CISPA HELMHOLTZ CENTER FOR INFORMATION SECURITY

MPC in the head using the subfield bilinear collision problem

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CISPA Helmholtz Center for Information Security

The Subfield Bilinear Collision (SBC) Problem

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Parameters: q and $k, n > 0$

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$$\mathbb{F}_{q^k}$$

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$$\mathbb{F}_{q^k} \qquad \vec{u}, \vec{v} \in \left(\mathbb{F}_{q^k}\right)^n$$

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$$\begin{array}{ll} \mathbb{F}_{q^k} & \vec{u}, \vec{v} \in \left(\mathbb{F}_{q^k}\right)^n \\ \Big| & \\ \mathbb{F}_q & \vec{x}, \vec{y} \in \left(\mathbb{F}_q\right)^n \end{array}$$

The Subfield Bilinear Collision (SBC) Problem

$$(\vec{u} \cdot \vec{x}) = \sum_{i=1}^{n} u_i x_i$$

Parameters: q and $k, n > 0$

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Conditions:

The <u>S</u>ubfield <u>B</u>ilinear <u>C</u>ollision (SBC) Problem

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$\vec{x}, \vec{y}$ non-colinear

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$$(\vec{x}, \vec{y}) \in \mathrm{SBC}[\vec{u}, \vec{v}]$$

$$n \approx \frac{k}{2}$$

Testing with polynomials

Testing with polynomials

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$$X_1, X_2, Y_1, Y_2 \stackrel{\$}{\leftarrow} \mathbb{F}_{q^k}$$

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$$X_1, X_2, Y_1, Y_2 \xleftarrow{\$} \mathbb{F}_{q^k}$$

$$F(t) = \left(X_1 + t\left(\vec{u} \cdot \vec{x}\right)\right)\left(Y_1 + t\left(\vec{v} \cdot \vec{y}\right)\right) - \left(X_2 + t\left(\vec{v} \cdot \vec{x}\right)\right)\left(Y_2 + t\left(\vec{u} \cdot \vec{y}\right)\right)$$

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$$F(t) = A + Bt + \left[\left(\vec{u} \cdot \vec{x}\right)\left(\vec{v} \cdot \vec{y}\right) - \left(\vec{u} \cdot \vec{y}\right)\left(\vec{v} \cdot \vec{x}\right)\right] t^2$$

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$$\boxed{\deg(F) < 2 \Leftrightarrow (\vec{x}, \vec{y}) \in \text{SBC}\left[\vec{u}, \vec{v}\right]}$$

MPC protocol for the polynomial testing

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$$\left(\vec{x}, \vec{y}\right) \in \mathrm{SBC}\left[\vec{u}, \vec{v}\right]$$

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$$[\![\vec{x}]\!] = \left(\vec{x}^{[\![1]\!]}, \ldots, \vec{x}^{[\![N]\!]}\right)$$

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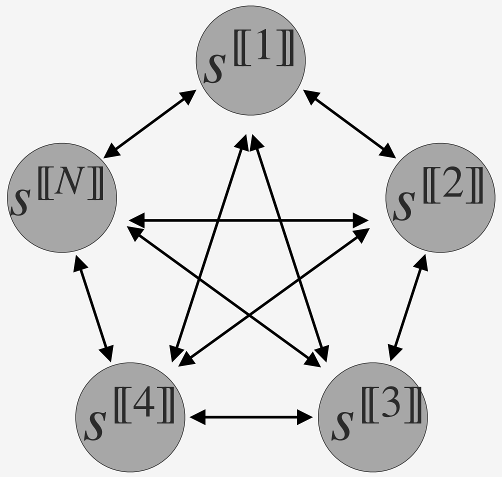

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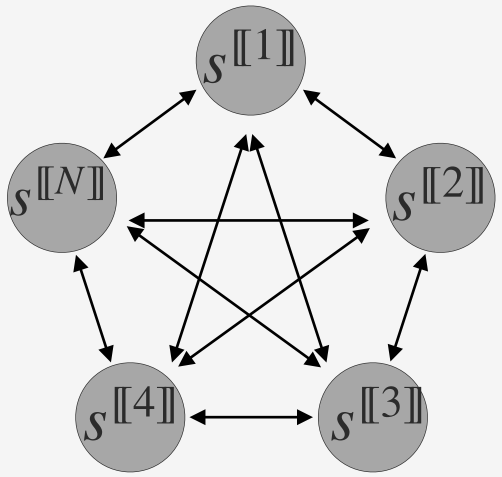

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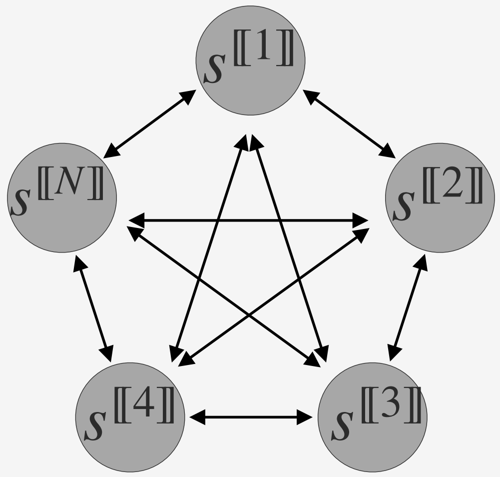

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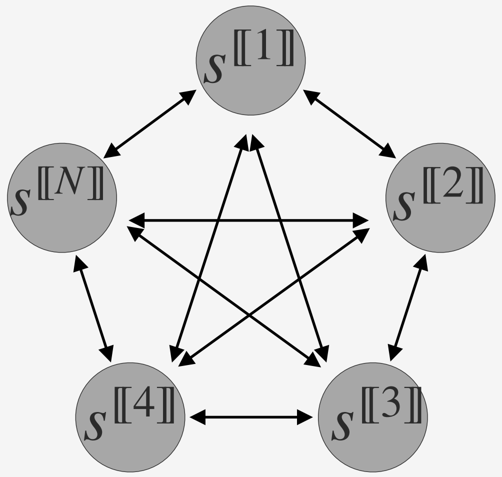

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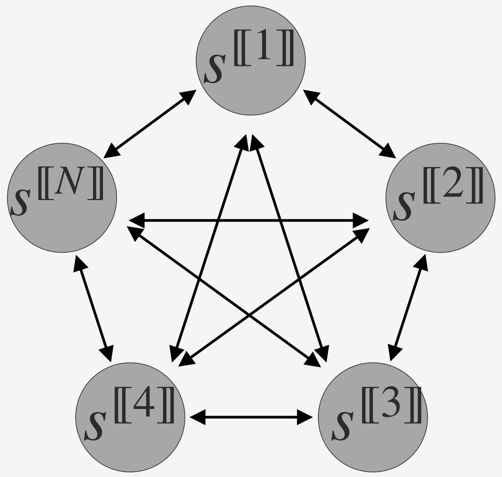

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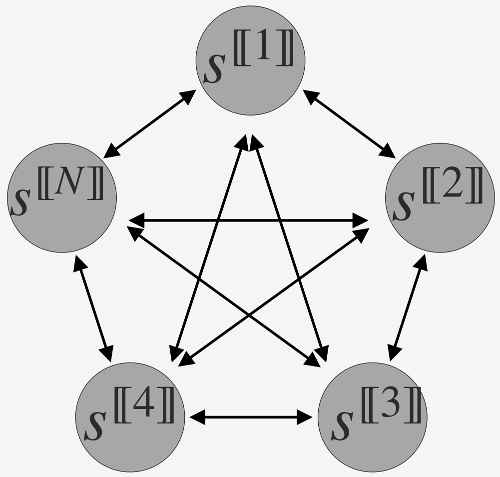

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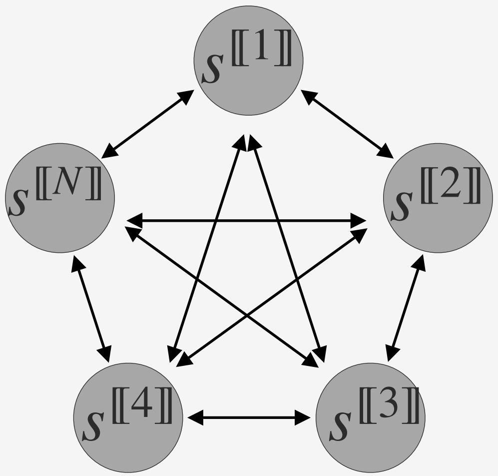

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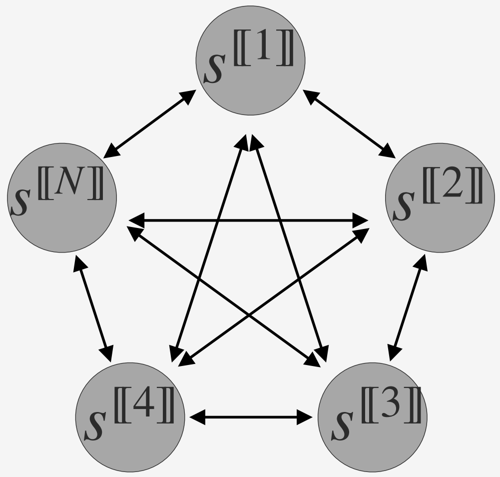

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The views of $N - 1$ parties reveal no information about $\left(\vec{x}, \vec{y}\right)$

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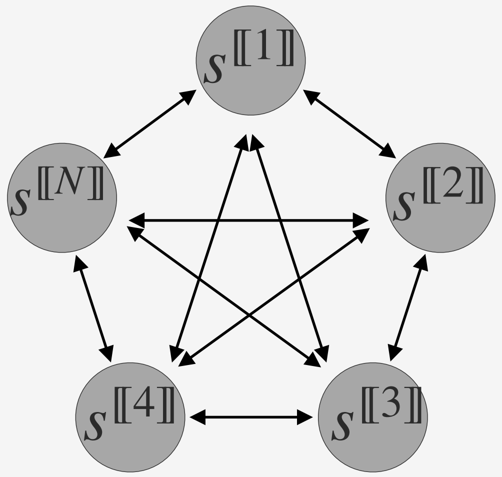

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False positive probability:

MPC protocol for the polynomial testing

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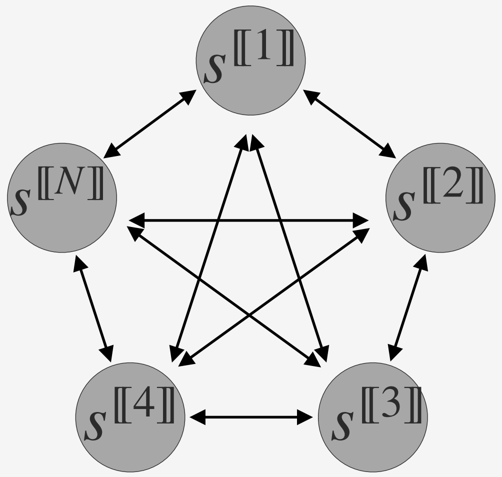

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False positive probability:

$$\Pr[\text{false positive}] \leq \frac{2}{q^k}$$

MPCitH protocol

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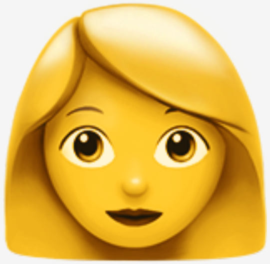

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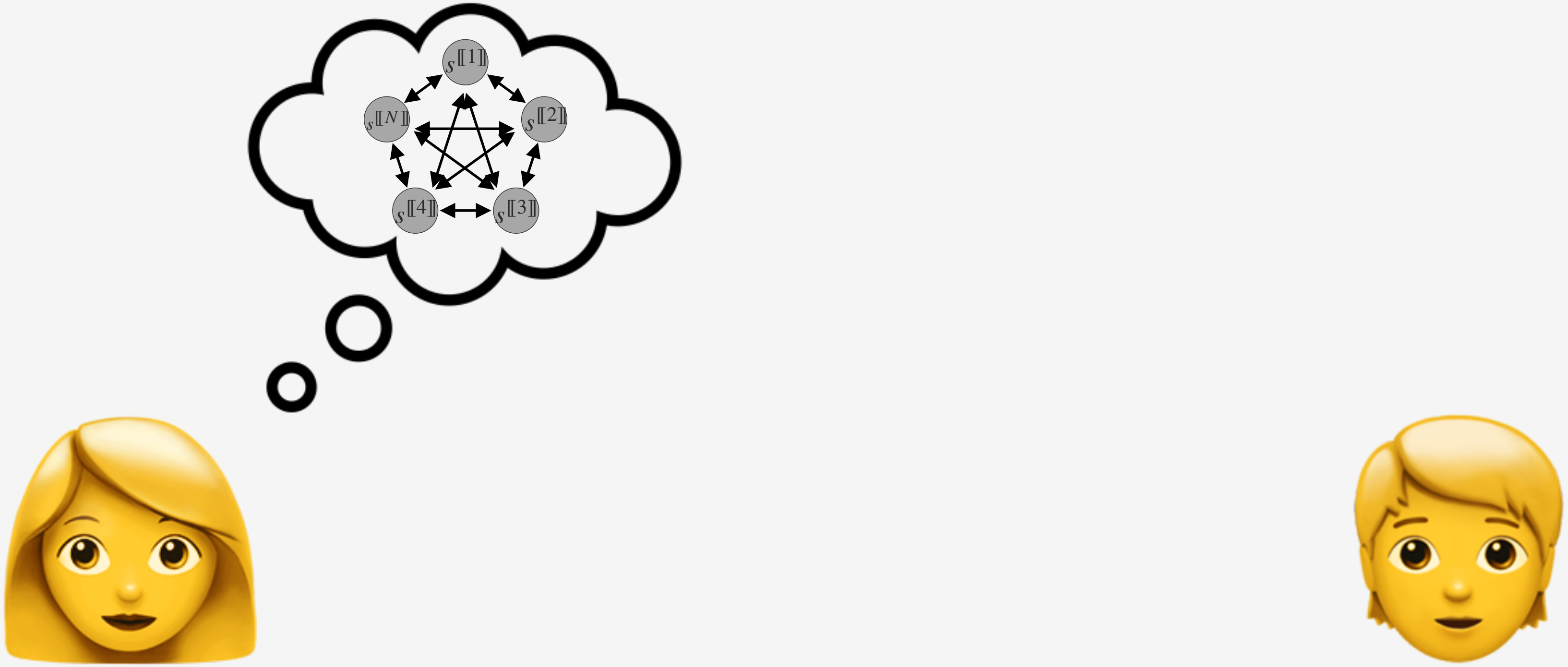

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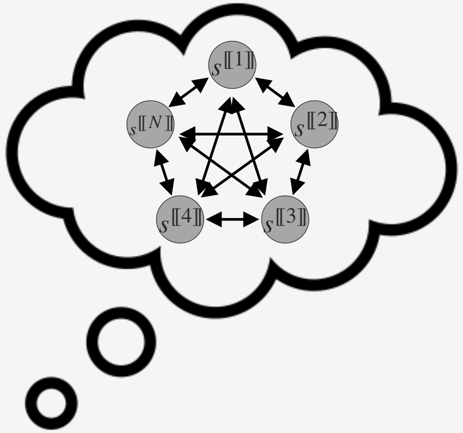
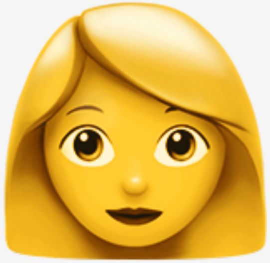
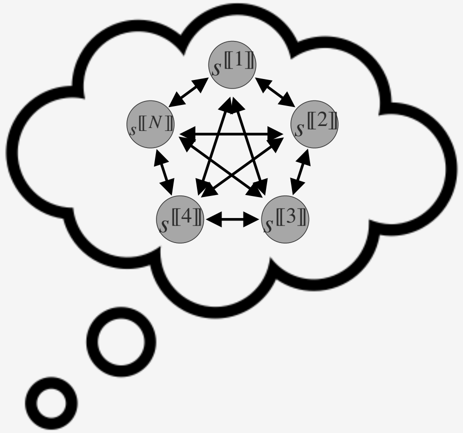
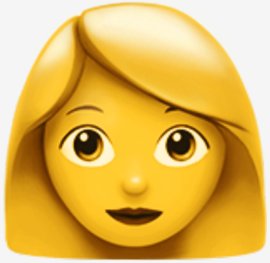




<u>commit:</u>

MPCitH protocol





<u>commit:</u>

- $\text{com}\left(\text{View}^{[\![i]\!]}\right)$

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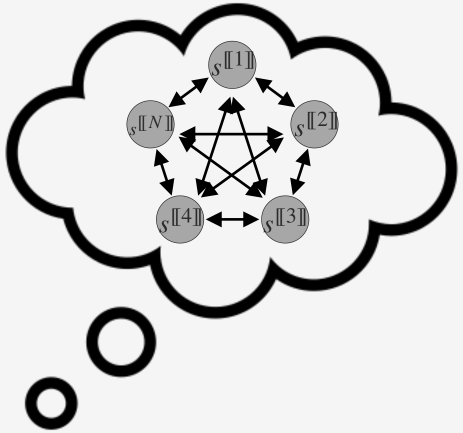
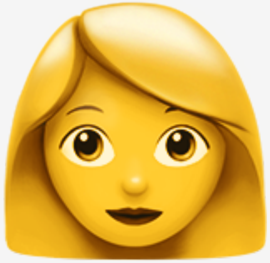
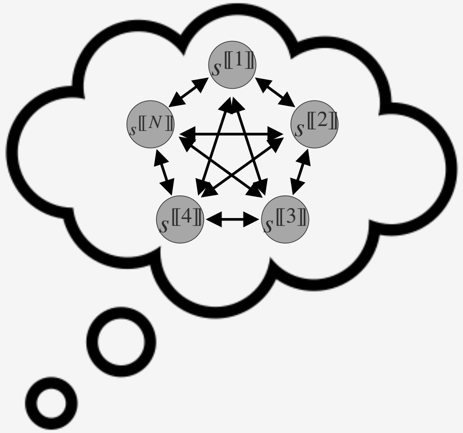


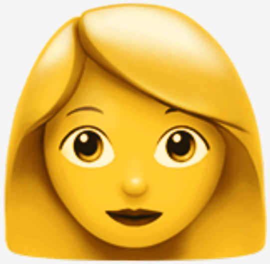


<u>commit:</u>

- $\mathrm{com}\left(\mathrm{View}^{[\![i]\!]}\right)$
- $\left(X_1 + t_0\left(\vec{u}\cdot\vec{x}\right)\right), \left(Y_1 + t_0\left(\vec{v}\cdot\vec{y}\right)\right)$
- $\left(X_2 + t_0\left(\vec{v}\cdot\vec{x}\right)\right), \left(Y_2 + t_0\left(\vec{u}\cdot\vec{y}\right)\right)$

MPCitH protocol





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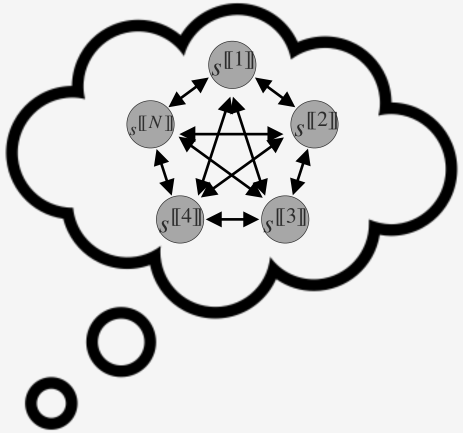
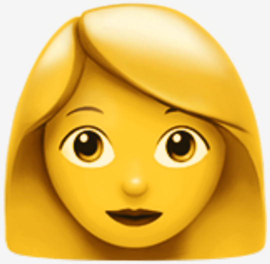
- $\left(X_1 + t_0\left(\vec{u}\cdot\vec{x}\right)\right), \left(Y_1 + t_0\left(\vec{v}\cdot\vec{y}\right)\right)$
- $\left(X_2 + t_0\left(\vec{v}\cdot\vec{x}\right)\right), \left(Y_2 + t_0\left(\vec{u}\cdot\vec{y}\right)\right)$
- t_0

MPCitH protocol





commit →

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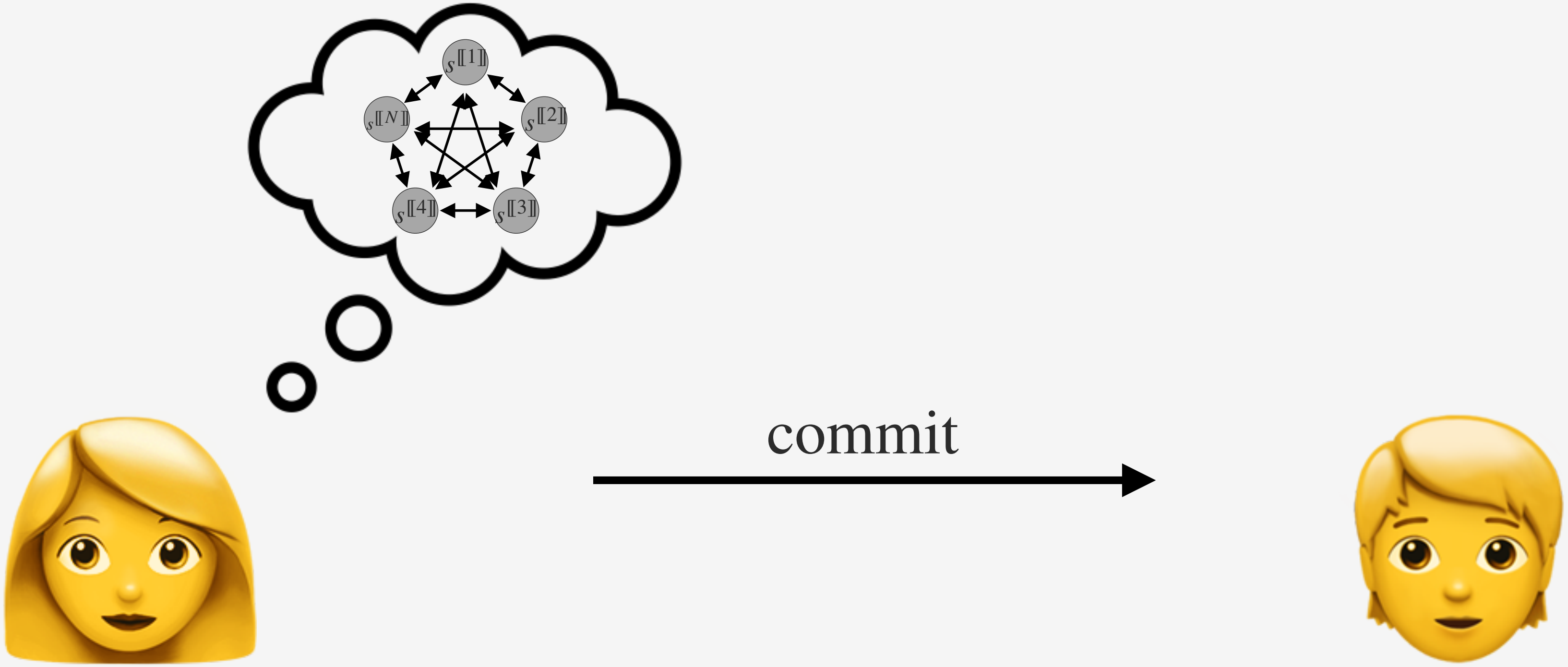
MPCitH protocol



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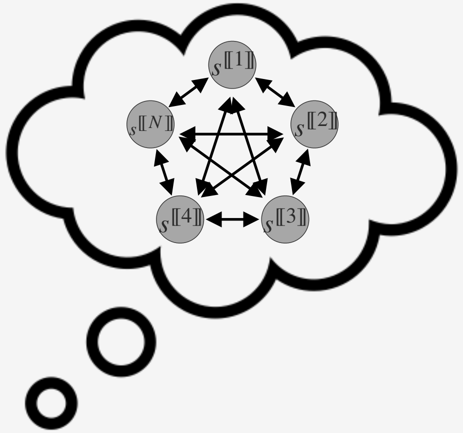
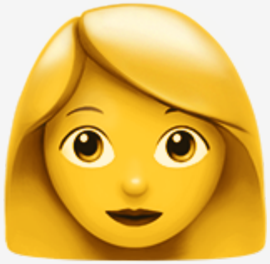
$i^* \xleftarrow{\$} \{1, \cdots, N\}$

MPCitH protocol





commit →

i^* ←

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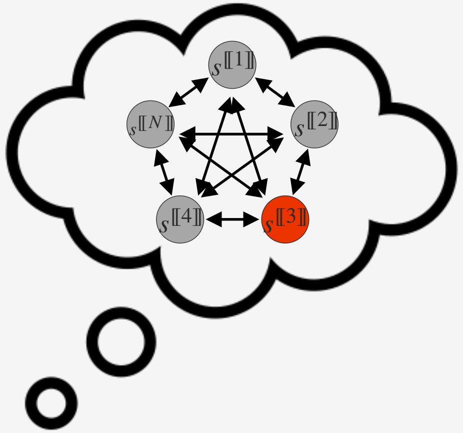

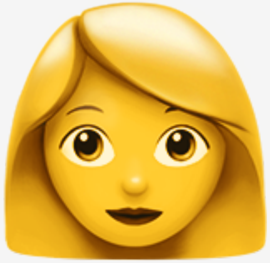

commit →

← i^*

$\text{View}^{[\![i]\!]}$ for $i \neq i^*$ →

$i^* \xleftarrow{\$} \{1, \cdots, N\}$

<u>commit:</u>

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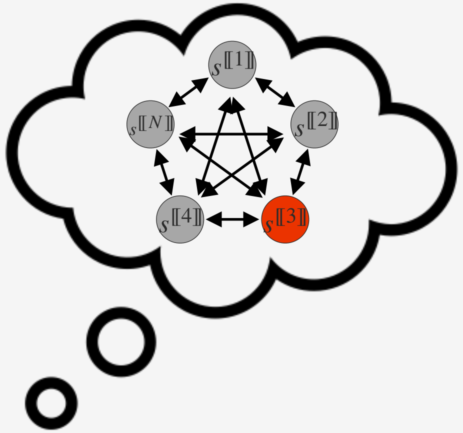
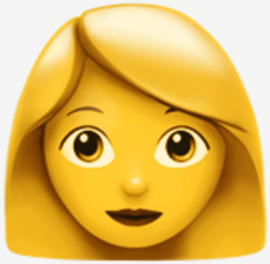
MPCitH protocol





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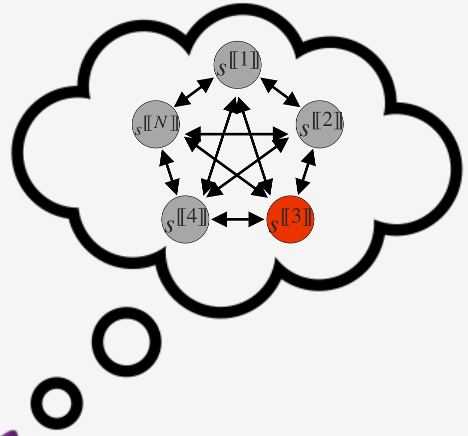
Check if $F(t_0) = A + Bt_0$

MPCitH protocol



commit →

i^* ←

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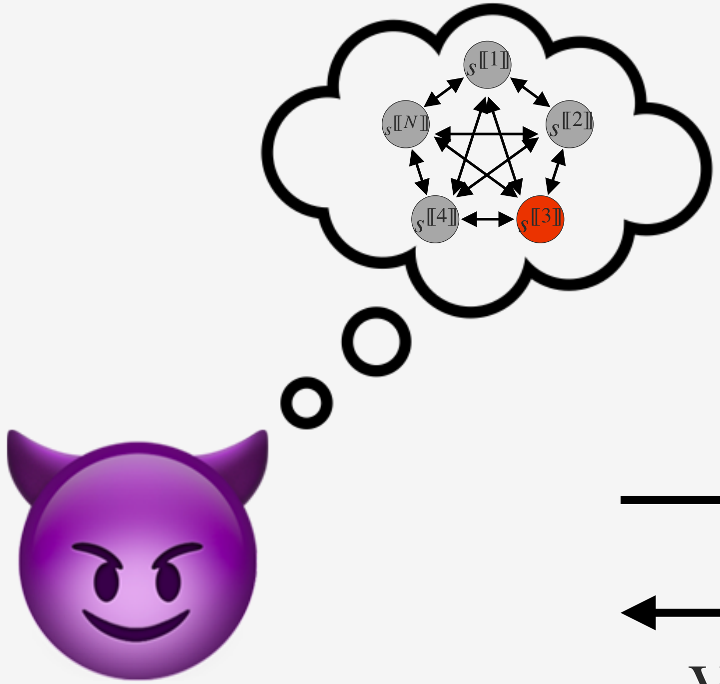
MPCitH protocol



commit →

← i^*

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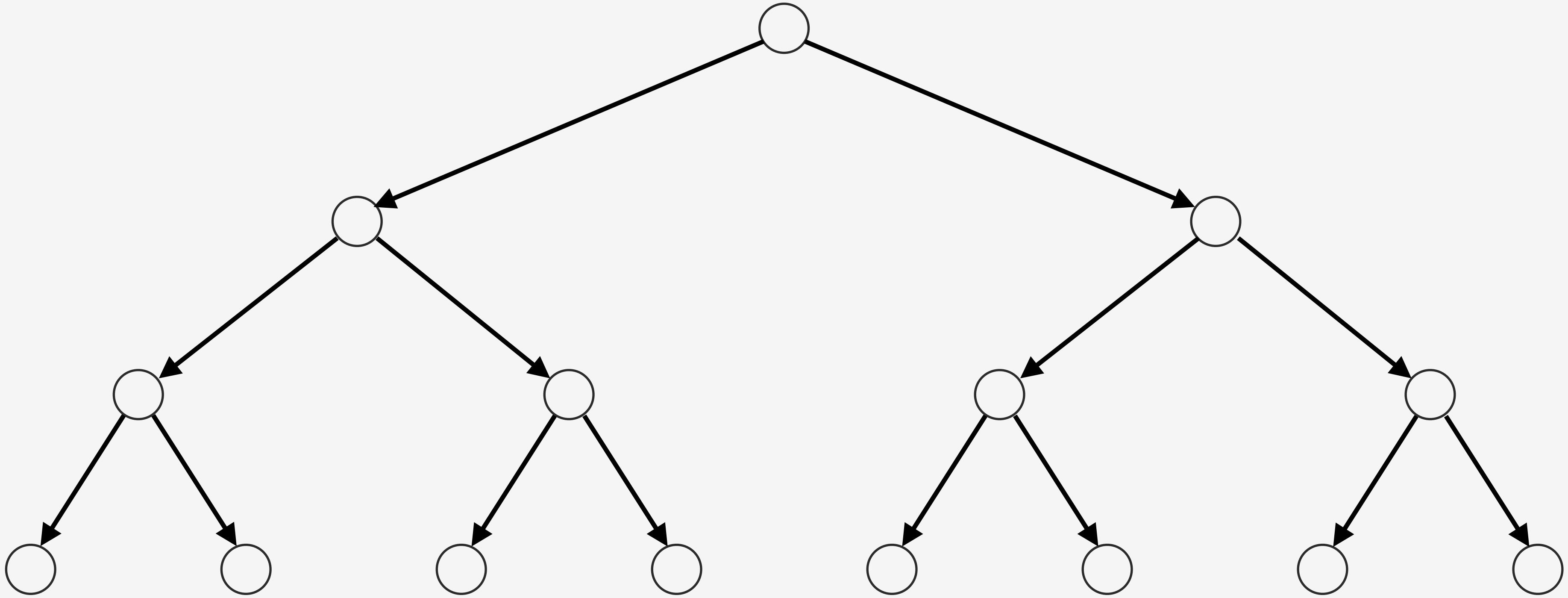
Soundness error $\dfrac{1}{N} + \dfrac{2}{q^k}$

Check if $F(t_0) = A + Bt_0$

Commit using GGM trees

Commit using GGM trees



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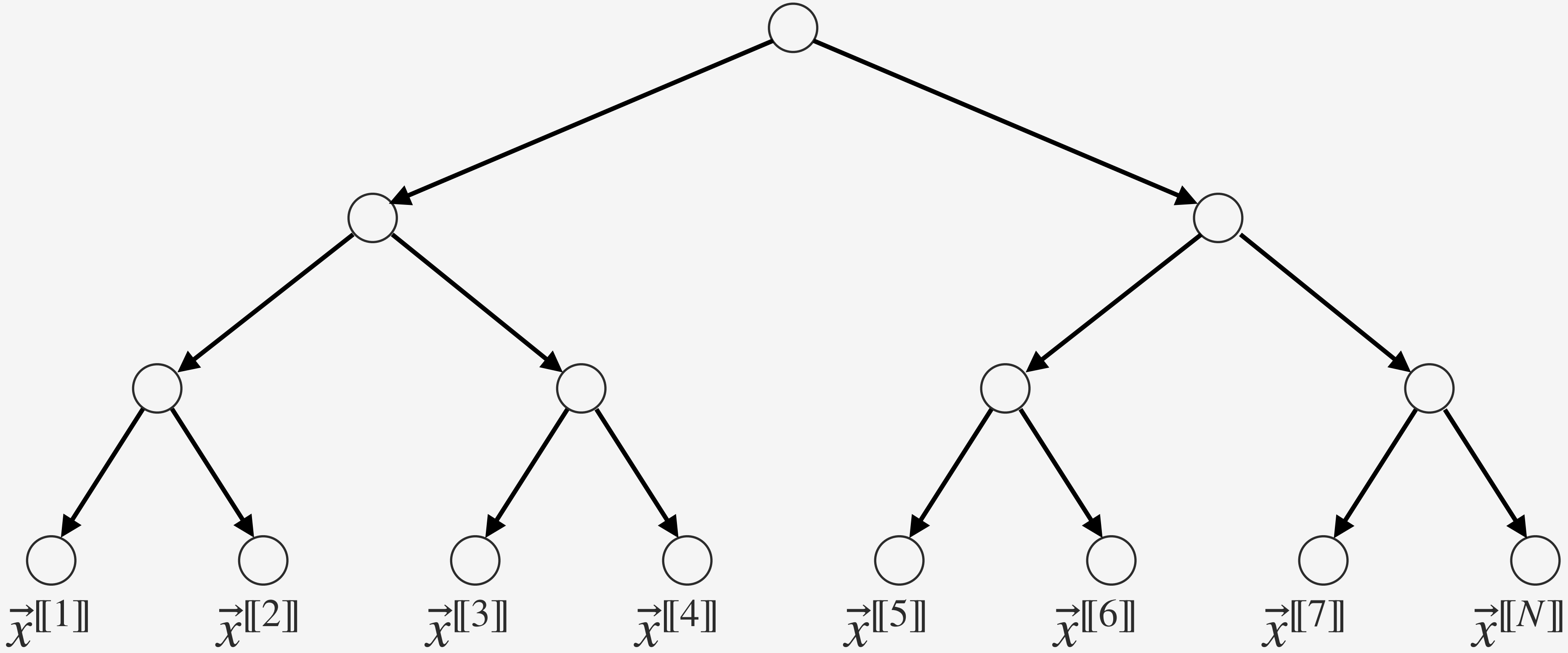
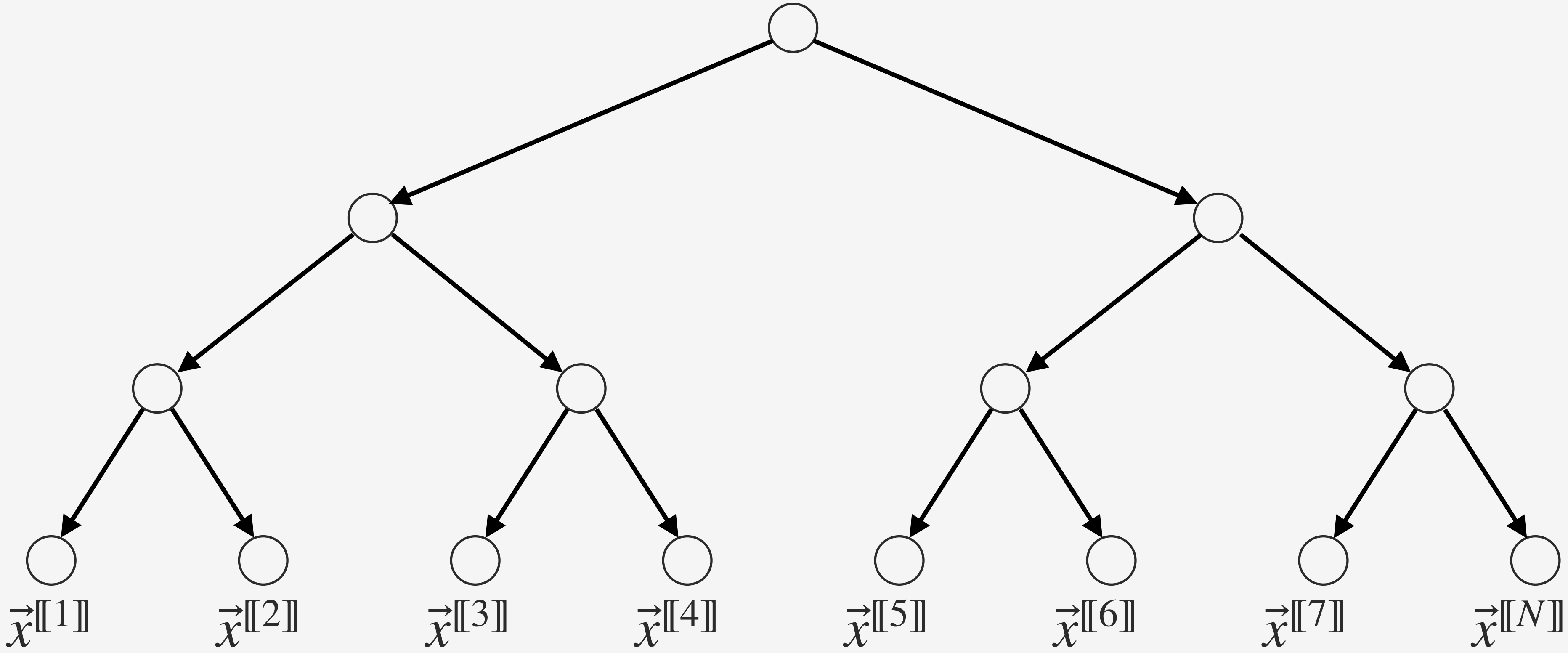


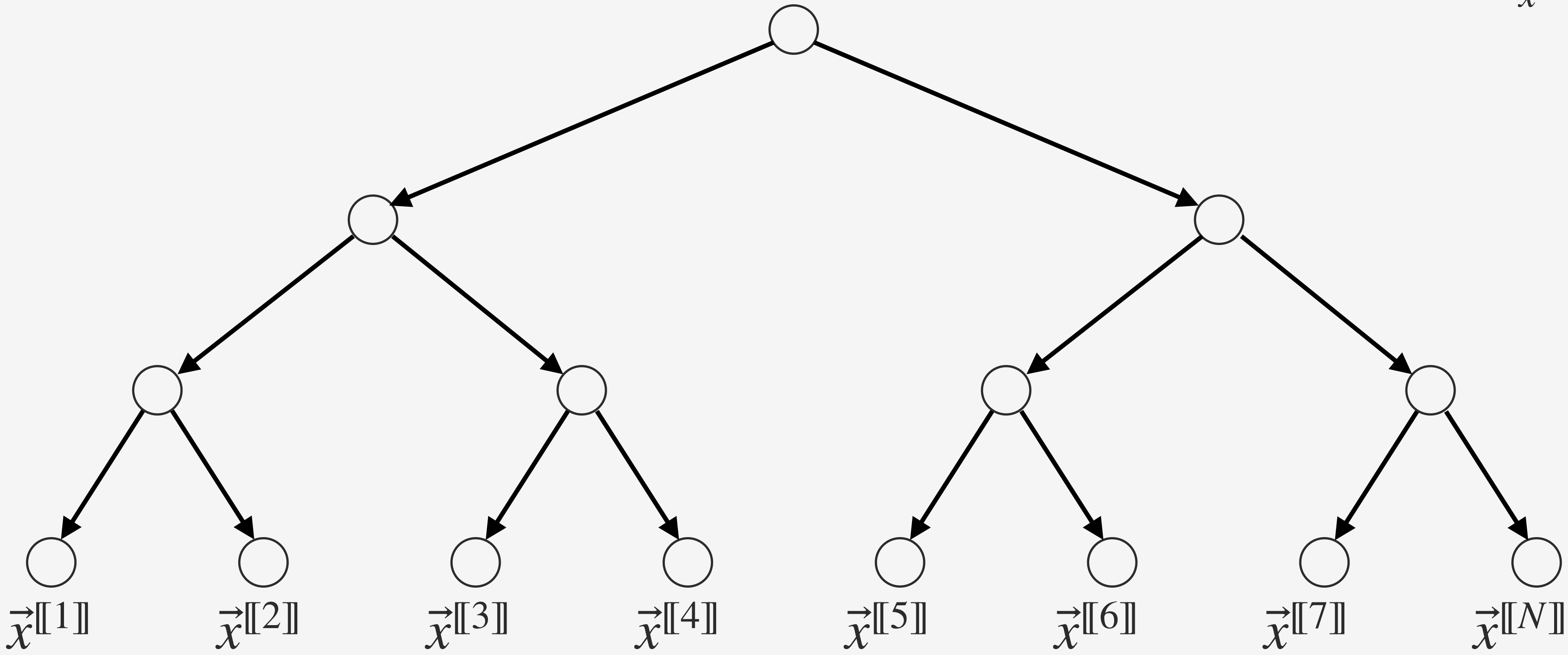
Commit using GGM trees



Derive $\vec{y}^{[\![i]\!]}, X_1^{[\![i]\!]}, X_2^{[\![i]\!]}, Y_1^{[\![i]\!]}, Y_2^{[\![i]\!]}, A^{[\![i]\!]}, B^{[\![i]\!]}$

Commit using GGM trees

$$\delta_{\vec{x}} = \vec{x} - \sum_{i=1}^{N} \vec{x}^{[\![i]\!]}$$



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Include offsets $\delta_{\vec{x}}, \delta_{\vec{y}}, \delta_A, \delta_B$ in commit

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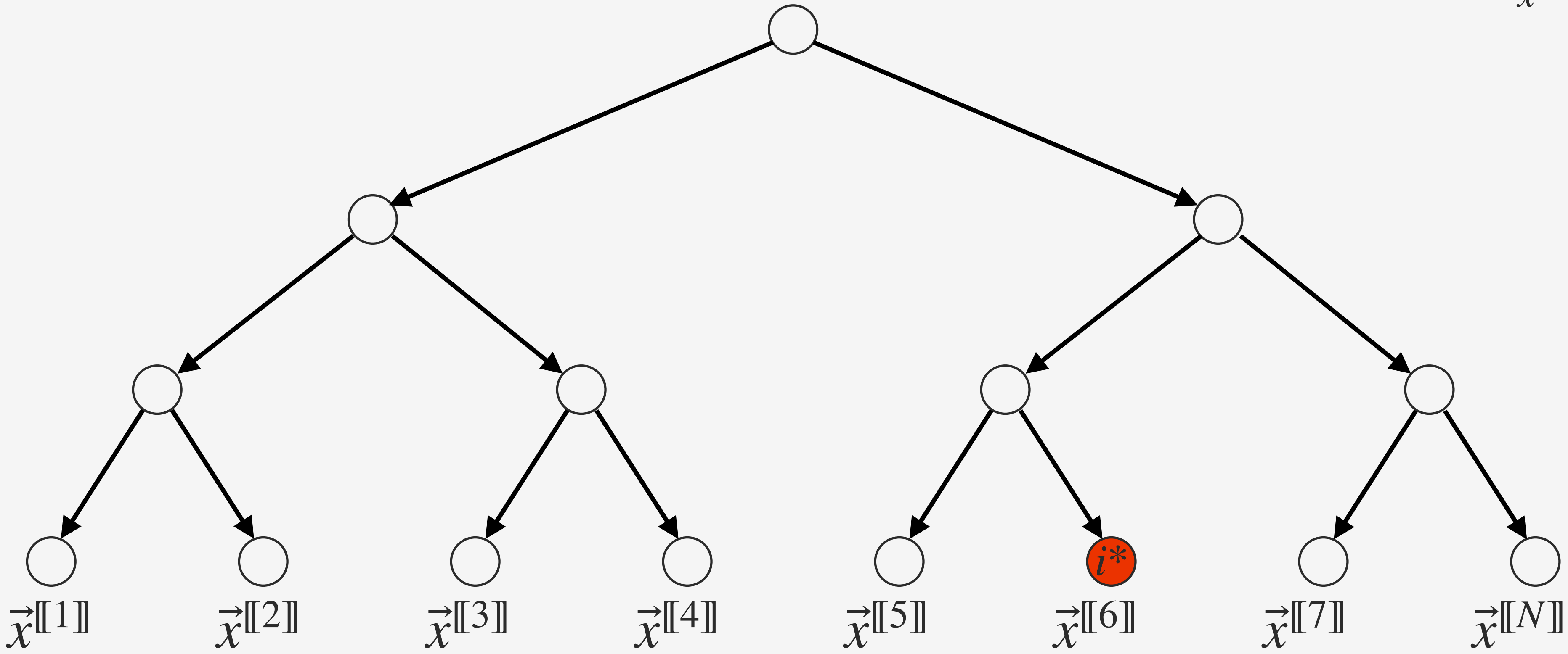

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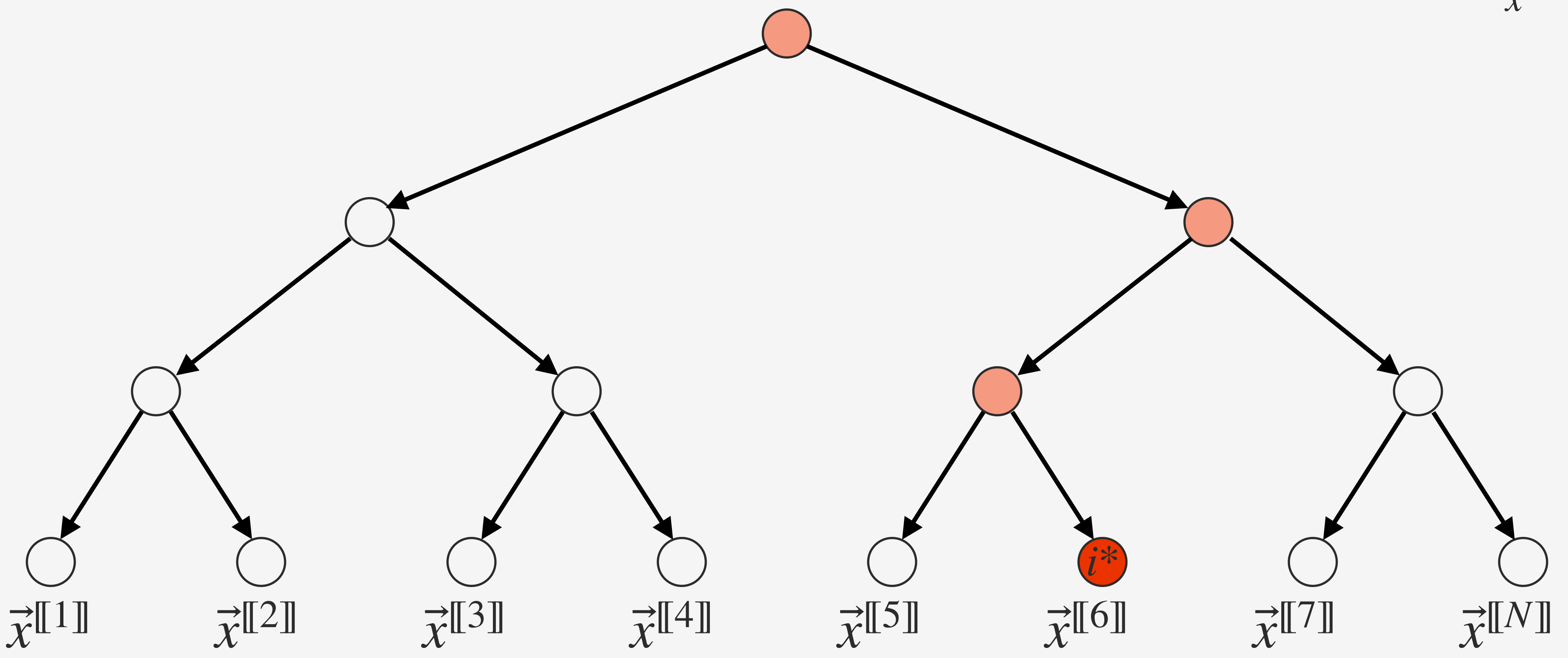

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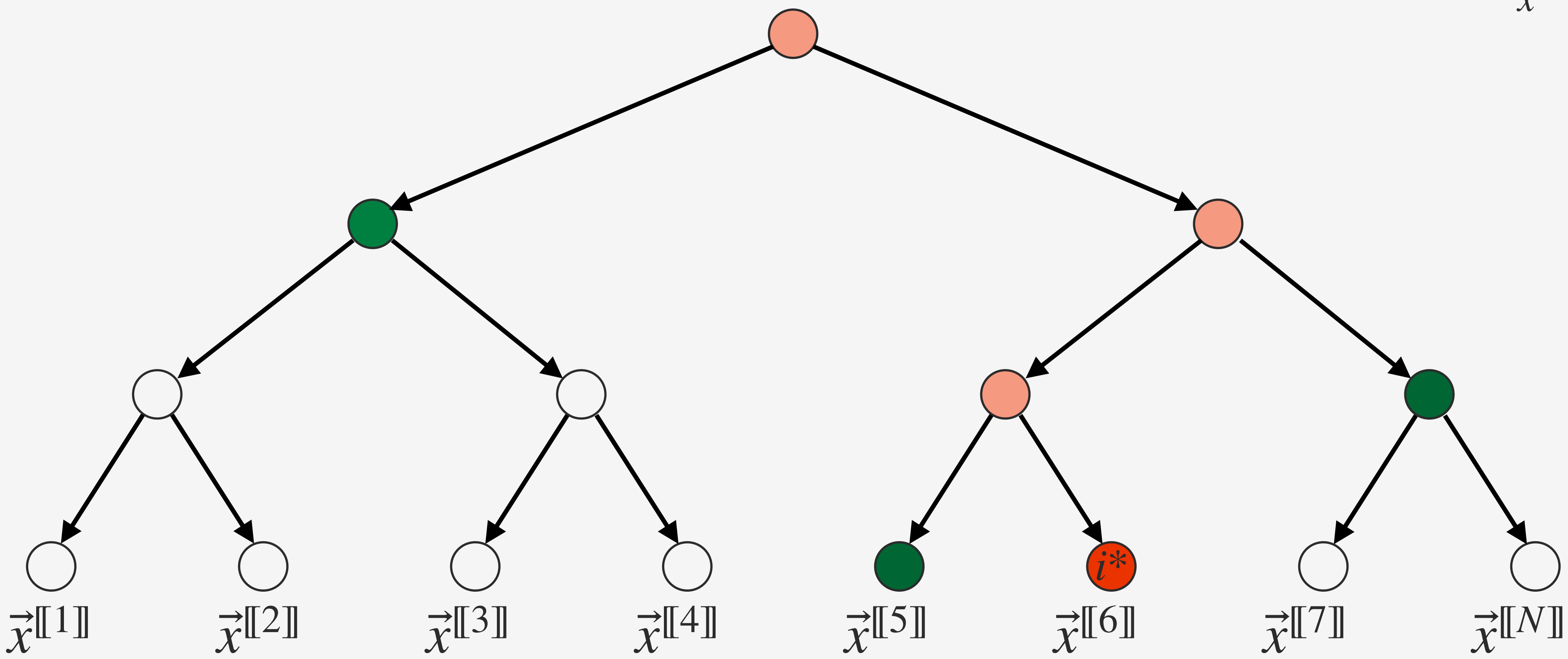

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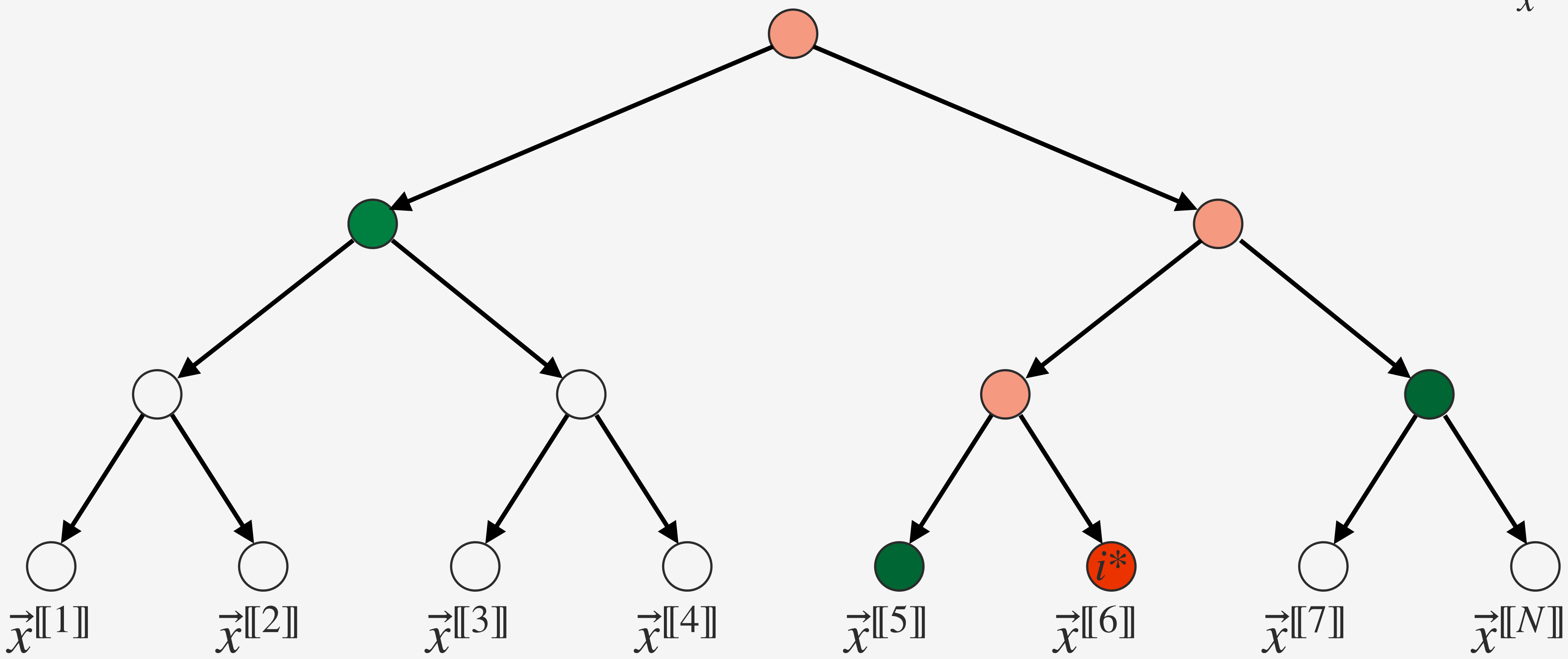

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Include offsets $\delta_{\vec{x}}, \delta_{\vec{y}}, \delta_A, \delta_B$ in commit

Open $\log_2(N)$ nodes

Signature scheme

Signature scheme

τ repetitions

Signature scheme

τ repetitions

Fiat-Shamir transform $\rightarrow$ signature scheme

Signature scheme

τ repetitions

Fiat-Shamir transform → signature scheme

h_{com}

Signature scheme

τ repetitions

Fiat-Shamir transform → signature scheme

h_{com}

For each round:

Signature scheme

τ repetitions

Fiat-Shamir transform $\rightarrow$ signature scheme

h_{com}

For each round:

- $\delta_{\vec{x}}, \delta_{\vec{y}}, \delta_A, \delta_B$

Signature scheme

τ repetitions

Fiat-Shamir transform → signature scheme

h_{com}

For each round:

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Signature scheme

τ repetitions

Fiat-Shamir transform → signature scheme

h_{com}

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Signature scheme

τ repetitions

Fiat-Shamir transform → signature scheme

h_{com}

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Signature scheme

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Fiat-Shamir transform → signature scheme

h_{com}

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- GGM tree opening

Signature scheme

τ repetitions

Fiat-Shamir transform → signature scheme

h_{com}

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- GGM tree opening

$\rightarrow \sigma$

Signature size and running times

Signature size and running times

$\underline{\lambda = 128}$

Signature size and running times

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$q = 2$
$n = 130$
$k = 257$

Signature size and running times

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N	τ	$\|\sigma\|$	t_{sign}	t_{verify}
2^8	16*	5.436 KB	0.76 ms	0.67 ms
2^9	15	5.340 KB	0.90 ms	0.81 ms
2^{10}	13	4.842 KB	1.05 ms	0.97 ms
2^{11}	12	4.665 KB	1.42 ms	1.34 ms
2^{12}	11	4.457 KB	2.10 ms	2.01 ms
2^{13}	10	4.216 KB	3.33 ms	3.23 ms
2^{15}	9	4.087 KB	11.85 ms	10.81 ms
2^{16}	8*	3.766 KB	19.32 ms	19.00 ms

Using an AMD EPYC 9374F running at 3.85 GHz
(*) $\lambda > 127.9999$

Signature size and running times

$\lambda = 128$

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Artifact versions:

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Artifact versions:
- Correlated GGM trees

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Artifact versions:
- Correlated GGM trees
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Signature size and running times

$\lambda = 128$

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Artifact versions:
- Correlated GGM trees
- AES
- Fast hypercube Folding

N	τ	$\|\sigma\|$	t_{sign}	t_{verify}
2^8	16*	5.436 KB	0.76 ms	0.67 ms
2^9	15	5.340 KB	0.90 ms	0.81 ms
2^{10}	13	4.842 KB	1.05 ms	0.97 ms
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Thank you

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Artifact