Twinkle: Threshold Signatures from DDH with Full Adaptive Security





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Threshold Signatures

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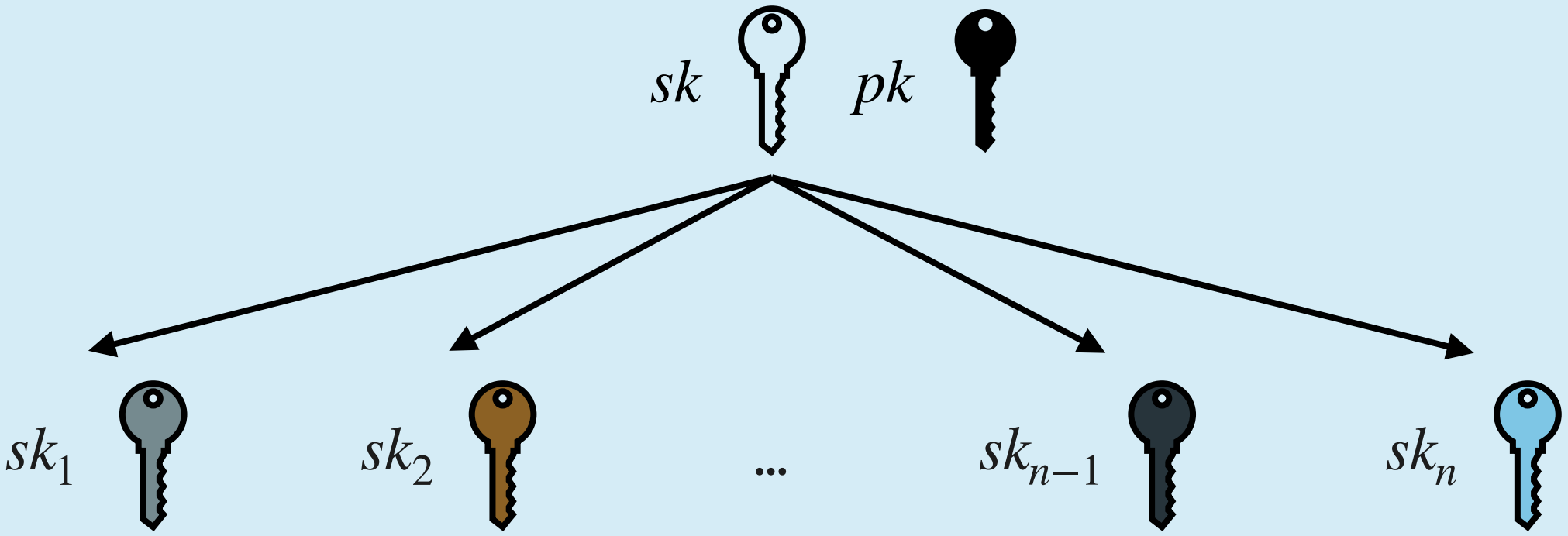

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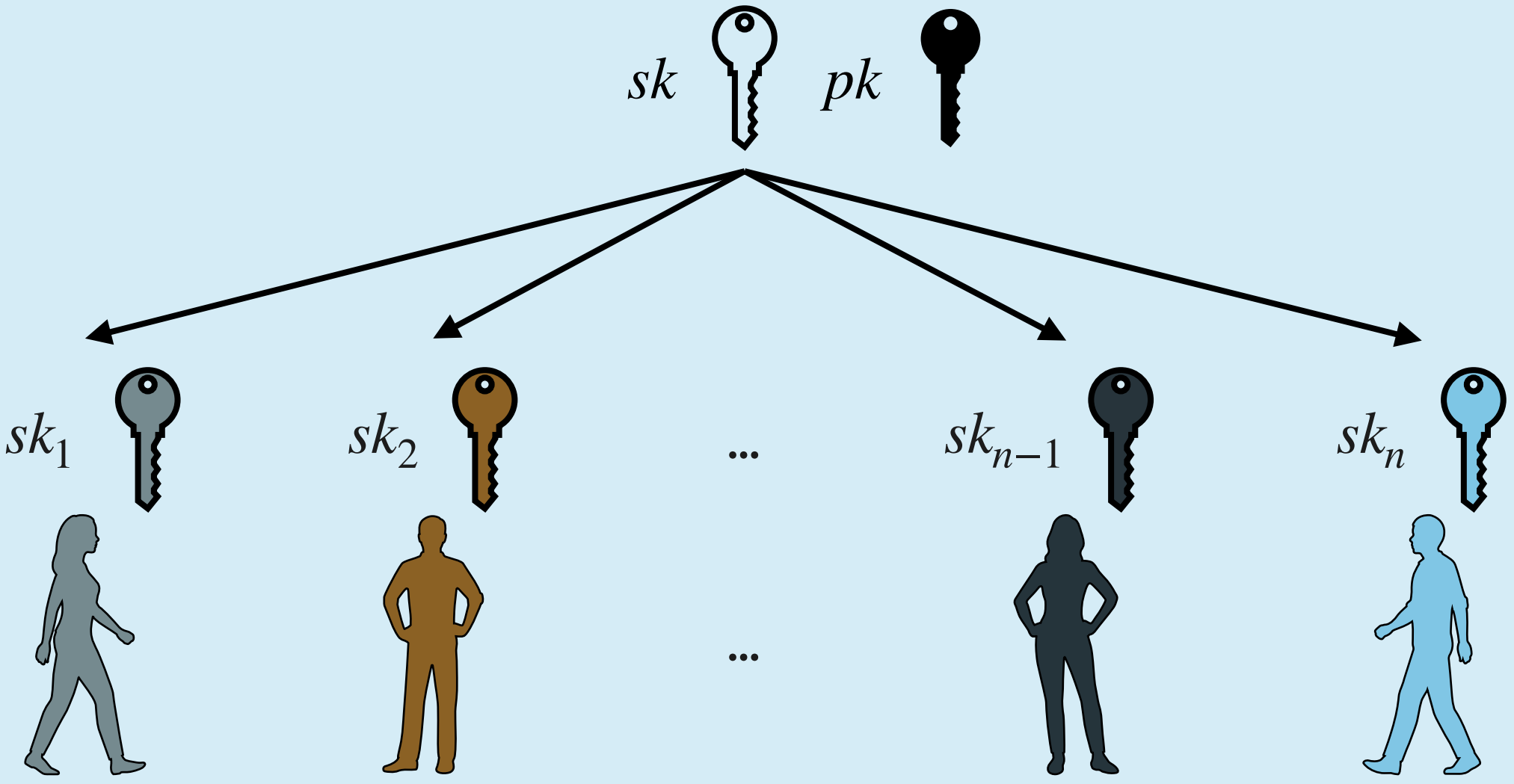

n Signers

Threshold Signatures with Threshold $t < n$

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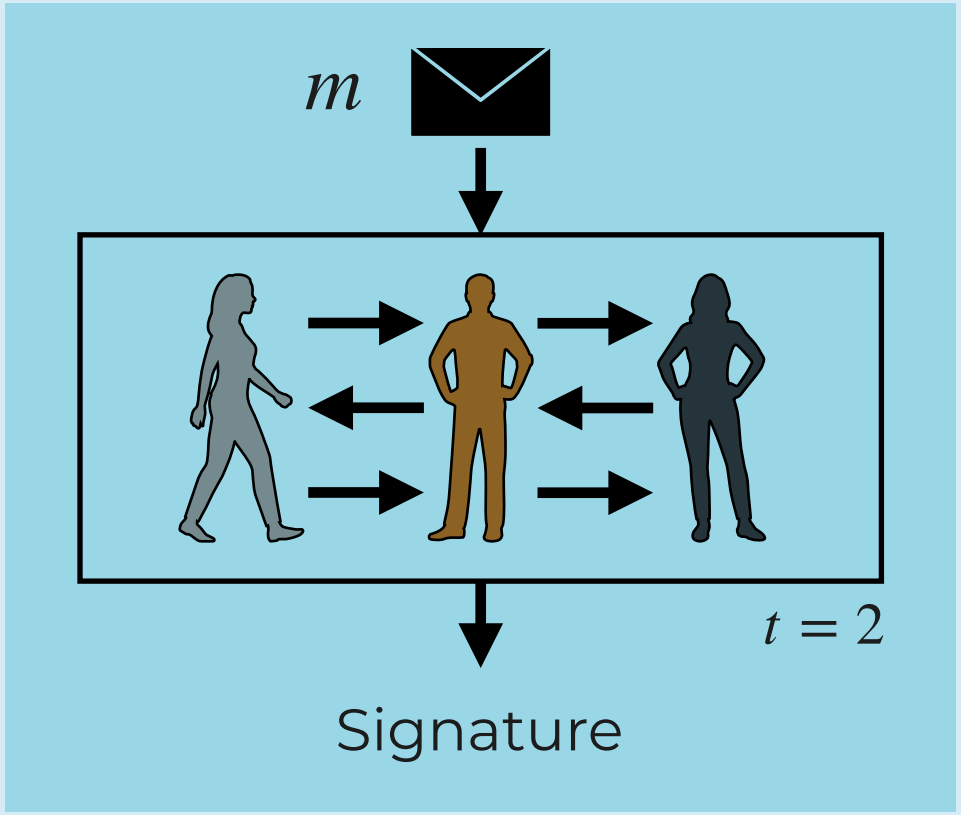
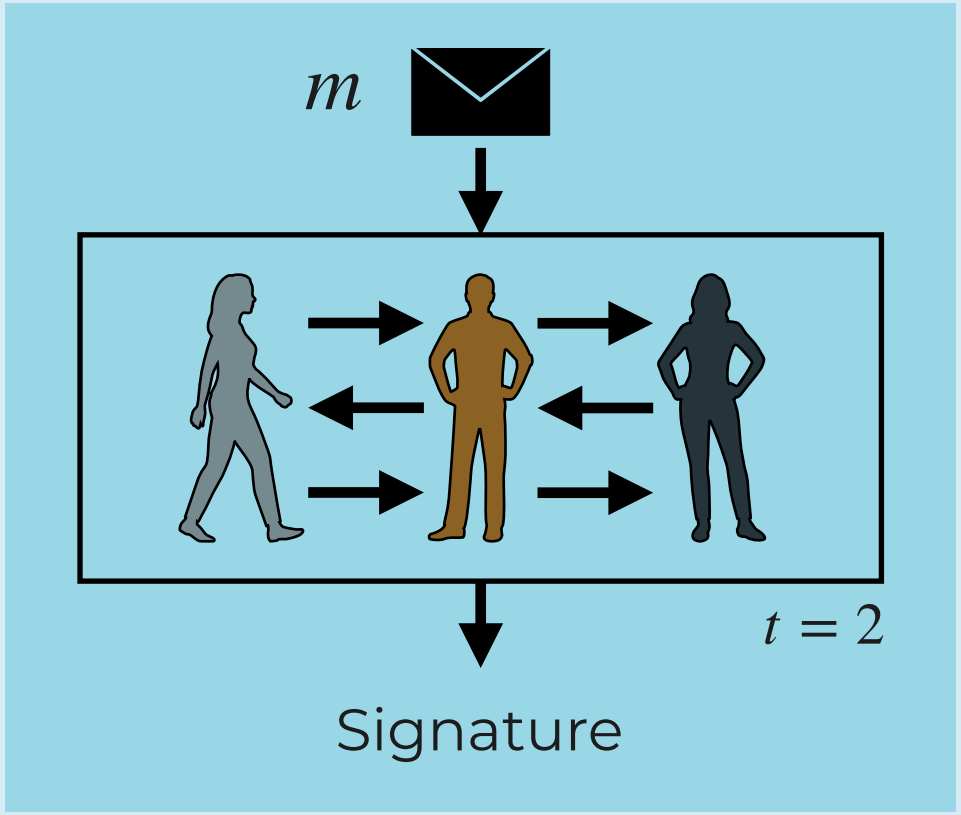
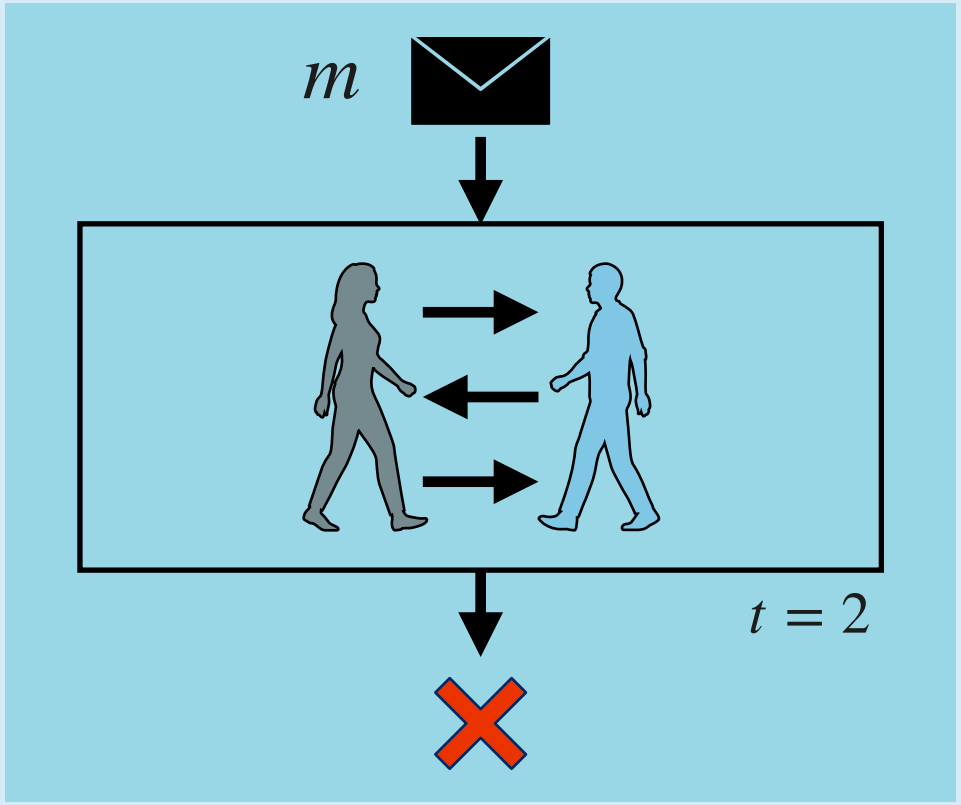


$> t$ Signers can sign

Threshold Signatures with Threshold $t < n$





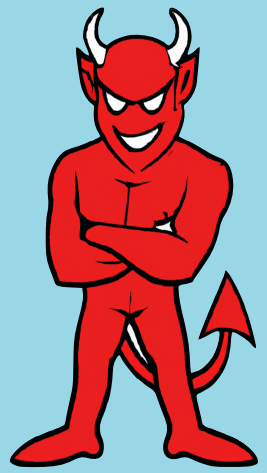
$> t$ Signers can sign

$\leq t$ Signers can not sign

Security of Threshold Signatures

Security of Threshold Signatures

Static Security

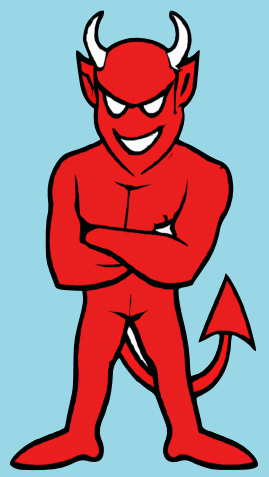


Security of Threshold Signatures

Static Security

Corruptions

$C \subset [n], \quad |C| \leq t$



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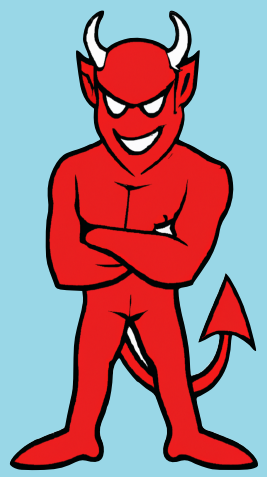
$C \subset [n], \quad |C| \leq t$

pk, sk_i for all $i \in C$



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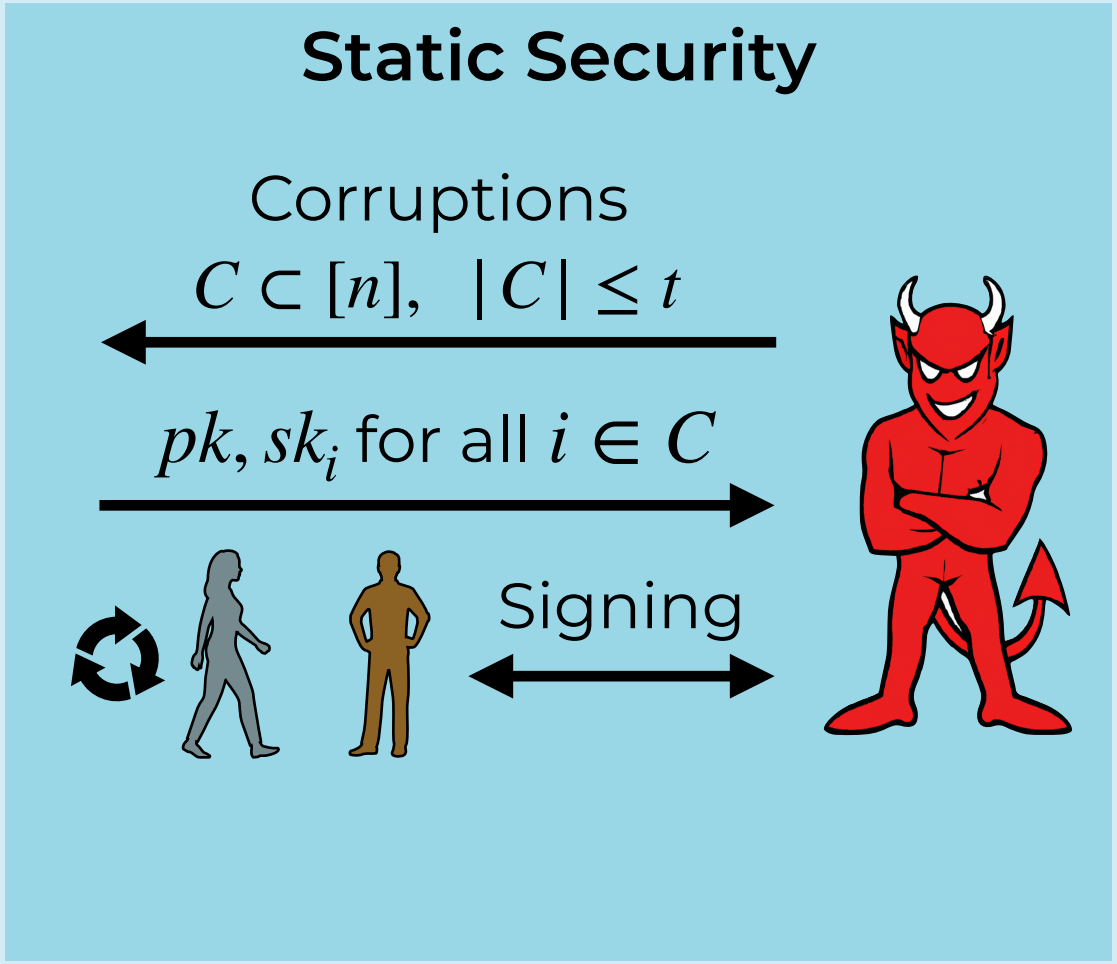

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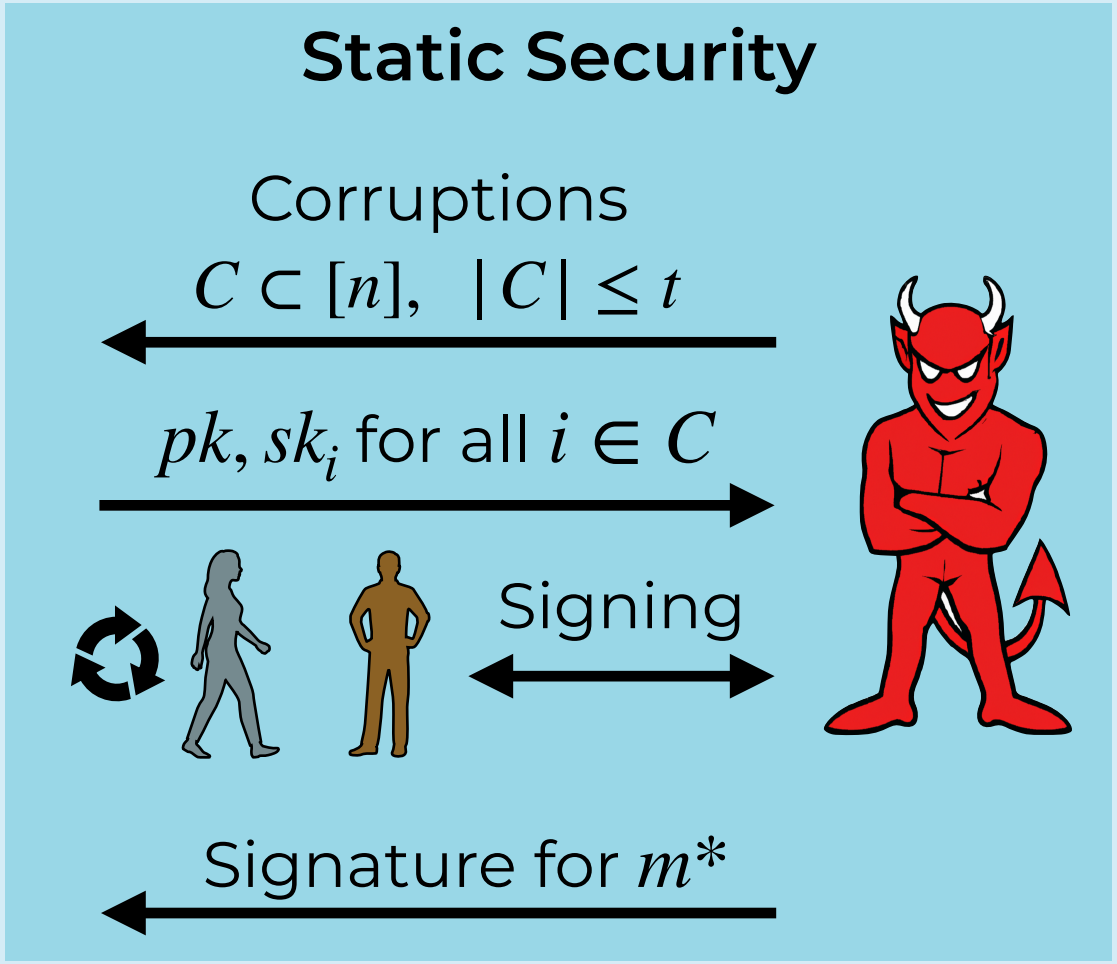

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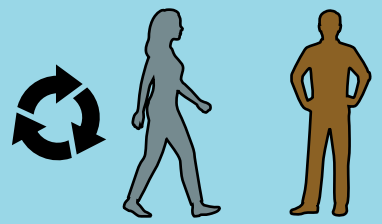

Signing

Signature for m^*

Adaptive Security

Security of Threshold Signatures

Static Security

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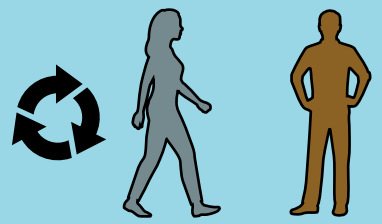

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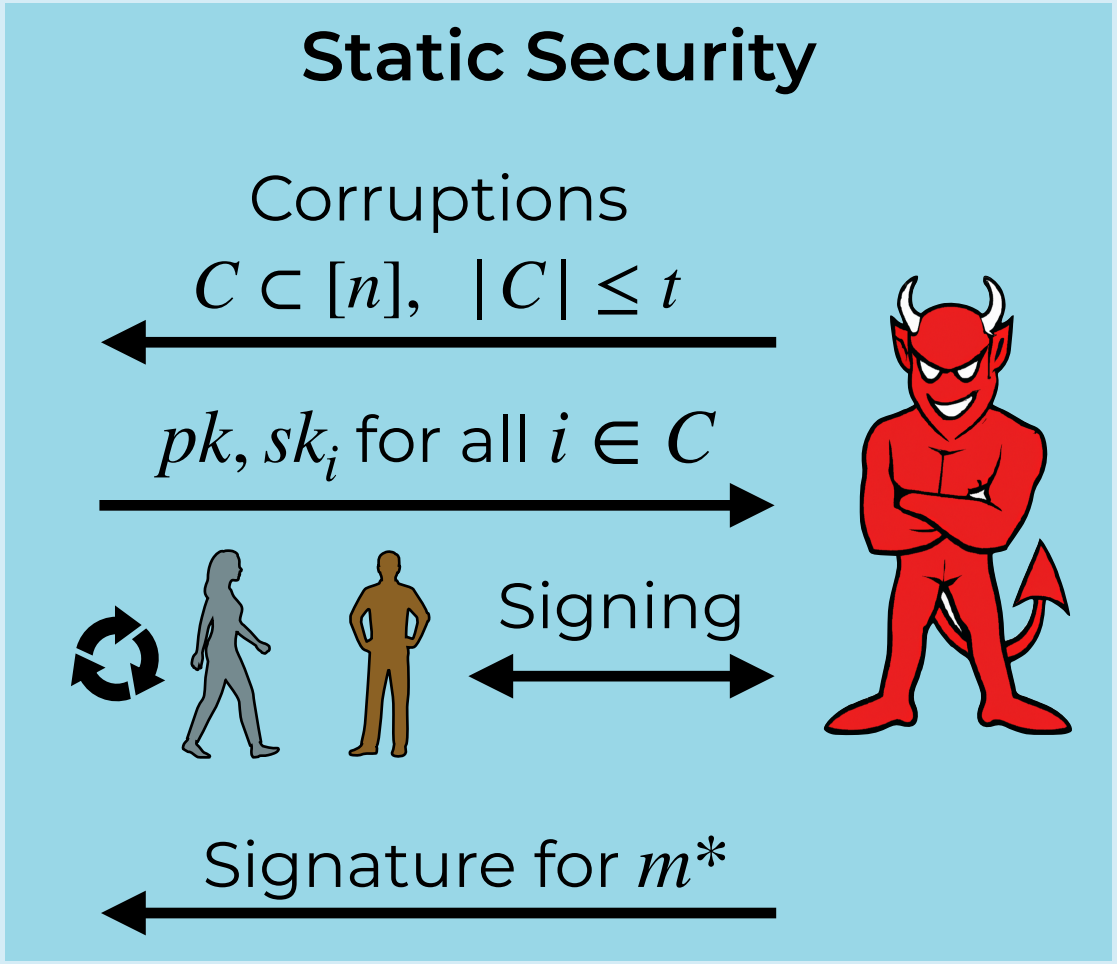

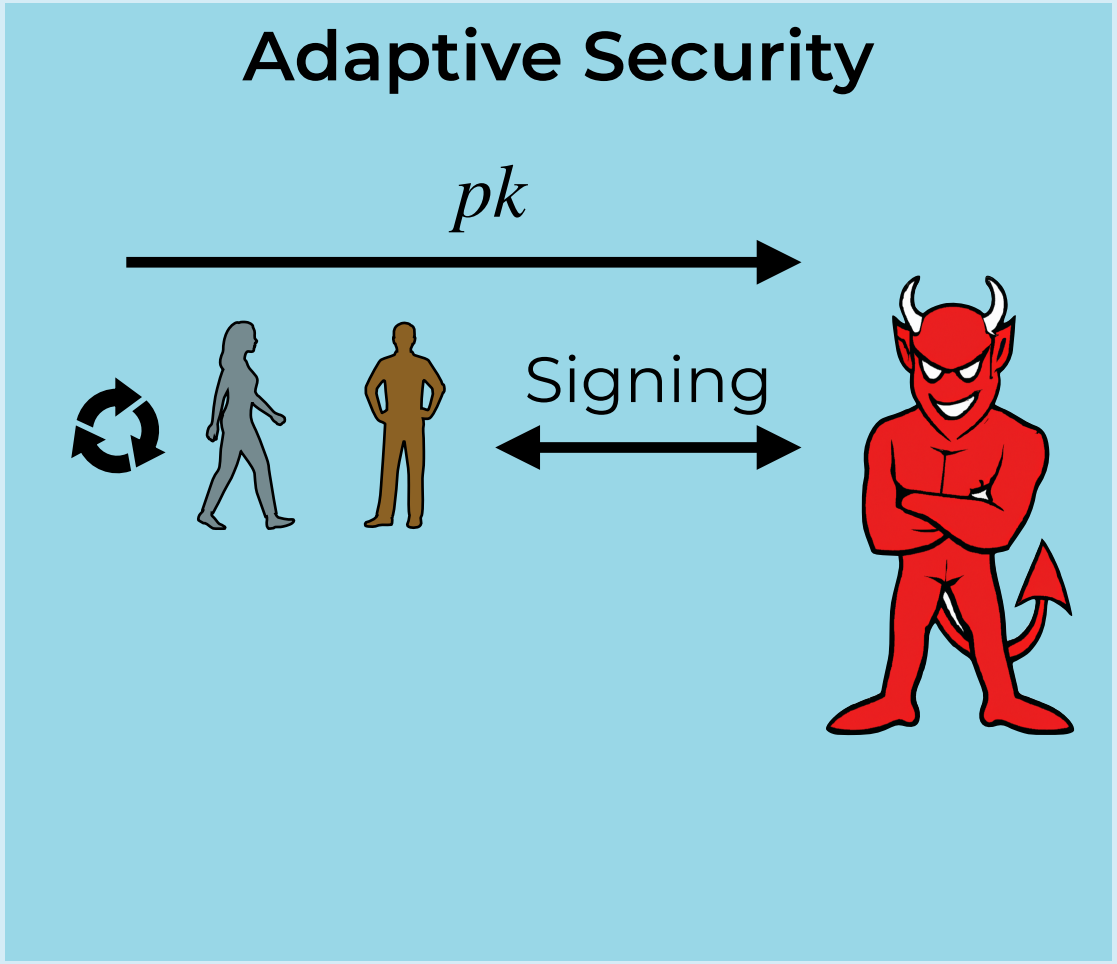

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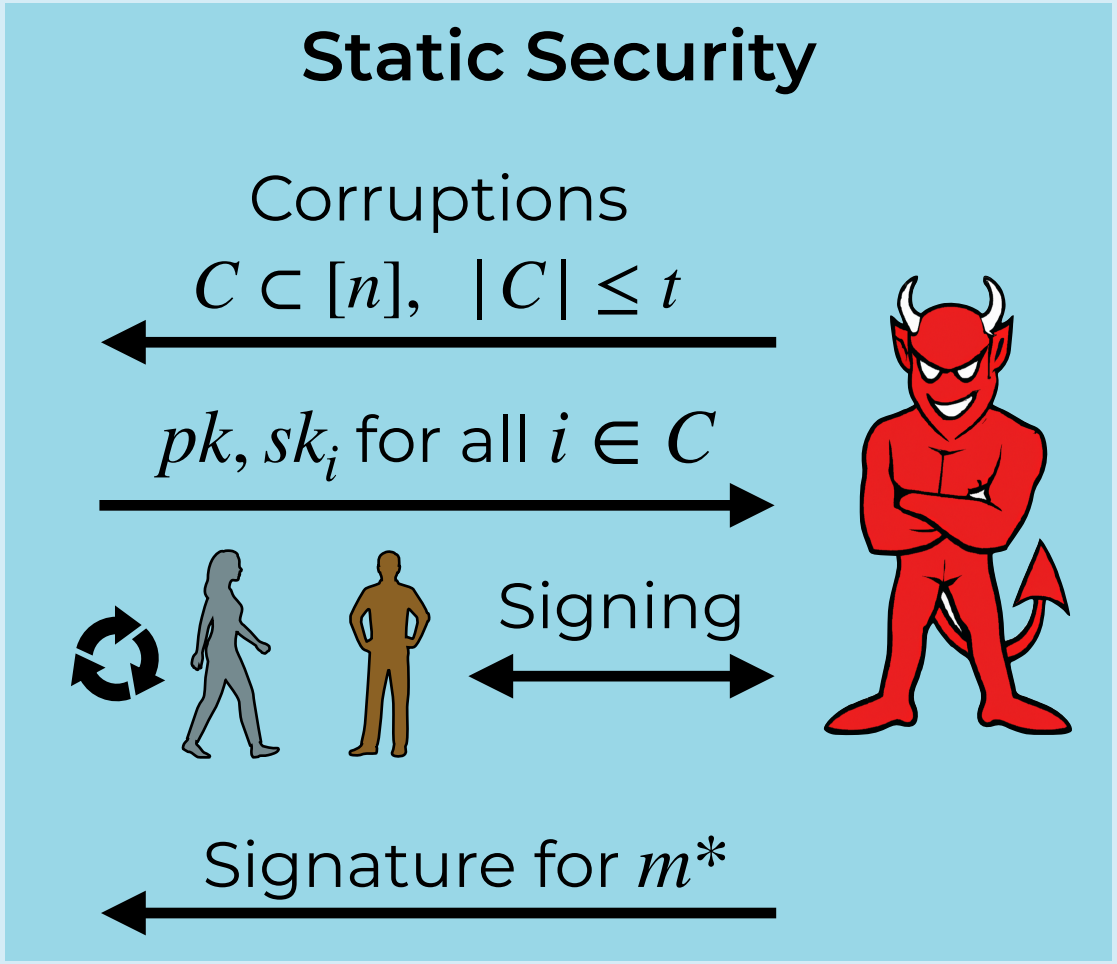

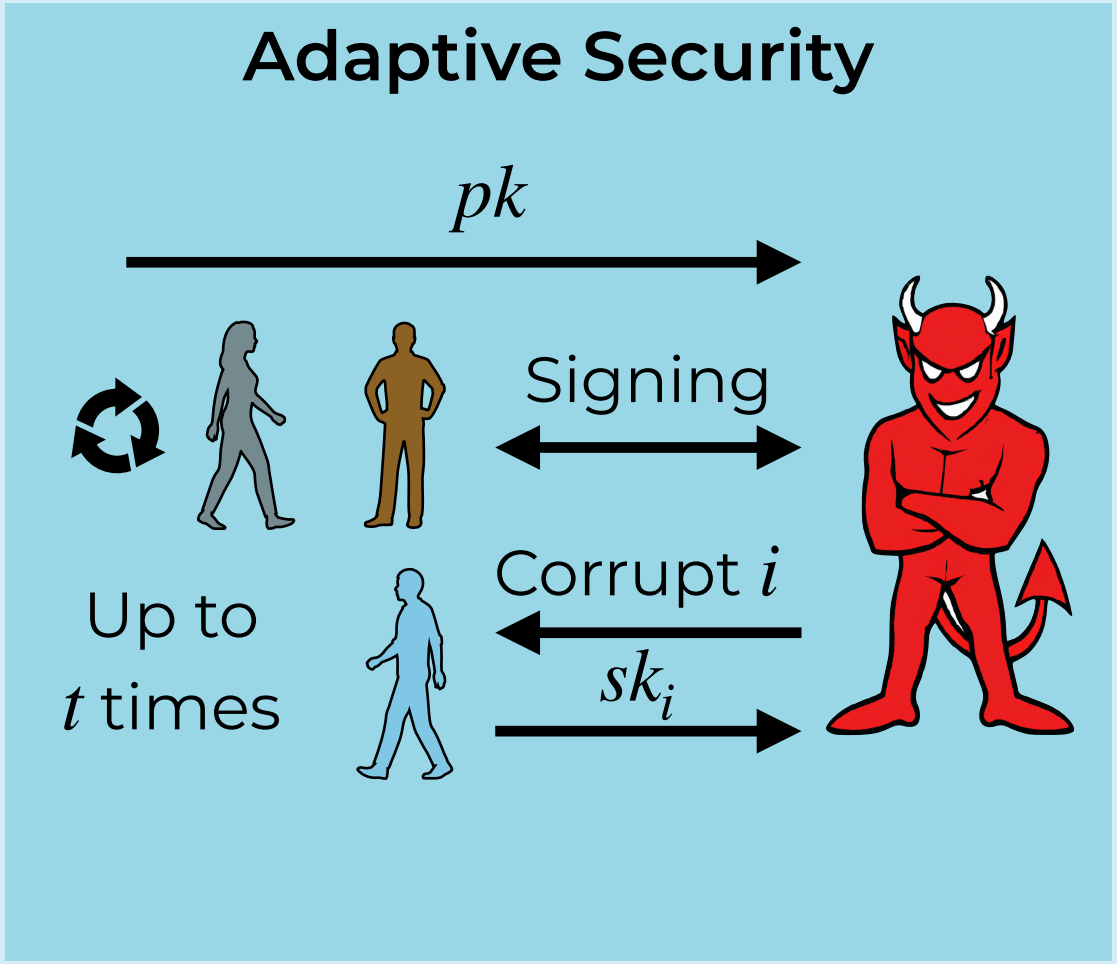

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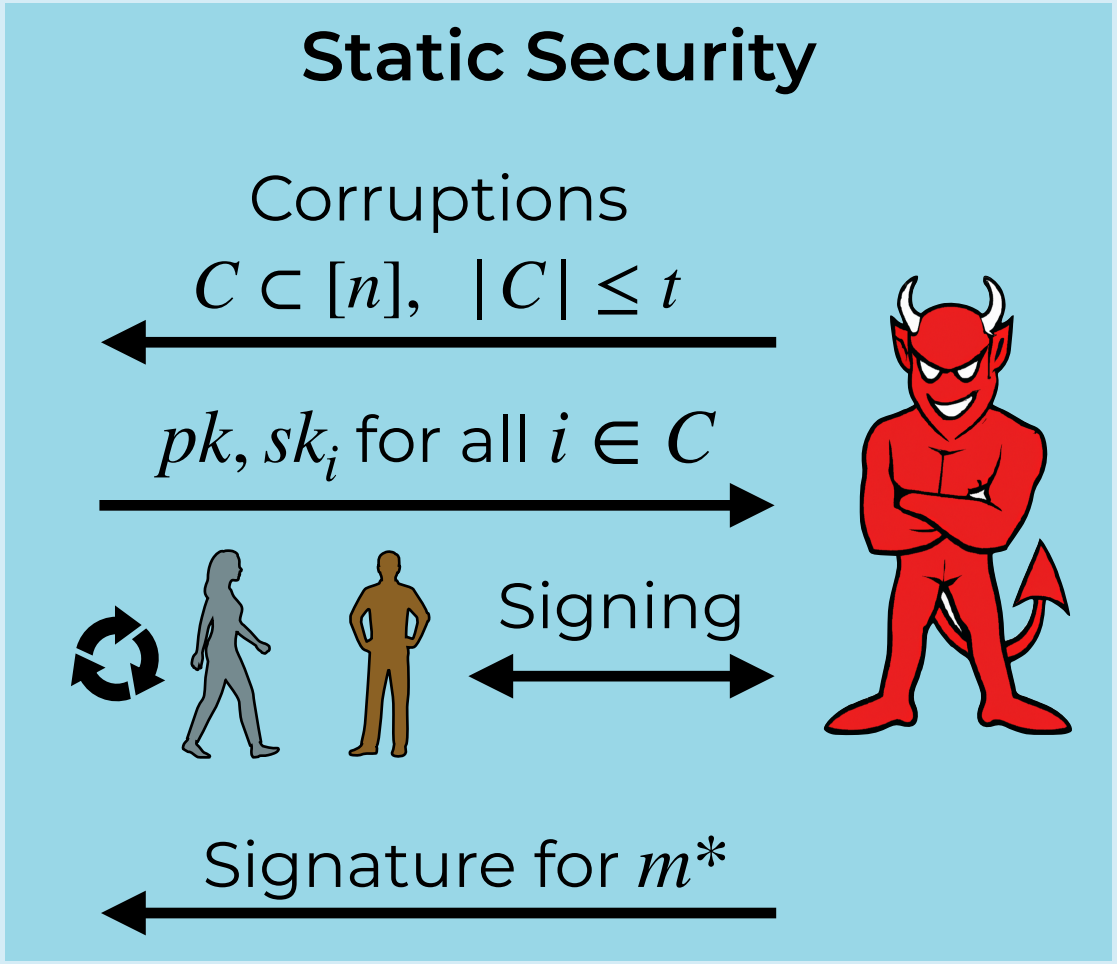

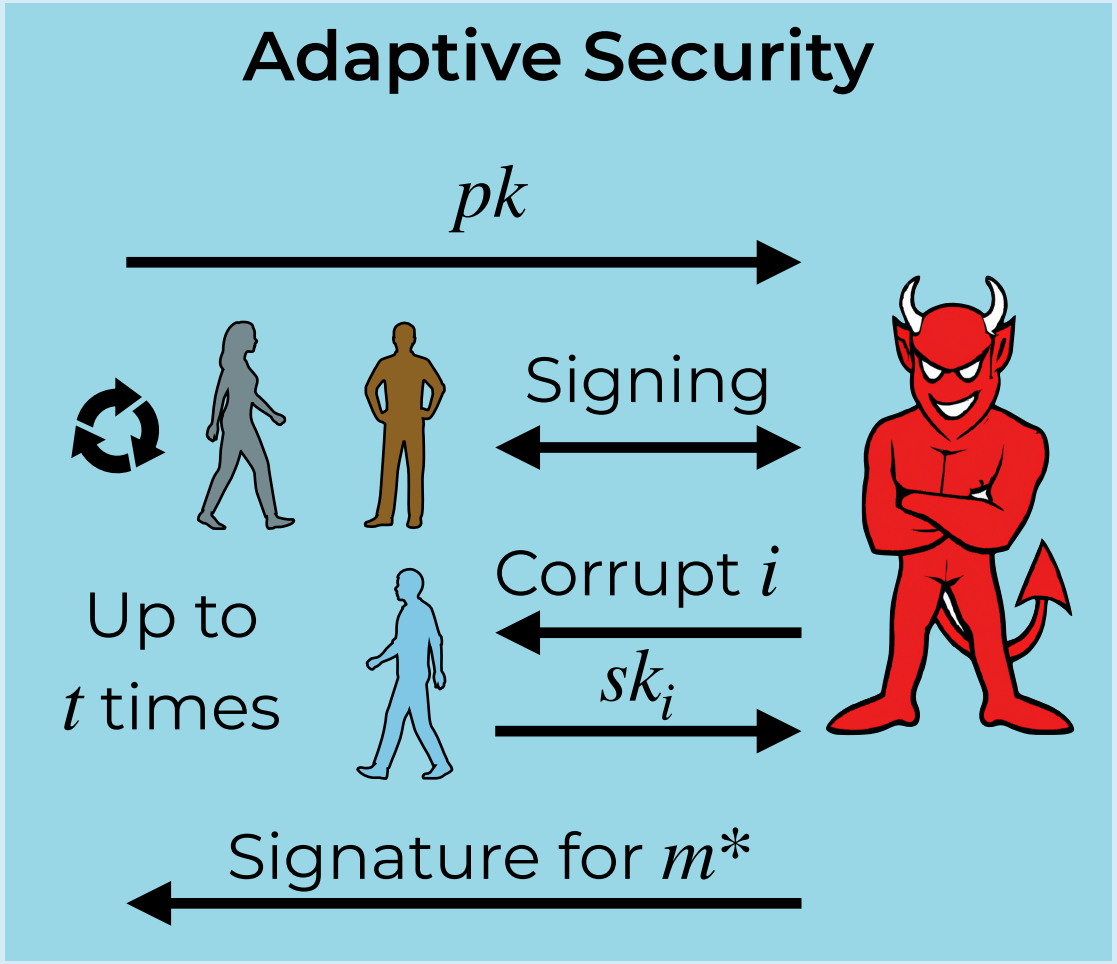

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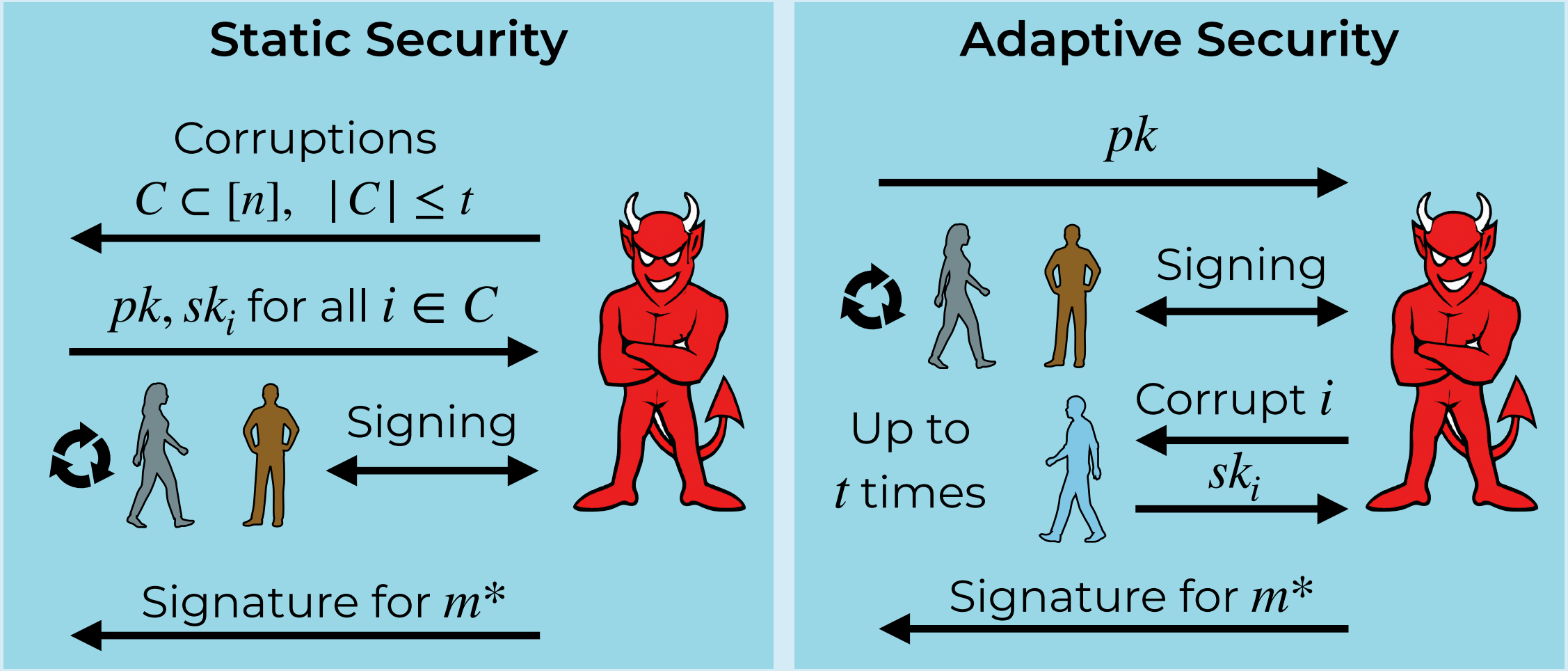

Goal: Adaptive Security

Threshold Signatures in Pairing-Free Groups

Threshold Signatures in Pairing-Free Groups

Frost Family

Threshold Signatures in Pairing-Free Groups

Frost Family

Frost 1-3	TZ
OMDL	DLOG
Static	Static

Frost [SAC 20, KG and Crypto 23, CKM+], Frost2 [Crypto 23, CKM+], Frost3 [CCS 22, RRJ+], TZ [Eurocrypt 23, TZ], Sparkle [Crypto 23, CKM]

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$\leq t/2$ **corruptions**

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Sparkle

Proof 1	Proof 2
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	$\leq t/2$ corruptions

Adaptive Security for $\leq t$ Corruptions?

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Threshold Signatures in Pairing-Free Groups

Frost Family		Sparkle	
Frost 1-3	**TZ**	**Proof 1**	**Proof 2**
OMDL	DLOG	DLOG	OMDL
Static	Static	Static	Adaptive
			$\leq t/2$ **corruptions**

Adaptive Security for $\leq t$ Corruptions?

Adaptive Security without Interactive Assumptions?

Frost [SAC 20, KG and Crypto 23, CKM+], Frost2 [Crypto 23, CKM+], Frost3 [CCS 22, RRJ+], TZ [Eurocrypt 23, TZ], Sparkle [Crypto 23, CKM]

Our Result: Twinkle Threshold Signatures

Frost Family		Sparkle	
Frost 1-3	**TZ**	**Proof 1**	**Proof 2**
OMDL	DLOG	DLOG	OMDL
Static	Static	Static	Adaptive
			$\leq t/2$ **corruptions**

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Our Result: Twinkle Threshold Signatures

Frost Family

Frost 1-3	TZ
OMDL	DLOG
Static	Static

Sparkle

Proof 1	Proof 2
DLOG	OMDL
Static	Adaptive

$\leq t/2$ **corruptions**

Twinkle

DDH Assumption

Full Adaptive Security

Frost [SAC 20, KG and Crypto 23, CKM+], Frost2 [Crypto 23, CKM+], Frost3 [CCS 22, RRJ+], TZ [Eurocrypt 23, TZ], Sparkle [Crypto 23, CKM]

Technical Challenges

Allow up to t Corruptions

Non-Interactive Assumption

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Non-Interactive Assumption

One-More Discrete Logarithm Assumption

Cyclic Group $\mathbb{G} = \{g^0, \ldots, g^{p-1}\}$

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OMDL Solver

OMDL Assumption: There is no Efficient OMDL Solver

One-More Discrete Logarithm Assumption

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$$f = a_0 + a_1 X + \cdots + a_t X^t$$

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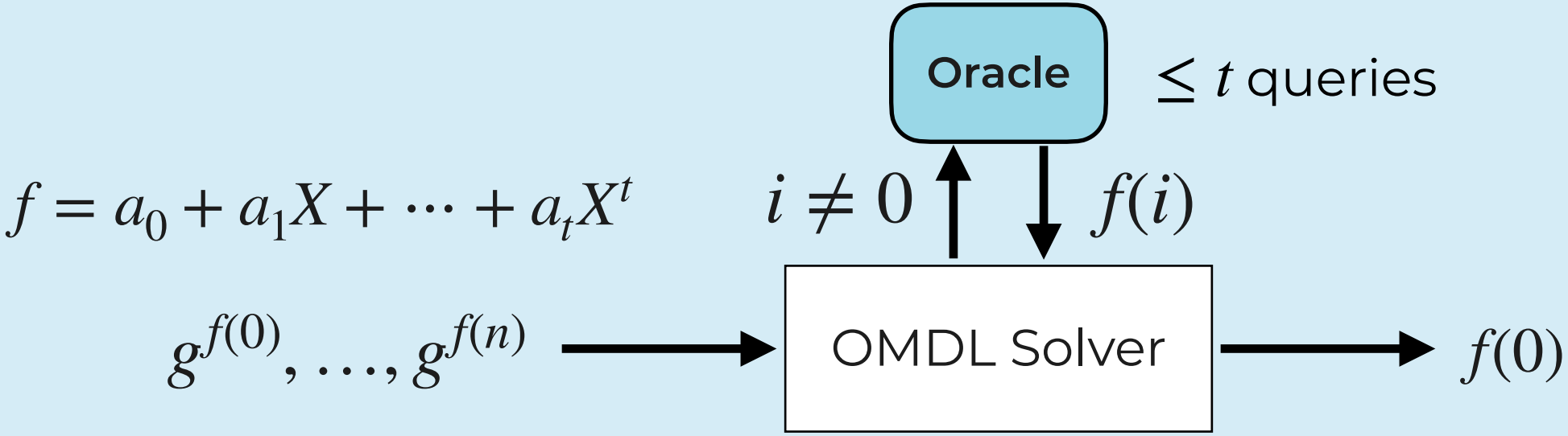

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Sparkle Threshold Signatures

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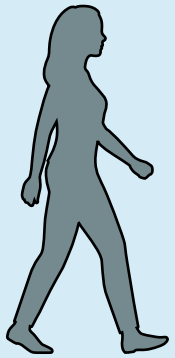
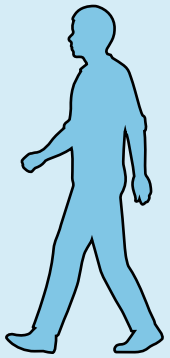


$$sk_i = f(i)$$



$$sk_j = f(j)$$

Sparkle Threshold Signatures

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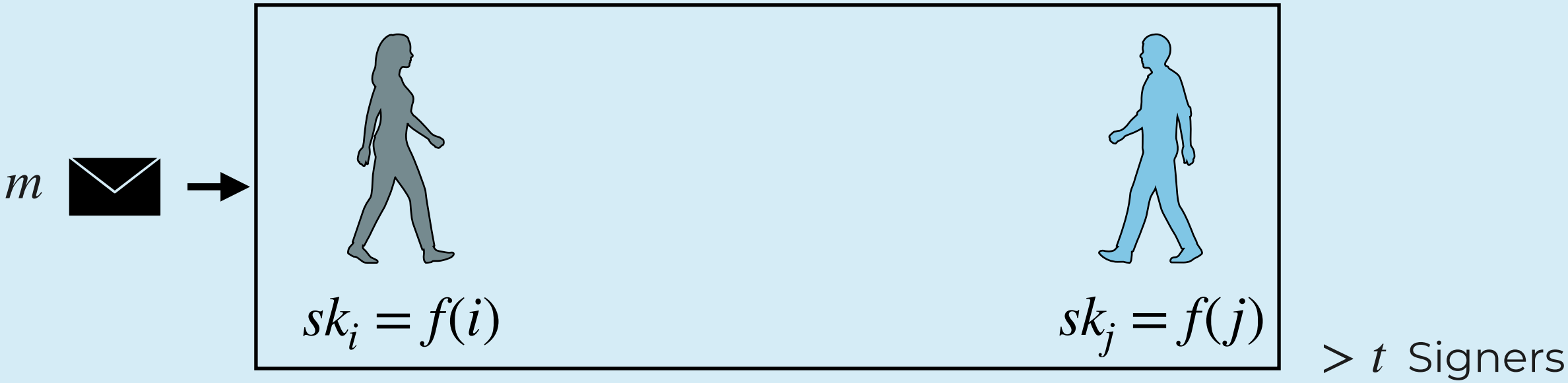

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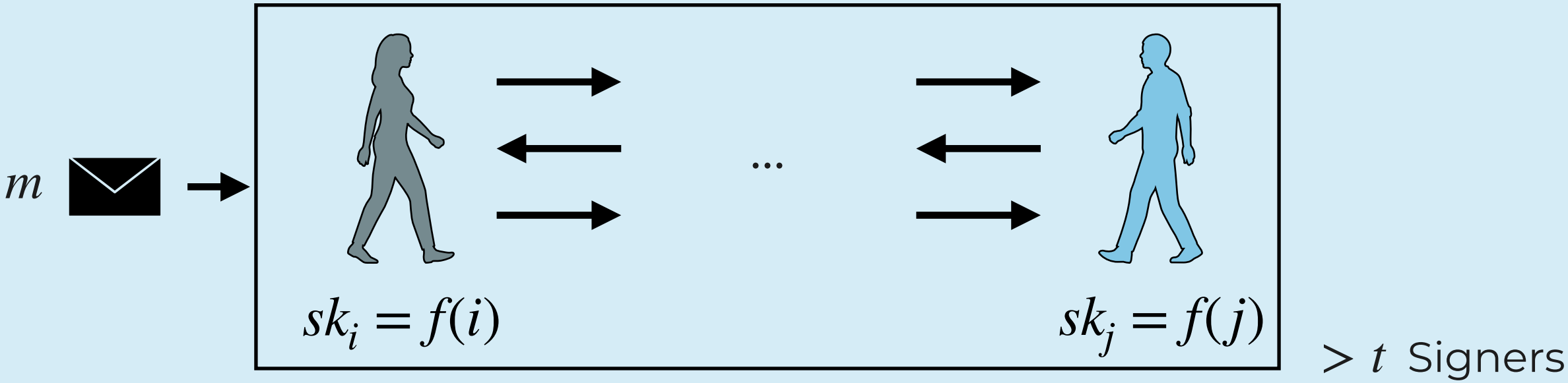

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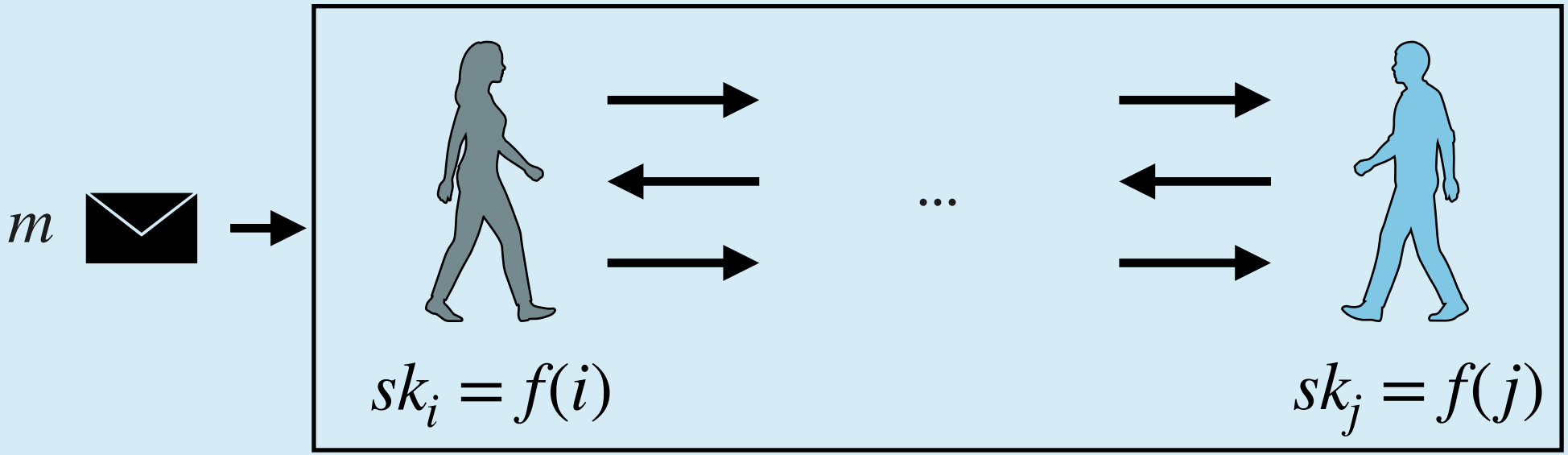

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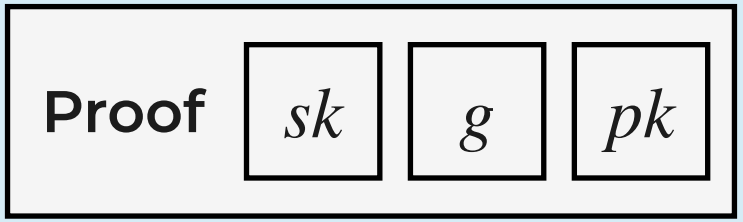

"We know sk such that $pk = g^{sk}$"

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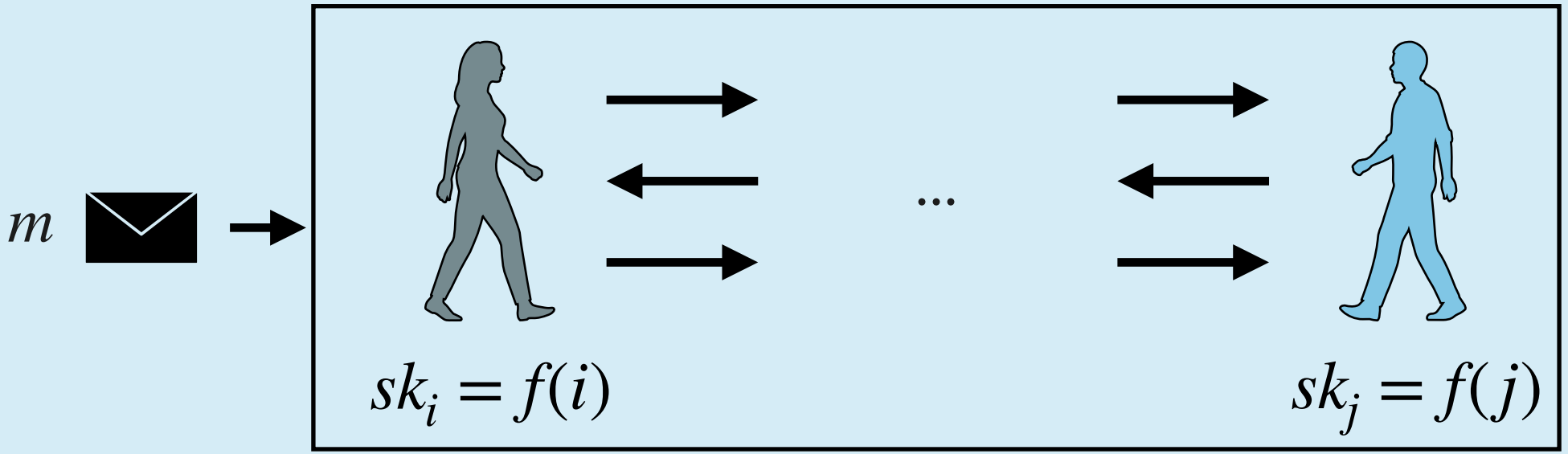

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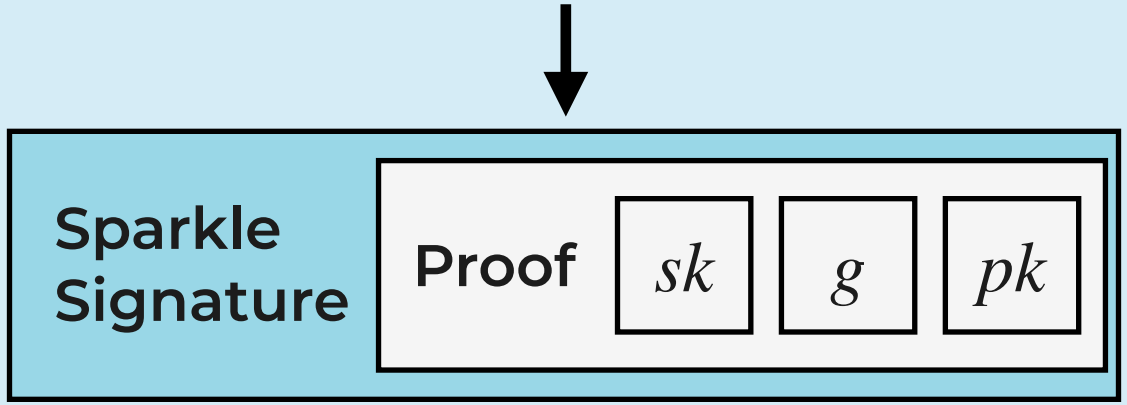

"We know sk such that $pk = g^{sk}$"

Sparkle’s Security Proof

Sparkle's Security Proof

One-More DLOG

Reduction

Sparkle Adversary

Sparkle's Security Proof

One-More DLOG

Reduction

Sparkle Adversary

Simplification: No Signing Query

Sparkle's Security Proof

One-More DLOG

$$X_0 = g^{f(0)}, \ldots, X_n = g^{f(n)}$$

Reduction

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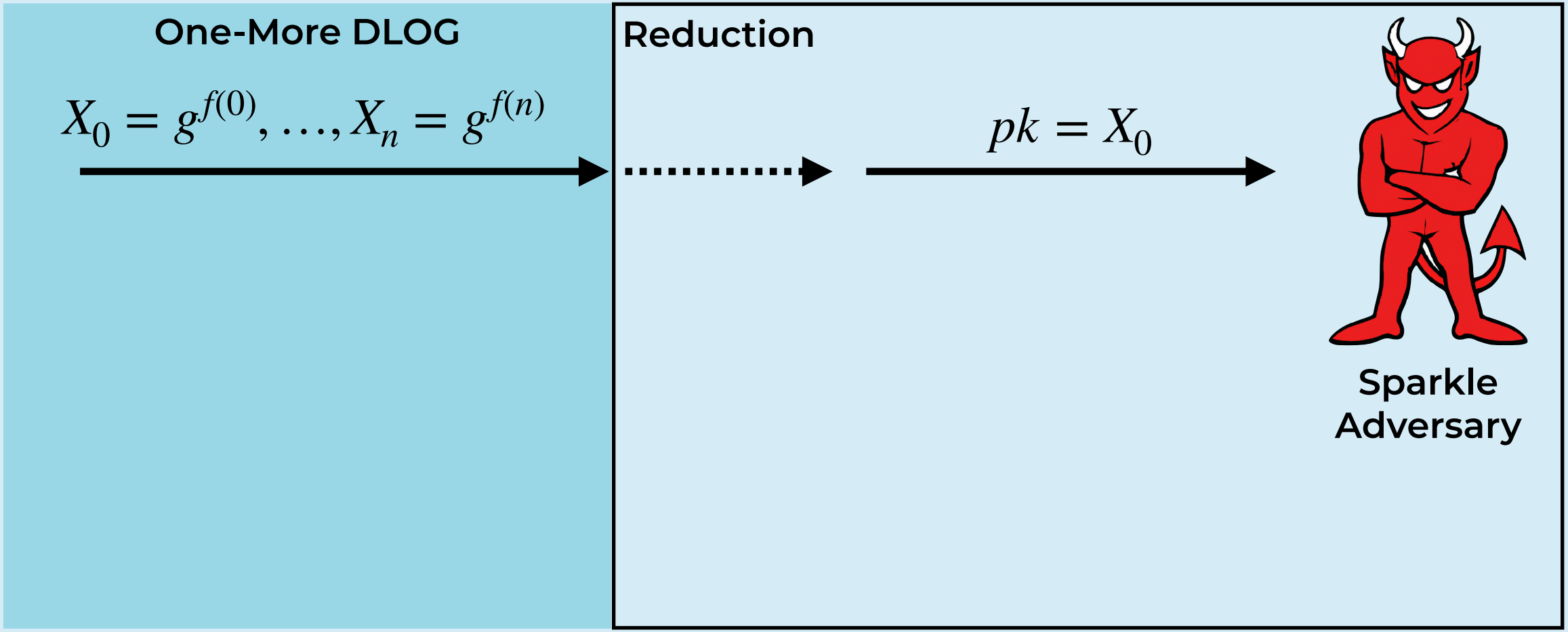
Sparkle's Security Proof



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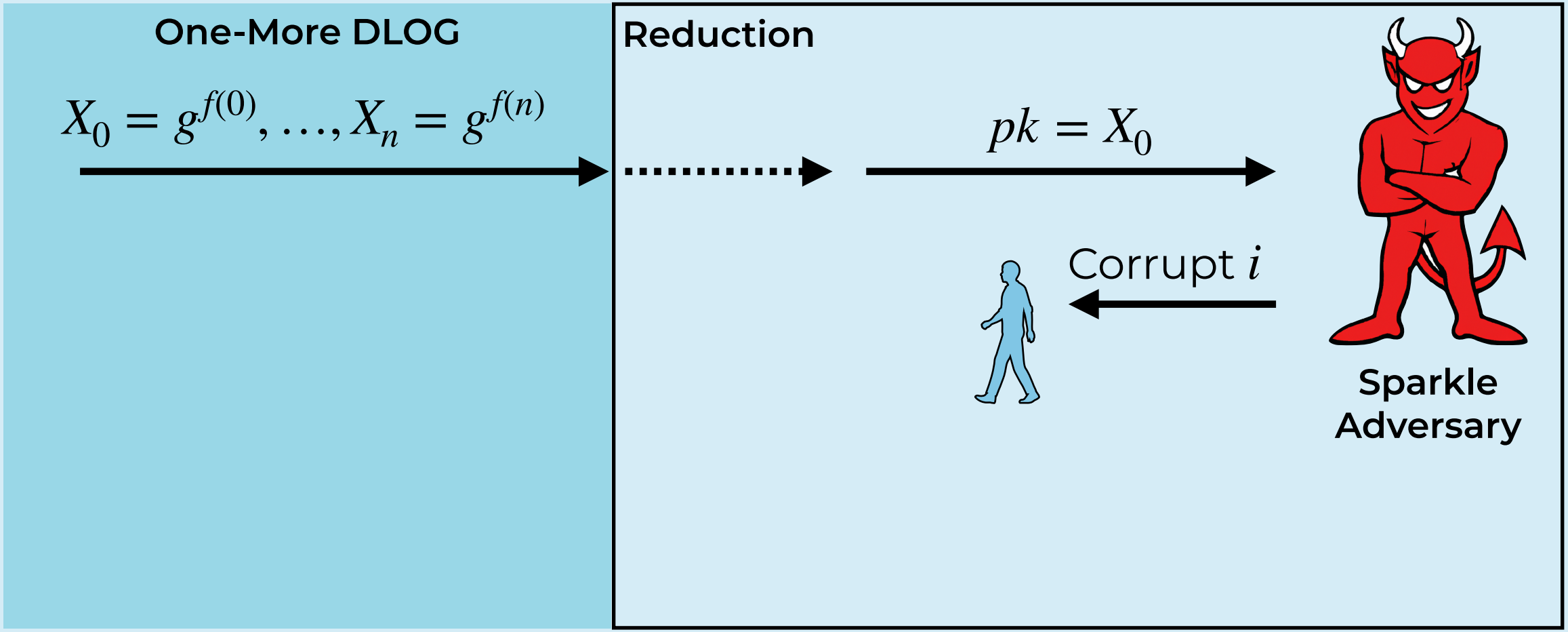

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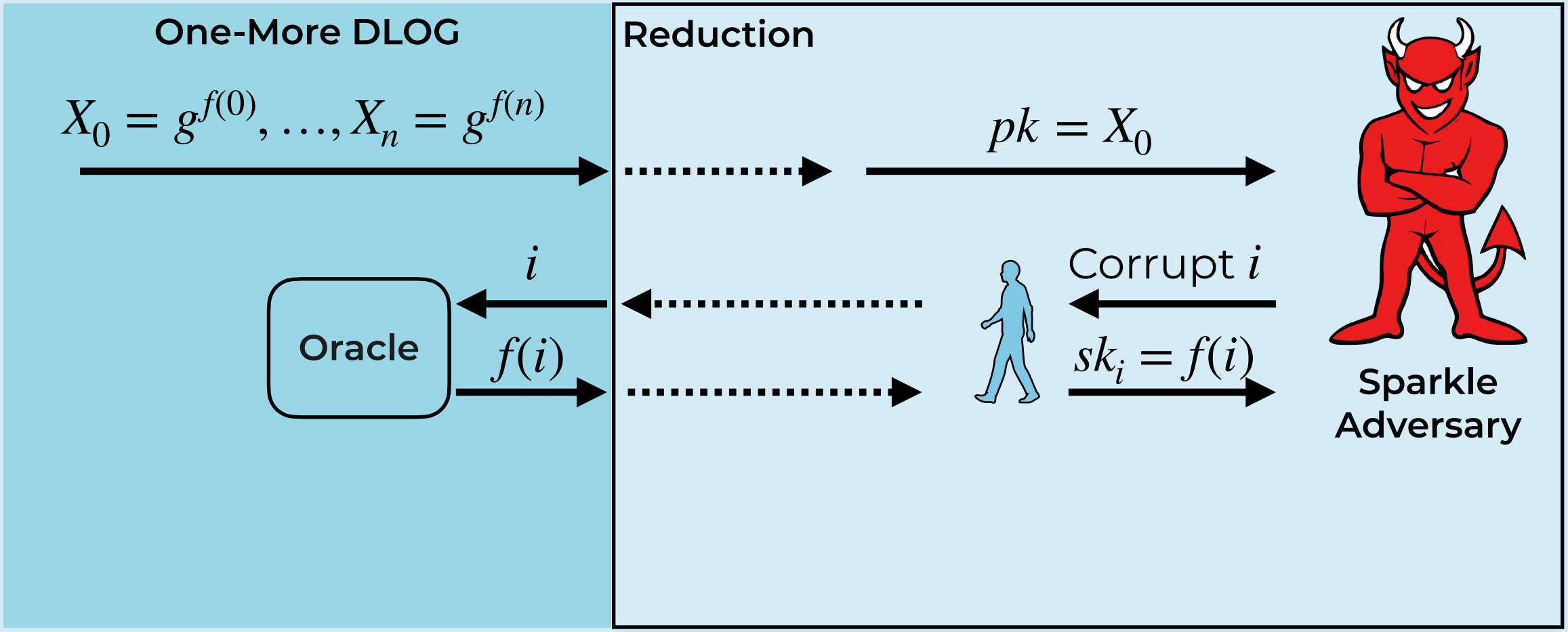

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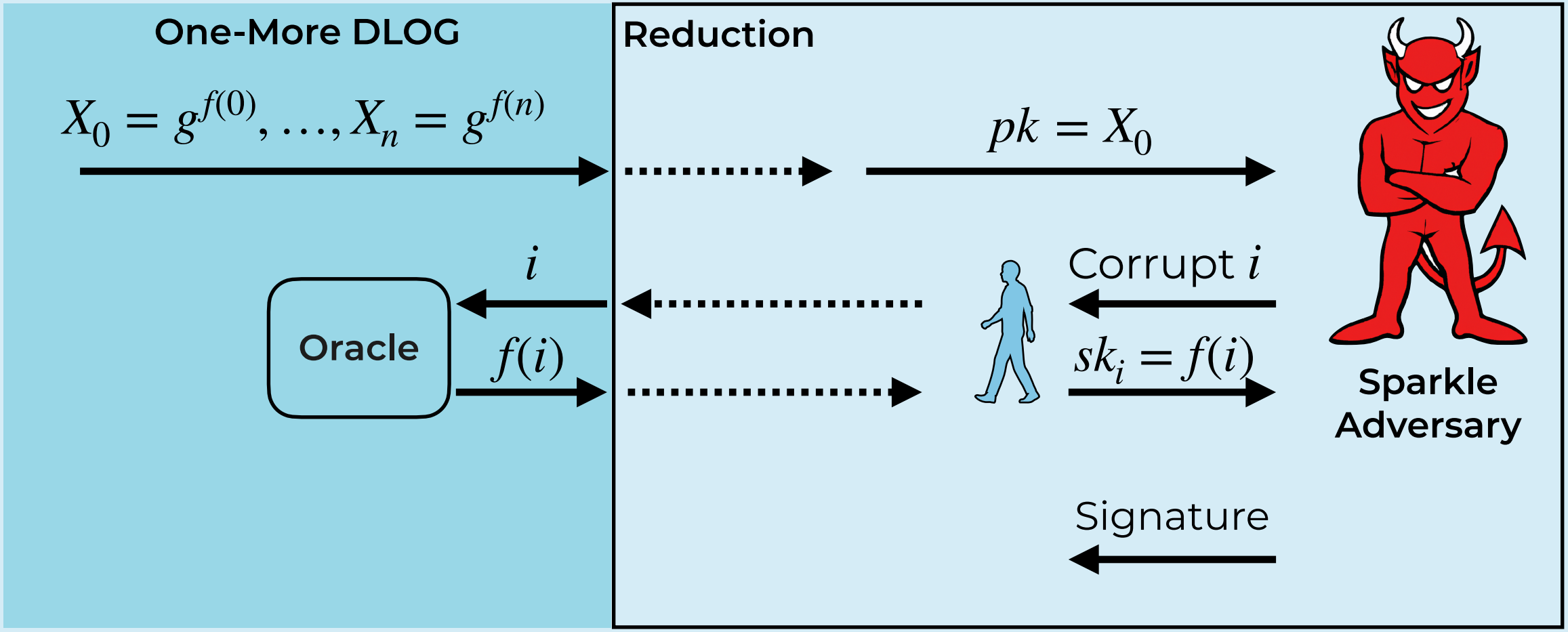

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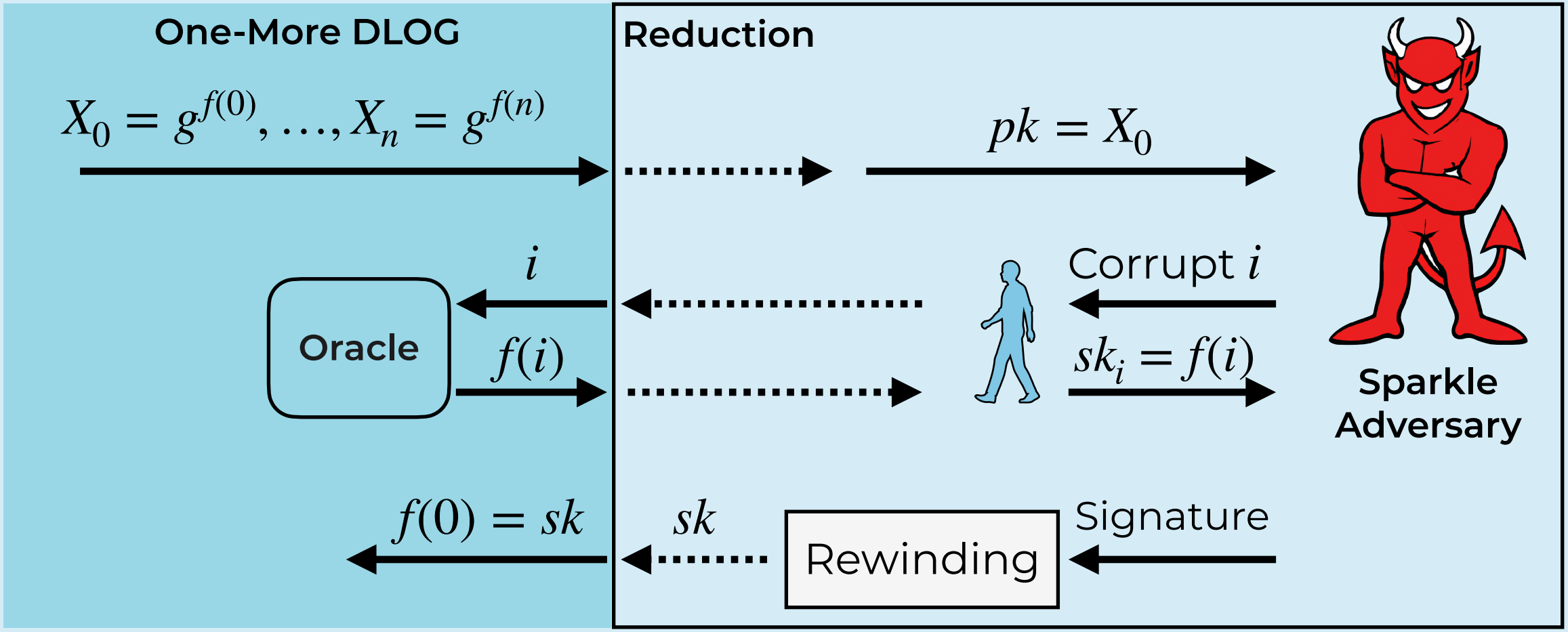

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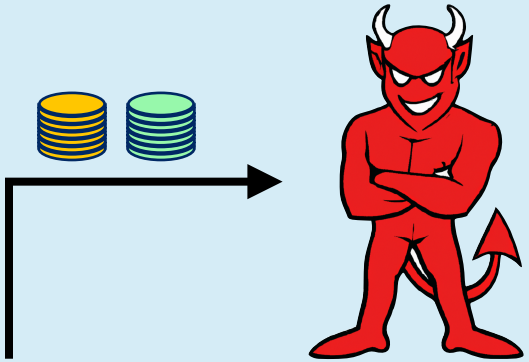
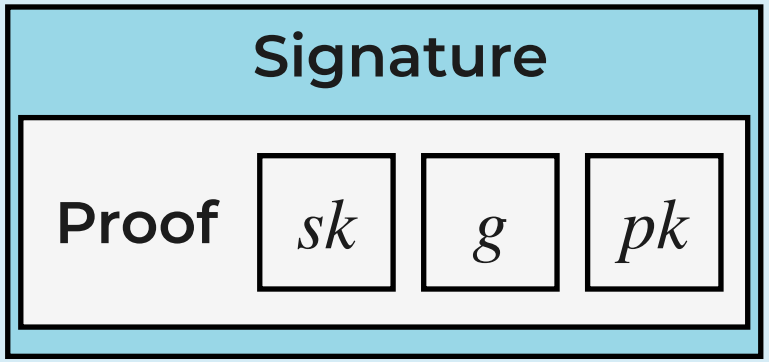


t_c corruptions



$pk = X_0$

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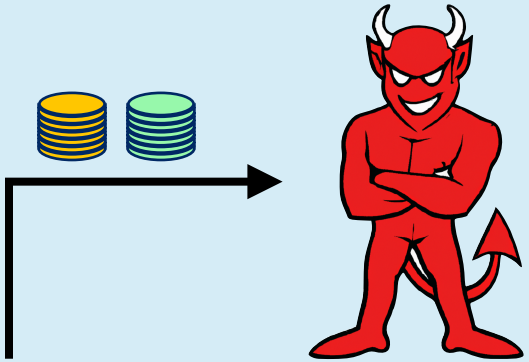

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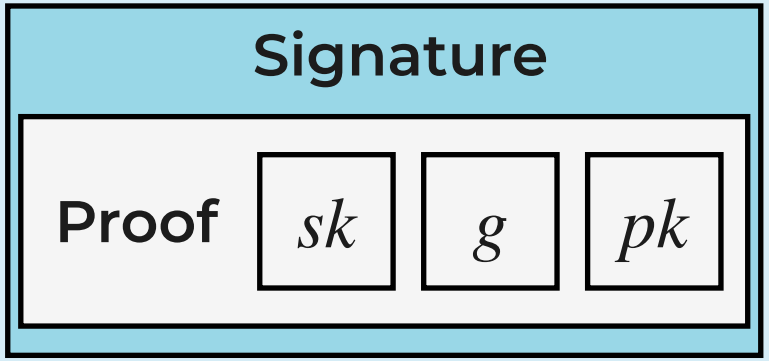

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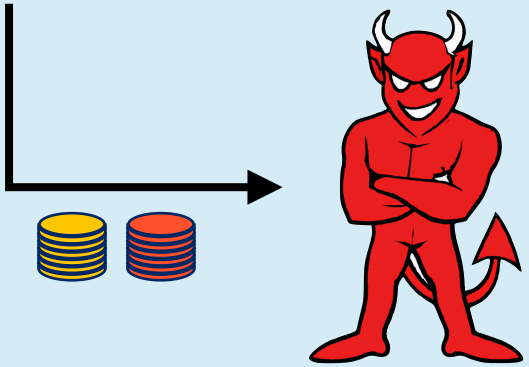

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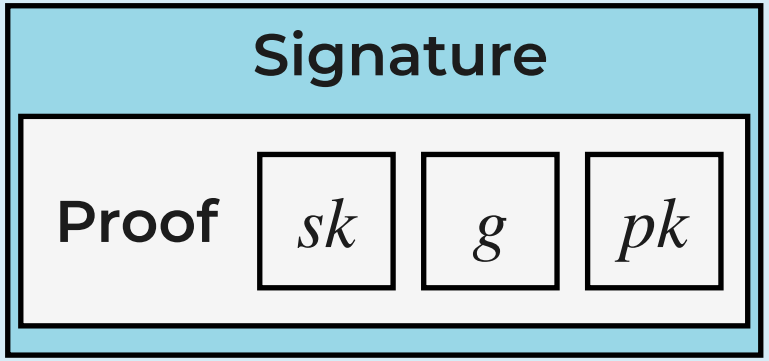
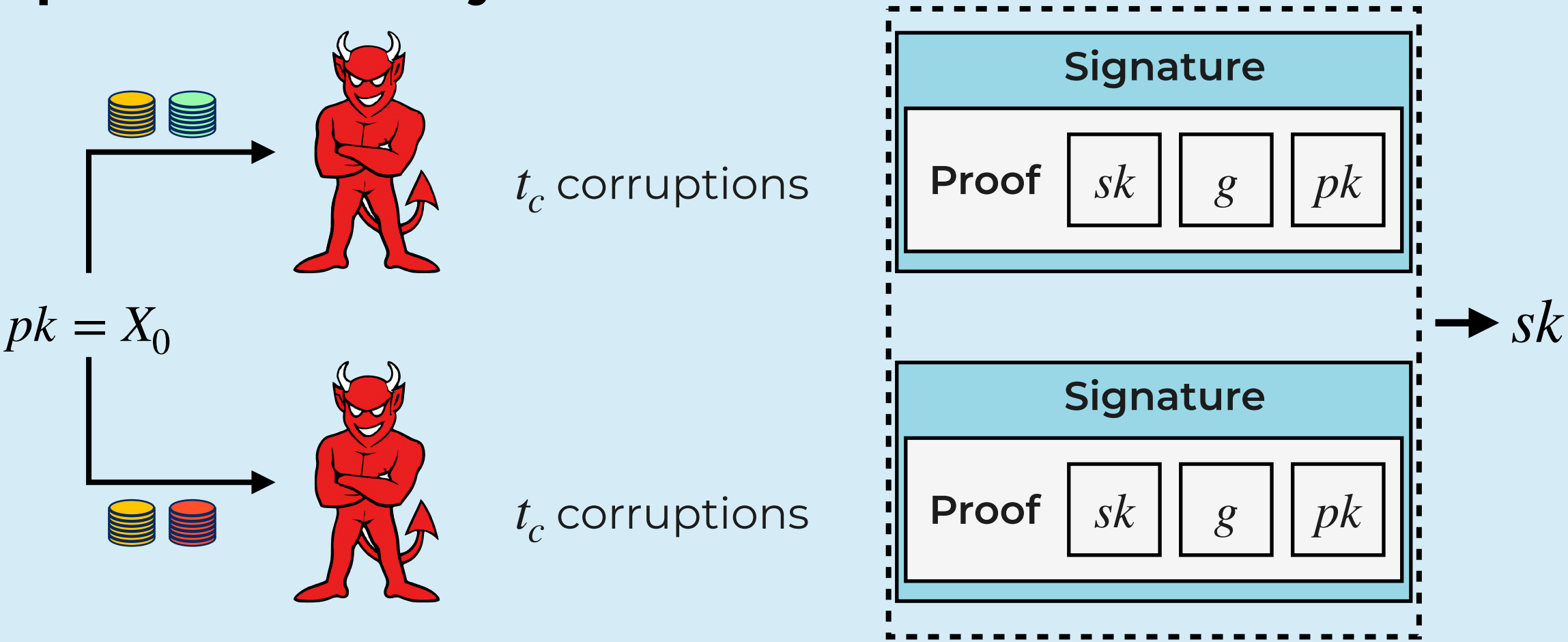
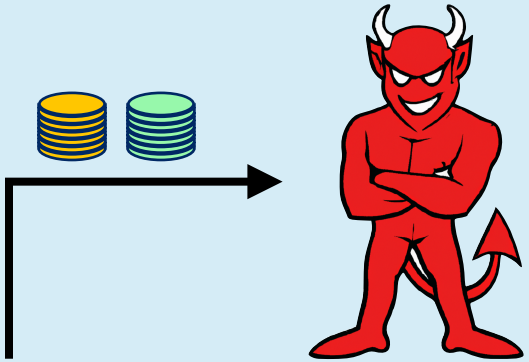


Sparkle's Security Proof

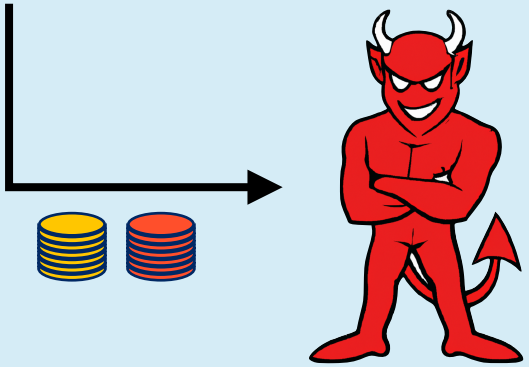


Sparkle's Security Proof



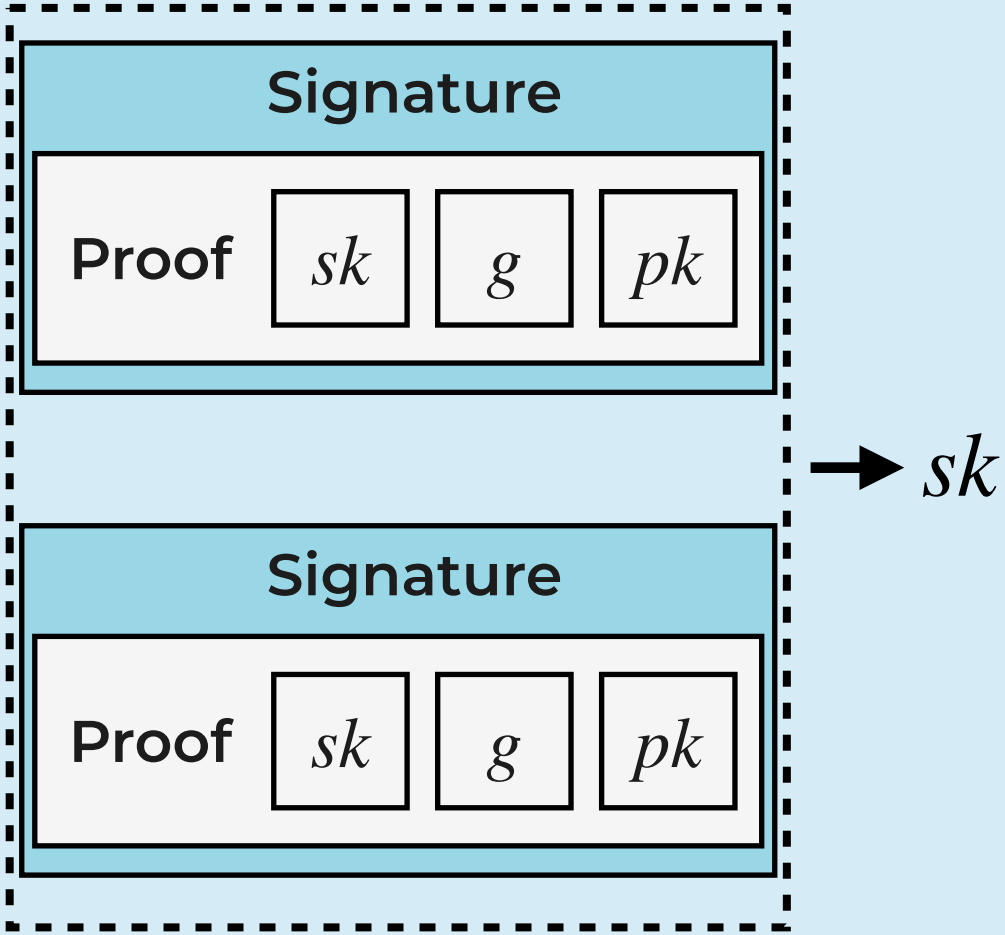
t_c corruptions

$pk = X_0$

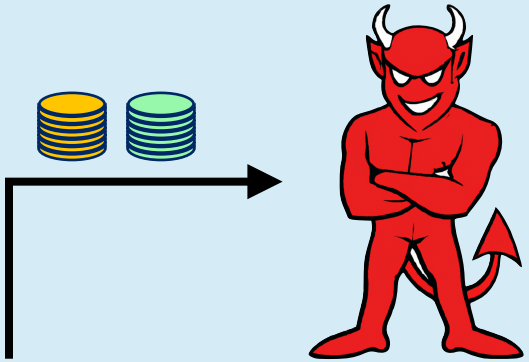


t_c corruptions

$\Longrightarrow 2t_c$ corruptions

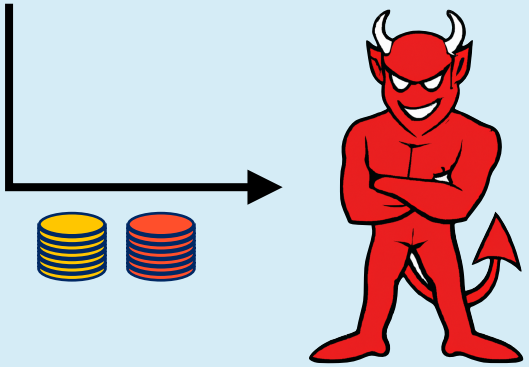


Sparkle's Security Proof

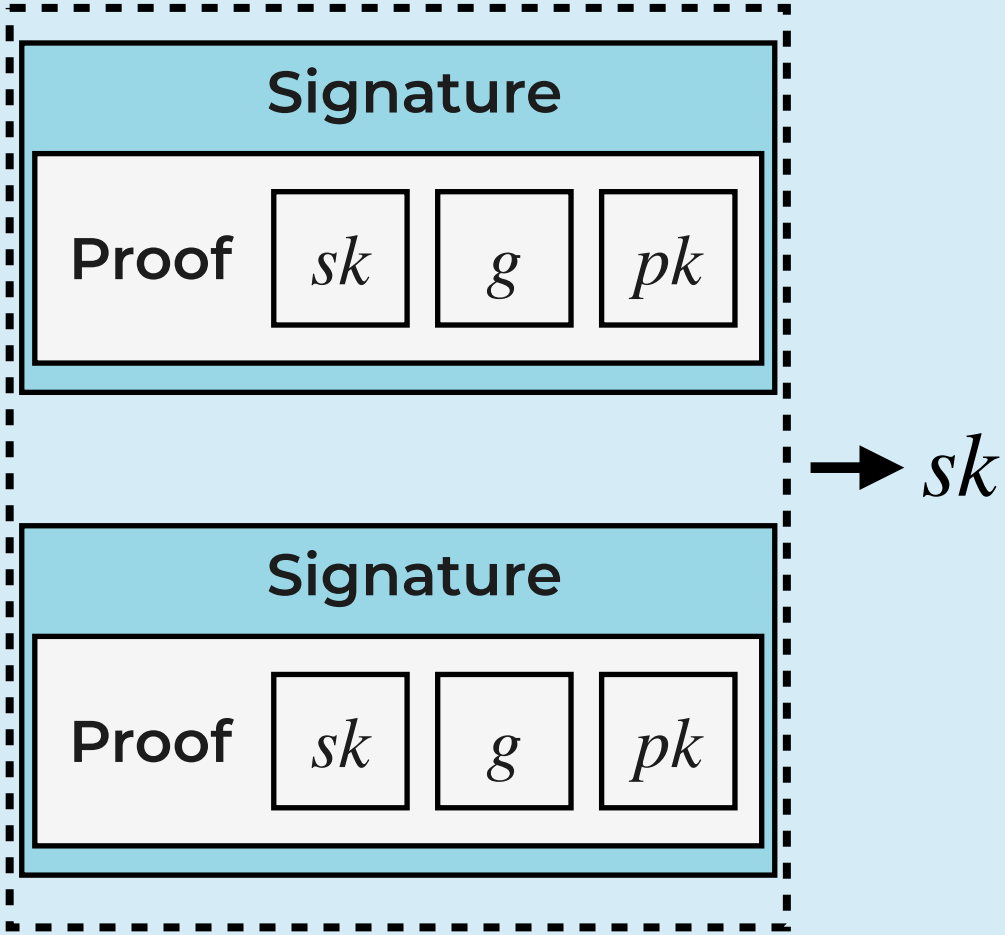


t_c corruptions

$pk = X_0$



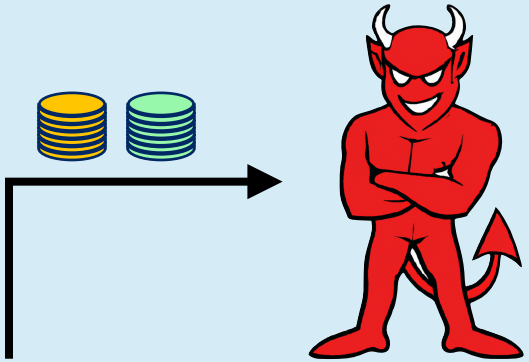
t_c corruptions



$\Longrightarrow 2t_c$ corruptions

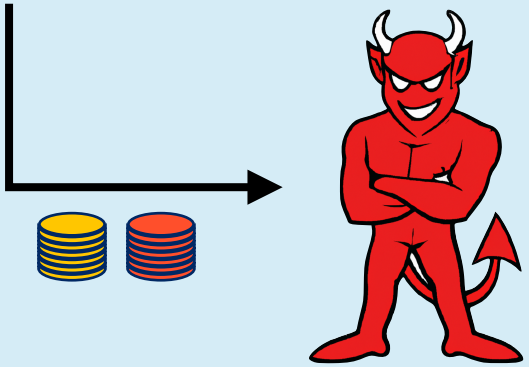
$\Longrightarrow 2t_c$ DLOG queries

Sparkle's Security Proof

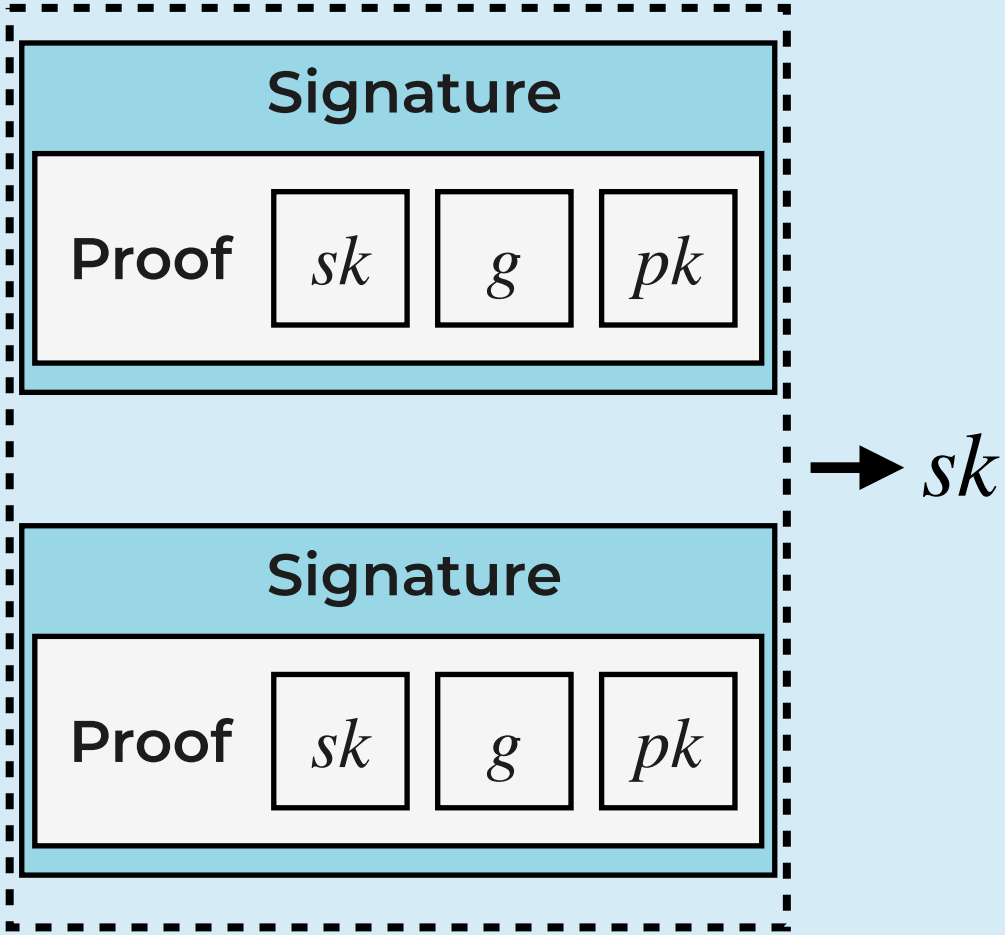


t_c corruptions

$pk = X_0$



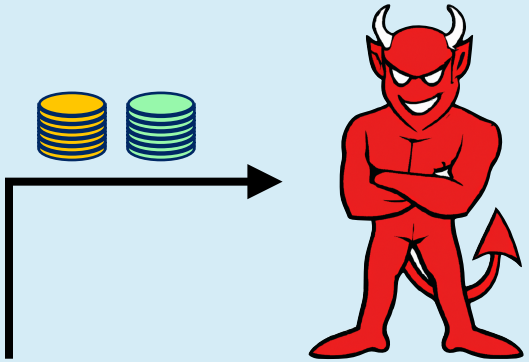
t_c corruptions



$\Longrightarrow 2t_c$ corruptions

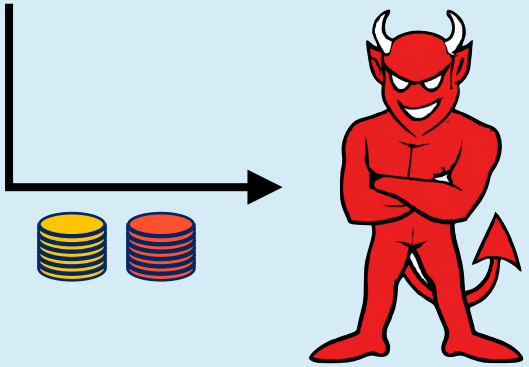
$\Longrightarrow 2t_c$ DLOG queries $\Longrightarrow 2t_c \leq t$

Sparkle's Security Proof

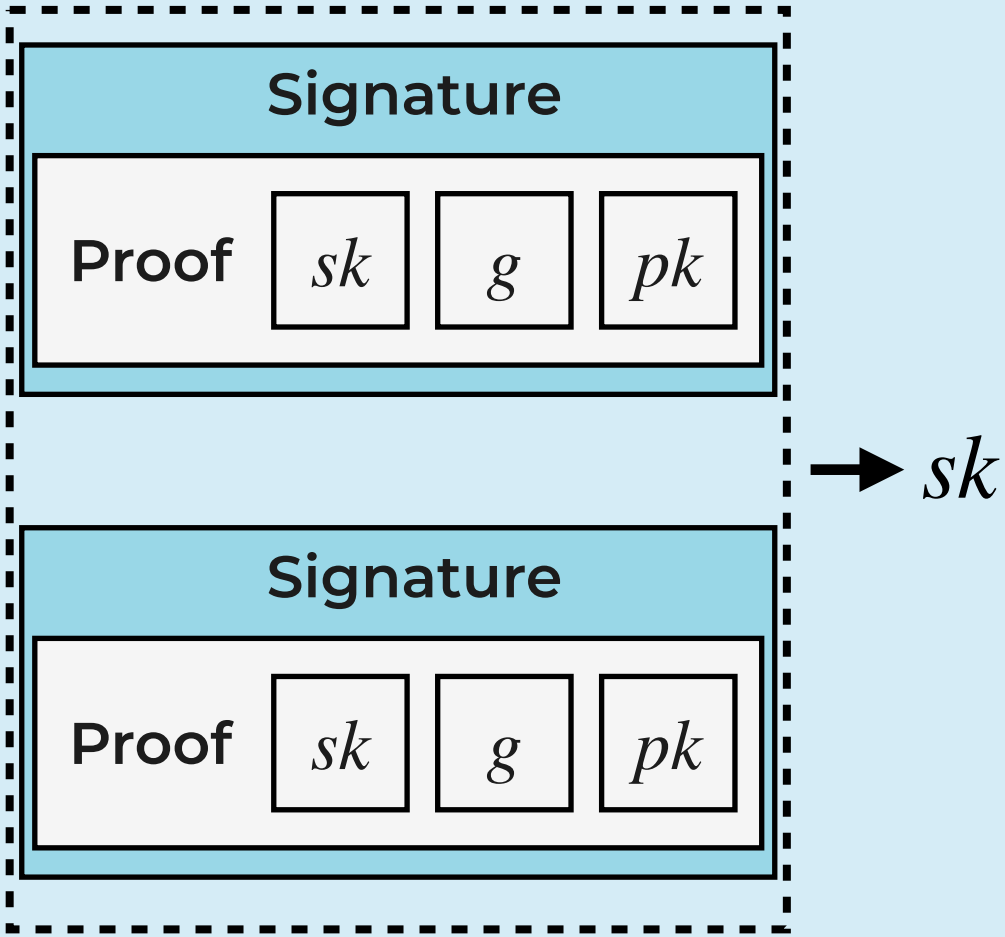


t_c corruptions

$pk = X_0$



t_c corruptions



$\Longrightarrow 2t_c$ corruptions

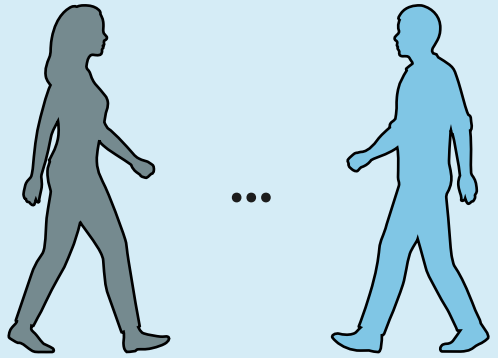
$\Longrightarrow 2t_c$ DLOG queries $\Longrightarrow 2t_c \leq t$

Goal: Avoid Rewinding

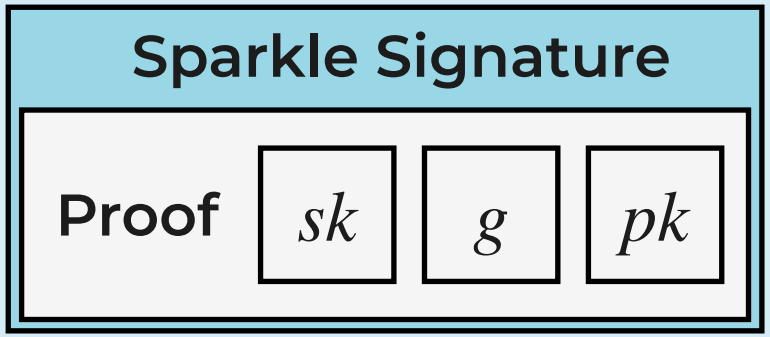
Our Solution

Our Solution

$$pk = g^{sk}$$



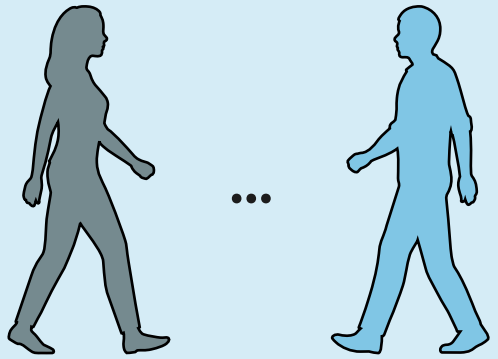
$sk_i = f(i)$ $sk_j = f(j)$



*"We **know** sk such that $pk = g^{sk}$"*

Our Solution

$$pk = g^{sk}$$



$sk_i = f(i)$ ... $sk_j = f(j)$

Sparkle Signature

Proof	sk	g	pk

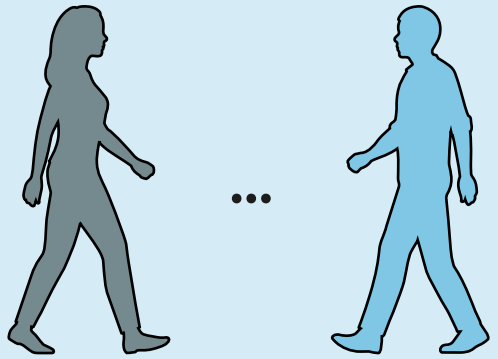
*"We **know** sk such that $pk = g^{sk}$"*

Twinkle Signature

$pk' = h^{sk}$ for $h = H(m)$

Our Solution

$$pk = g^{sk}$$



$sk_i = f(i)$ ... $sk_j = f(j)$

Sparkle Signature

Proof	sk	g	pk

*"We **know** sk such that $pk = g^{sk}$"*

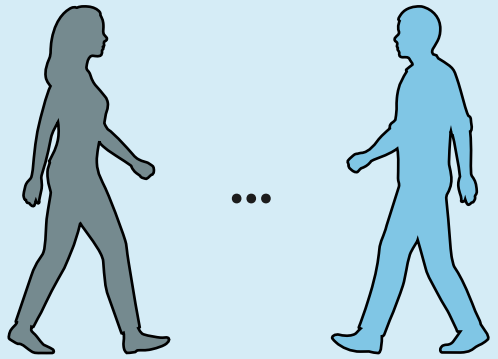
Twinkle Signature

$pk' = h^{sk}$ for $h = H(m)$

Proof	sk	g	pk
		h	pk'

Our Solution

$$pk = g^{sk}$$



$sk_i = f(i)$ ... $sk_j = f(j)$

Sparkle Signature

Proof	sk	g	pk

*"We **know** sk such that $pk = g^{sk}$"*

Twinkle Signature

$pk' = h^{sk}$ for $h = H(m)$

Proof	sk	g	pk
		h	pk'

*"There **is** sk such that $pk = g^{sk}$ and $pk' = h^{sk}$"*

Our Solution

Our Solution

$$pk = g^{sk}$$

Twinkle
Adversary

Our Solution

$$pk = g^{sk}$$

Twinkle Adversary

Twinkle Signature

pk' for $h = H(m^*)$

Proof

Our Solution

$pk = g^{sk}$

Twinkle Adversary

Twinkle Signature

pk' for $h = H(m^*)$

Proof

Case 1. $pk' = h^{sk}$

Our Solution

$pk = g^{sk}$

Twinkle Adversary

Twinkle Signature

pk' for $h = H(m^*)$

Proof

Case 1. $pk' = h^{sk}$

Solve CDH for $pk = g^{sk}, h$

Our Solution

$pk = g^{sk}$

Twinkle Adversary

Twinkle Signature

pk' for $h = H(m^*)$

Proof

Case 1. $pk' = h^{sk}$

Solve CDH for $pk = g^{sk}, h$

No Rewinding!

Our Solution

$pk = g^{sk}$

Twinkle Adversary

Twinkle Signature

pk' for $h = H(m^*)$

Proof

Case 1. $pk' = h^{sk}$

Solve CDH for $pk = g^{sk}, h$

No Rewinding!

Case 2. $pk' \neq h^{sk}$

Our Solution

$pk = g^{sk}$

Twinkle Adversary

Twinkle Signature

pk' for $h = H(m^*)$

Proof

Case 1. $pk' = h^{sk}$

Solve CDH for $pk = g^{sk}, h$

No Rewinding!

Case 2. $pk' \neq h^{sk}$

Generating Proof statistically hard

Our Solution

$pk = g^{sk}$

Twinkle Adversary

Twinkle Signature

pk' for $h = H(m^*)$

Proof

Case 1. $pk' = h^{sk}$

Solve CDH for $pk = g^{sk}, h$

No Rewinding!

Case 2. $pk' \neq h^{sk}$

Generating Proof statistically hard

No Reduction Needed!

Our Solution

CDH

No Corruptions

No Signing

No Rewinding

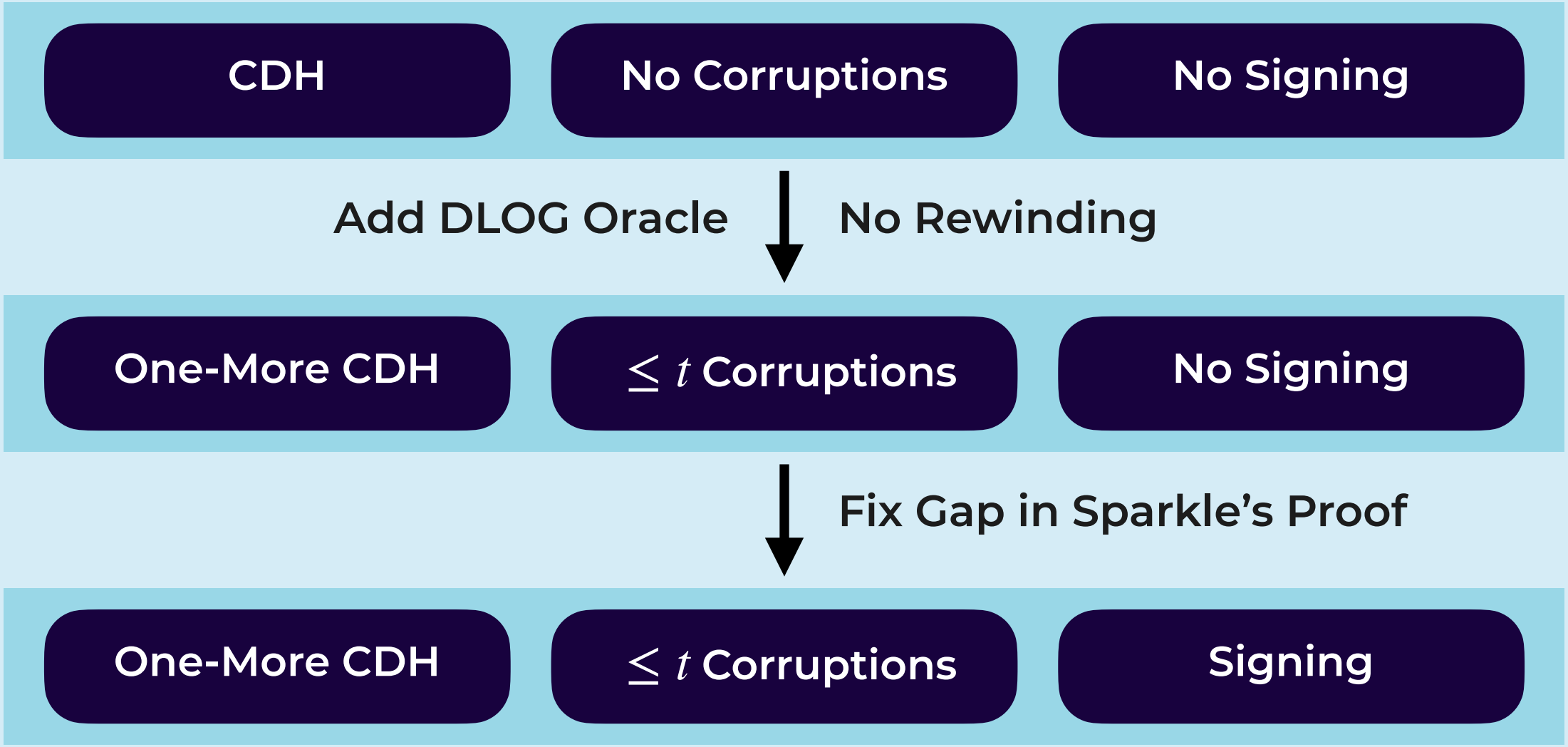
Our Solution

CDH | No Corruptions | No Signing

Add DLOG Oracle ↓ No Rewinding

One-More CDH | $\leq t$ Corruptions | No Signing

Our Solution



Summary and Future Work

Summary and Future Work

Twinkle

DDH Assumption

Full Adaptive Security

Summary and Future Work

Twinkle

DDH Assumption

Full Adaptive Security

Two-Round Construction?

Summary and Future Work

Twinkle

DDH Assumption

Full Adaptive Security

Two-Round Construction?

Non-Interactive Search Assumption?

Twinkle: Threshold Signatures from DDH with Full Adaptive Security

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Abstract

Sparkle is the first threshold signature scheme in the pairing-free discrete logarithm setting (Crites, Komlo, Maller, Crypto 2023) to be proven secure under adaptive corruptions. However, without using the algebraic group model, Sparkle's proof imposes an undesirable restriction on the adversary. Namely, for a signing threshold $t < n$, the adversary is restricted to corrupt at most $t/2$ parties. In addition, Sparkle's proof relies on a strong one-more assumption.

In this work, we propose Twinkle, a new threshold signature scheme in the pairing-free setting which overcomes these limitations. Twinkle is the first pairing-free scheme to have a security proof under up to t adaptive corruptions without relying on the algebraic group model. It is also the first such scheme with a security proof under adaptive corruptions from a well-studied non-interactive assumption, namely, the Decisional Diffie-Hellman (DDH) assumption.

We achieve our result in two steps. First, we design a generic scheme based on a linear function that satisfies several abstract properties and prove its adaptive security under a suitable one-more assumption related to this function. In the context of this proof, we also identify a gap in the security proof of Sparkle and develop new techniques to overcome this issue. Second, we give a suitable instantiation of the function for which the corresponding one-more assumption follows from DDH.

Keywords: Threshold Signatures, Adaptive Security, Pairing-Free, Non-Interactive Assumptions

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