require Ramp: [Micali-Reyzin-Vlachos-Wahby-Zeldovich'21]

# Technical Highlights

Prover wants to prove  $C(\llbracket x \rrbracket) = \llbracket y \rrbracket$ , where  $\llbracket \cdot \rrbracket$  supports some homomorphism.

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Homomorphism on top of SNARK

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### SNARK on top of Homomorphism

$C(\llbracket x \rrbracket) = \llbracket y \rrbracket$  is a **public relation**

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Homomorphic Evaluation on  $\llbracket x \rrbracket$   
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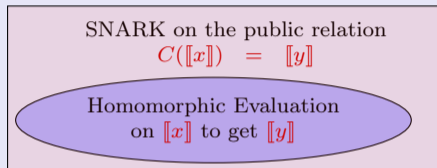
SNARK on the private relation  
 $C(x) = y$

- Black-box
- Verifiability?

Prover wants to prove  $C(\llbracket x \rrbracket) = \llbracket y \rrbracket$ , where  $\llbracket \cdot \rrbracket$  supports some homomorphism.

### SNARK on top of Homomorphism

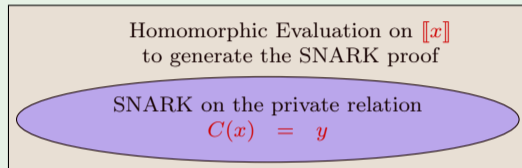
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- Non-black-box

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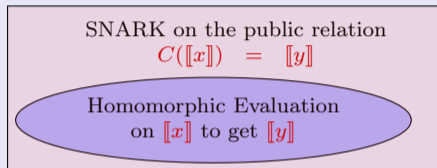


- Black-box
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- What Homomorphism needed beyond  $C$ ?

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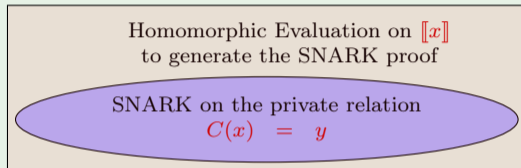
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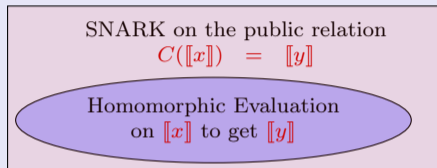
What kind of operations does a prover need to perform to prove  $C$ ?

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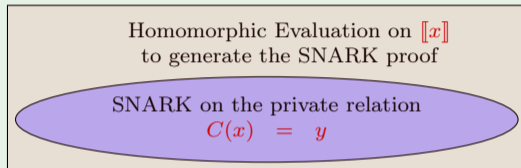
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What kind of operations does a prover need to perform to prove  $C$ ?

- Feasibility:  $\llbracket x \rrbracket$  may only support linear homomorphism:  $g^x$
- Efficiency: The efficiency of FHE depends on the homomorphism supported.



# Interactive Oracle Proof (IOP) [Ben-Sasson-Chiesa-Spooner'16]

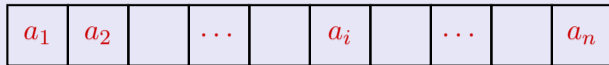


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# Interactive Oracle Proof (IOP) [Ben-Sasson-Chiesa-Spooner'16]



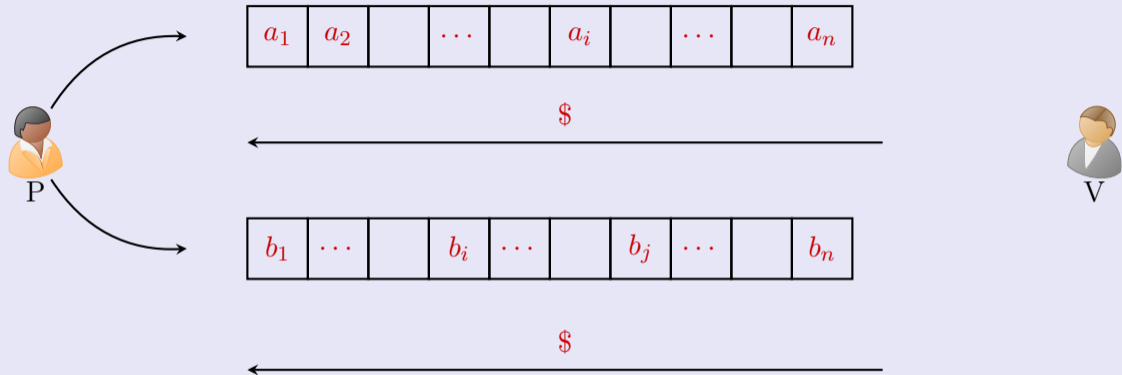
# Interactive Oracle Proof (IOP) [Ben-Sasson-Chiesa-Spooner'16]



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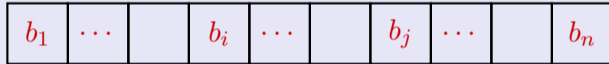
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$$3 \cdot a_2 - a_i \stackrel{?}{=} 0$$



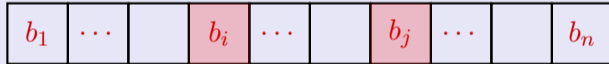
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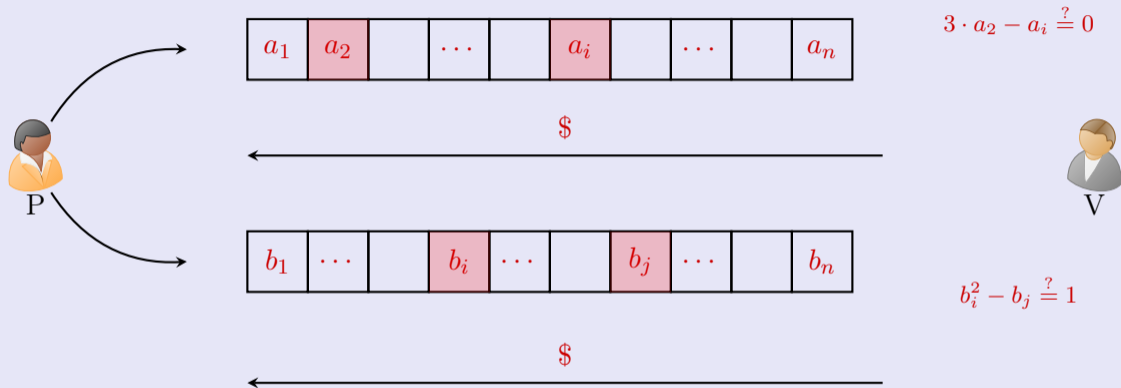
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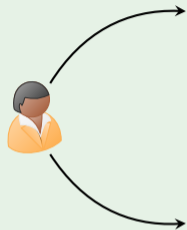
$$b_i^2 - b_j \stackrel{?}{=} 1$$

# Interactive Oracle Proof (IOP) [Ben-Sasson-Chiesa-Spooner'16]

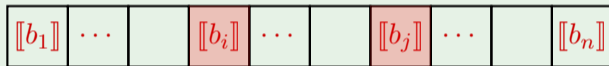


- Can be compiled into SNARKs using [Merkle's commitment](#) and [Fiat-Shamir](#).
  - Proof size grows with # queries

# IOP on Hidden Values



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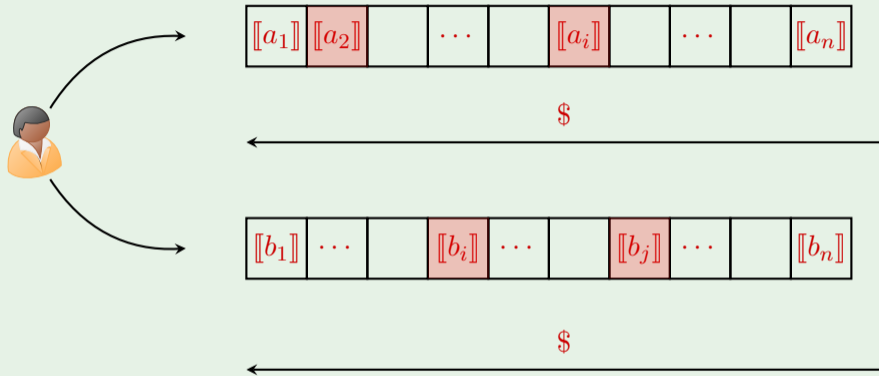
$$3 \cdot [[a_2]] - [[a_i]] \stackrel{?}{=} 0$$



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## IOP on Hidden Values



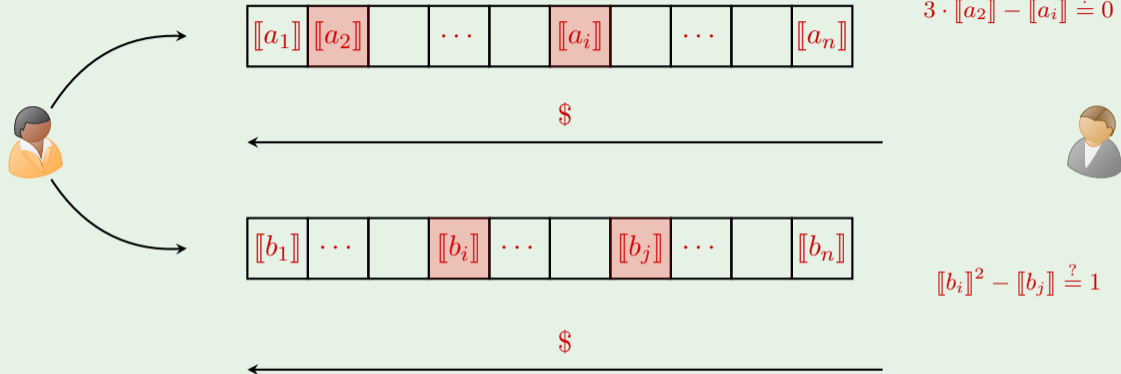
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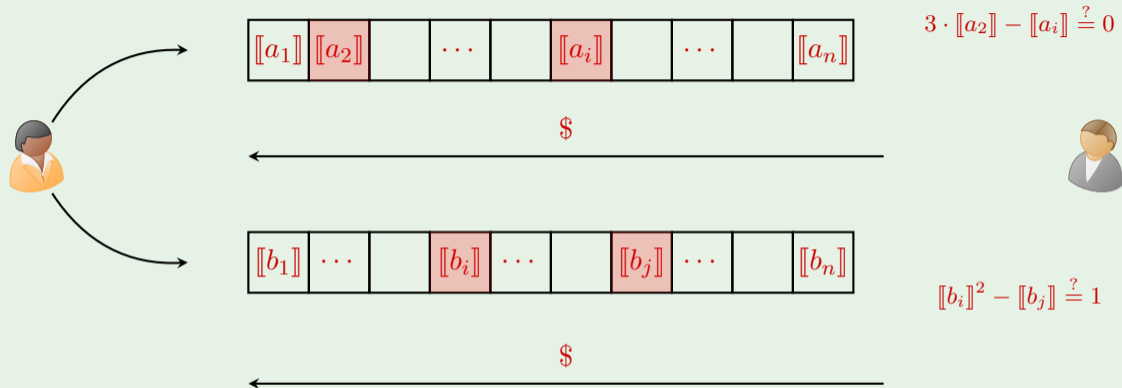
- Privately verifiable if decryptable

## IOP on Hidden Values



- **Privately** verifiable if **decryptable**
- How to compile IOP to SNARKs?

## IOP on Hidden Values



- **Privately** verifiable if **decryptable**
- How to compile IOP to SNARKs?
  - directly apply **Merkle's commitment** and **Fiat-Shamir** on  $[[x]]$
  - No need for leveraging **homomorphism** on **random oracle computation**

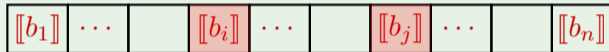
# FRI on Hidden Values



$[[f_0], \dots, [f_n]]$



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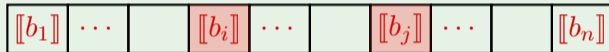
## FRI on Hidden Values



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\$



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$$3 \cdot [[a_2]] - [[a_i]] \stackrel{?}{=} 0$$



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- Polynomial Commitment for hidden polynomial.  $f(x) := f_0 + f_1 \cdot x + \dots + f_n \cdot x^n$

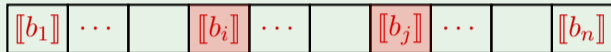
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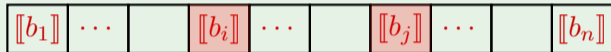
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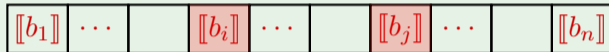
## FRI on Hidden Values



$[f_0], \dots, [f_n]$



\$



\$



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  - Homomorphic in randomness  $\implies$  Check relation at encapsulation level

$$(g^{a_2})^3 / g^{a_i} \stackrel{?}{=} 1$$



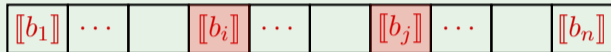
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FRI + Polynomial IOP:  $|C| \cdot \log |C|$  operations with multiplication depth  $\text{depth}(C) + O(1)$

## Summary

FRI on hidden values enables oblivious proof generation:

- Verifiable Delegation of Computation
- Delegation of the generation of zkSNARKs to untrusted server
- (Weighted) Threshold Signature without DKG

Future direction: more applications?



Thanks!

Questions?

[ia.cr/2023/1609](https://ia.cr/2023/1609)