Generic MitM Attack Frameworks on Sponge Constructions

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Outline

Hash Function

Hash Function

A cryptographic hash function H maps a message M of arbitrary length into a short fixed-length h-bit target T.

Security Properties

- Preimage resistance: given T, find x such that $H(x) = T$ by querying at least 2^h H.
- Second preimage resistance: given x, find $x' \neq x$ such that $H(x) = H(x')$ by querying at least 2^h H.
- Collision resistance: find $x \neq x'$, such that $H(x) = H(x')$ by querying at least $2^{h/2}$ H.

Application

Signatures $\left(Sign_{key}(H(m))\right)$, Block Chain, Integrity $(H(m))$, MAC $(H(key, m))$, $\cdots$

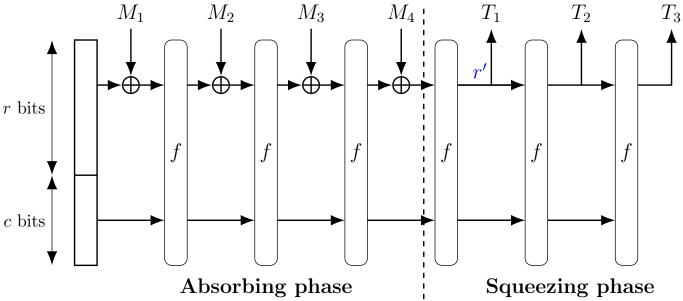
Secure Hash Algorithm-3 (SHA-3)

In 2004-2005, several cryptographic hash algorithms were successfully attacked, like MD5 and SHA-1. Hence, NIST held the SHA-3 competition in 2007.

Timeline

- 2008/10: 64 algorithms were submitted, and 51 algorithms were selected as the first-round candidates.
- 2009/07: 14 algorithms were selected as the second-round candidates.
- 2010/12: 5 third-round candidates: BLAKE, Grøstl, JH, Keccak and Skein.
- 2012/10: Keccak was selected as the winner.
- 2015/08: Keccak was standardized as SHA-3.

The sponge construction of SHA-3

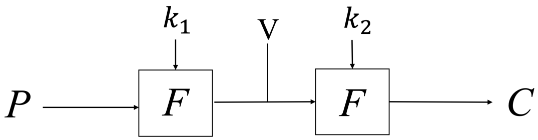


- b-bit Keccak-f permutation, with r-bit outer part (rate) and c-bit inner part (capacity).
- **Absorbing phase**: Given message is padded and divided into several r-bit blocks, i.e., M_i. Each M_i is XOR-ed into the outer part.
- **Squeezing phase**: Output h-bit digest $T_1 \| T_2 \| \ldots$, $h = 224, 256, 384, 512$.

Outline

Meet-in-the-Middle (MitM) Attack

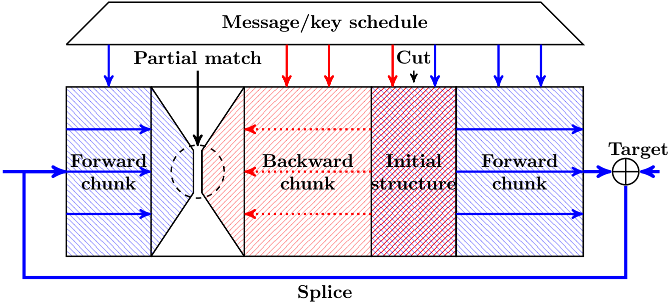
- MitM was first introduced by Diffie and Hellman in 1977 to attack Double-DES.
- Example: $C = E_K(P) = F_{k_2}(F_{k_1}(P))$, $K = k_1 \| k_2$.
  - Neutral sets: k_1 and k_2 are independent of each other.
  - Match: $F_{k_1}(P)$ and $F^{-1}_{k_2}(C)$.
- Time complexity: $2^{|k_1|+|k_2|} \rightarrow 2^{|k_1|+|k_2|-n}$.



- Enhanced techniques: splice-and-cut, initial structure, automated tools, $\cdots$.
- Application to MD constructions: MD4, MD5, SHA-1, Whirpool, AES-MMO, Simpira-DM, $\cdots$.

Splice-and-Cut MitM Attack Framework on MD Hash Functions

- At SAC 2008, Aoki and Sasaki proposed the splice-and-cut technique [AS08].



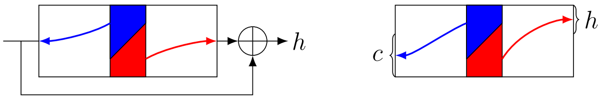
- Splice-and-Cut
- Initial Structure
- Partial Matching

1. For $2^{d_{\mathcal{R}}}$ values of ■, compute backward to the matching points and store them in L_1.
2. For $2^{d_{\mathcal{B}}}$ values of ■, compute forward to the matching points and store them in L_2.
3. Find m-bit partial match between L_1 and L_2.

- Time complexity: $Time = 2^{h-(d_{\mathcal{R}}+d_{\mathcal{B}})} \cdot (2^{\max(d_{\mathcal{R}},d_{\mathcal{B}})} + 2^{d_{\mathcal{R}}+d_{\mathcal{B}}-m}) \simeq 2^{h-\min(d_{\mathcal{R}},d_{\mathcal{R}},m)}$

The Limitation of MitM Attack on Sponge Construction

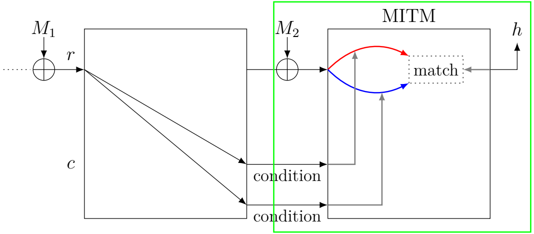
Open problem: How to mount an MitM attack on sponge constructions, like SHA-3, Ascon?



(a) MITM on DM (b) MITM on Sponge

- For DM hashing mode,
  - MitM attack starts at an n-bit internal state in the middle.
  - Two independent chunks meet at the matching point to filter the wrong internal states through the given h-bit target.
  - If $n > h$, at most 2^h internal states are searched to find the preimage.
- For sponge construction, if MitM attack starts at an internal state in the middle,
  - h-bit target in forward computation and c-bit inner part in backward computation should both be satisfied.
  - The search space is 2^{h+c} (preimage security bound usually $\leq 2^h$).

Conditional MitM Attack (EUROCRYPT 2023)



- Two independent neutral sets are divided from the starting state M_2.
- Some conditions determined by M_1 are set to reduce the diffusion of ■ and ■ bits.
  - For the non-linear operation $\chi : b_i = a_i \oplus (a_{i+1} \oplus 1) \cdot a_{i+2}$.
  - If (a_i, a_{i+1}) is (■, ■), then b_i depends on both ■ and ■.
  - If $a_{i+2} = 0$, then b_i only depends on a_i ■.
- Compute backward with the known h-bit target to derive an m-bit matching.

[QHD+23] Lingyue Qin, Jialiang Hua, Xiaoyang Dong, Hailun Yan, Xiaoyun Wang: Meet-in-the-Middle Preimage Attacks on Sponge-Based Hashing. EUROCRYPT 2023

Time Complexity of Conditional MitM Attack

- After finding one proper M_1 satisfying all bit conditions, an MitM episode is performed as follows:
  1. For each of $2^{d_{\mathcal{R}}}$ ■, compute forward to the matching point.
  2. For each of $2^{d_{\mathcal{B}}}$ ■, compute forward to the matching point.
  3. Given the h-bit target, compute backward to derive an m-bit matching point.
  4. Filter states.
- The complexity of one MitM episode is $2^{\max(d_{\mathcal{R}},d_{\mathcal{B}})} + 2^{d_{\mathcal{R}}+d_{\mathcal{B}}-m}$

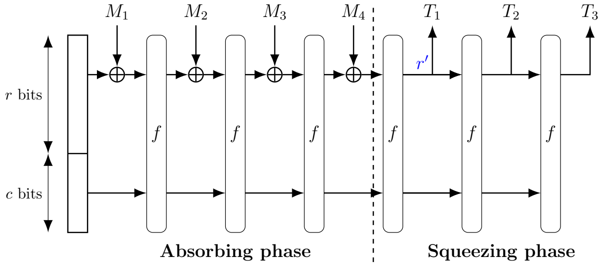
Time complexity

In order to find a h-bit target preimage, the episode should be repeated $2^{h-(d_{\mathcal{R}}+d_{\mathcal{B}})}$ times,

$$Time \simeq C + 2^{h-(d_{\mathcal{R}}+d_{\mathcal{B}})} \times \left(2^{\max(d_{\mathcal{R}},d_{\mathcal{B}})} + 2^{d_{\mathcal{R}}+d_{\mathcal{B}}-m}\right) = C + 2^{h-\min\{d_{\mathcal{R}},d_{\mathcal{B}},m\}}$$

where C is the time complexity to find M_1.

Tight Preimage Security Bound of Sponge Construction [LM22]



- For SHA-3, $h = c/2$, the general bound of preimage attack is 2^h.
- For other sponge constructions, like Ascon-Hash, the general bound was proved to be $\min\{\max\{2^{h-r'}, 2^{c/2}\}, 2^h\}$ [LM22].

How to Attack General Sponge Construction

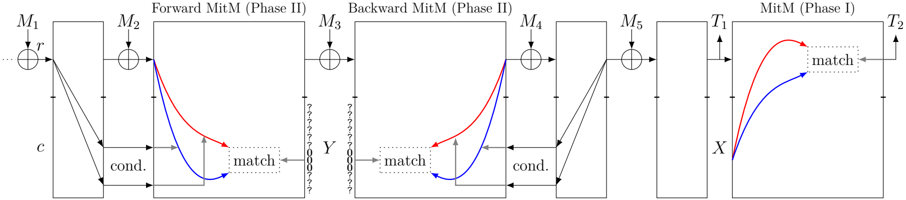
- E.g., SPHINCS$^+$-Haraka, with $b = 512, h = c = r = r' = 256$, then $\min\{\max\{2^{h-r'}, 2^{c/2}\}, 2^h\} = 2^{128}$.
- The time complexity of Qin's model is at least $2^{h-\min\{d_{\mathcal{R}}, d_{\mathcal{B}}, m\}}$.
- At least one MitM episode should be performed, the optimal complexity is achieved when $d_{\mathcal{R}} = d_{\mathcal{B}} = m = h/2$, i.e., $Time \simeq 2^{h/2}$.
- For SPHINCS$^+$-Haraka, $2^{h/2} = 2^{128}$. Qin's MitM model can not achieve preimage attack with complexity better than $2^{h/2}$.

Analysis

With $b = 512, h = c = r = r' = 256$, it leads to $h - r' < c/2 < h$. Hence, $2^{c/2}$ becomes the security bound.

Outline

Attack Framework



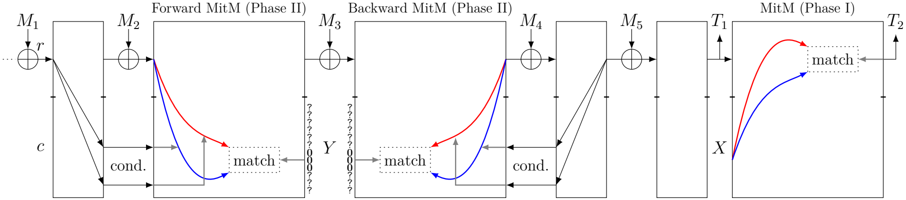
- Phase I: To find a capacity state X, such that $squeeze(T_1 \| X) = T_2 \| T_3 \| \cdots$. With Qin's MitM model [QHD+23], the time to find X can be reduced to

$$2^{|T|-|T_1|-\min(d^l_{\mathcal{R}},d^l_{\mathcal{B}},m^l)} = 2^{h-r'-\min(d^l_{\mathcal{R}},d^l_{\mathcal{B}},m^l)}, \quad (1)$$

- Phase II: To find an inner collision at Y, the time is $2^{c/2}$ trivially. Suppose t-bit of Y are fixed to be 0. Qin's MitM model can find $2^{\frac{c-t}{2}}$ $M_1 \| M_2$ for the forward path where the corresponding t-bit are 0, the time cost is:

$$\mathcal{C}_1 + 2^{\frac{c-t}{2}} \cdot 2^{t-\min(d^{L1}_{\mathcal{R}},d^{L1}_{\mathcal{B}},m^{L1})} = \mathcal{C}_1 + 2^{\frac{c}{2}+\frac{t}{2}-\min(d^{L1}_{\mathcal{R}},d^{L1}_{\mathcal{B}},m^{L1})} \quad (2)$$

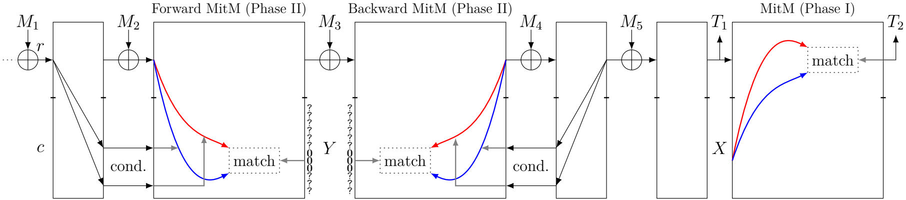
Attack Framework



The tight preimage security bound was proved to be $\min\{\max\{2^{h-r'}, 2^{c/2}\}, 2^h\}$ [LM22]. To beat the general bound, the following three cases are considered:

- Case I: If $h - r' < c/2 < h$, the general bound is $2^{c/2}$. Therefore, **Phase II** is only needed to derive better result than general bound. X in **Phase I** is a fixed constant. E.g., SPHINCS$^+$-`Haraka` with $h = 256$, $r = 256$, $c = 256$, $r' = 256$.

Attack Framework



- Case II: If $h - r' = c/2$, the general bound are $2^{h-r'}$ and $2^{c/2}$. Therefore, **Phase I** and **Phase II** are both needed. E.g., `Gimli-Hash`, `Xoodyak-Hash`, with $h = 256$, $c = 256$, $r = r' = 128$.
- Case III: If $h - r' > c/2$, the general bound is $2^{h-r'}$. Therefore, **Phase I** is only needed. The inner collision in **Phase II** can be performed in time of $2^{c/2}$. E.g., `Ascon-Hash`, `PHOTON`, `SPONGENT` and ACE-$\mathcal{H}$-256.

Description of SPHINCS$^+$-Haraka

- SPHINCS$^+$ is one of the selected Post-Quantum Digital Signature by NIST.
- SPHINCS$^+$-Haraka is instantiated with a sponge-based hashing based on the 512-bit permutation of Haraka v2.
- The 512-bit internal state is the concatenation of 4 AES states.

0	4	8	12
1	5	9	13
2	6	10	14
3	7	11	15

X_0

16	20	24	28
17	21	25	29
18	22	26	30
19	23	27	31

X_1

32	36	40	44
33	37	41	45
34	38	42	46
35	39	43	47

X_2

48	52	56	60
49	53	57	61
50	54	58	62
51	55	59	63

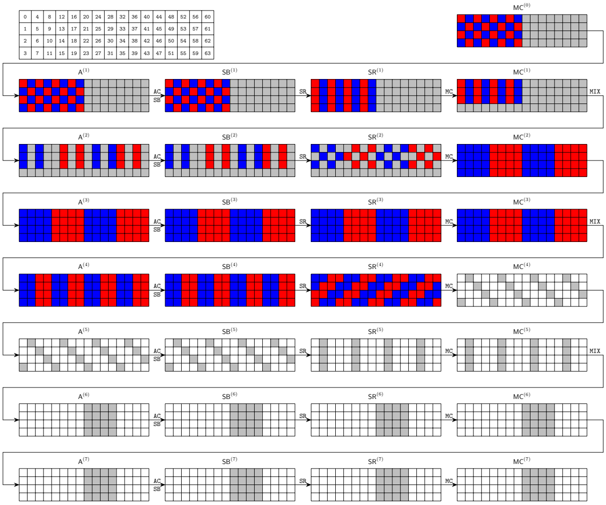
X_3

Two AES rounds are applied individually in each round (total 5 rounds), followed by an MIX operation:

$$0, \cdots, 15 \rightarrow (3, 11, 7, 15), (8, 0, 12, 4), (9, 1, 13, 5), (2, 10, 6, 14)$$

- Our target is the 4-round SPHINCS$^+$-Haraka without the last MIX operation.

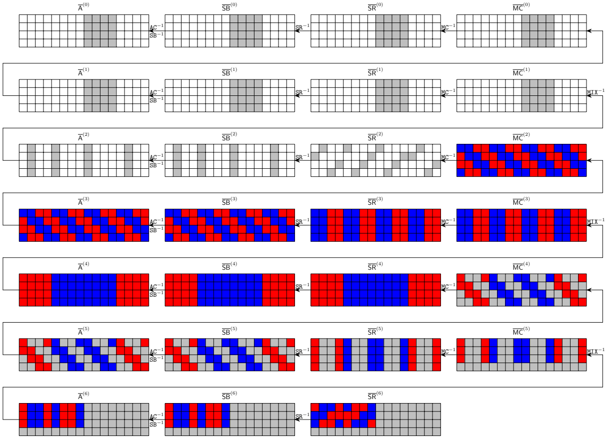
Preimage Attack on 4-round SPHINCS$^+$-Haraka



(a) Forward MitM

1. 16 bytes $\mathrm{MC}^{(7)}[32-47]$ are fixed to be 0 as matching points.
2. Starting state:
   $\mathrm{MC}^{(1)} = \mathtt{AES}(\mathtt{AES}(\mathrm{A}^{(0)}))$.
   $\mathrm{MC}^{(1)}[3,7,11,15,19,23,27,31] \leftarrow 0$,
   $\mathrm{MC}^{(1)}[32\text{-}63]$ is determined by M_1.
3. $d_{\mathcal{R}} = d_{\mathcal{B}} = 12,\ m = 16$.
4. With time of 2^{96}, $2^{96+96-128} = 2^{64}$ M_2 are stored in L_1 indexed by $\mathrm{MC}^{(7)}[48-63]$.

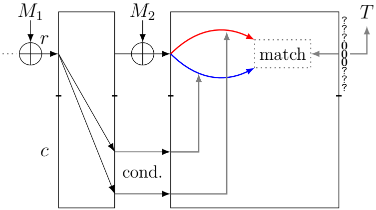
Preimage Attack on 4-round SPHINCS$^+$-Haraka



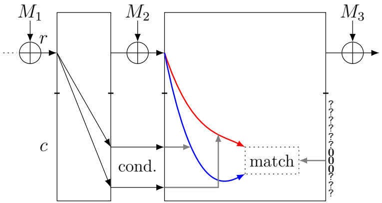
(b) Backward MitM

1. 16 bytes $\overline{A}^{(0)}[32-47]$ are fixed to be 0 as matching points.
2. Starting state: $\overline{SR}^{(6)}$. $\overline{SR}^{(6)}[3,7,11,15,19,23,27,31] \leftarrow 0$, $\overline{SR}^{(6)}$[32-63] is determined by M_5.
3. $d_{\mathcal{R}} = d_{\mathcal{B}} = 12,\ m = 16$.
4. With time of 2^{96}, $2^{96+96-128} = 2^{64}$ M_4 are stored in L_2 indexed by $\overline{A}^{(0)}[48-63]$.
5. Find a collision between L_1 and L_2.
6. $\overline{A}^{(0)}[0-31] = \text{MC}^{(7)}[0-31]$ can be modified by free M_3.

Outline



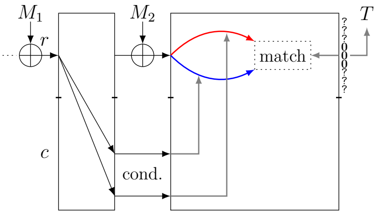
Collision Framework I (h-bit target collision)



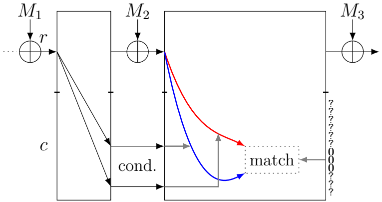
Collision Framework II (c-bit inner collision)

- Collision Framework I: The attack procedure is to find $2^{(h-t)/2}$ messages leading to the same t-bit pre-fixed constants. Time complexity: $\mathcal{C}_I \cdot 2^{(h-t)/2} < 2^{h/2}$.
- Collision Framework II: The attack procedure is to find $2^{(c-t)/2}$ messages leading to the same t-bit pre-fixed constants. The r-bit outer part can be modified by free message. Time complexity: $\mathcal{C}_{II} \cdot 2^{(c-t)/2} < 2^{c/2}$.

Attack Framework



Collision Framework I (h-bit target collision)



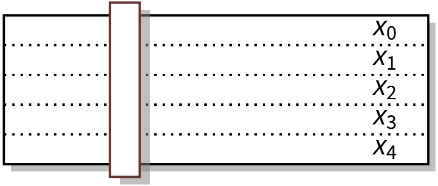
Collision Framework II (c-bit inner collision)

General bound of sponge construction: $\min\{2^{c/2}, 2^{h/2}\}$

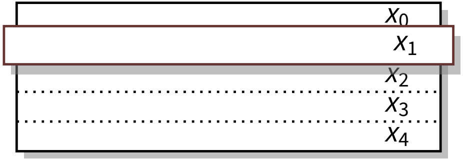
- If $h > c$, the **Collision Framework II** is applied. E.g., `Ascon-XOF`, `Xoodyak-XOF`.
- If $h = c$, the **Collision Framework I** or **II** is applied. E.g., `Ascon-Hash`, `Xoodyak-Hash`.
- If $h < c$, the **Collision Framework I** is applied. E.g., XOF with $h < c$.

Application to Ascon-Hash with Collision Framework II

- Winner of the NIST Lightweight Cryptography Project.
- Description of Ascon permutation: $p_L \circ p_S \circ p_C$



p_S: each column is updated with 5-bit S-box



p_C: each row is diffused with linear functions

- Parameters for Ascon-Hash: $b = 320, h = c = 256, r = r' = 64$.
- x_0 is the r-bit outer part.
- Since $h = c$, **Collision Framework II** can be applied.

Application to Ascon-Hash with Collision Framework II

- p_S applies the 5-bit Ascon S-Box column-wise as $(b_0, b_1, b_2, b_3, b_4) \leftarrow S(a_0, a_1, a_2, a_3, a_4)$. The algebraic normal form (ANF) of the Sbox is as follows:

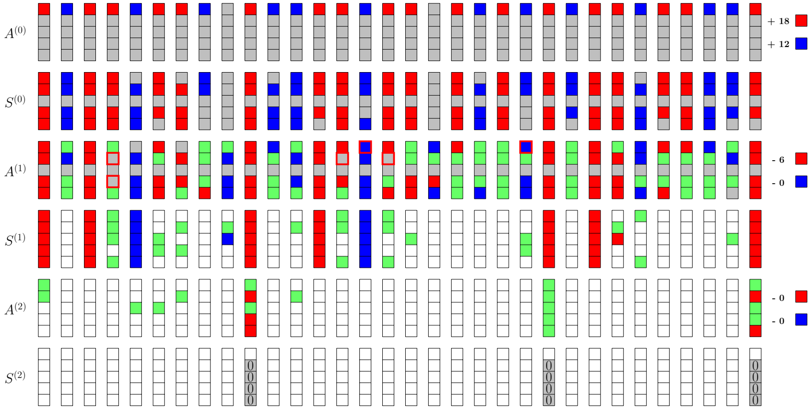
$$\begin{cases} b_0 = a_4a_1 + a_3 + a_2a_1 + a_2 + a_1a_0 + a_1 + a_0 \\ b_1 = a_4 + a_3a_2 + a_3a_1 + a_3 + a_2a_1 + a_2 + a_1 + a_0 \\ b_2 = a_4a_3 + a_4 + a_2 + a_1 + 1 \\ b_3 = a_4a_0 + a_4 + a_3a_0 + a_3 + a_2 + a_1 + a_0 \\ b_4 = a_4a_1 + a_4 + a_3 + a_1a_0 + a_1 \end{cases} \quad (3)$$

Observation 1 (Matching Strategy for Collision Framework II)

If $b_1 = b_2 = b_3 = b_4 = 0$, then $a_0 = 1,\ a_1 \oplus a_2 = 1,\ a_3 = 0,\ a_4 = 0$ can be derived. Therefore, 4 matching equations can be immediately obtained if there are no unknown $\square$ bit in $(a_0, a_1, a_2, a_3, a_4)$.

Application to Ascon-Hash with Collision Framework II

- Collision attack on 3-round Ascon-Hash. The attack parameters are: $d_{\mathcal{R}} = d_{\mathcal{B}} = 24$, $m = t = 24$. The time cost is $2^{\frac{c}{2} - \min\{d_{\mathcal{R}} - \frac{t}{2}, d_{\mathcal{B}} - \frac{t}{2}, m - \frac{t}{2}, \frac{t}{2}\}} = 2^{128-12} = 2^{116}$.



Outline

Summary of applications to preimage and collision attacks

Target	Attacks	Methods	Rounds	Time	Memory	Claim	Generic	Ref.
Ascon-Hash	Collision	Diff. Diff. MitM MitM	2/12 2/12 3/12 4/12	2^{125} 2^{103} $2^{116.74}$ $2^{124.85}$	- - 2^{116} 2^{124}	2^{128}	2^{128}	[ZDW19] [GPT21] ours ours
SPHINCS$^+$-Haraka	Preimage	MitM MitM	3.5/5 4/5	$2^{64.6}$ Q 2^{98}	- 2^{96}	- 2^{128}	$2^{85.3}$ Q 2^{128}	[SS22] ours
PHOTON-80/20/16	Preimage	MitM	4.5/12	2^{60}	2^{24}	2^{64}	2^{64}	ours
ACE-$\mathcal{H}$-256	Preimage	MitM	9/16	2^{160}	2^{128}	2^{192}	2^{192}	ours
Subterranean 2.0	Preimage	MitM	Full	2^{160}	2^{100}	2^{112}	2^{224}	ours
Xoodyak-XOF	Preimage	Neural MitM MitM	1/12 3/12 3/12	- $2^{125.06}$ $2^{121.77}$	- 2^{97} 2^{118}	2^{128}	2^{128}	[LLL$^+$21] [QHD$^+$23] ours
Xoodyak-Hash	Collision	MitM	3/12	$2^{125.23}$	2^{124}	2^{128}	2^{128}	ours

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