# Unconditionally Secure Commitments with Quantum Auxiliary Inputs / Preprocessing

*Merged talk based on concurrent works by*

Barak Nehoran (Princeton University)
Tomoyuki Morimae (Yukawa Institute for Theoretical Physics, Kyoto University)
Takashi Yamakawa (NTT Social Informatics Labs & YITP, Kyoto University)

*and*

Luowen Qian (Boston University & NTT Research)

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Barak Nehoran (Princeton U), Luowen Qian (Boston U & NTT), Tomoyuki Morimae (YITP), Takashi Yamakawa (NTT & YITP)

# Unconditionally Secure

Barak Nehoran (Princeton U), Luowen Qian (Boston U & NTT), Tomoyuki Morimae (YITP), Takashi Yamakawa (NTT & YITP)

# Unconditionally Secure

$\neq$

# Information Theoretic Security

Barak Nehoran (Princeton U), Luowen Qian (Boston U & NTT), Tomoyuki Morimae (YITP), Takashi Yamakawa (NTT & YITP)

# Unconditional ≠ Information-Theoretic Security

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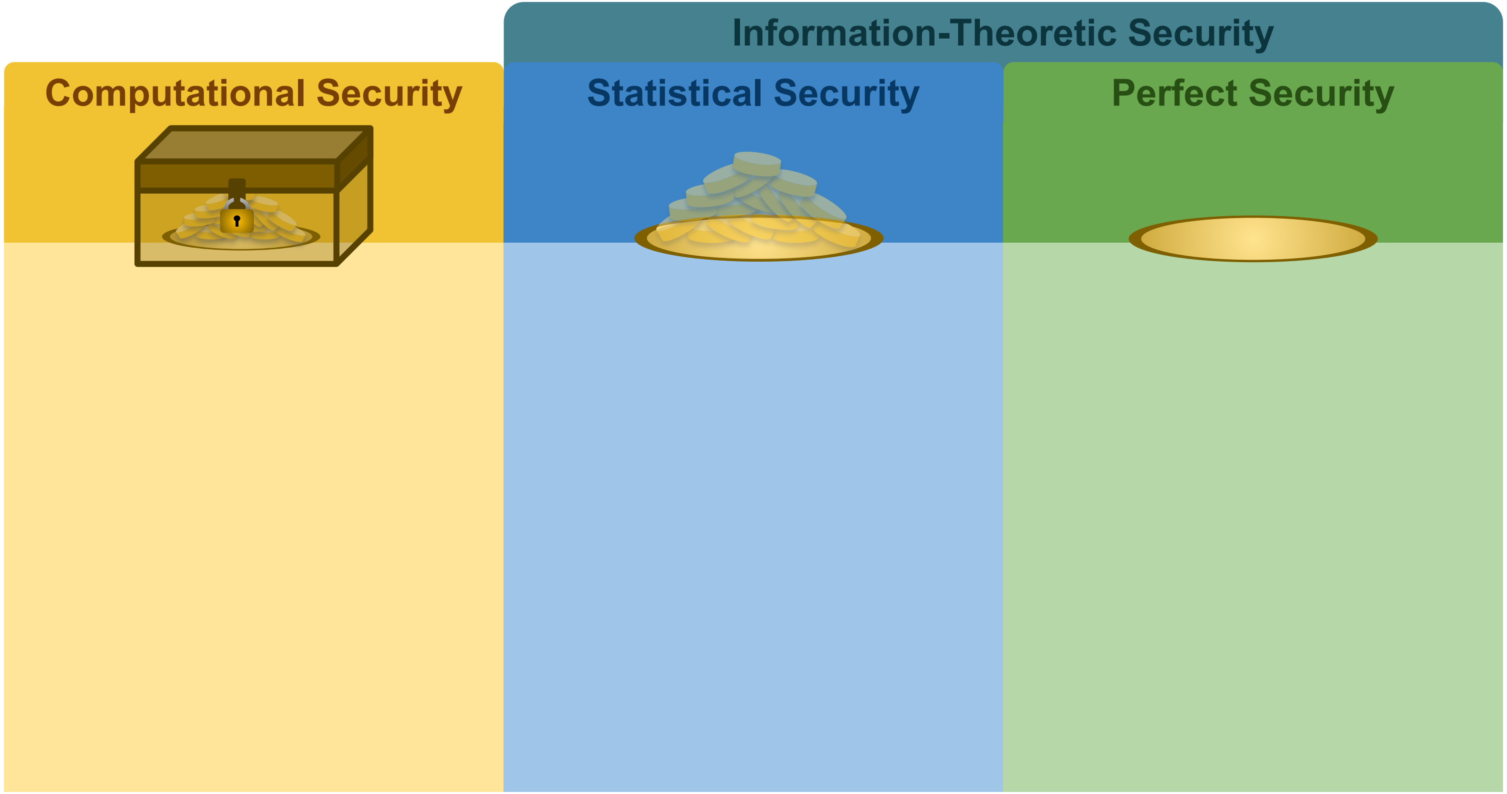
# Unconditional ≠ Information-Theoretic Security



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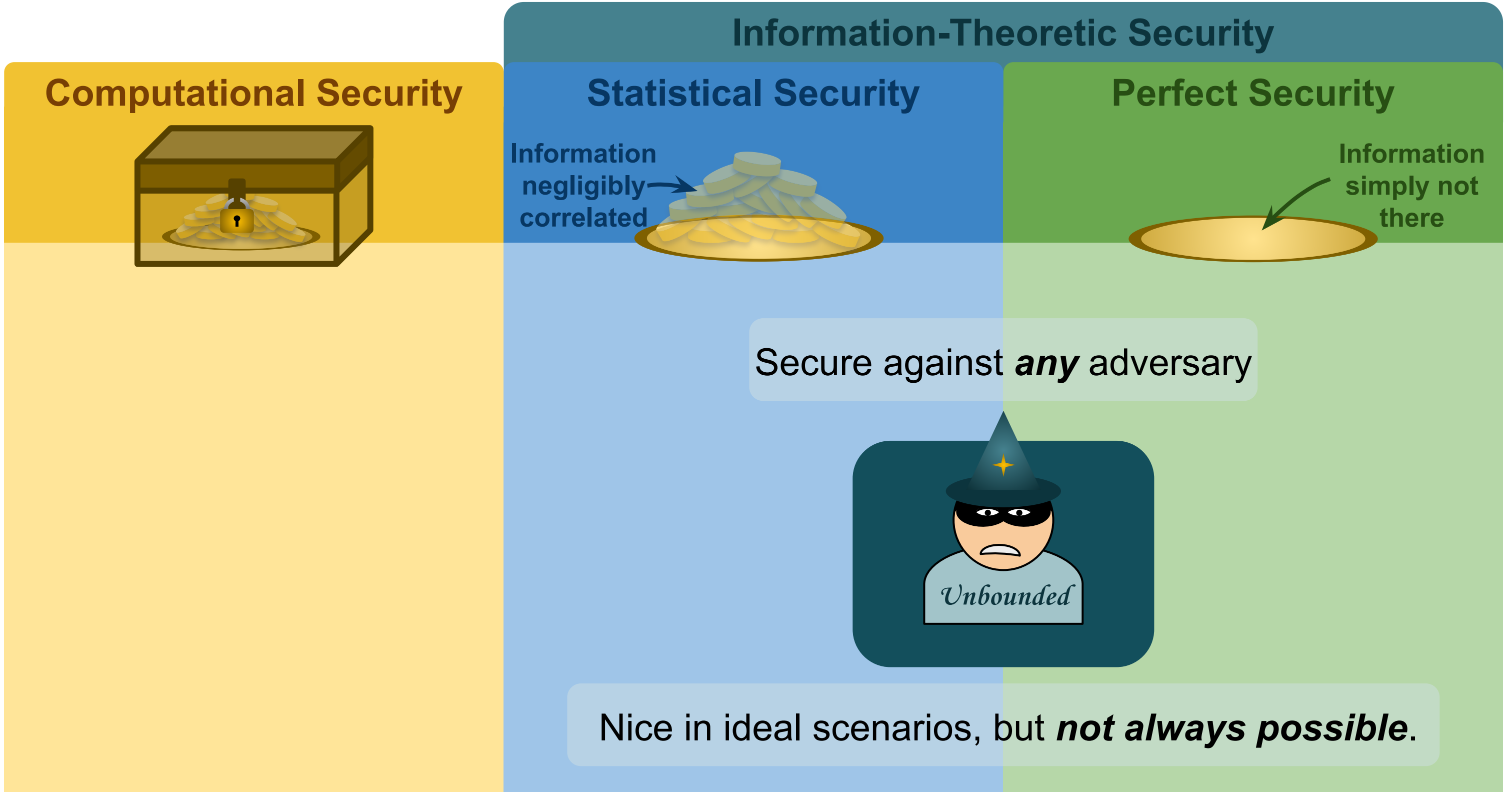
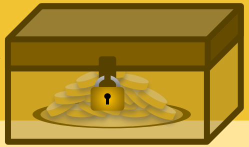


# Unconditional ≠ Information-Theoretic Security

Information is there, but secured behind hard computation

## Information-Theoretic Security

| Computational Security | Statistical Security | Perfect Security |
|---|---|---|



Information negligibly correlated

Information simply not there

Secure against a specified "***efficient***" adversary


Secure against ***any*** adversary


Nice in ideal scenarios, but ***not always possible***.

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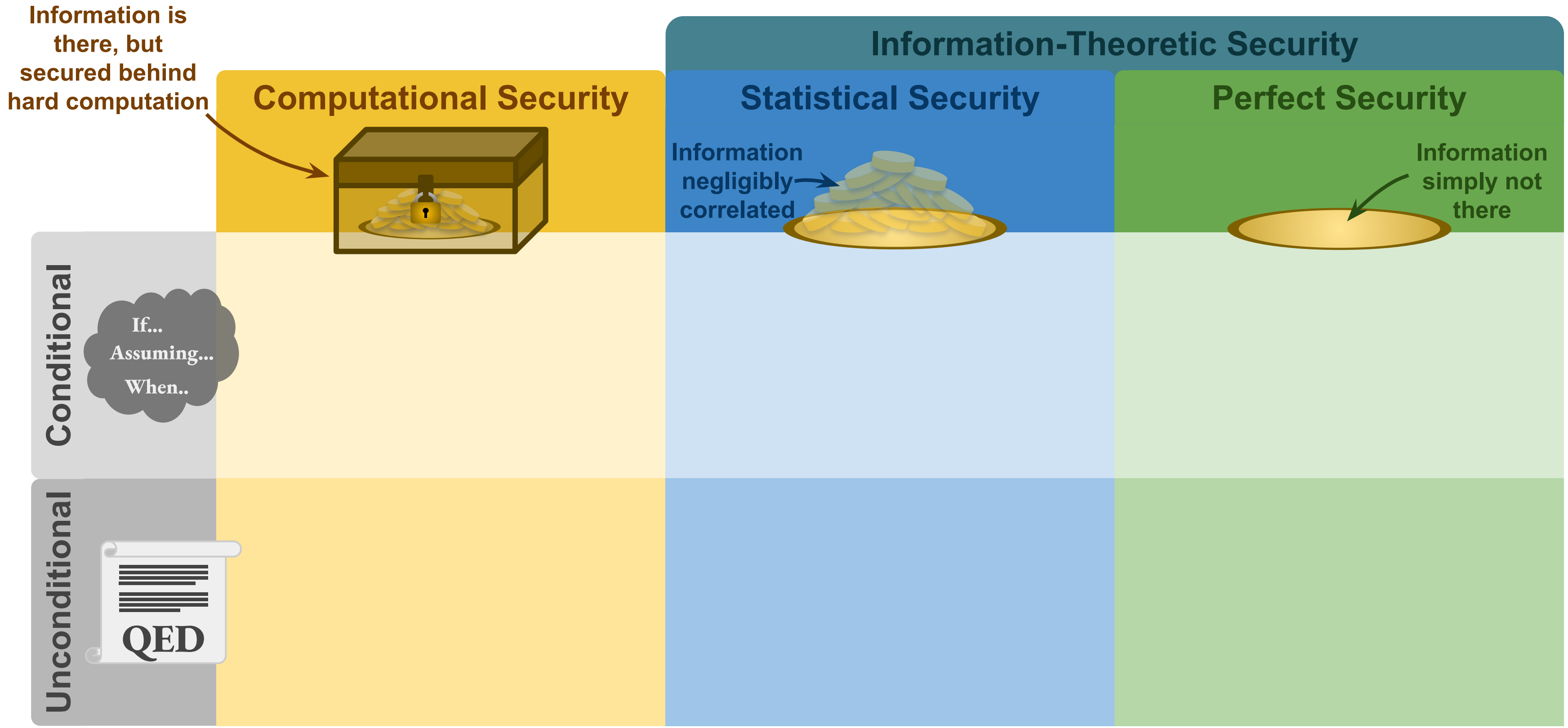

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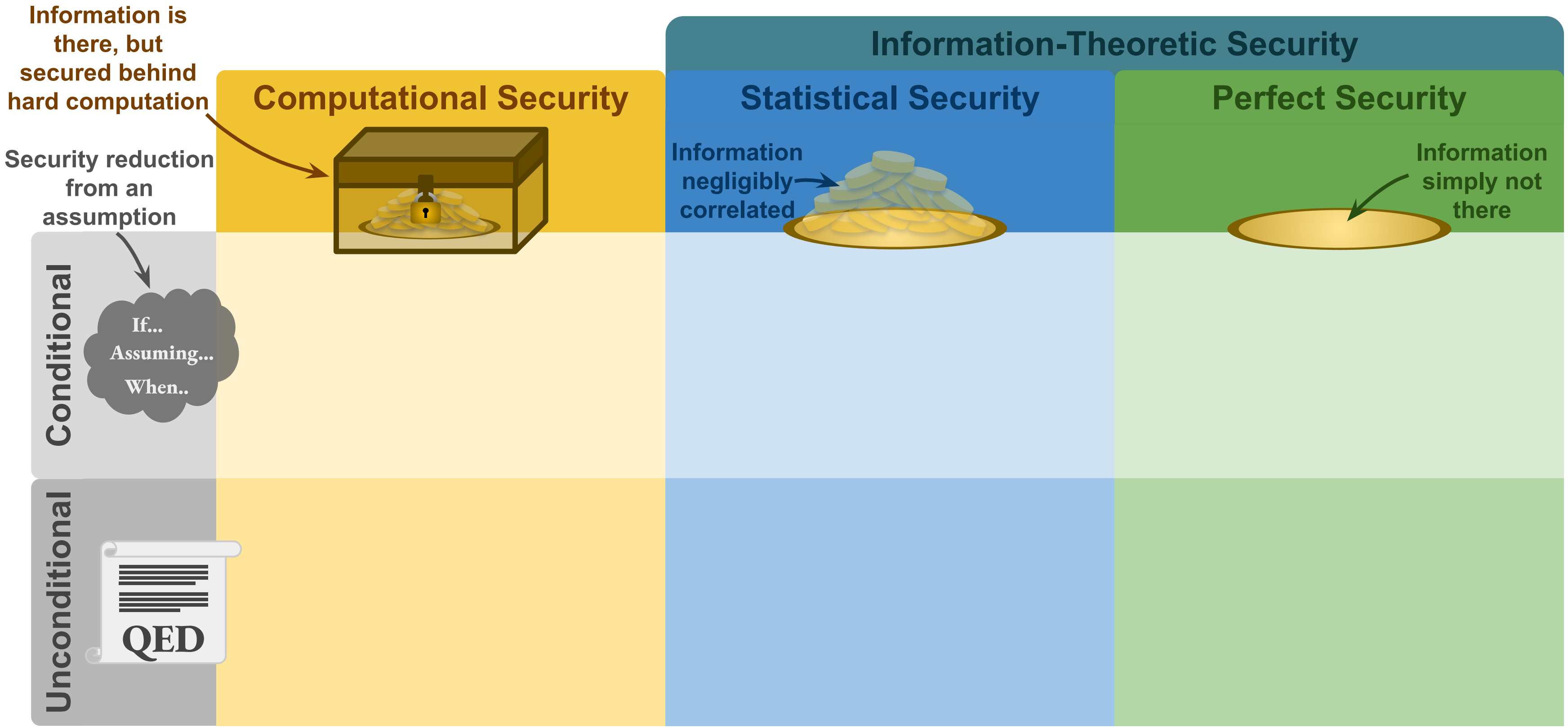

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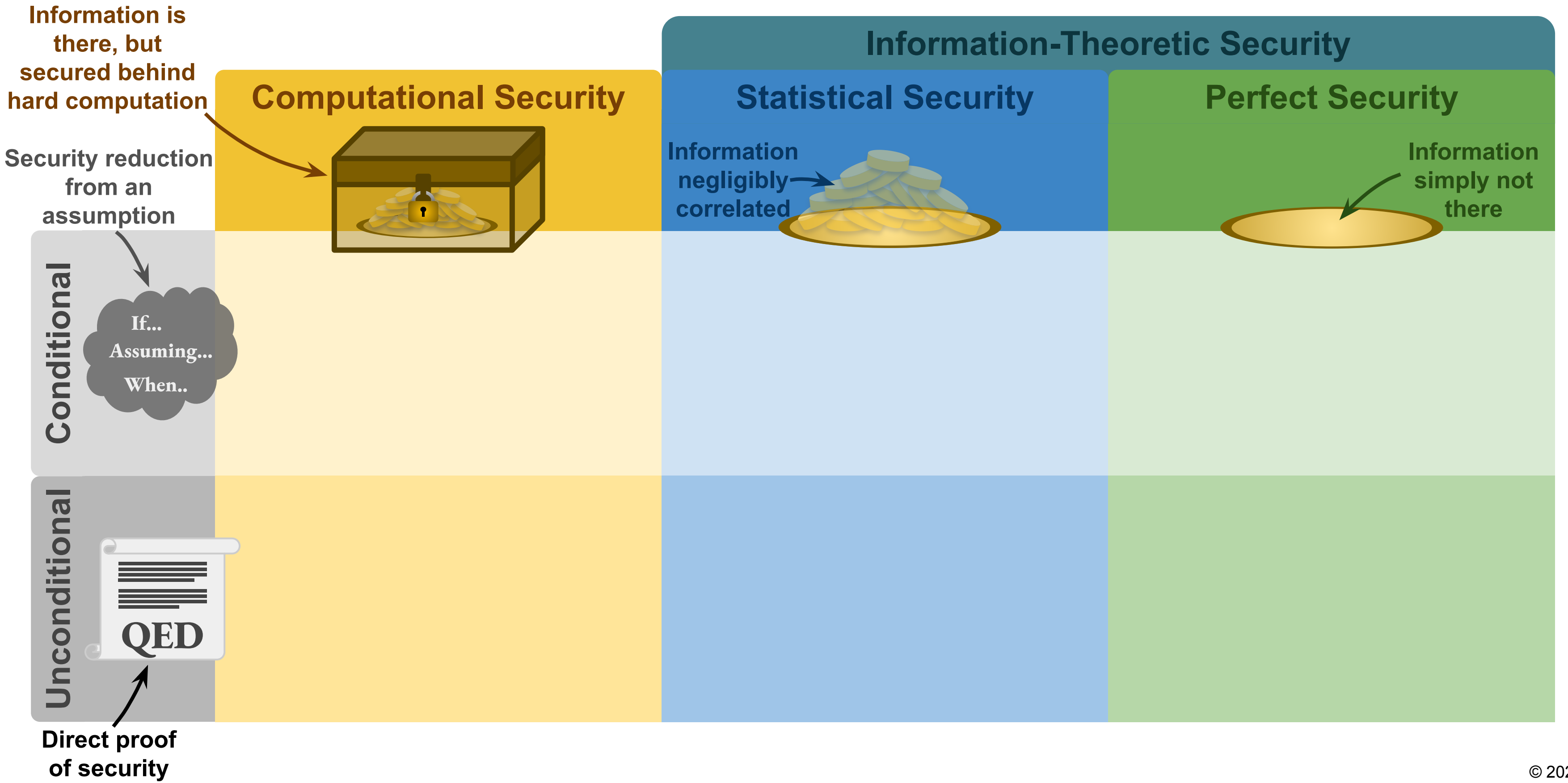

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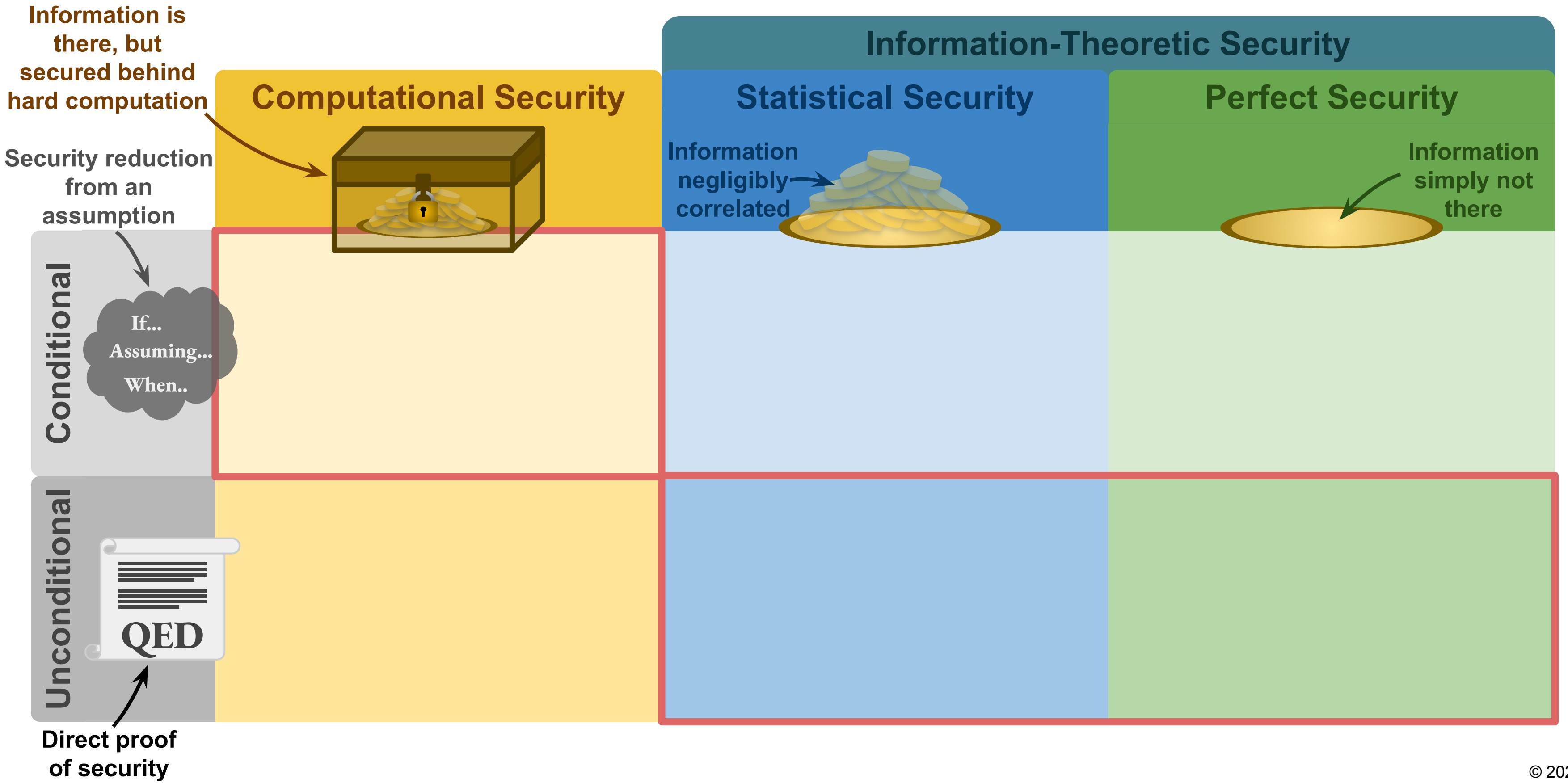

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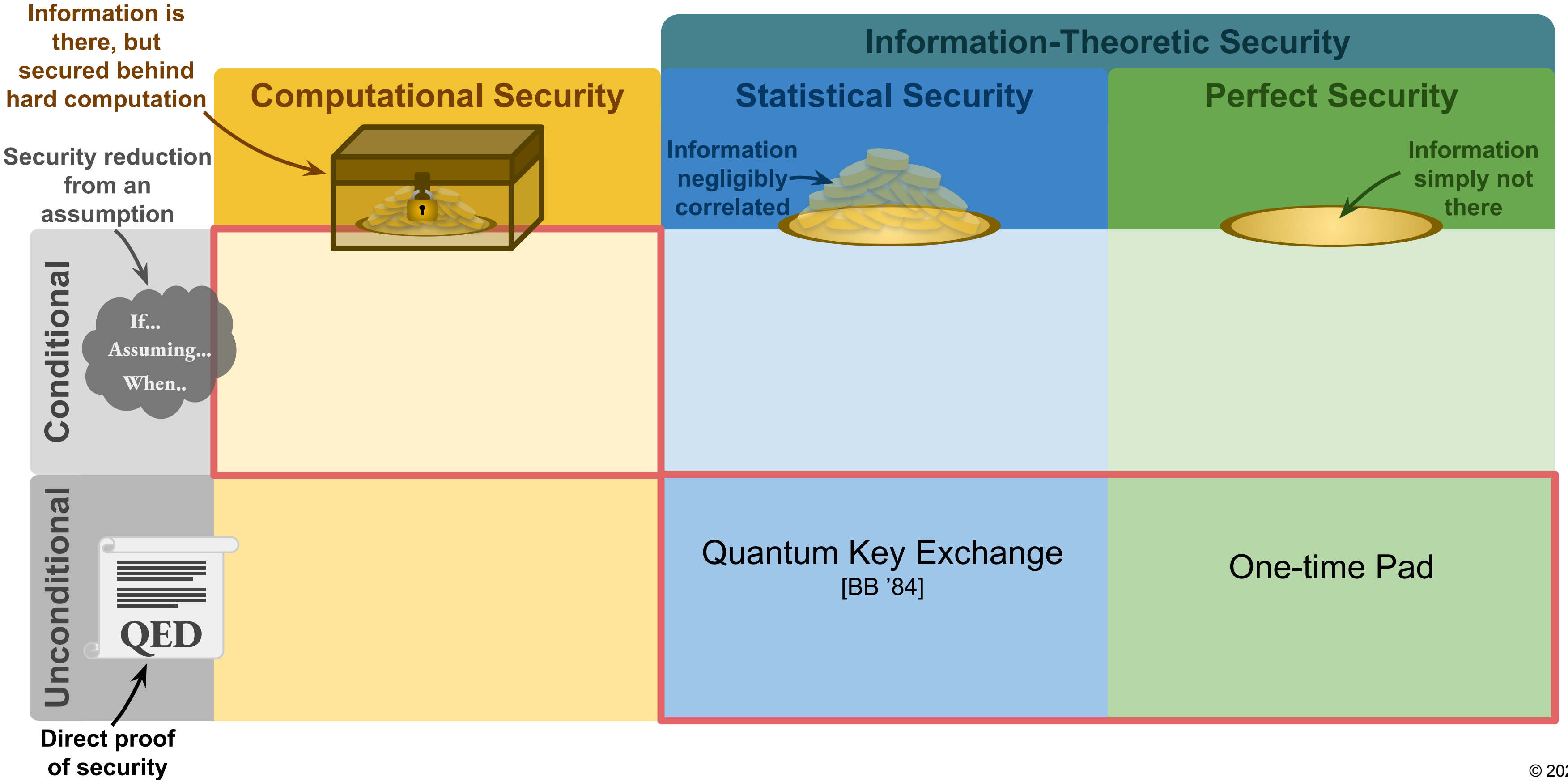

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# Unconditional ≠ Information-Theoretic Security

Information is there, but secured behind hard computation

Security reduction from an assumption

Information negligibly correlated

Information simply not there

| | | Information-Theoretic Security | |
|---|---|---|---|
| | Computational Security | Statistical Security | Perfect Security |
| Conditional (If… Assuming… When..) | Classical Key Exchange<br>→ Against *classical* poly time adversaries<br>→ Assuming DDH assumption<br>[DH '76] | | |
| Unconditional (QED) | | Quantum Key Exchange<br>[BB '84] | One-time Pad |

Direct proof of security

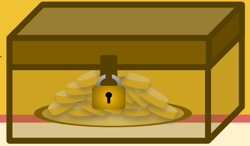
Barak Nehoran (Princeton U), Luowen Qian (Boston U & NTT), Tomoyuki Morimae (YITP), Takashi Yamakawa (NTT & YITP)

# Unconditional ≠ Information-Theoretic Security

Information is there, but secured behind hard computation

Security reduction from an assumption

## Computational Security



## Information-Theoretic Security

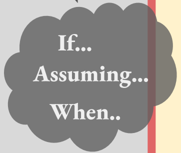
### Statistical Security

Information negligibly correlated

### Perfect Security

Information simply not there

**Conditional**




Classical Key Exchange
→ Against *classical* poly time adversaries
→ Assuming DDH assumption
[DH '76]

(examples of this are rare)

**Unconditional**


Direct proof of security

Quantum Key Exchange
[BB '84]

One-time Pad

# Unconditional ≠ Information-Theoretic Security

Information is there, but secured behind hard computation

Security reduction from an assumption

| | Computational Security | Information-Theoretic Security: Statistical Security | Information-Theoretic Security: Perfect Security |
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Direct proof of security

Barak Nehoran (Princeton U), Luowen Qian (Boston U & NTT), Tomoyuki Morimae (YITP), Takashi Yamakawa (NTT & YITP)

# Unconditional ≠ Information-Theoretic Security

|  | **Computational Security** | **Statistical Security** | **Perfect Security** |
|---|---|---|---|
|  |  | **Information-Theoretic Security** | **Information-Theoretic Security** |
| **Conditional** | Classical Key Exchange<br>→ Against *classical* poly time adversaries<br>→ Assuming DDH assumption<br>[DH '76] | (examples of this are rare) | (examples of this are rare) |
| **Unconditional** | existence of<br>Hard Functions | Quantum Key Exchange<br>[BB '84] | One-time Pad |

**Information is there, but secured behind hard computation**

**Information negligibly correlated**

**Information simply not there**

**Security reduction from an assumption**

If...
Assuming...
When..

QED

**Direct proof of security**

Barak Nehoran (Princeton U), Luowen Qian (Boston U & NTT), Tomoyuki Morimae (YITP), Takashi Yamakawa (NTT & YITP)

# Unconditional ≠ Information-Theoretic Security

Information is there, but secured behind hard computation

Security reduction from an assumption

Direct proof of security

| | Computational Security | Information-Theoretic Security: Statistical Security | Information-Theoretic Security: Perfect Security |
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| Unconditional (QED) | ~~Hard Functions~~ Nothing cryptographically useful existence of | Quantum Key Exchange [BB '84] | One-time Pad |

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Direct proof of security

Can we get ***unconditional computational*** *cryptography?*

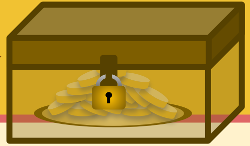
Barak Nehoran (Princeton U), Luowen Qian (Boston U & NTT), Tomoyuki Morimae (YITP), Takashi Yamakawa (NTT & YITP)

# Unconditional ≠ Information-Theoretic Security

Information is there, but secured behind hard computation

Security reduction from an assumption

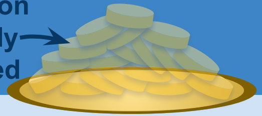
## Computational Security



## Information-Theoretic Security

### Statistical Security

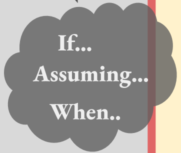
Information negligibly correlated



### Perfect Security

Information simply not there

**Conditional**




Classical Key Exchange
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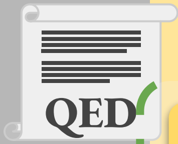
**Unconditional**




Direct proof of security

existence of ~~Hard Functions~~ Nothing cryptographically useful

Commitments with Quantum Auxiliary Inputs

This work

Quantum Key Exchange
[BB '84]

One-time Pad

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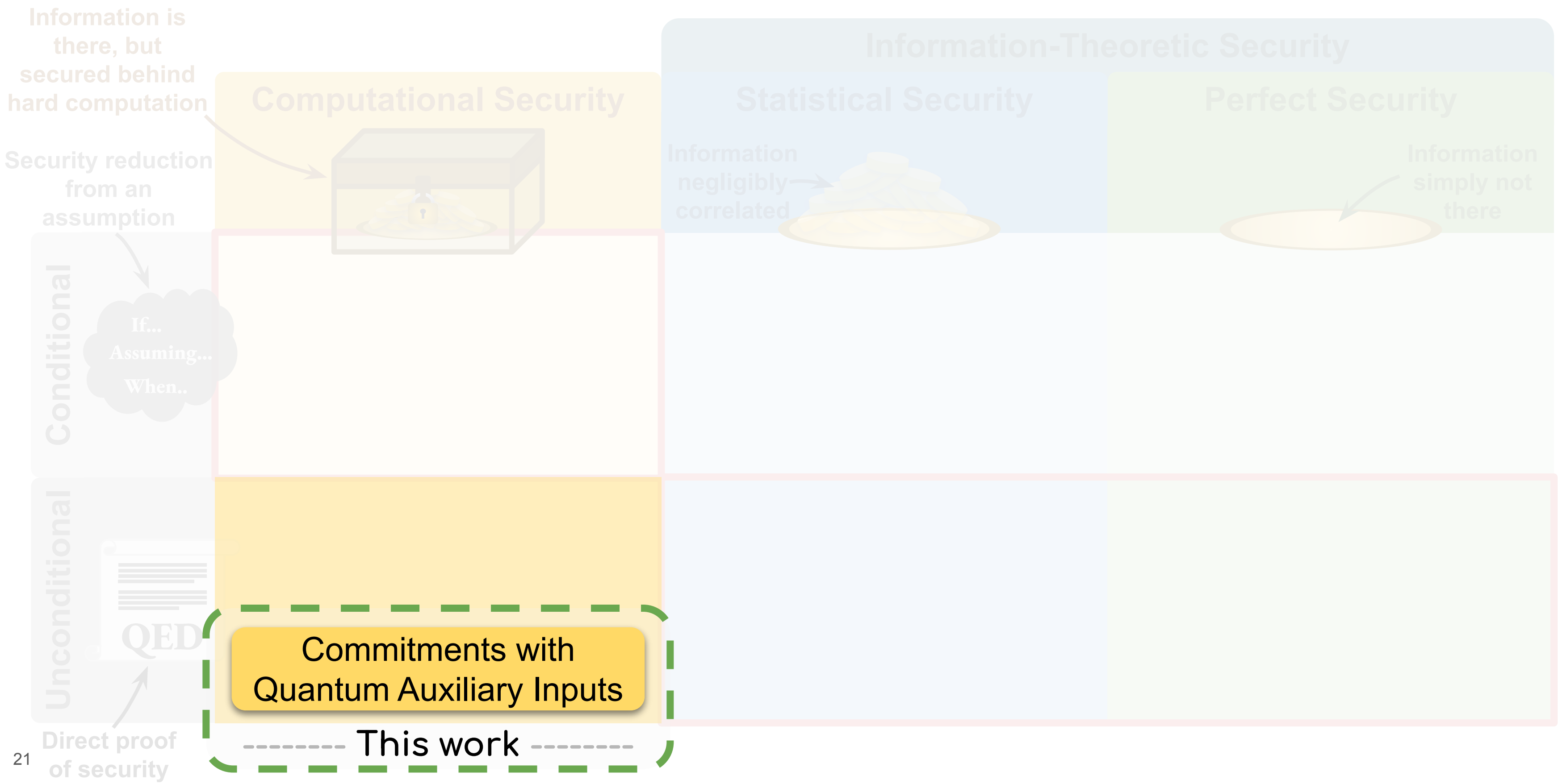

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# Unconditional ≠ Information-Theoretic Security

Information is there, but secured behind hard computation

Computational Security

Security reduction from an assumption

Conditional

If... Assuming... When..

Unconditional

QED

Direct proof of security

## Quantum Commitments

1. **Commit** to a *hidden* message without revealing it
2. **Reveal** the original message, which is *bound* to what you committed


Commitments with Quantum Auxiliary Inputs

This work

Barak Nehoran (Princeton U), Luowen Qian (Boston U & NTT), Tomoyuki Morimae (YITP), Takashi Yamakawa (NTT & YITP)

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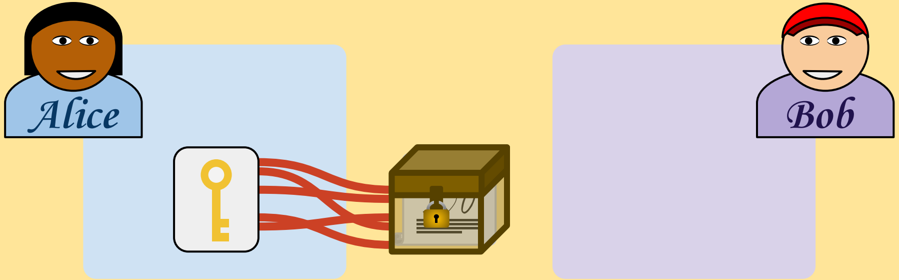


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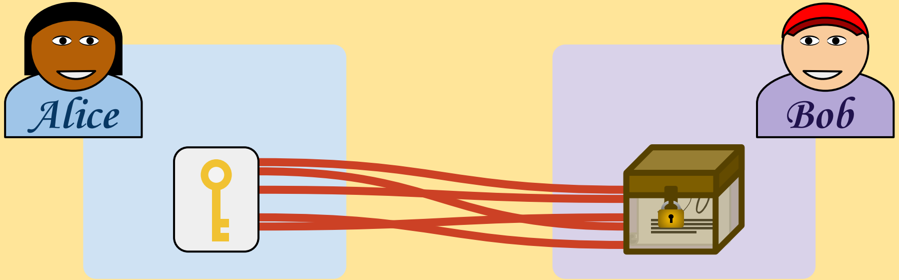

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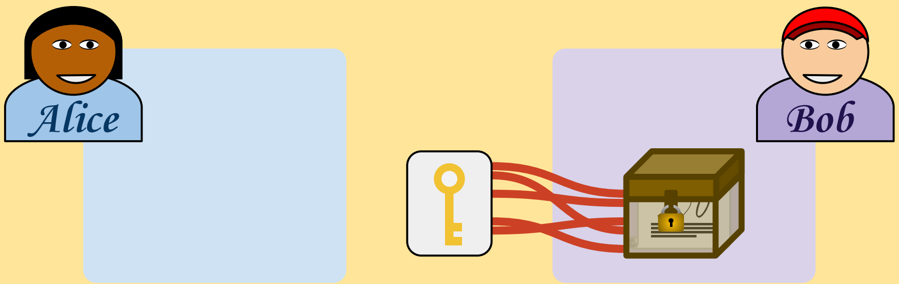


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## Quantum Commitments with Quantum Auxiliary Inputs

First defined by [Chailloux, Kerenidis, Rosgen '16]

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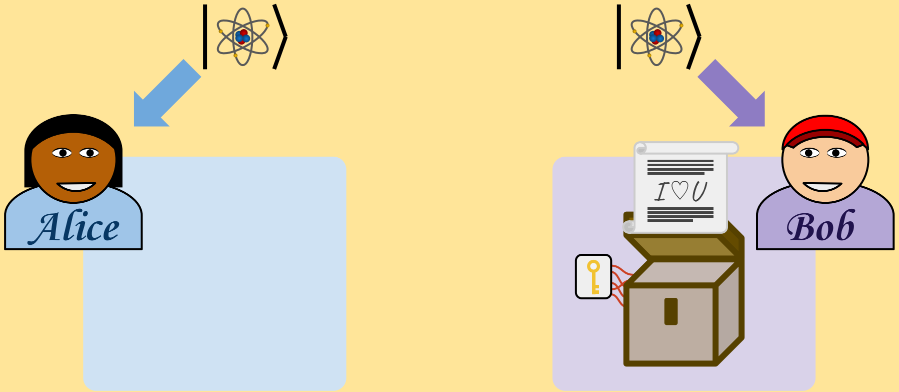


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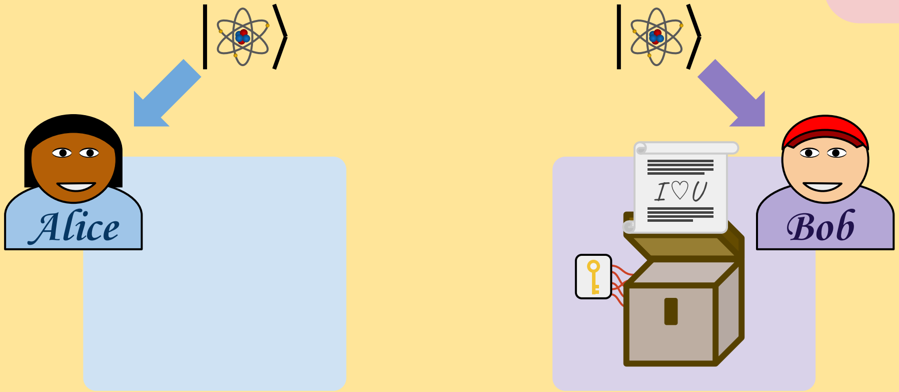


showed security from an **assumption.** **(QIP ≠ QMA)**

Commitments with Quantum Auxiliary Inputs

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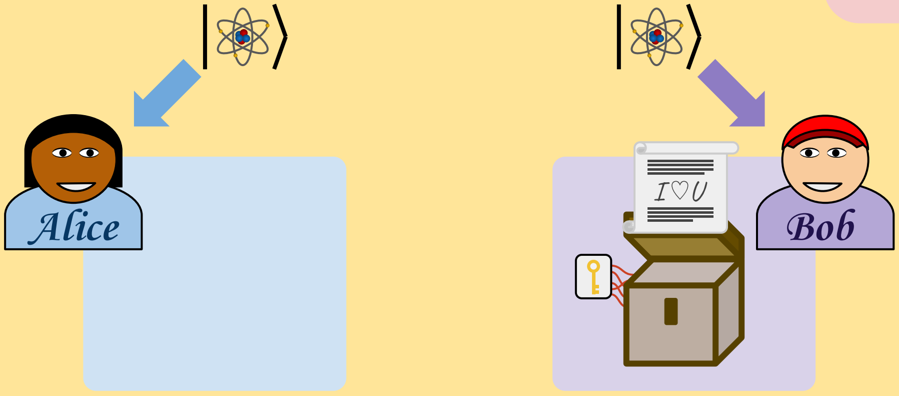


We show they exist ***unconditionally***

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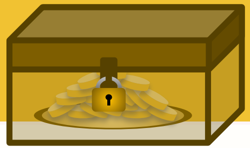
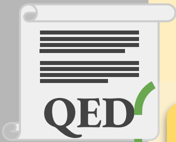
## Computational Security






Commitments with Quantum Auxiliary Inputs

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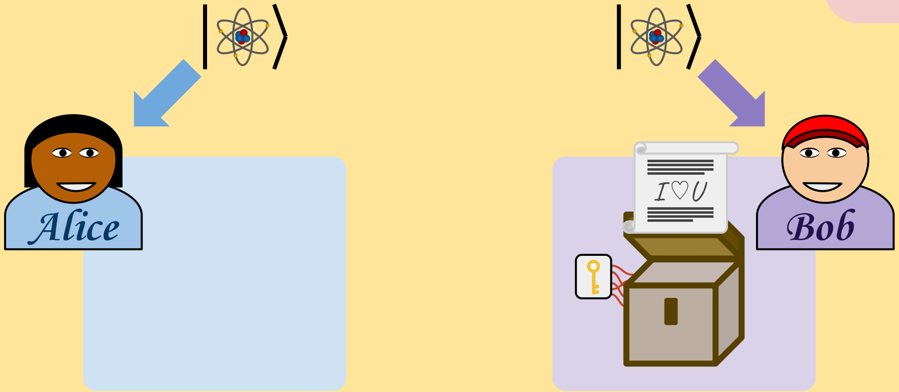


We show they exist ***unconditionally***

with ***computational*** security!

(computational hiding and statistical binding)

Barak Nehoran (Princeton U), Luowen Qian (Boston U & NTT), Tomoyuki Morimae (YITP), Takashi Yamakawa (NTT & YITP)

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Security reduction from an assumption

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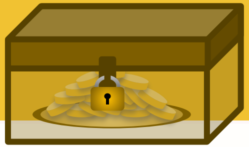
## Computational Security



This is the ***first unconditional computationally secure*** cryptographic scheme.

Unconditional


Commitments with Quantum Auxiliary Inputs

--------- This work ---------

Direct proof of security

## Quantum Commitments with Quantum Auxiliary Inputs

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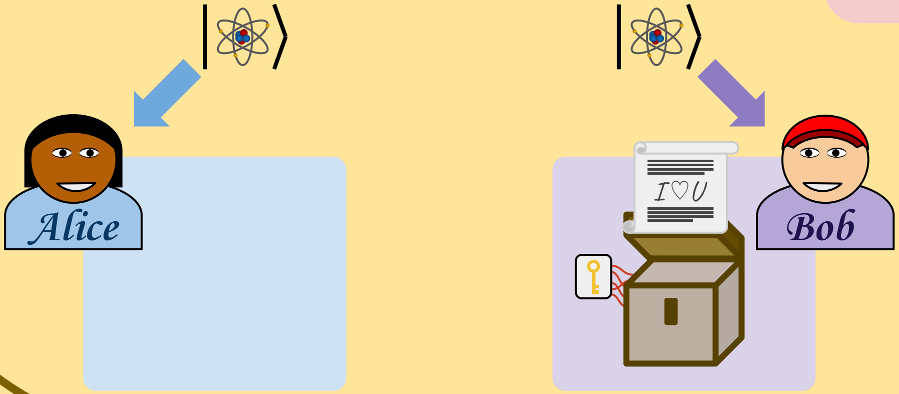


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# Quantum Commitments

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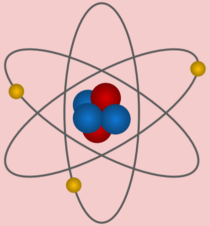
# Quantum Commitments

$|\Psi_0\rangle$



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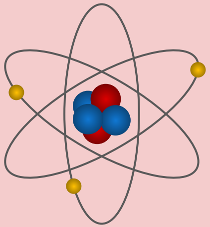
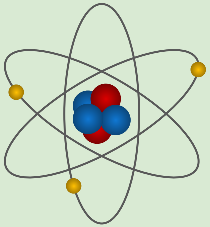
# Quantum Commitments

$$|\Psi_0\rangle$$



$$|\Psi_1\rangle$$

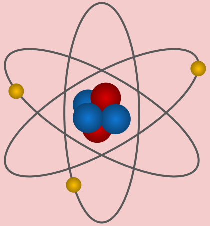


Barak Nehoran (Princeton U), Luowen Qian (Boston U & NTT), Tomoyuki Morimae (YITP), Takashi Yamakawa (NTT & YITP)

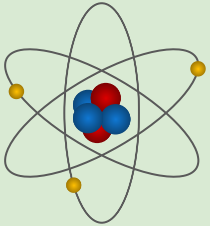
# Quantum Commitments

**Commitment to 0**

$$|\Psi_0\rangle$$



$$|\Psi_1\rangle$$







**(This is without loss of generality)**

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# Quantum Commitments

**Commitment to 0**

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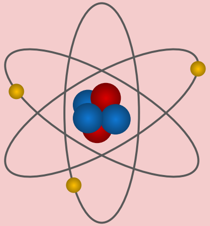
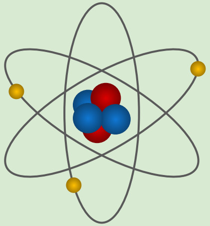


**Commitment to 1**

$|\Psi_1\rangle$







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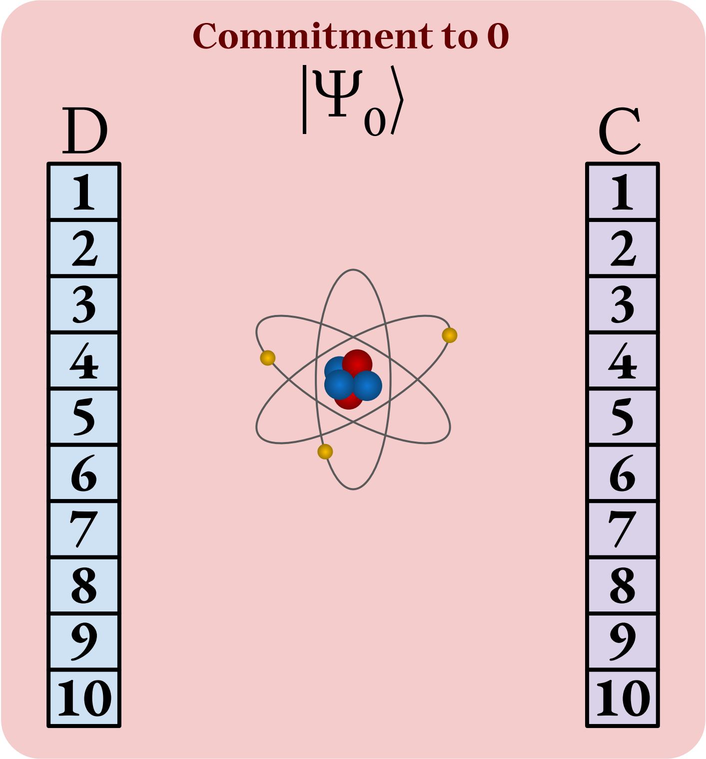


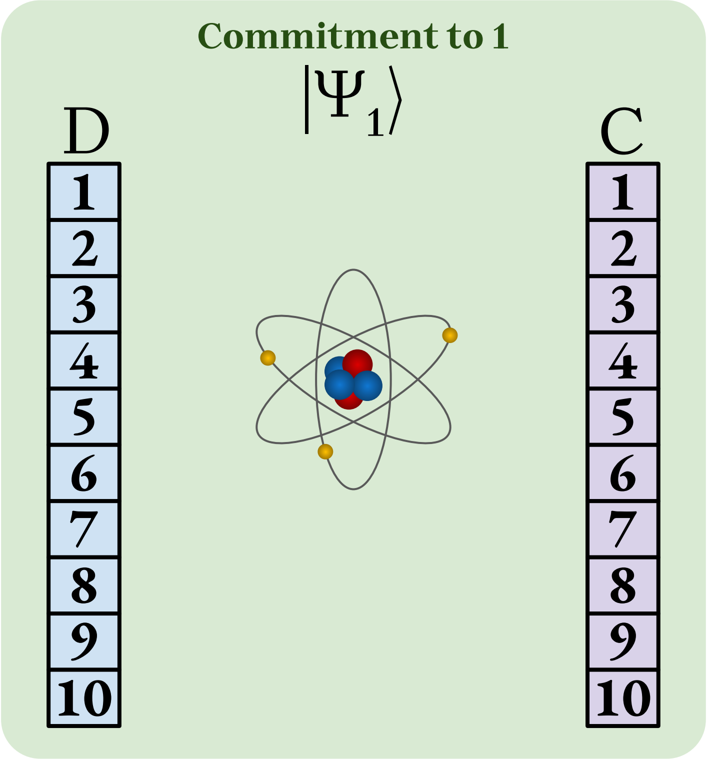


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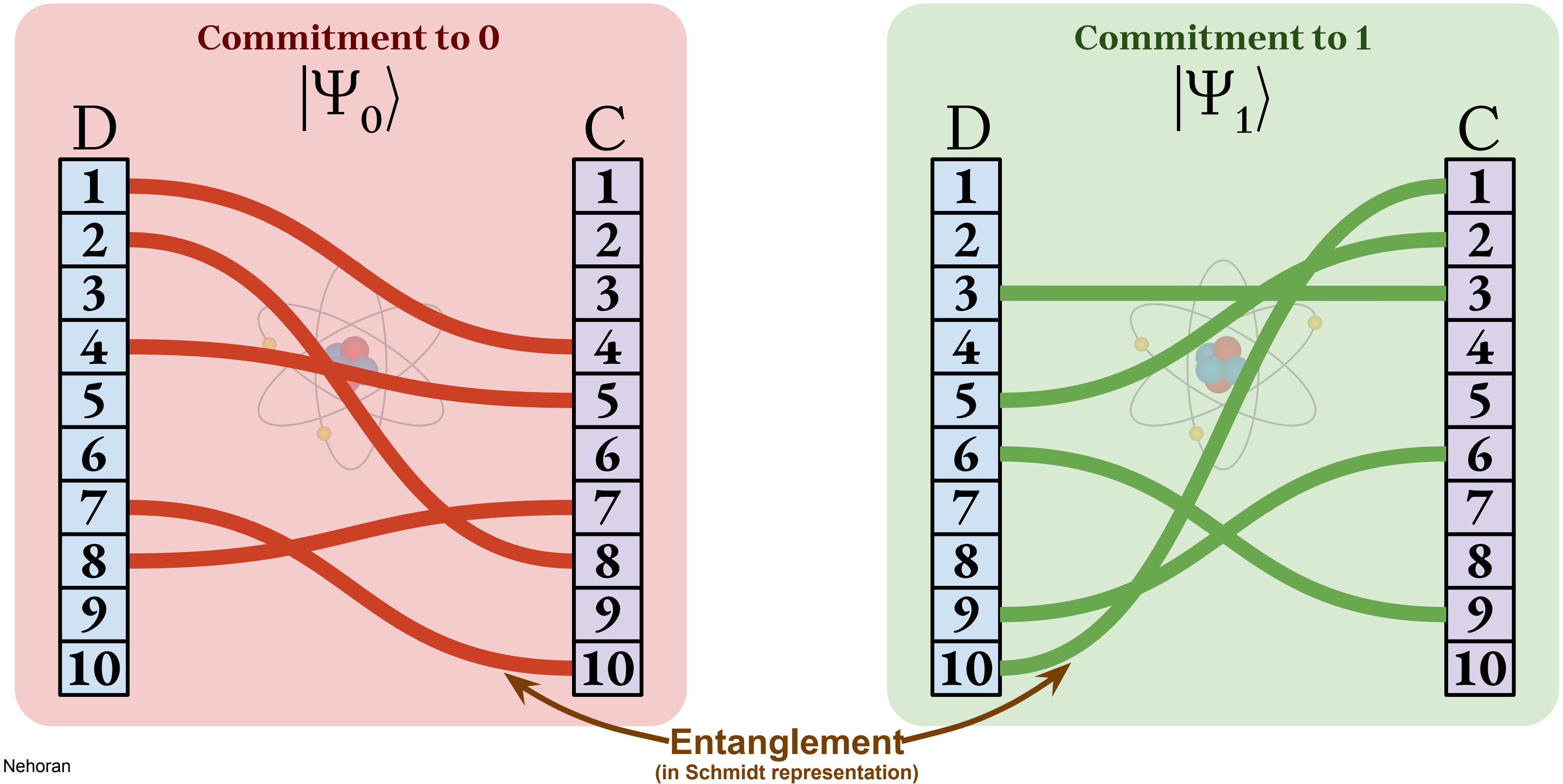

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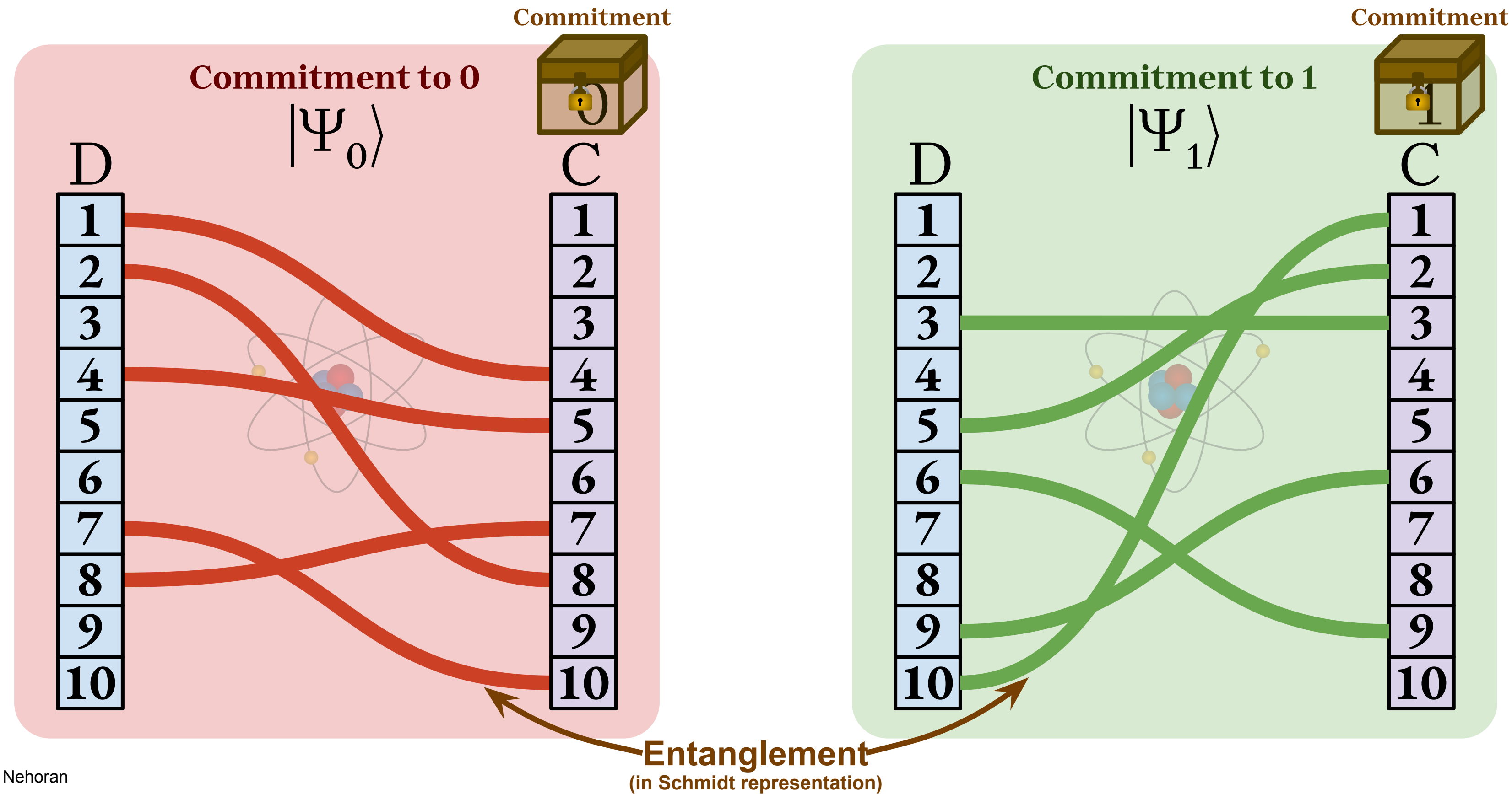

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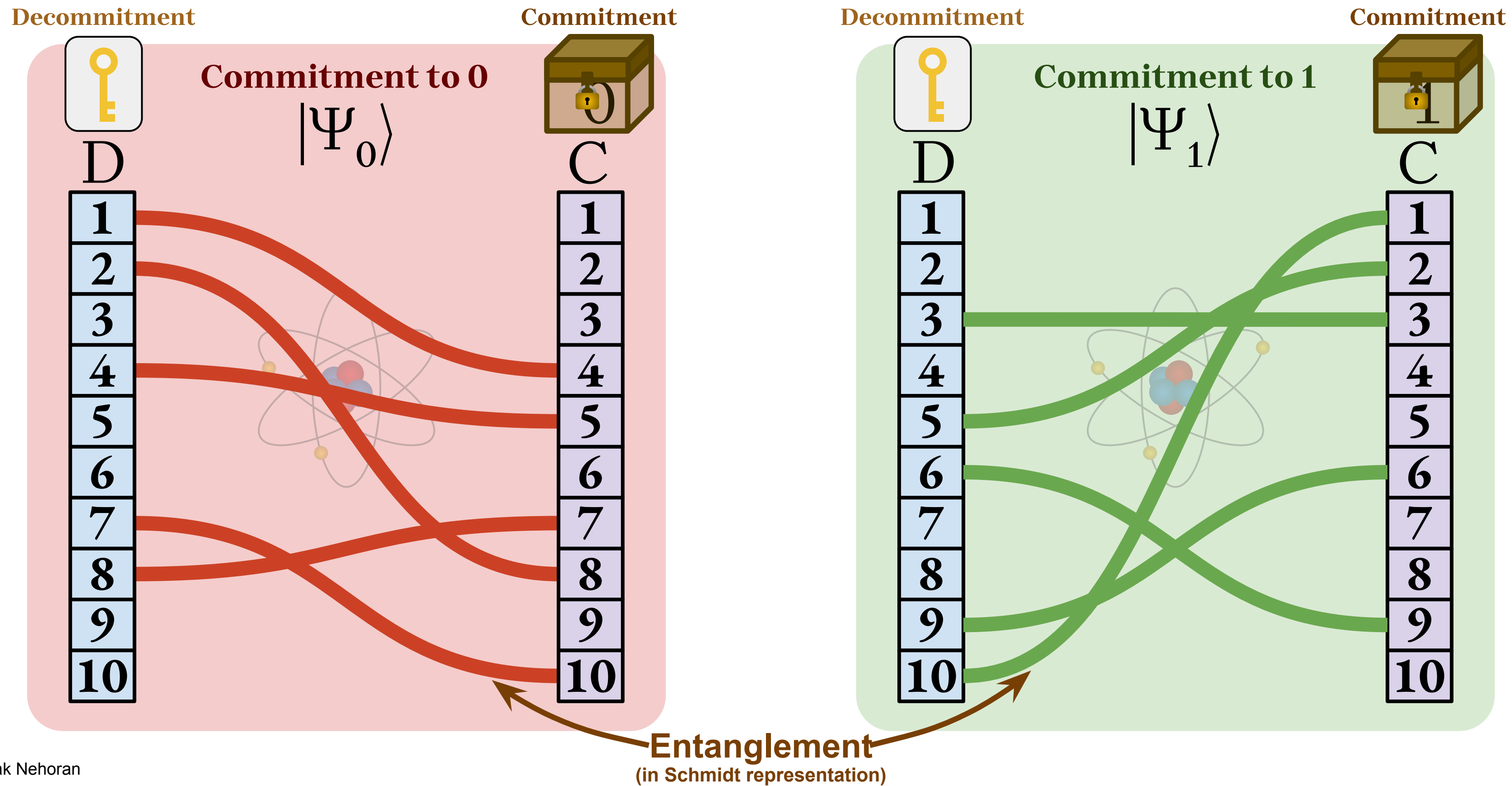

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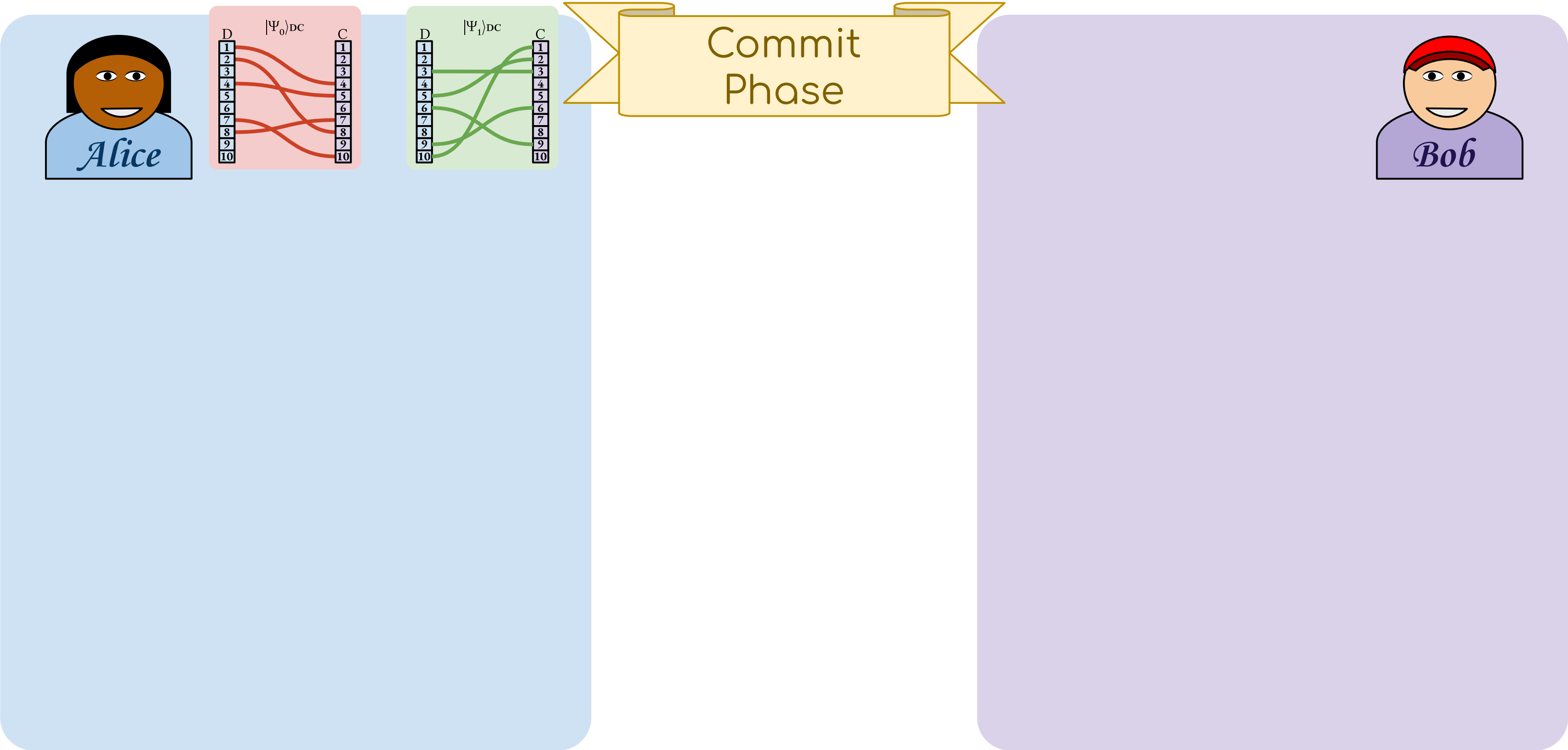

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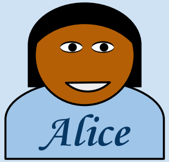
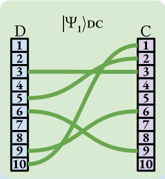
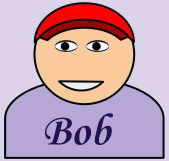
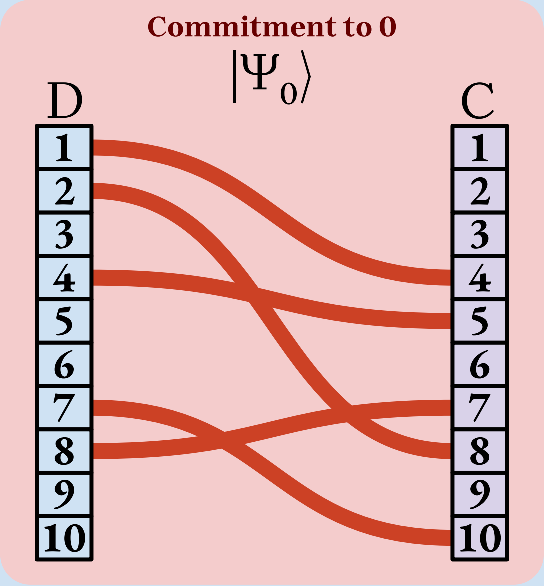



Commit Phase



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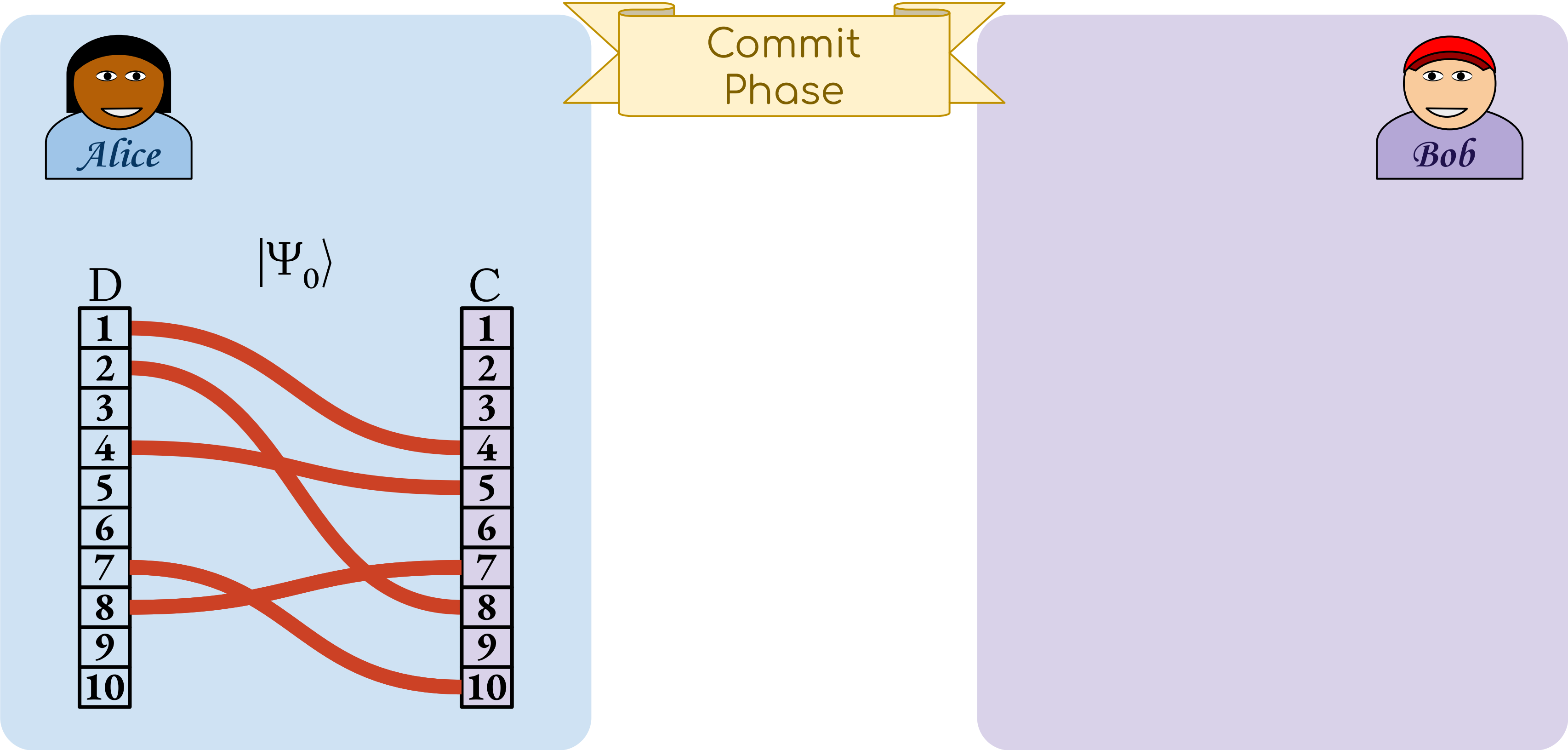

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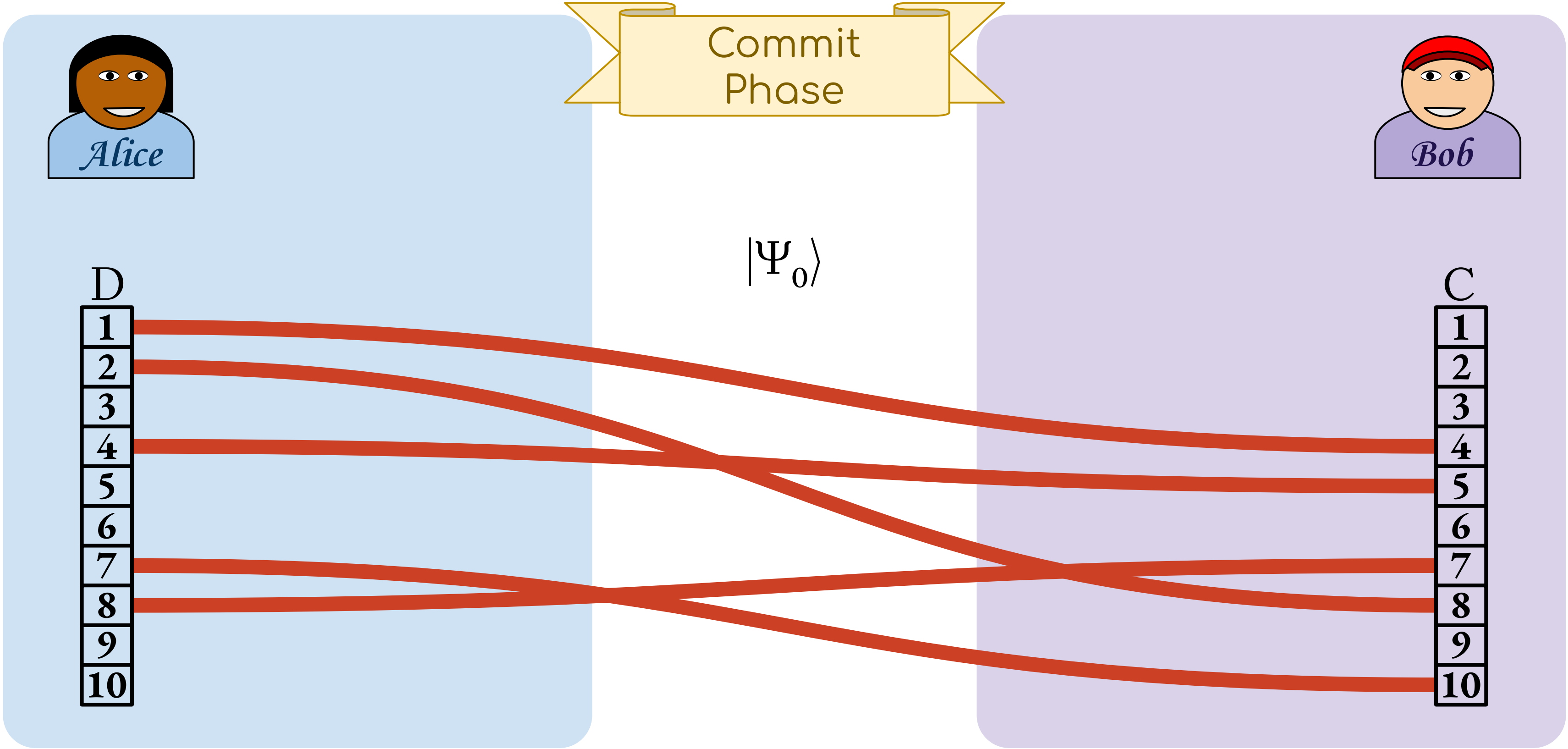

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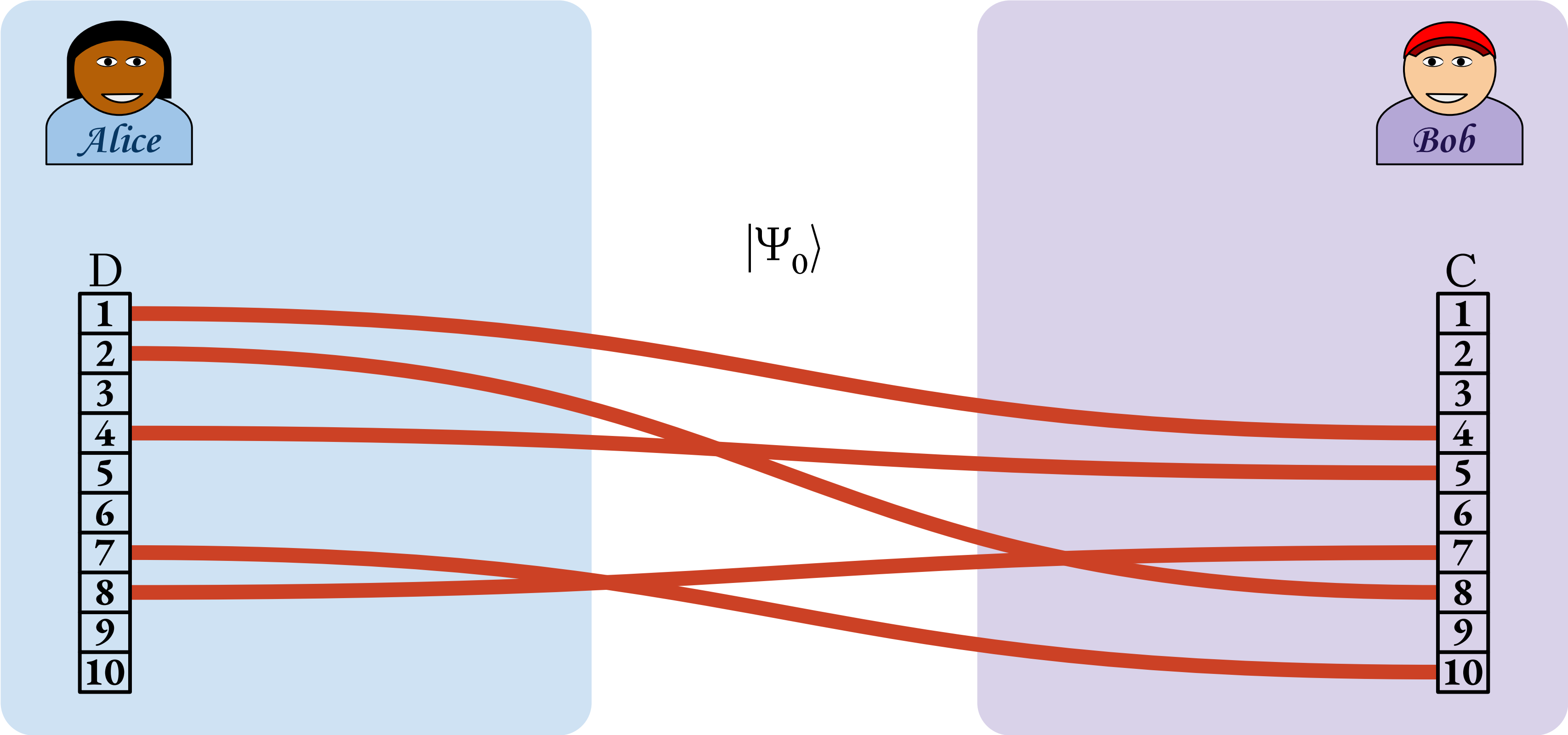

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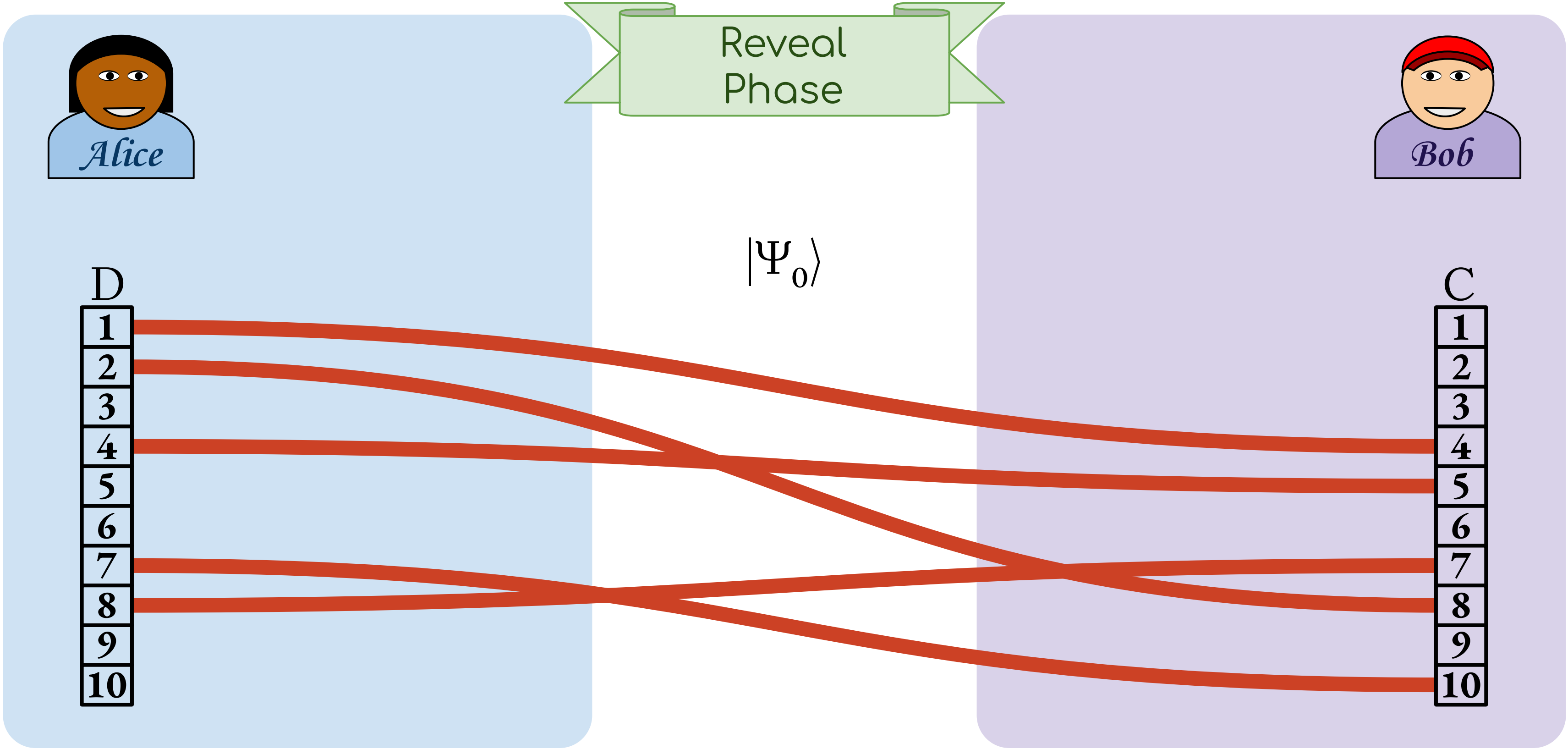

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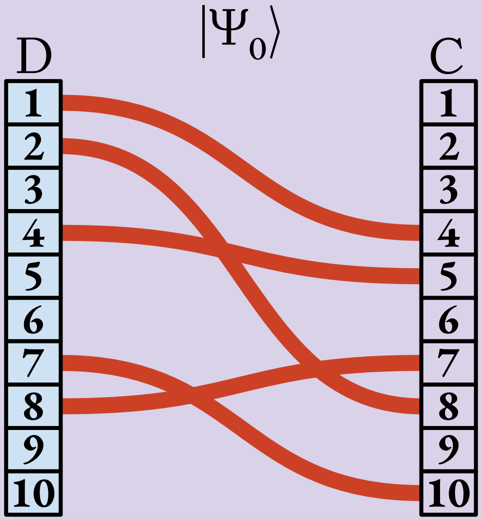
# Quantum Commitments

Reveal Phase

Alice

Bob



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# Quantum Commitments

Reveal Phase

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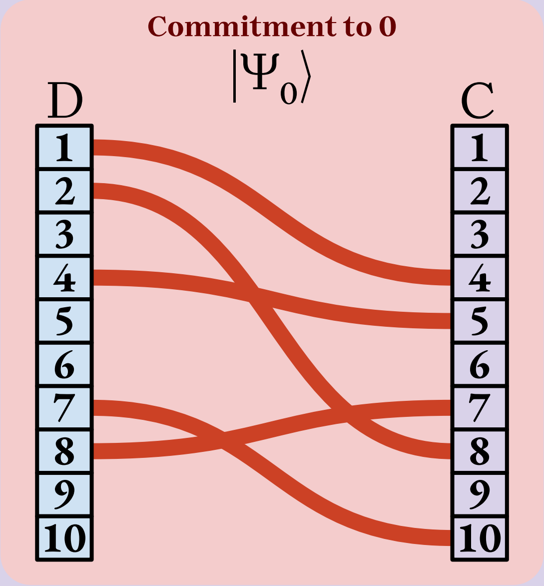
Bob



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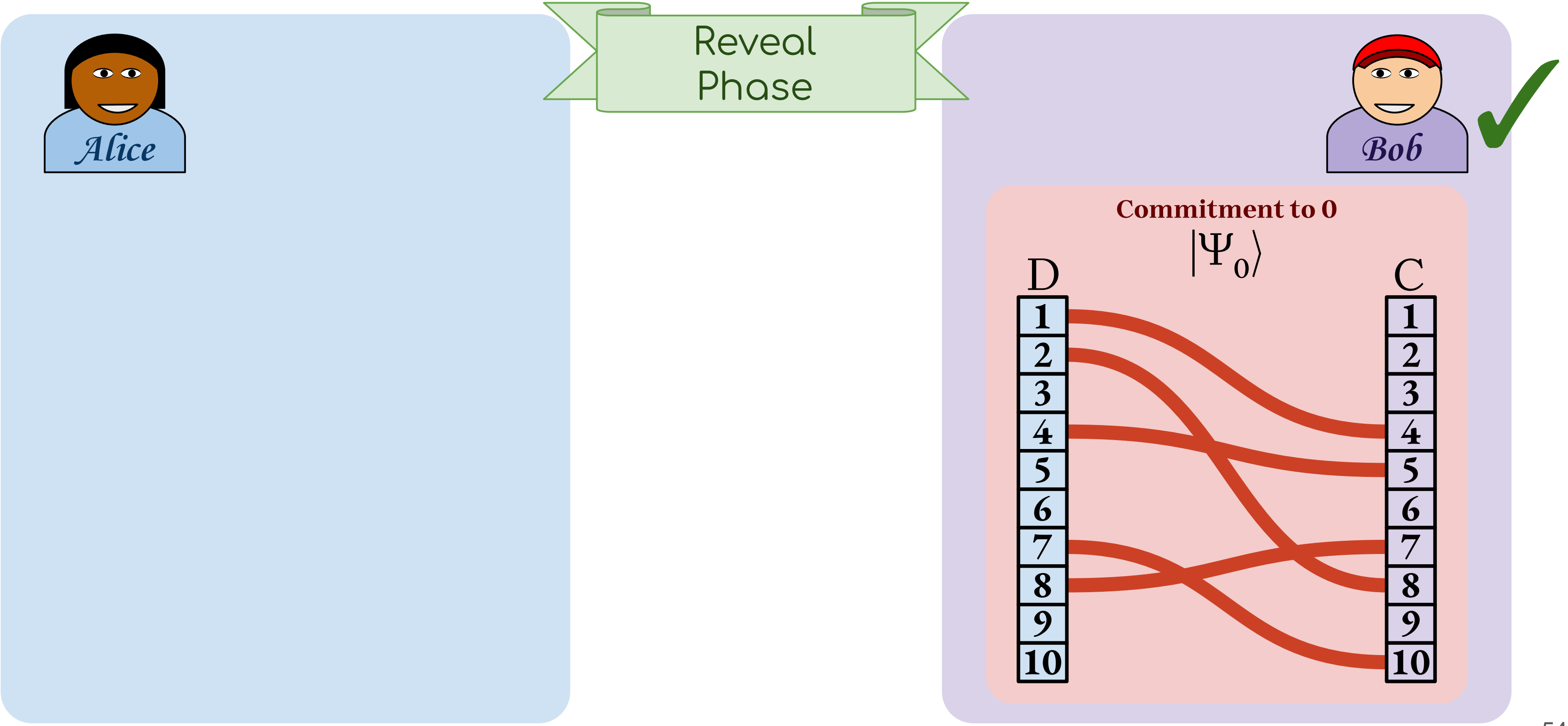

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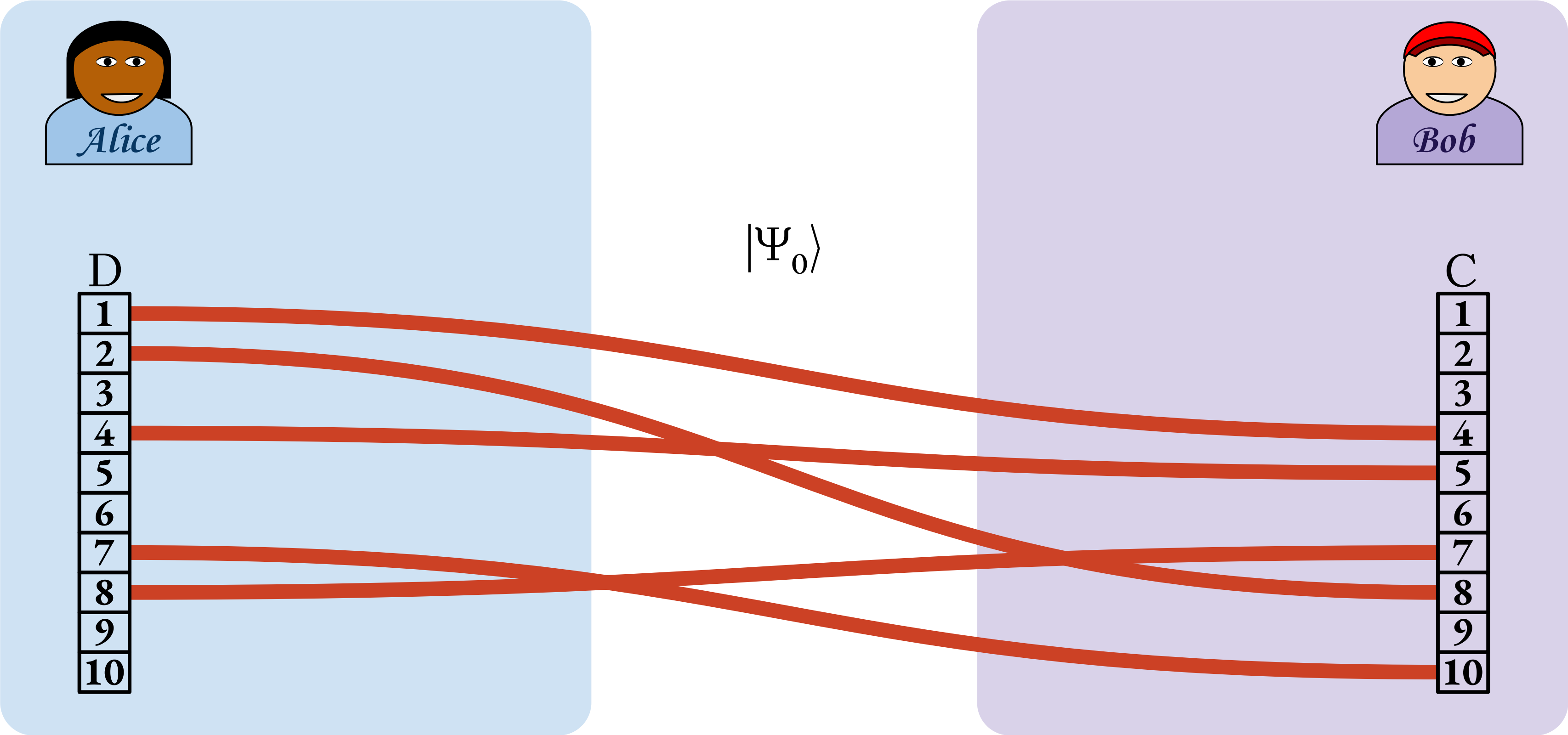
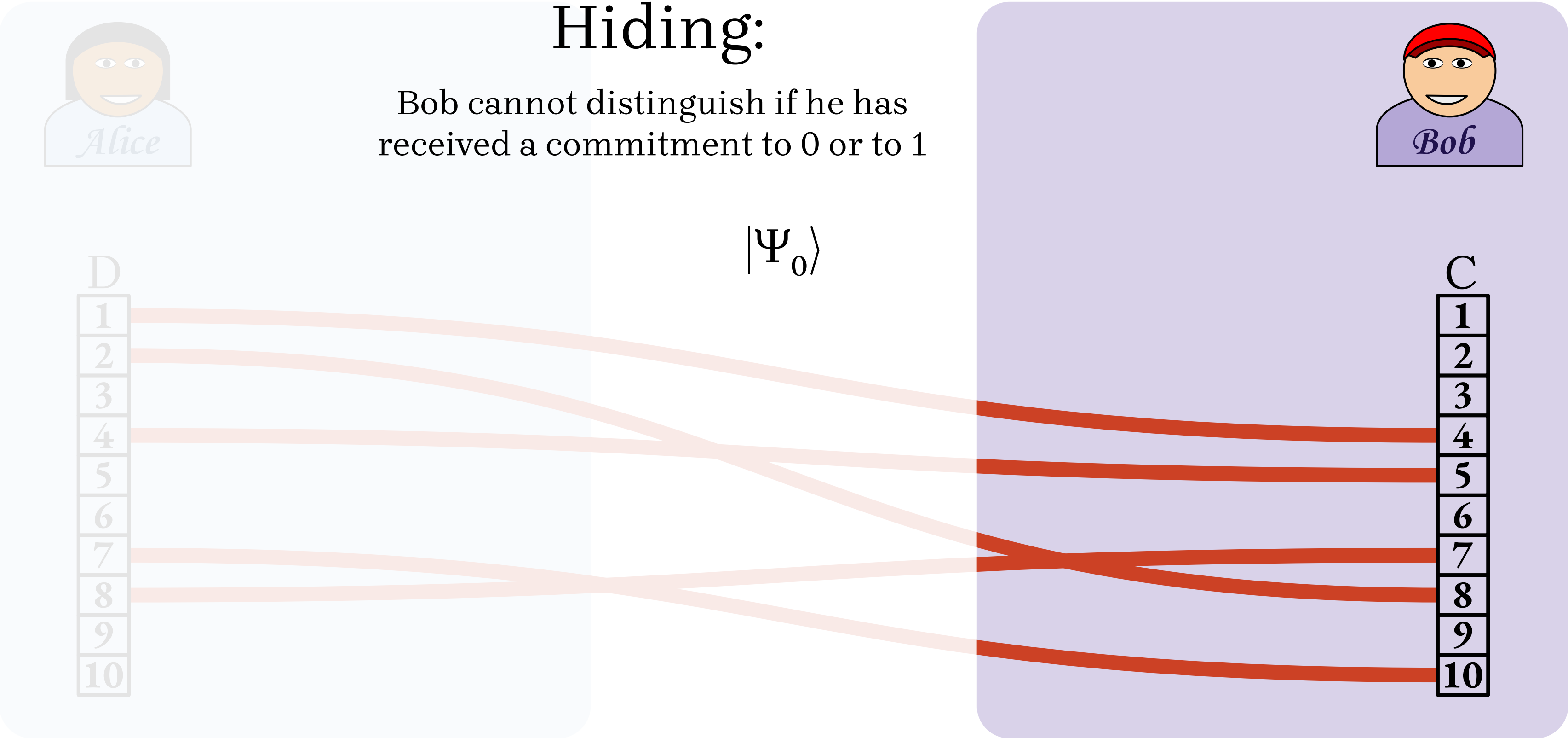


Barak Nehoran (Princeton U), Luowen Qian (Boston U & NTT), Tomoyuki Morimae (YITP), Takashi Yamakawa (NTT & YITP)

# Quantum Commitments

## Hiding:

Bob cannot distinguish if he has received a commitment to 0 or to 1



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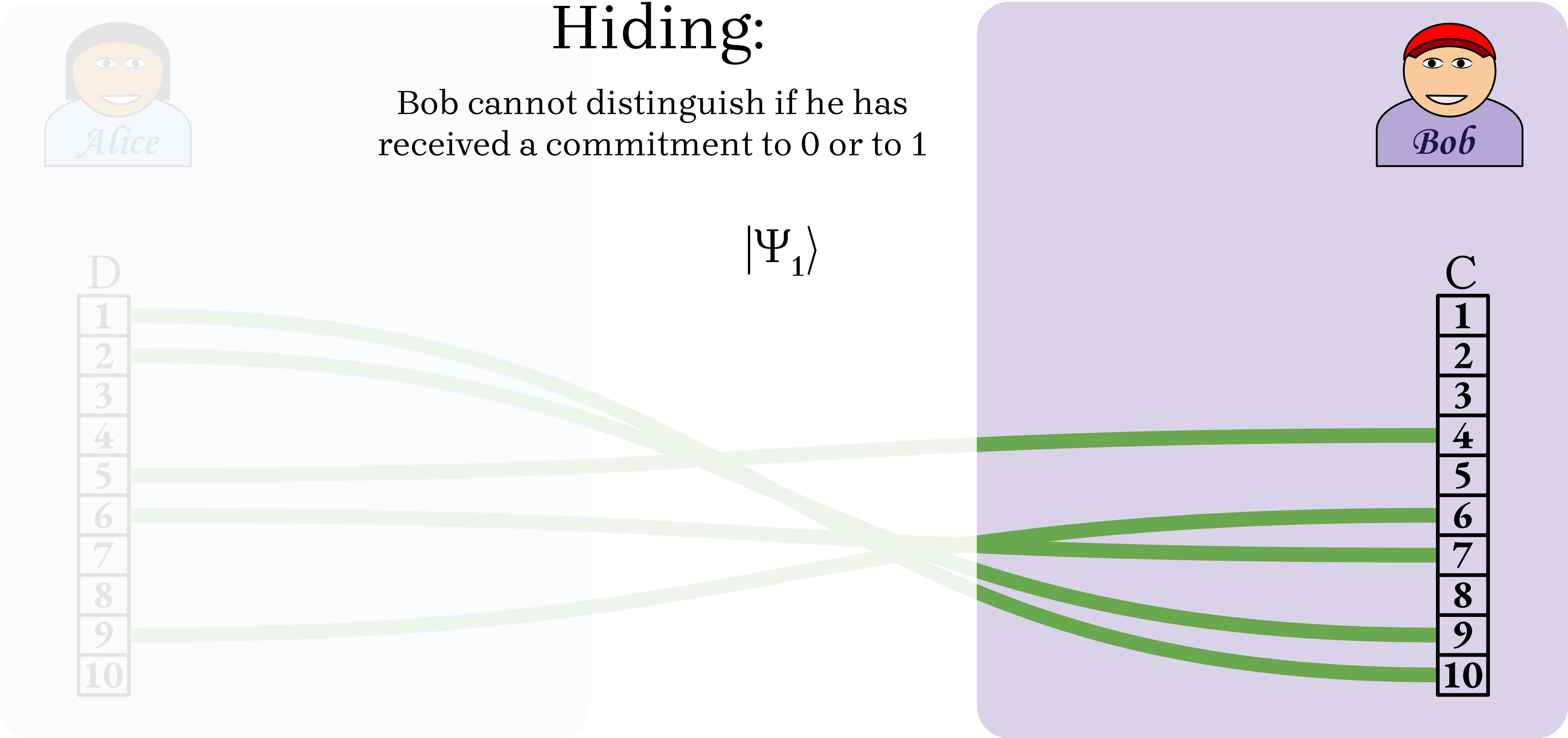

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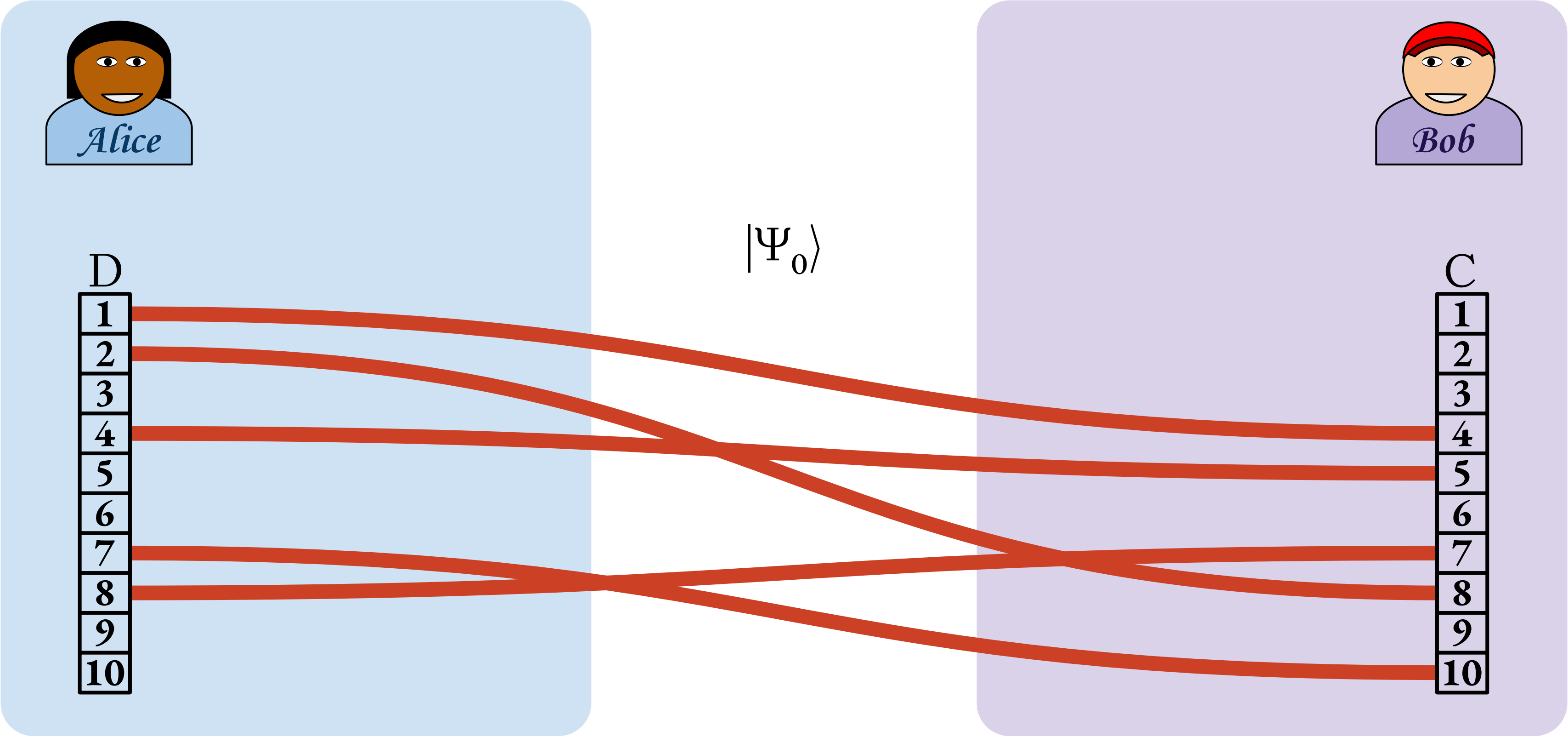

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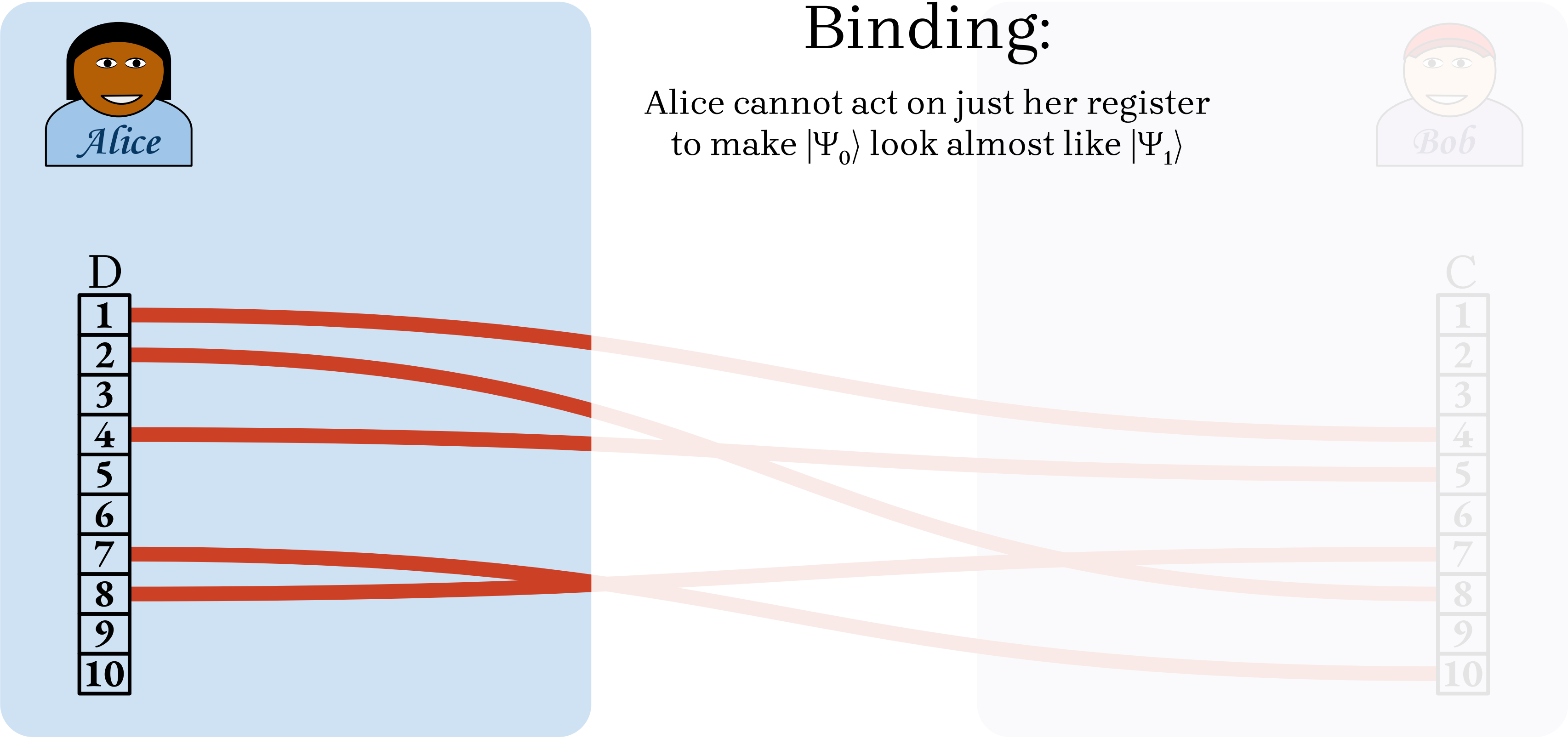


## Binding:

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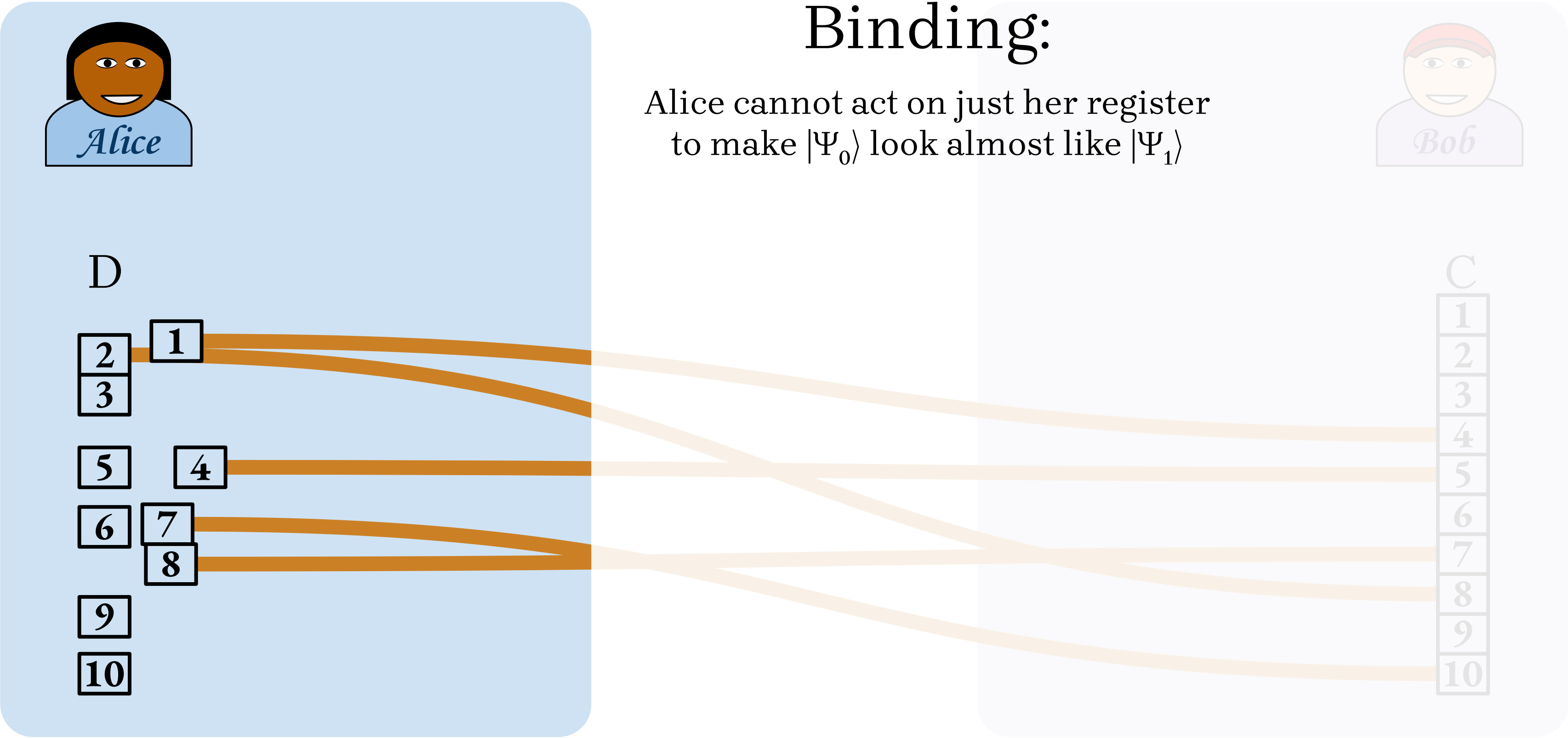

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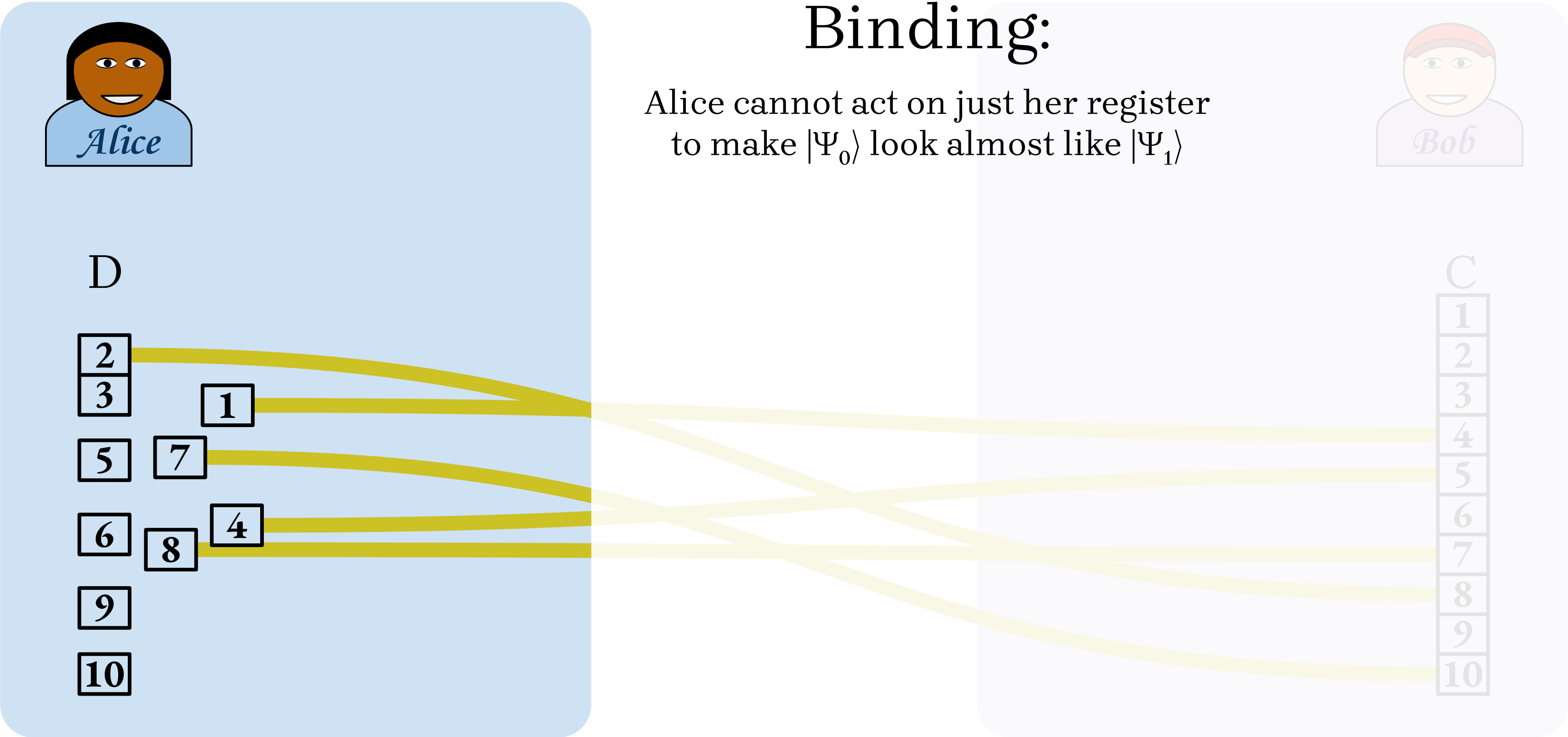

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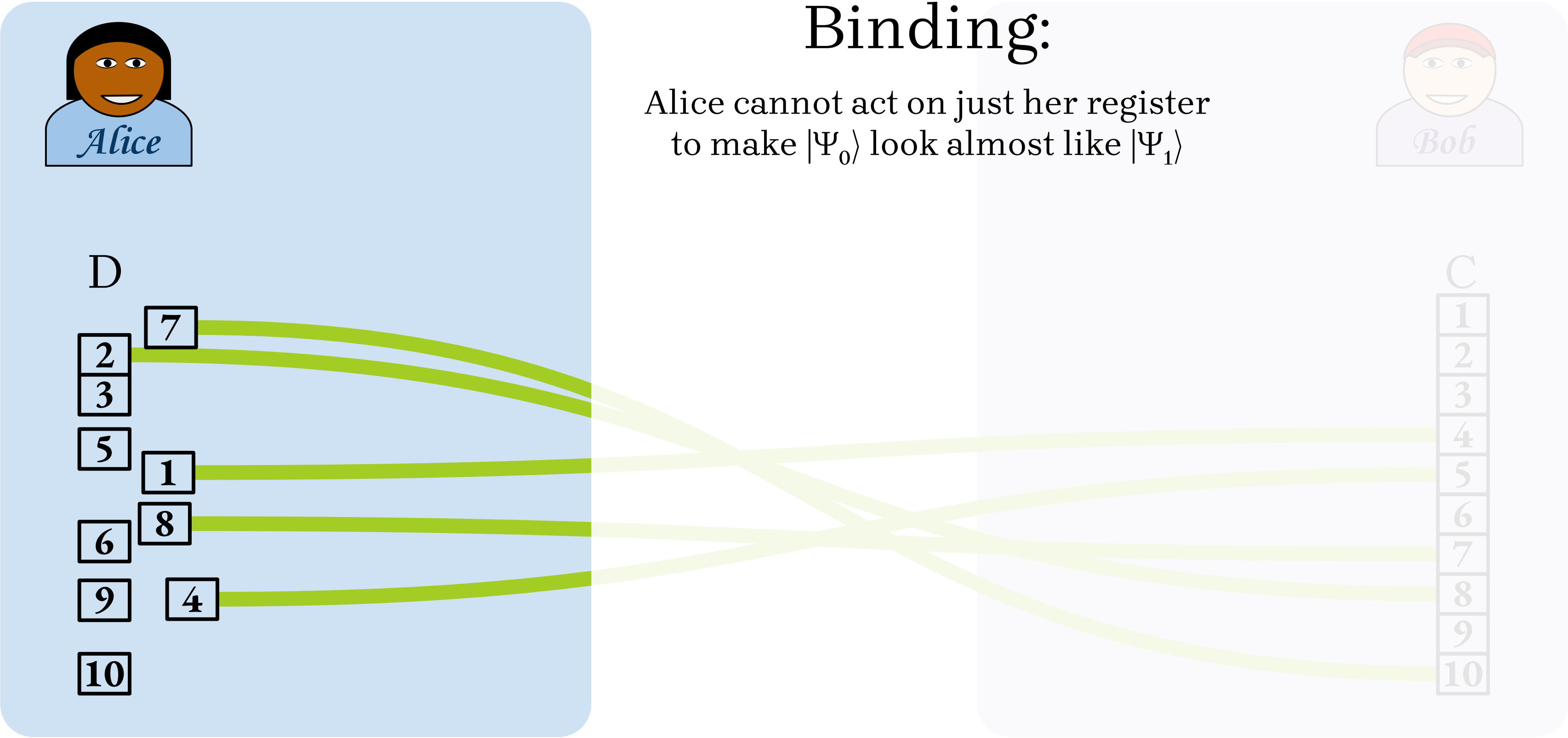

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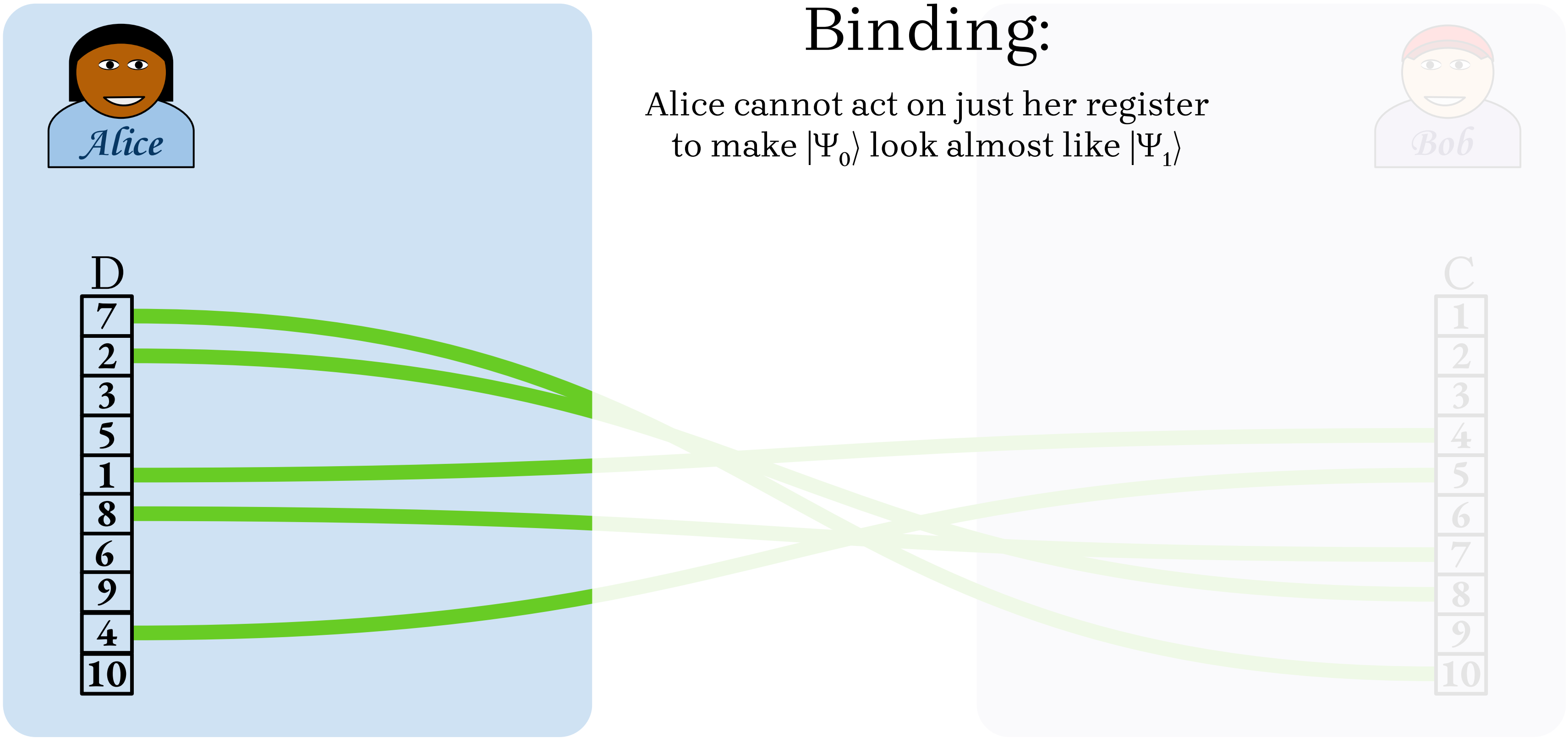


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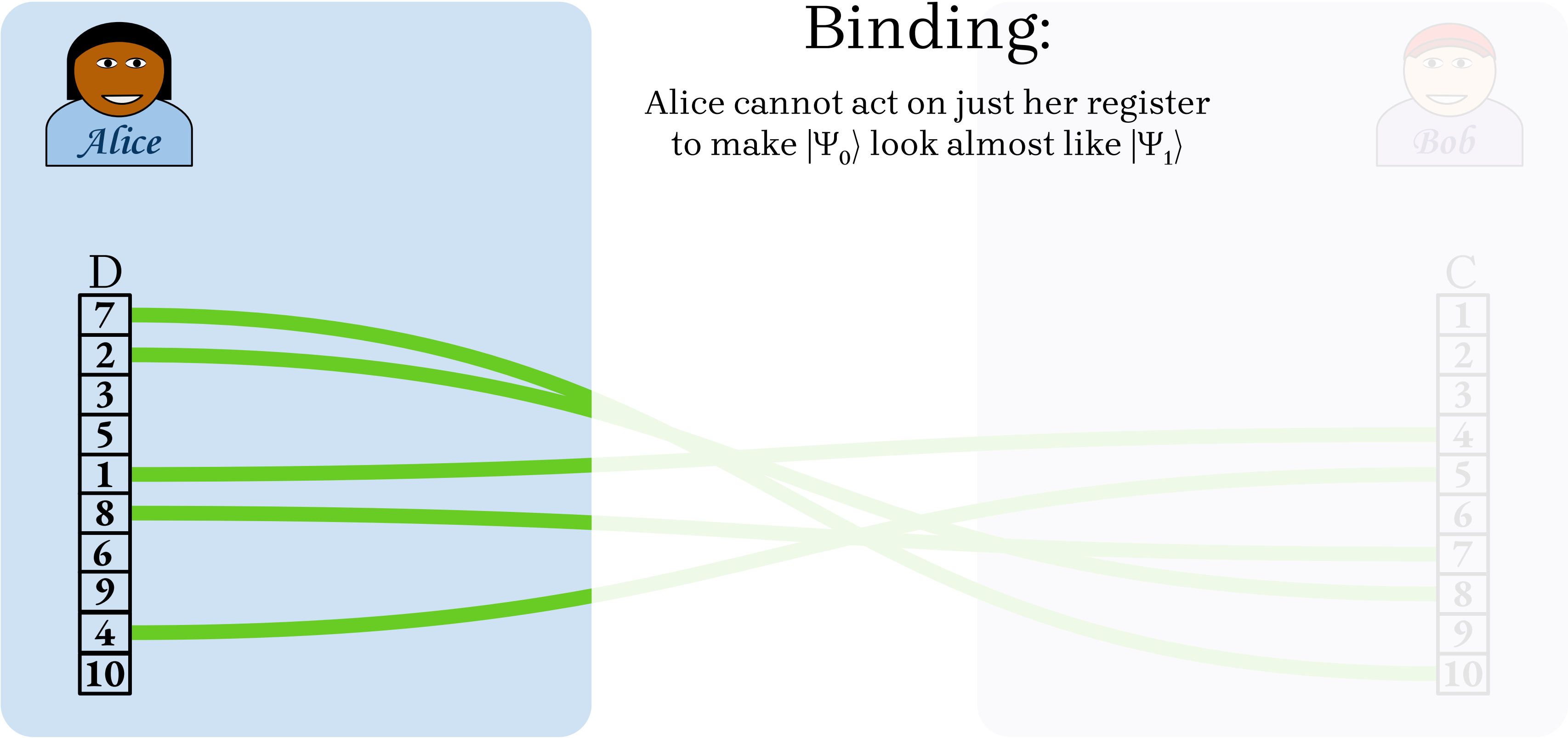


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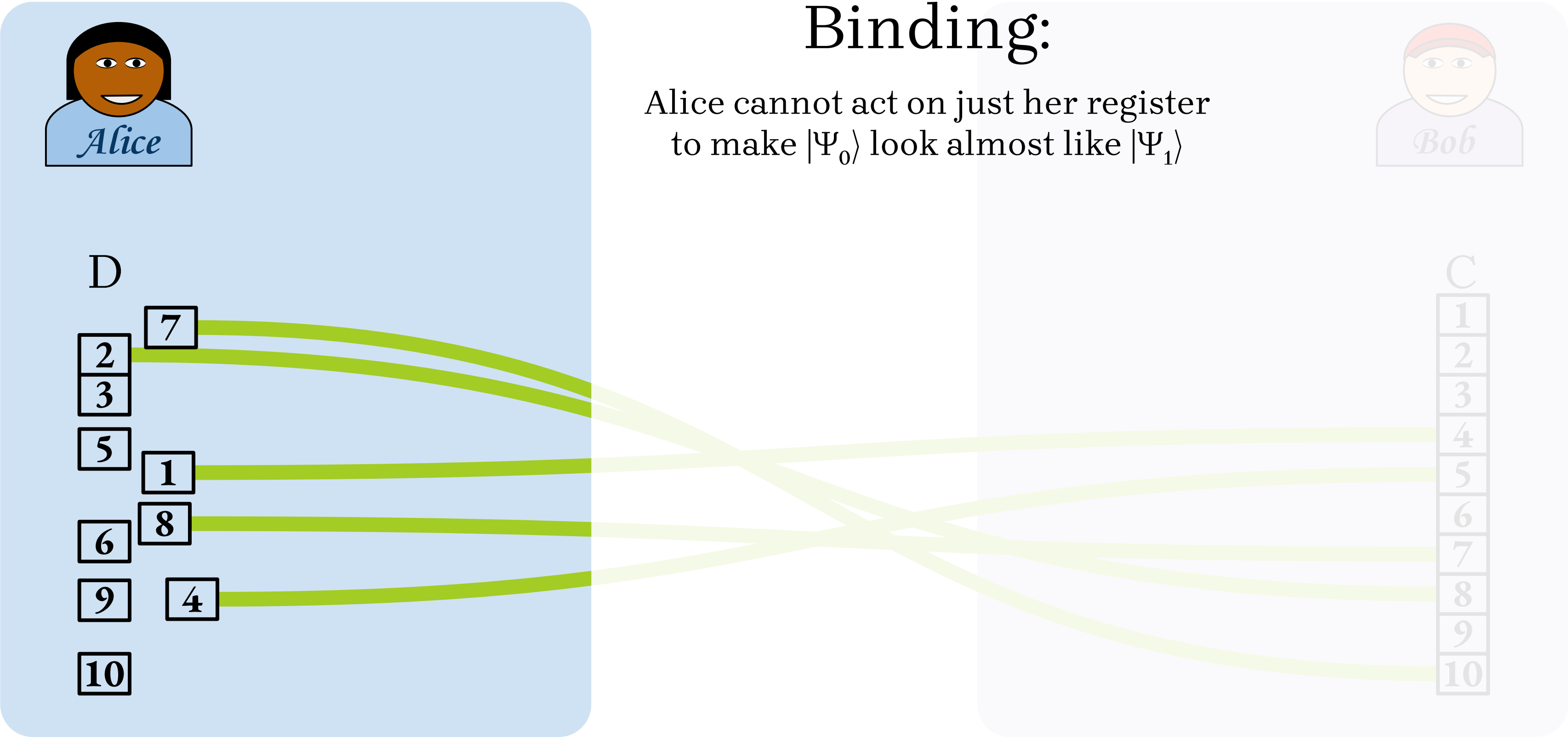


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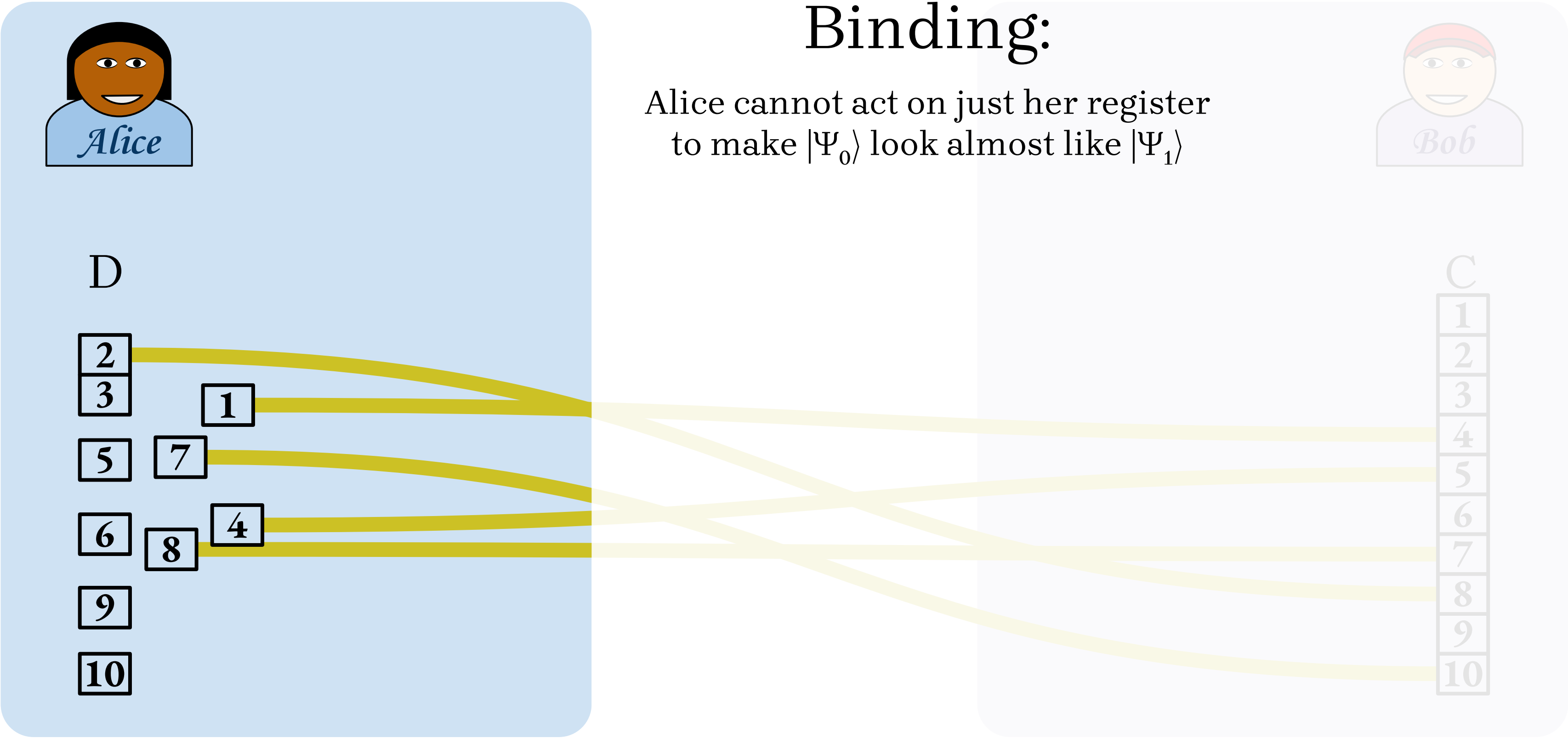

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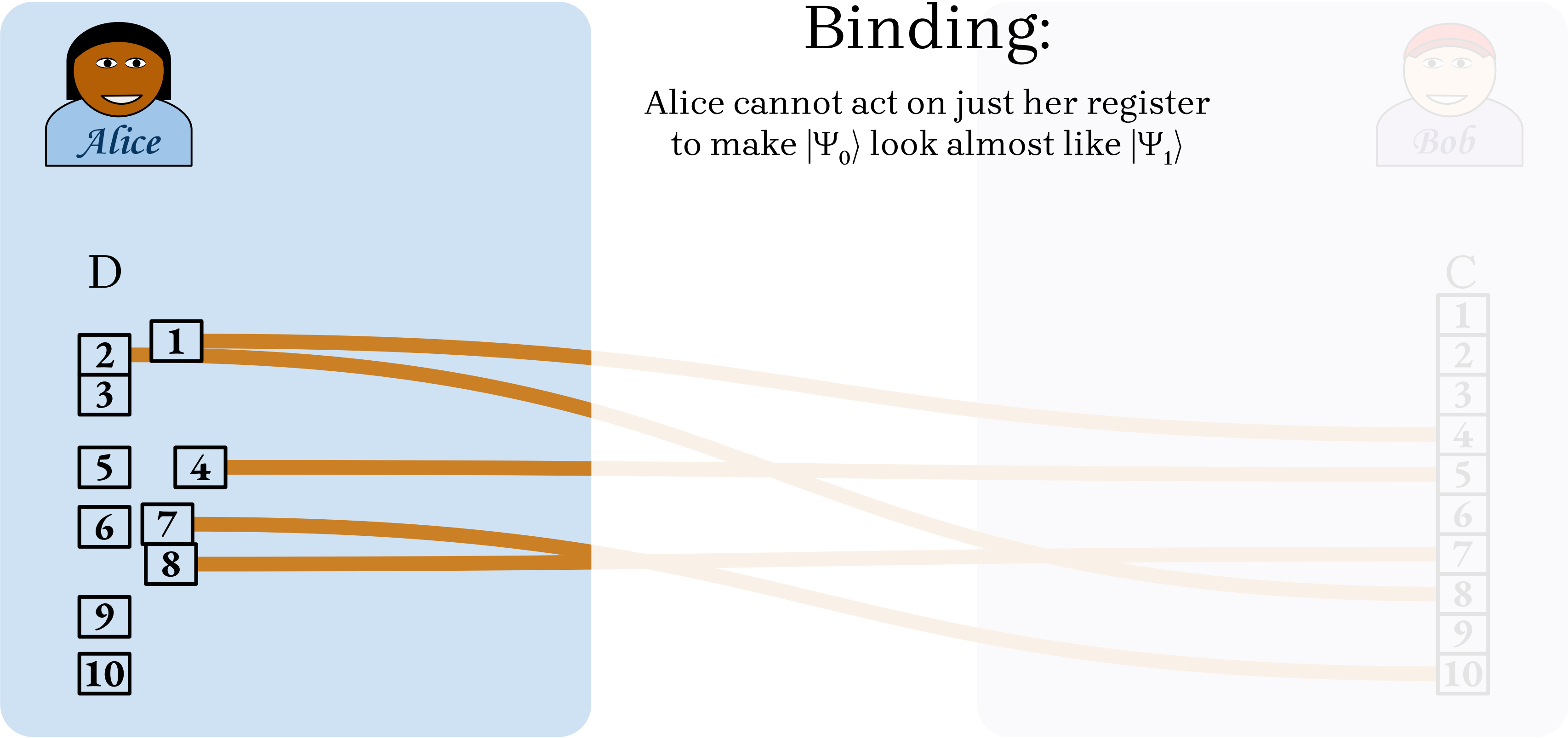

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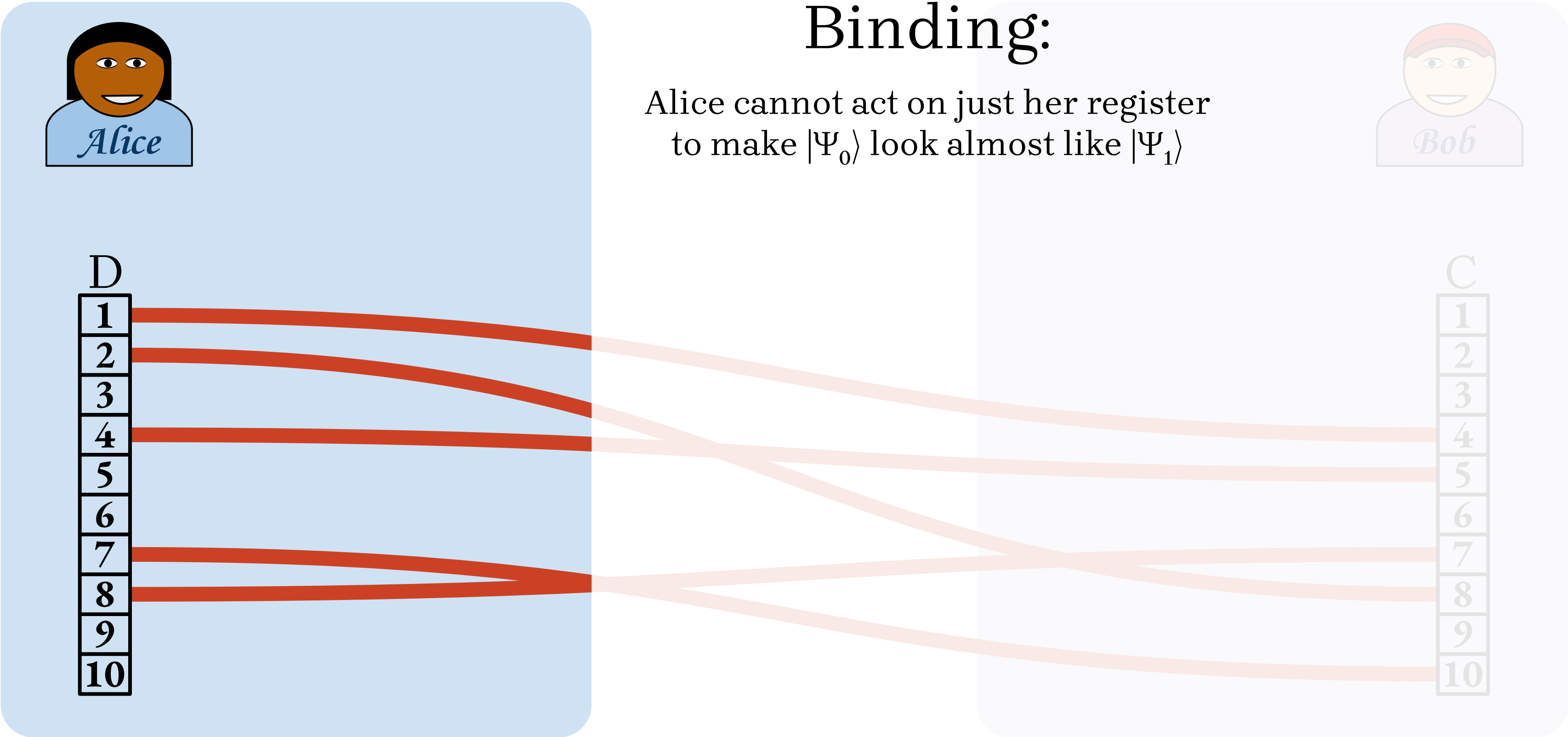

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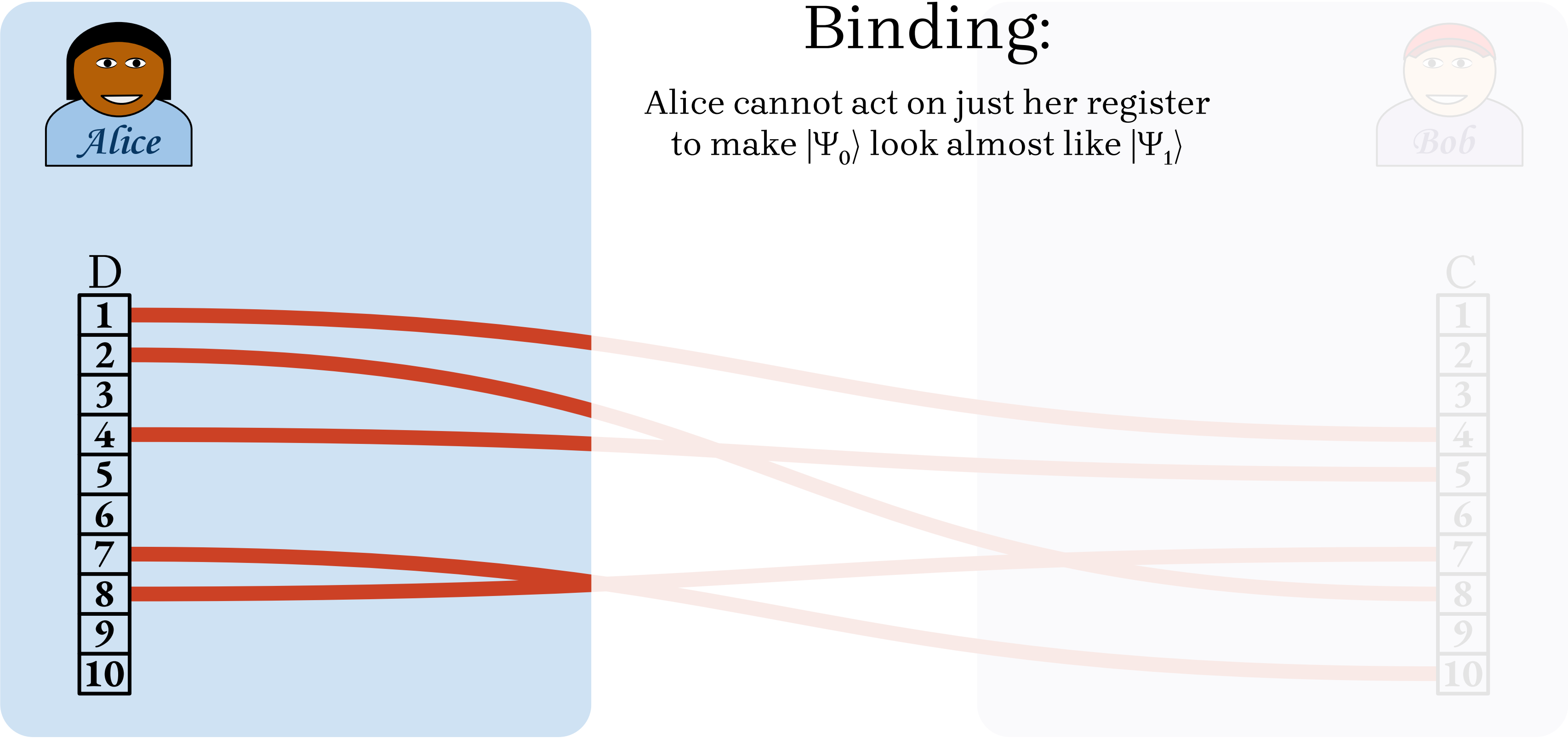

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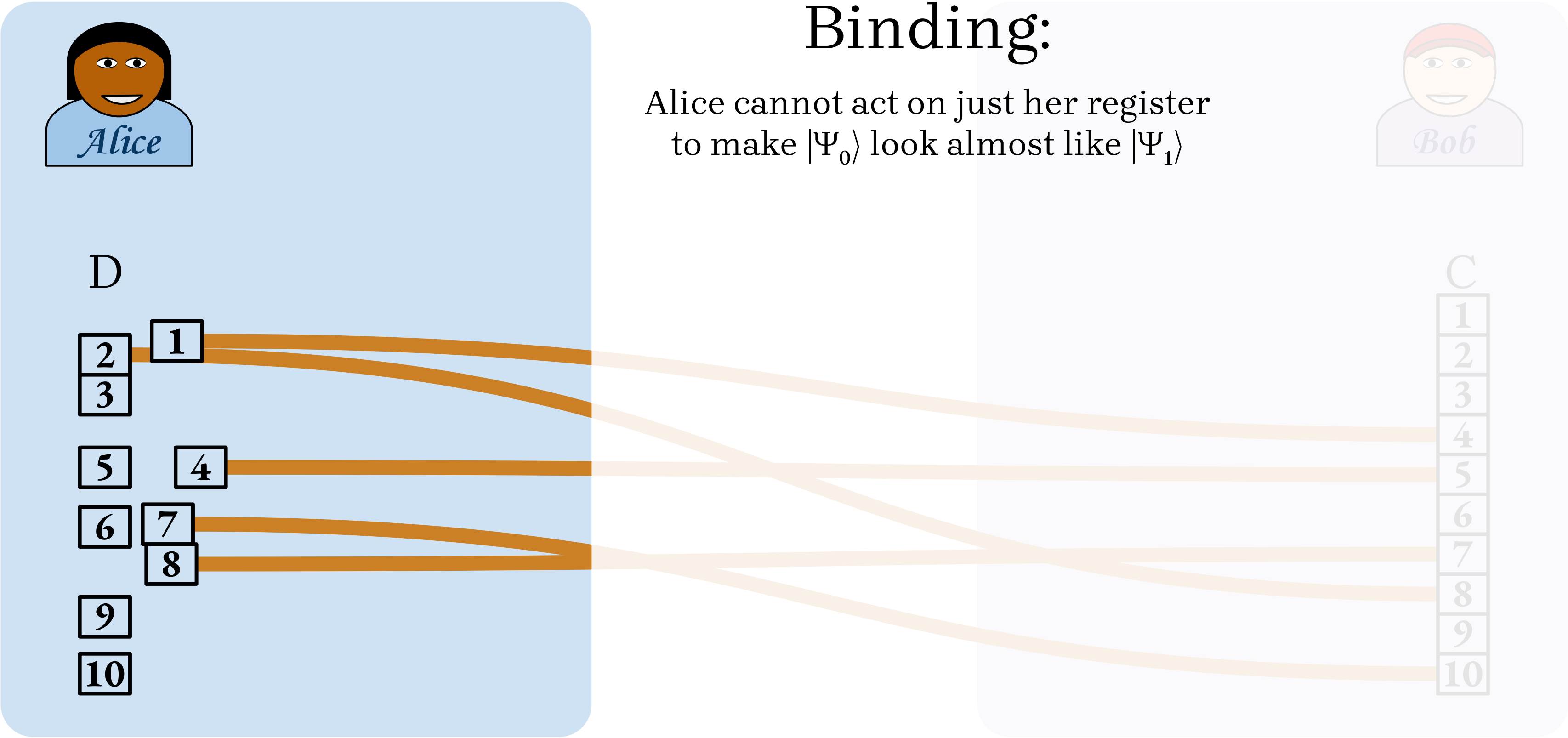

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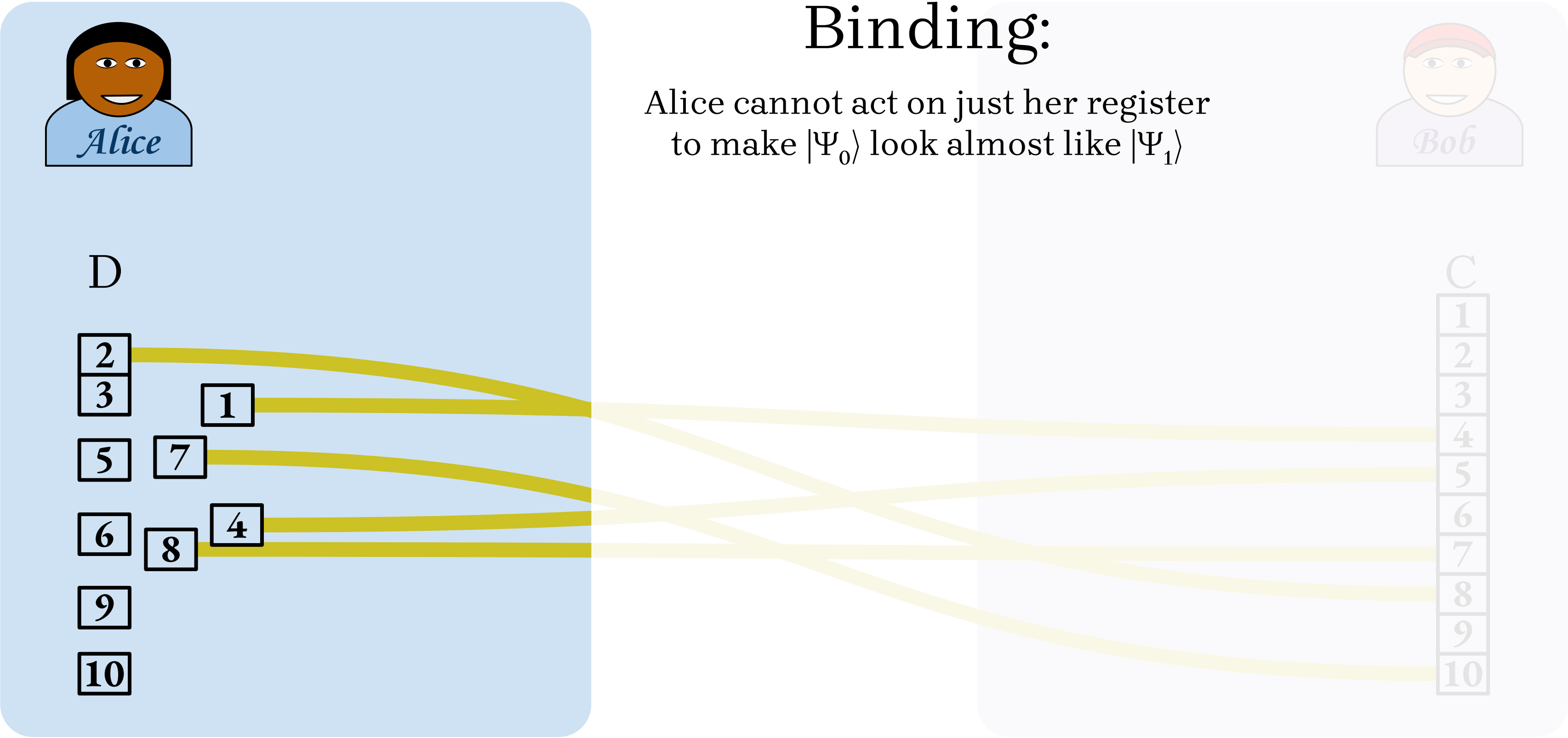

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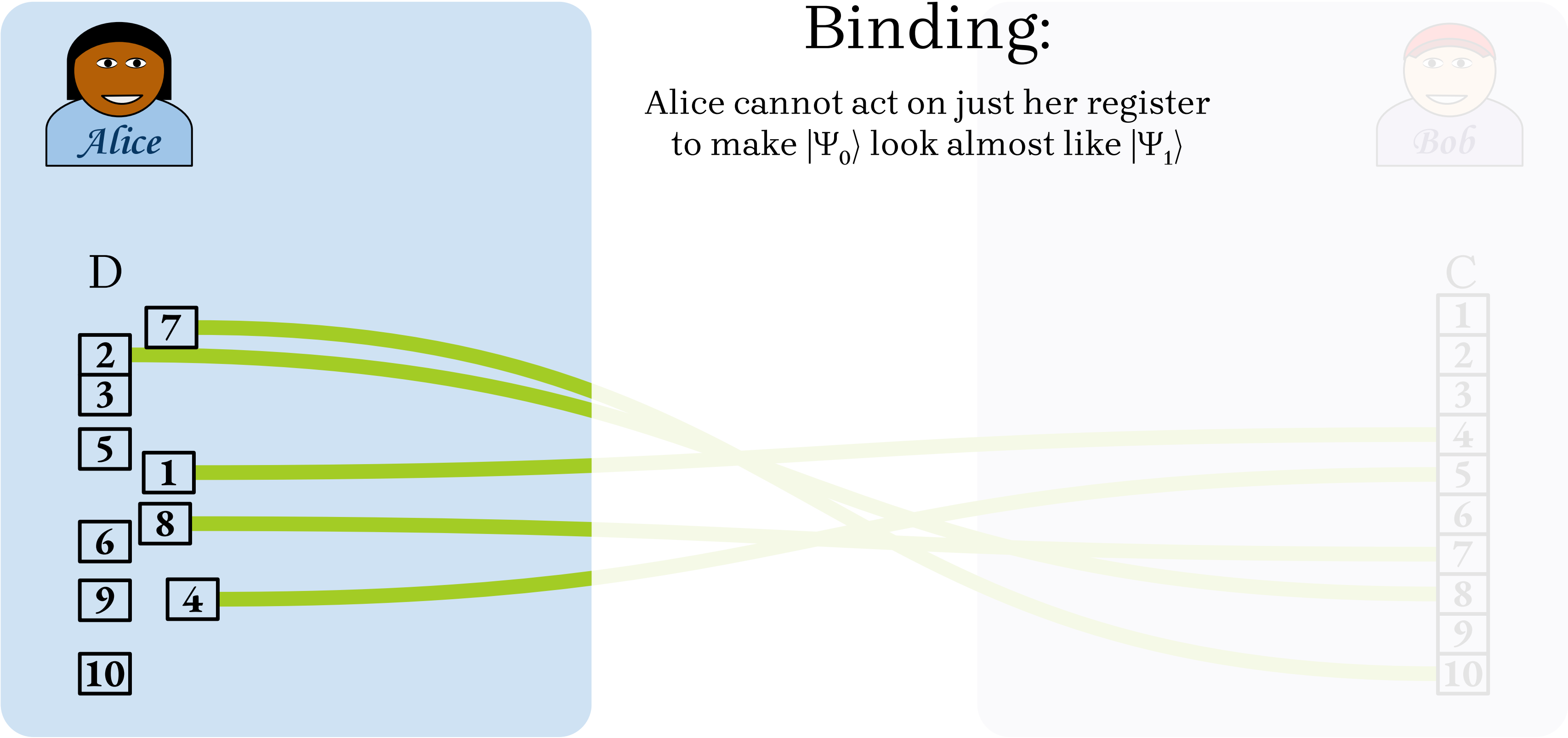

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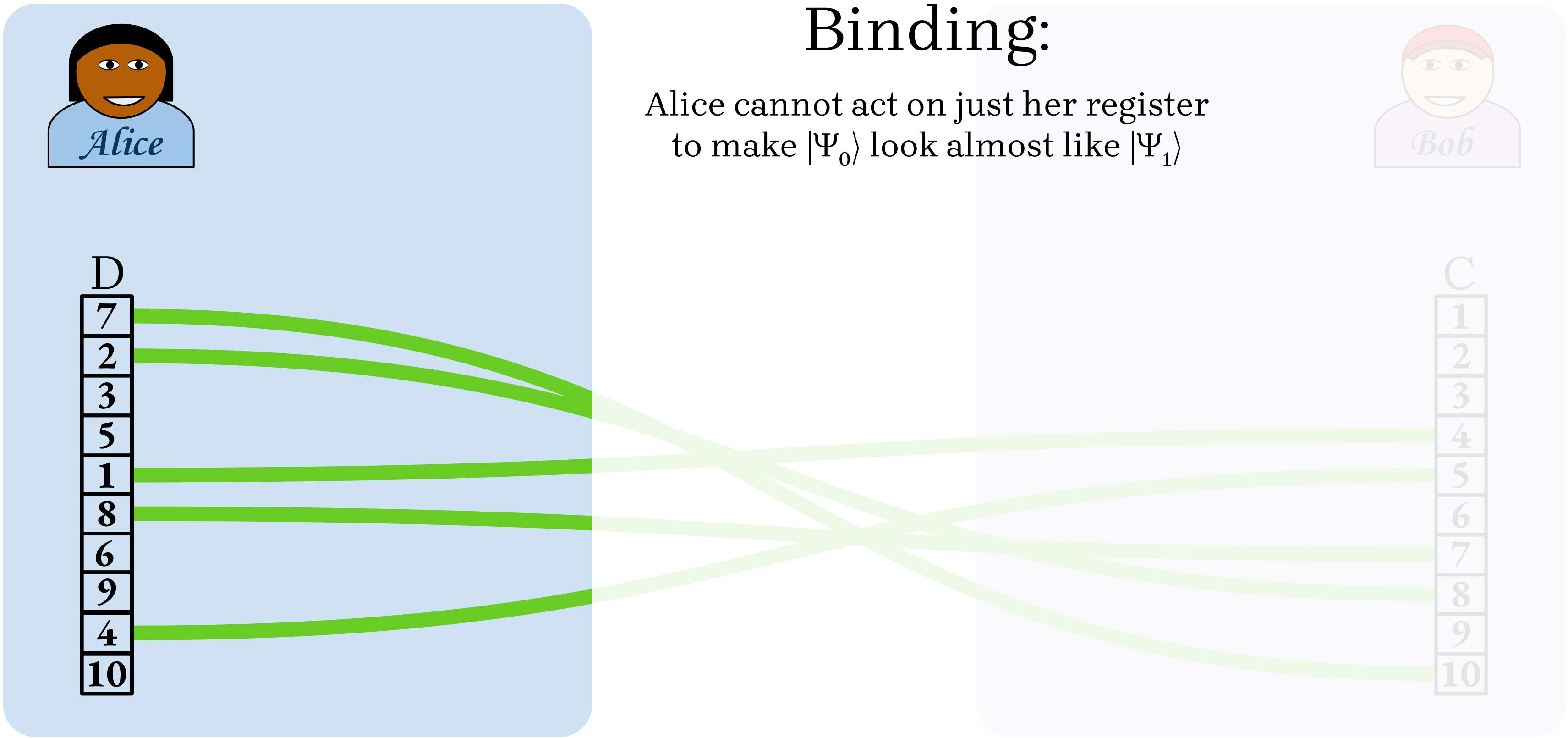

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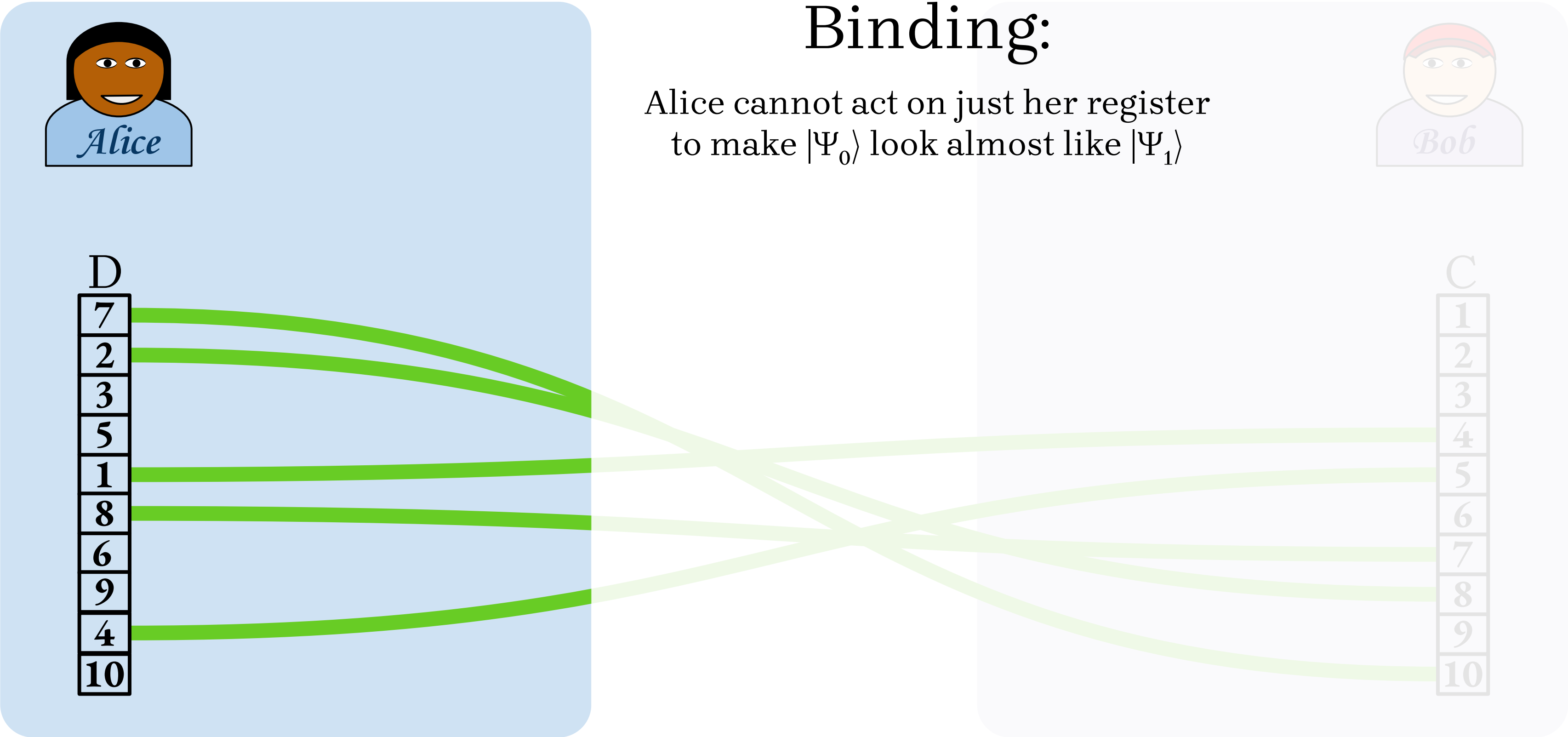

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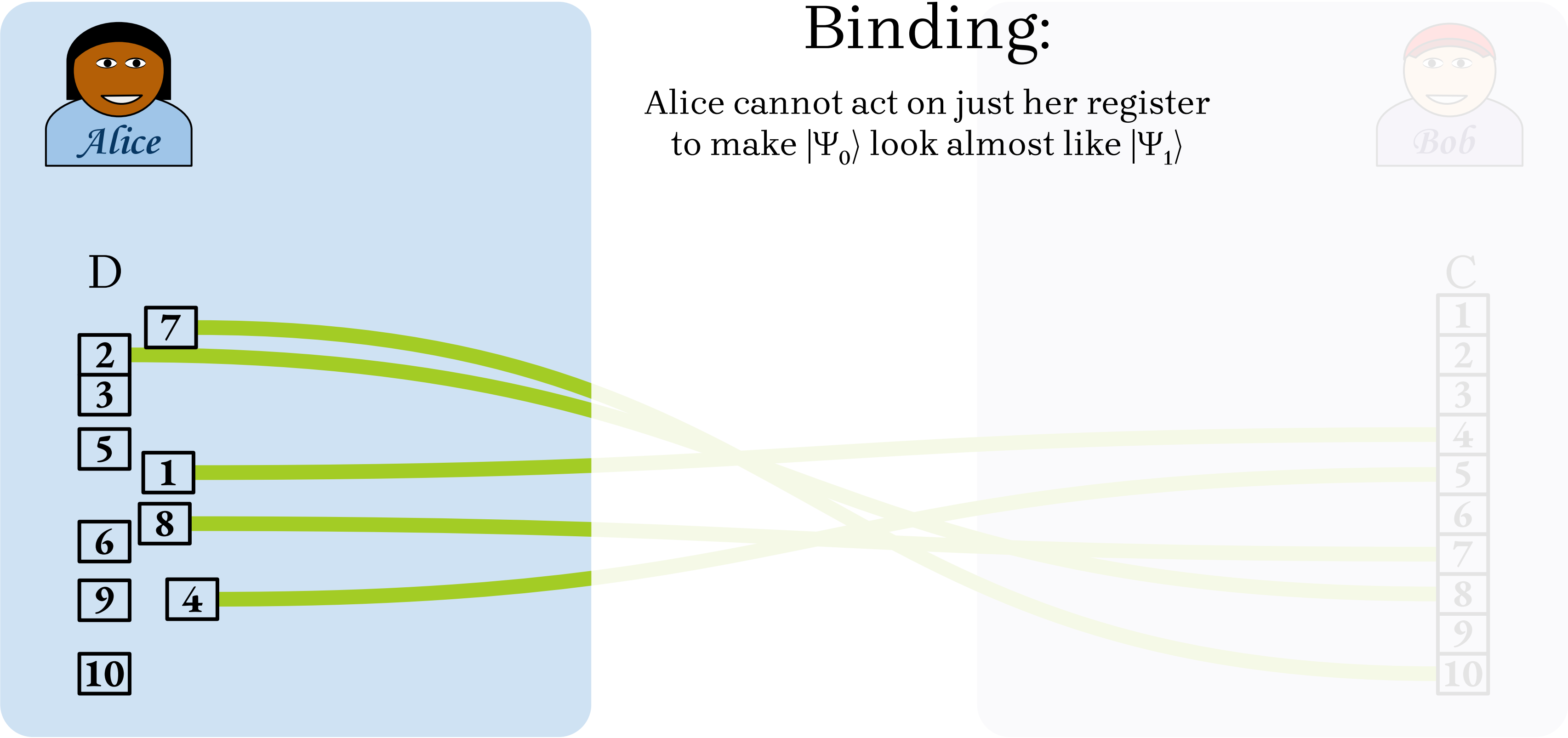


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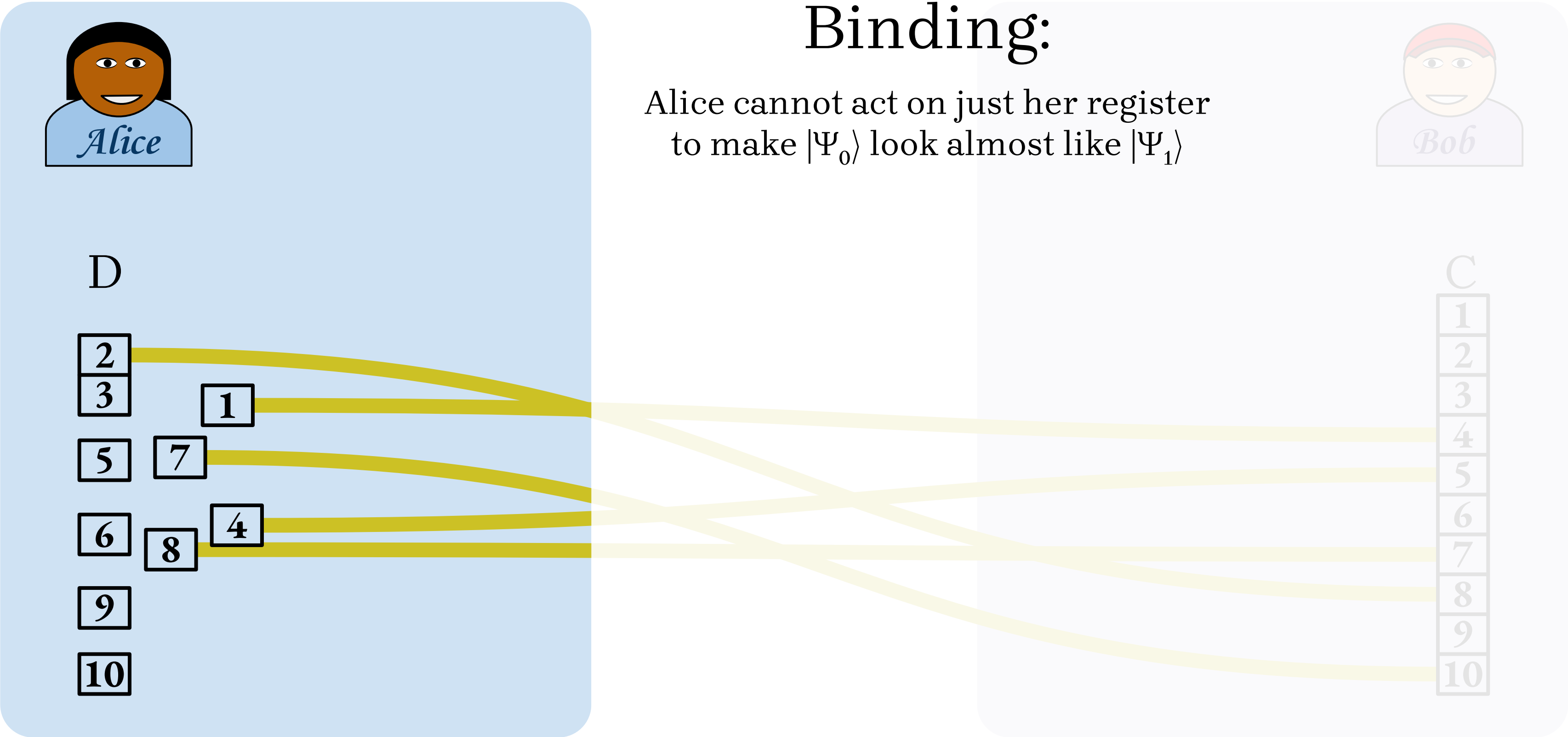

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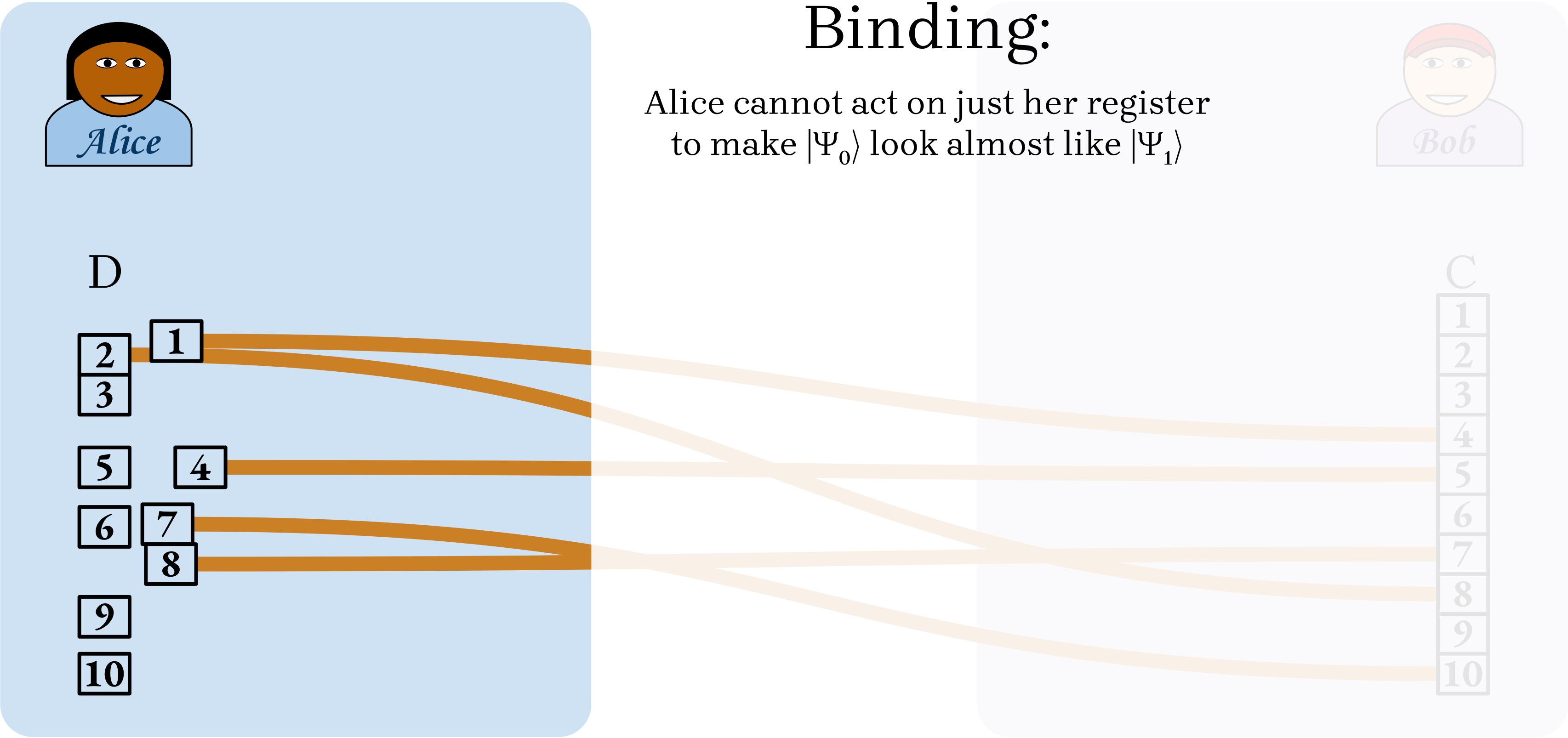

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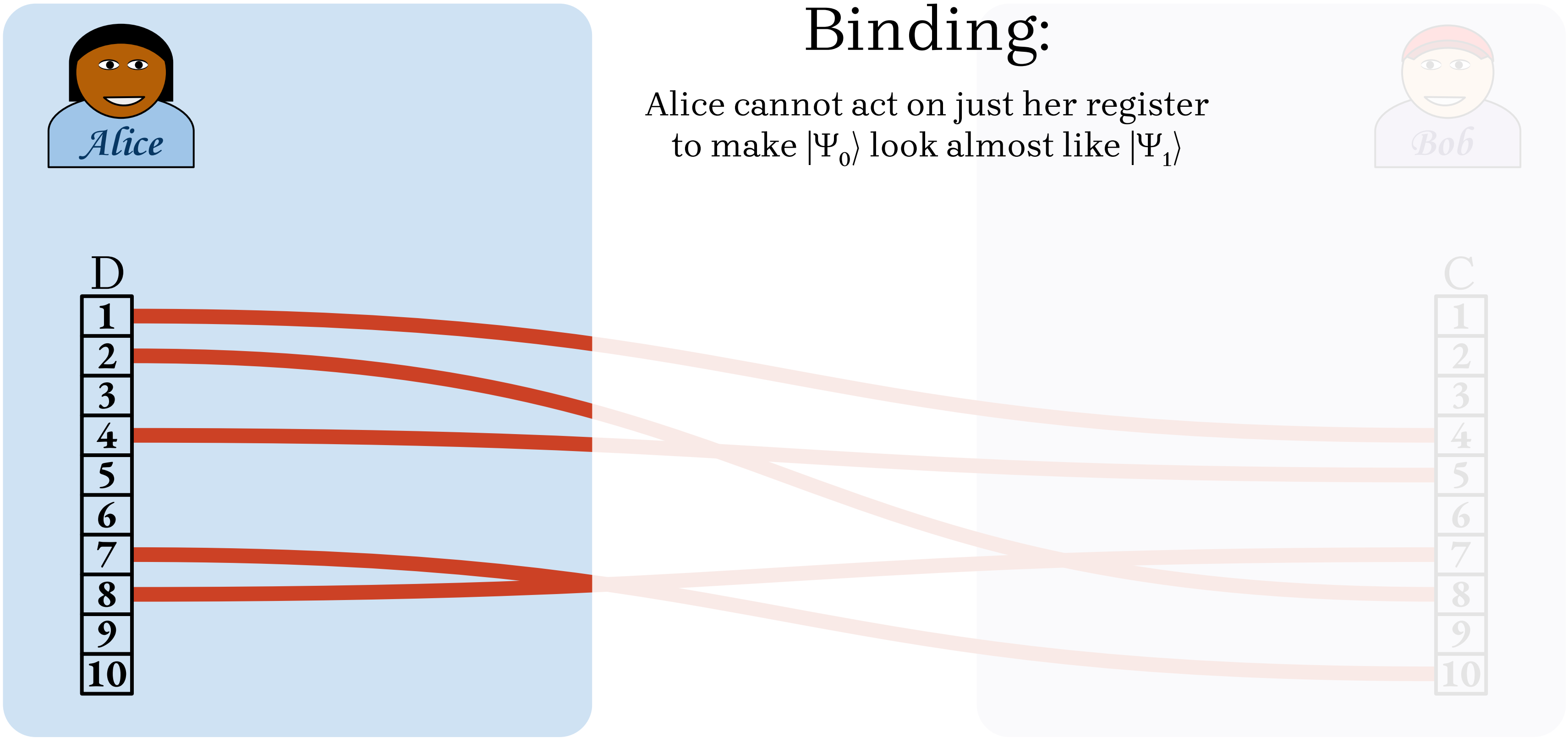

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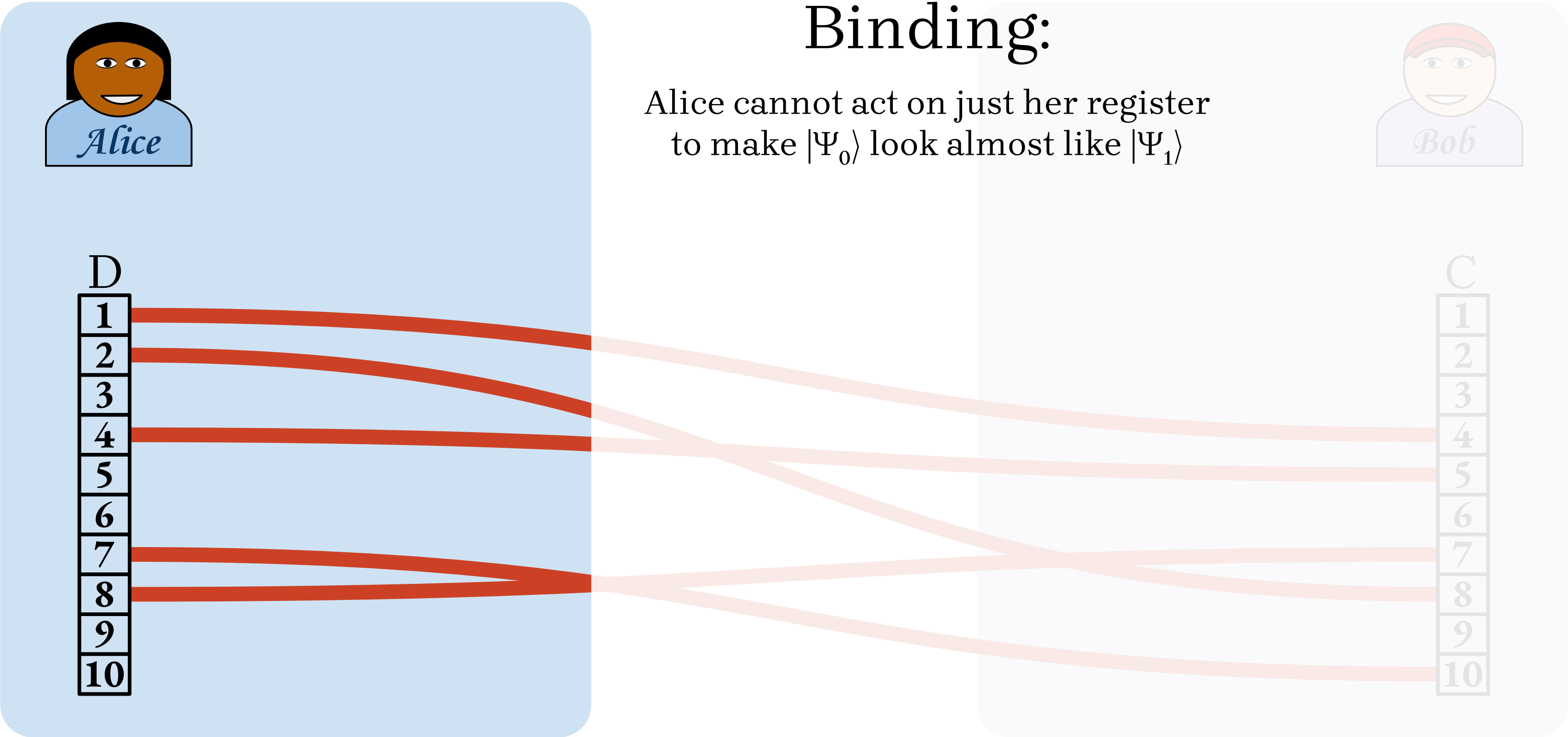


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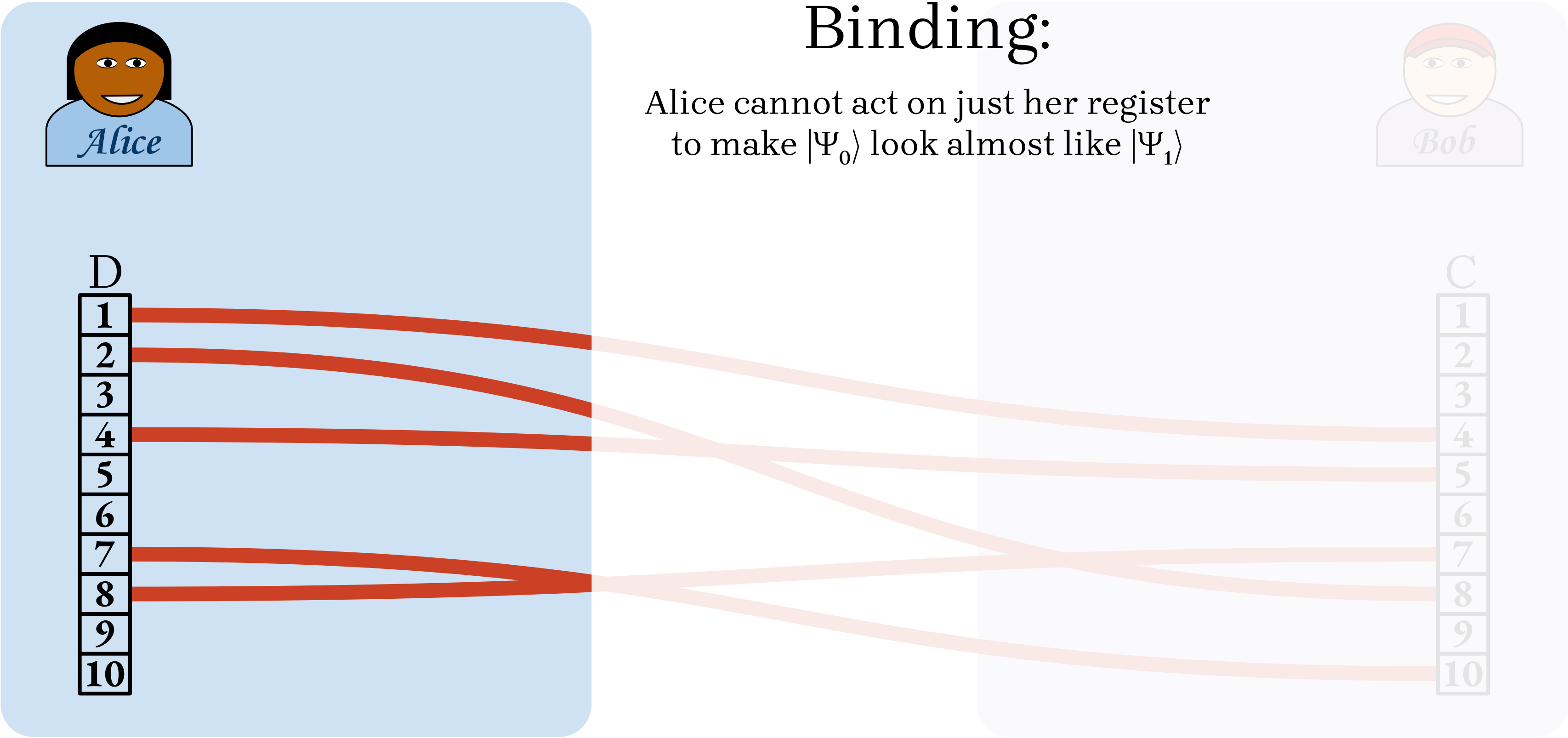


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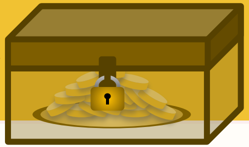
Information-Theoretic Security

| | Computational Security | Statistical Security | Perfect Security |
|---|---|---|---|
| Conditional | | | |
| Unconditional | Commitments with Quantum Auxiliary Inputs | | |

Information is there, but secured behind hard computation



Information negligibly correlated



Information simply not there



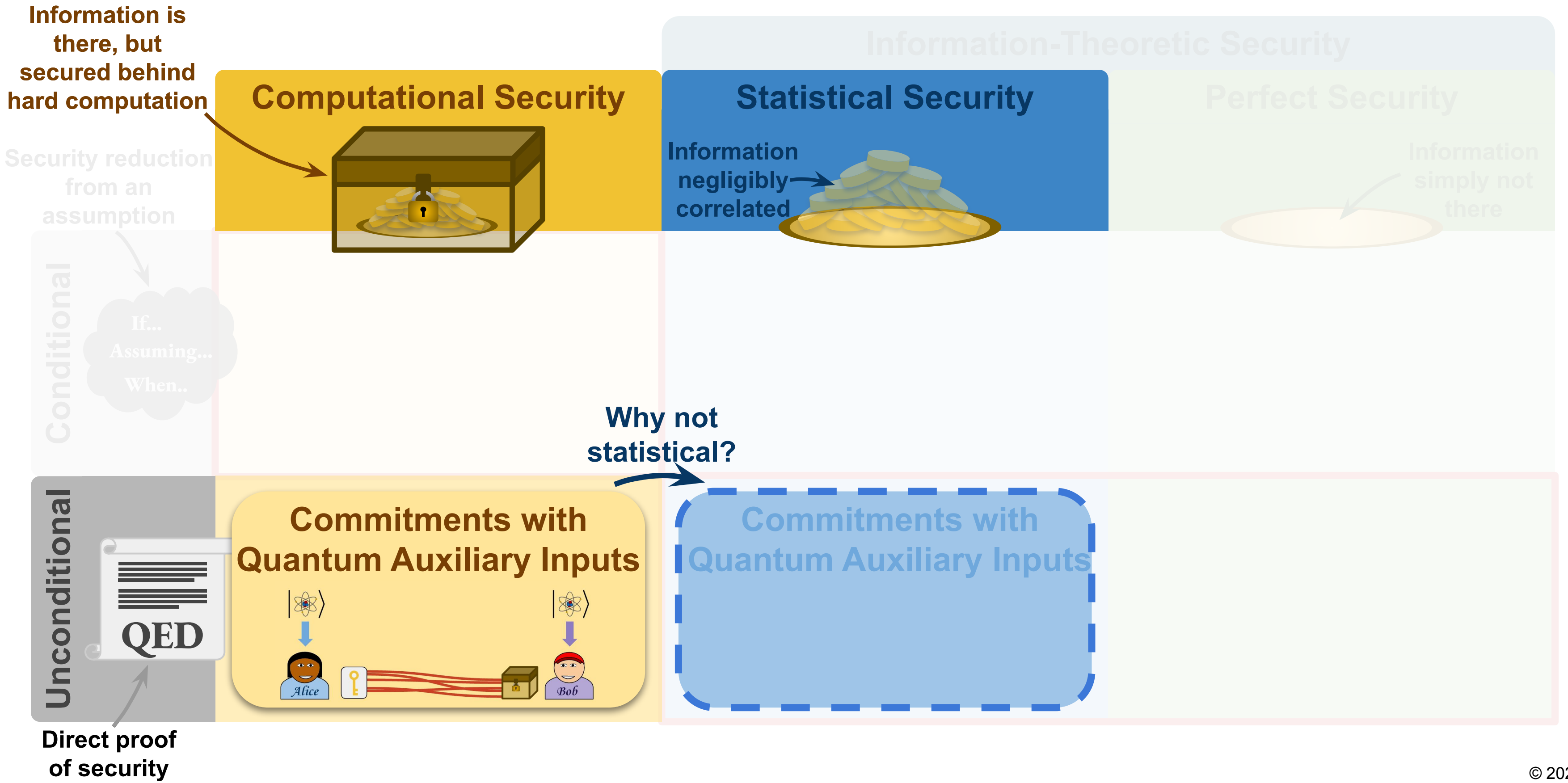
Security reduction from an assumption





Direct proof of security

Barak Nehoran (Princeton U), Luowen Qian (Boston U & NTT), Tomoyuki Morimae (YITP), Takashi Yamakawa (NTT & YITP)



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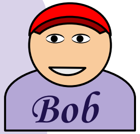
# Commitments Cannot Be Statistically Secure!

**[Mayers, Lo, Chau '97]**

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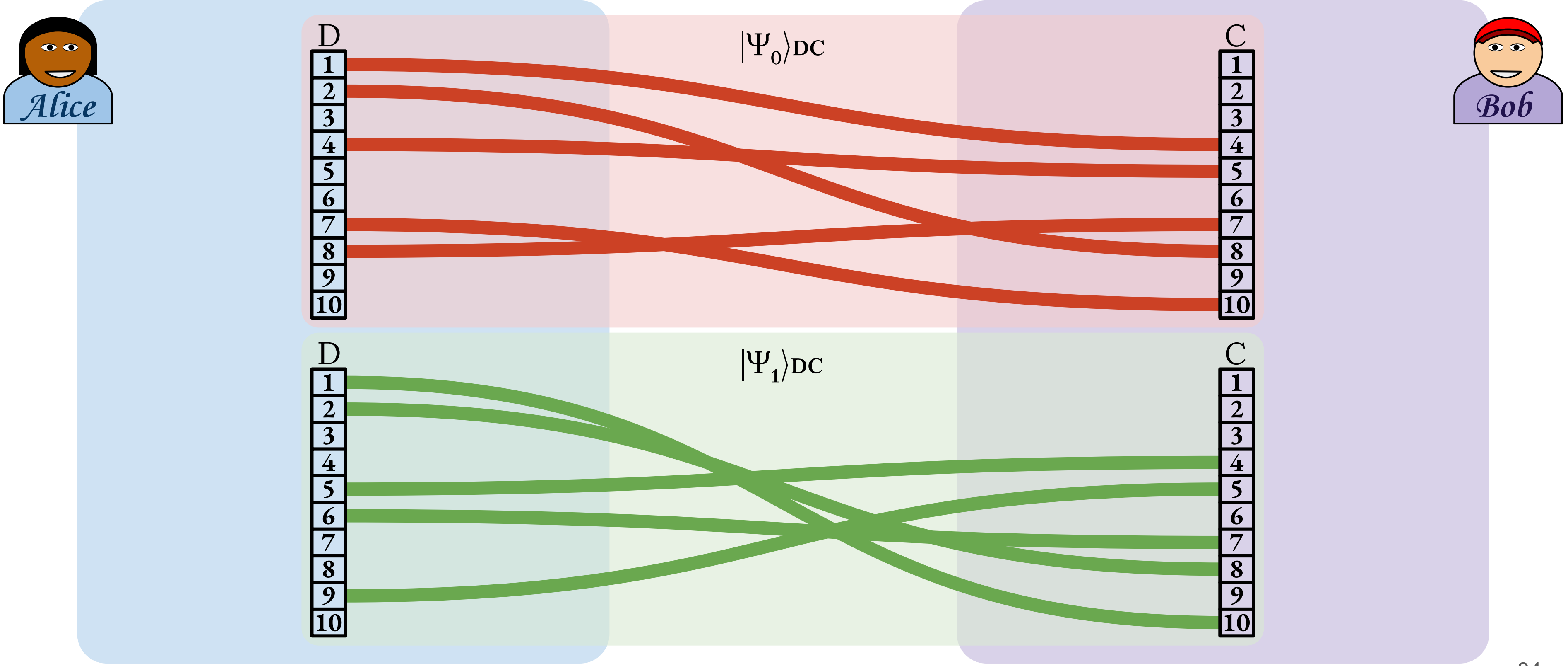

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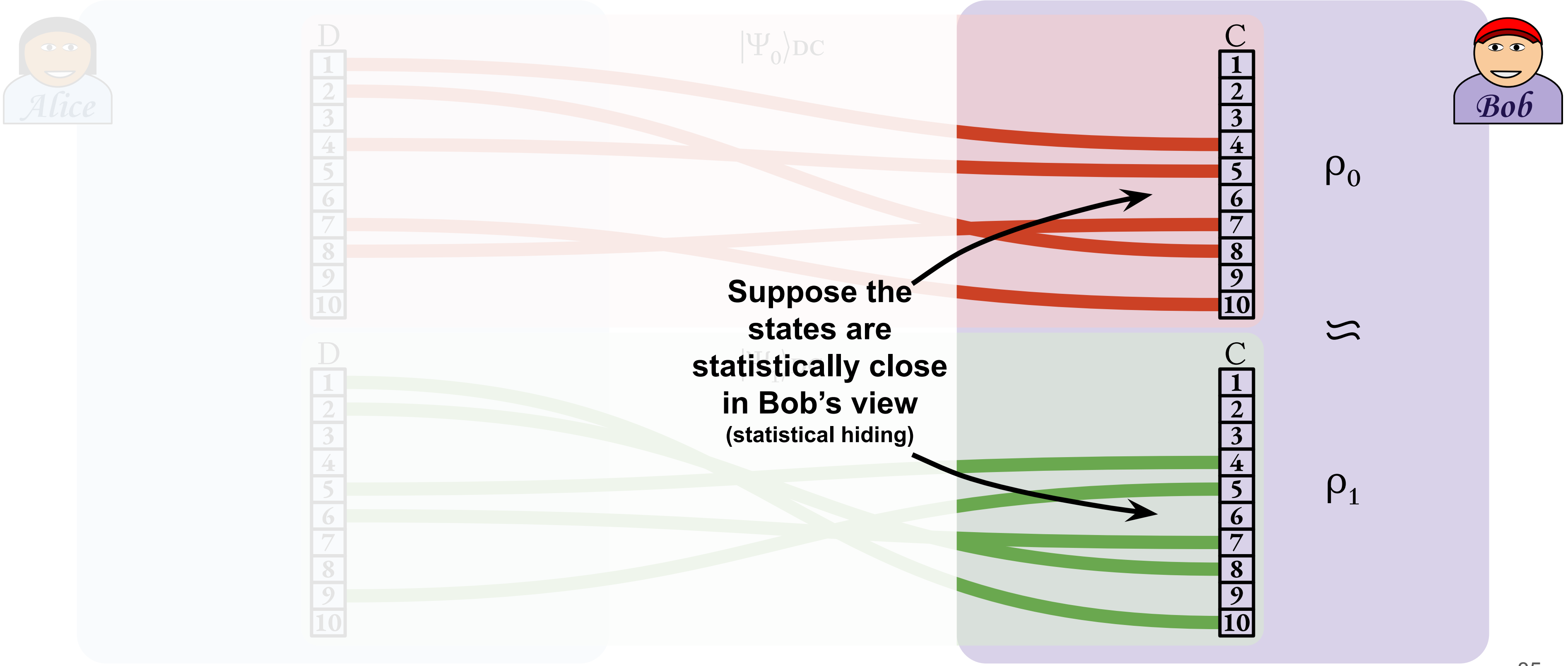

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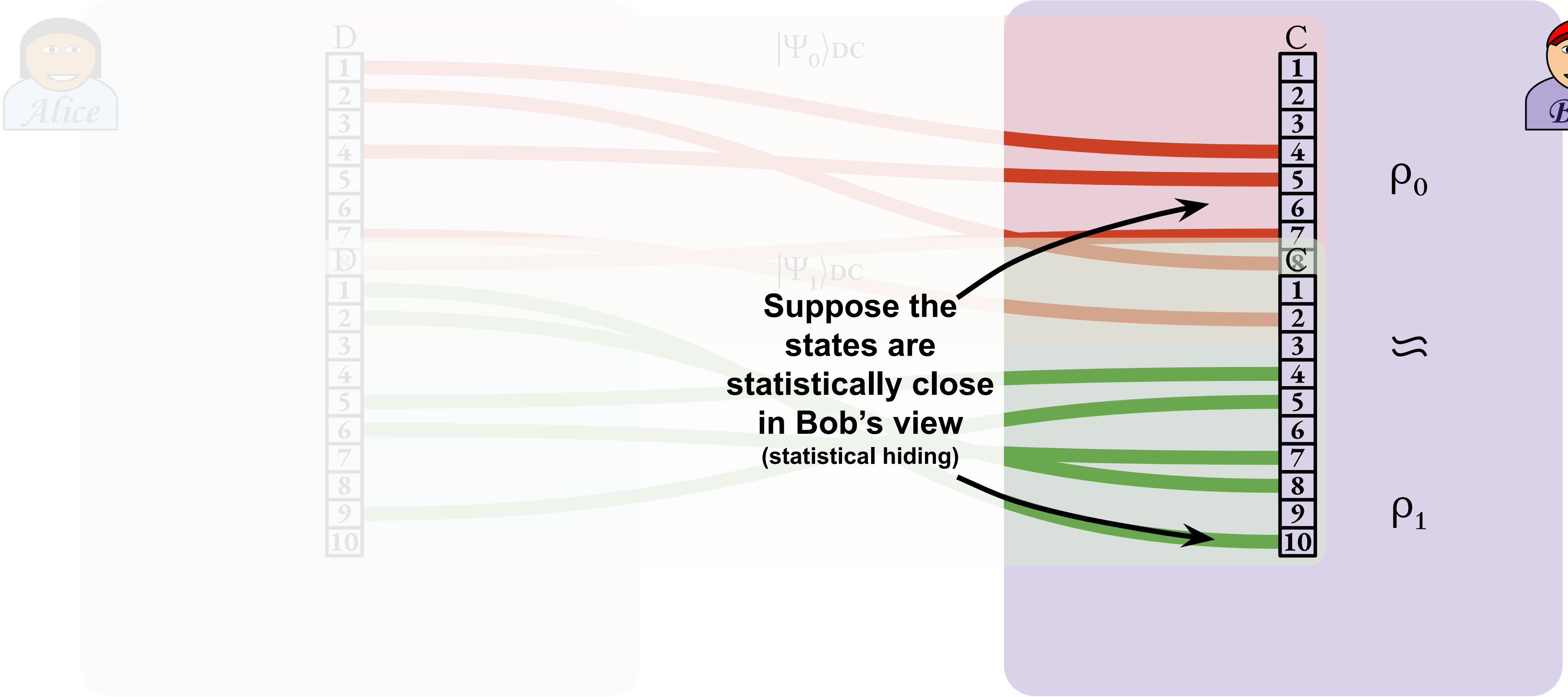

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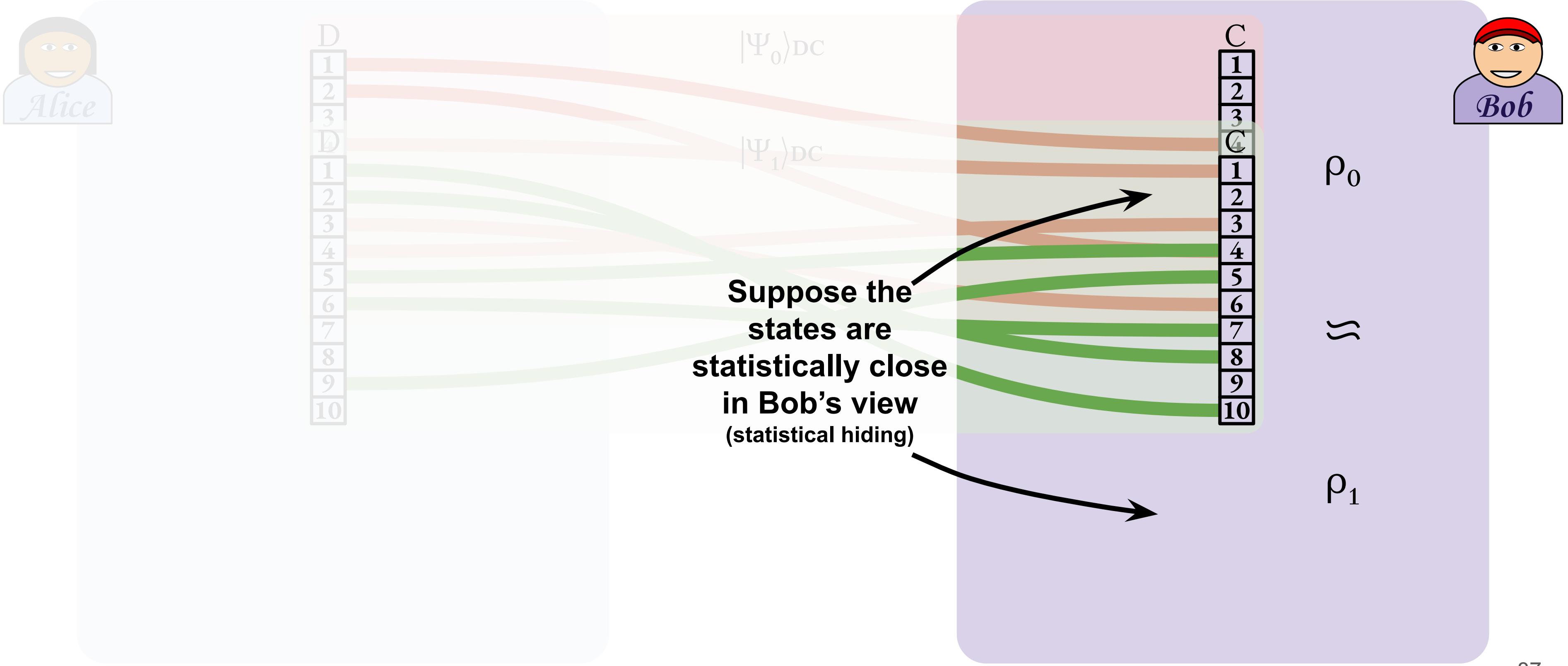


**Suppose the states are statistically close in Bob's view**
**(statistical hiding)**

Barak Nehoran (Princeton U), Luowen Qian (Boston U & NTT), Tomoyuki Morimae (YITP), Takashi Yamakawa (NTT & YITP)

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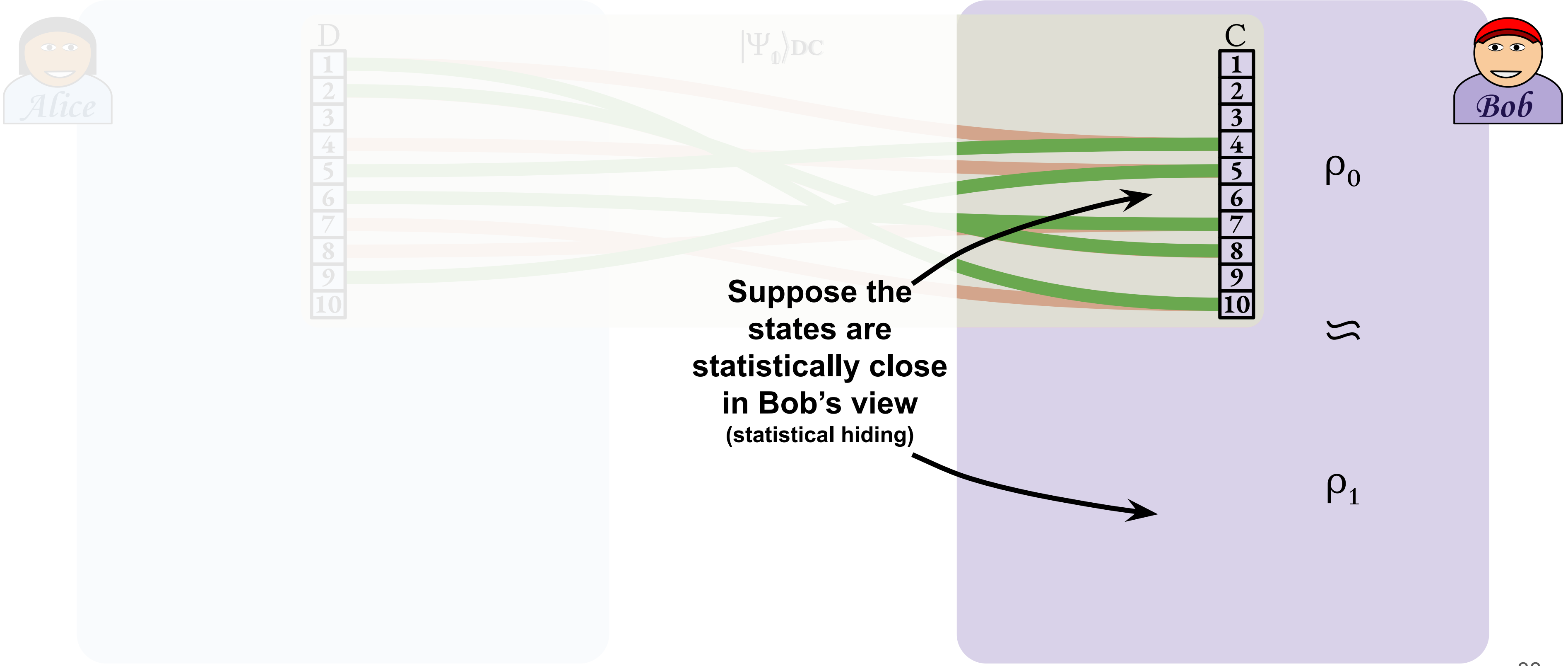

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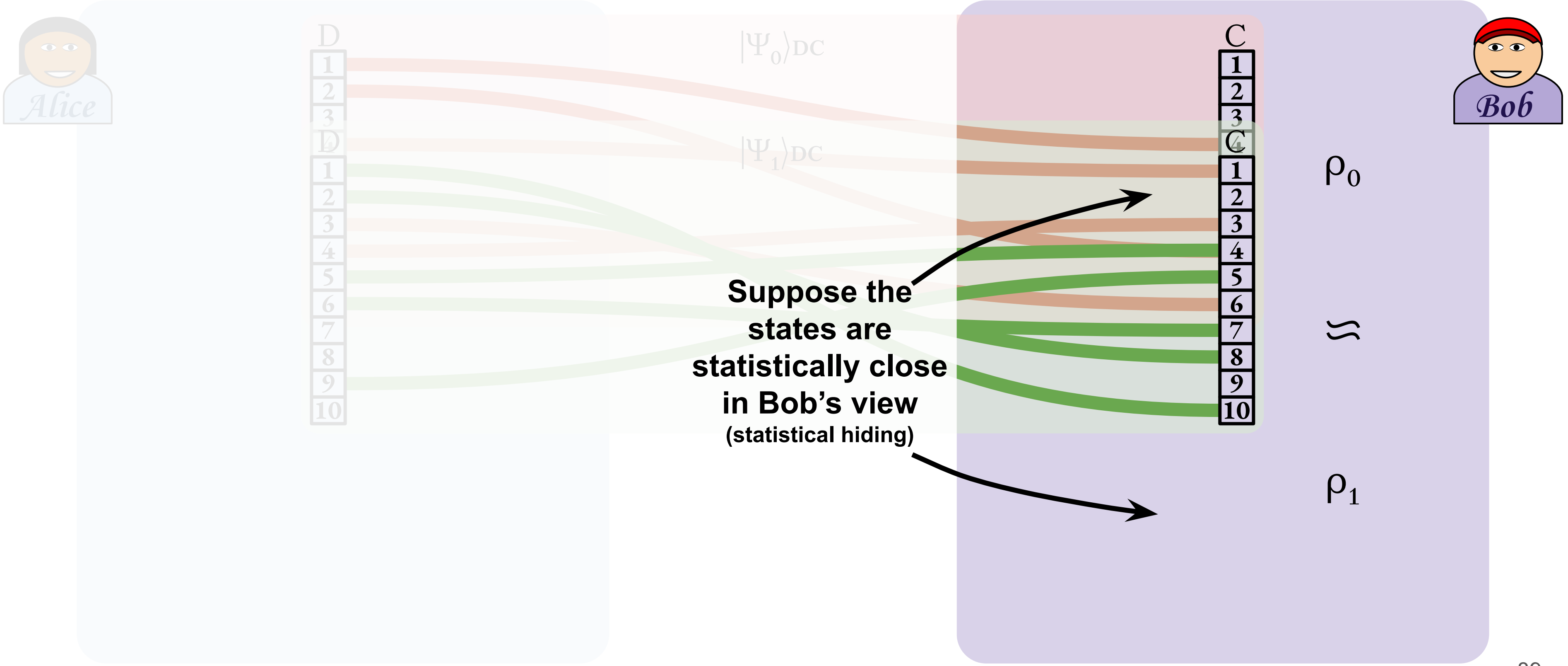

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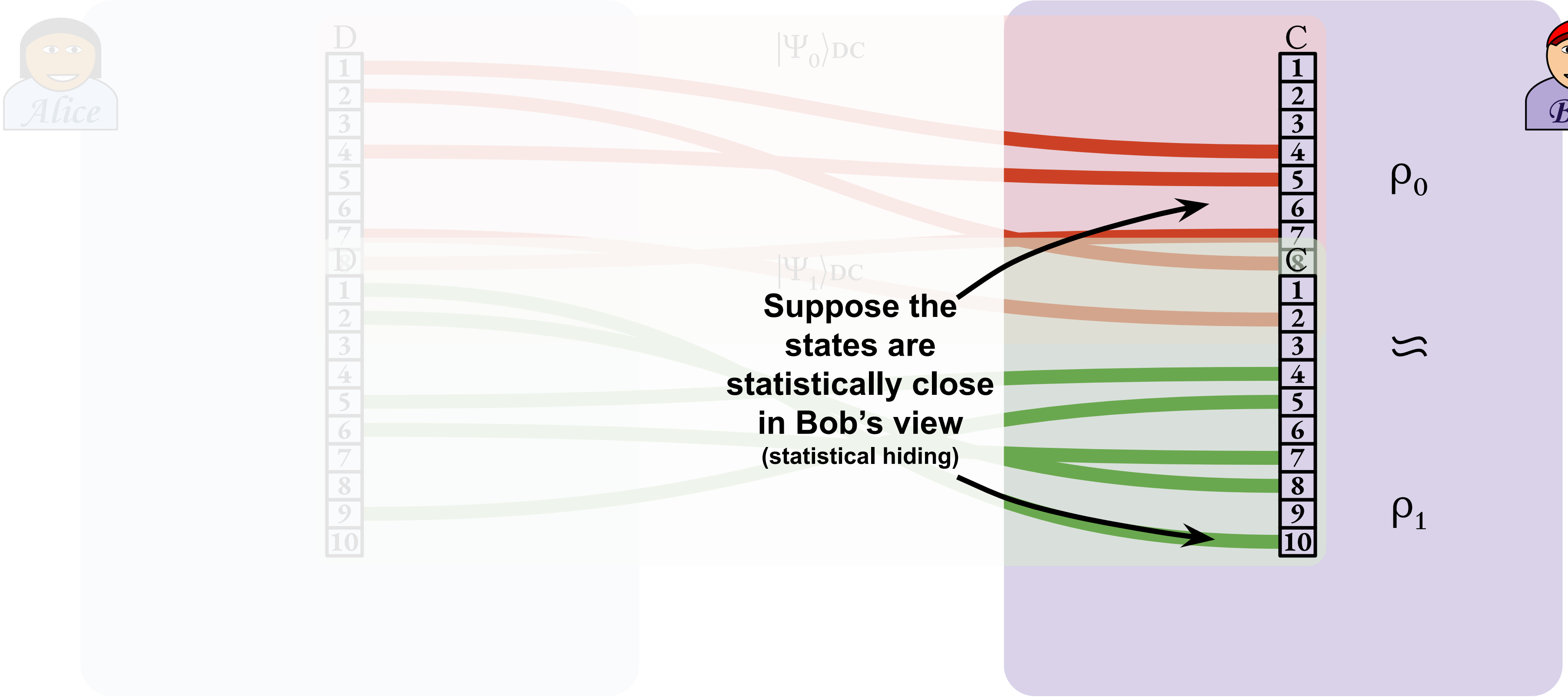

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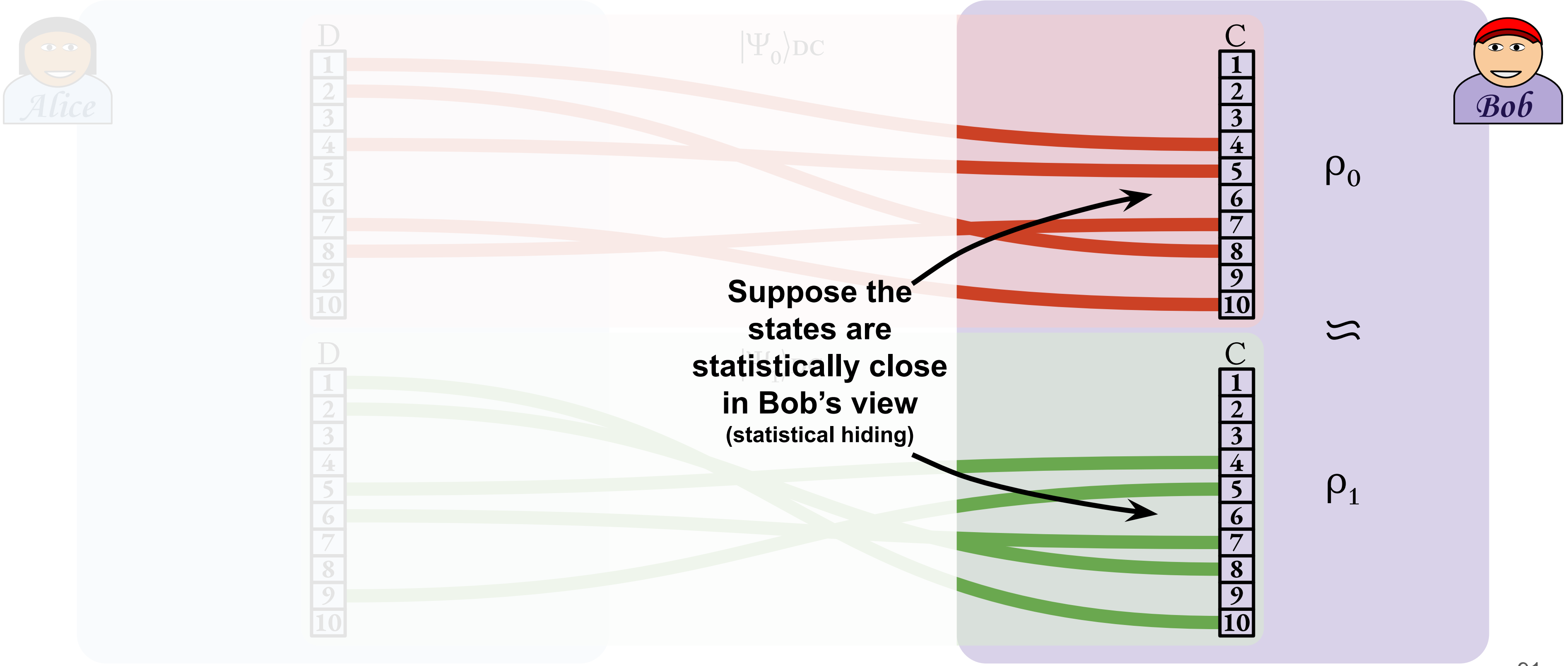

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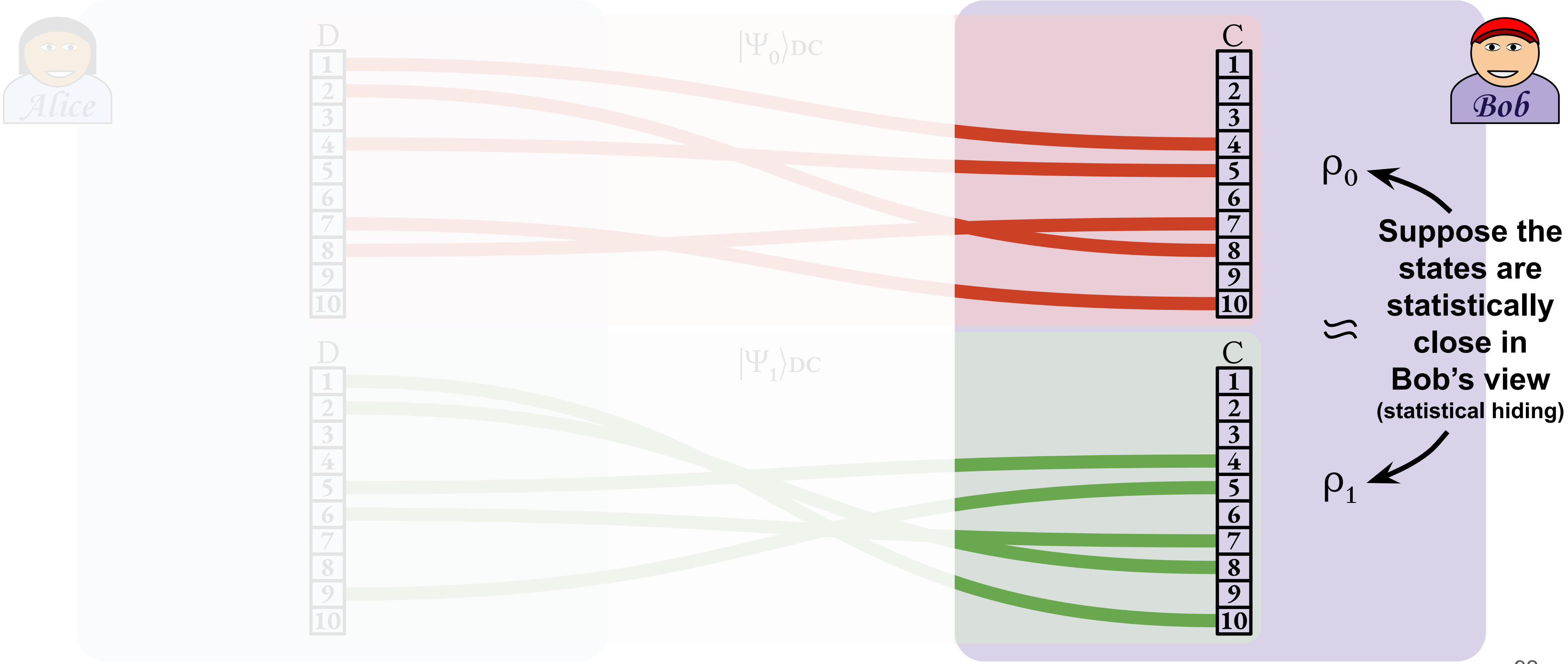

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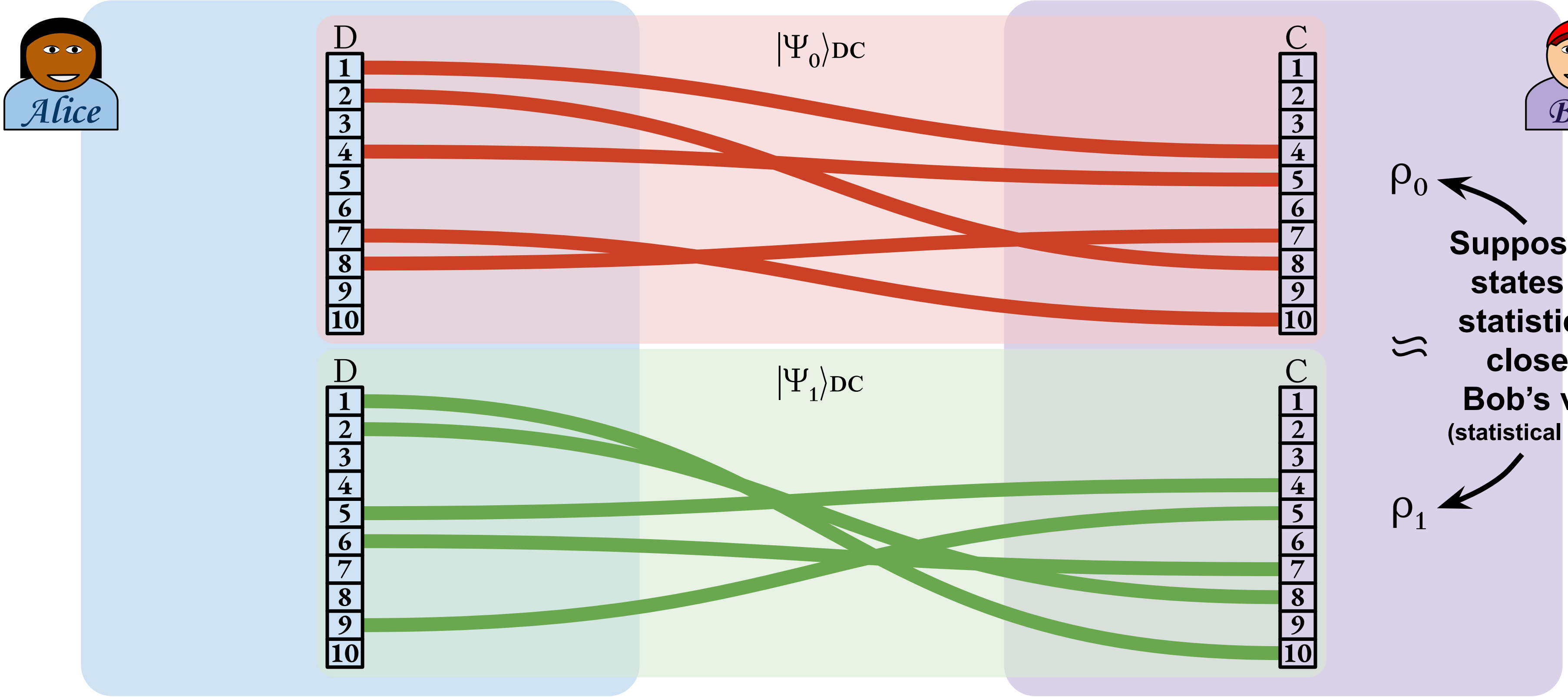


Suppose the states are statistically close in Bob's view (statistical hiding)

Barak Nehoran (Princeton U), Luowen Qian (Boston U & NTT), Tomoyuki Morimae (YITP), Takashi Yamakawa (NTT & YITP)

# Commitments Cannot Be Statistically Secure!

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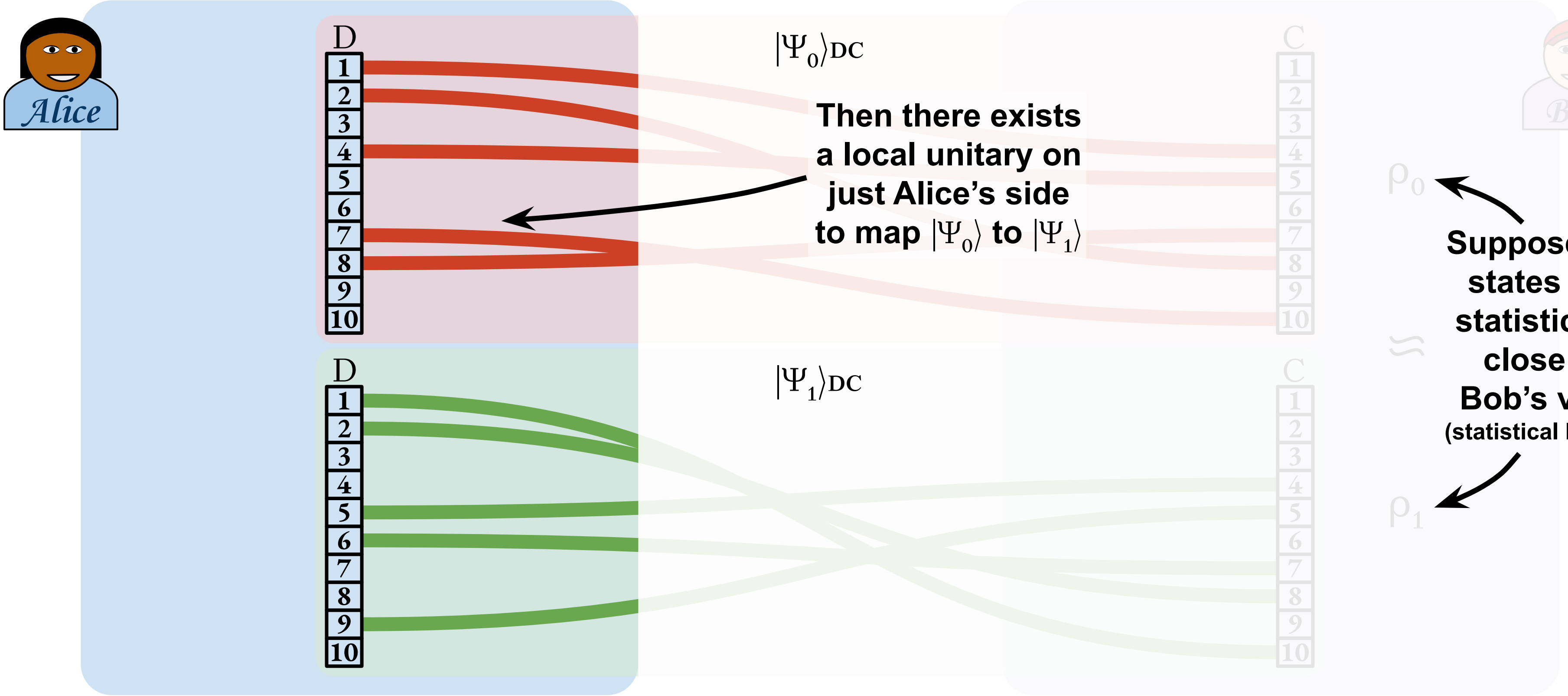


Then there exists a local unitary on just Alice's side to map $|\Psi_0\rangle$ to $|\Psi_1\rangle$

Suppose the states are statistically close in Bob's view

(statistical hiding)

Barak Nehoran (Princeton U), Luowen Qian (Boston U & NTT), Tomoyuki Morimae (YITP), Takashi Yamakawa (NTT & YITP)

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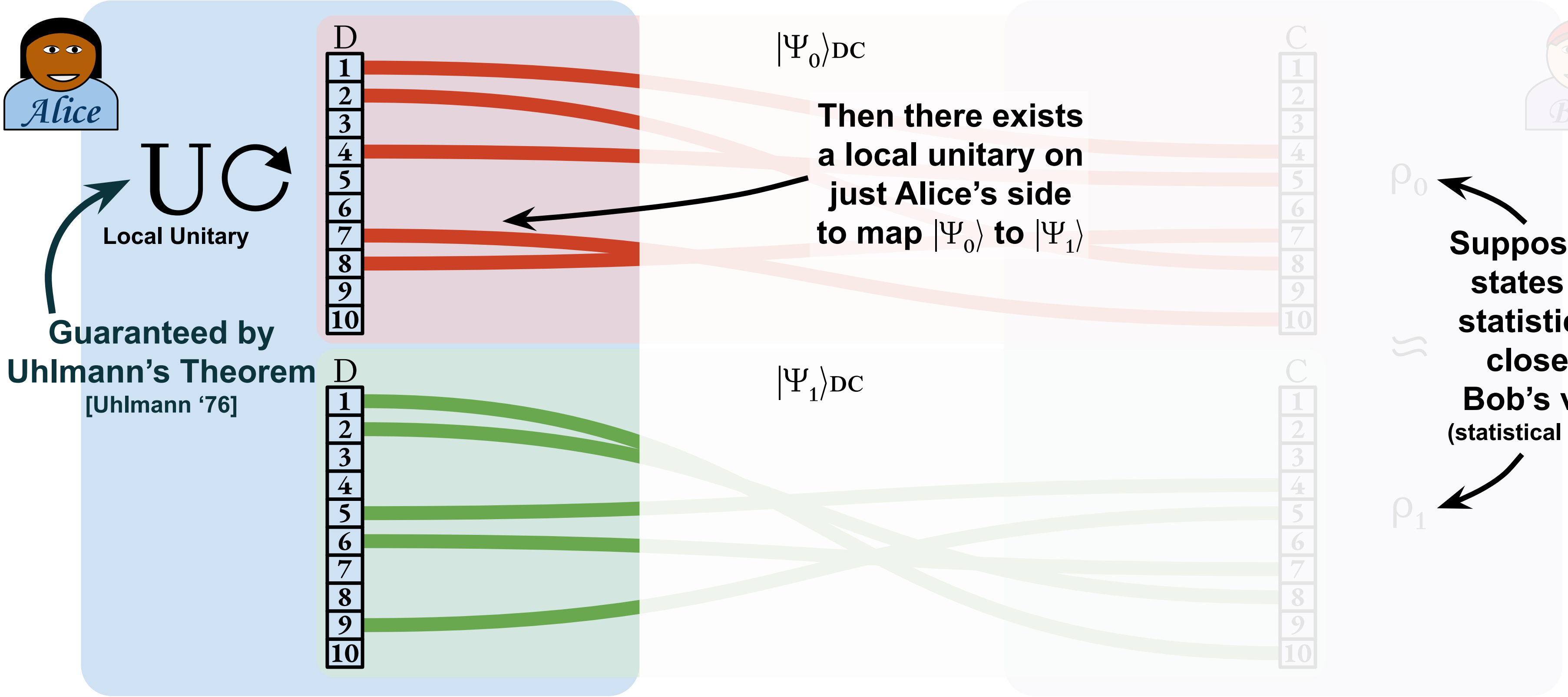


Then there exists a local unitary on just Alice's side to map $|\Psi_0\rangle$ to $|\Psi_1\rangle$

Guaranteed by Uhlmann's Theorem [Uhlmann '76]

Suppose the states are statistically close in Bob's view (statistical hiding)

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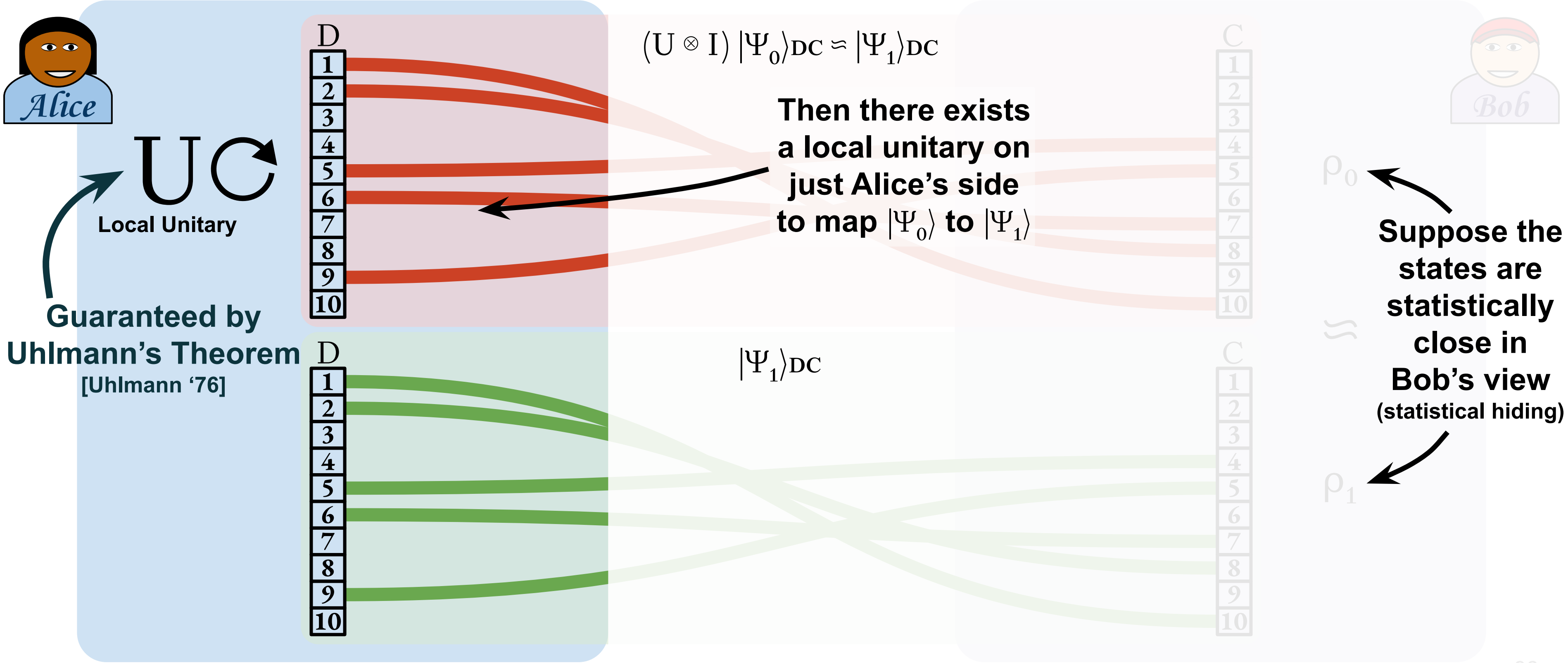

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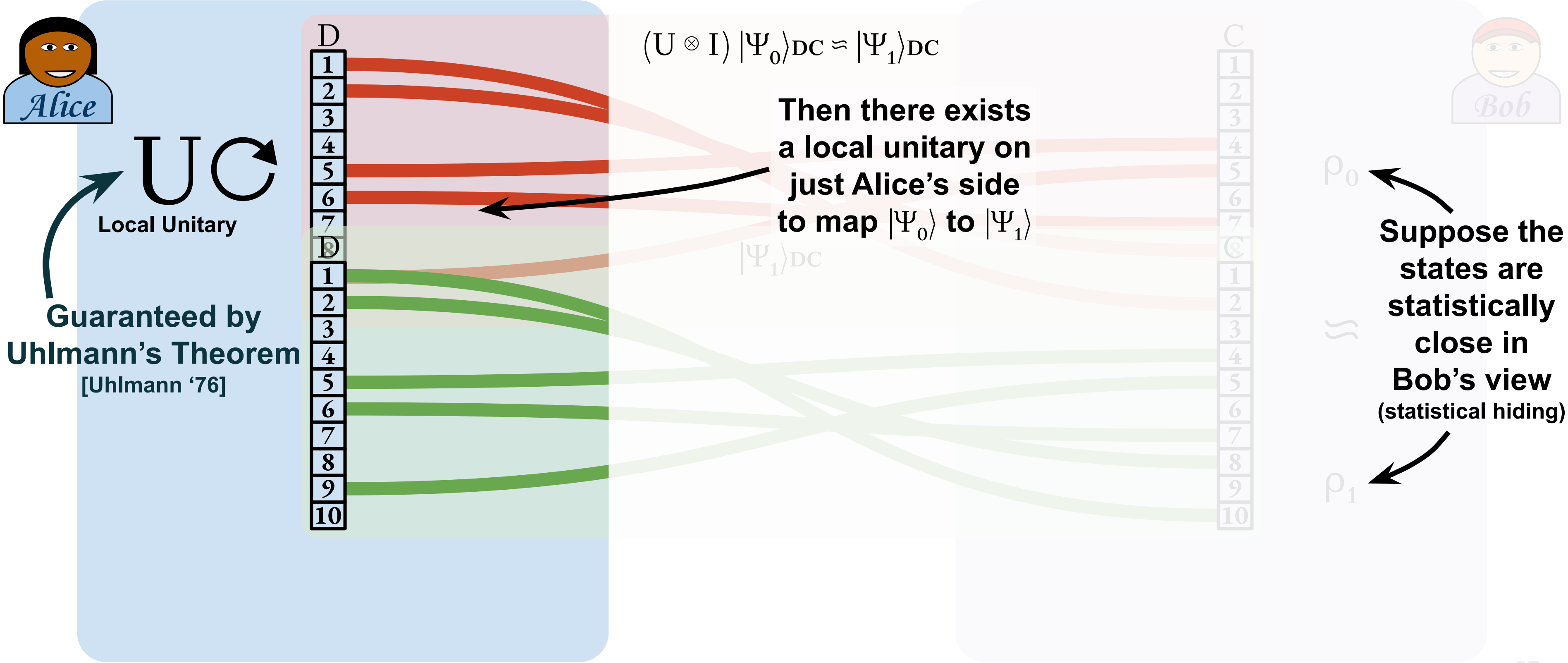


$U$ Local Unitary

Guaranteed by Uhlmann's Theorem [Uhlmann '76]

$(U \otimes I)\,|\Psi_0\rangle_{DC} \approx |\Psi_1\rangle_{DC}$

**Then there exists a local unitary on just Alice's side to map $|\Psi_0\rangle$ to $|\Psi_1\rangle$**

**Suppose the states are statistically close in Bob's view (statistical hiding)**

Barak Nehoran (Princeton U), Luowen Qian (Boston U & NTT), Tomoyuki Morimae (YITP), Takashi Yamakawa (NTT & YITP)

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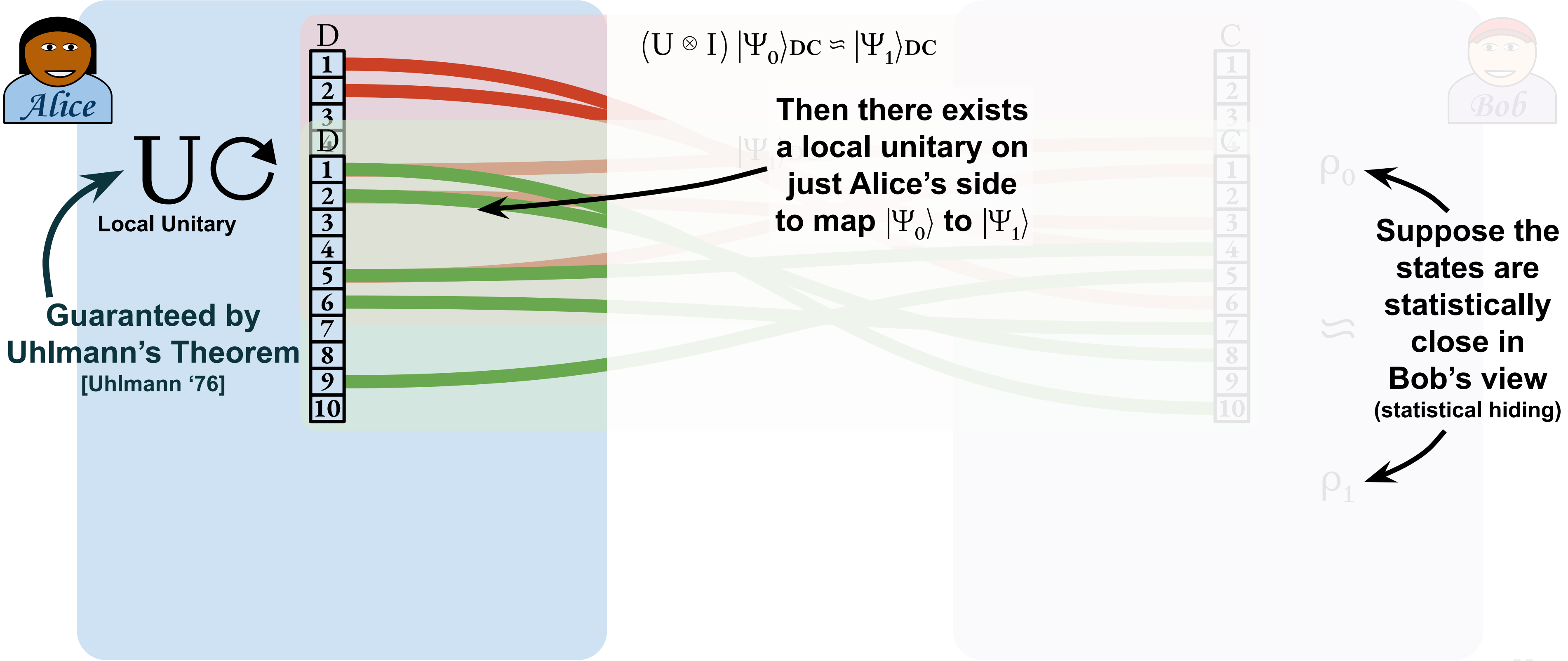

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# Commitments Cannot Be Statistically Secure!

[Mayers, Lo, Chau '97]

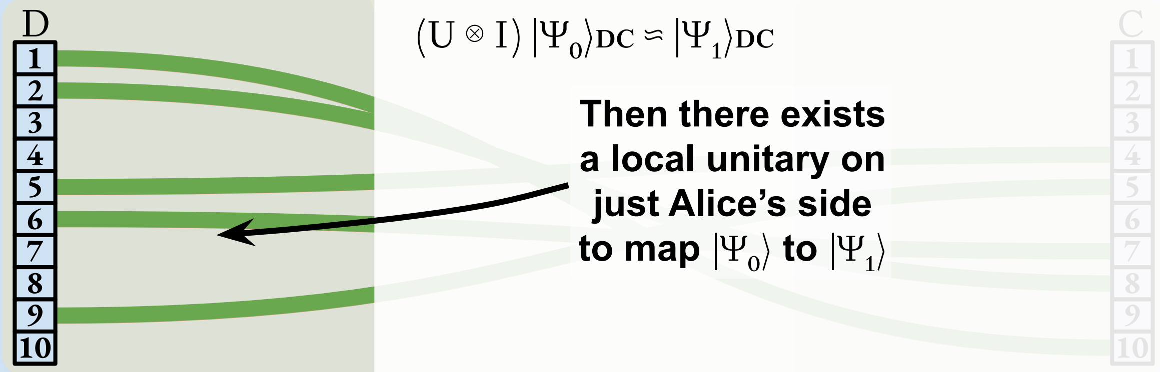
Alice

U

Local Unitary

Guaranteed by Uhlmann's Theorem

[Uhlmann '76]




$(U \otimes I)\,|\Psi_0\rangle_{DC} \approx |\Psi_1\rangle_{DC}$

**Then there exists a local unitary on just Alice's side to map $|\Psi_0\rangle$ to $|\Psi_1\rangle$**

Bob

$\rho_0$

$\approx$

$\rho_1$

**Suppose the states are statistically close in Bob's view**

**(statistical hiding)**

Barak Nehoran (Princeton U), Luowen Qian (Boston U & NTT), Tomoyuki Morimae (YITP), Takashi Yamakawa (NTT & YITP)

# Commitments Cannot Be Statistically Secure!

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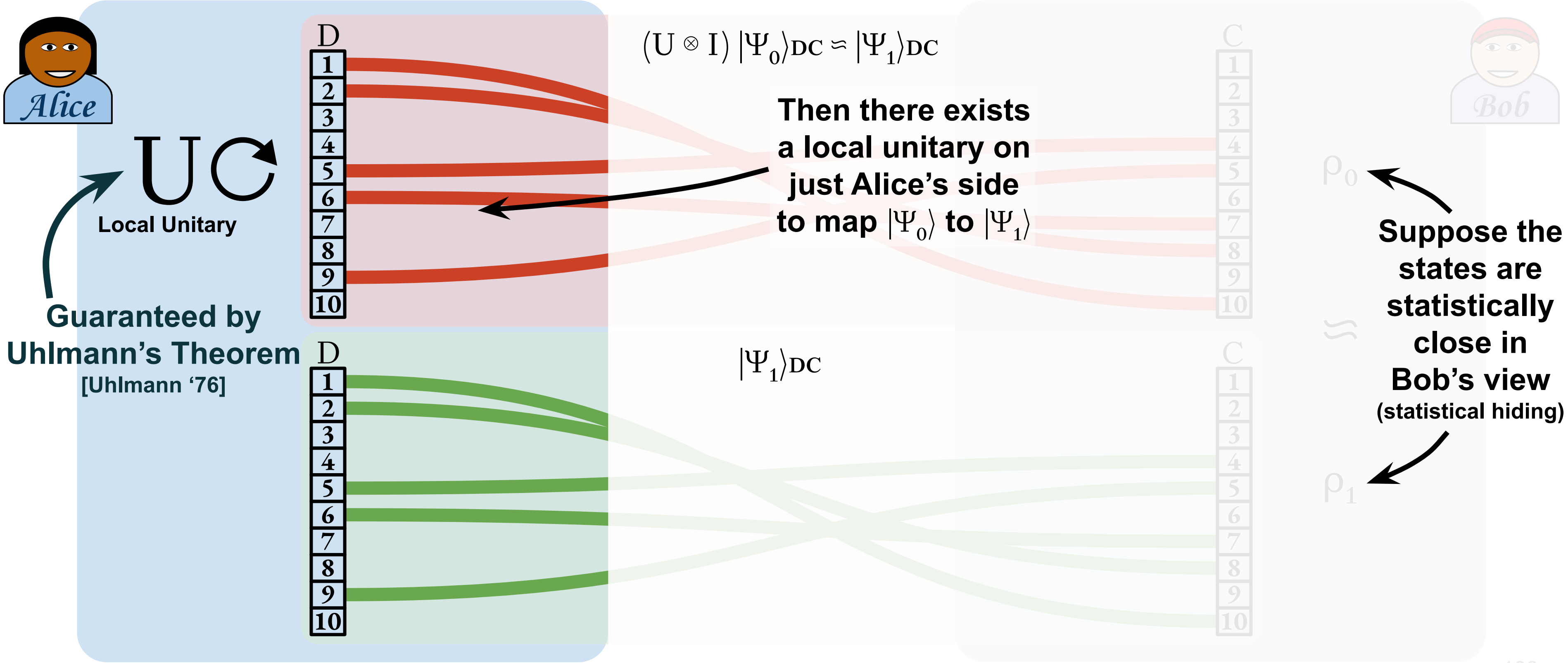

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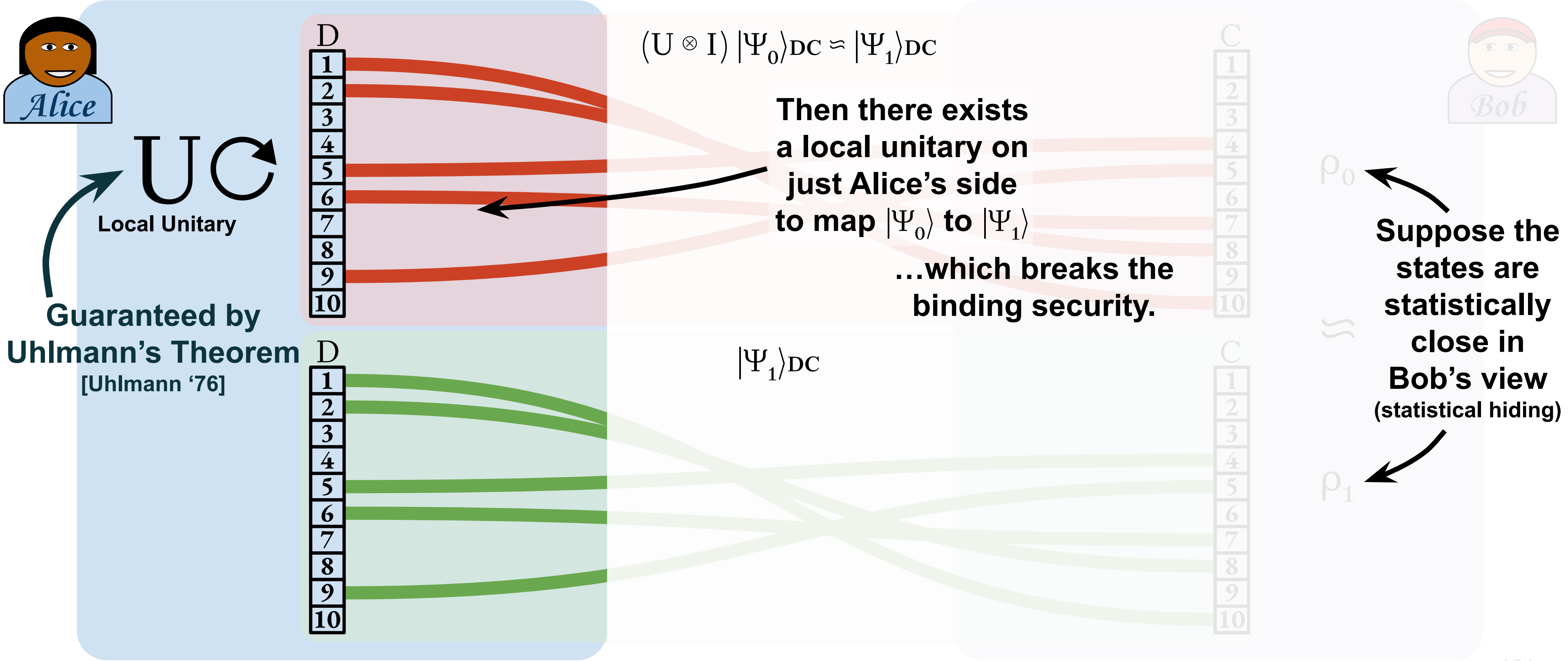

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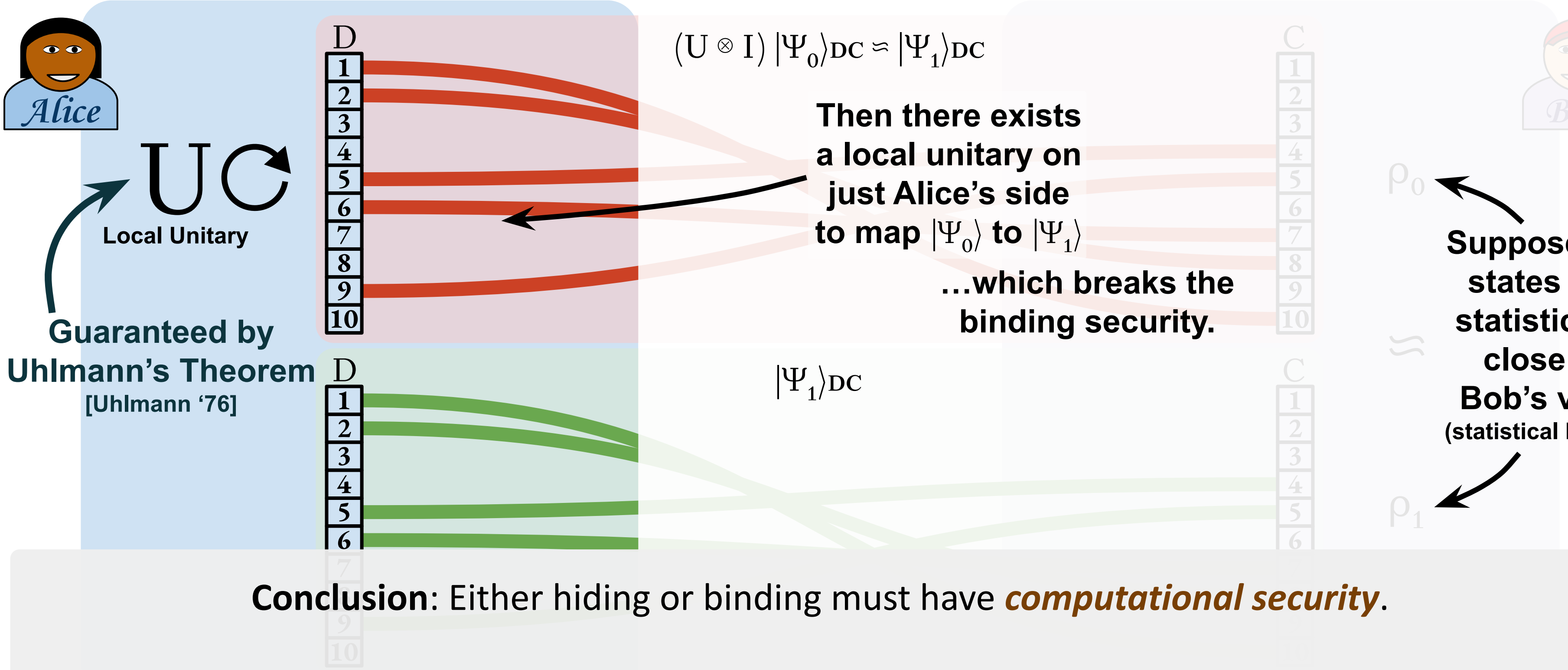


Guaranteed by Uhlmann's Theorem [Uhlmann '76]

Then there exists a local unitary on just Alice's side to map $|\Psi_0\rangle$ to $|\Psi_1\rangle$

…which breaks the binding security.

Suppose the states are statistically close in Bob's view (statistical hiding)

**Conclusion**: Either hiding or binding must have ***computational security***.

Barak Nehoran (Princeton U), Luowen Qian (Boston U & NTT), Tomoyuki Morimae (YITP), Takashi Yamakawa (NTT & YITP)

# Commitments Cannot Be Statistically Secure!

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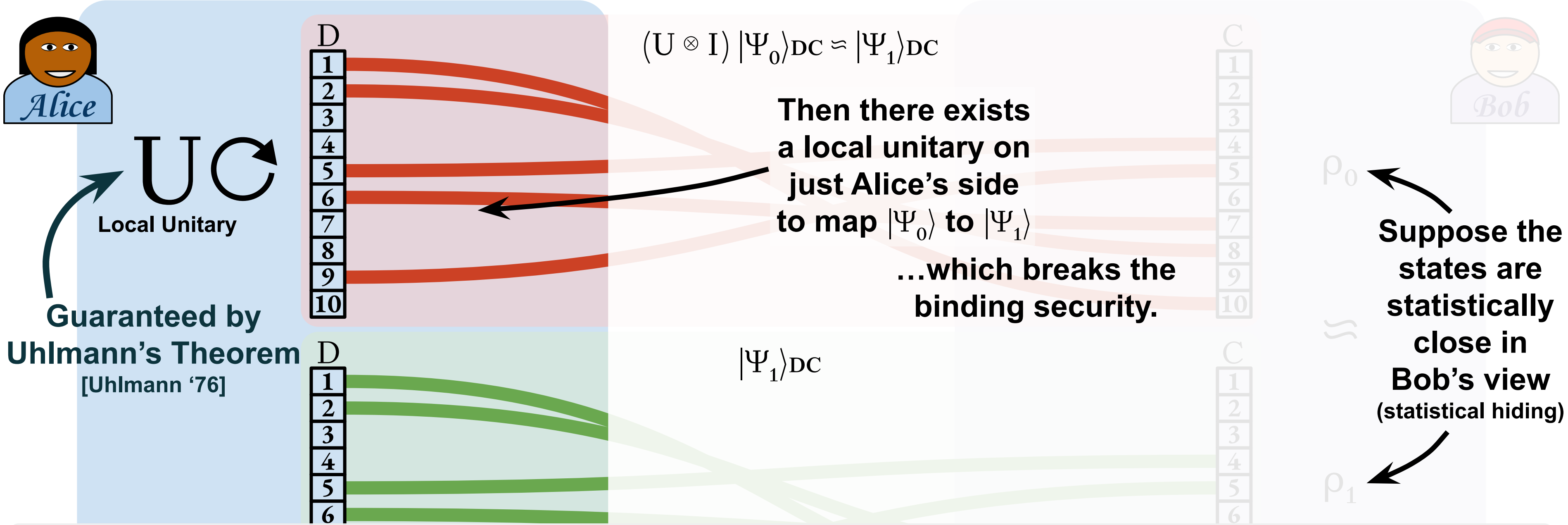
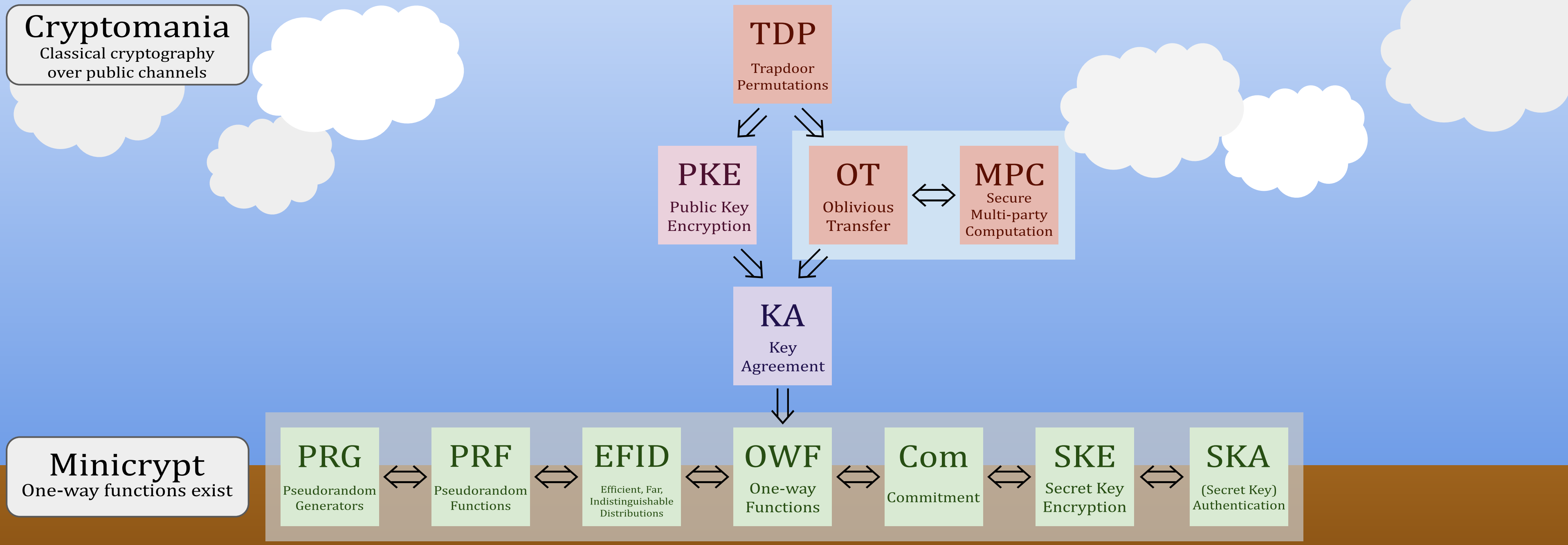



**Conclusion**: Either hiding or binding must have ***computational security***.

But classical barriers **don't** rule out showing it ***unconditionally***.

Barak Nehoran (Princeton U), Luowen Qian (Boston U & NTT), Tomoyuki Morimae (YITP), Takashi Yamakawa (NTT & YITP)

# A (More) Minimal Assumption For Cryptography



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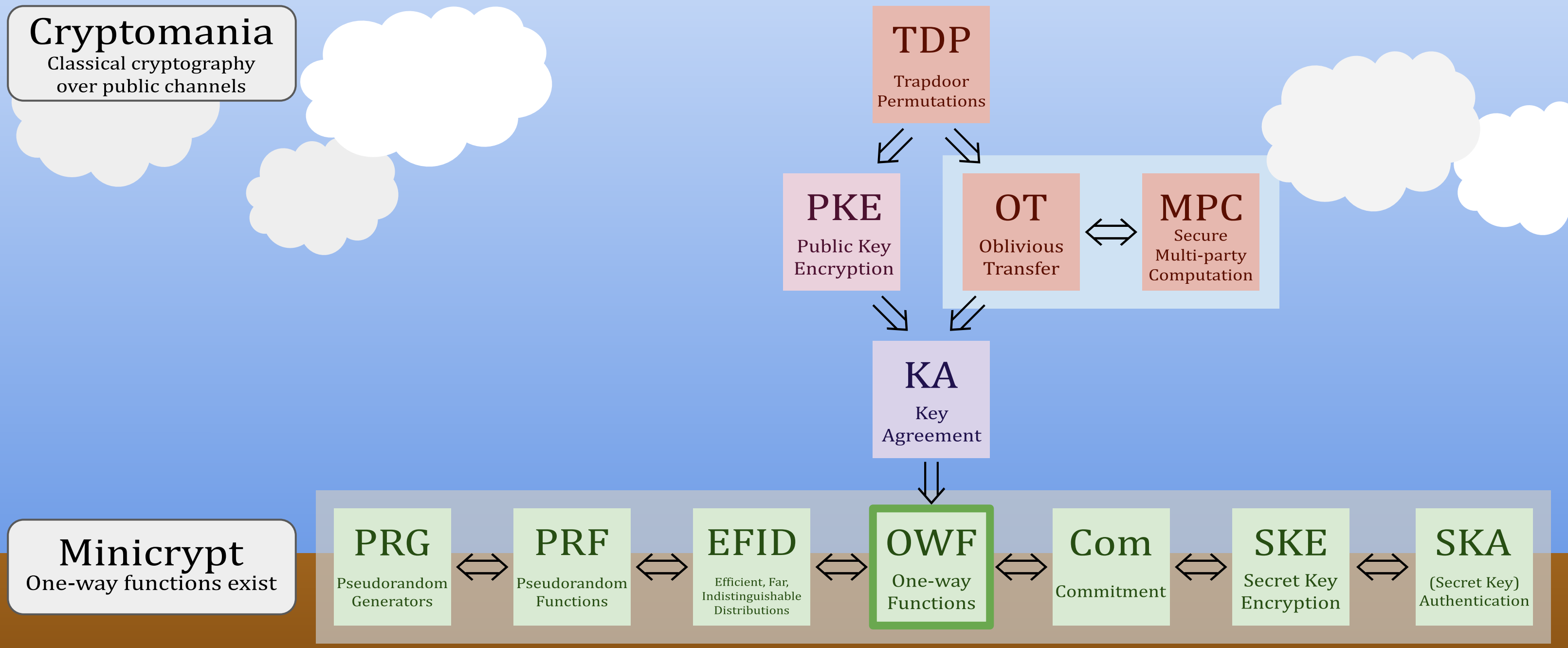

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# A (More) Minimal Assumption For Cryptography

**Cryptomania**
Classical cryptography over public channels

**TDP** Trapdoor Permutations

**PKE** Public Key Encryption

**OT** Oblivious Transfer ⇔ **MPC** Secure Multi-party Computation

**KA** Key Agreement

⇓

**Minicrypt**
One-way functions exist

**PRG** Pseudorandom Generators ⇔ **PRF** Pseudorandom Functions ⇔ **EFID** Efficient, Far, Indistinguishable Distributions ⇔ **OWF** One-way Functions ⇔ **Com** Commitment ⇔ **SKE** Secret Key Encryption ⇔ **SKA** (Secret Key) Authentication

**Unconditionally Secure Commitments with Quantum Auxiliary Inputs / Preprocessing**

Barak Nehoran (Princeton U), Luowen Qian (Boston U & NTT), Tomoyuki Morimae (YITP), Takashi Yamakawa (NTT & YITP)

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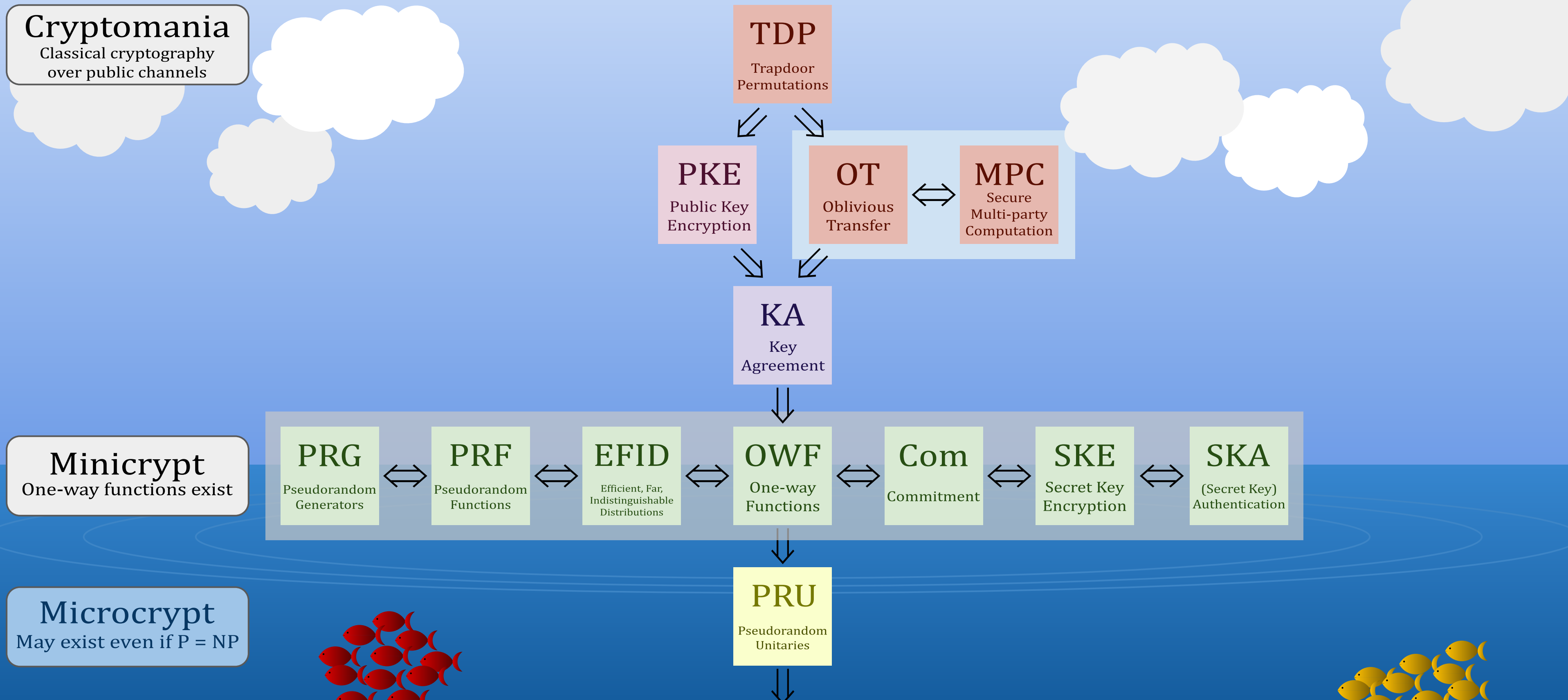

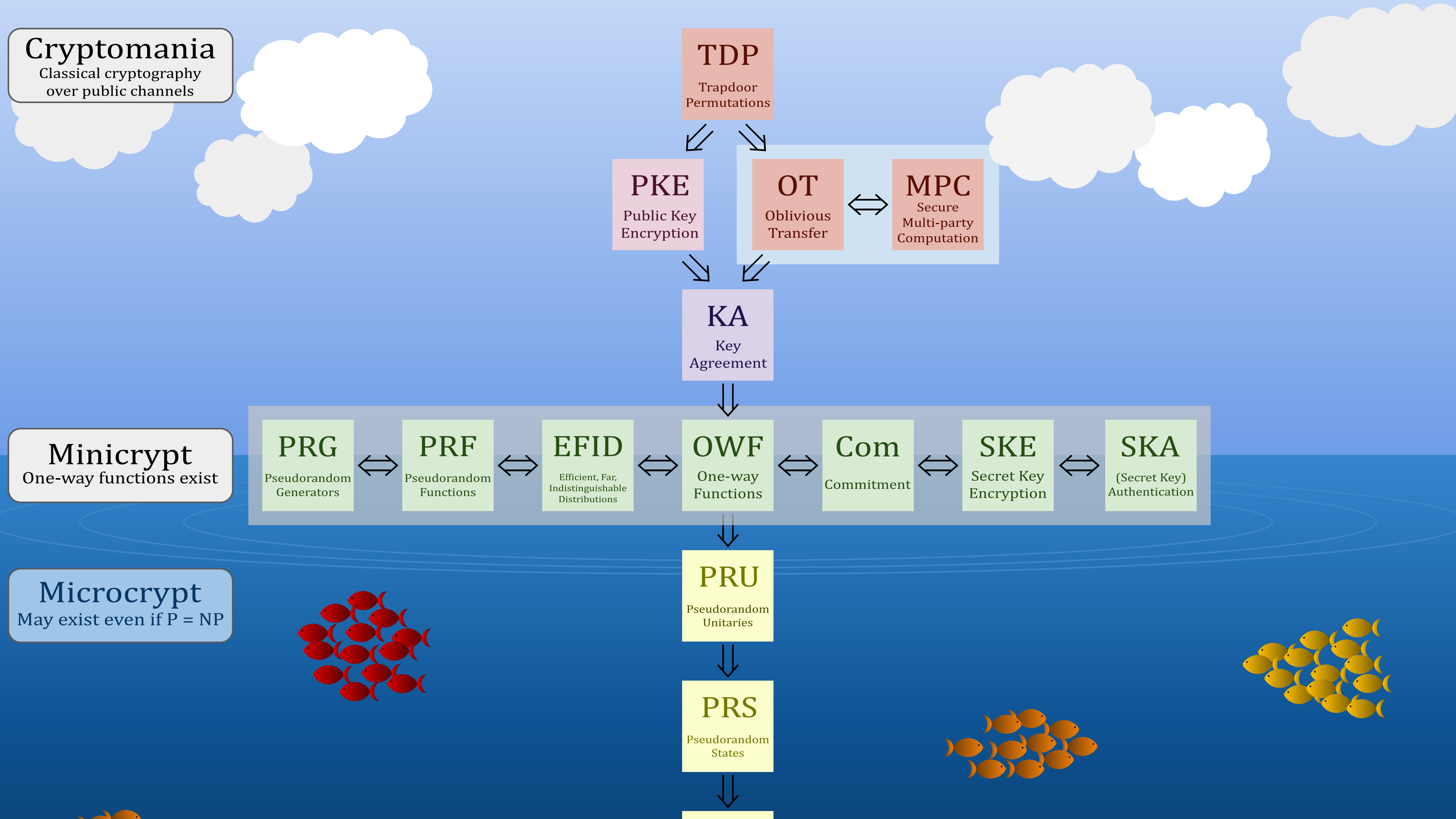

Cryptomania
Classical cryptography over public channels
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Microcrypt
May exist even if P = NP
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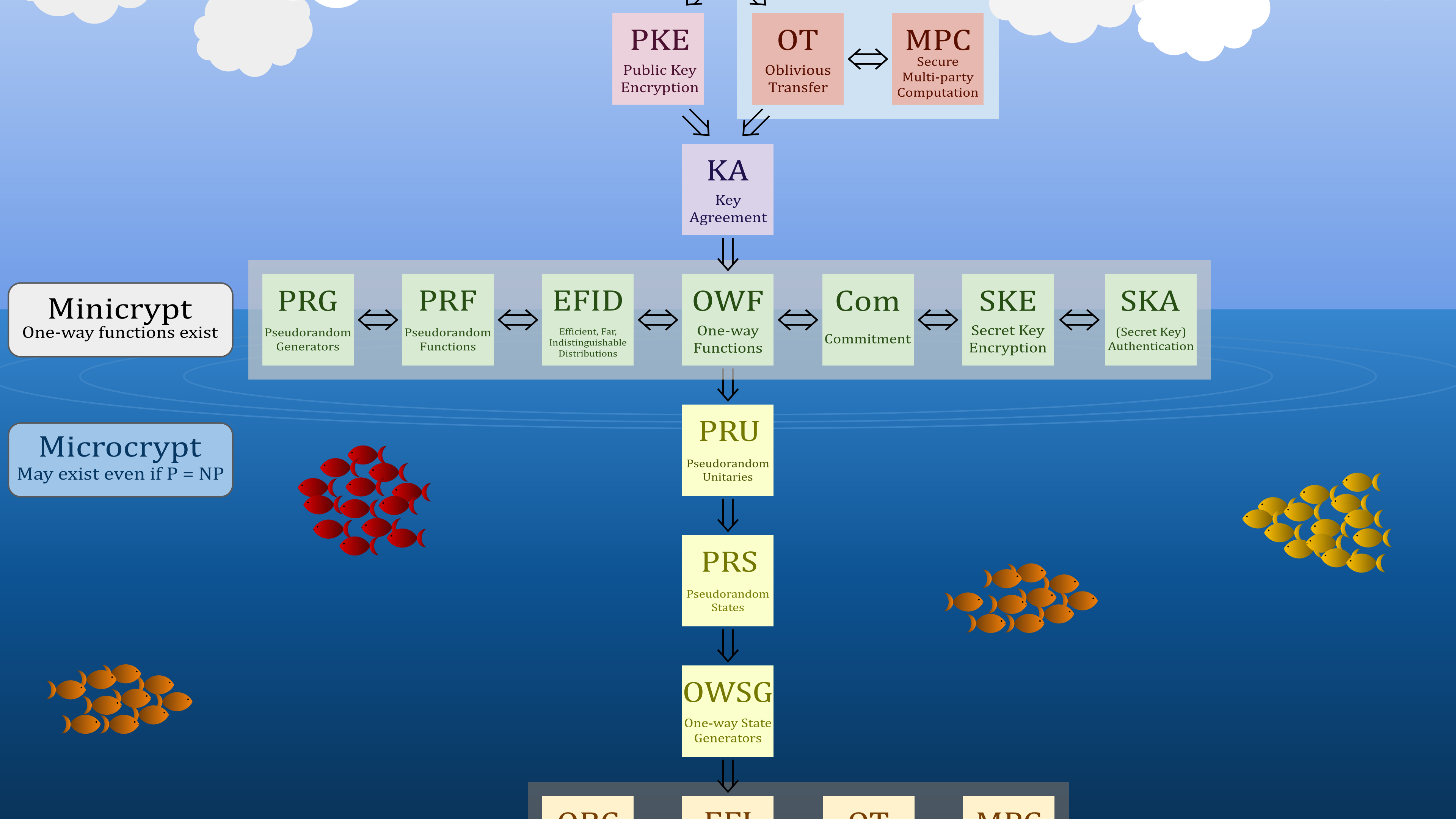
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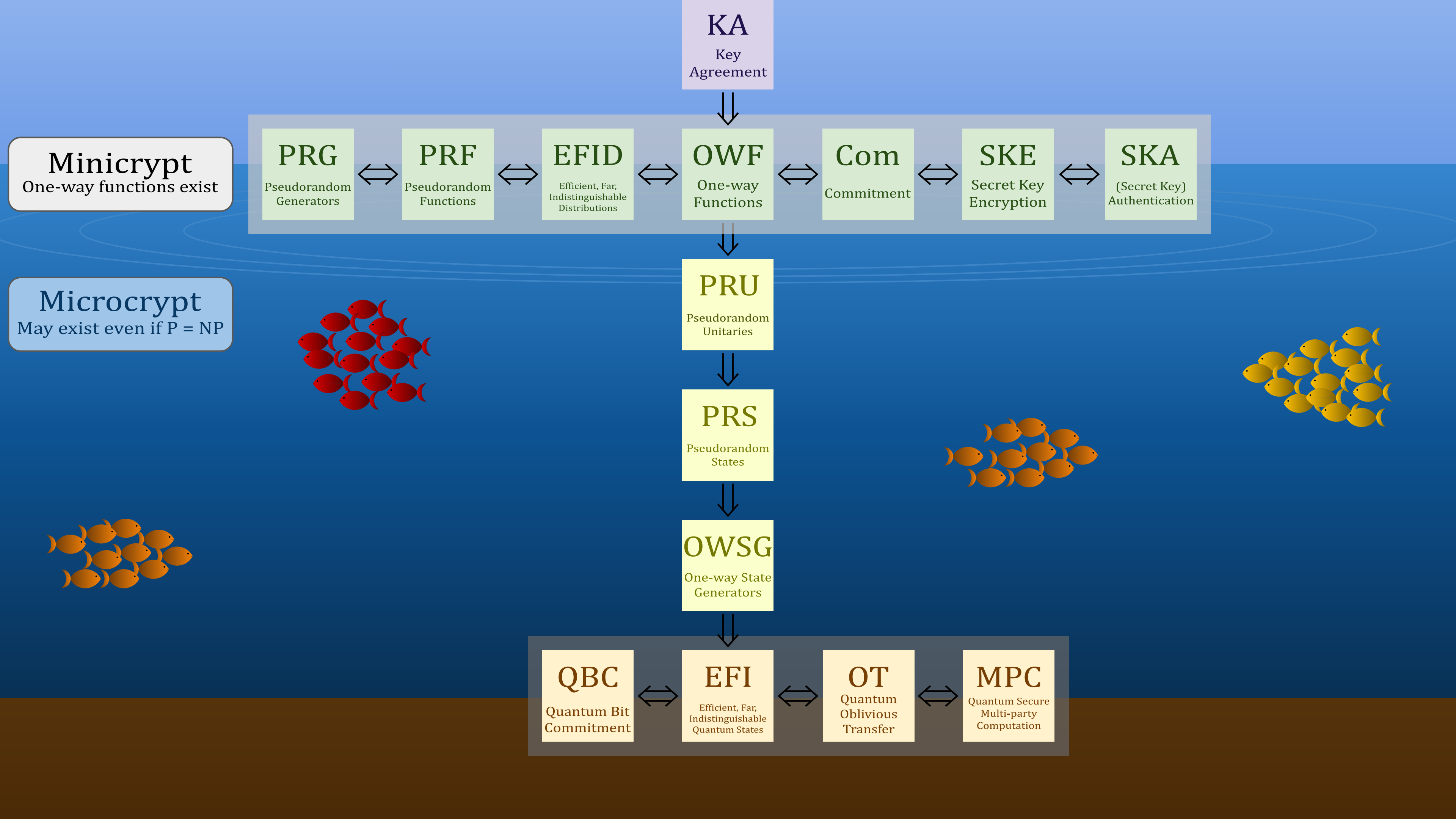

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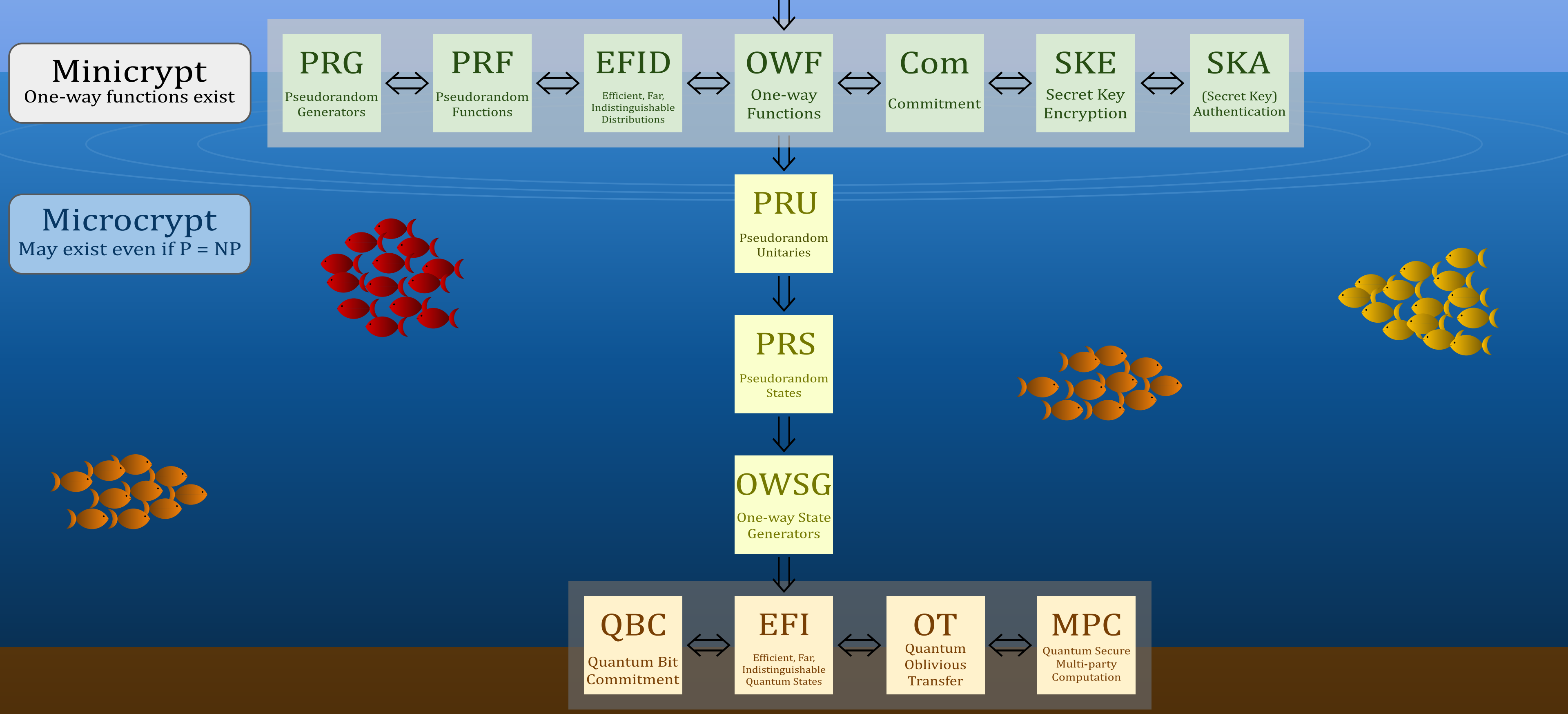
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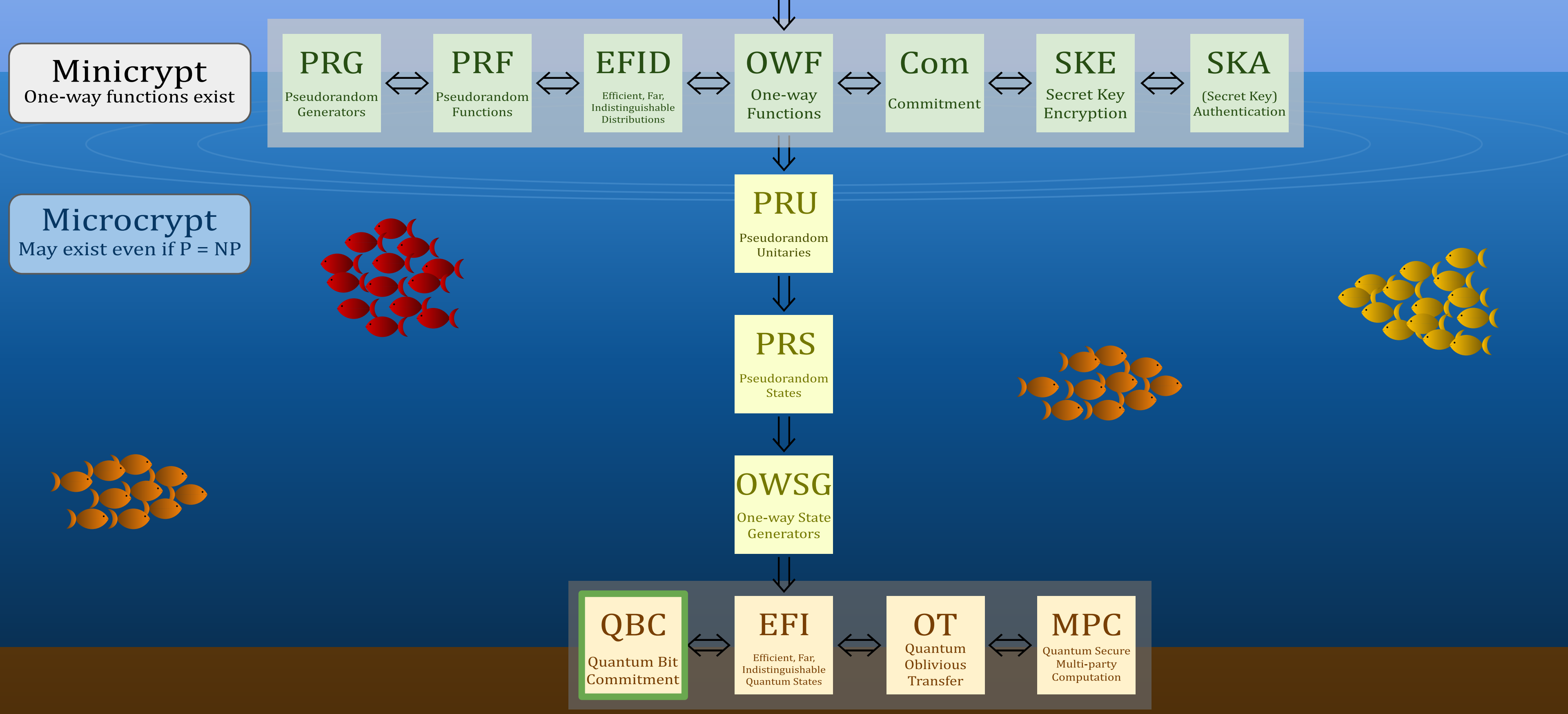
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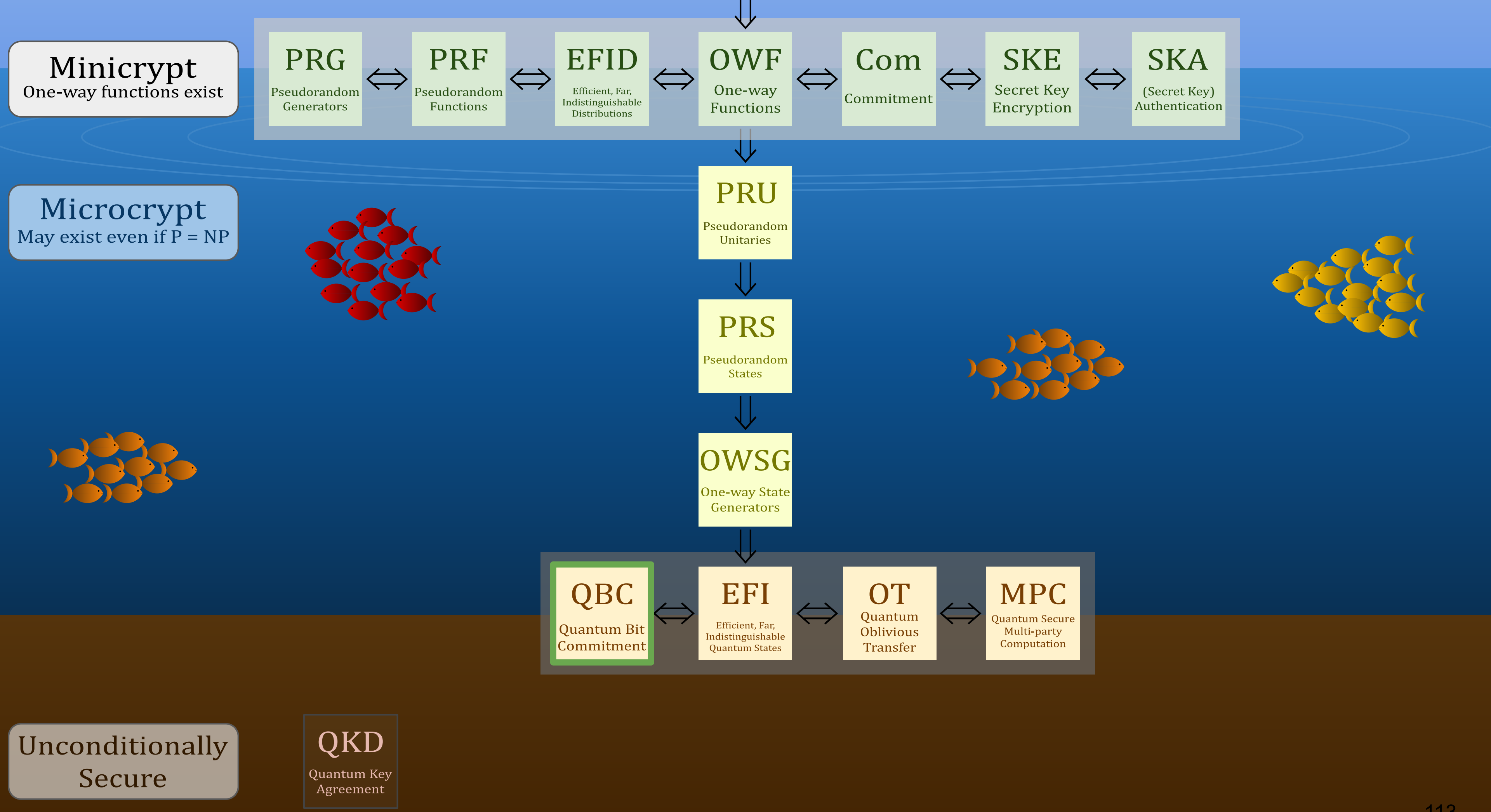

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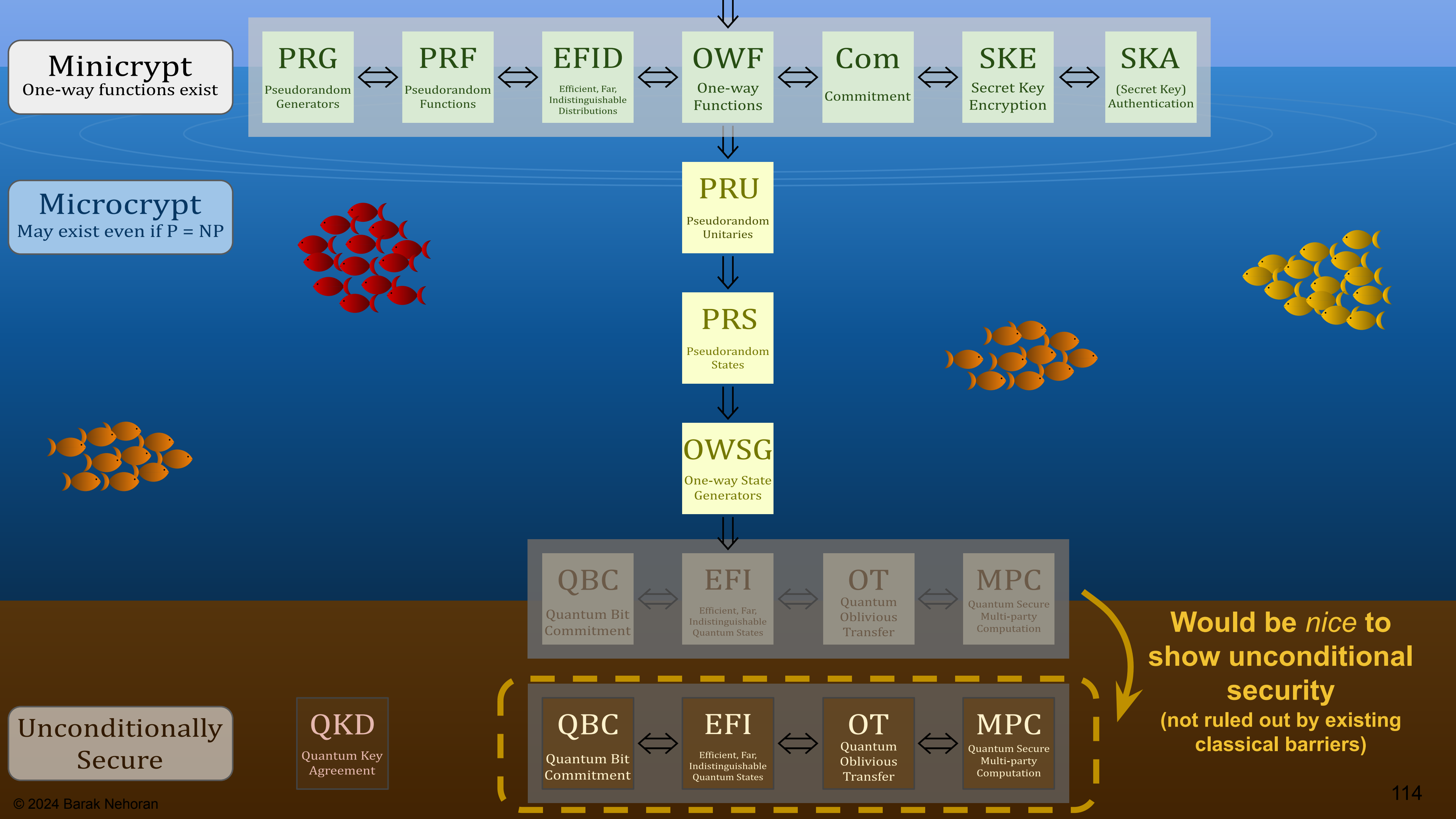
Quantum Key Agreement



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Would be *nice* to show unconditional security
(not ruled out by existing classical barriers)

# Unconditionally secure commitments with quantum auxiliary inputs/preprocessing

Barak Nehoran, Tomoyuki Morimae, Takashi Yamakawa

Princeton University; Yukawa Institute for Theoretical Physics, Kyoto University; NTT Social Informatics Laboratories

merged with

**Luowen Qian**

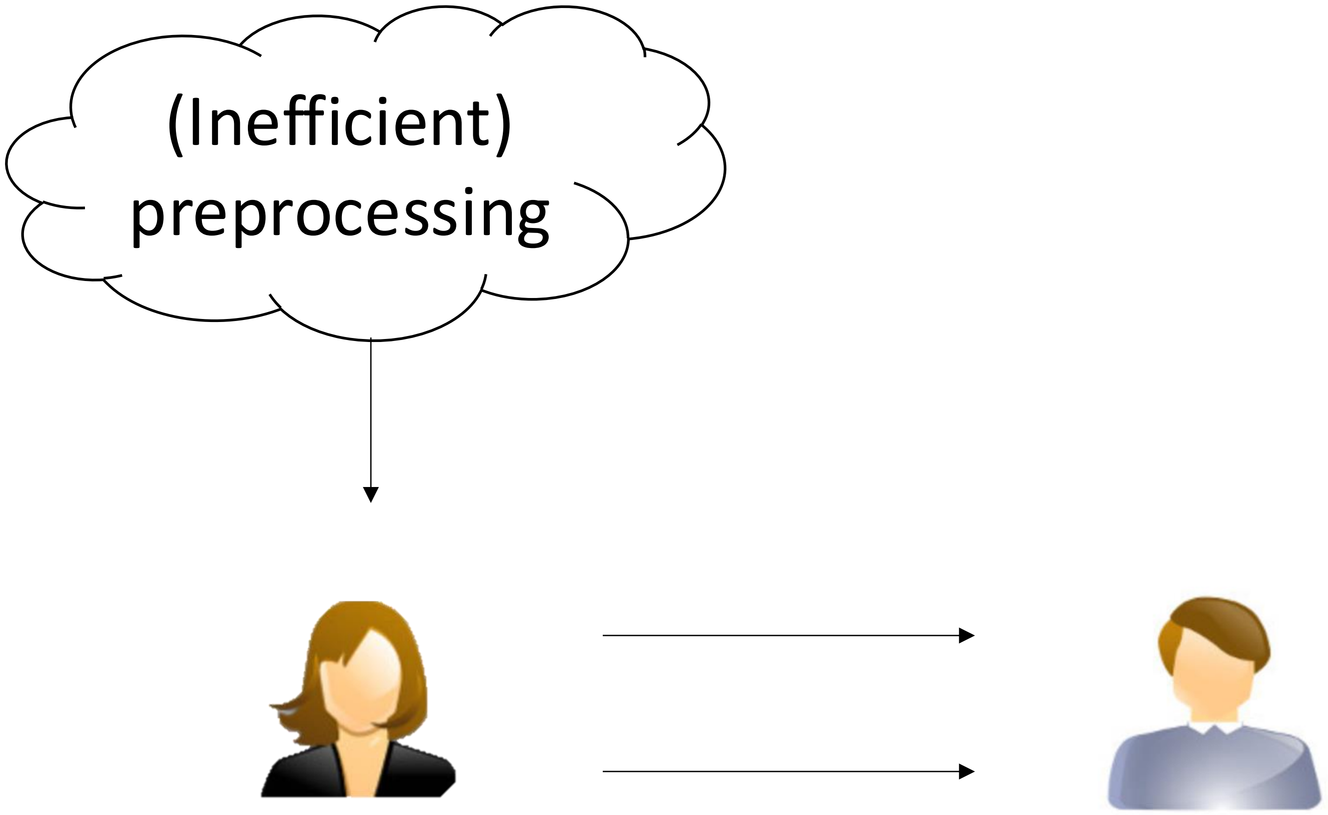
Boston University → NTT Research, Inc.

# Auxiliary-input (non-uniform) commitment

[Ostrovsky-Widgerson'93, …]

# Auxiliary-input (non-uniform) commitment

[Ostrovsky-Widgerson'93, …]



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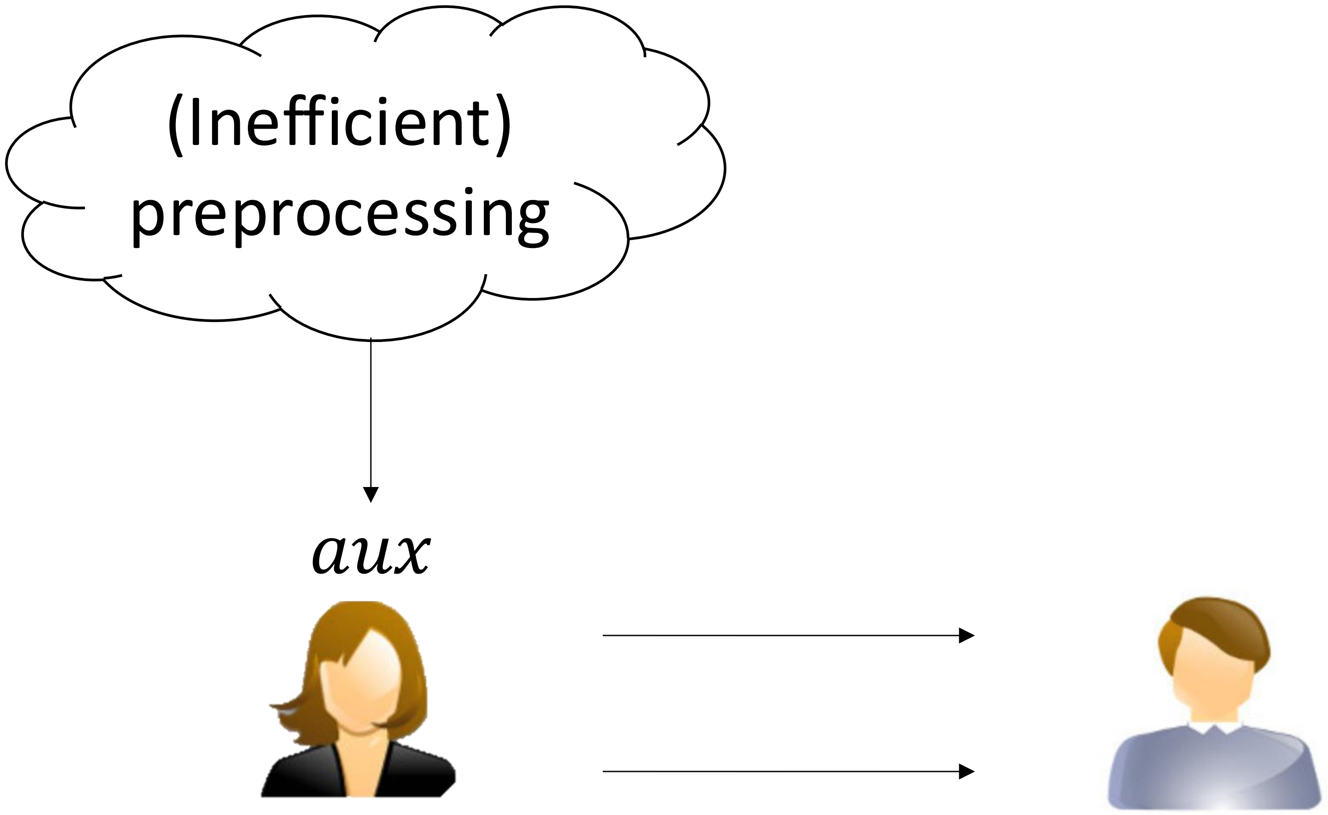
[Ostrovsky-Widgerson'93, …]



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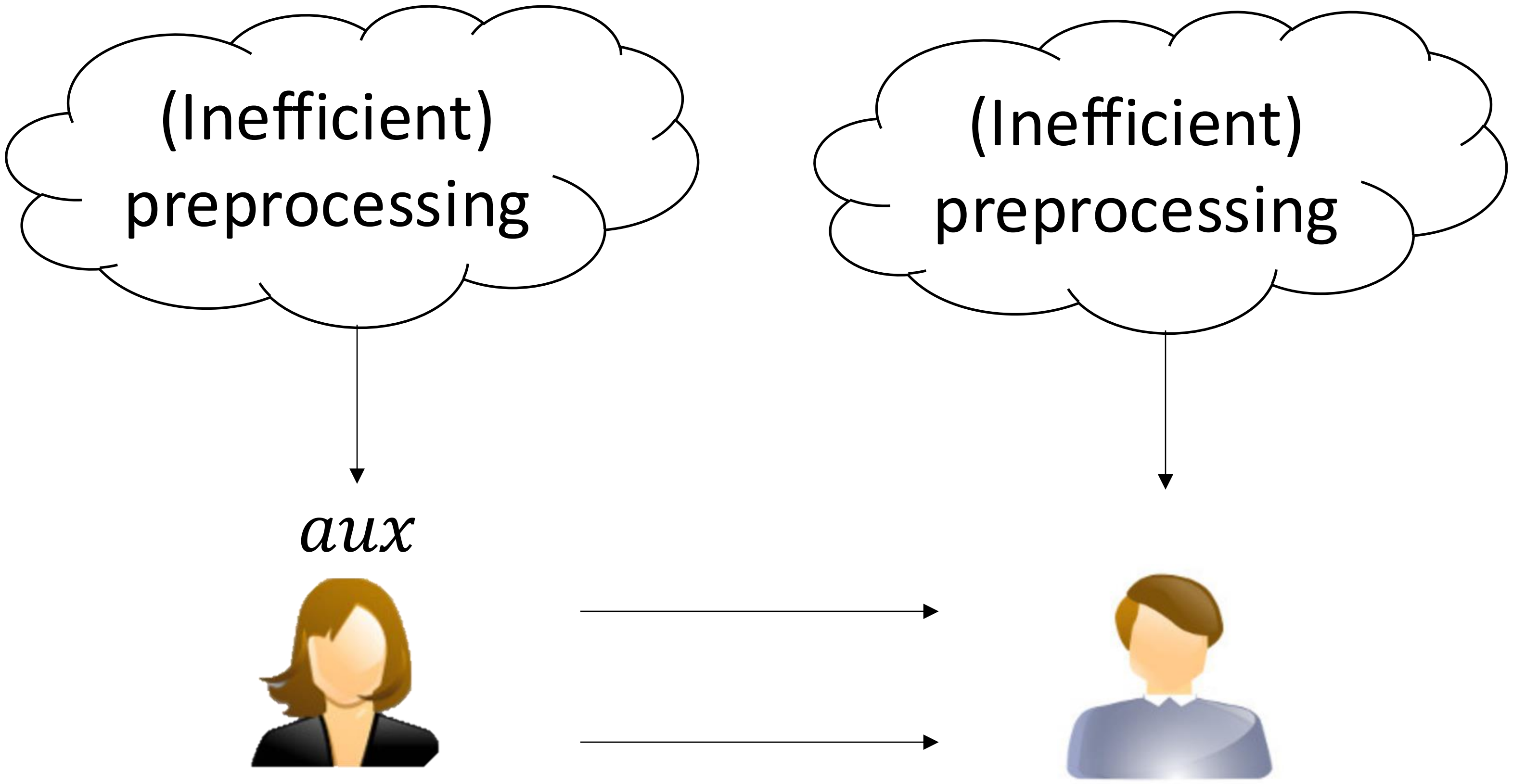
[Ostrovsky-Widgerson’93, …]



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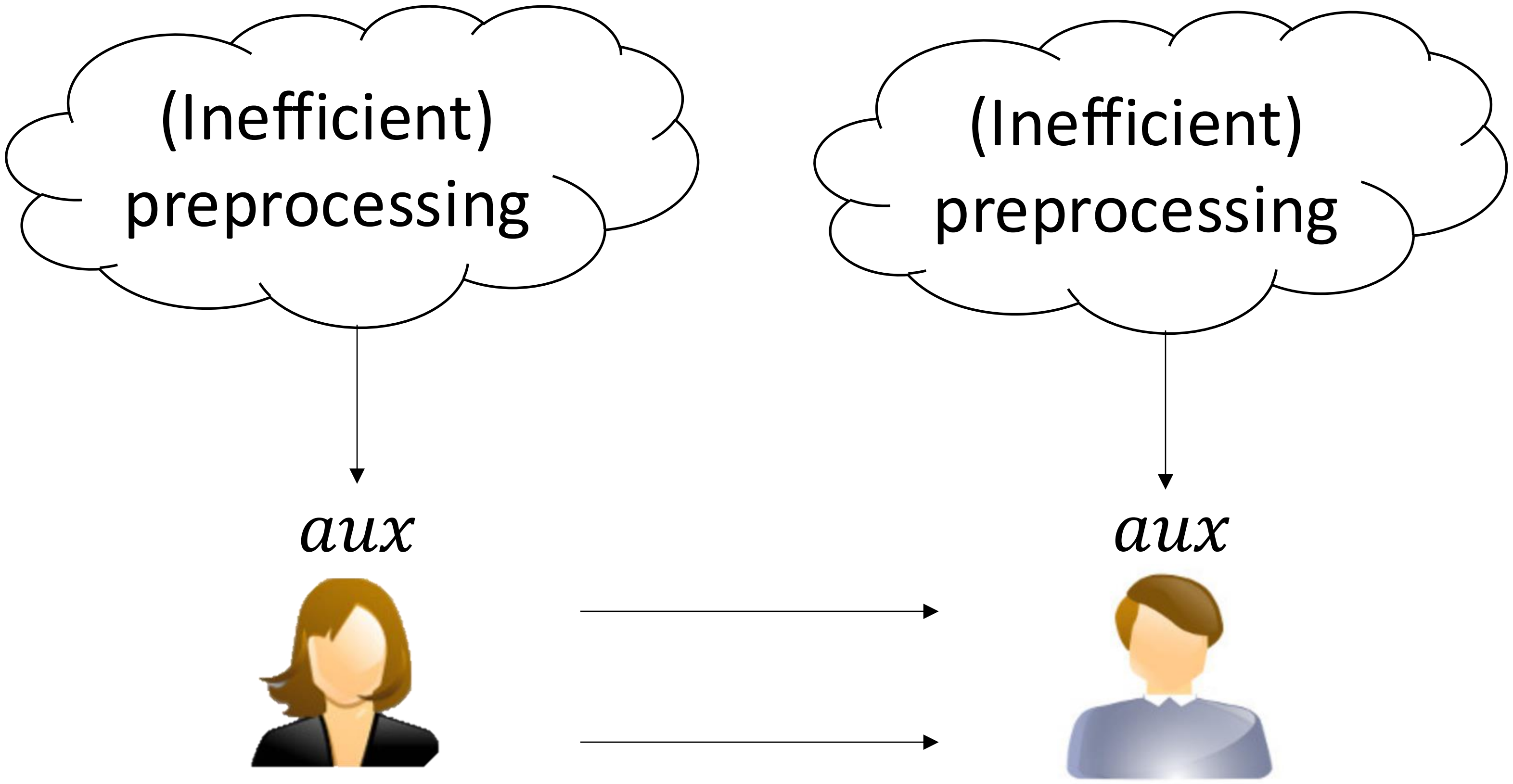

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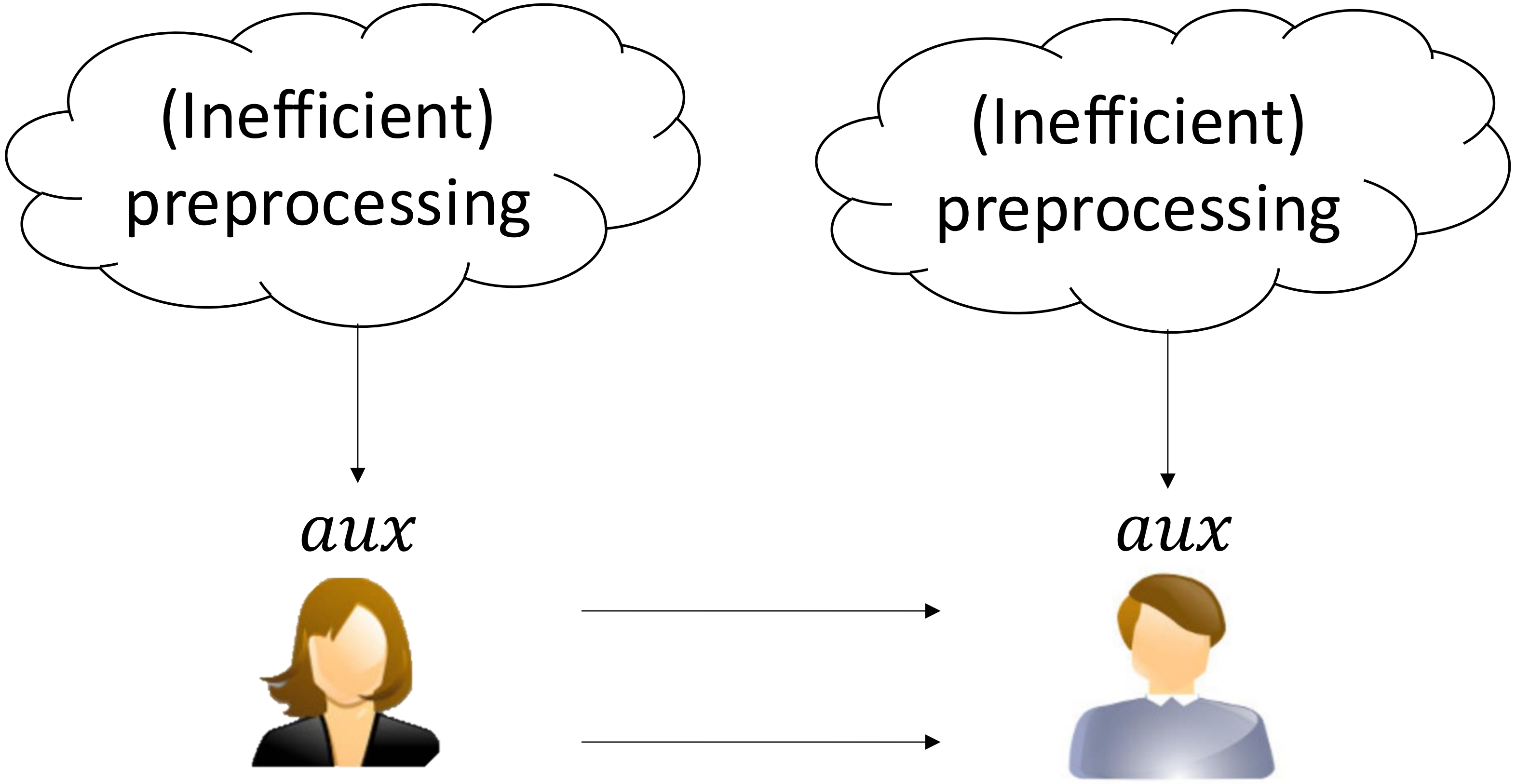


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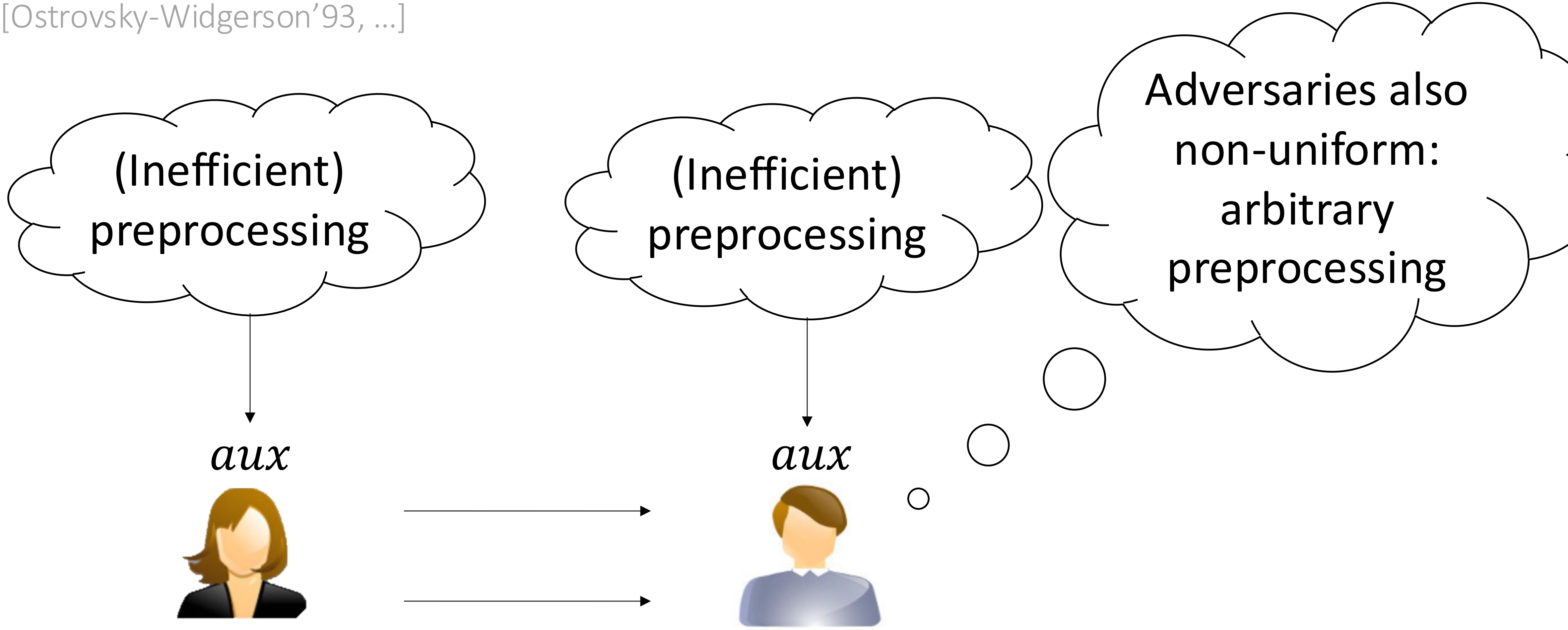
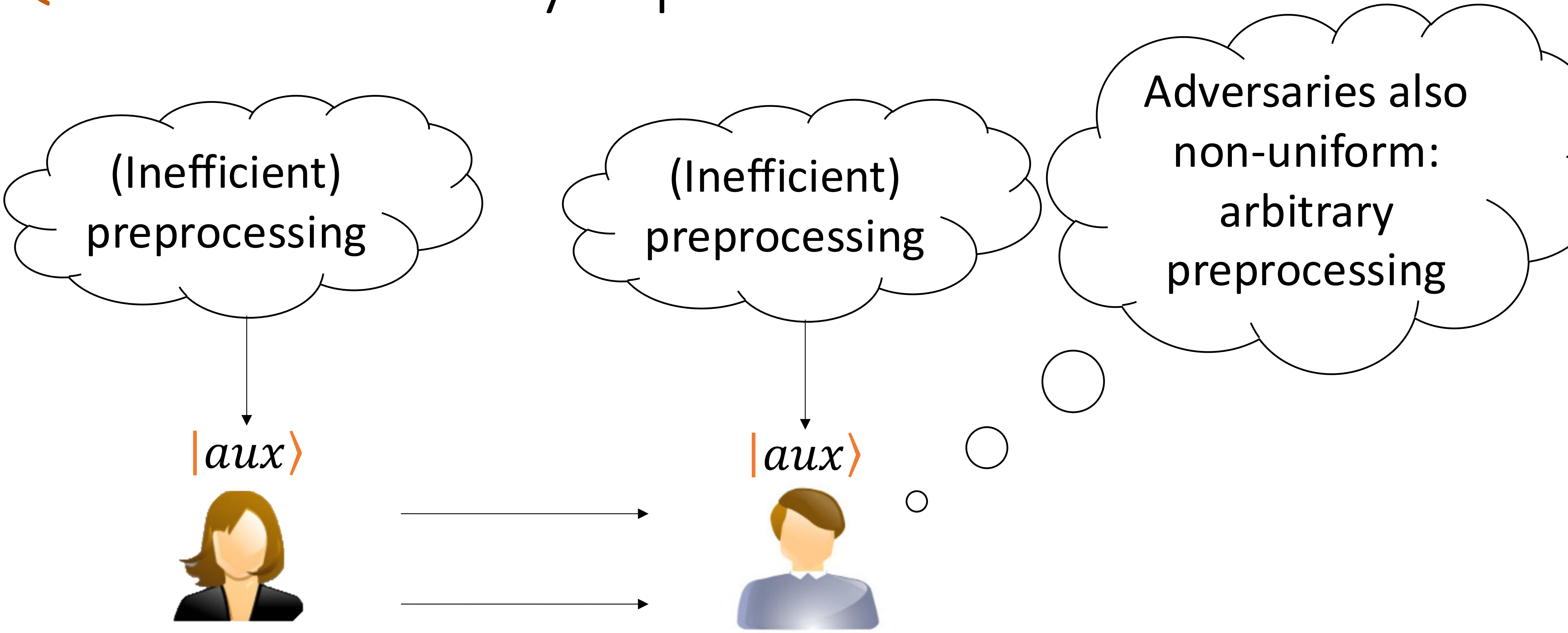



(“$\mathrm{P} \stackrel{?}{=} \mathrm{NP}$” barrier still applies)

# Quantum auxiliary-input commitment



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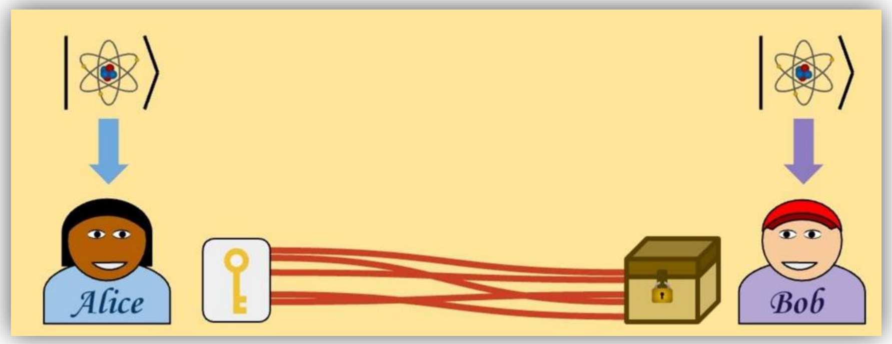

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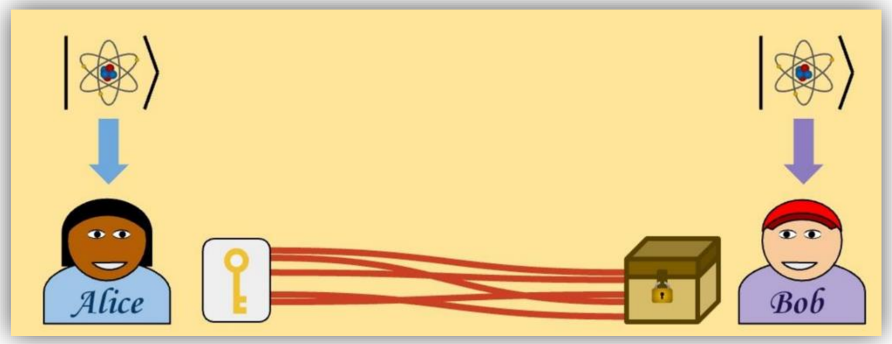


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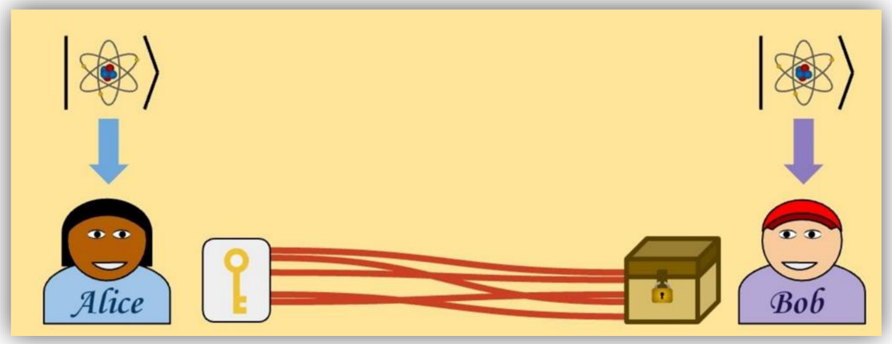


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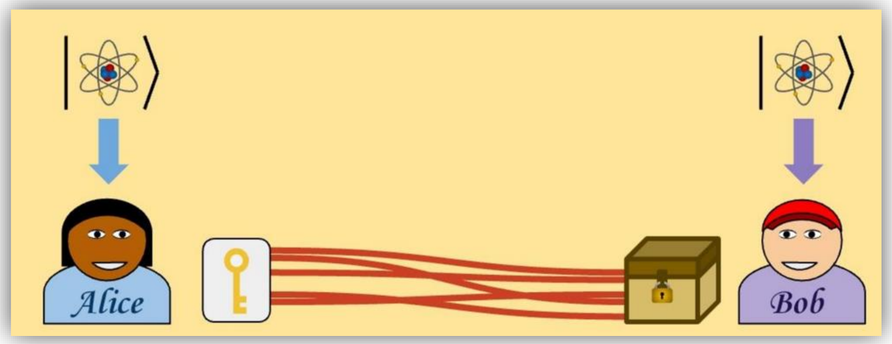


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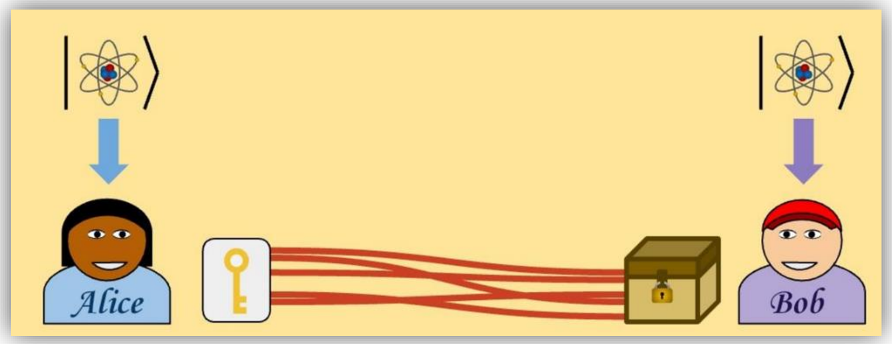


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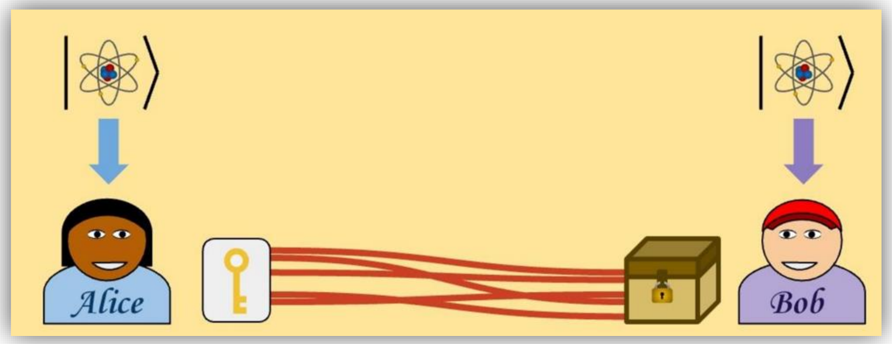


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❖ Preprocessing time can be reduced to single exponential either with communication or assuming $\mathrm{BQP} = \mathrm{QMA}$

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Well, you could pick a smaller security parameter... (48? so that preprocessing time is at most 2 years)

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(see papers for details)

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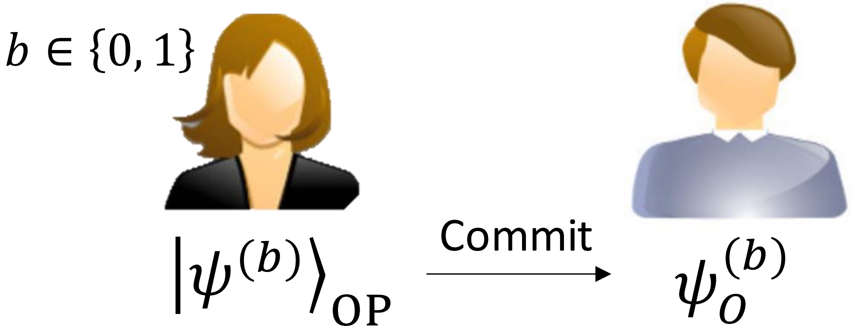

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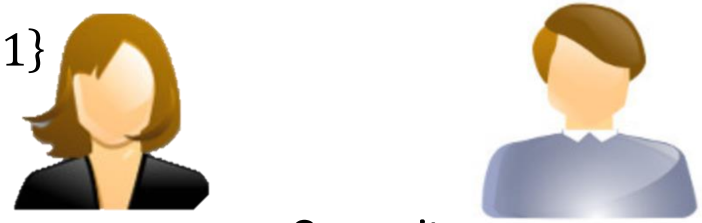

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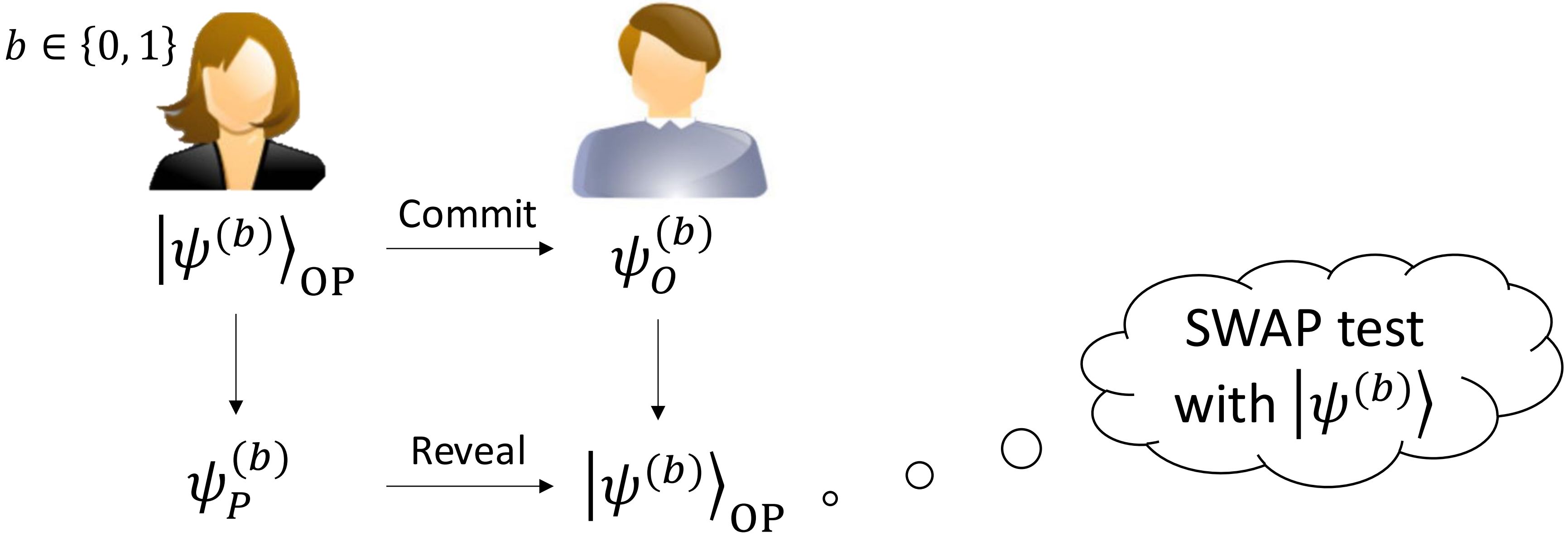

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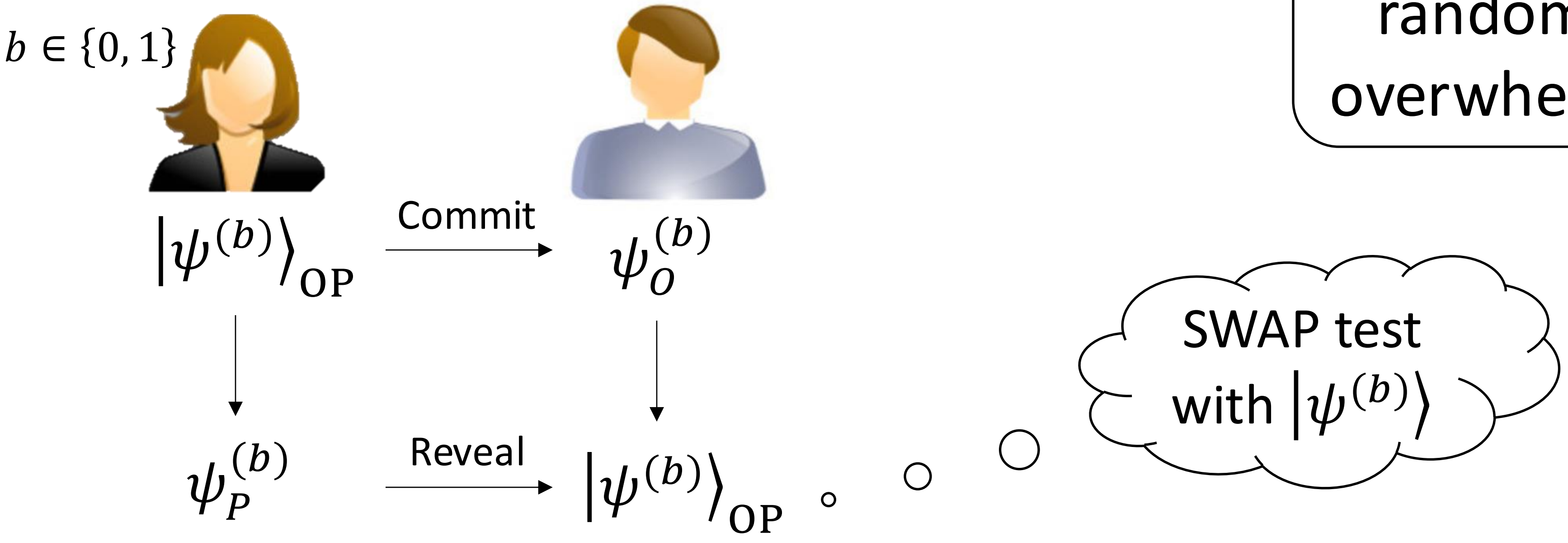

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**Goal**: Prove $H(x)$ is indistinguishable from random $y$ even against non-uniform adversary

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- There are $\exp(2^S)$ approximately-different quantum states of size $S$

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   - Similar idea but use a matrix Hoeffding bound for operator norm
   - Less general but better security: $\sqrt{6S/2^\lambda}$ instead of $12\sqrt[3]{S/2^\lambda}$ asymptotically matches classical attack $\Omega\left(\sqrt{S/2^\lambda}\right)$

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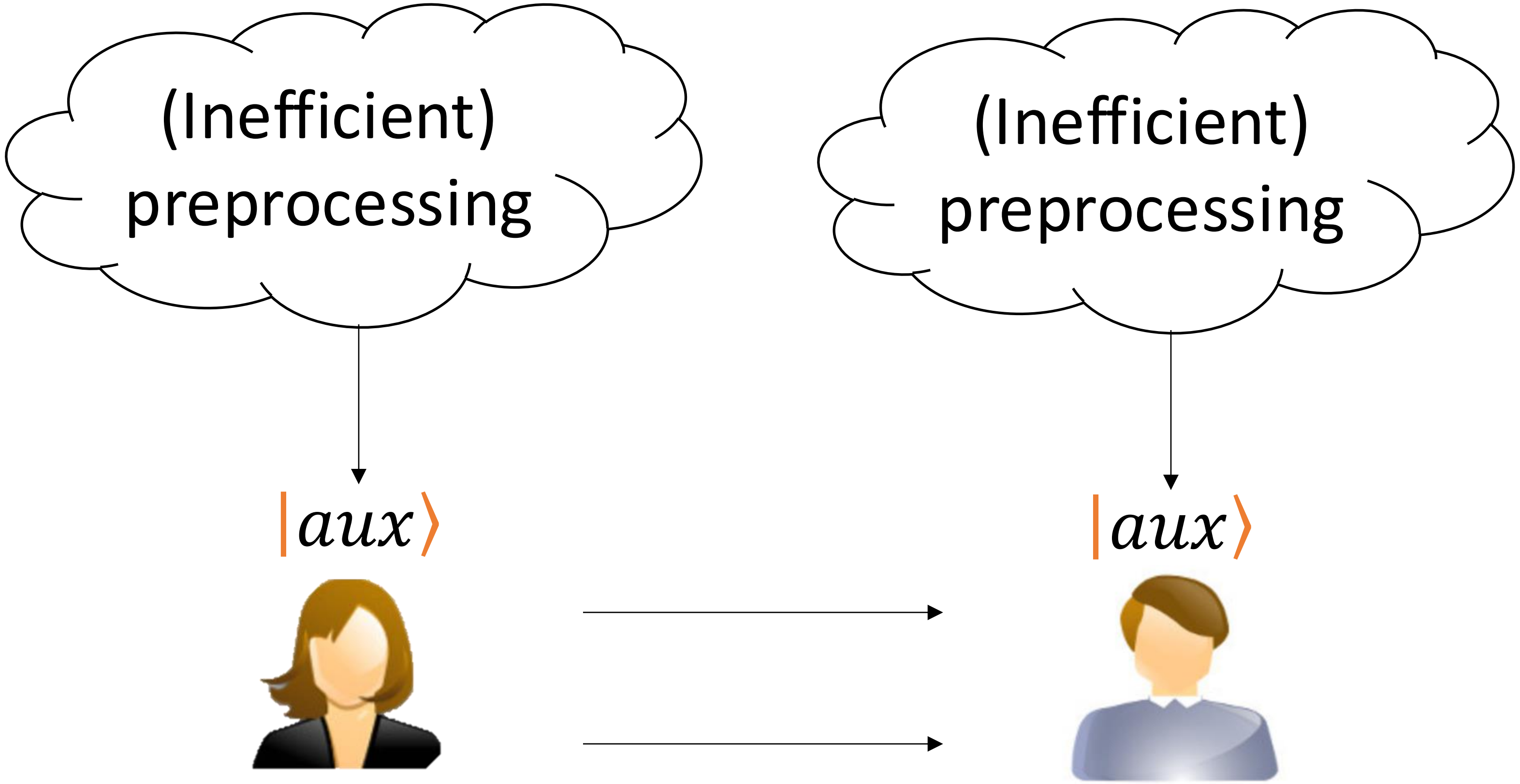

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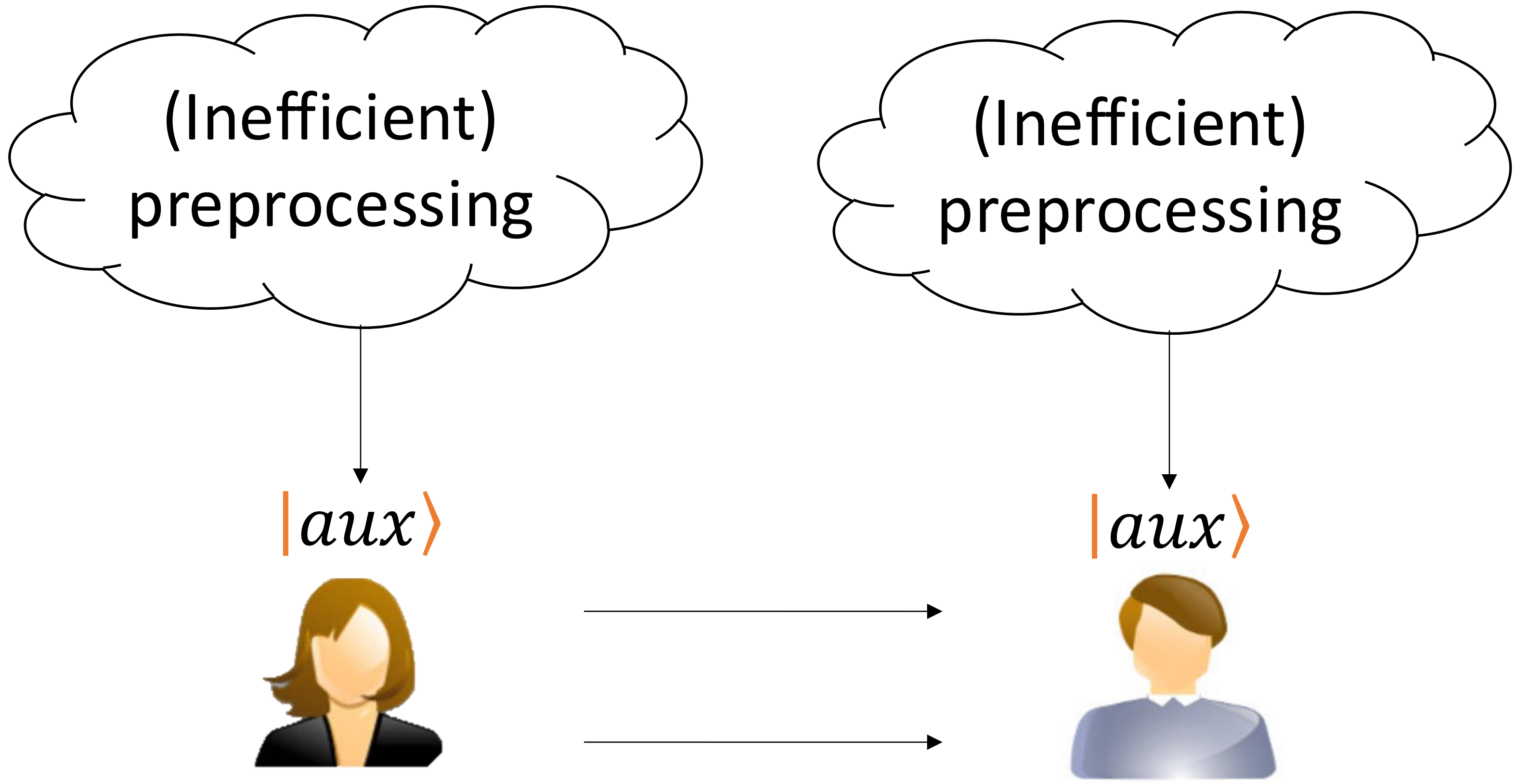


Unconditionally possible!

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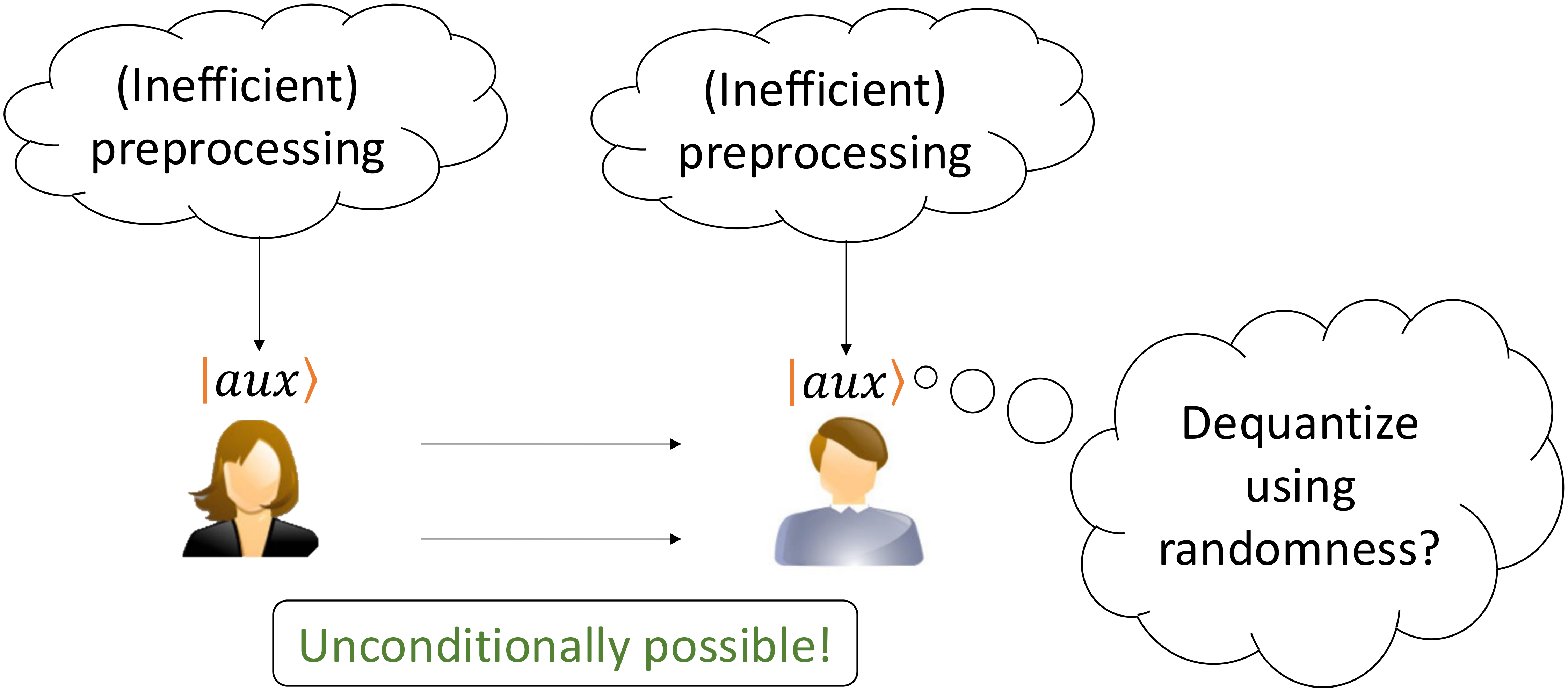

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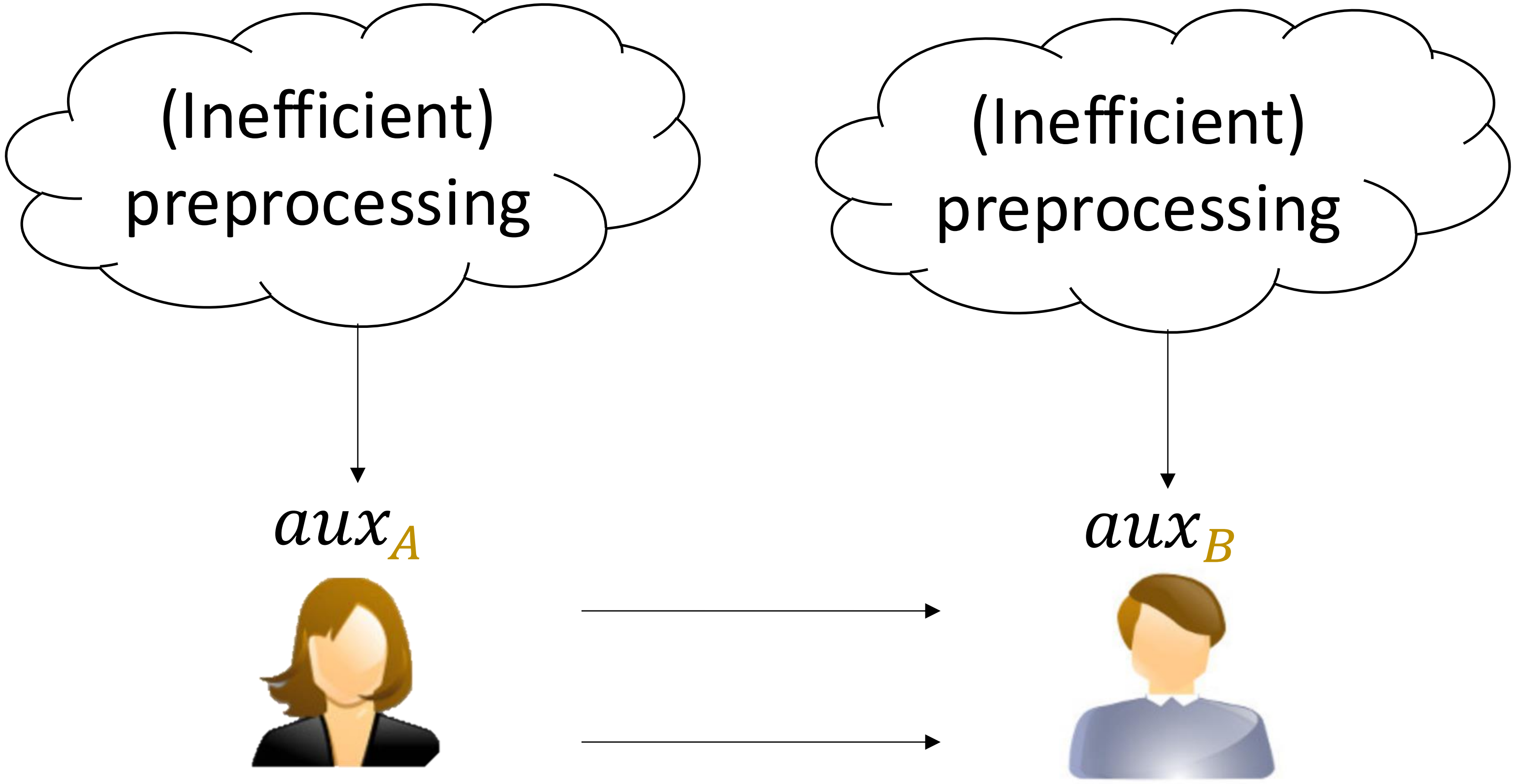

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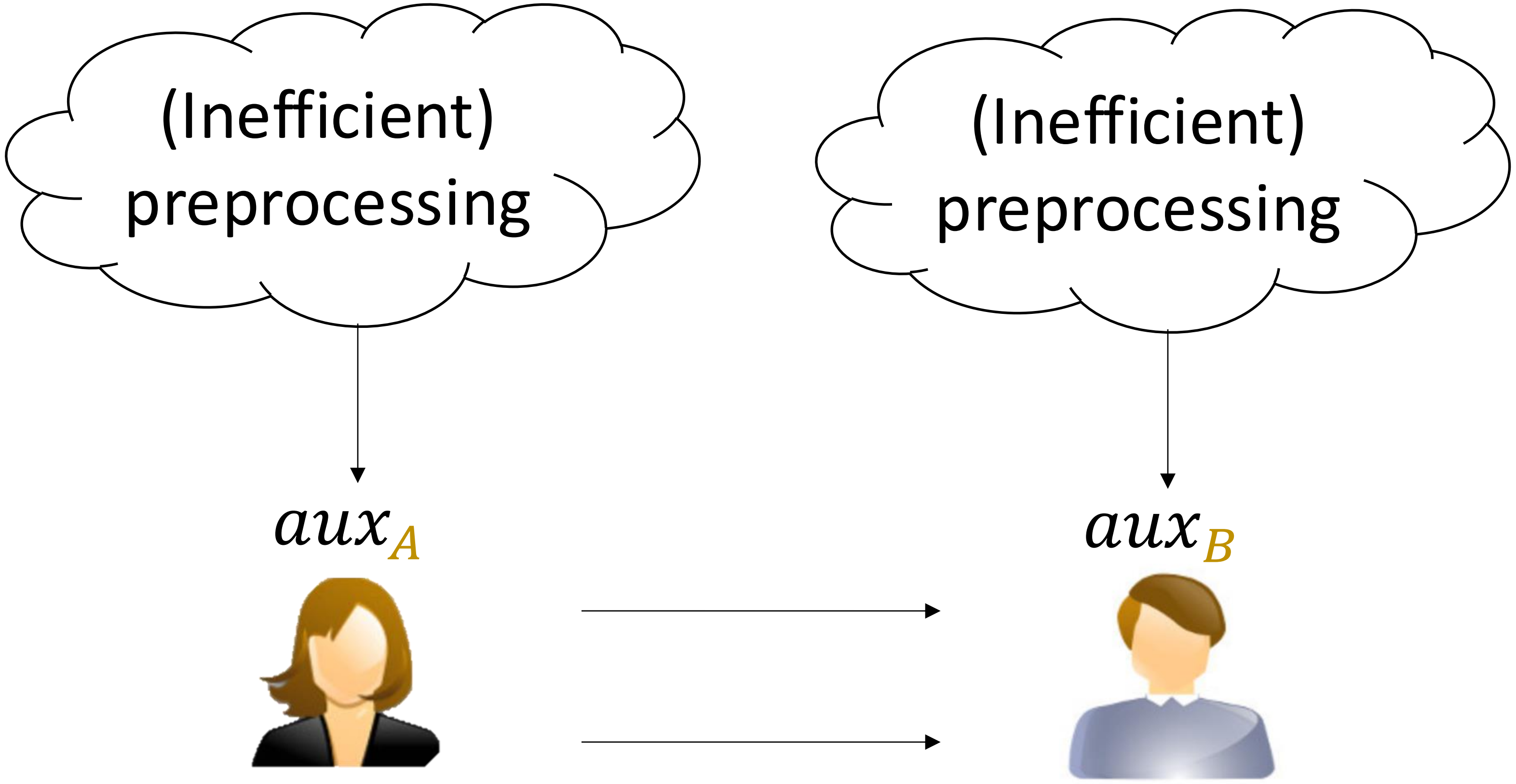
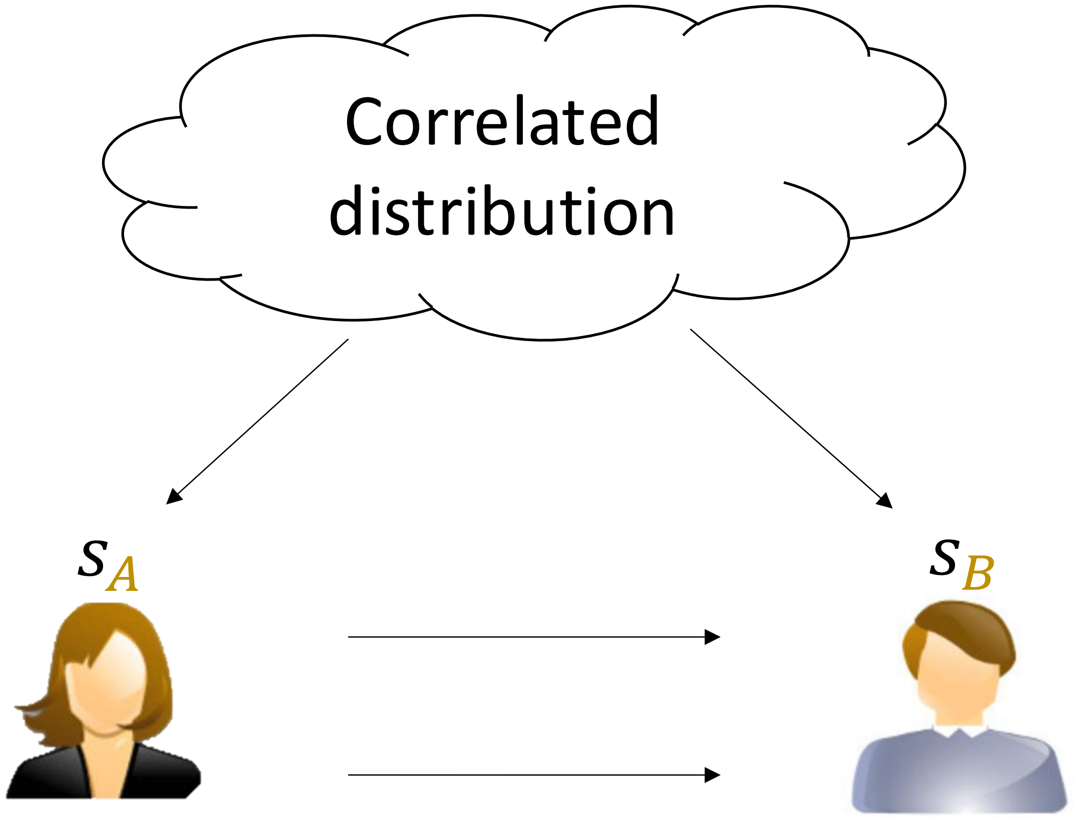



Q24: classical commitments with sampling oracles $\Rightarrow \mathrm{P} \neq \mathrm{NP}$

# Secret parameter model (trusted preprocessing)



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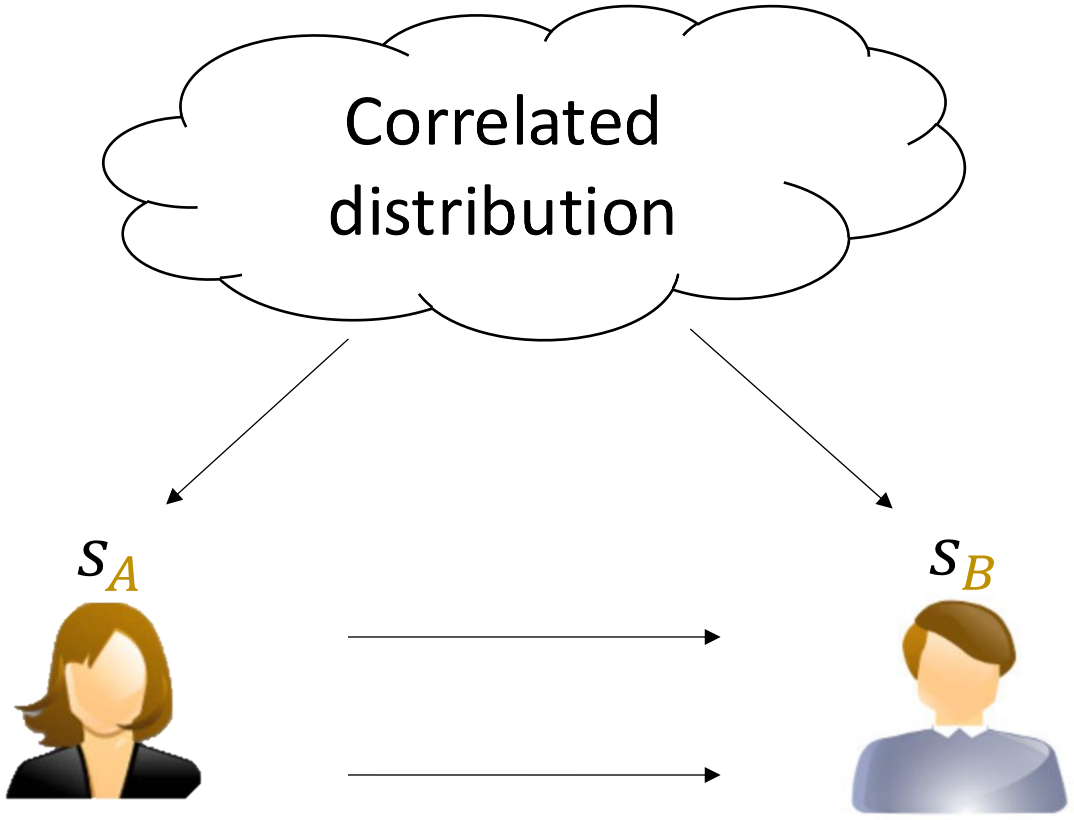
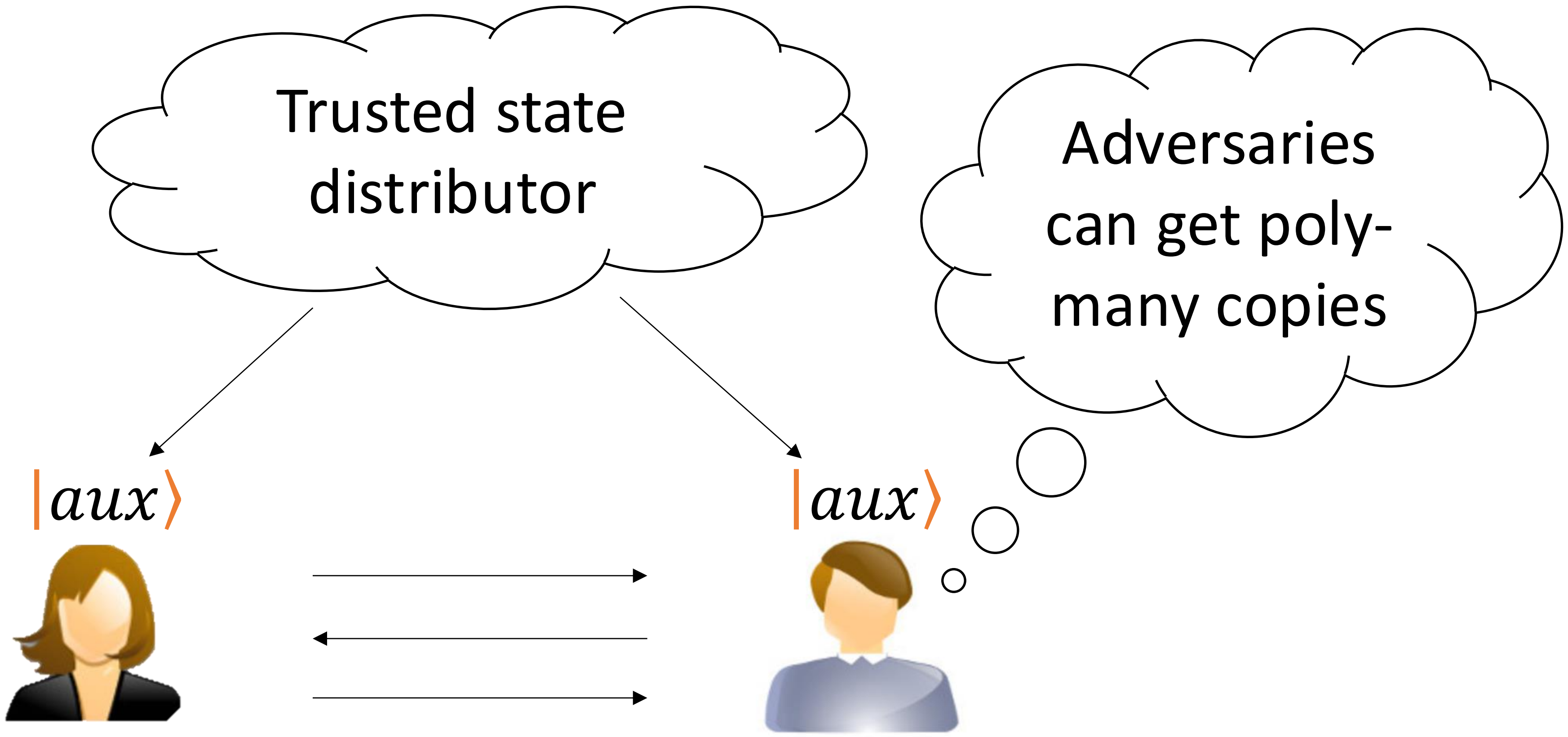



Statistical (even classical) commitment is possible

MNY24: commitments impossible if correlation < .127

# (Unclonable) common reference quantum state model



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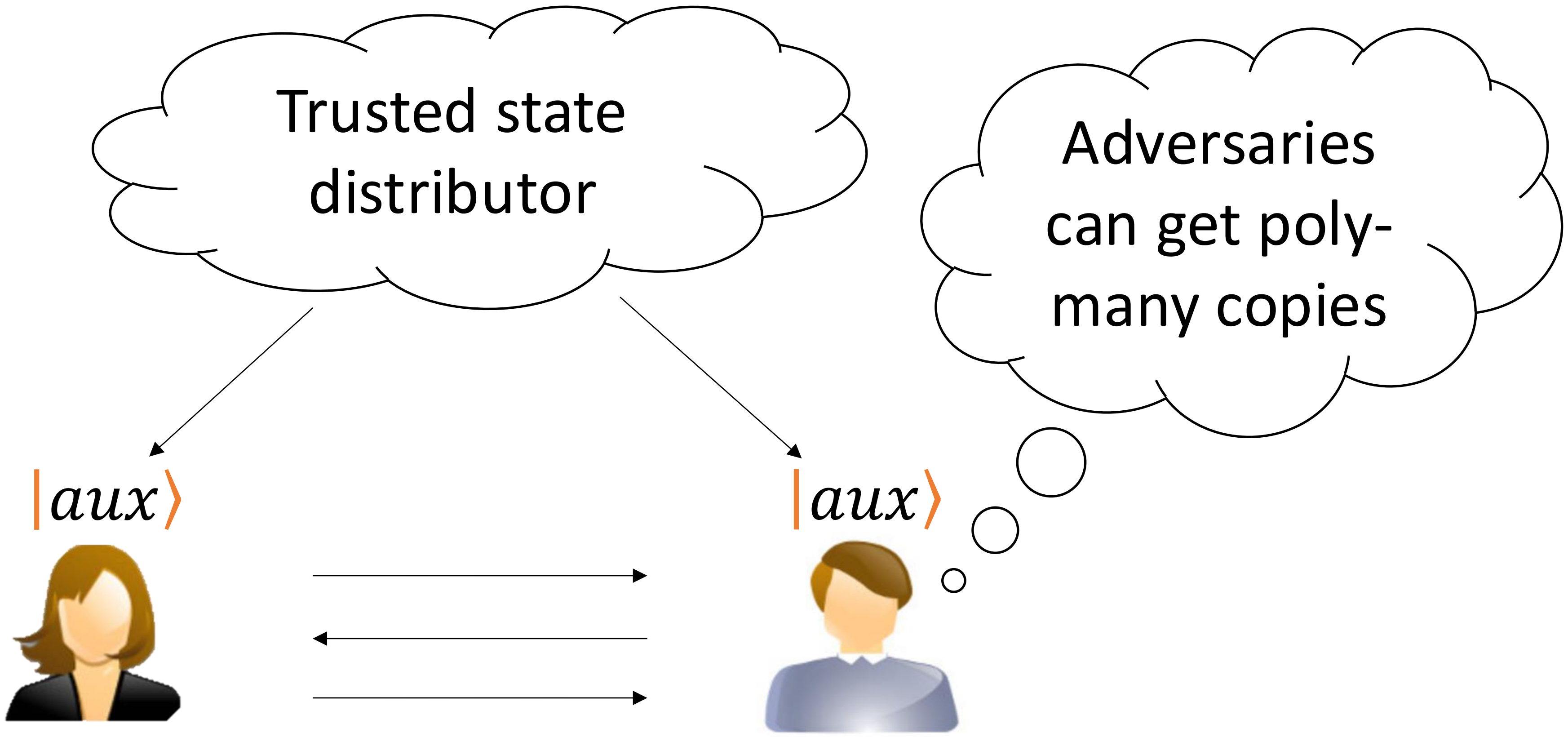


Statistically secure <u>completely-efficient</u> commitments is possible in this model
Q24: impossible in the common reference classical distribution model

# Conclusions

Image: Wikipedia

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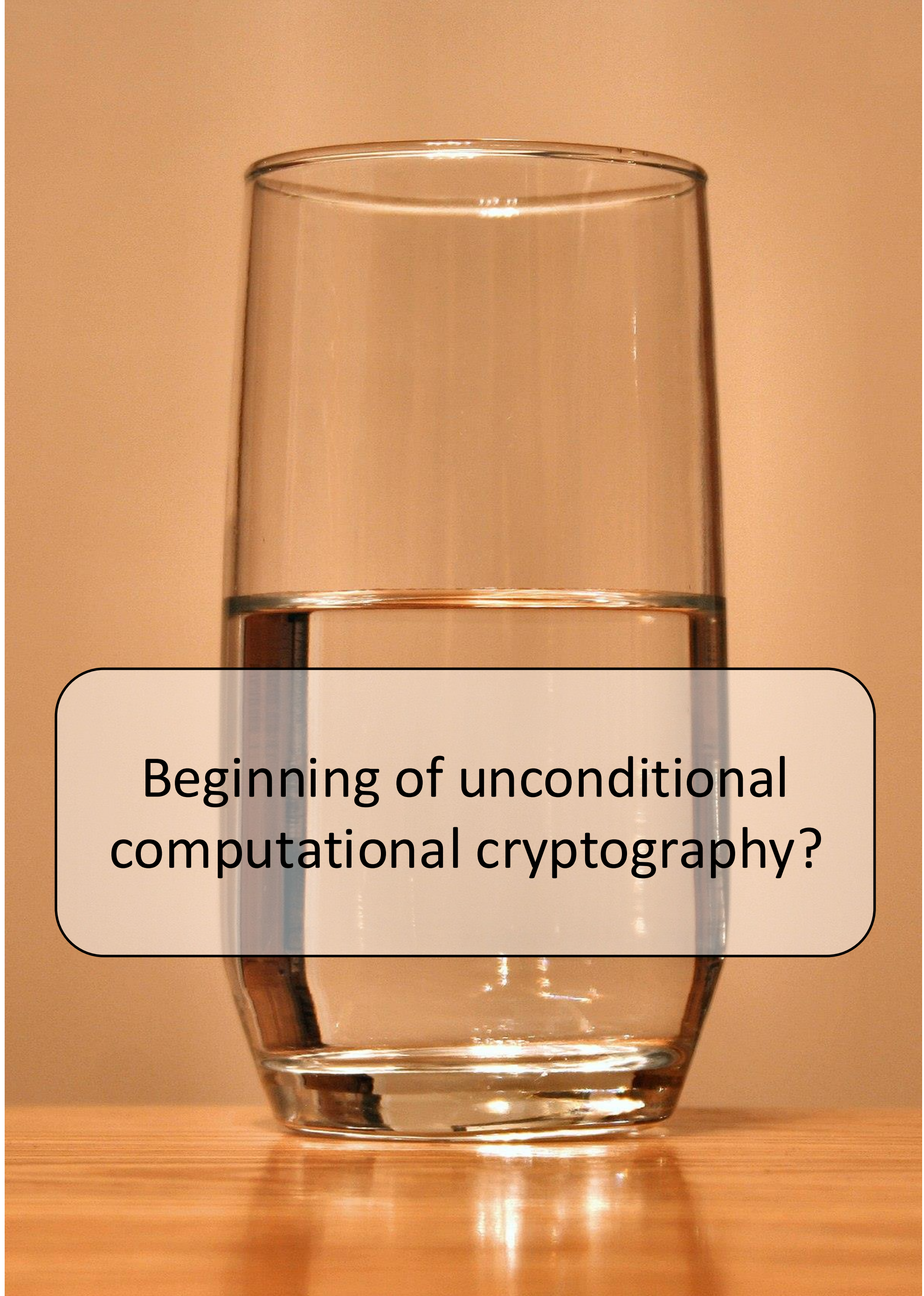


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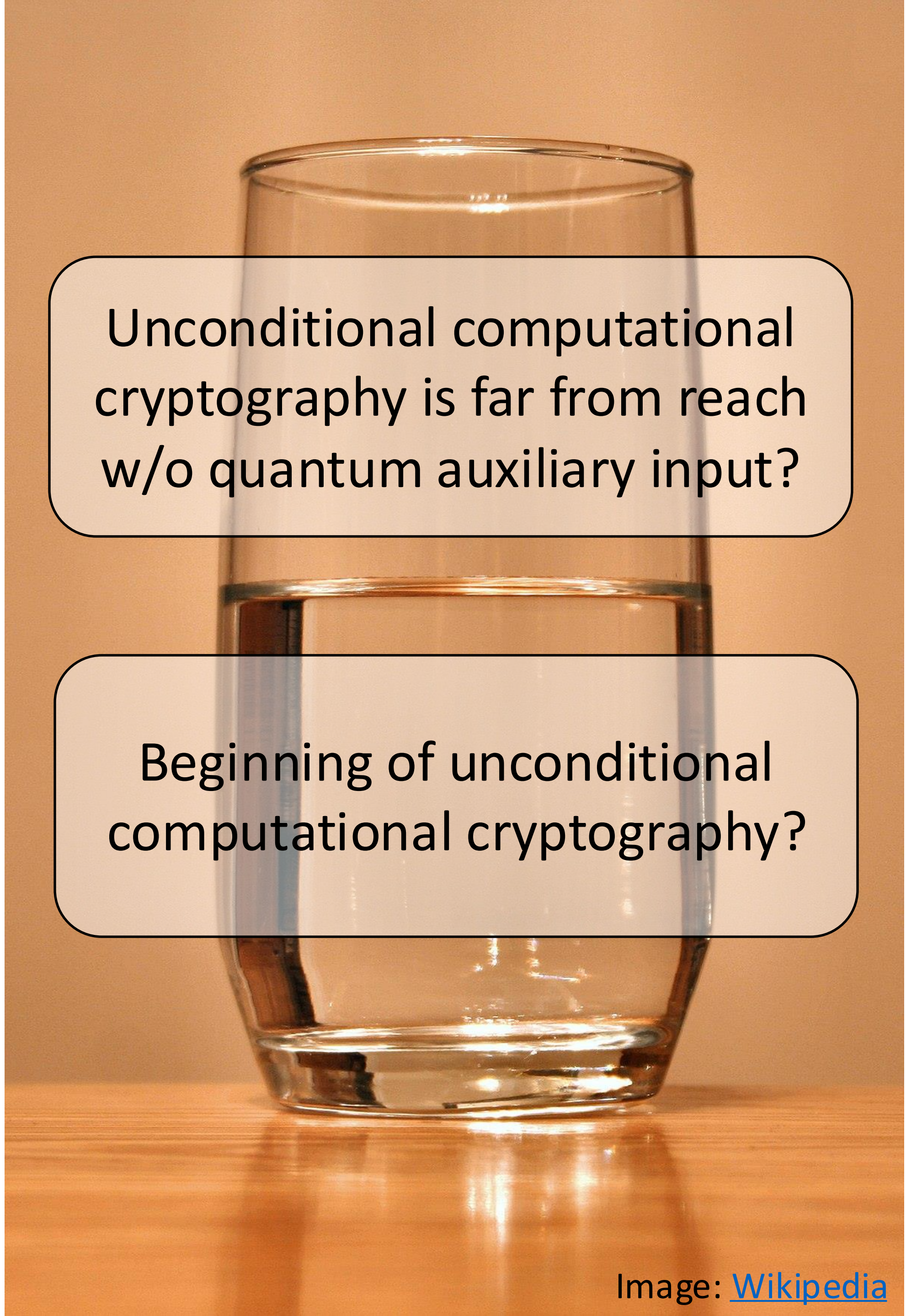

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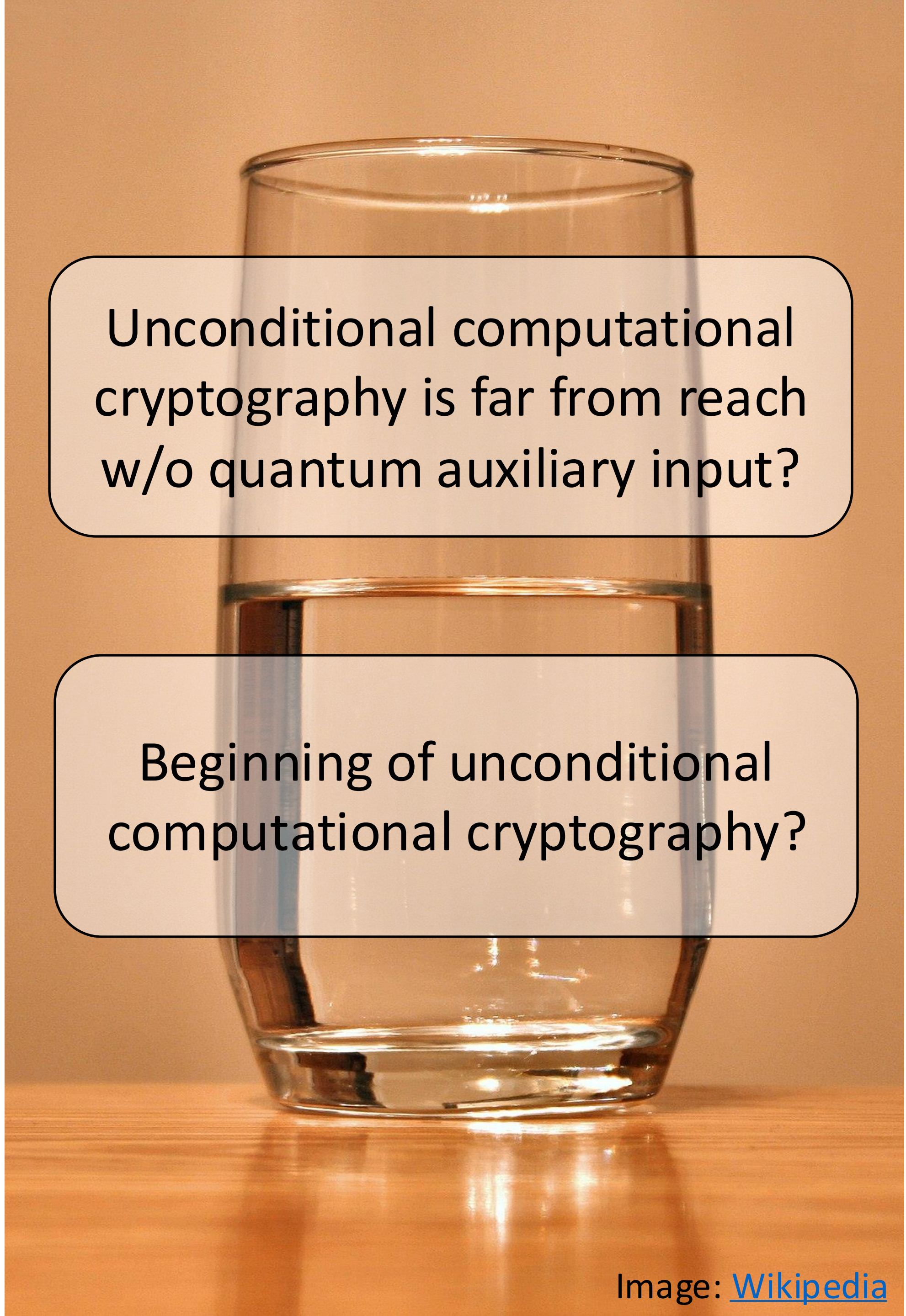

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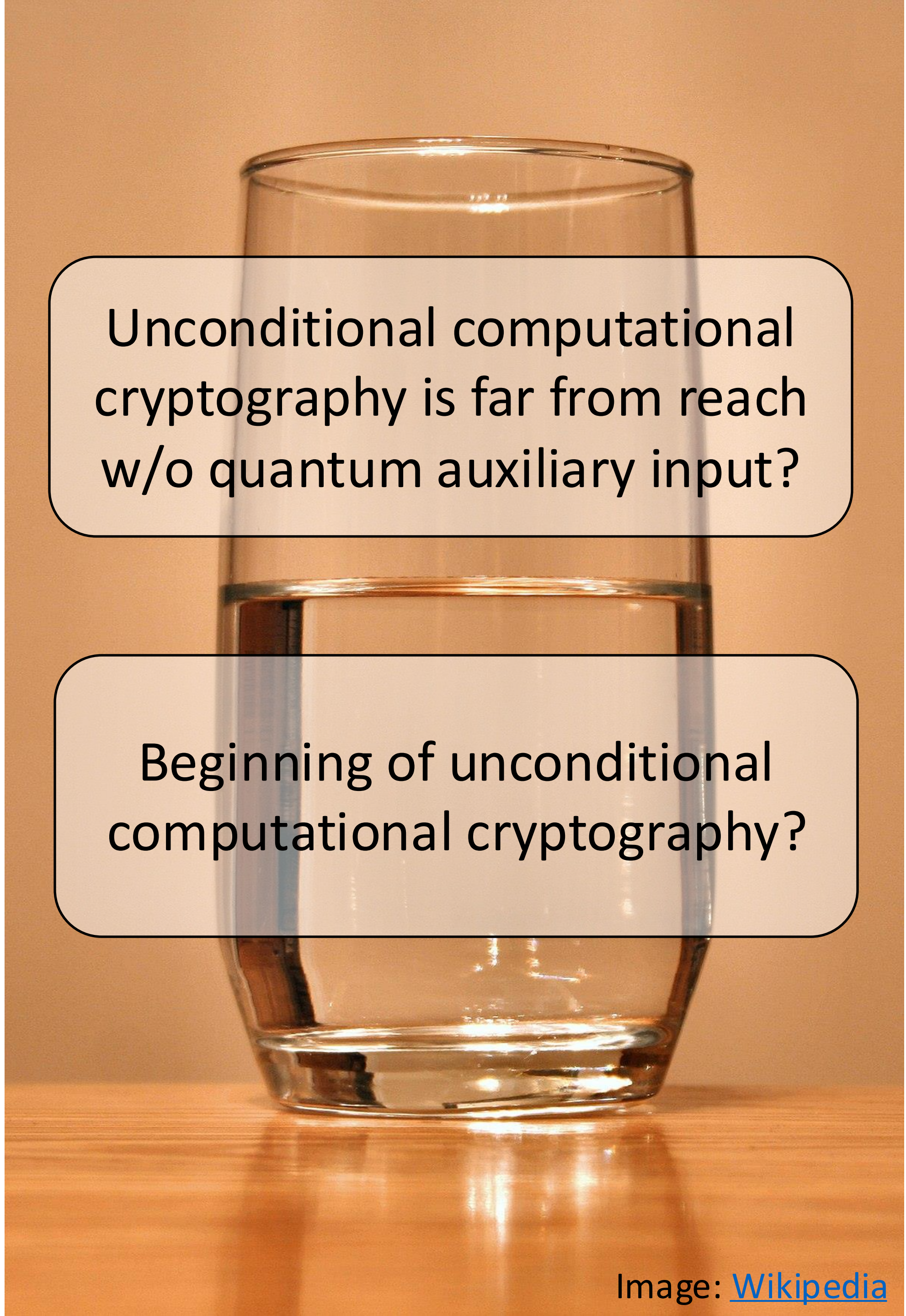
Thank you! Questions?



Unconditional computational cryptography is far from reach w/o quantum auxiliary input?

Beginning of unconditional computational cryptography?

Image: Wikipedia