Publicly Verifiable Secret Sharing over Class Groups and Applications to DKG and YOSO

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- Correct reconstruction of secret (by *reconstructing parties*).

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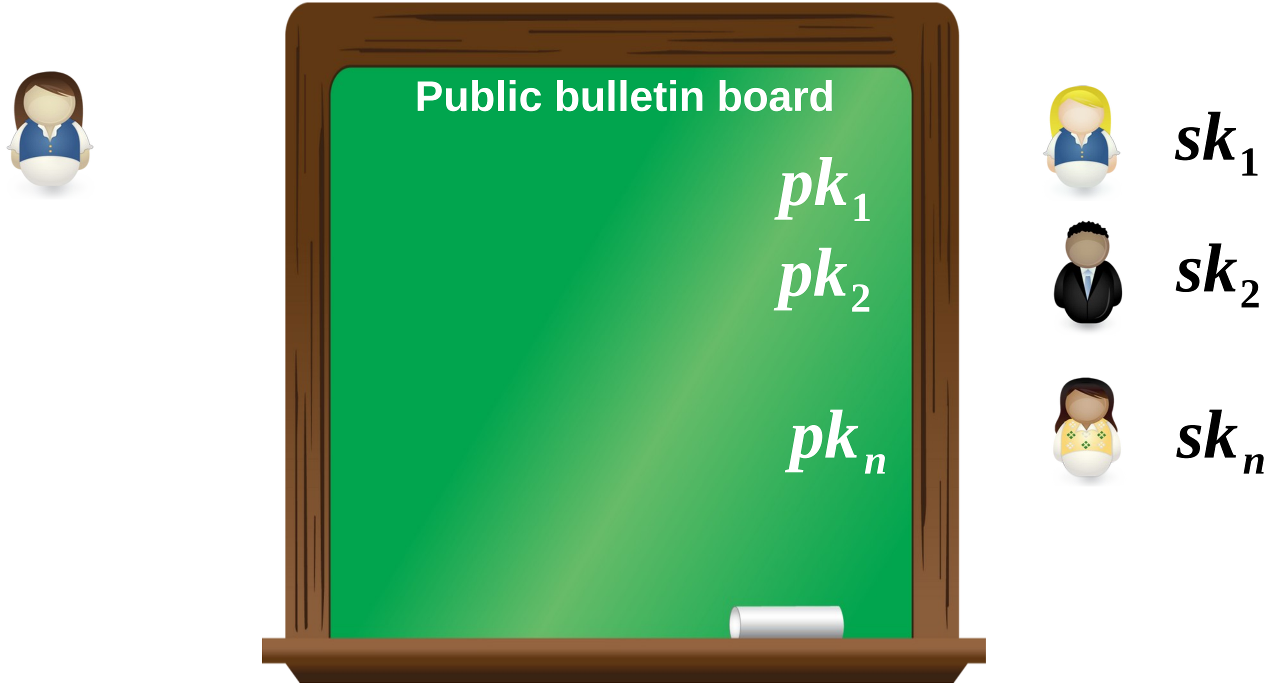

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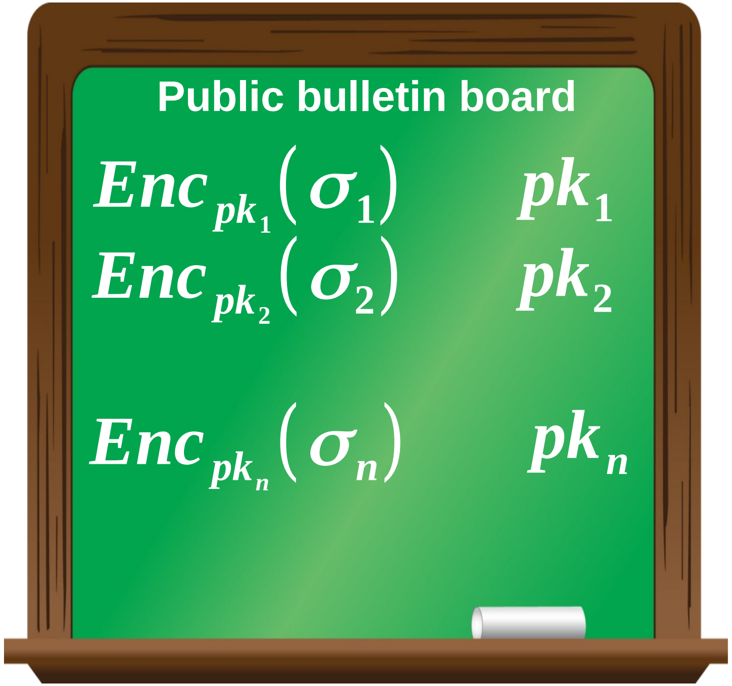

sk_1

sk_2

sk_n

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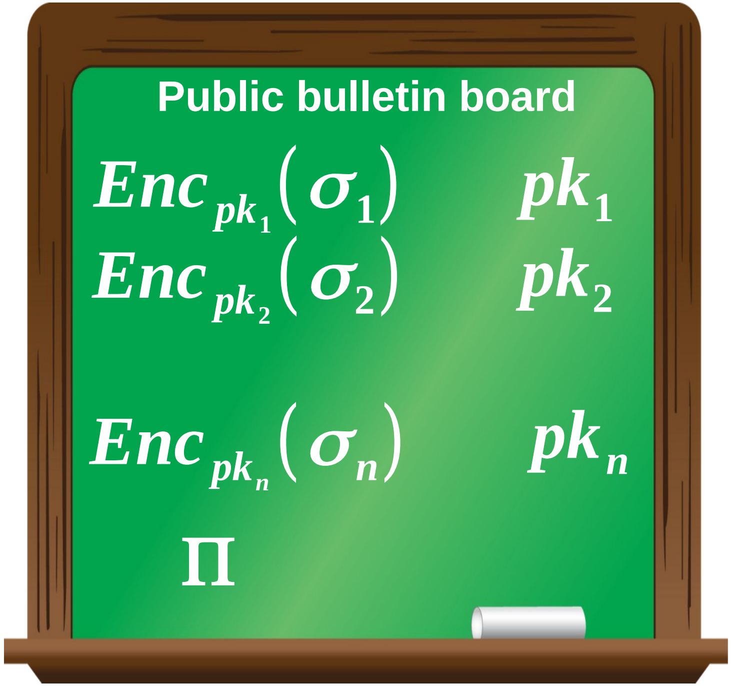
- Dealer publishes NIZK that plaintexts are a correct sharing.

Secret: **s**

Shamir shares: $(\sigma_1,\ldots,\sigma_n)$

Proof :

Π=NIZK($\exists$ f , deg f $\leq$ t,
f(α_i)=σ_i , $\forall$i $\in$ [n])



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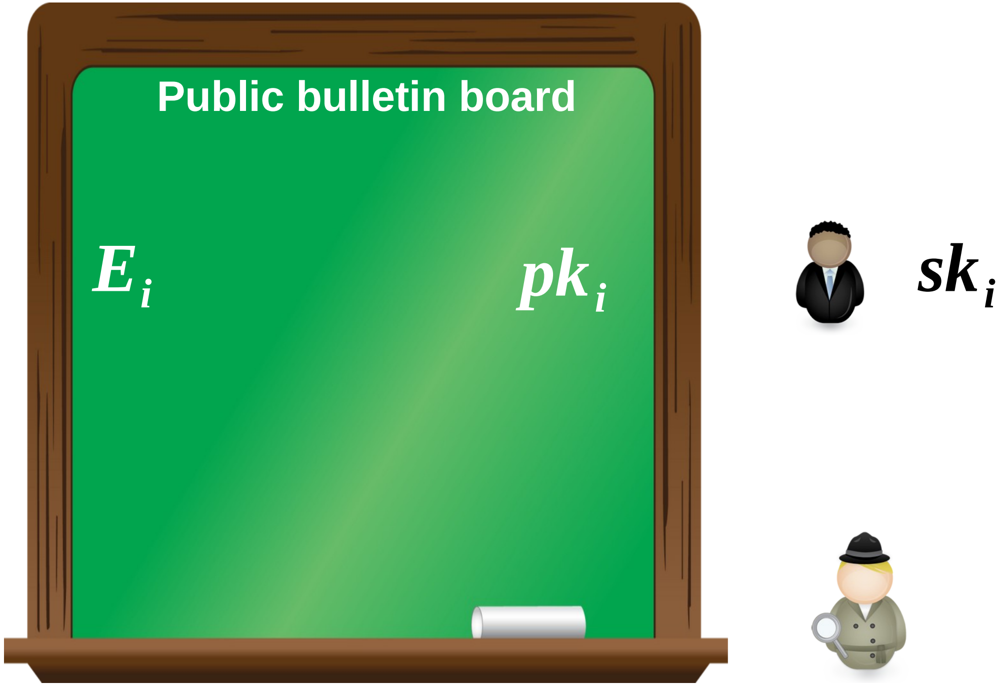
Publicly Verifiable Secret Sharing (PVSS)

- Parties proves correct opening of σ_i, given encryption and pk_i



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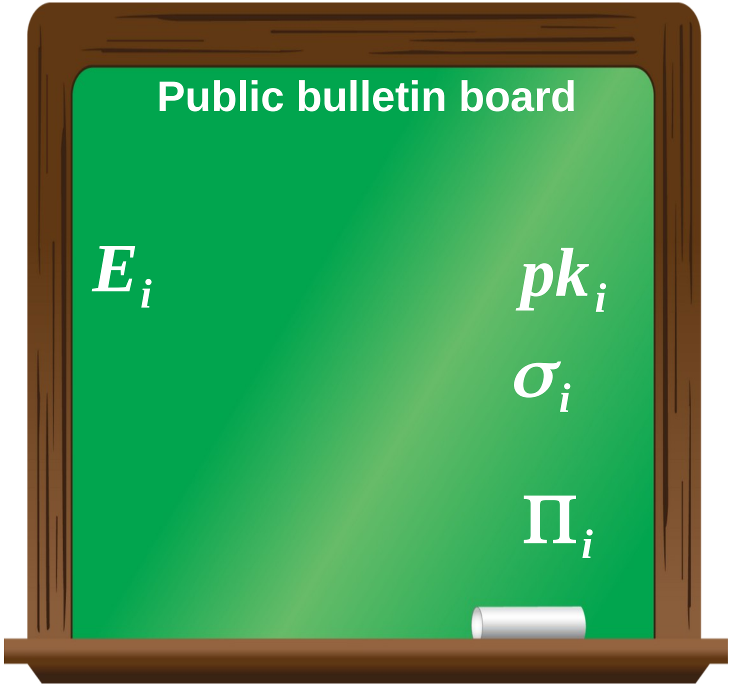

sk_i

Proof :

$\Pi_i = \mathrm{NIZK}(\sigma_i = \mathrm{Dec}_{sk}(E_i))$

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Common features:

- Cyclic group $\mathbb{G} = \langle g \rangle$ of prime order q with hard DL
- Parties get (after decryption) only g^{σ_i} (***not the Shamir shares*** σ_i)
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Some applications:

- MPC linear functions with **small output**, e.g. elections [Sch99]
- Randomness beacons [SCRAPE, Albatross]
- **Non-linear** PVSS of **r** in $\mathbb{Z}_q$ [YOLO YOSO]:

Dealer PVSSs random g^s and broadcasts $r - H(g^s)$, (for H: $\mathbb{G} \rightarrow \mathbb{Z}_q$ random oracle)

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Drawbacks:
Parties do not learn σ_i

- Bad for Distributed Key Generation (DKG).
- Bad for MPC.

DKG (for DL Key Pairs) via PVSS

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- Parties determine **set Q of *correctly shared*** $\mathbf{r^{(j)}}$
- Aggregate correct shares:
  - Party i defines $\mathbf{tsk_i} = \Sigma_{j\in Q}(\boldsymbol{\sigma}^{(j)})_i$
  - Party i publishes $\mathbf{tpk_i} = \mathbf{g}^{tsk_i}$ and proof of correctness
  - Parties reconstruct **tpk** from correct $\mathbf{tpk_i}$

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- Efficient YOSO MPC with transparent setup based on class groups.

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Decryptor recovers f^m as in El Gamal, **solves DL in** F, gets m.

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- Obstacle: We need to change our proof of sharing.

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Sharing proof from [CDGK22] uses “SCRAPE trick” [CD17]:

Linear check $(\sigma_1,\ldots,\sigma_n)$ is a Shamir sharing:

Sample $(w_1,\ldots,w_n)$ uniformly in corresponding **dual code**

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In [CDGK22], dealer publishes $R = g^r$ and $B_i = (pk_i)^r \cdot g^{\sigma_i}$

Sharing proof uses SCRAPE to reduce to DL equality proof:

Sample random $(w_1,\ldots,w_n)$, prove $\Pi\, B_i^{w_i} = \Pi\, (pk_i^{w_i})^r$ for same r s.t. $g^r = R$

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- DL-EQ PoKs are more expensive.

  Alternatively we show we **can** use **sound** proofs for sharing and reconstruction correctness.

  [BDO23] provides more efficient sound DL-EQ proofs

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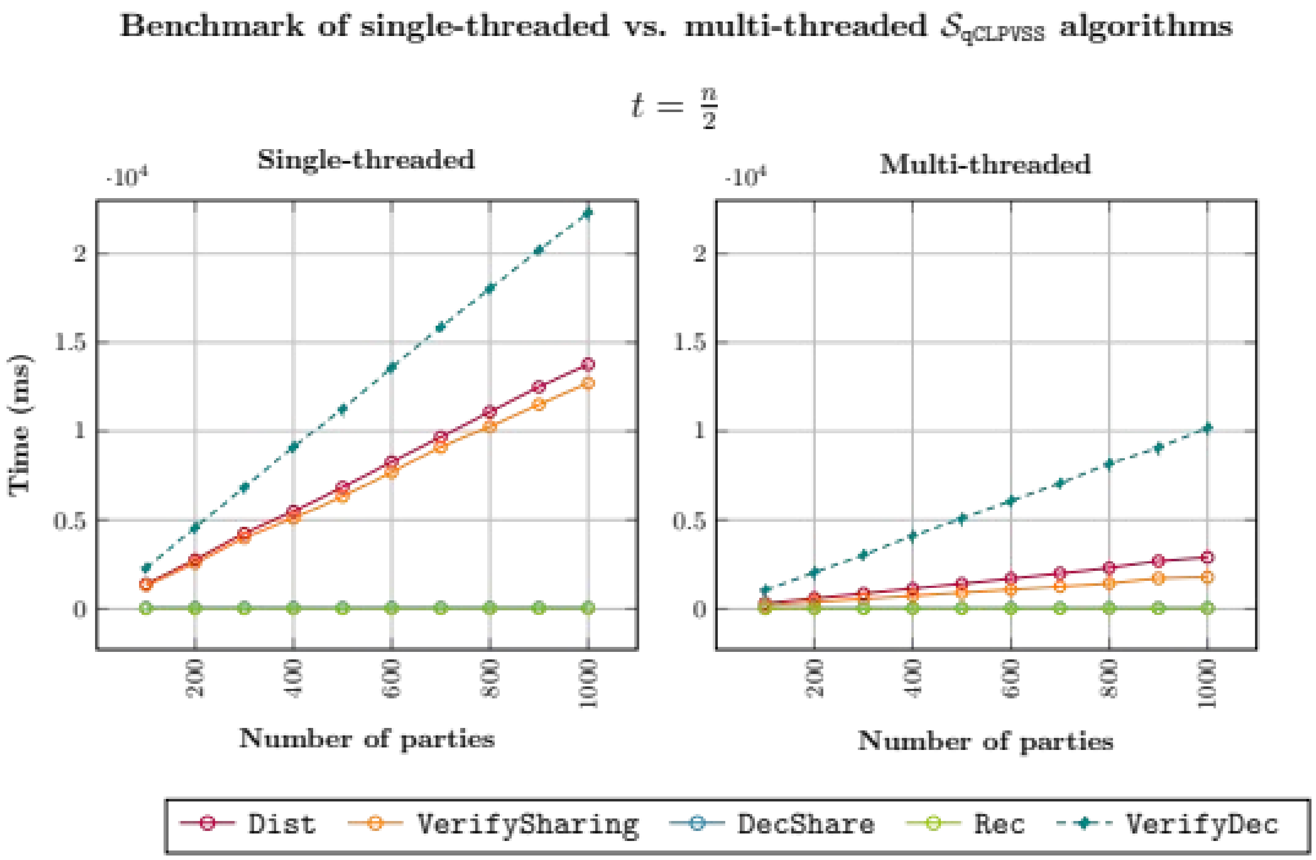
Comparison:

- Our PVSS sharing is more communication-efficient and we achieve a stronger notion of security (they leak g^s).
- For 1-round DKG: Their scheme is more efficient than ours in communication and computation.

Implementation



Implementation by
Rasmus Føgh Sørensen

Main bottleneck:

Verification of share
opening proofs.

Conclusions

- We present an **efficient PVSS over class groups**, counterpart to CDGK22
- We present **2-round DKG** (unbiasable key) and **1-round DKG**
- We also instantiate **MPC in the YOSO model** based on our PVSS
- Implementation is fast, main bottleneck verification of (many) DLEQ proofs.

Thank you!

https://eprint.iacr.org/2023/1651

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