SLAP 👋

Succinct Lattice-Based Polynomial Commitment Schemes from Standard Assumptions (2023/1469)

Giacomo Fenzi @ EPFL

Joint work with:
Martin Albrecht
Ngoc Khanh Nguyen

Oleksandra Lapiha

Motivation

SNARKs

SNARKs

(Succinct Non-Interactive ARguments of Knowledge)

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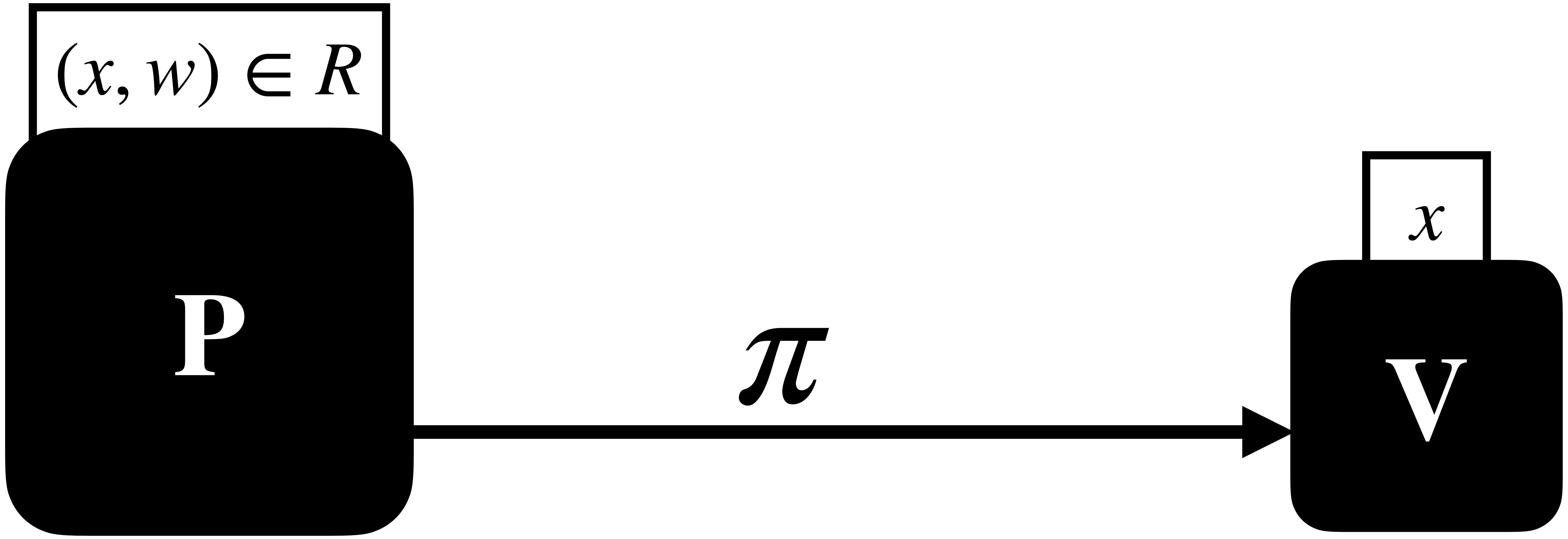

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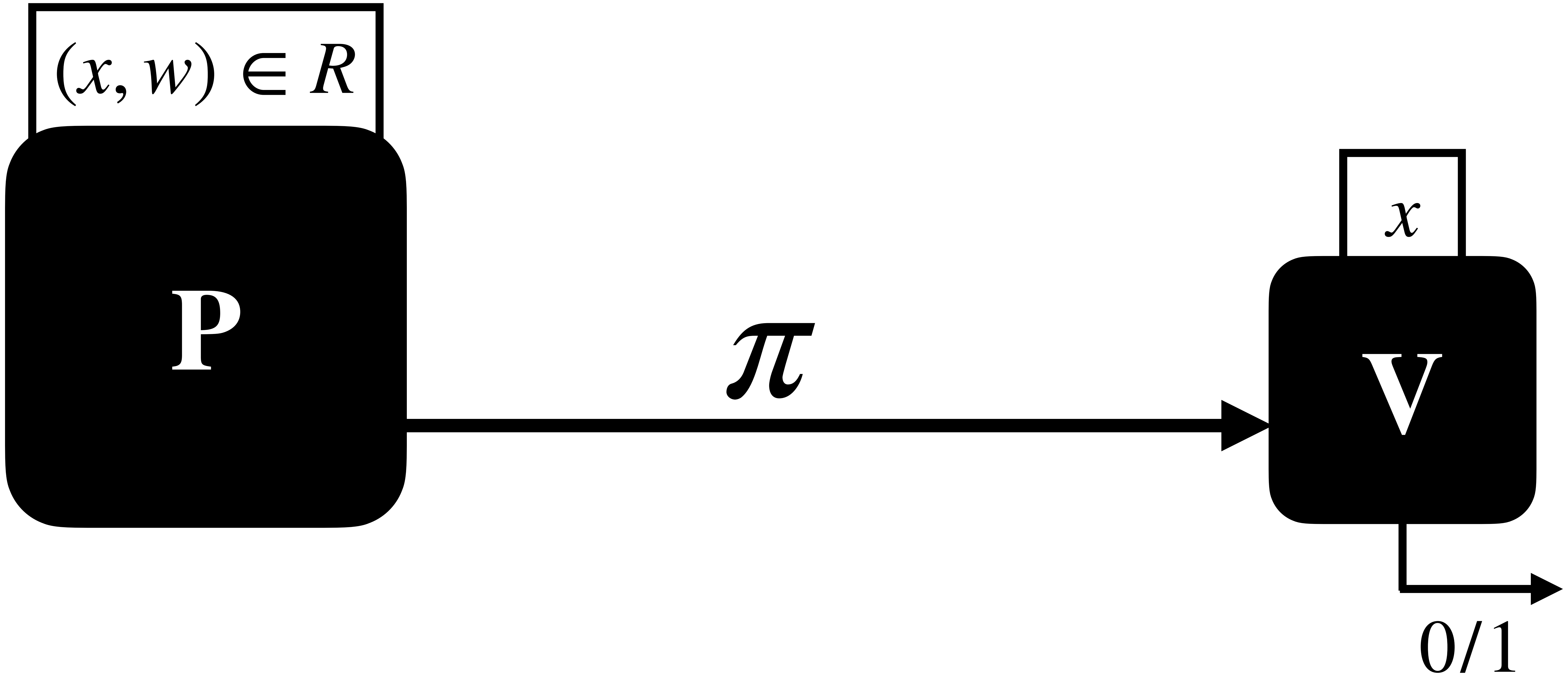

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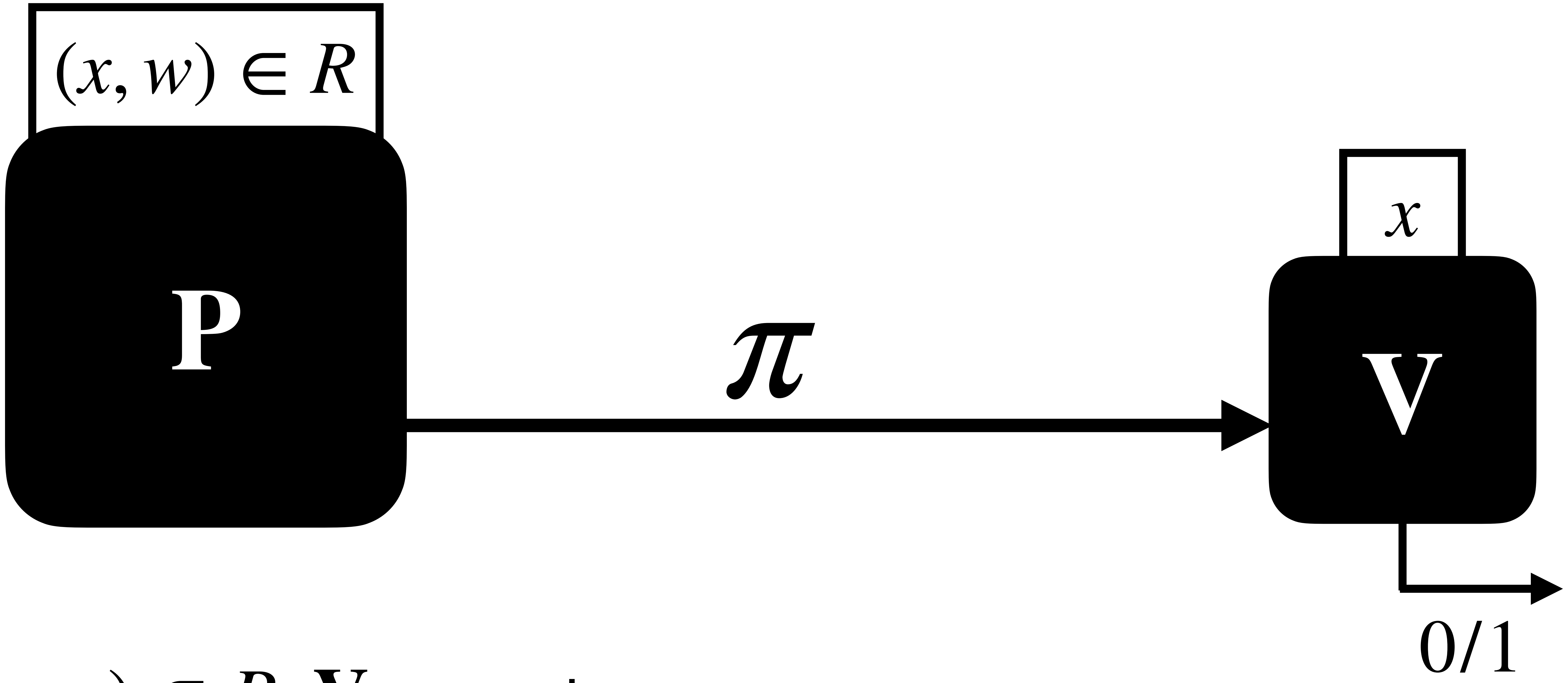

Complete: if $(x, w) \in R$, $\mathbf{V}$ accepts.

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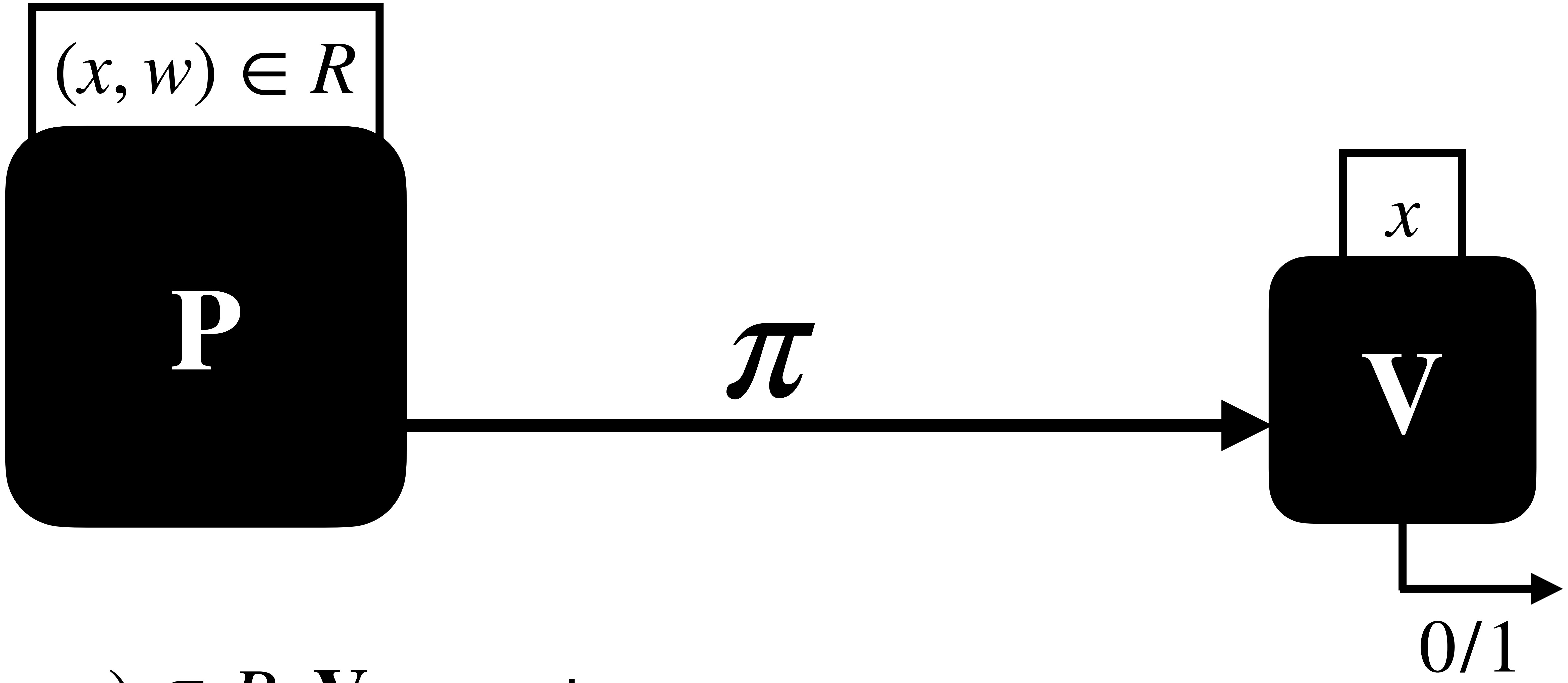

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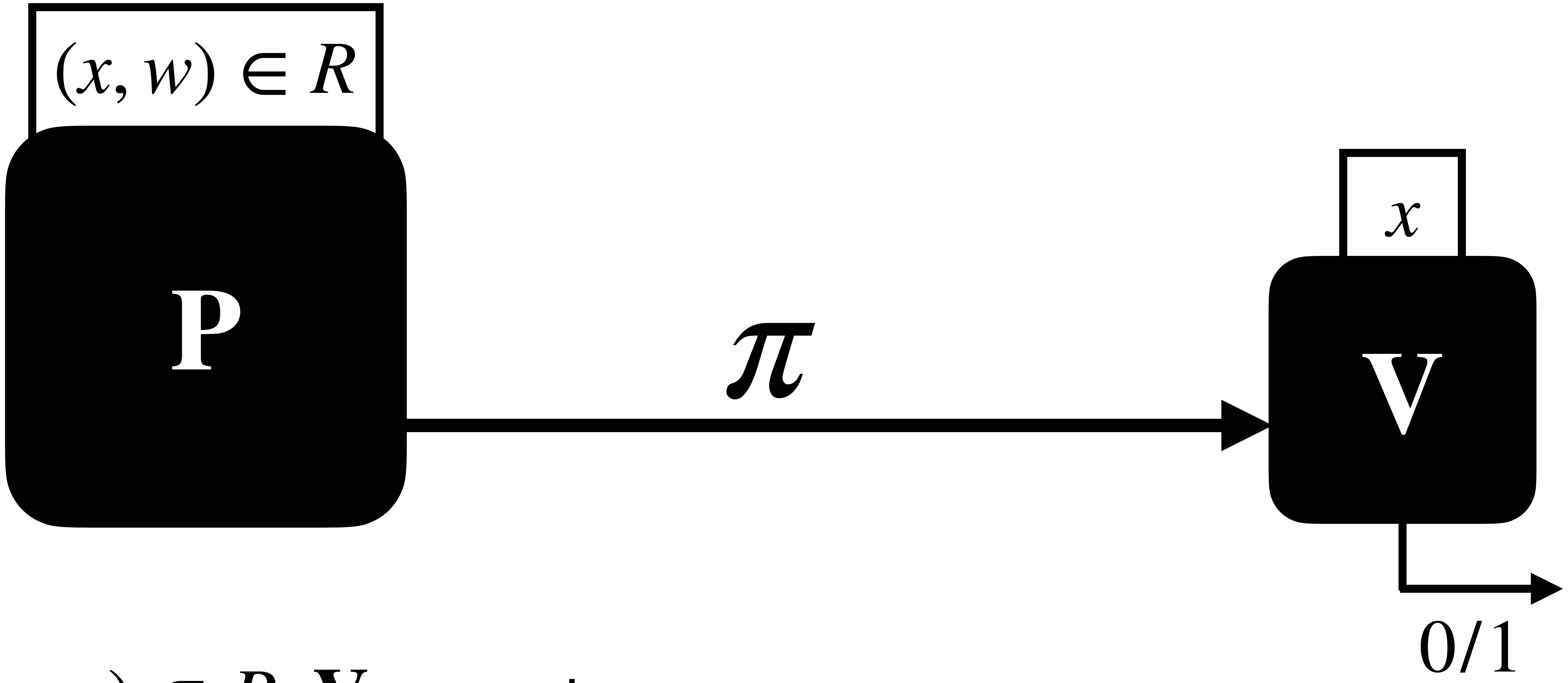

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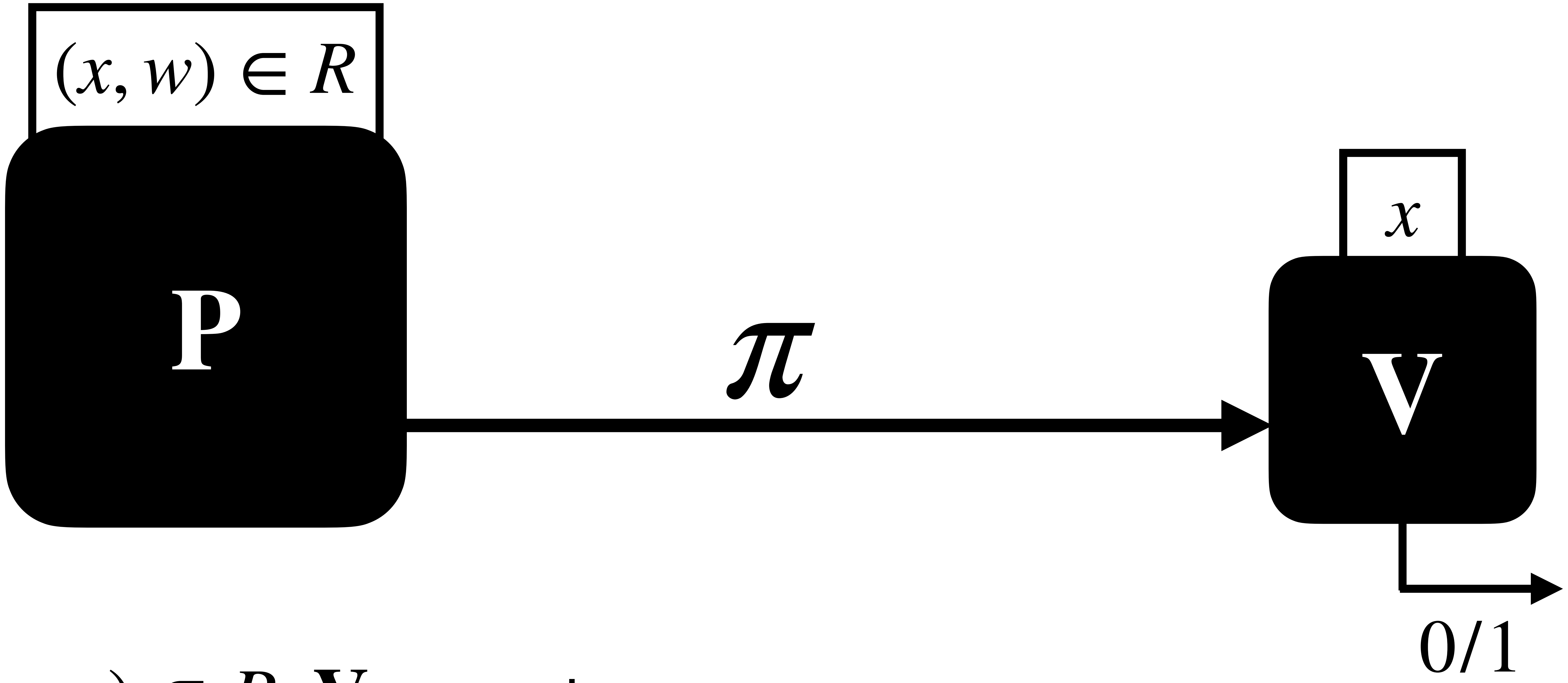

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Knowledge Sound: if $\mathbf{V}(x, \pi) = 1$, can extract w such that $(x, w) \in R$

Constructing SNARKs

The modular way™

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PIOP

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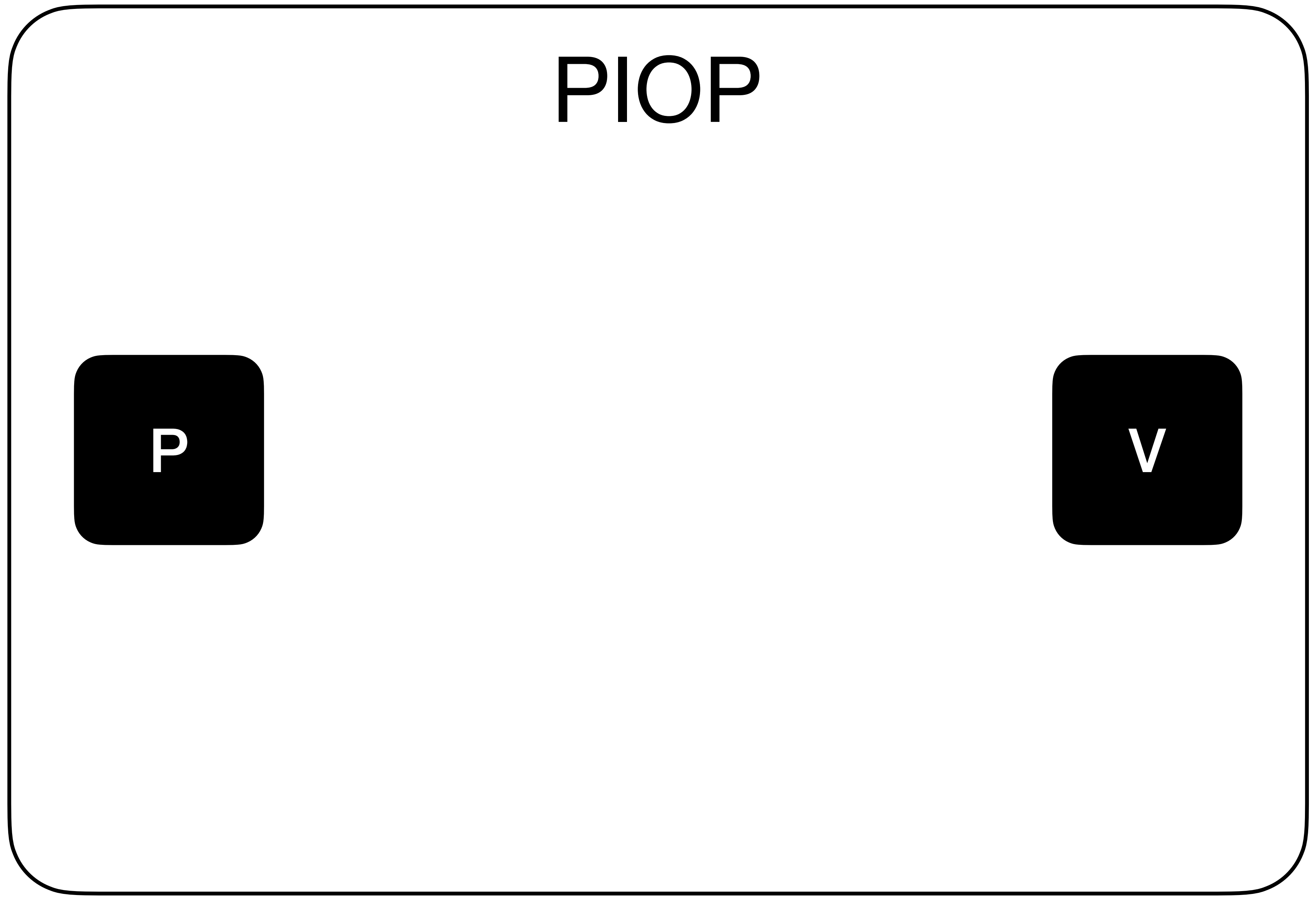

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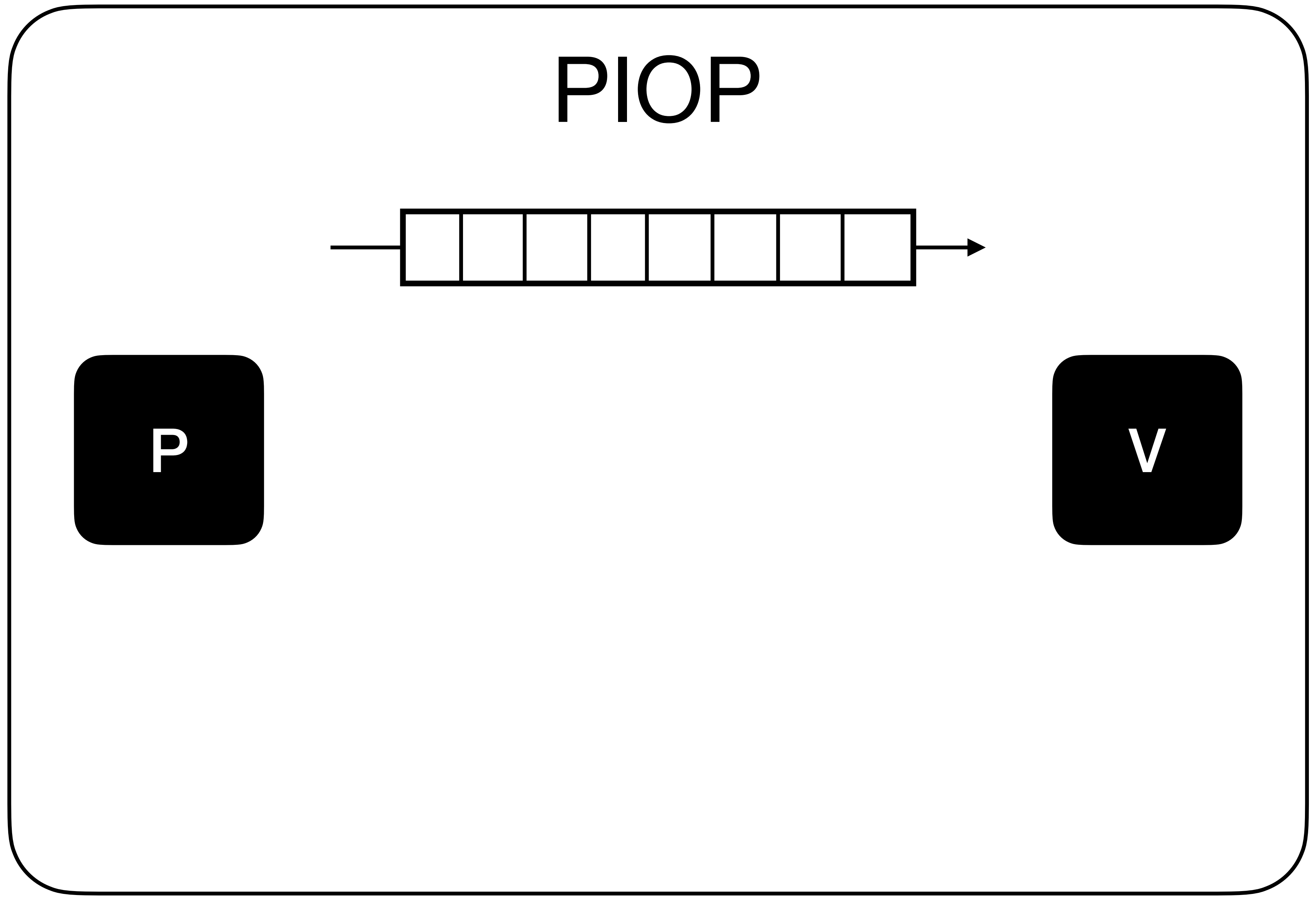

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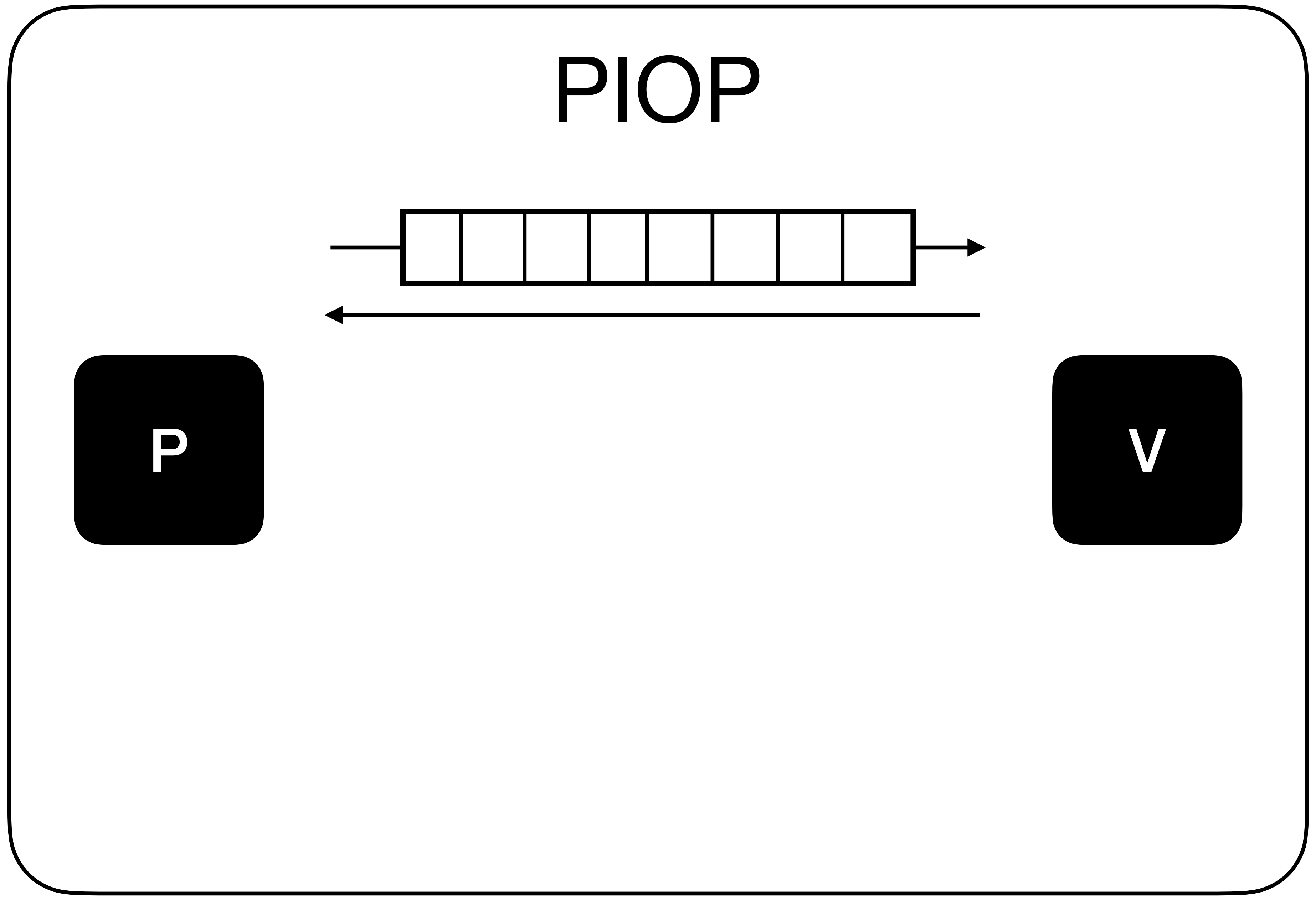

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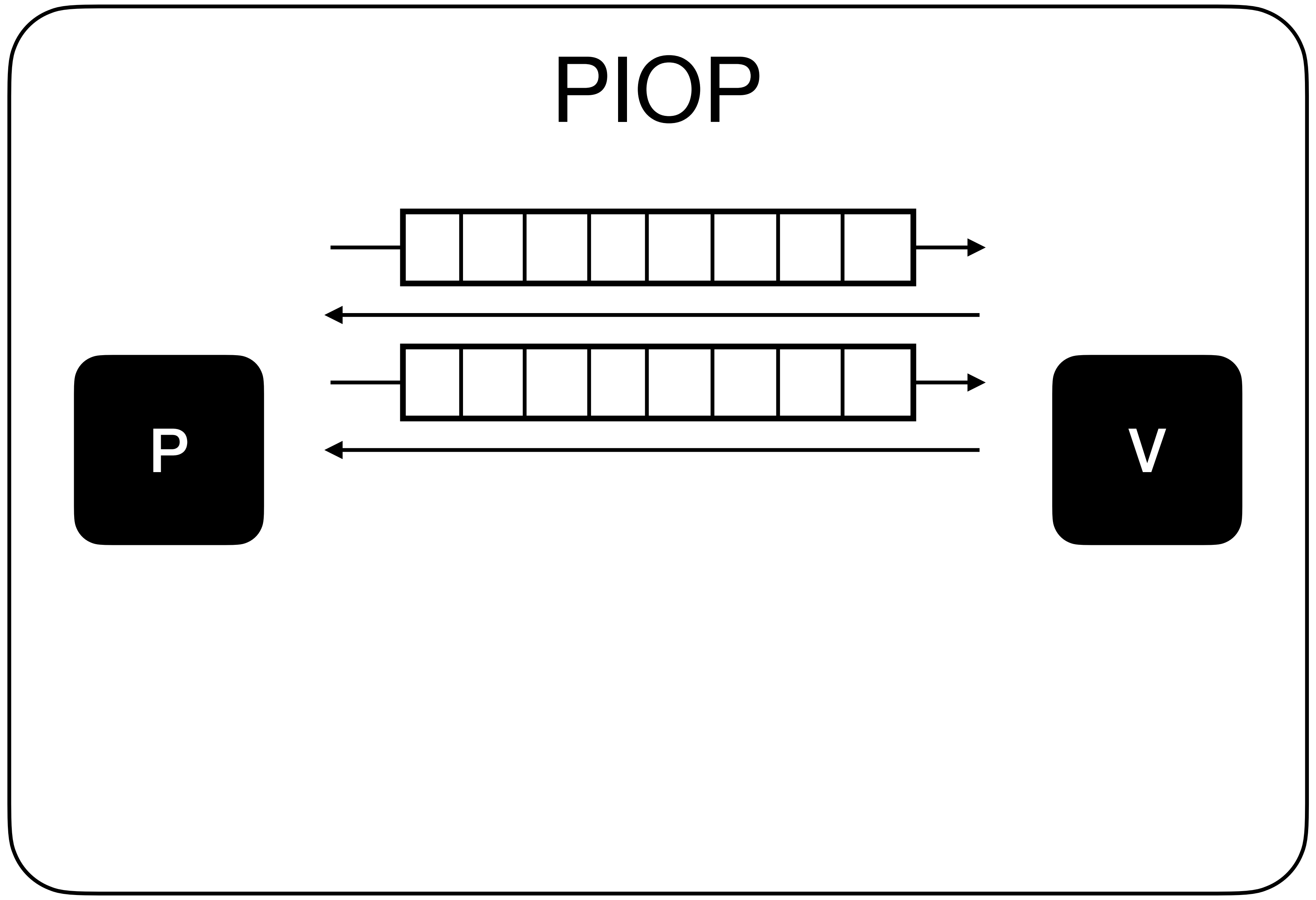

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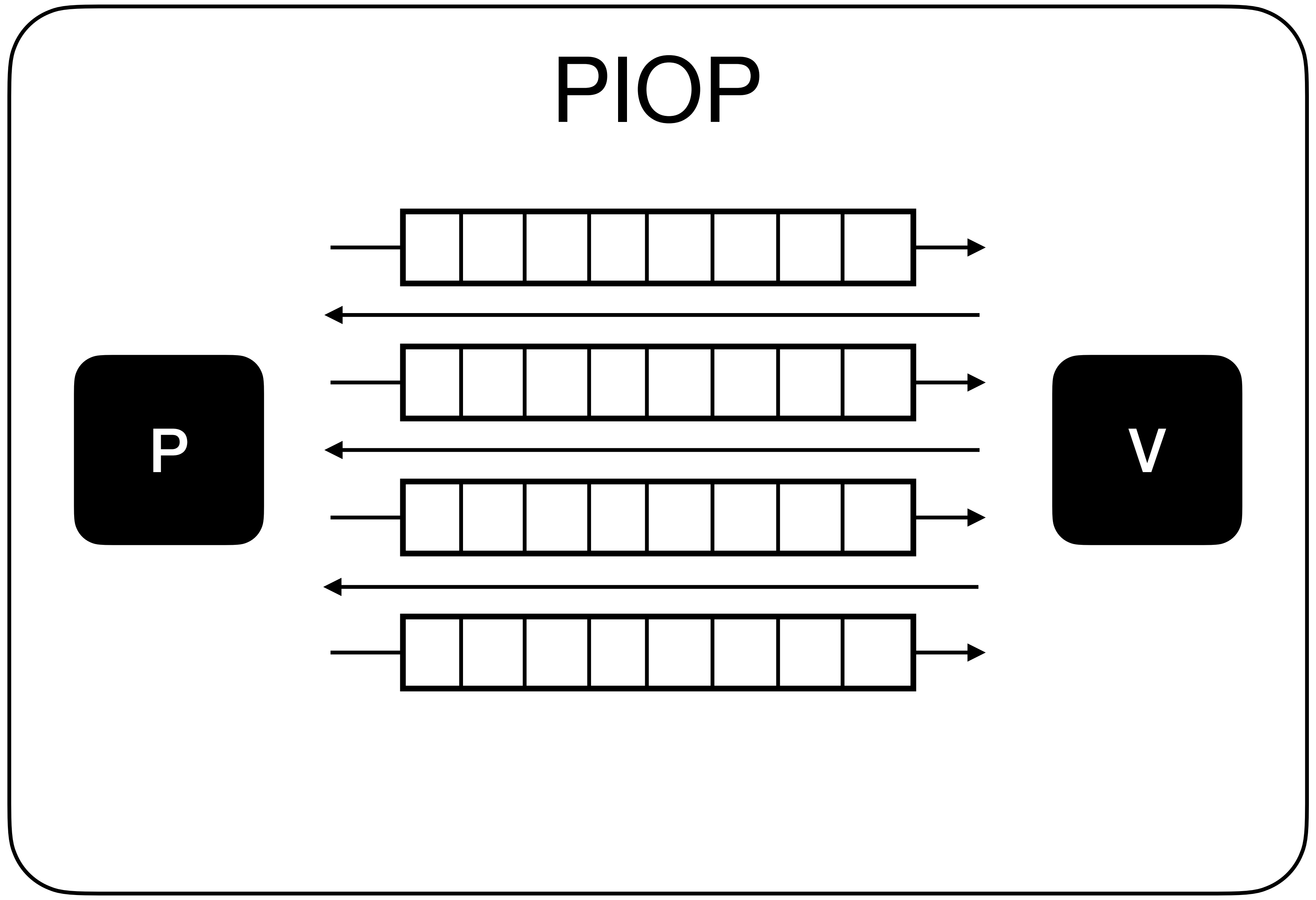

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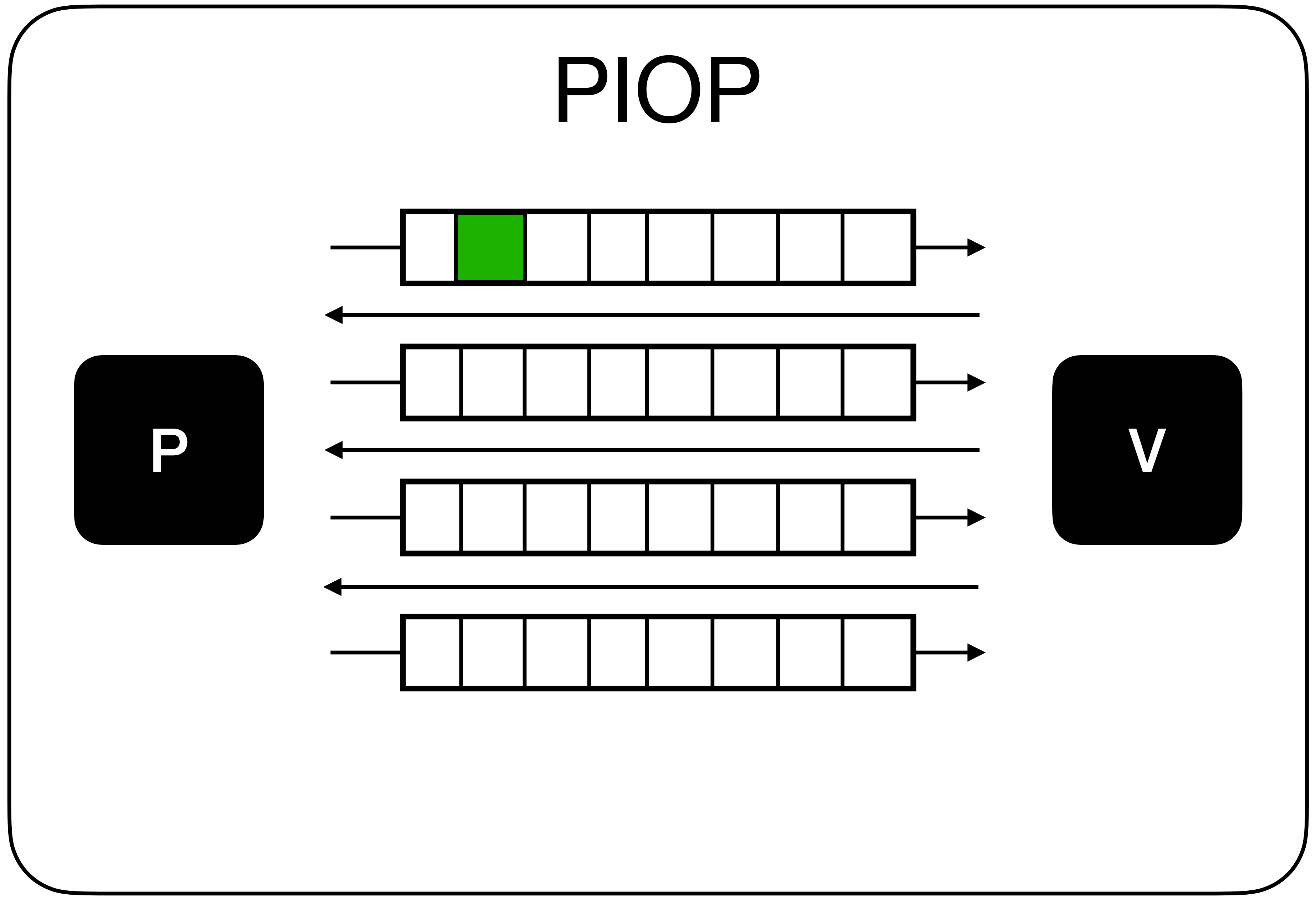

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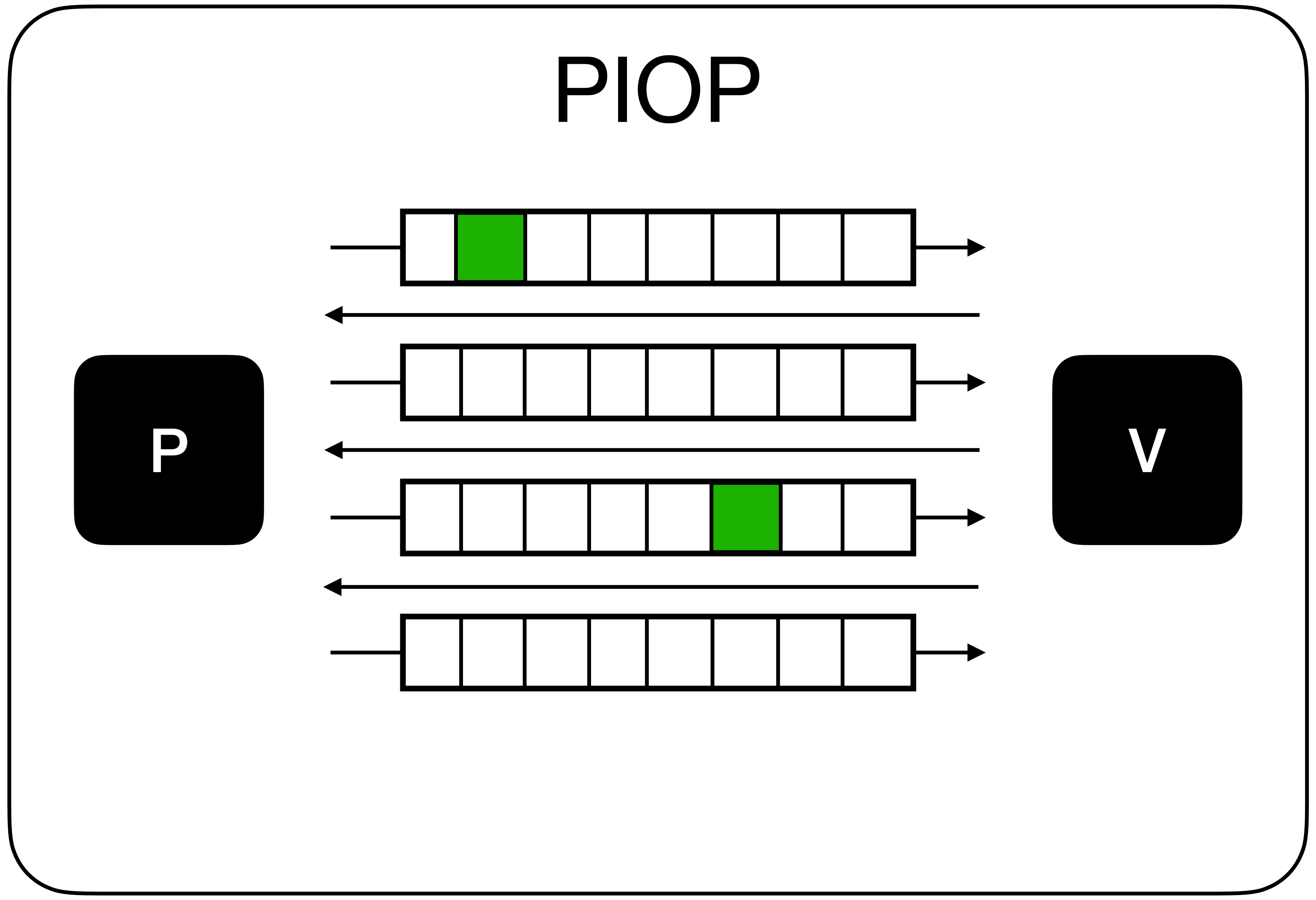

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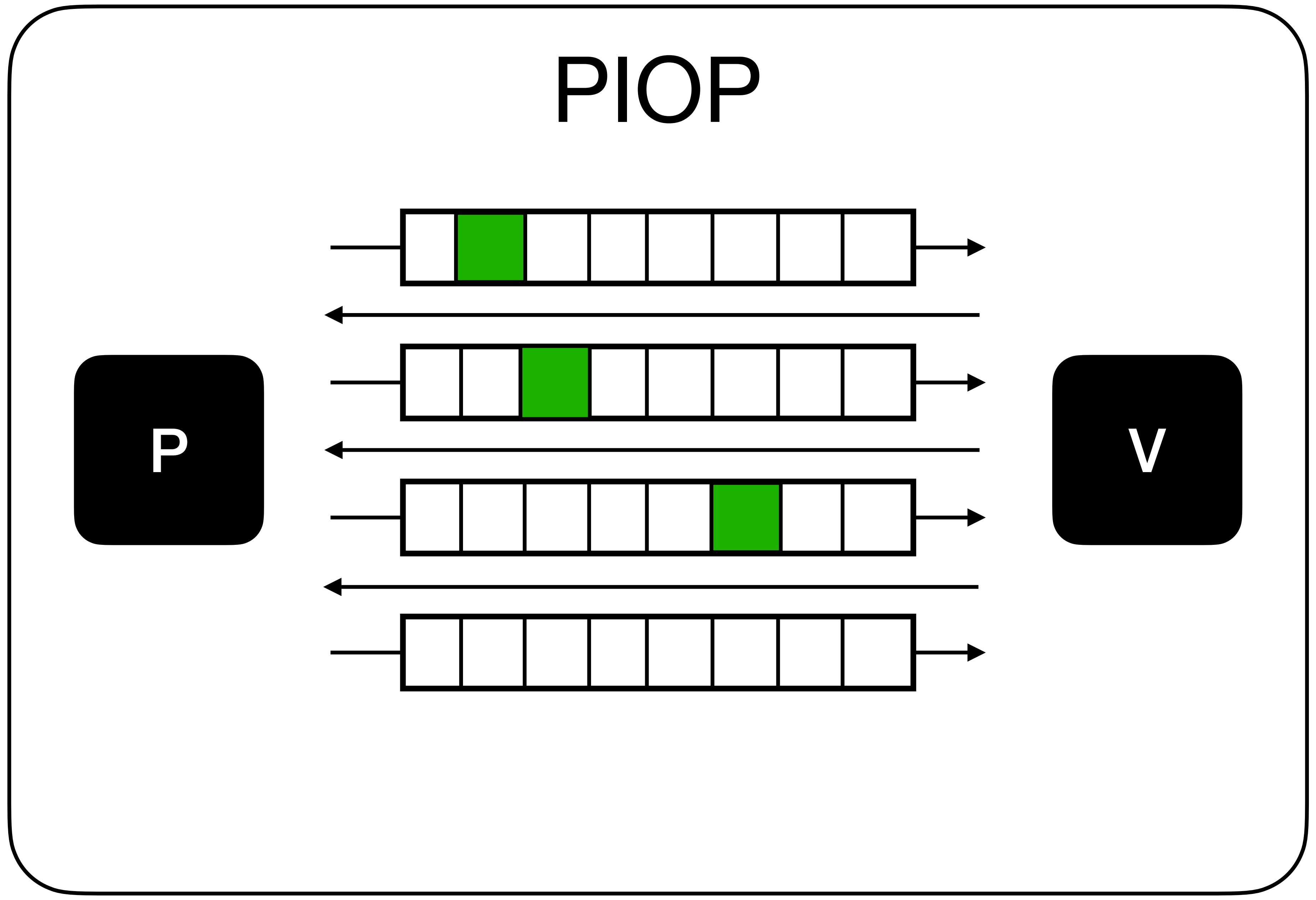

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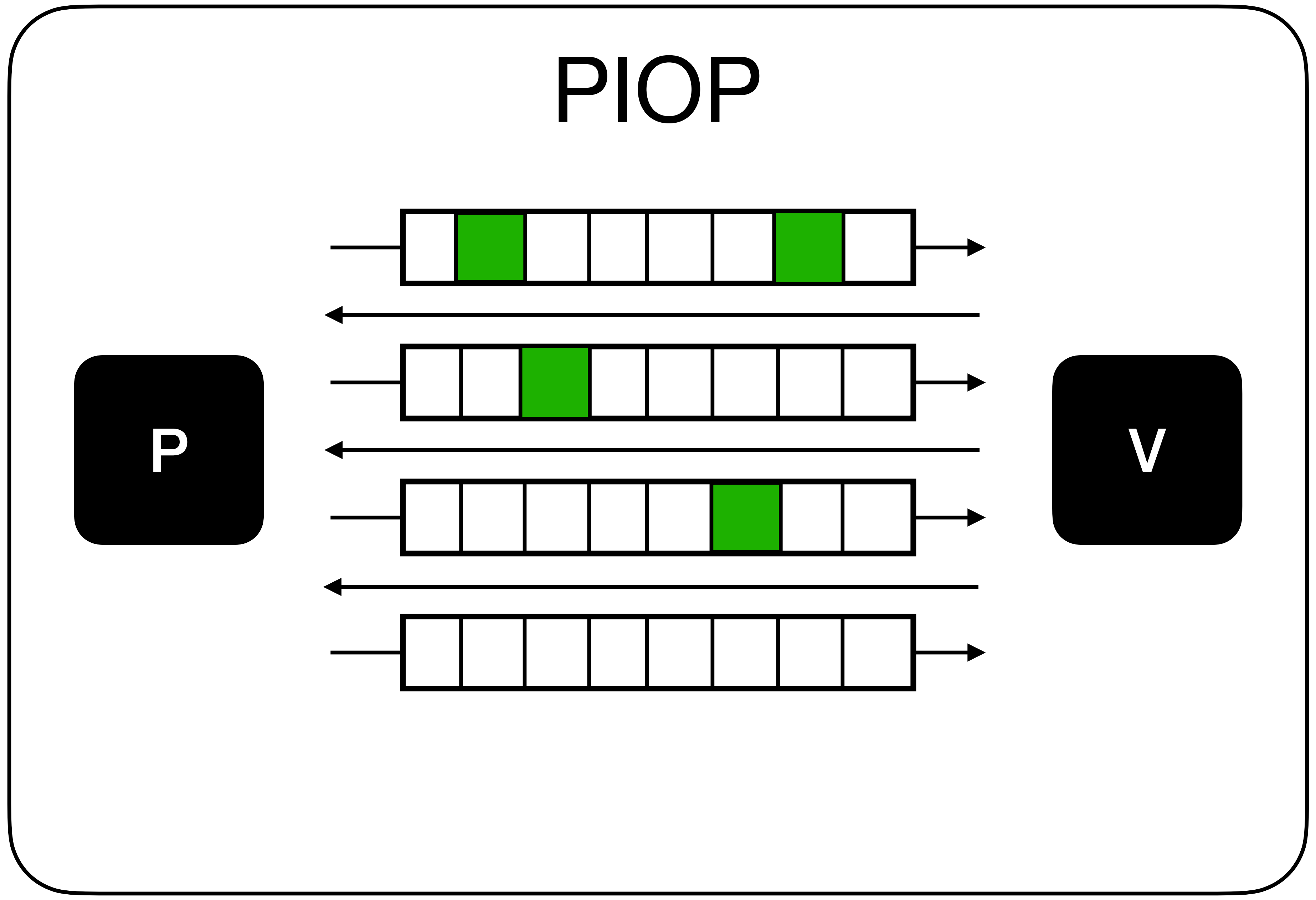

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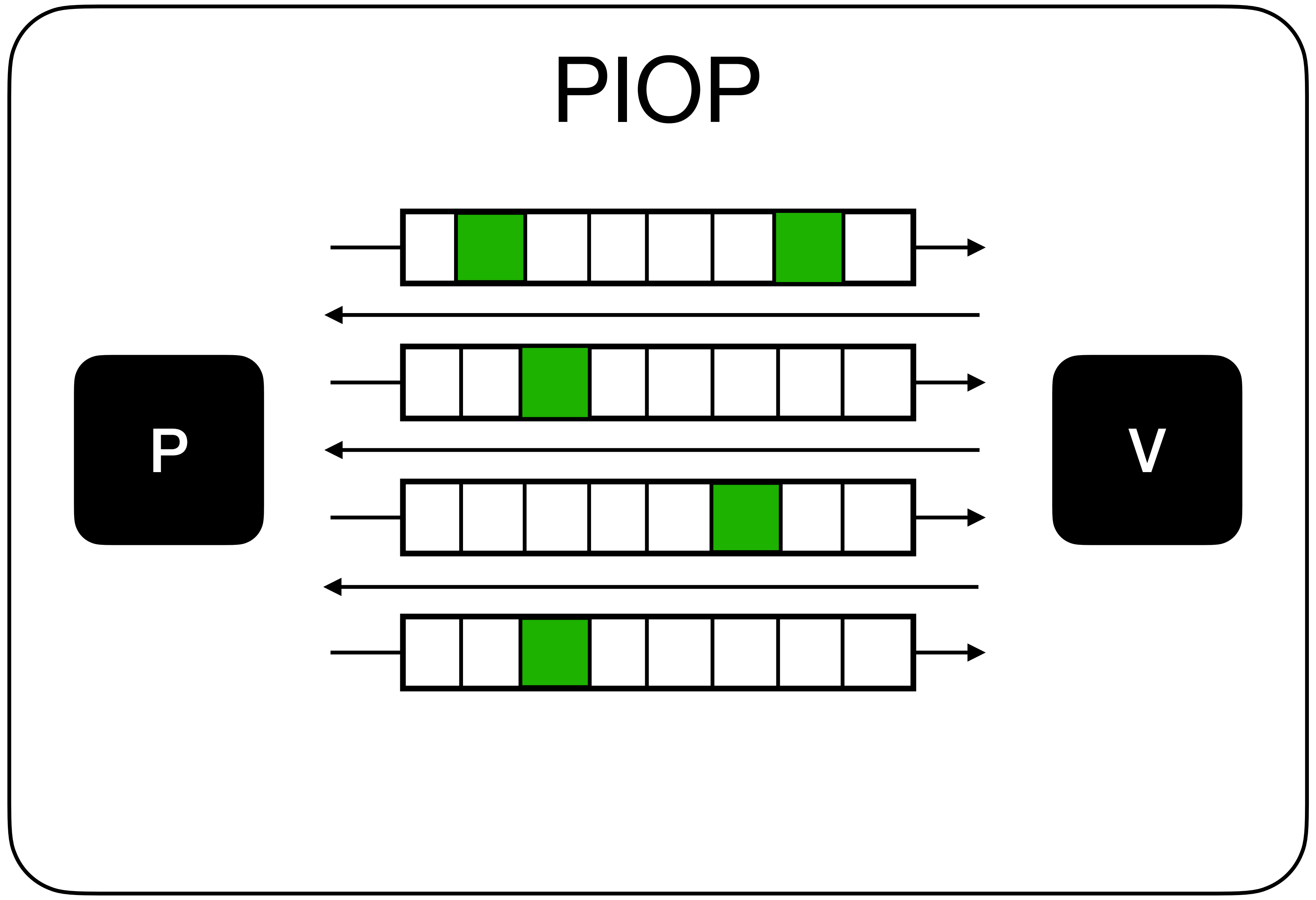

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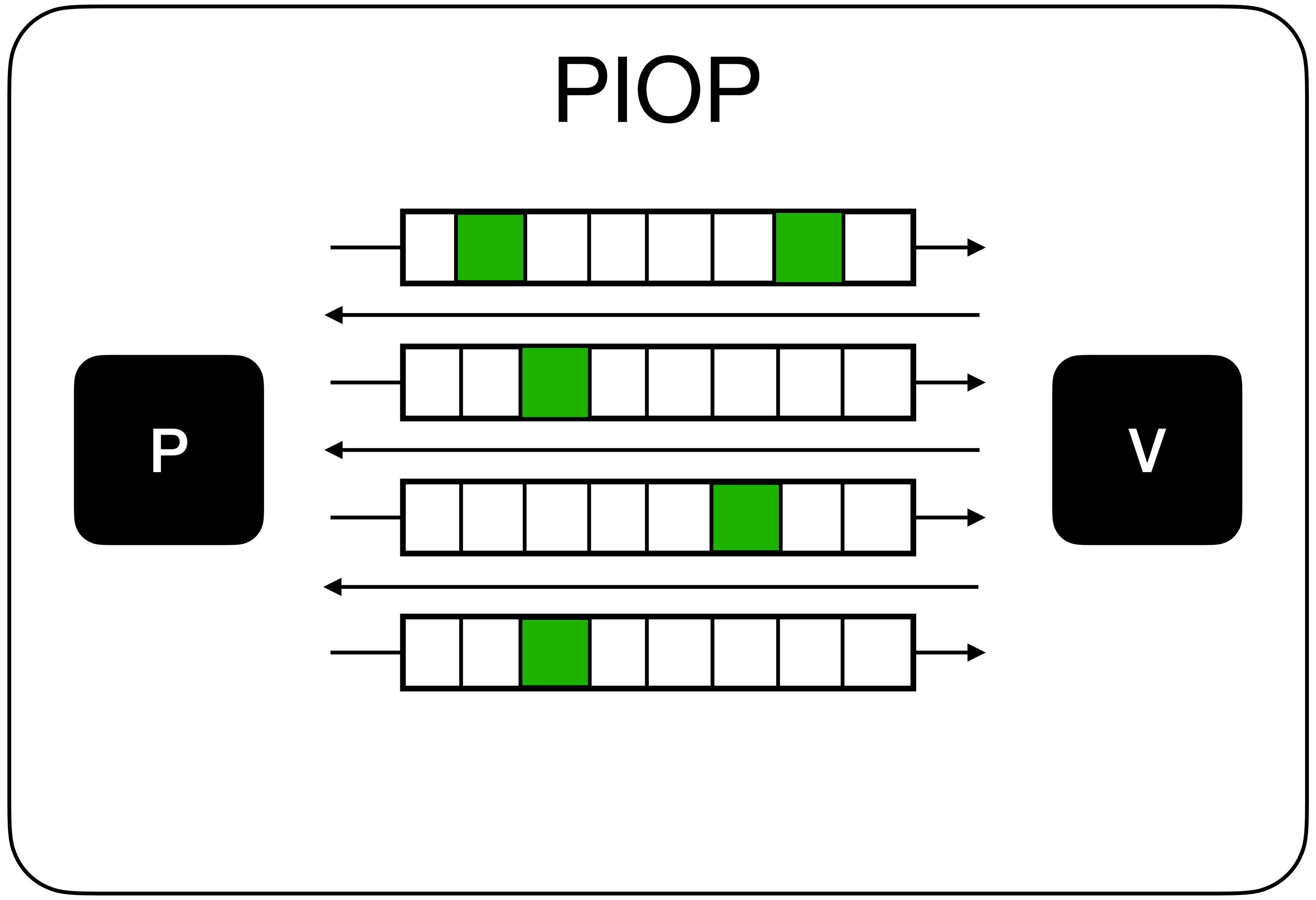

- Oracles are polynomials

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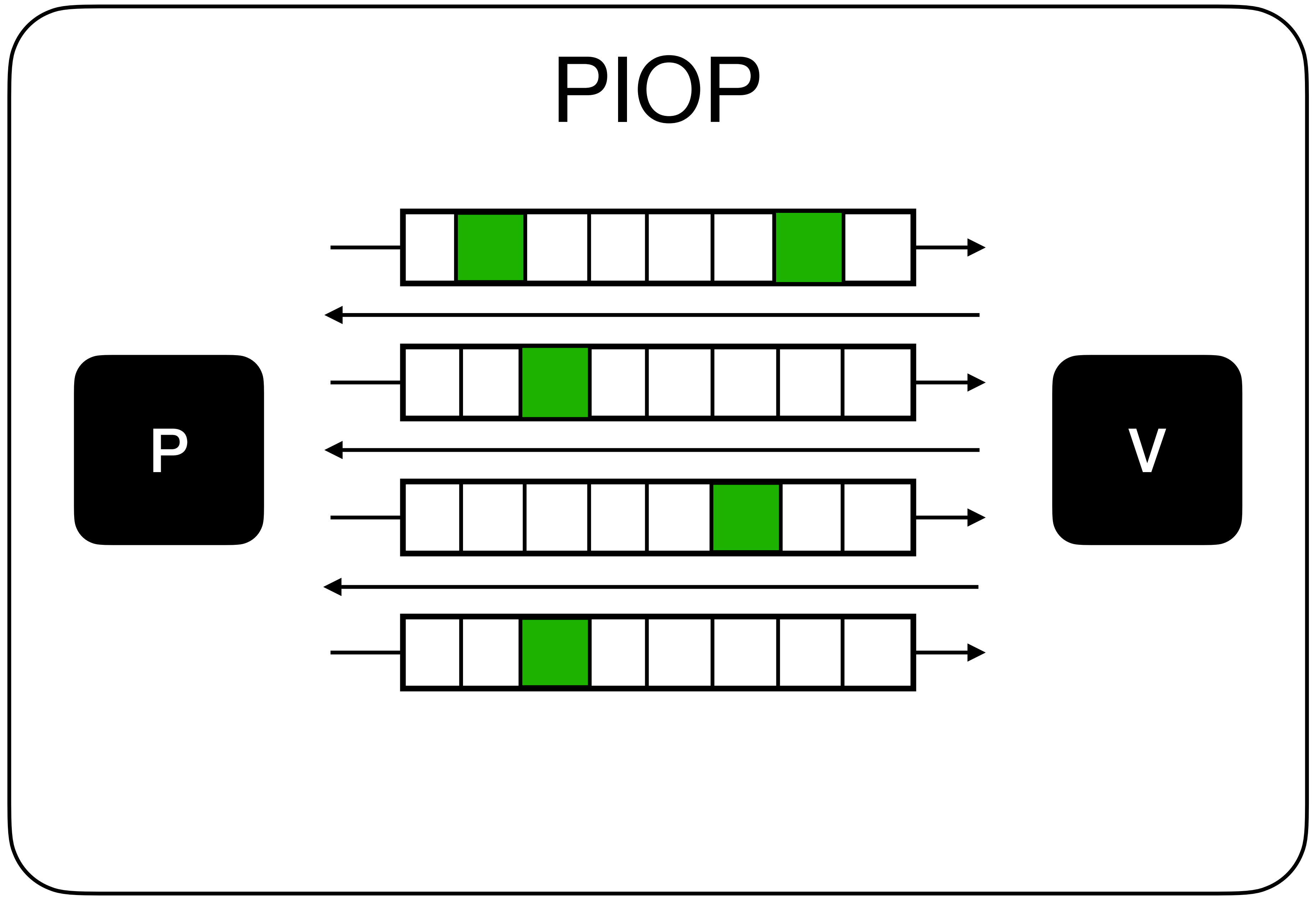

- Oracles are polynomials
- Security is information-theoretical

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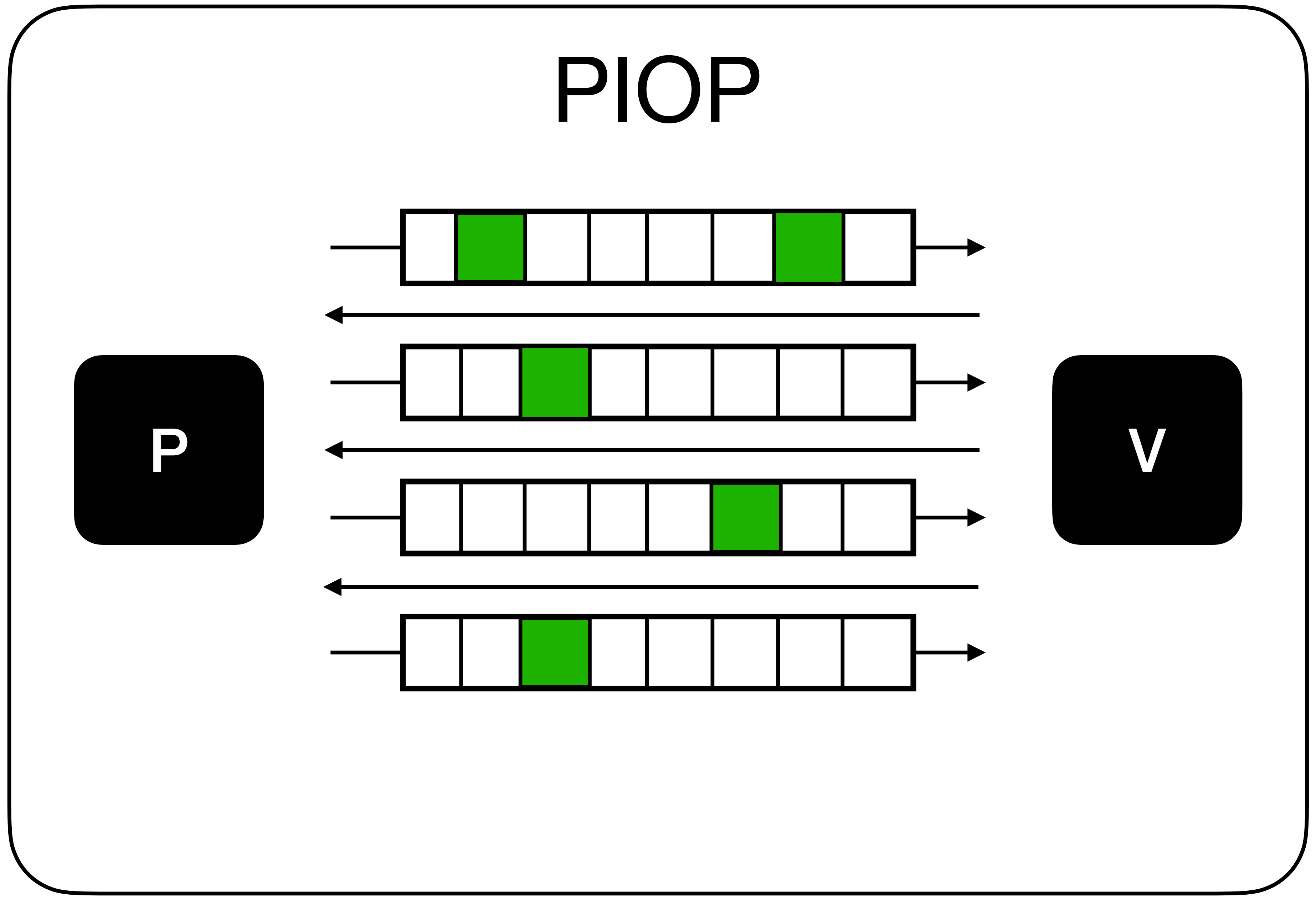

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- Security is information-theoretical
- Proof length is $\Omega(n)$ (not succinct)

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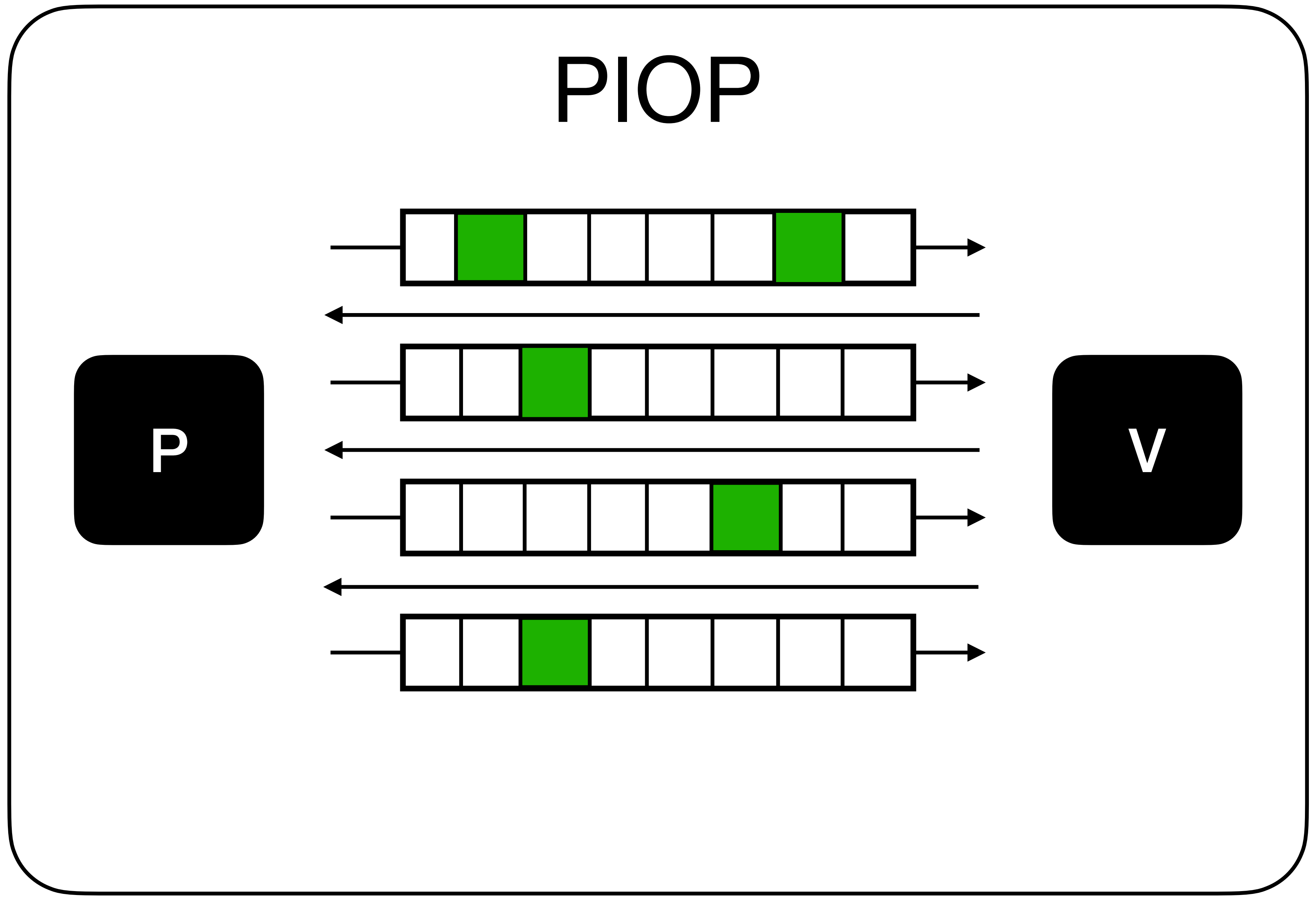

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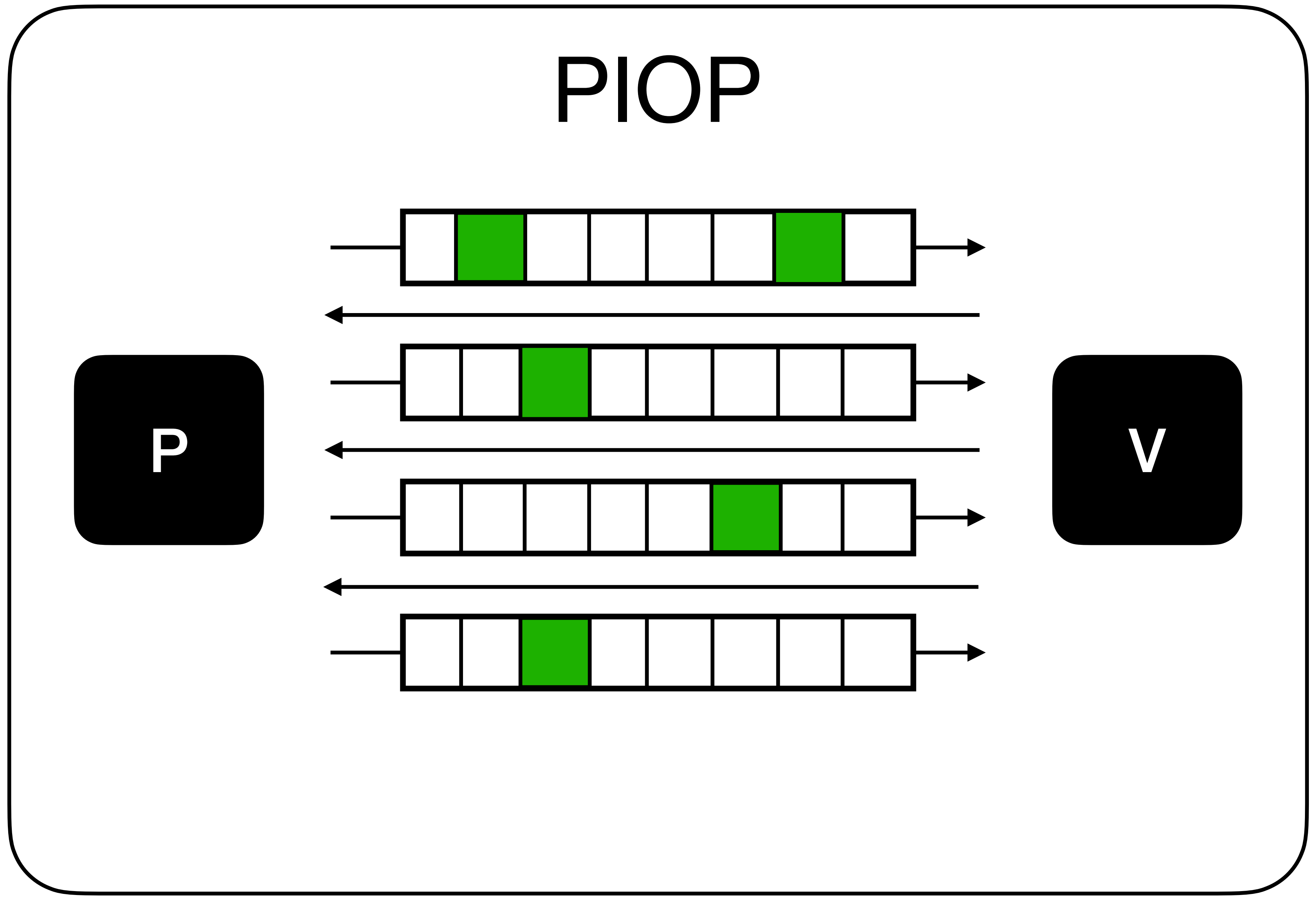

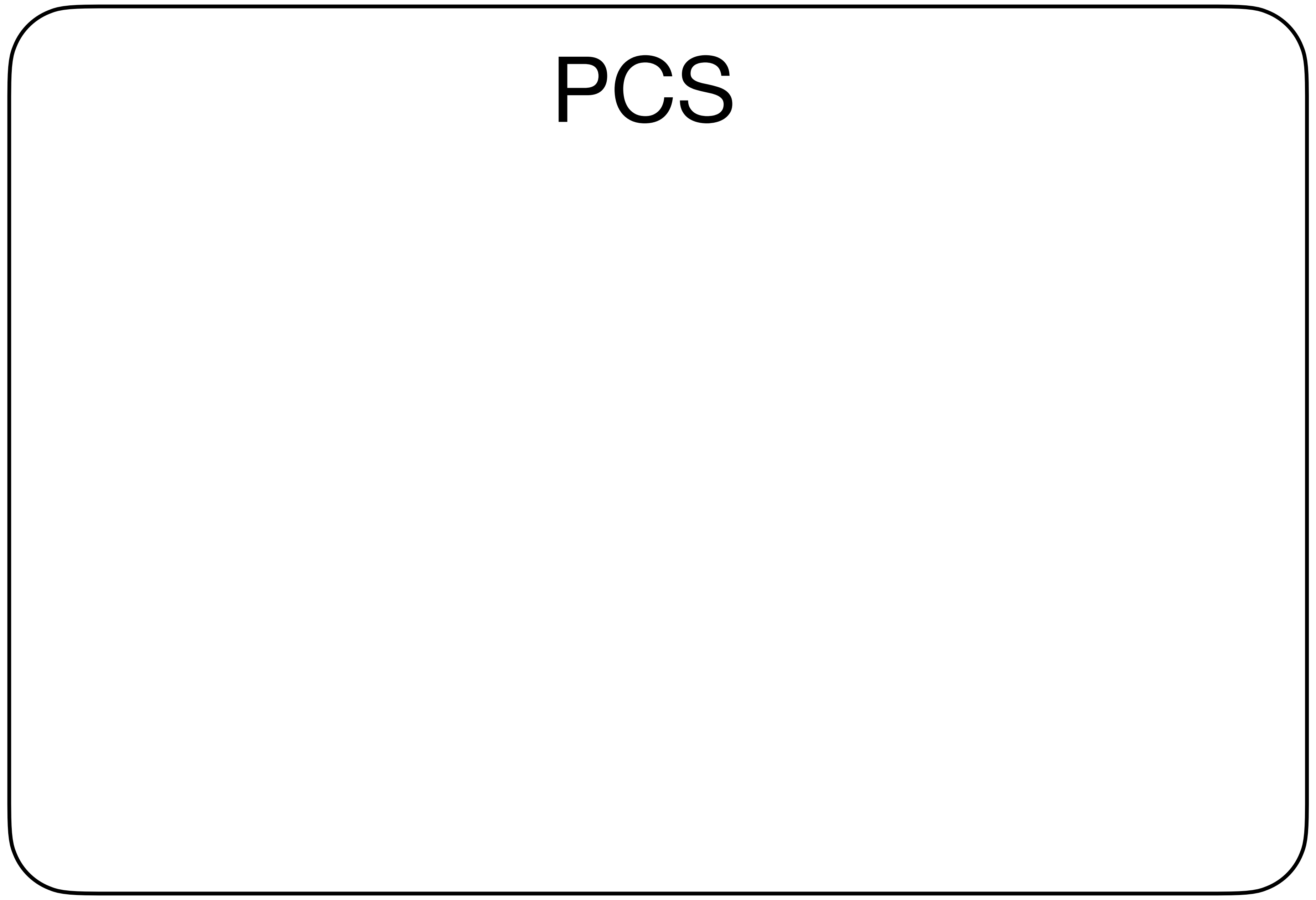

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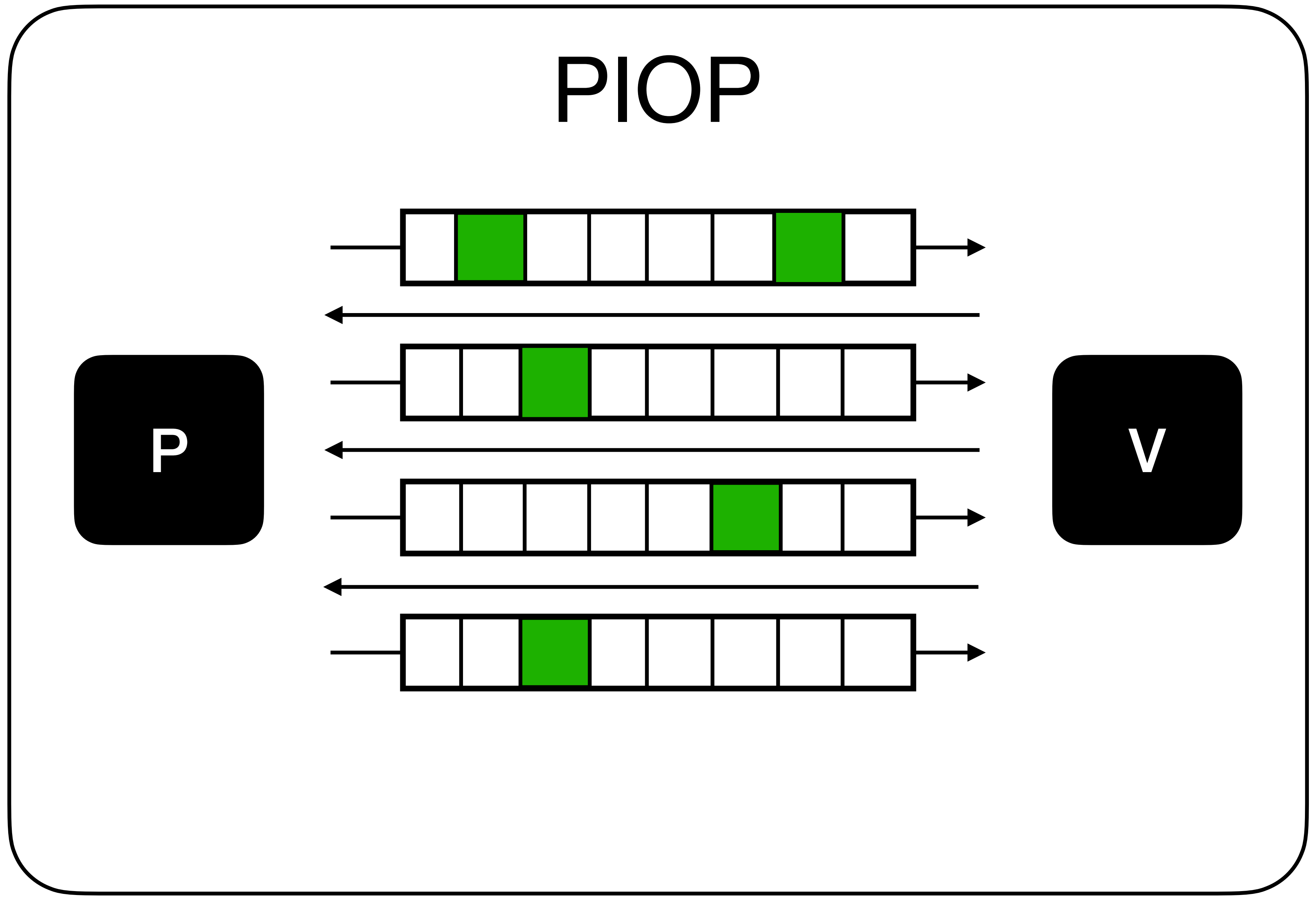

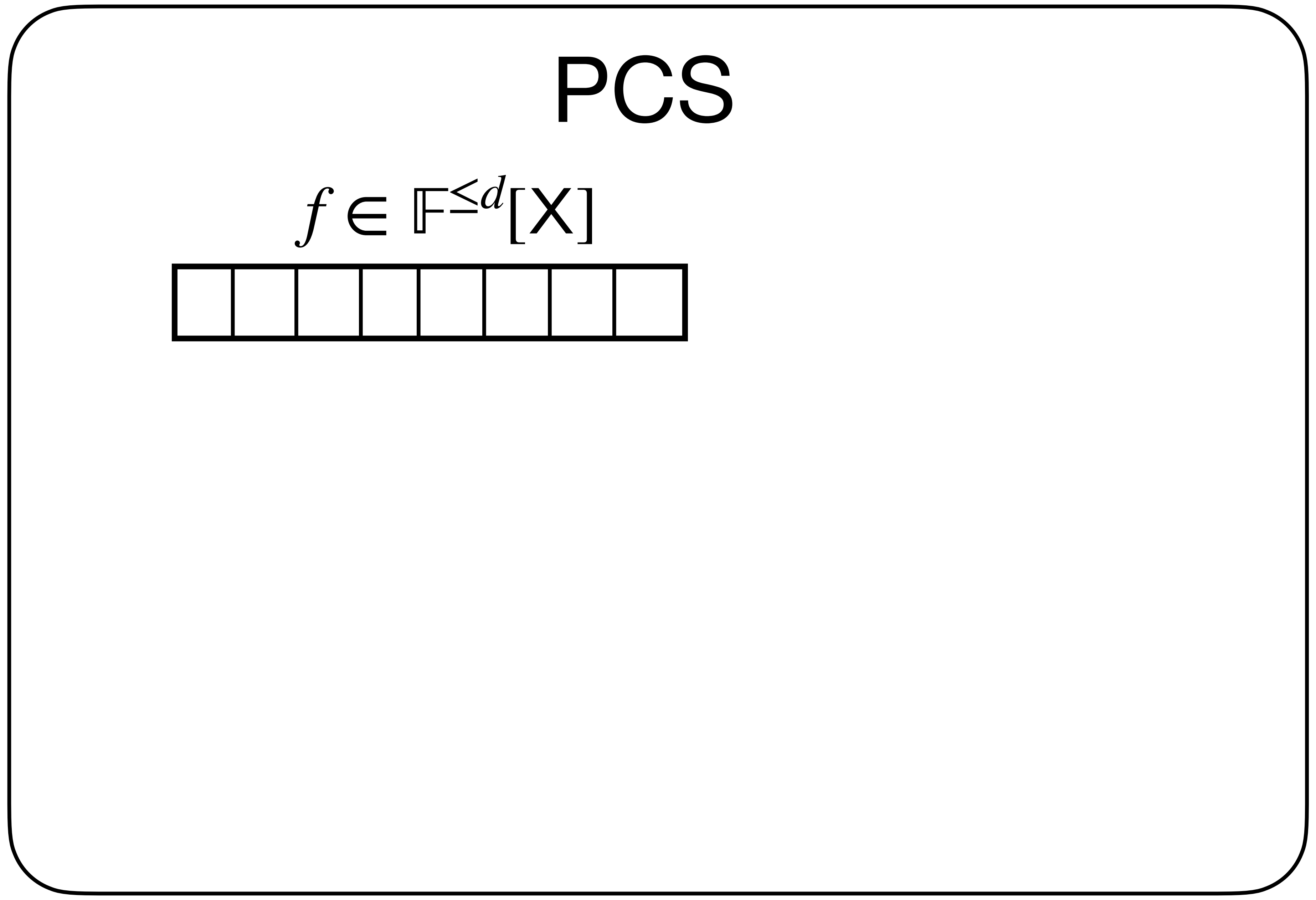

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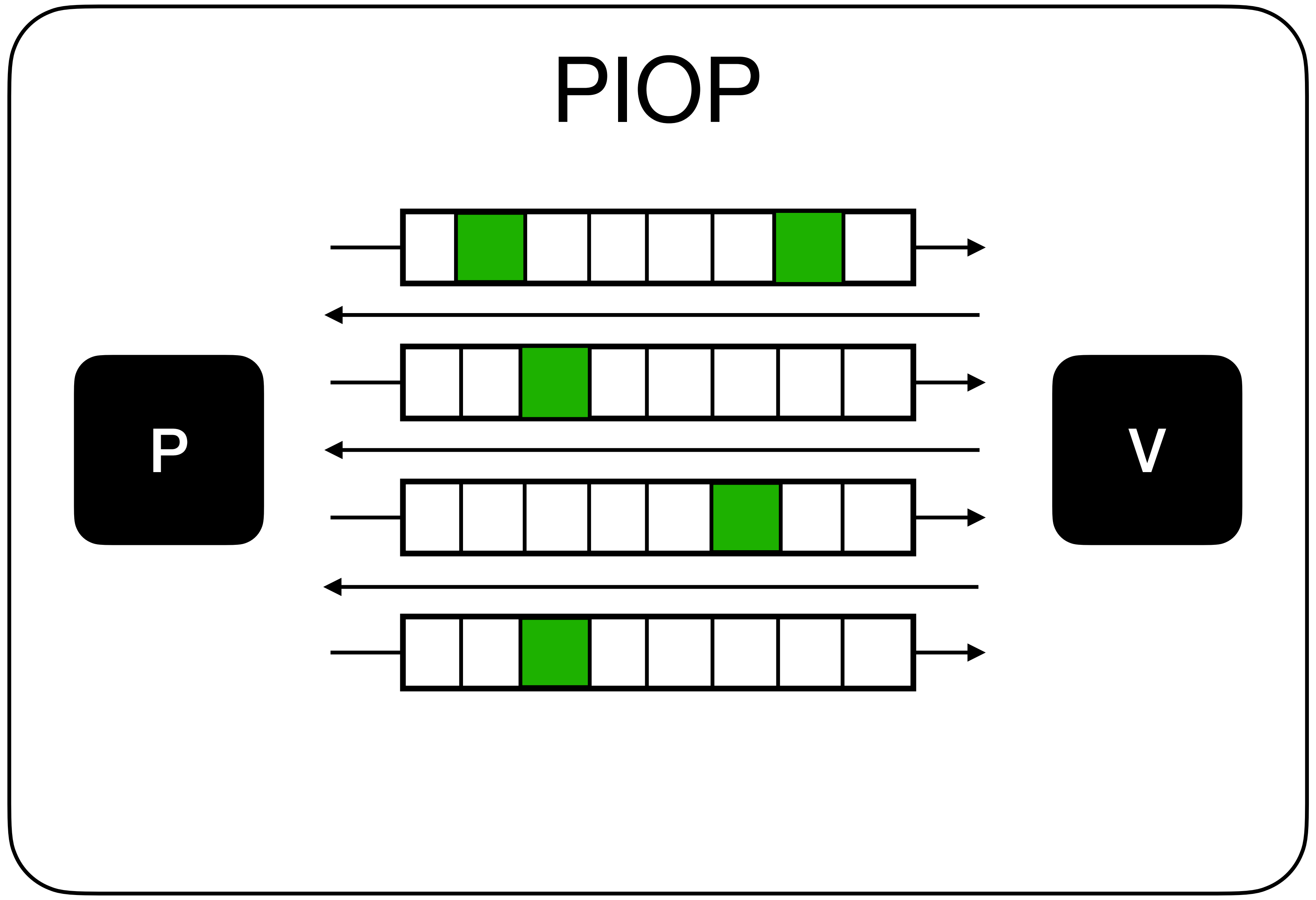

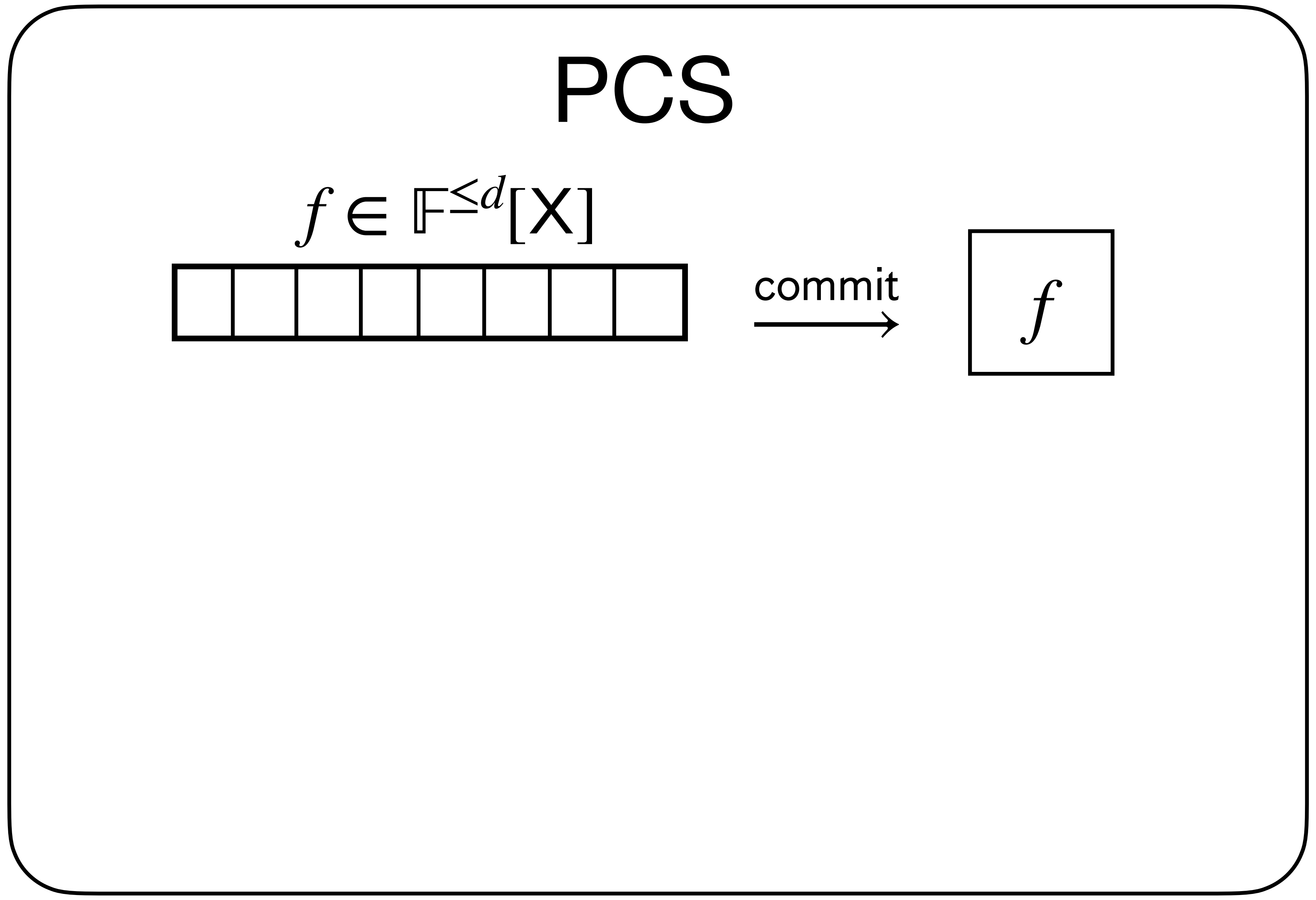

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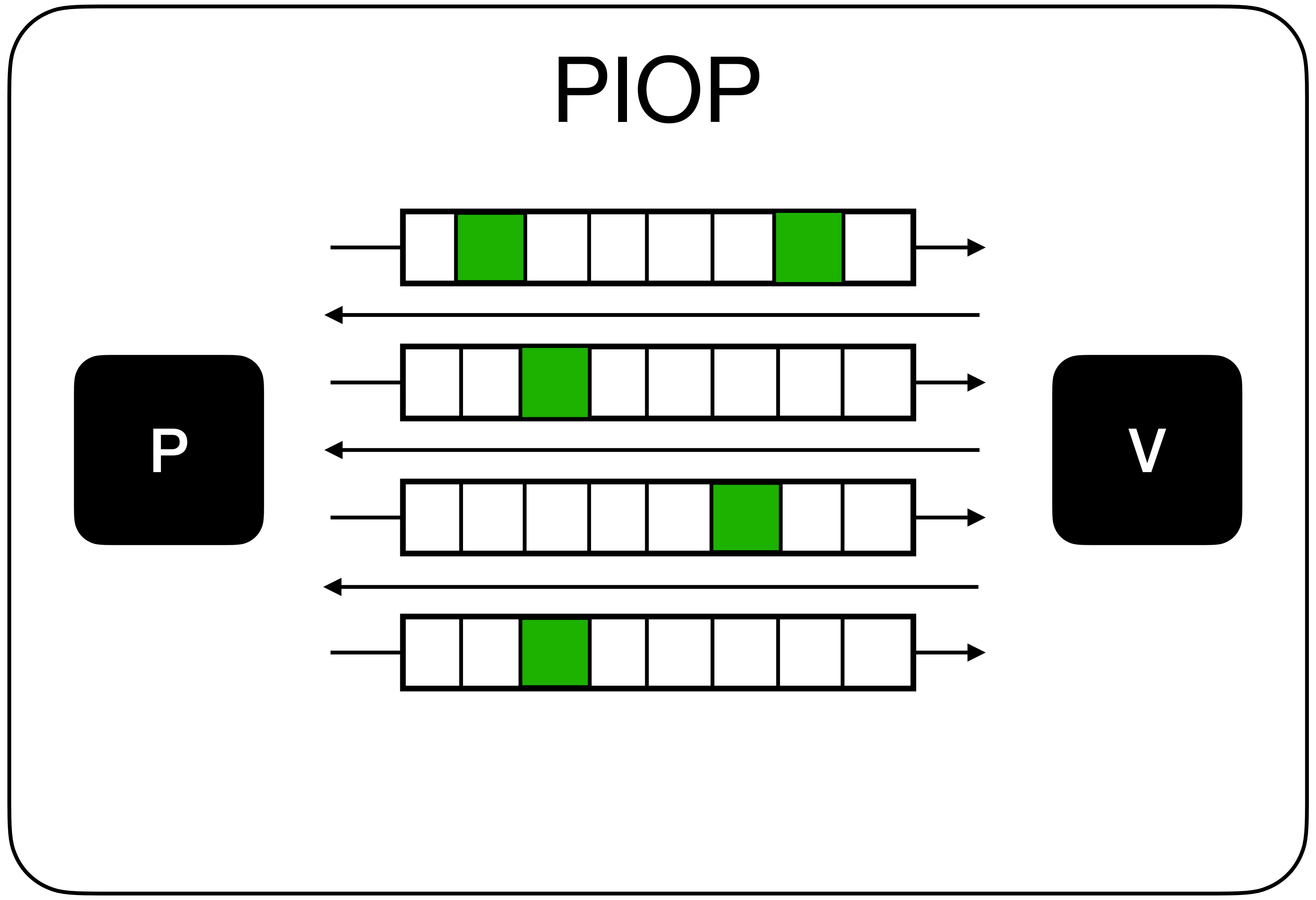

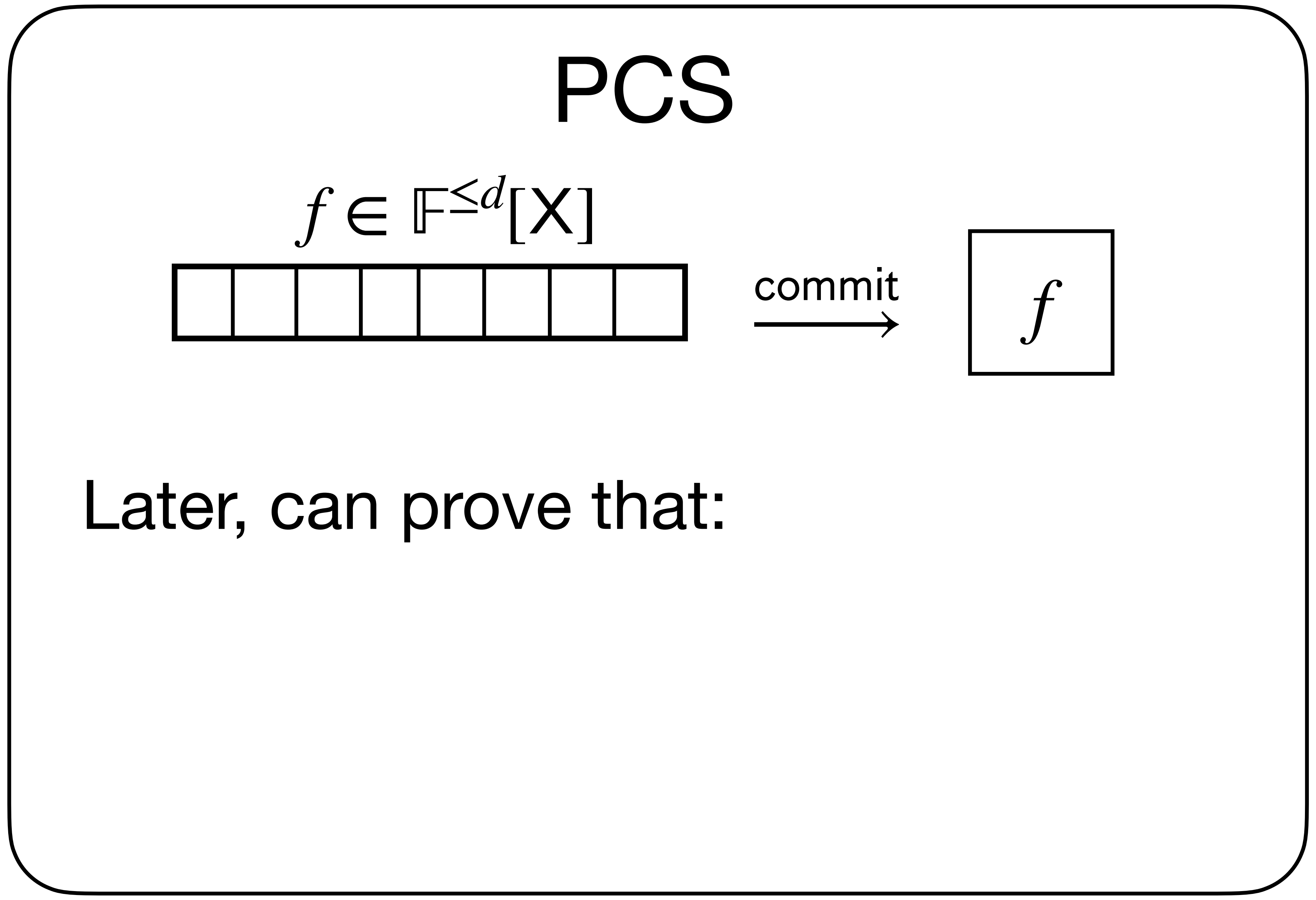

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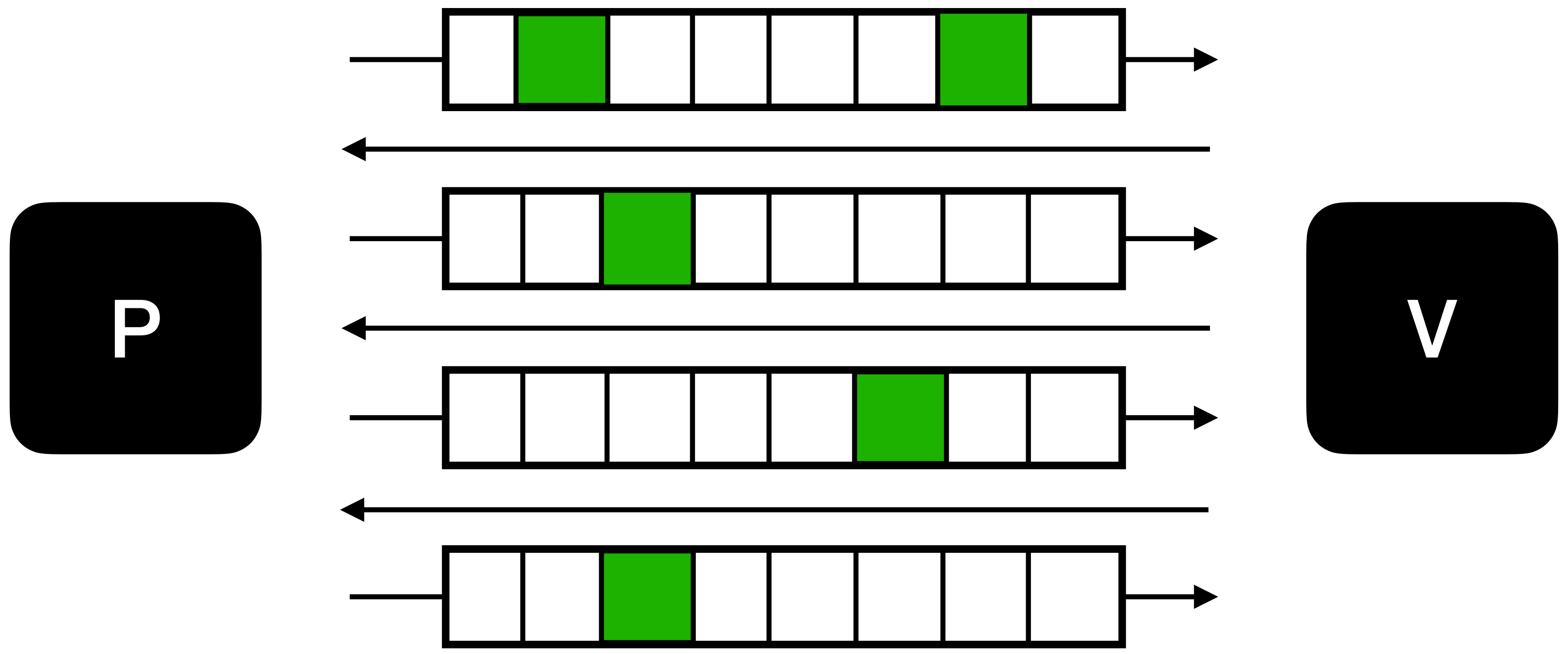
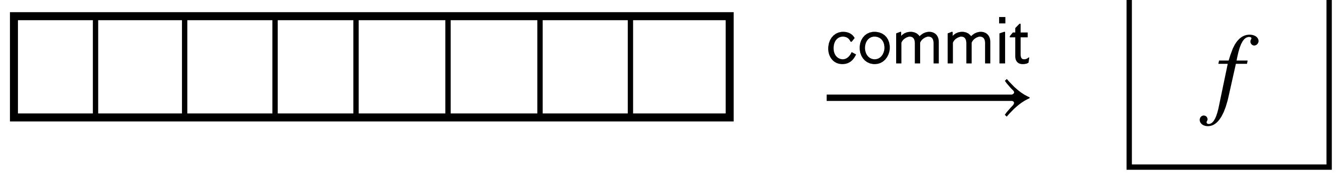


PCS

$f \in \mathbb{F}^{\leq d}[X]$



Later, can prove that:

$$f(x) = y, \text{ for } x, y \in \mathbb{F}$$

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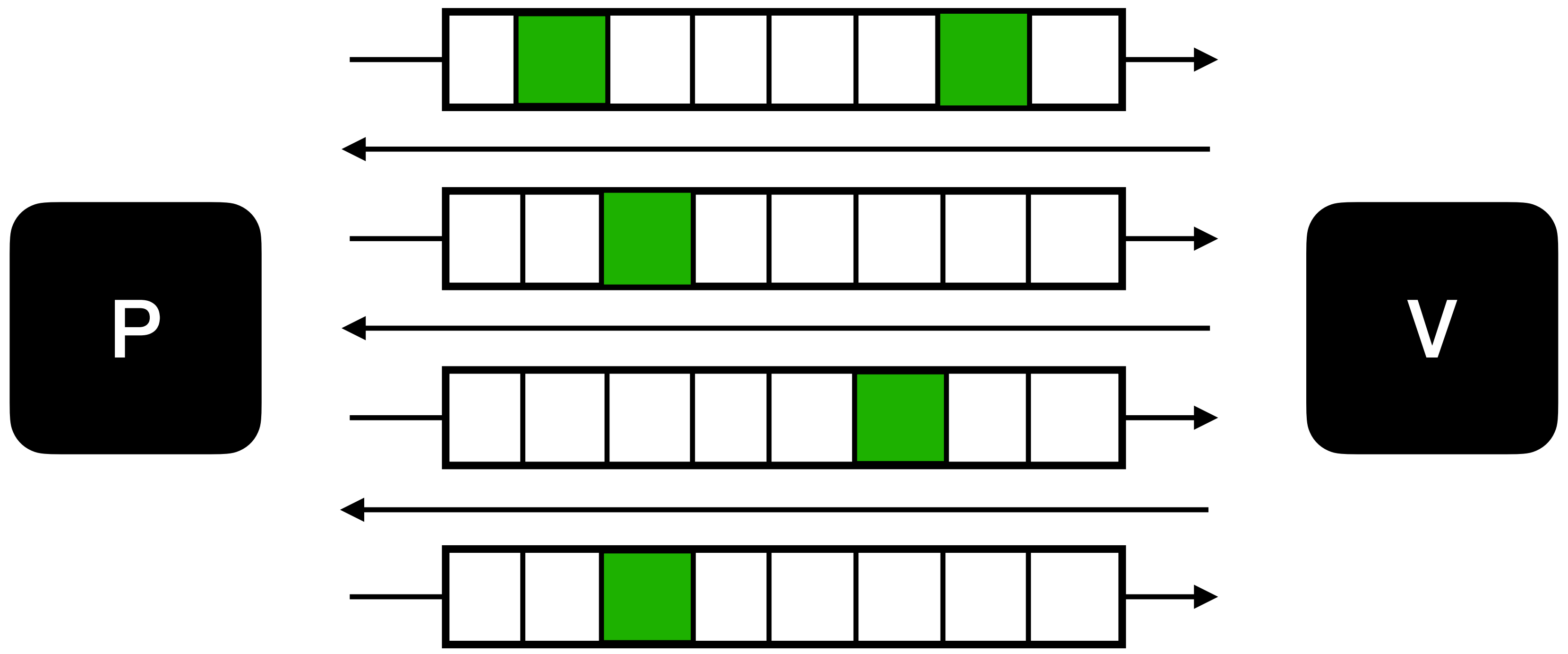

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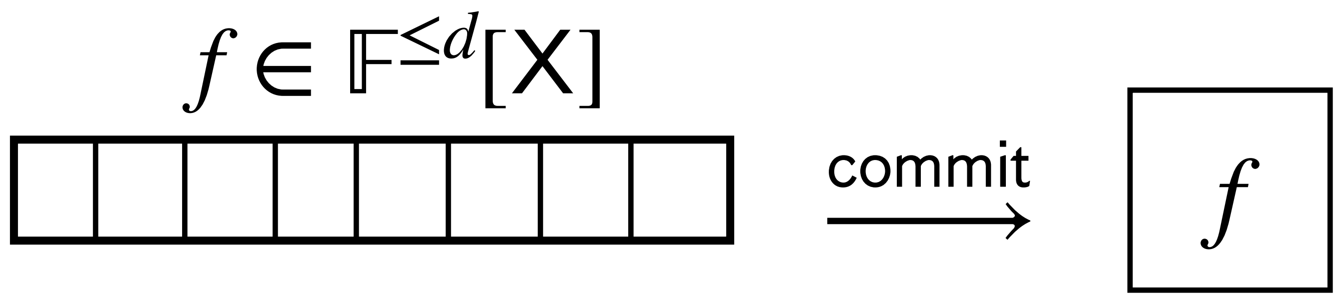

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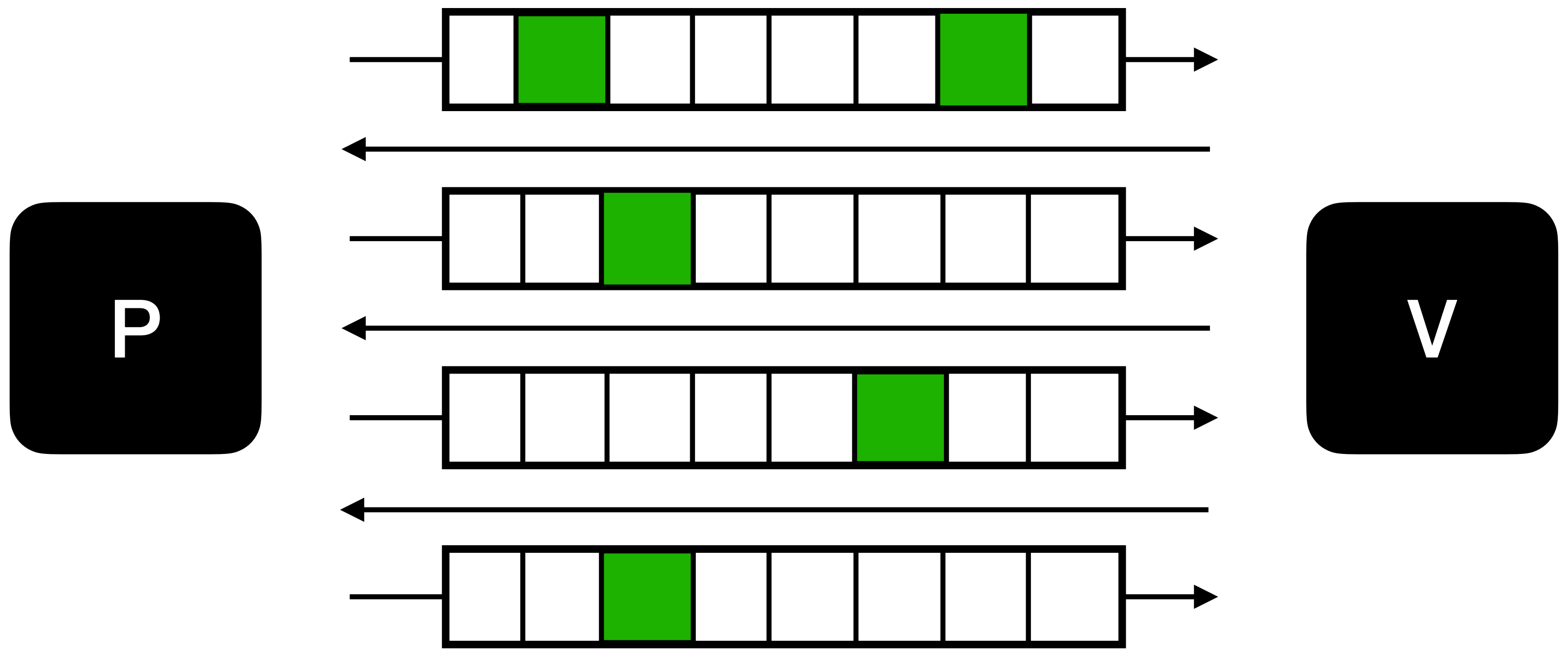

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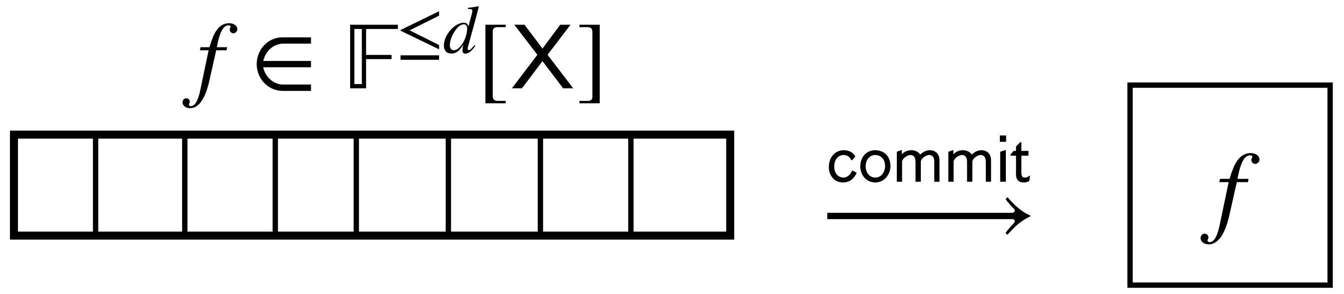

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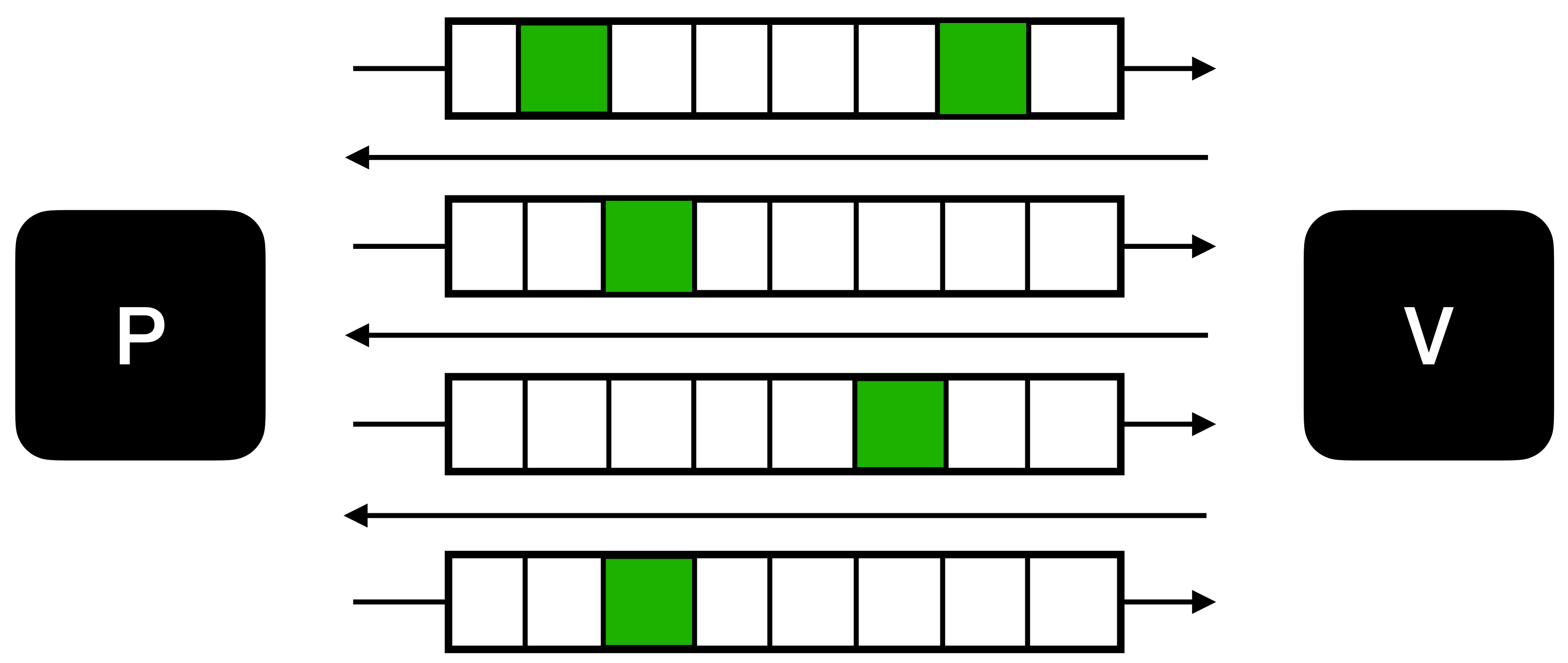

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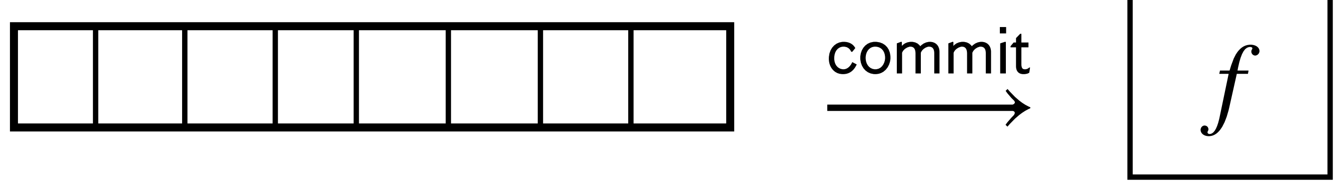

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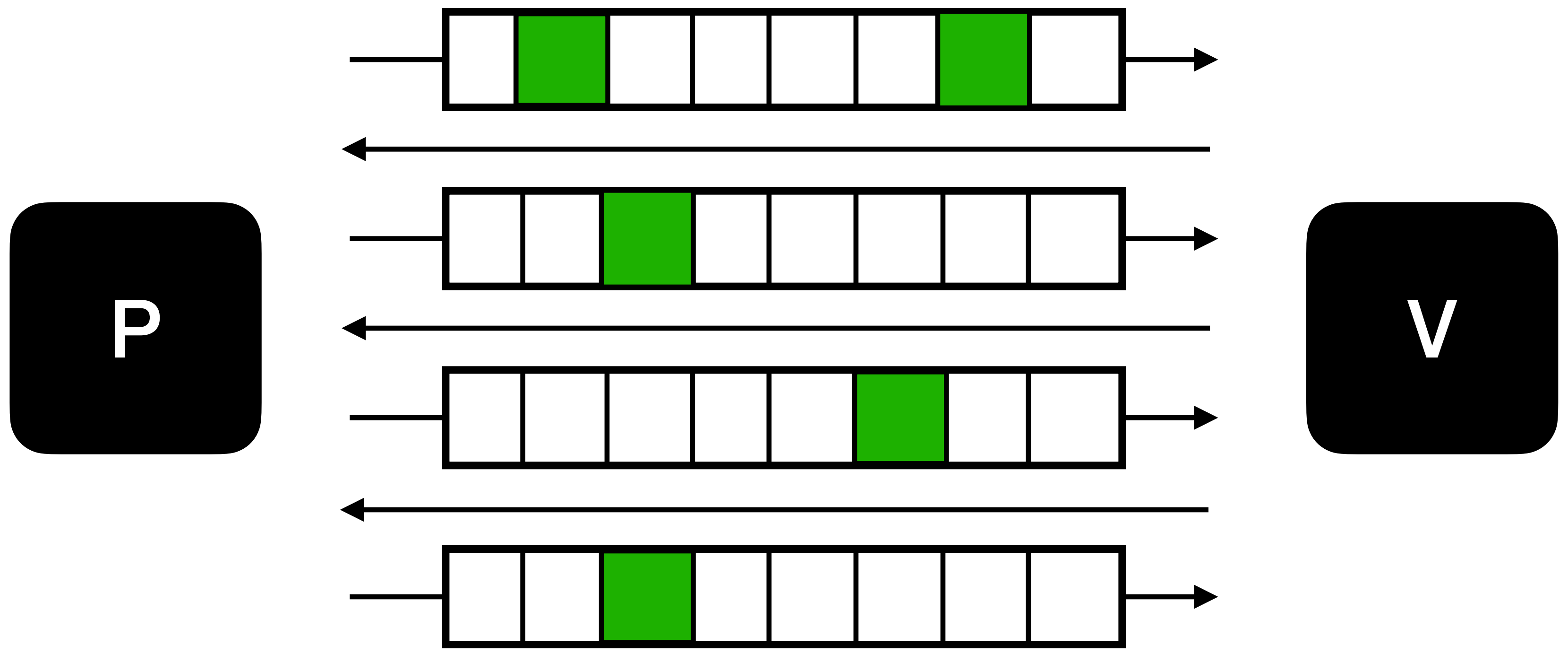

+

FS

PCS

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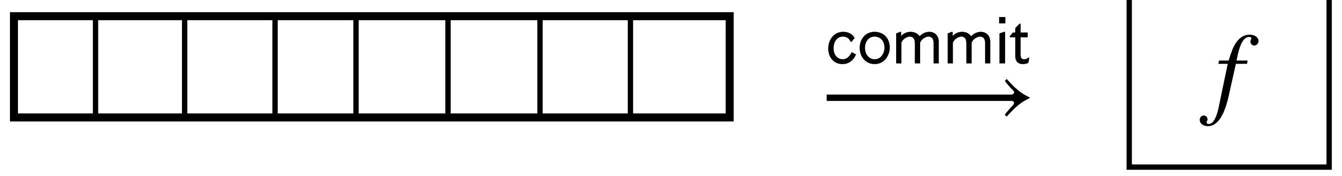

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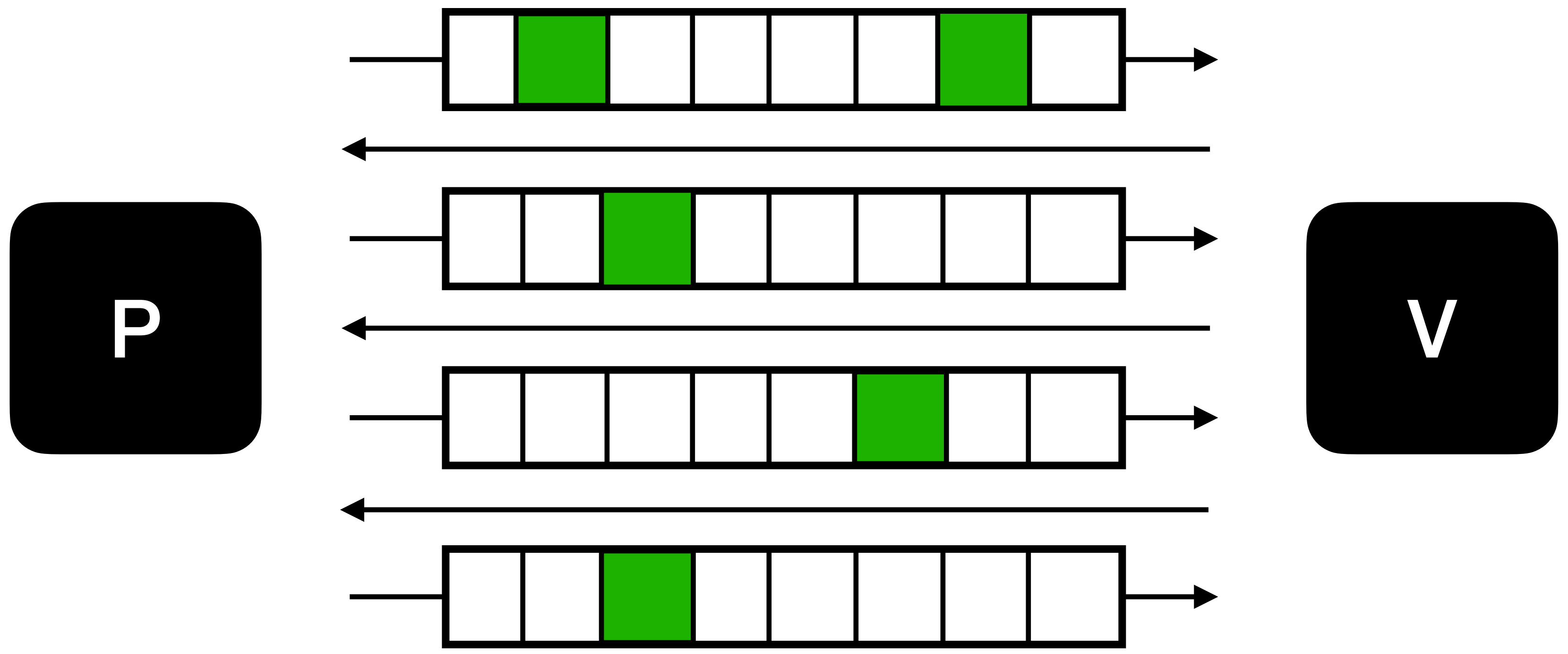
We focus on this!

PIOP



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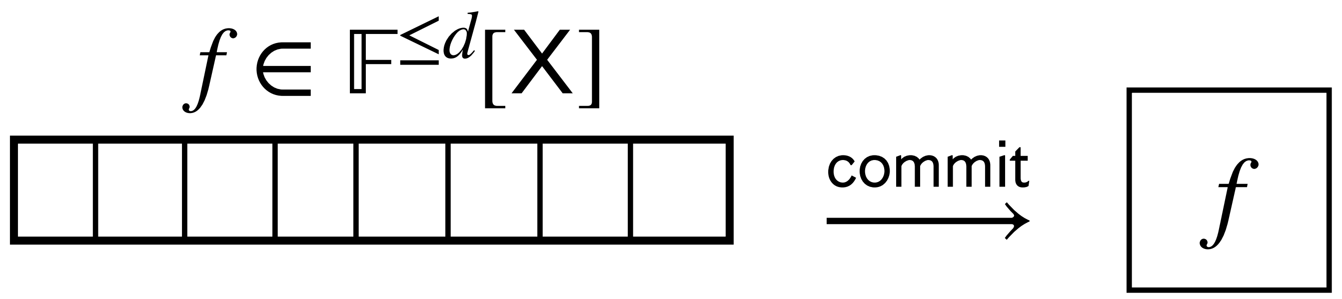

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Zoo of Polynomial Commitments

A very incomplete list…

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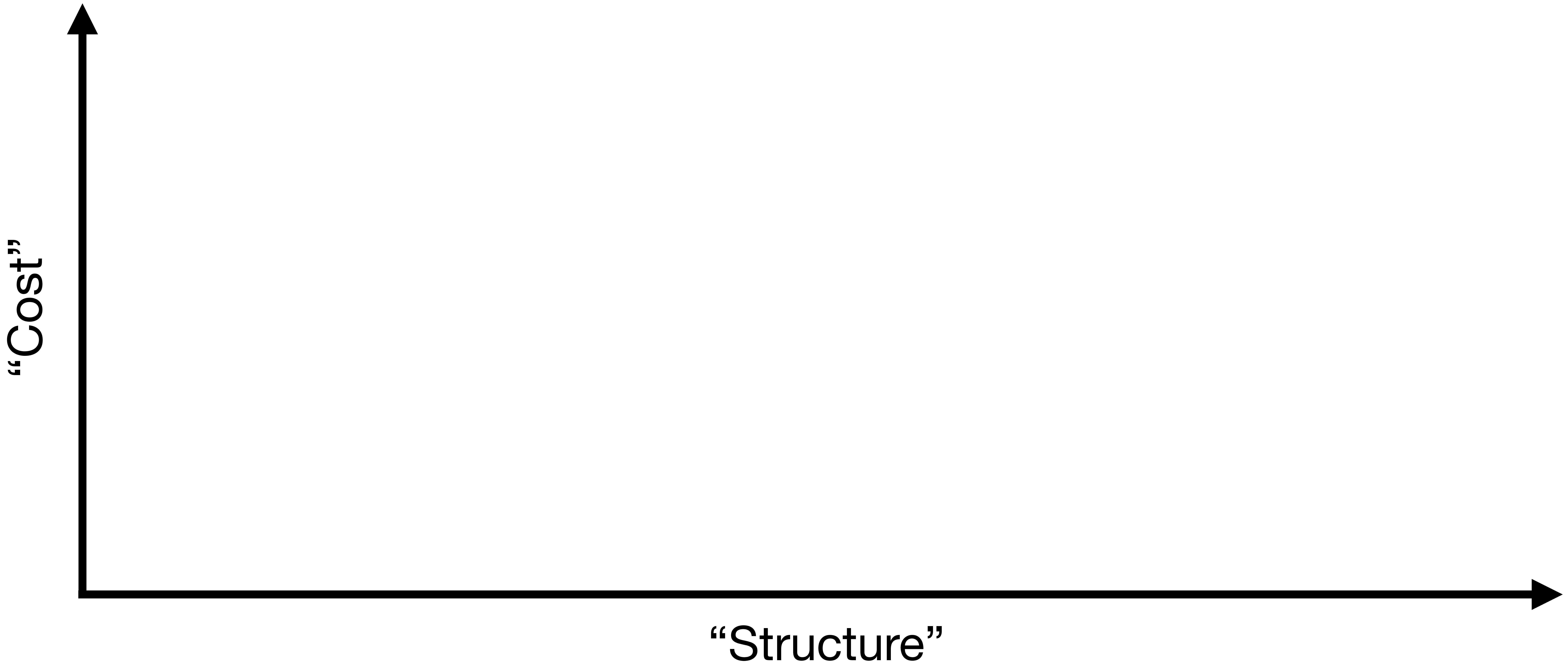

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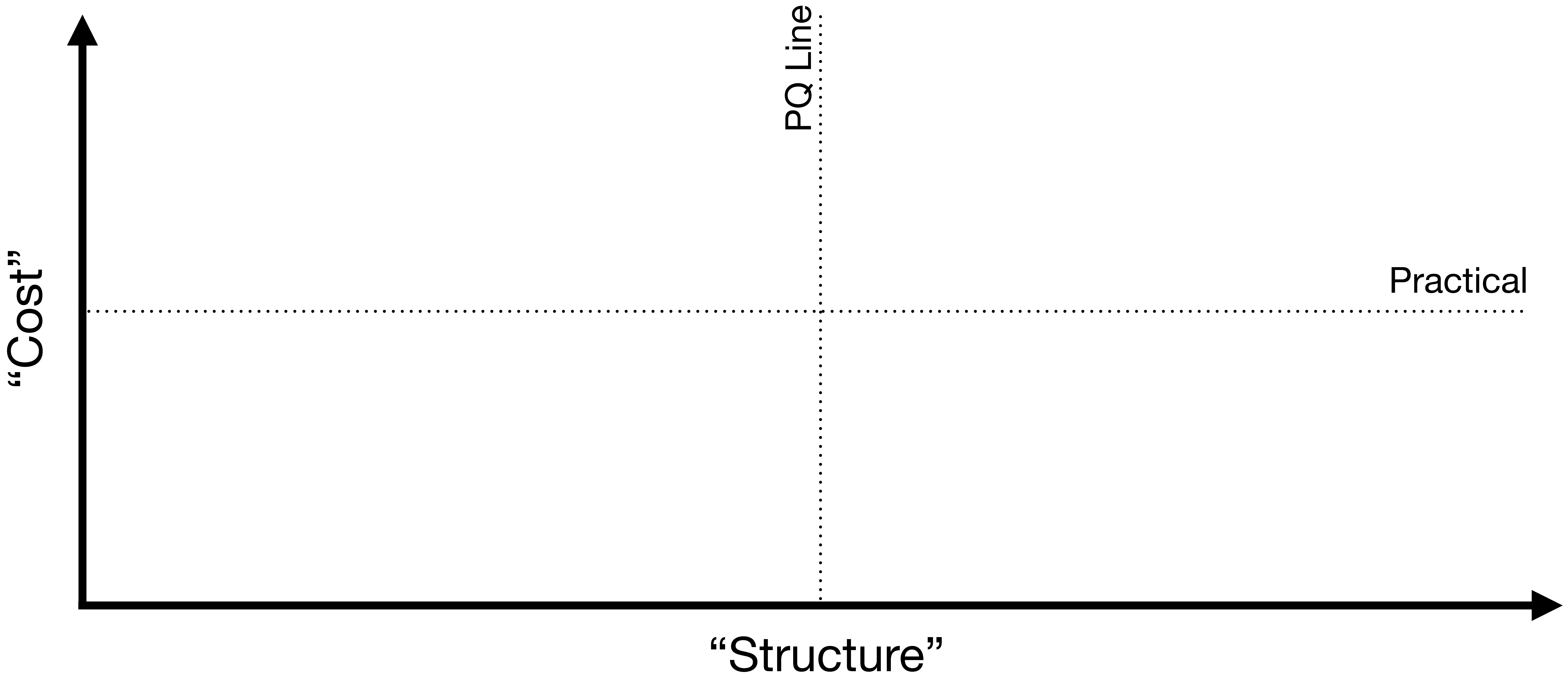

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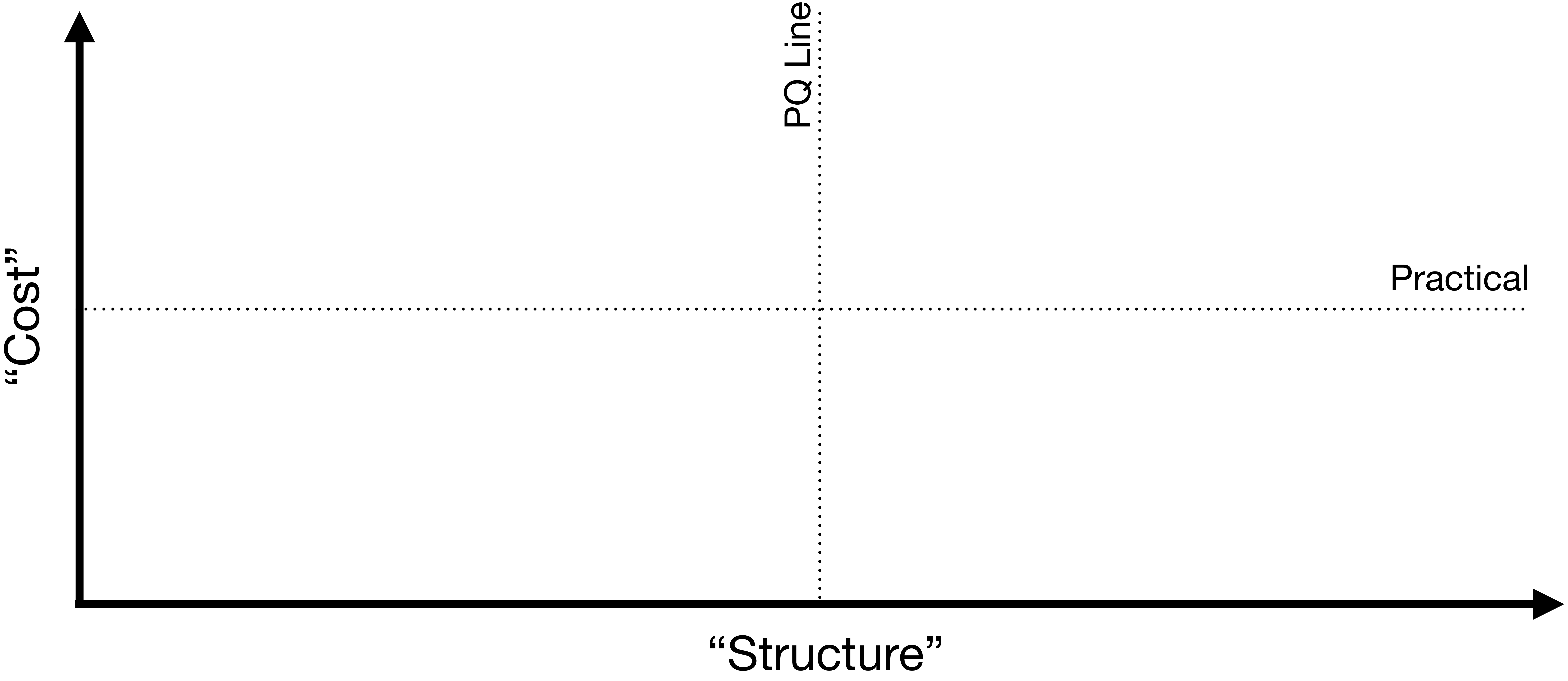
<u>Underlined:</u> succinct verification
*: interactive (no FS)
(T): trusted setup



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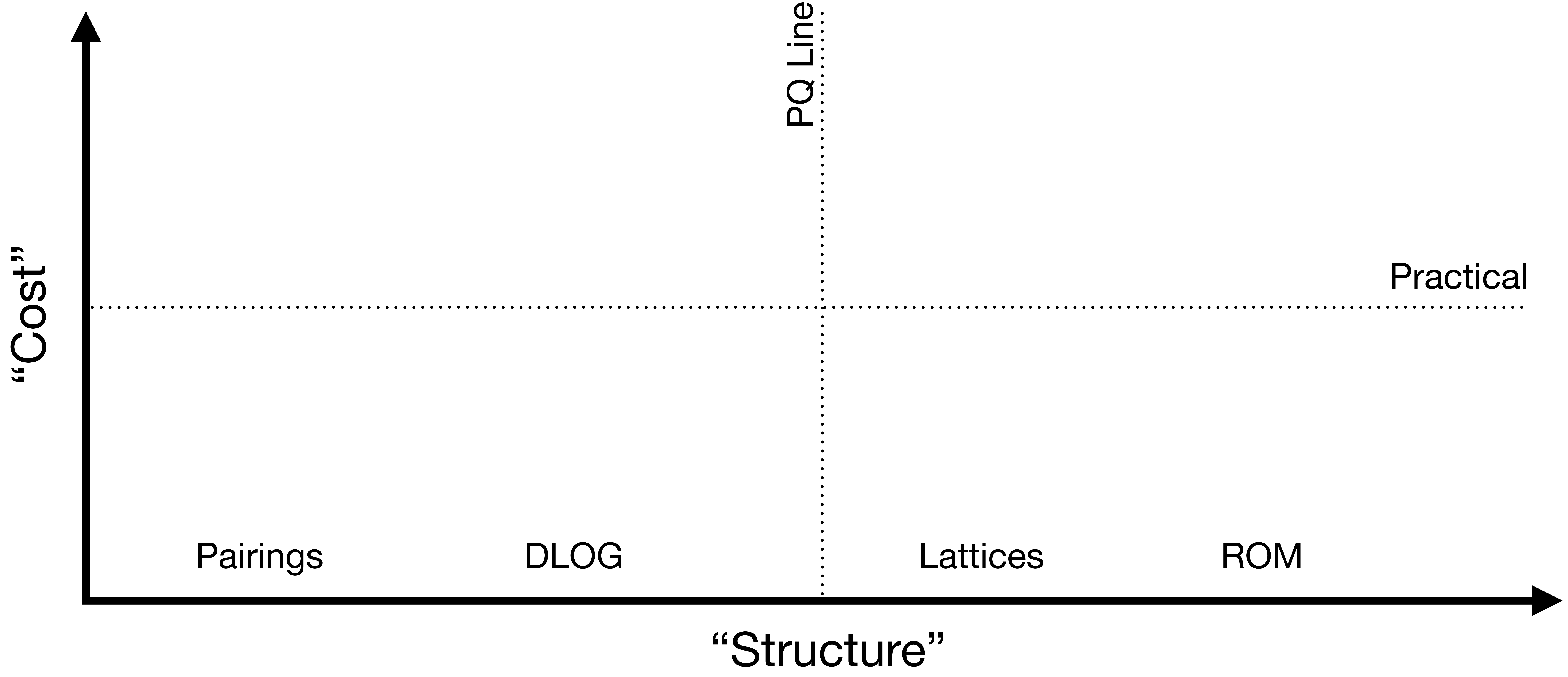

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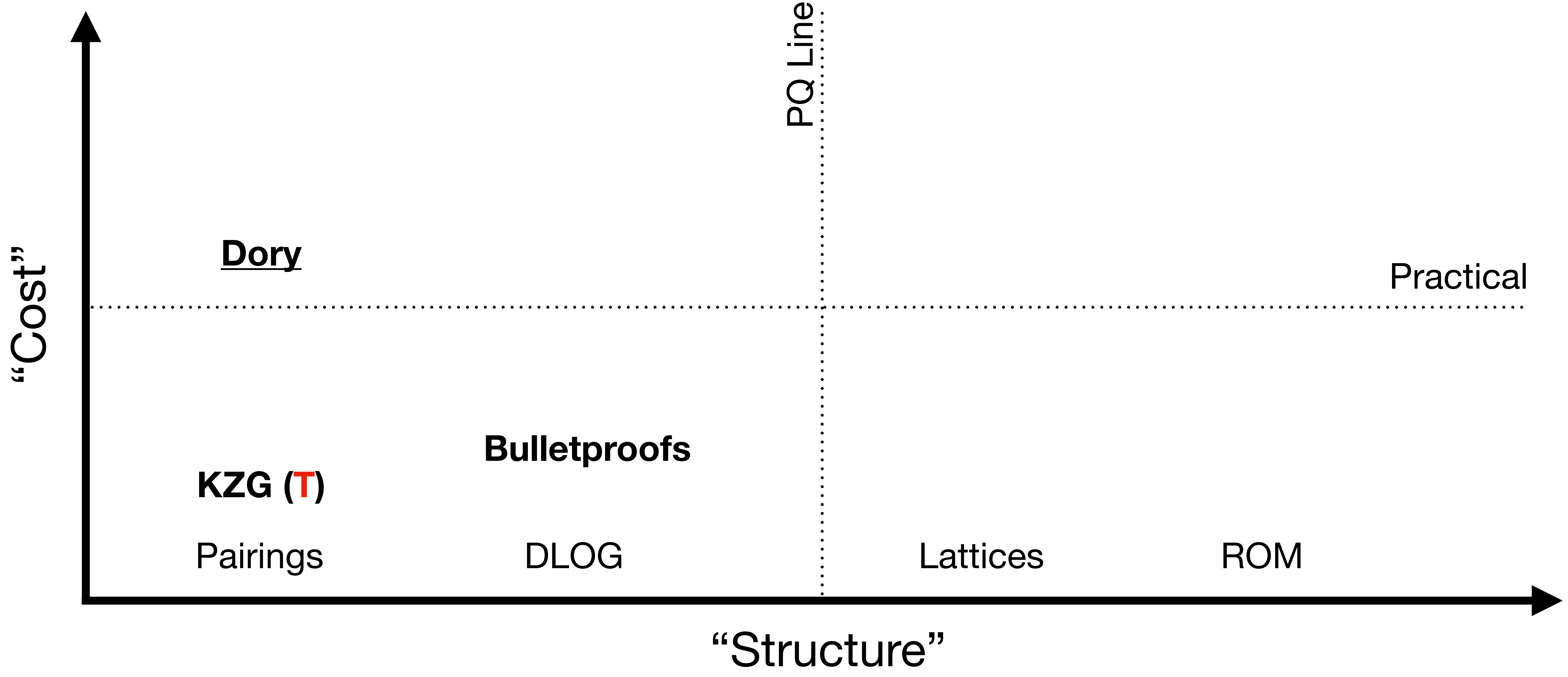

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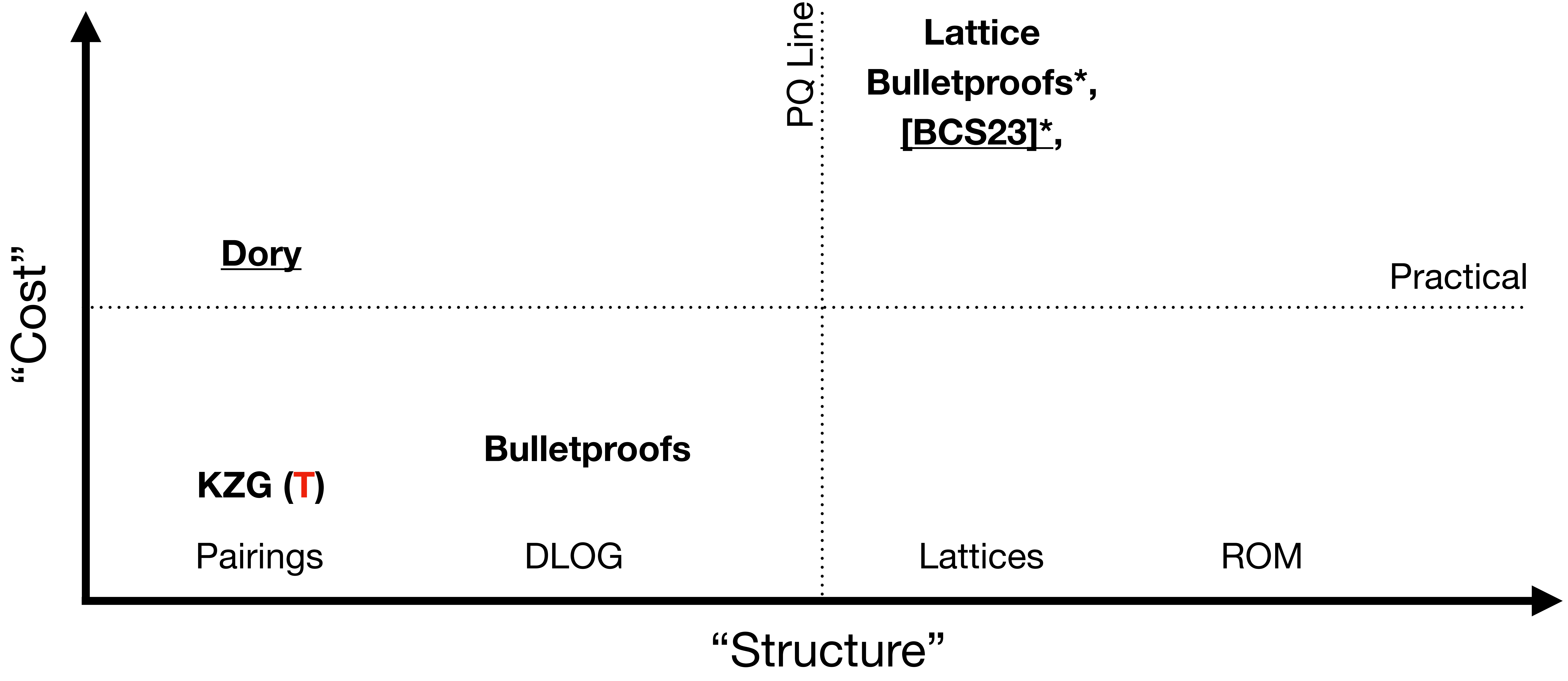

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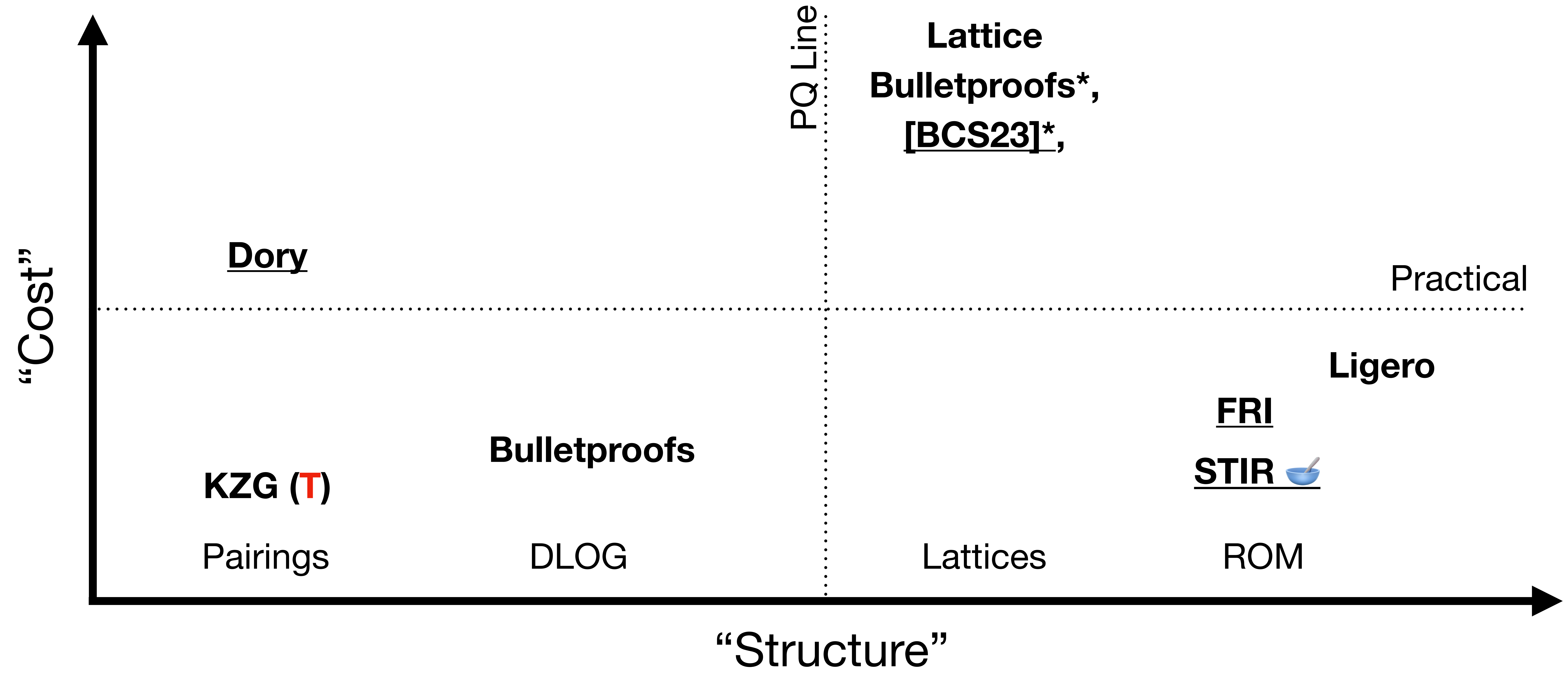

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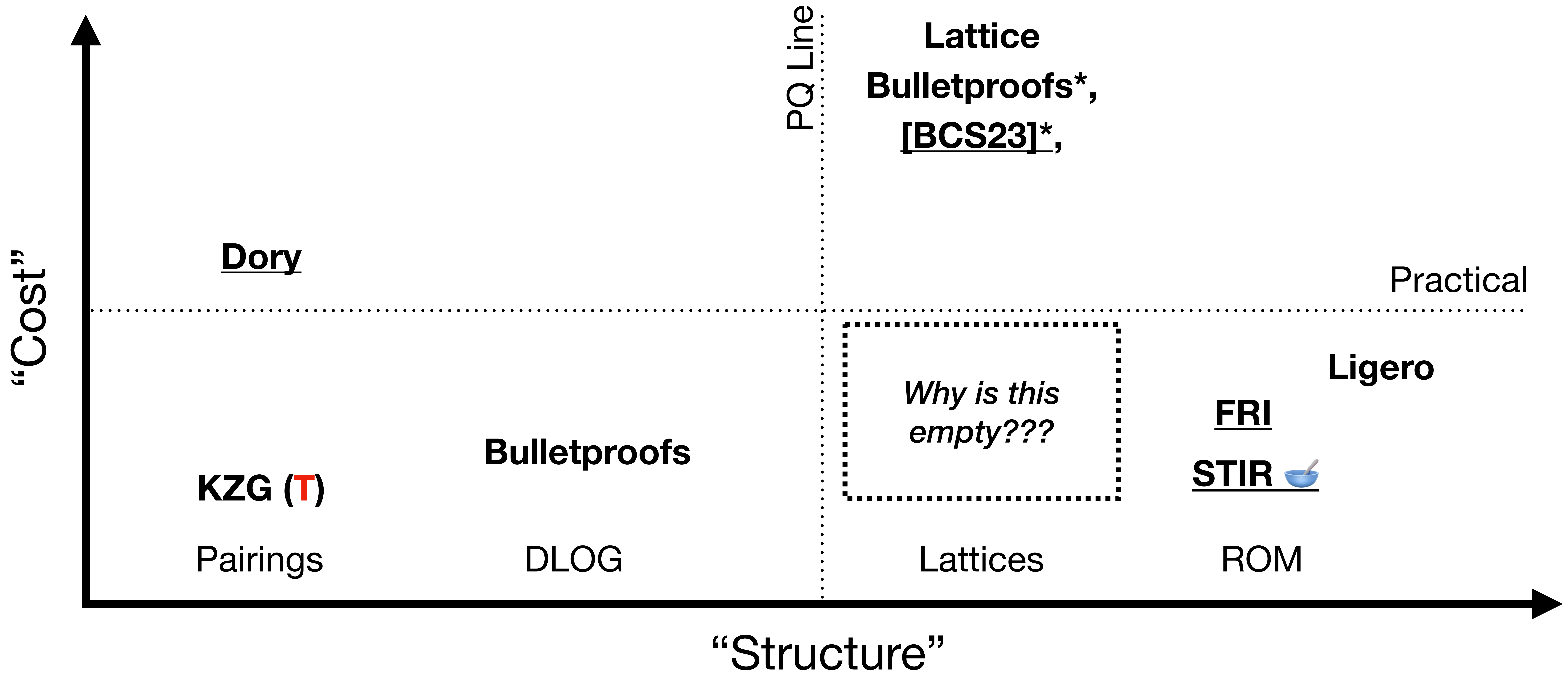

Our Results

SLAP: Succinct Lattice-Based Polynomial Commitments from Standard Assumptions

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We construct a non-interactive lattice-based polynomial commitment with:

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We construct a non-interactive lattice-based polynomial commitment with:

1. **Succinct** proofs

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We construct a non-interactive lattice-based polynomial commitment with:

1. **Succinct** proofs
2. **Succinct** verification time

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We construct a non-interactive lattice-based polynomial commitment with:

1. **Succinct** proofs
2. **Succinct** verification time
3. Binding under **(M)SIS**

Techniques

Lattice-Based SNARKs

How to get around [GW11]?

Lattice-Based SNARKs

How to get around [GW11]?

[GW11] - You cannot get **SN**ARG from falsifiable assumptions.

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Knowledge Assumptions

Lattice-Based SNARKs

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Knowledge Assumptions

Oblivious LWE Sampling

Lattice-Based SNARKs

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[GW11] - You cannot get **SN**ARG from falsifiable assumptions.

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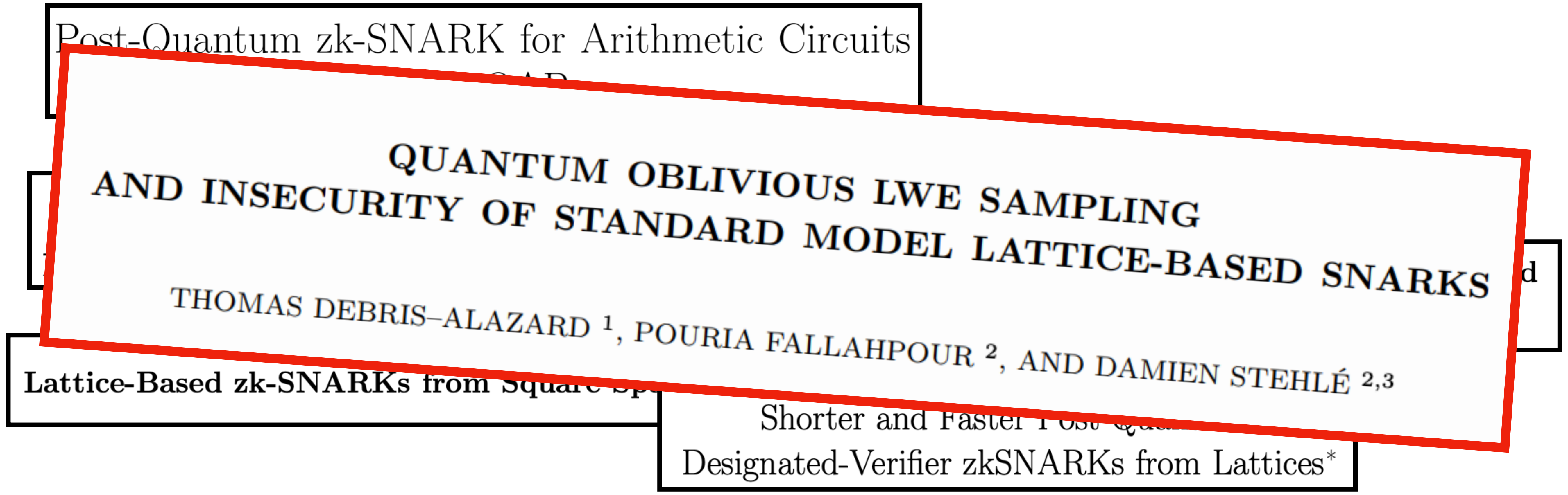

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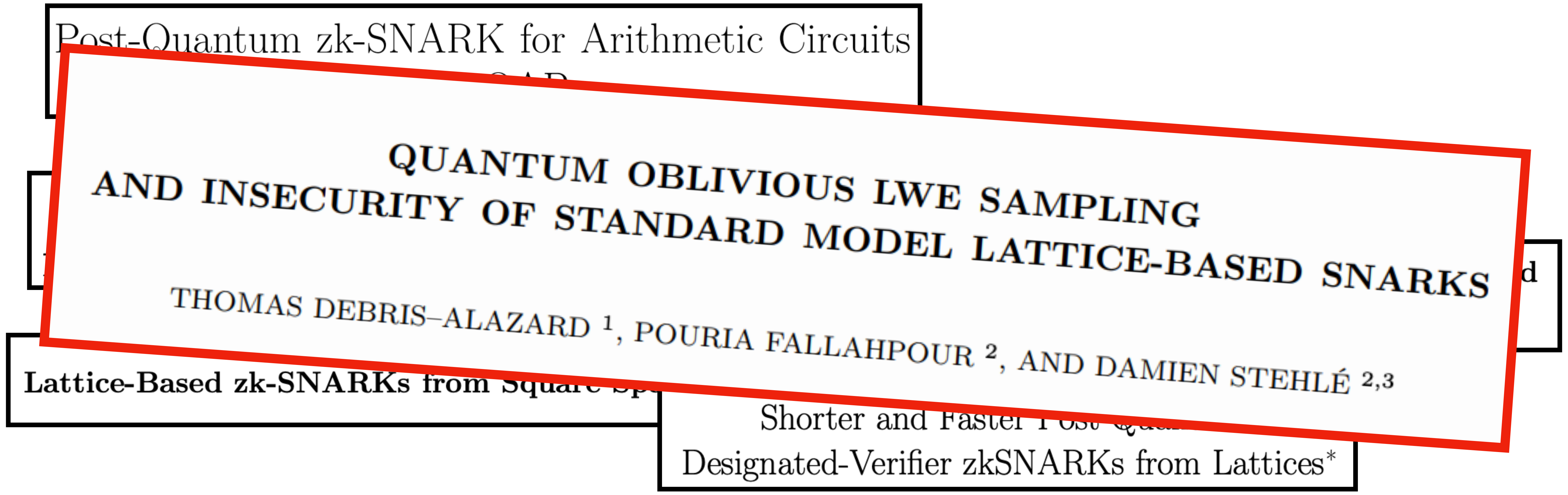

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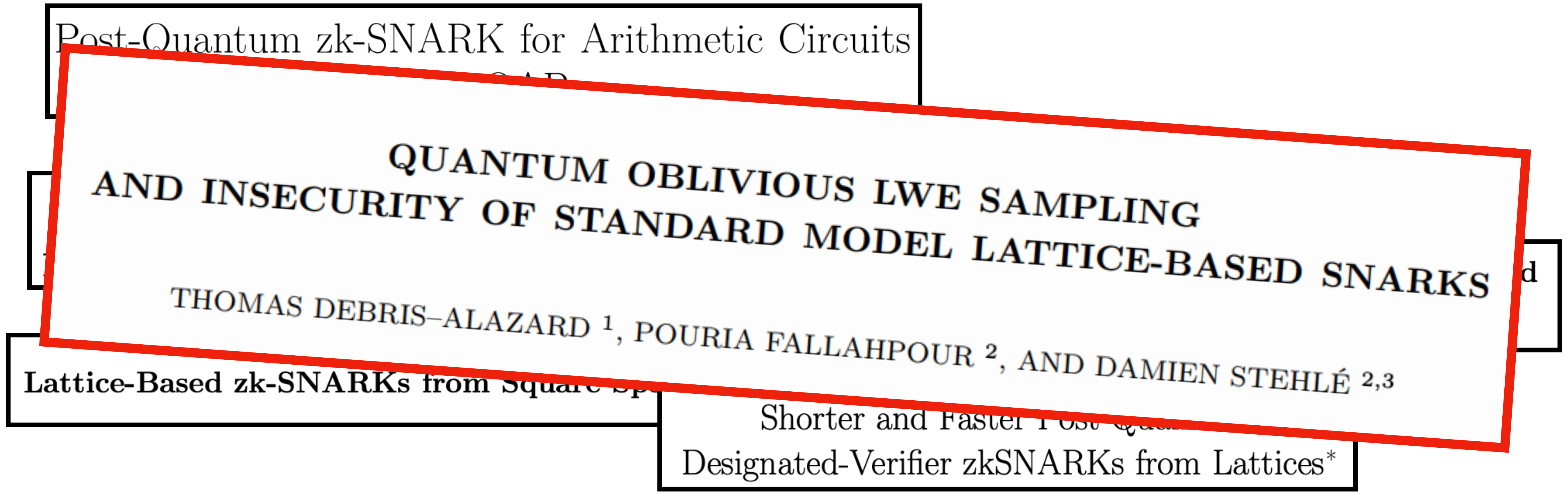

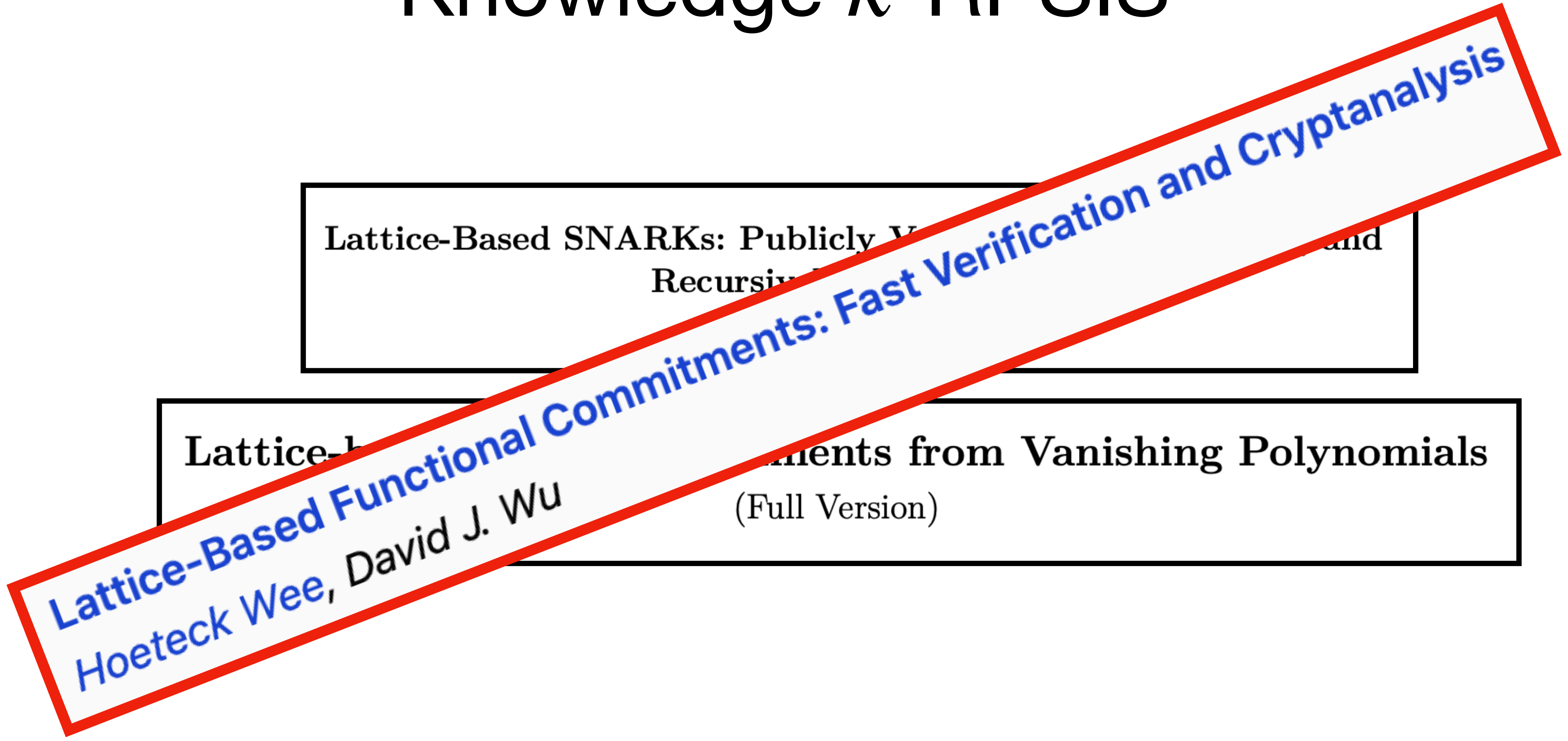

Lattice Assumptions ❤️ ROM

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Special Sound Interactive Protocol + Fiat-Shamir Transform = Knowledge Sound PCS

- Use lattices to get succinctness in the interactive protocol.
- **Open Question**: ROM alone is sufficient for efficient PCS (e.g. STIR), can we gain by using lattices?

Building succinct PCS

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Commitment Scheme

Building succinct PCS

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- Commit to a vector $\mathbf{f} \in \mathscr{R}_q^d$

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$\mathbf{t}, u, v$

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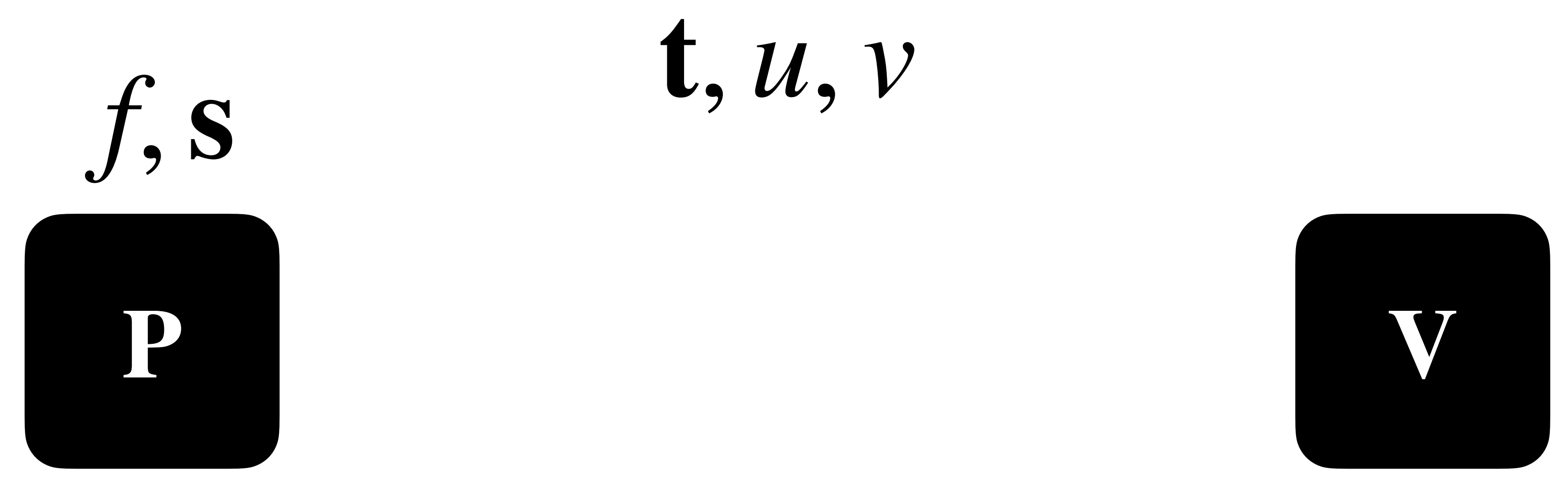

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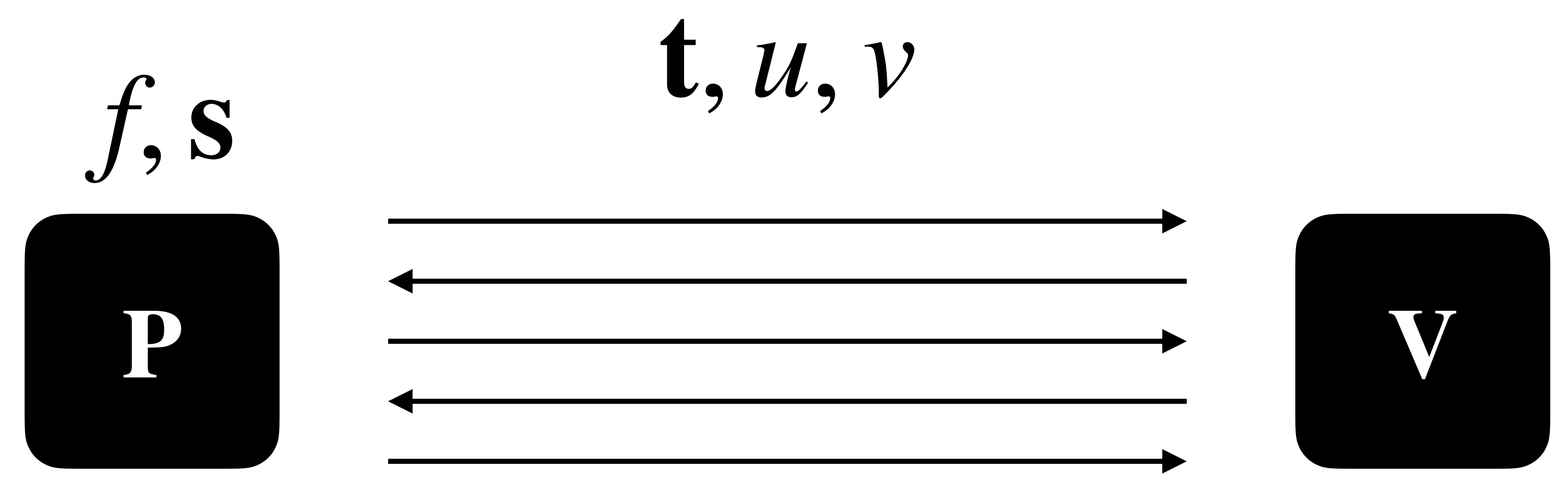

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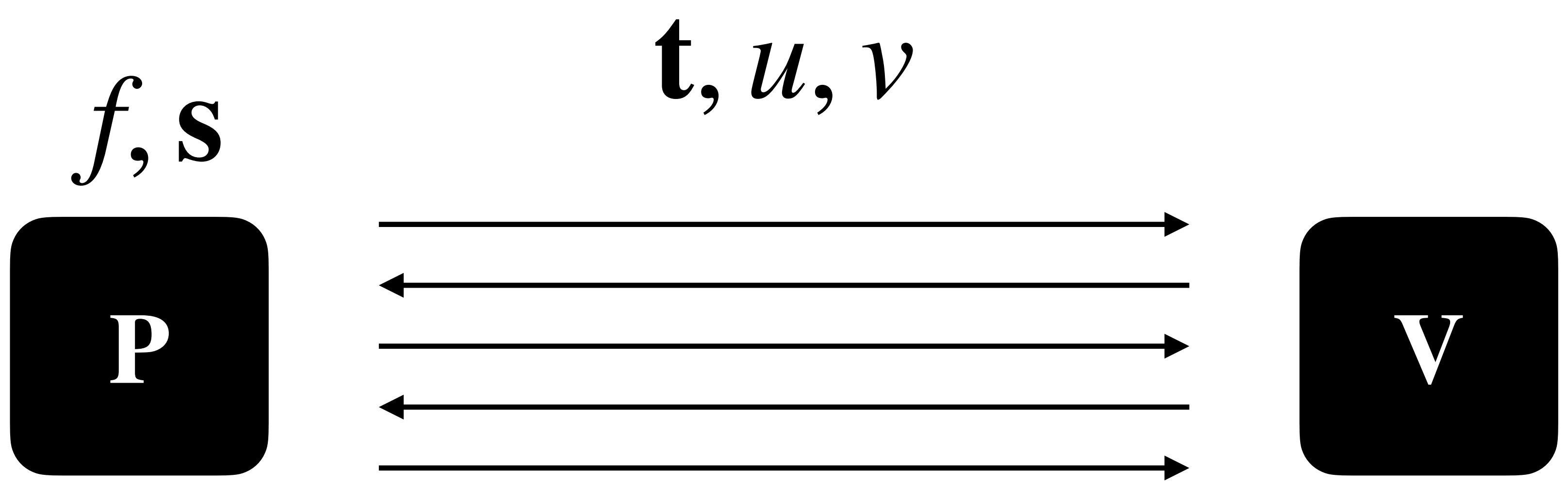

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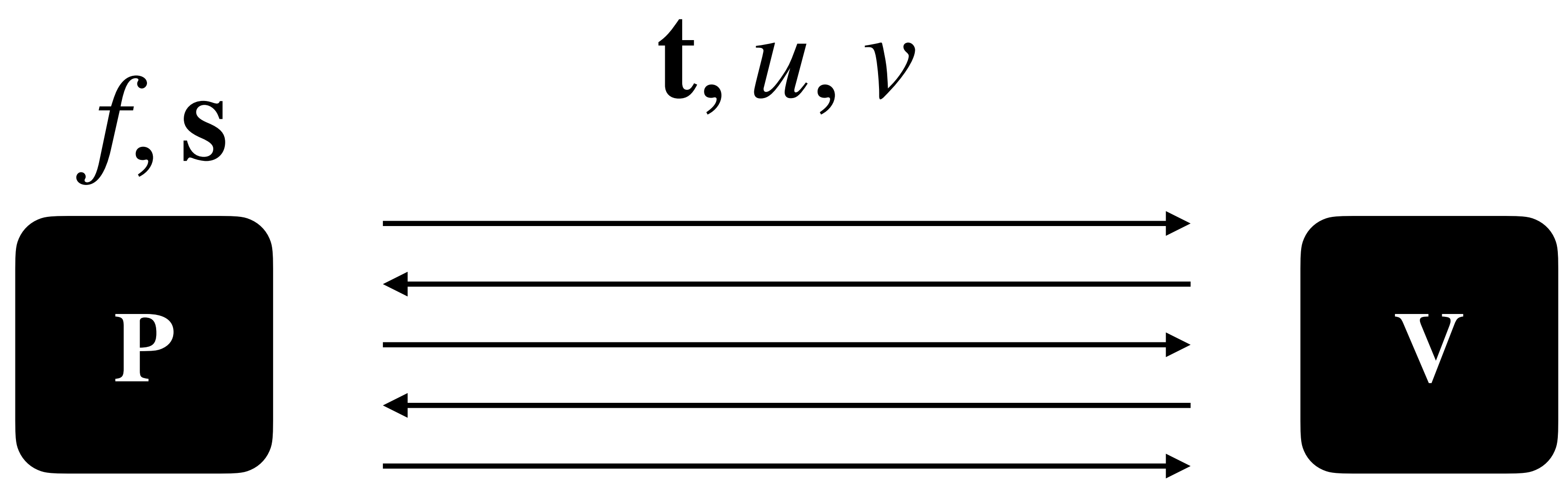

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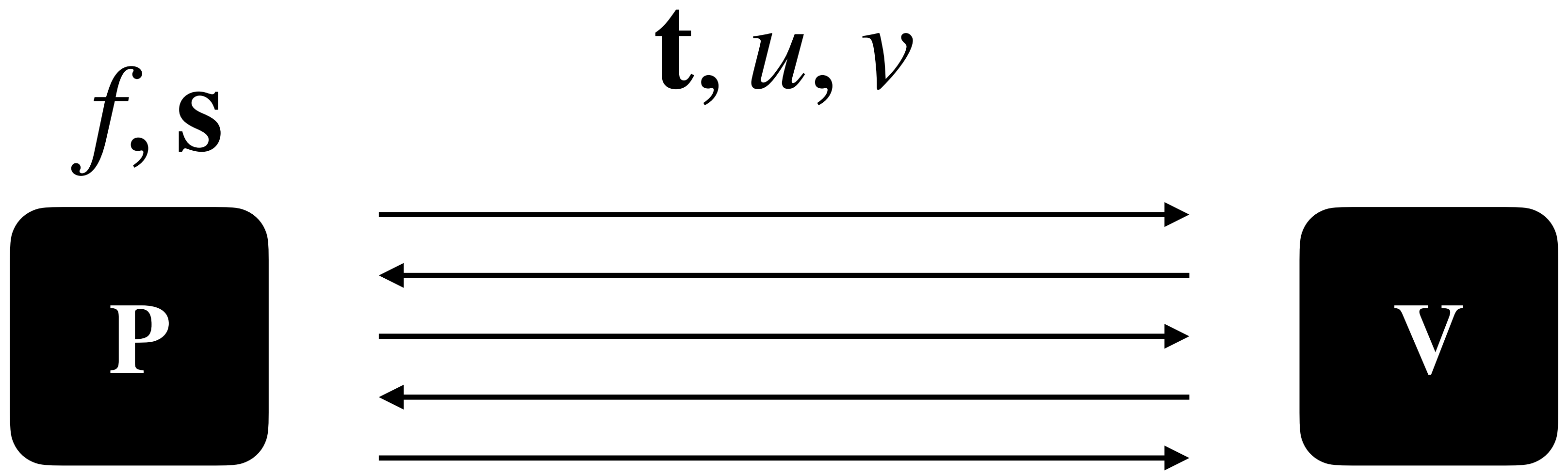

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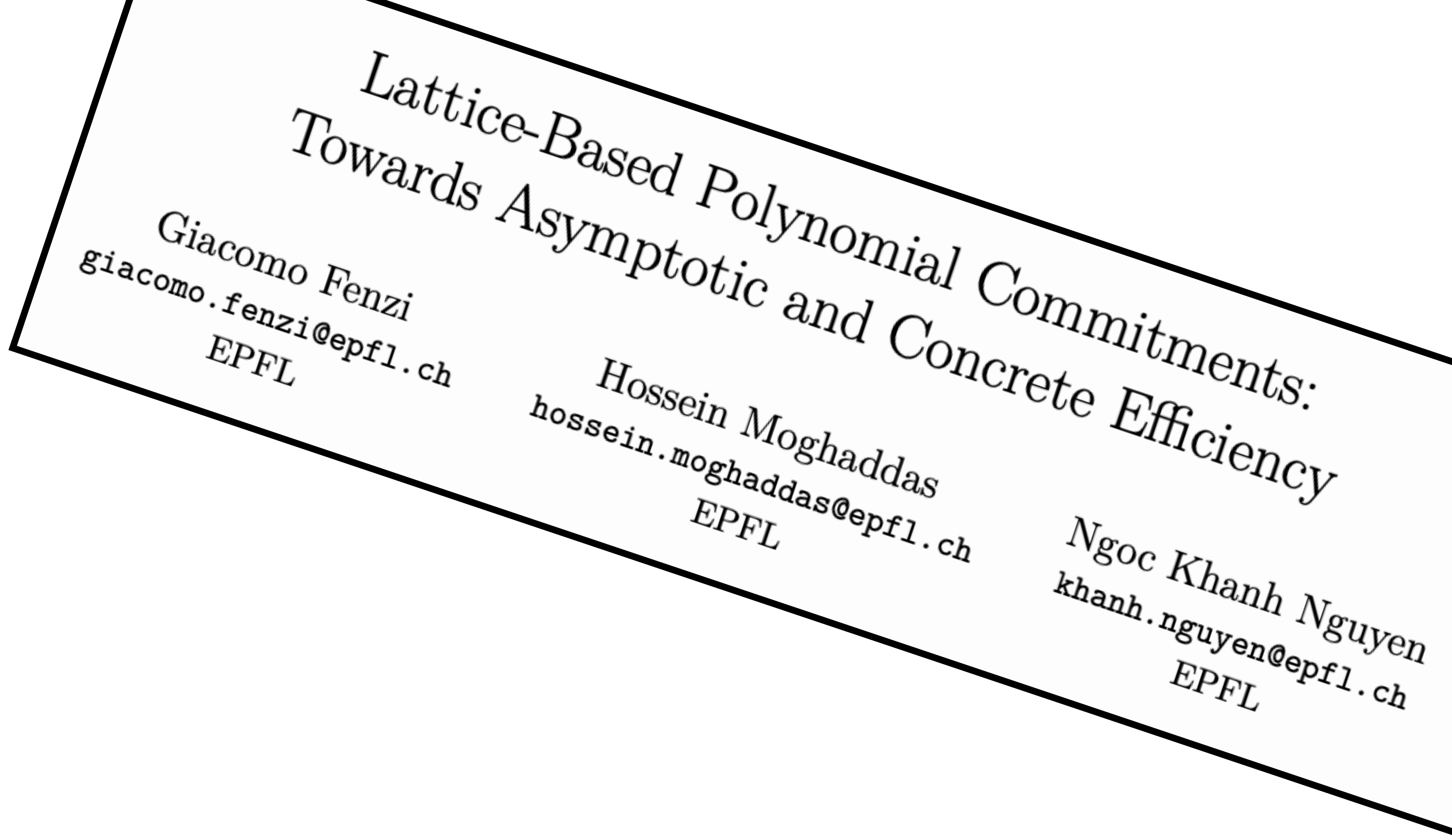
PRISIS Commitments I

A starting point [FMN23]



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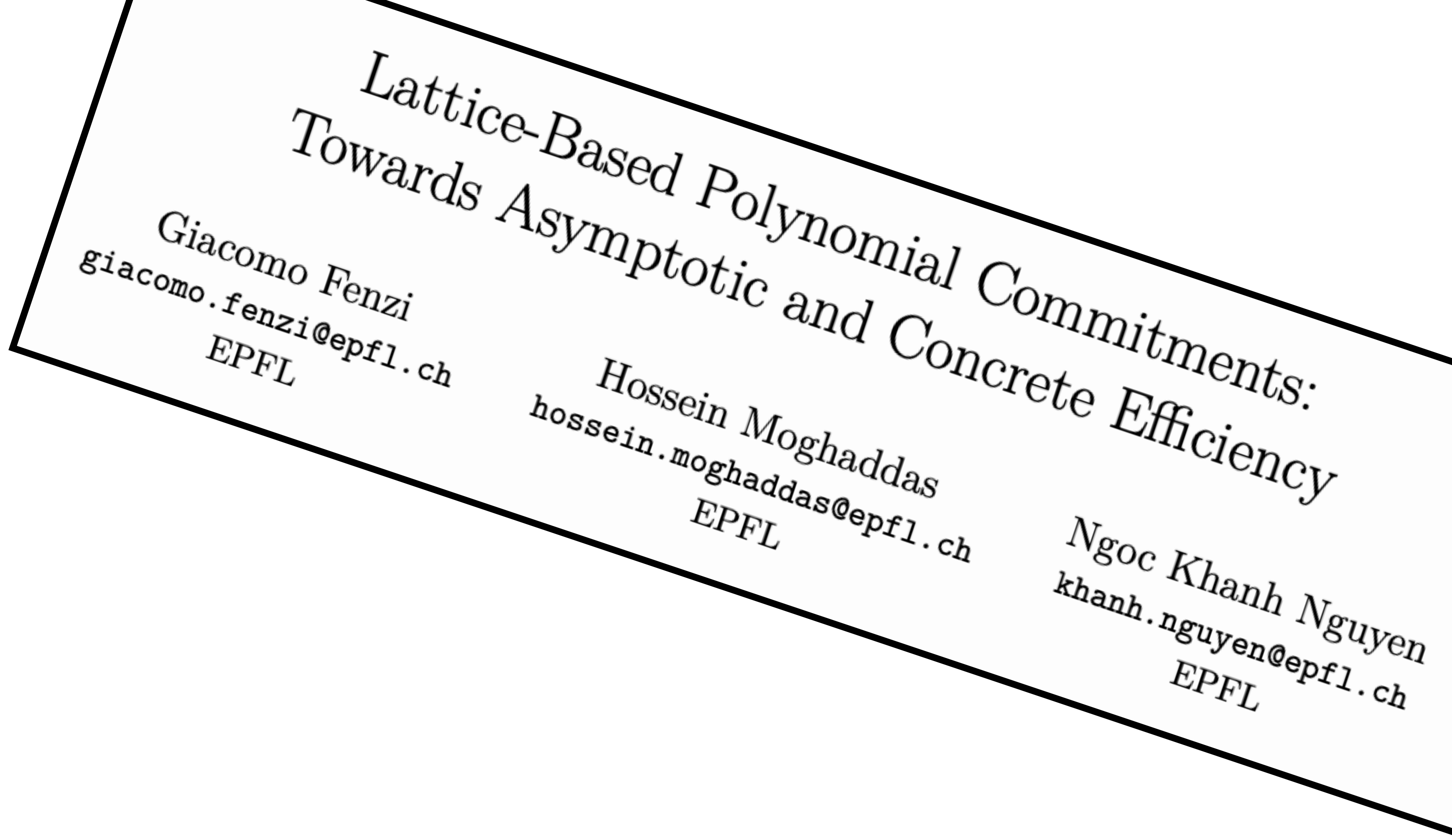
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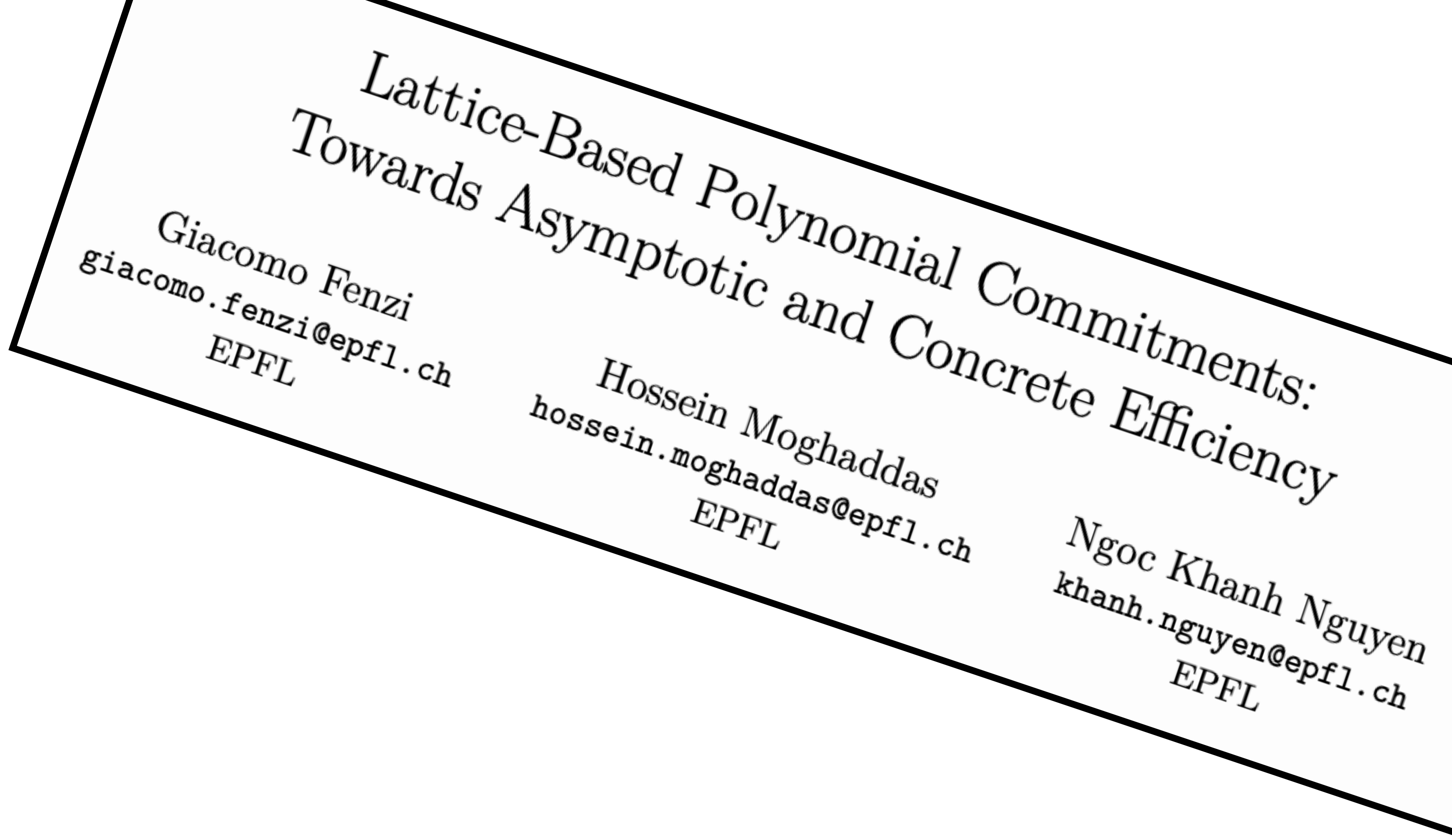
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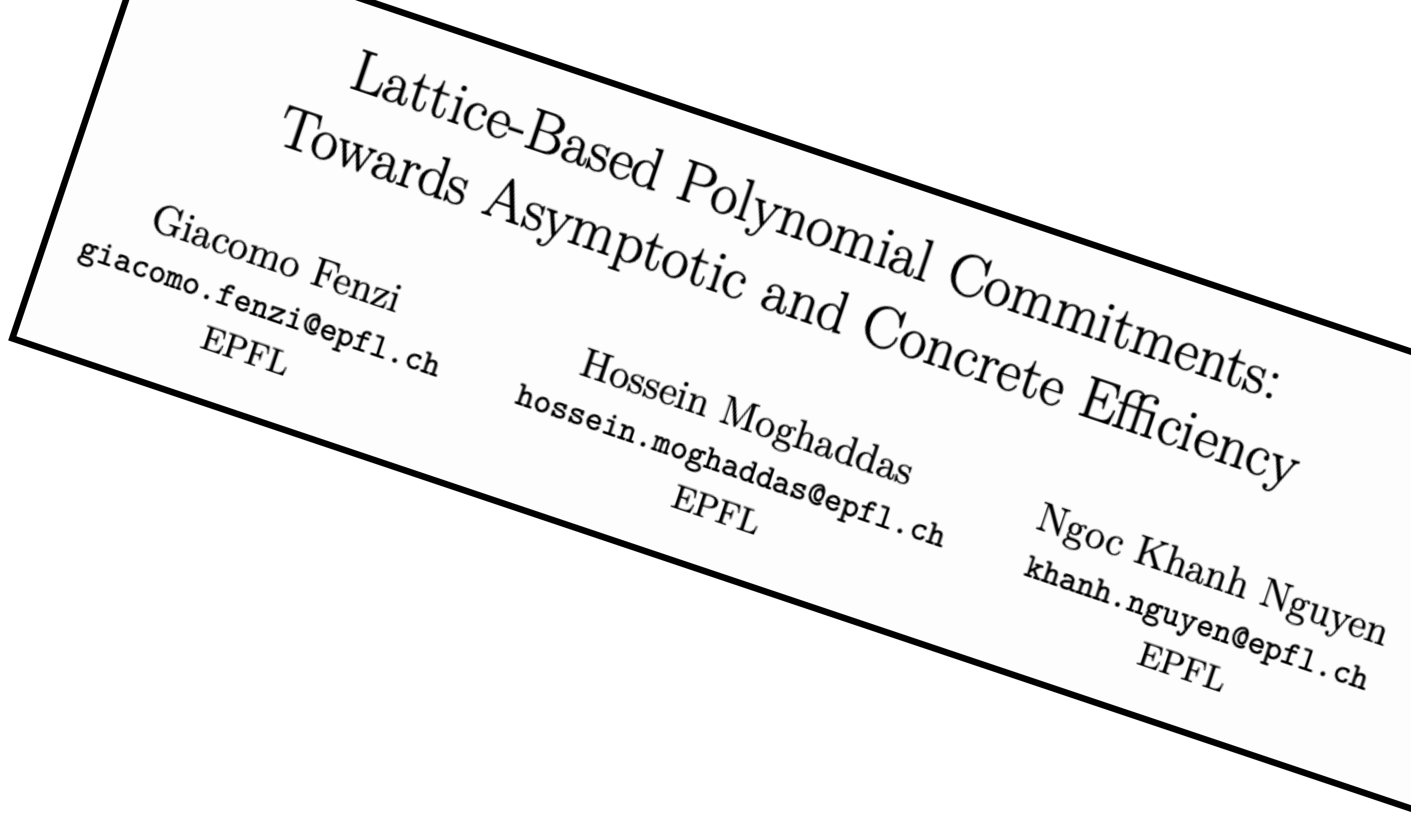
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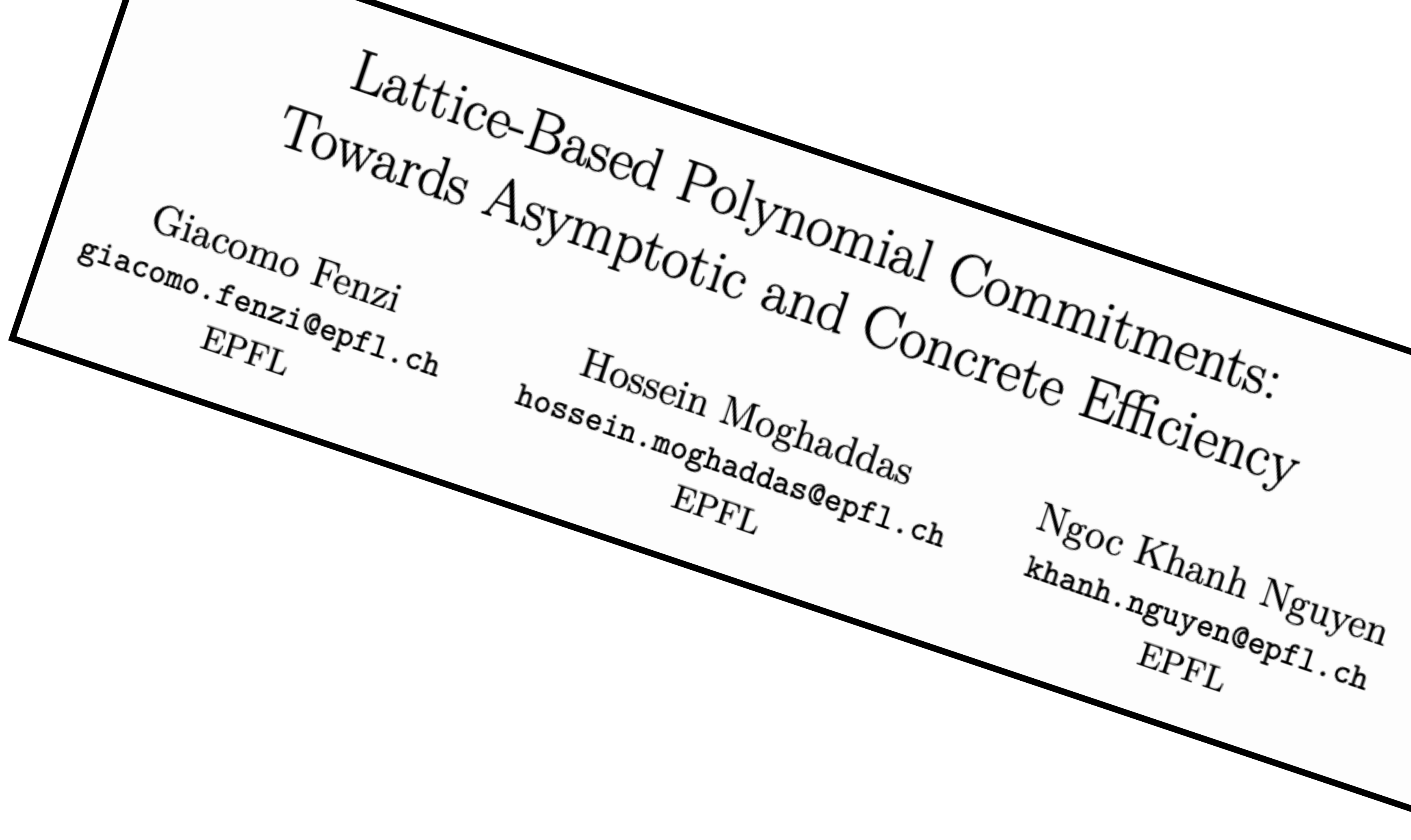
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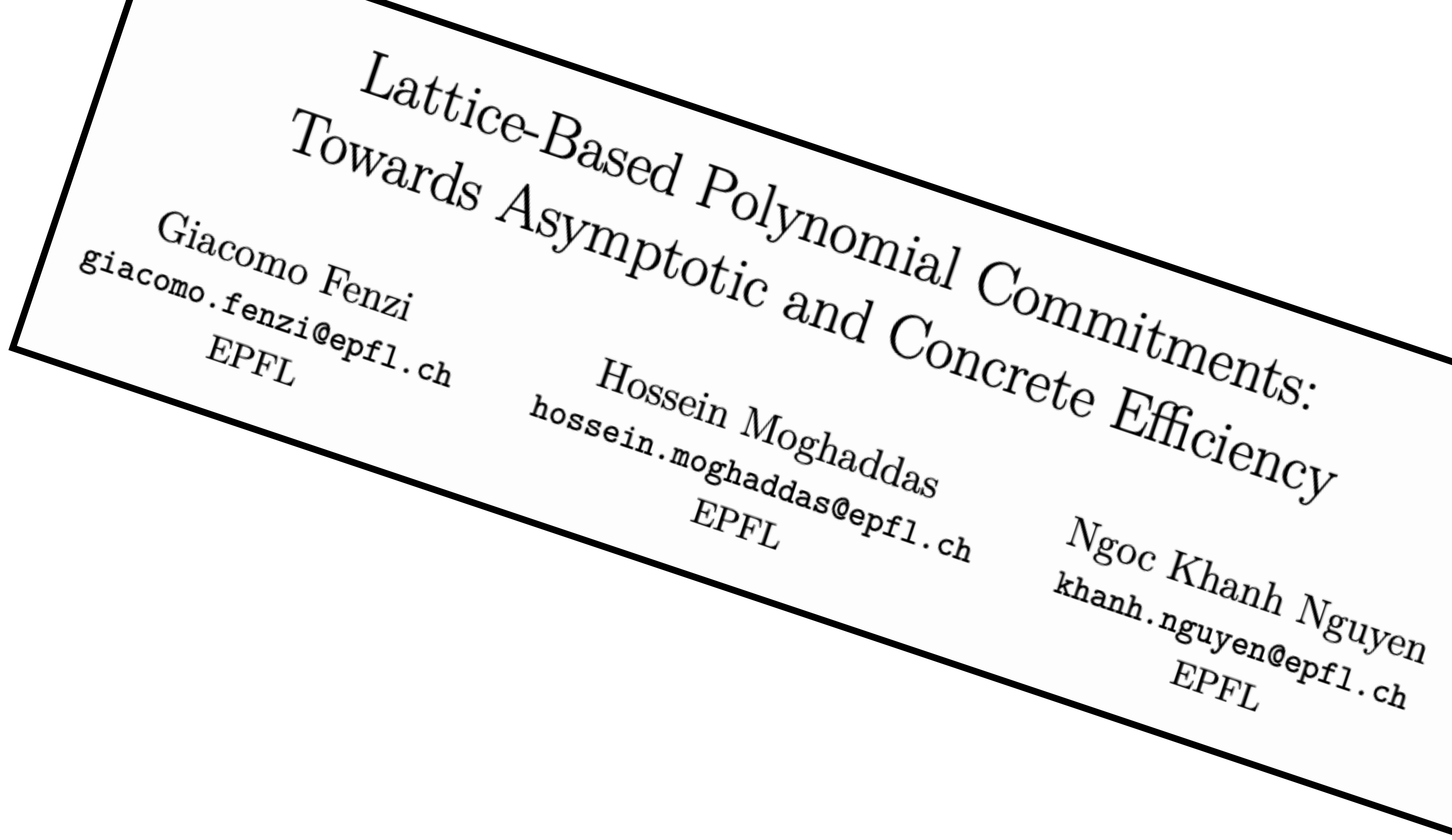

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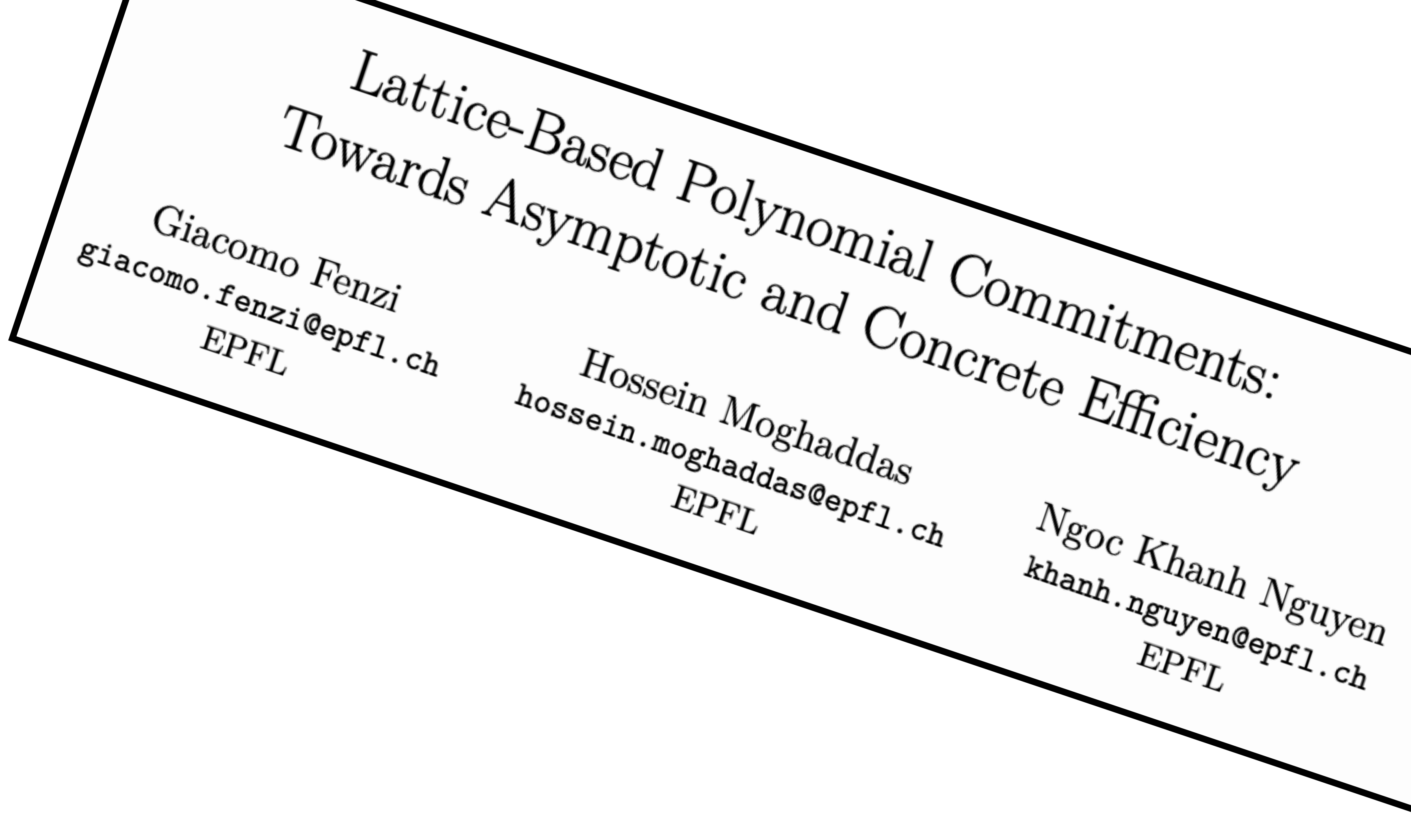
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Can we do better?

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$$\sigma \geq \delta \sqrt{tN \cdot (N^2 \mathfrak{s}^2 m + 2t)} \cdot \omega(\sqrt{N \log nN}).$$

Then, $\mathsf{PRISIS}_{n,m,N,q,2,\sigma,\beta}$ *is hard under the* $\mathsf{MSIS}_{n,m,N,q,\beta}$ *assumption.*

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Multi-Instance BASIS

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h-instance BASIS Game

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For $\ell = O(1)$, if PRISIS_ℓ is hard so is h-PRISIS_ℓ!

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Example with $d = 8$

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f_{000}

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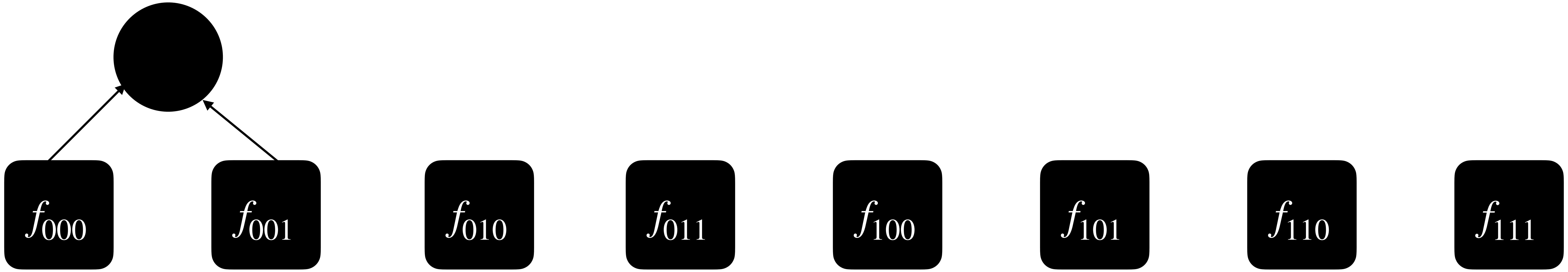

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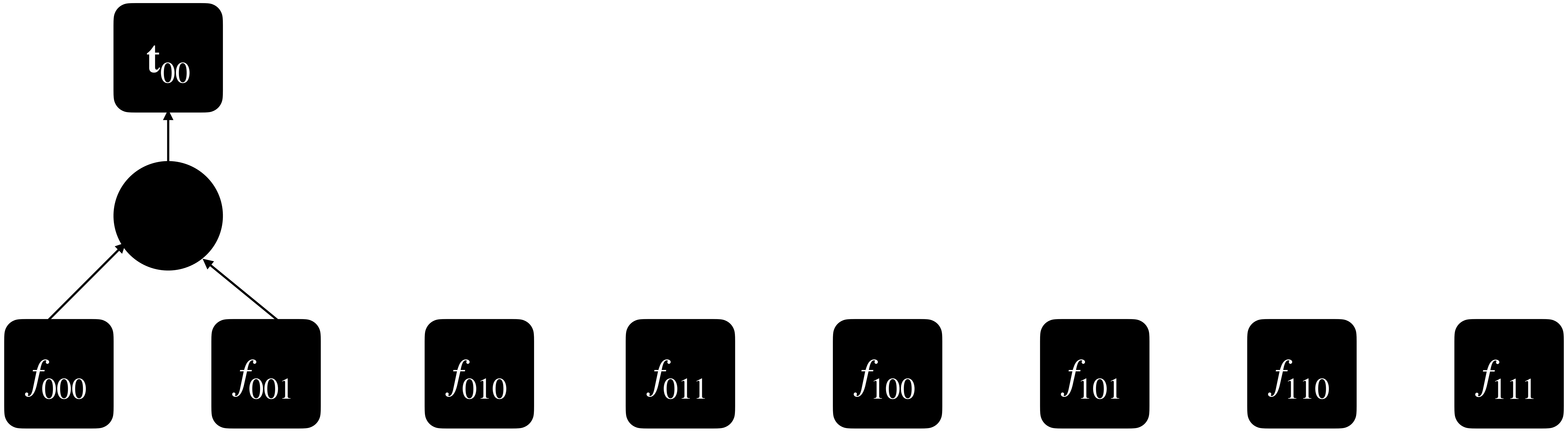

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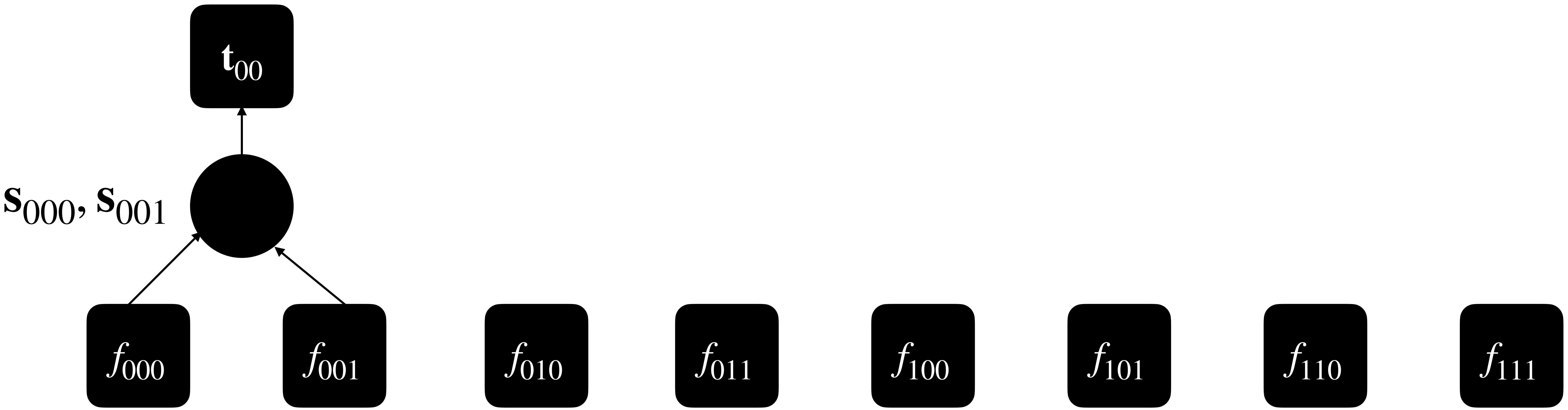

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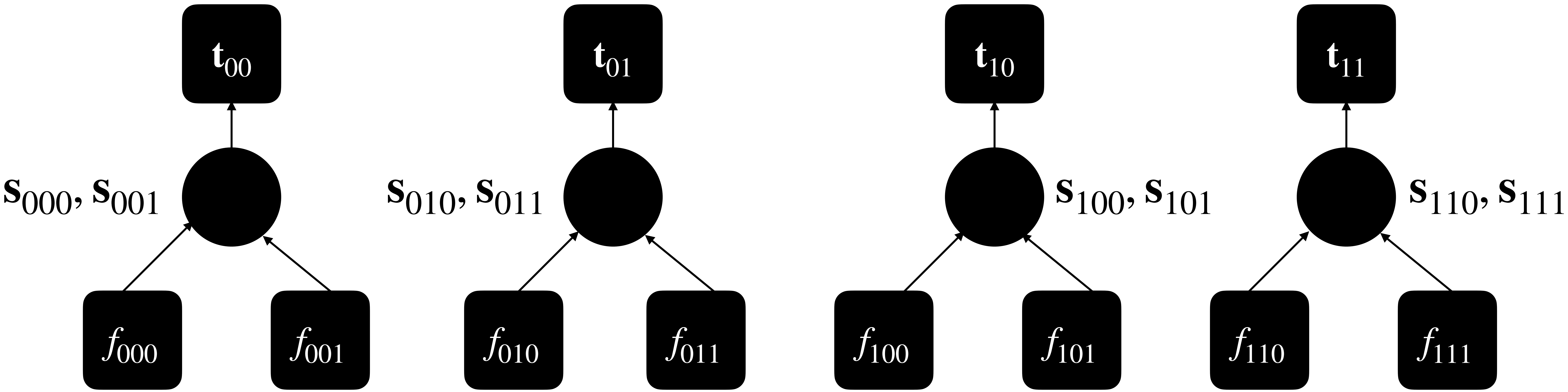

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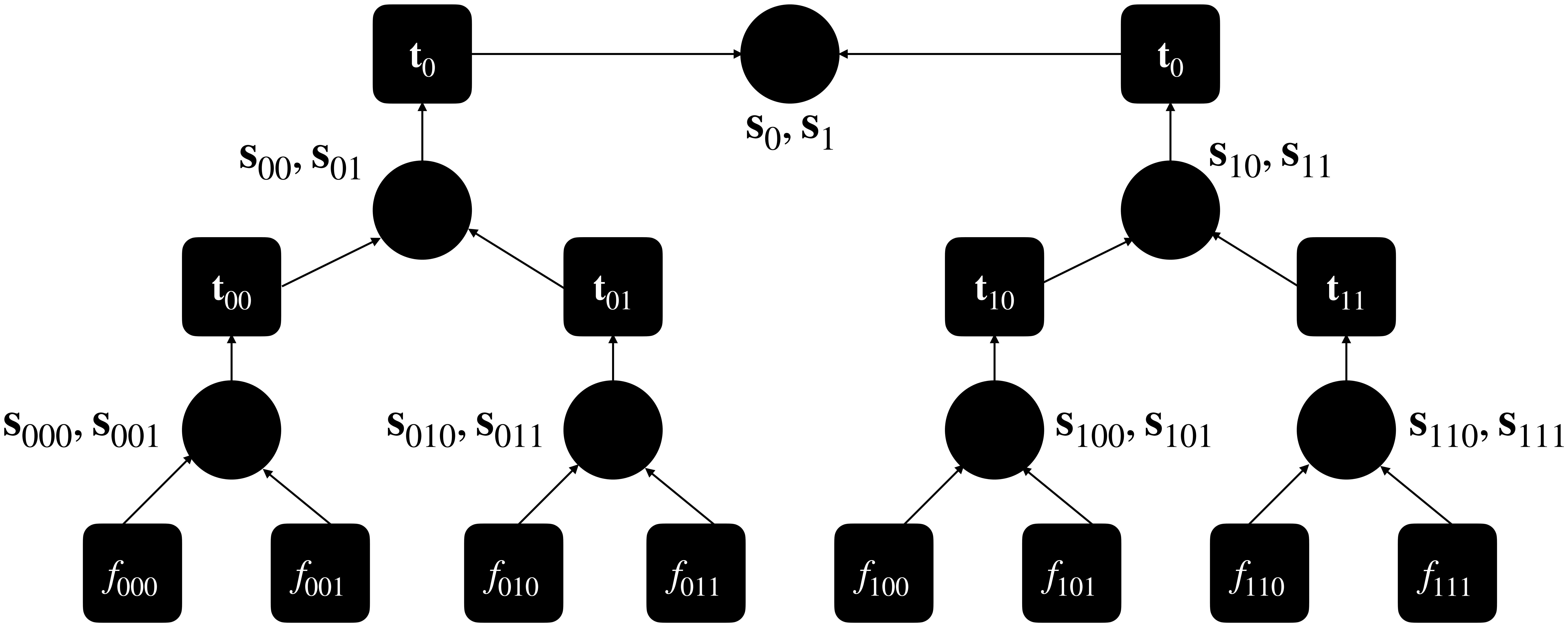

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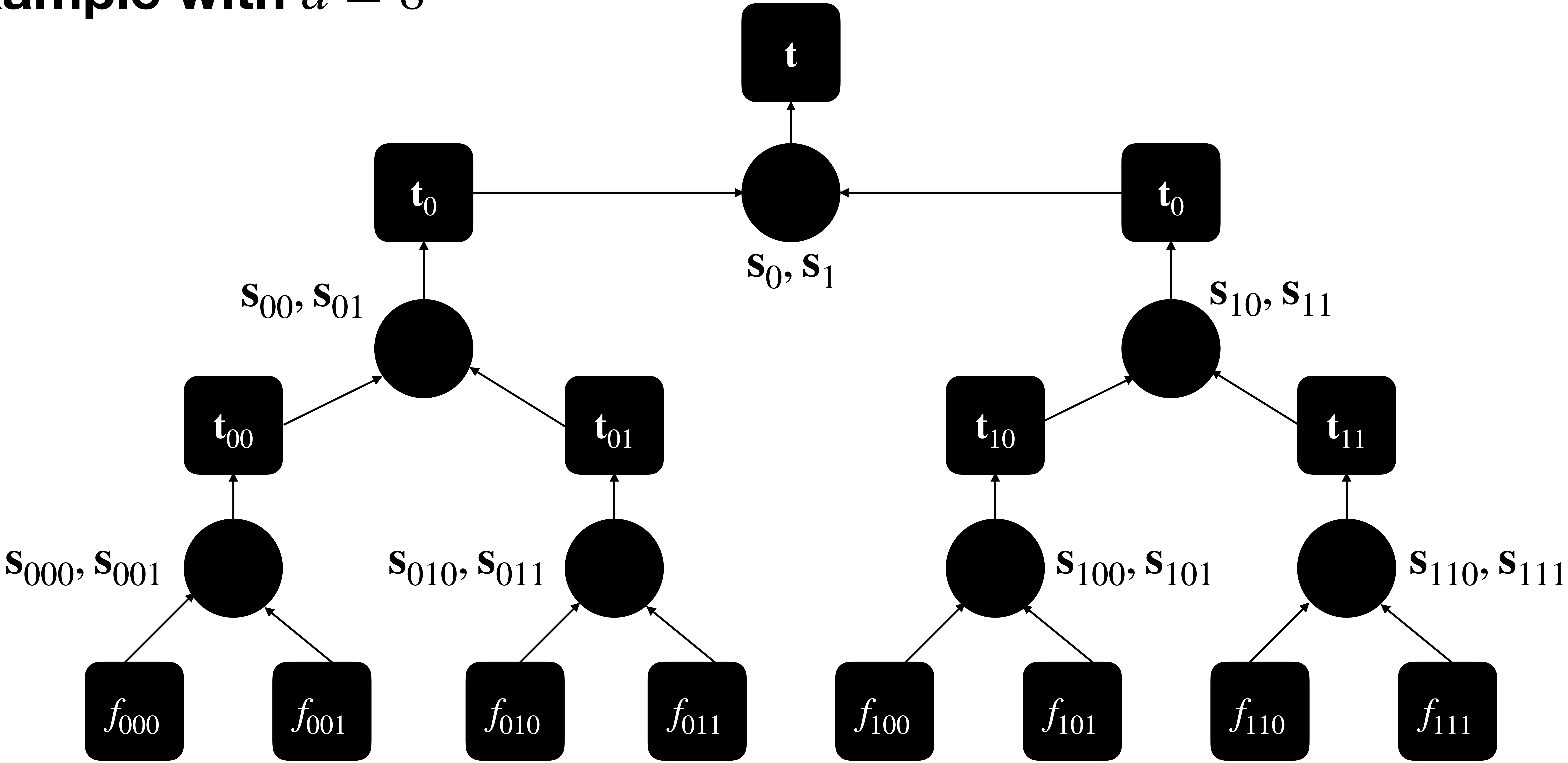

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  reduces to h-PRISIS$_\ell$ i.e. **MSIS**!

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Can we do an efficient evaluation protocol?

Evaluation Protocol

FRI Inspired folding + CWSS

Basic Σ-Protocol

Prover		**Verifier**
$f(\mathsf{X}) = f_0(\mathsf{X}^2) + \mathsf{X} f_1(\mathsf{X}^2)$		
$z_i \coloneqq f_i(u^2)$ for $i \in \mathbb{Z}_2$	$\xrightarrow{z_0, z_1, \mathbf{s}_0, \mathbf{s}_1}$	Check: $z_0 + u z_1 =_? z$; Check: $\mathbf{s}_0, \mathbf{s}_1$ short
$g(\mathsf{X}) \coloneqq \alpha_0 f_0(\mathsf{X}) + \alpha_1 f_1(\mathsf{X})$	$\xleftarrow{\alpha_0, \alpha_1}$	$\alpha_0, \alpha_1 \leftarrow \{ X^i : i \in \mathbb{Z} \}$
$\mathbf{z_b} \coloneqq \alpha_0 \mathbf{s}_{\mathbf{b},0} + \alpha_1 \mathbf{s}_{\mathbf{b},1}$ for $\mathbf{b} \in \mathbb{Z}_2^{\leq h-1}$	$\xrightarrow{g, (\mathbf{z_b})_\mathbf{b}}$	$\mathsf{crs}' \coloneqq (\mathbf{A}_{1+t}, w_{1+t}, \mathbf{T}_{1+t})_{t \in [h-1]}$
		$\mathbf{t}' \coloneqq \alpha_0 \cdot (\mathbf{t} - w_1^0 \mathbf{A}_1 \mathbf{s}_0) + \alpha_1 \cdot (\mathbf{t} - w_1^1 \mathbf{A}_1 \mathbf{s}_1)$
		$u' \coloneqq u^2; z' \coloneqq \alpha_0 \cdot z_0 + \alpha_1 \cdot z_1$
		Check: $g(u') = z'$
		Check: $\mathsf{Open}(\mathsf{crs}', \mathbf{t}', g, (\mathbf{z_b})_\mathbf{b}) = 1$

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  - Subtractive challenge space => Challenge space of size at most $\text{poly}(\lambda)$ [AL21]

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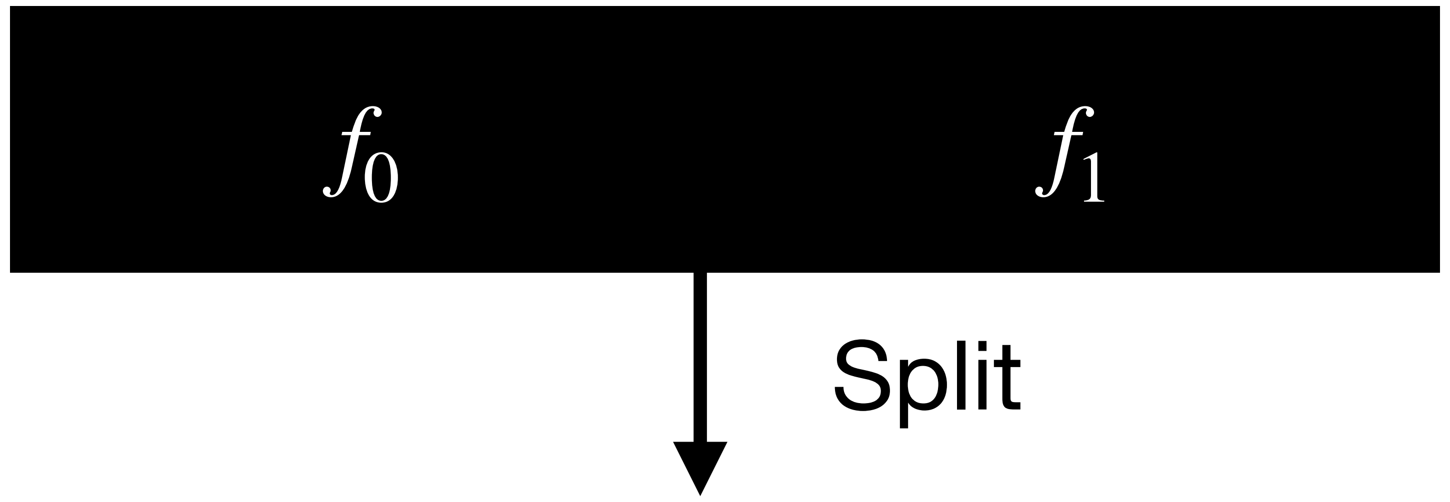

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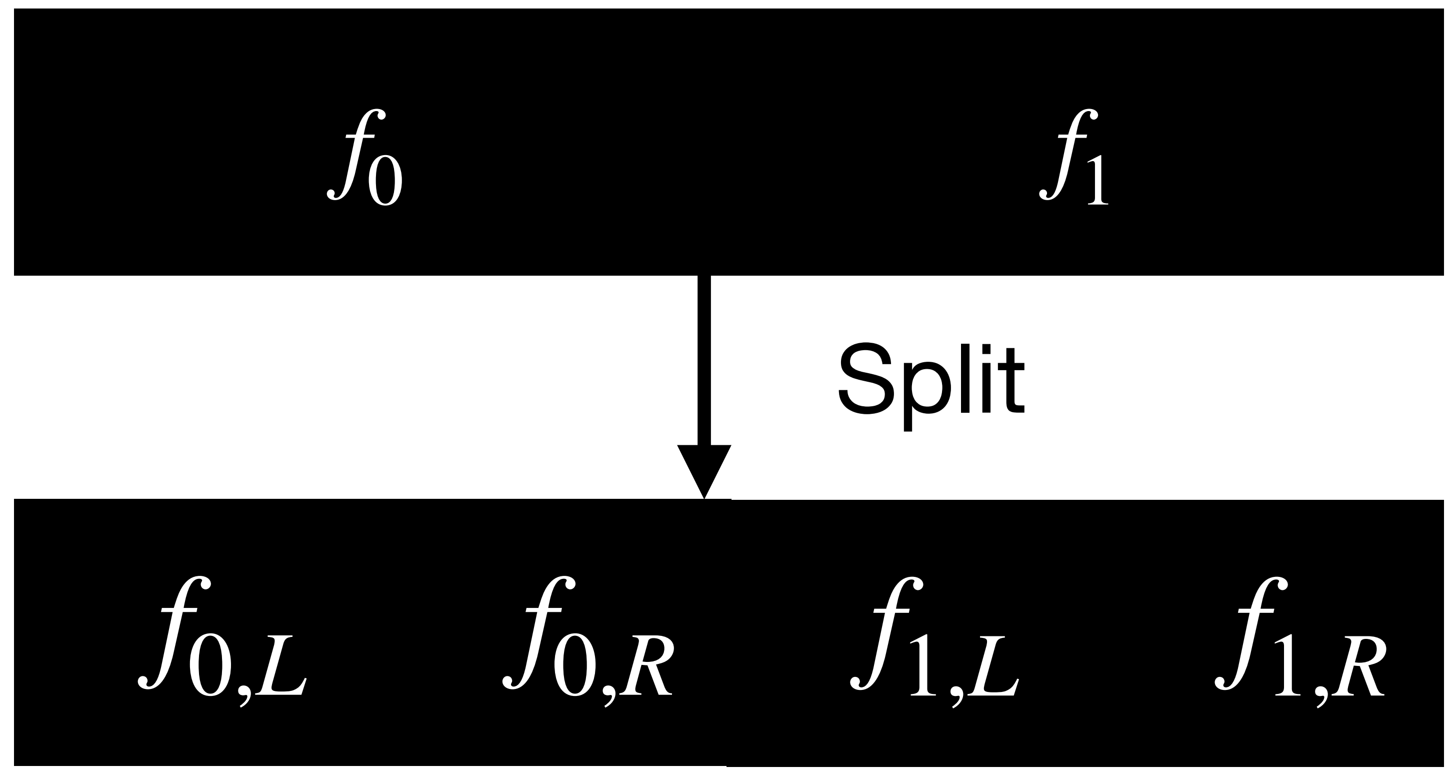

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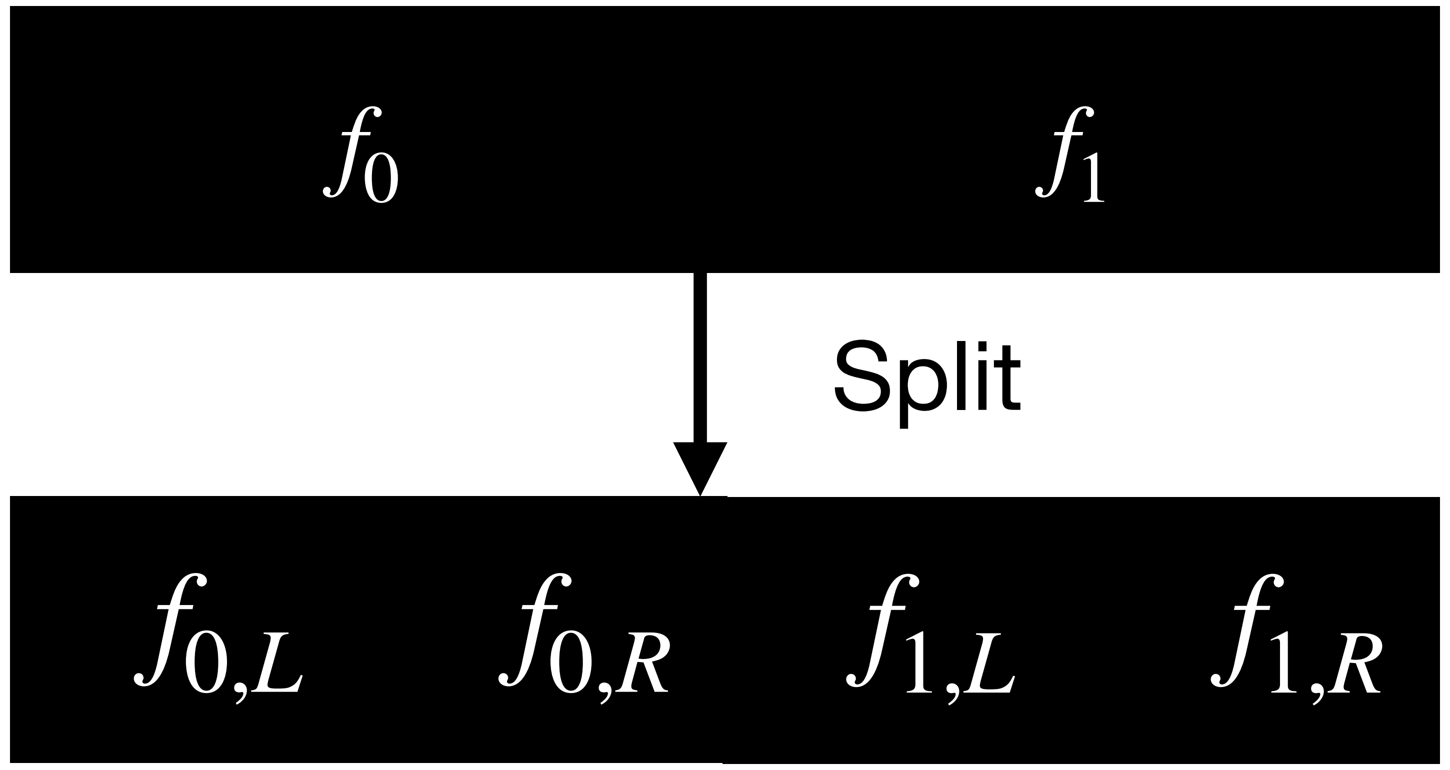

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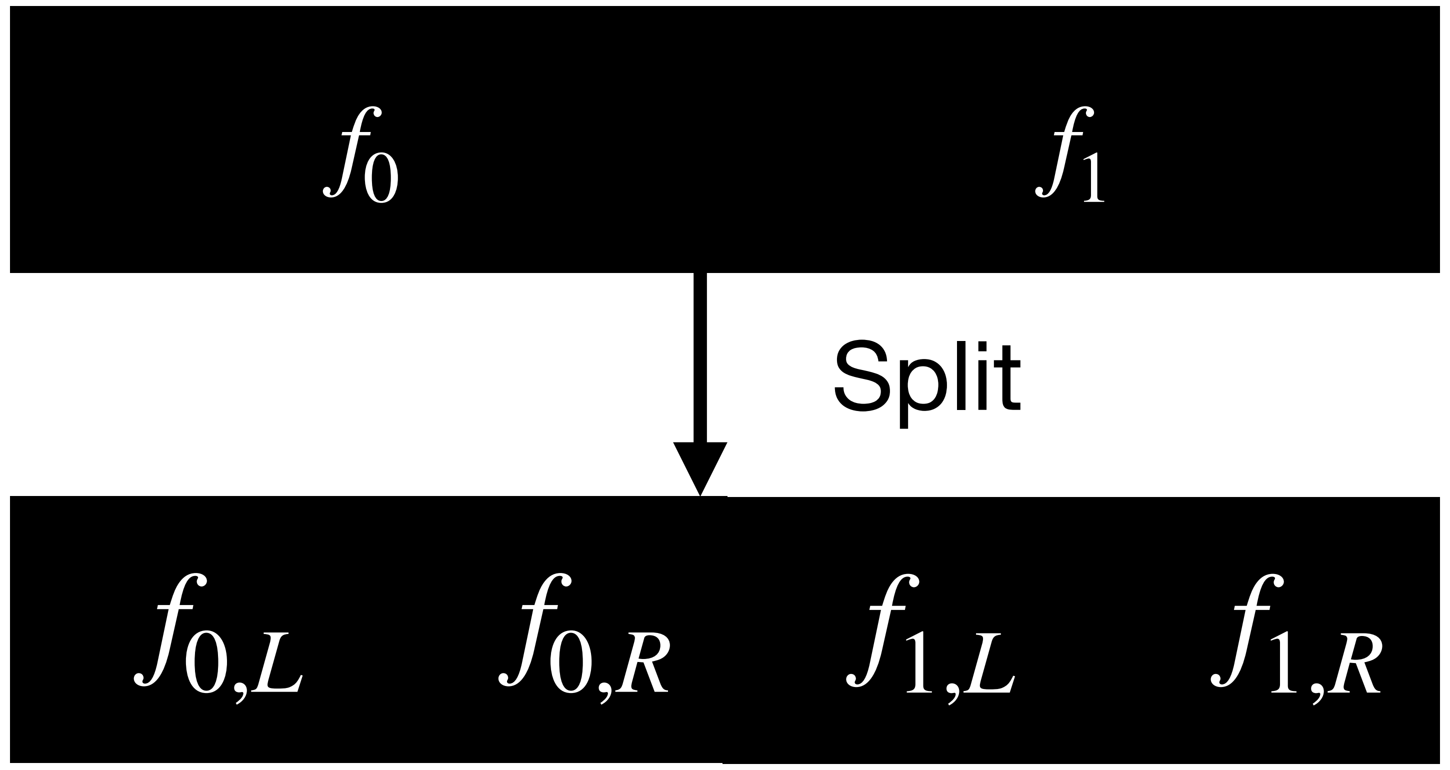

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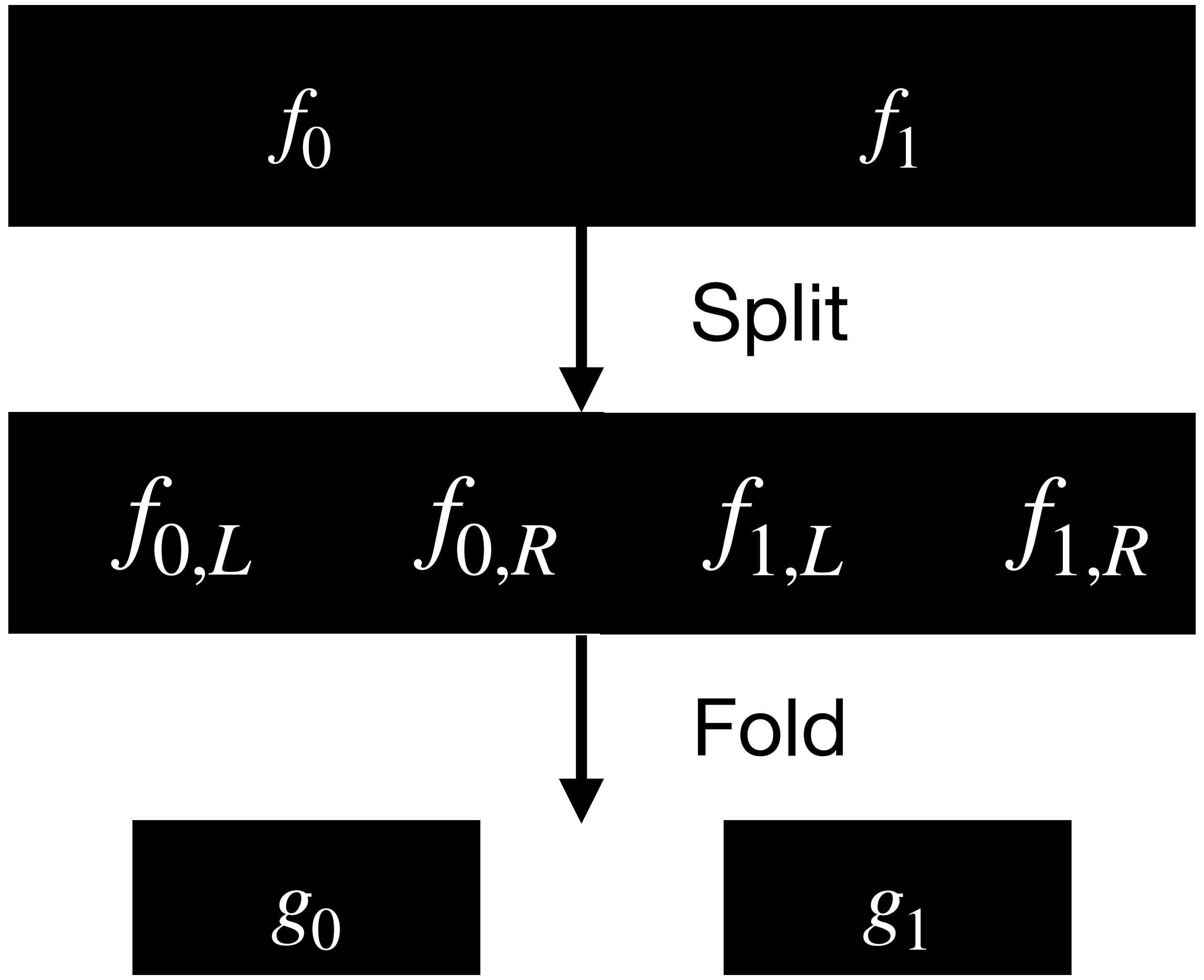

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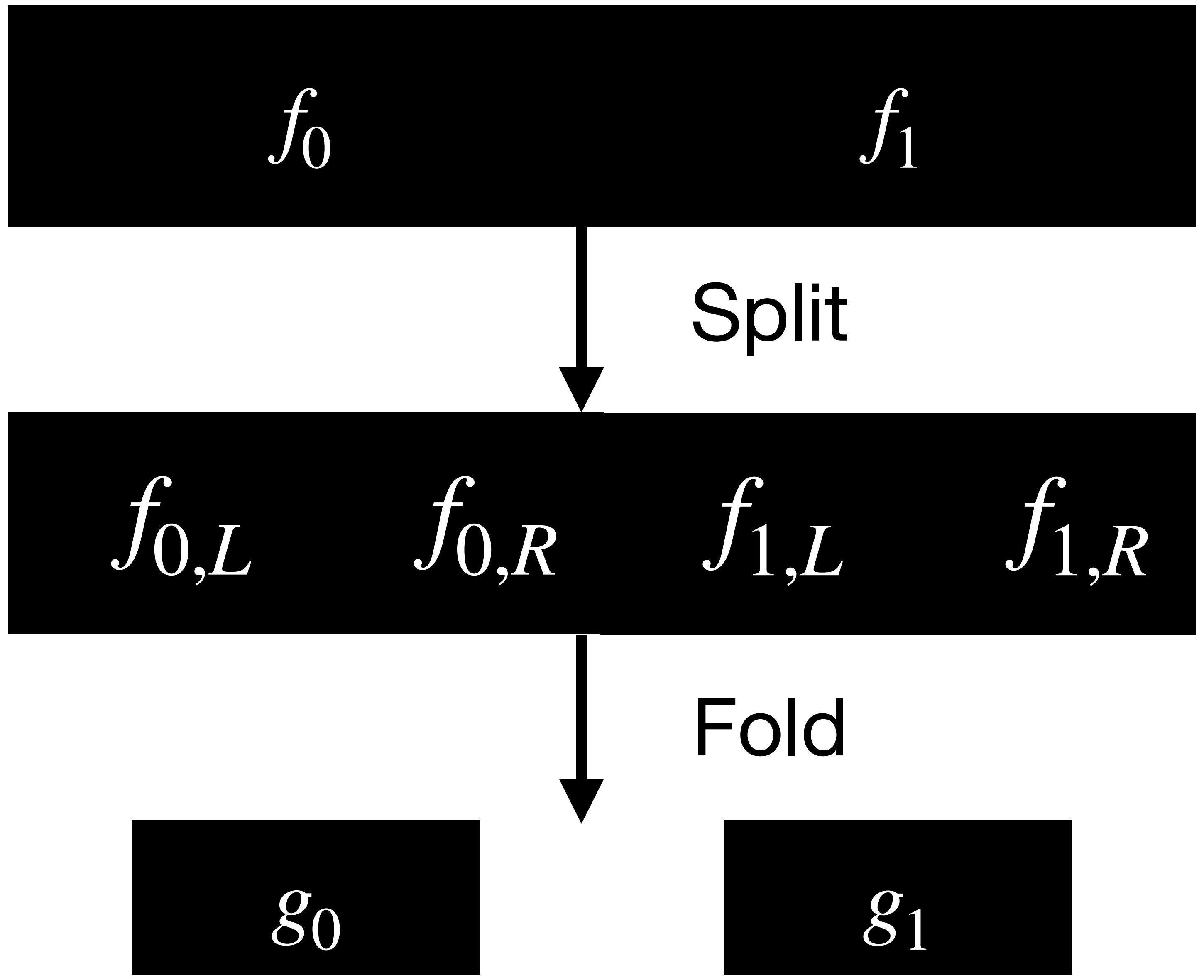

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Folded polynomial:

$$g_0 := \alpha_{0,L,0} f_{0,L} + \alpha_{0,R,0} f_{0,R} + \alpha_{1,L,0} f_{1,L} + \alpha_{1,R,0} f_{1,R}$$

$$g_1 := \alpha_{0,L,1} f_{0,L} + \alpha_{0,R,1} f_{0,R} + \alpha_{1,L,1} f_{1,L} + \alpha_{1,R,1} f_{1,R}$$

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- To prove a single claim $f(u) = v$, simply set $f_1, \ldots, f_r = f$ and $v_1, \ldots, v_r = v$.

Conclusion

👋 - SLAP

A non-interactive lattice-based polynomial commitment with succinct proofs and verification time, from standard lattice assumptions.

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SLAP: Succinct Lattice-Based Polynomial Commitments from Standard Assumptions

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Details here!

SLAP: Succinct Lattice-Based Polynomial Commitments from Standard Assumptions

September 2023 · Martin R. Albrecht, Giacomo Fenzi, Oleksandra Lapiha, Ngoc Khanh Nguyen · EUROCRYPT 2024 - ePrint: 2023/1469

This blog-post is a short introduction to our new work: "SLAP: Succinct Lattice-Based Polynomial Commitments from Standard Assumptions". This is joint work with Martin Albrecht, Oleksandra Lapiha and Ngoc Khanh Nguyen, and the full version is available on ePrint . Here are also some slides that might be helpful.

gfenzi.io/papers/slap

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Open Questions

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Thank you!

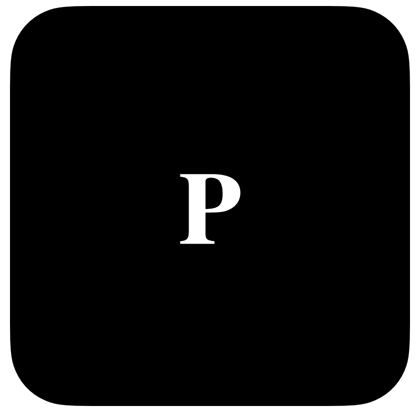
Extra slides

Evaluation Protocol I

Strategy

Evaluation Protocol I

Strategy



Evaluation Protocol I

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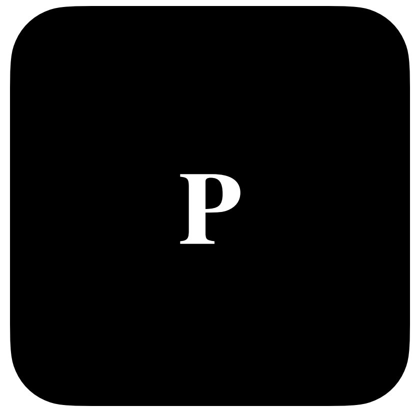
Prover knows:



Evaluation Protocol I

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Prover knows:

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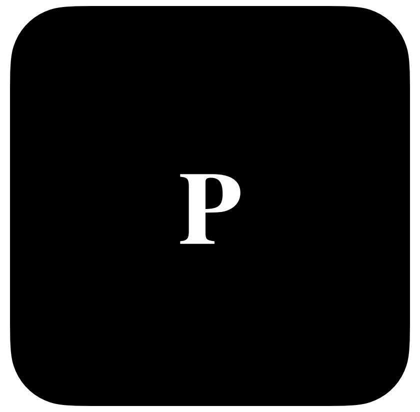

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P

V

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Evaluation Protocol I

Strategy

Verifier knows:

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- Commitment $\mathbf{t}$

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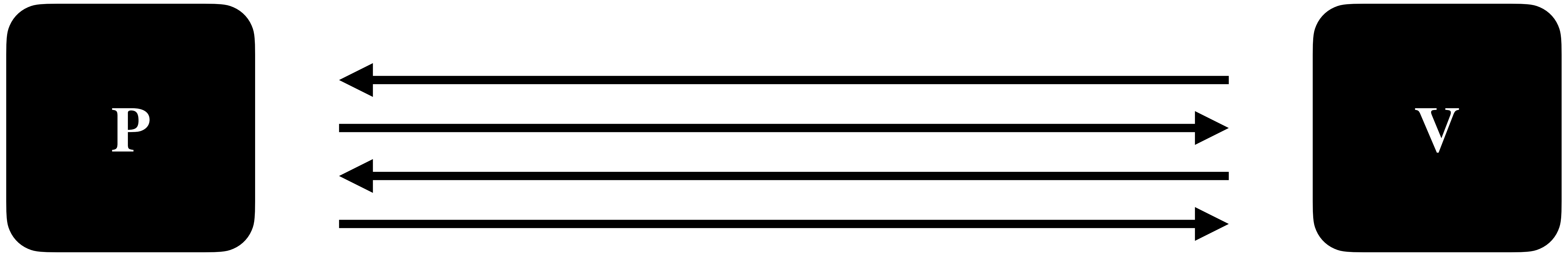

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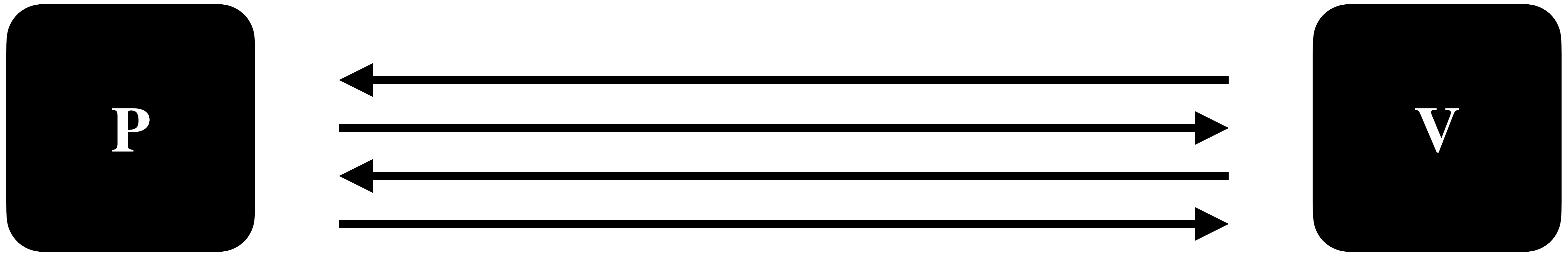

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Evaluation Protocol I

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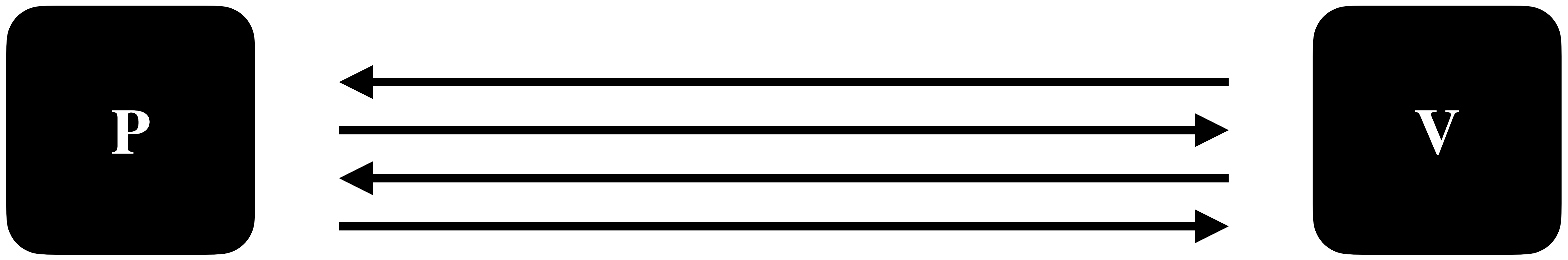

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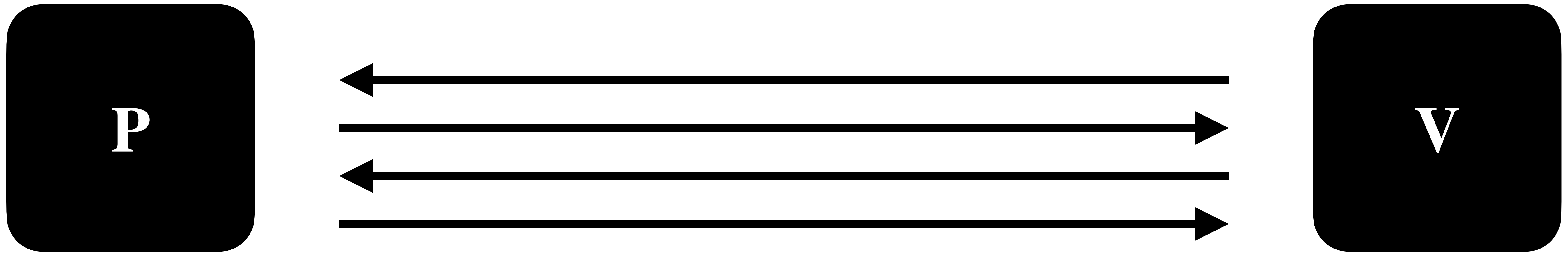

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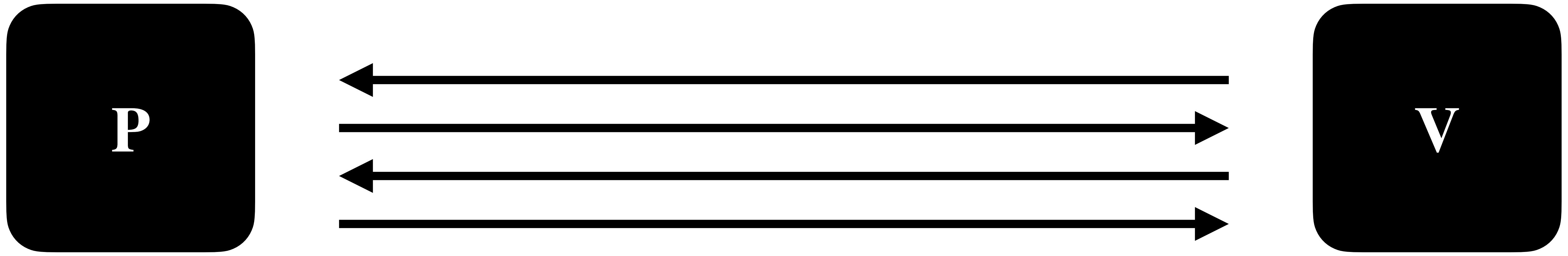

Verifier now knows:

- Common reference string crs'

Prover now knows:

- Polynomial $g \in \mathcal{R}_q^{<d/2}[X]$ and openings $(\mathbf{z_b})_\mathbf{b}$

Evaluation Protocol I

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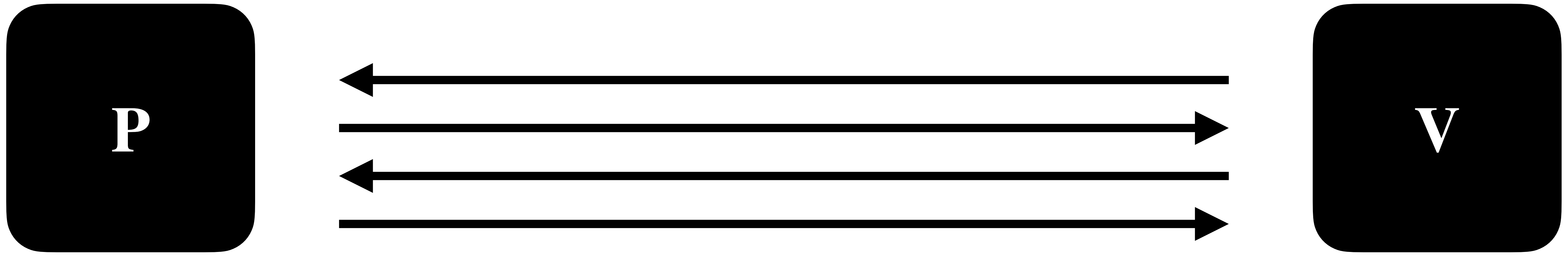
- Claim: $f(u) = v$ and $\mathsf{Open}(\mathsf{crs}, \mathbf{t}, f, (\mathbf{s_b})_\mathbf{b}) = 1$



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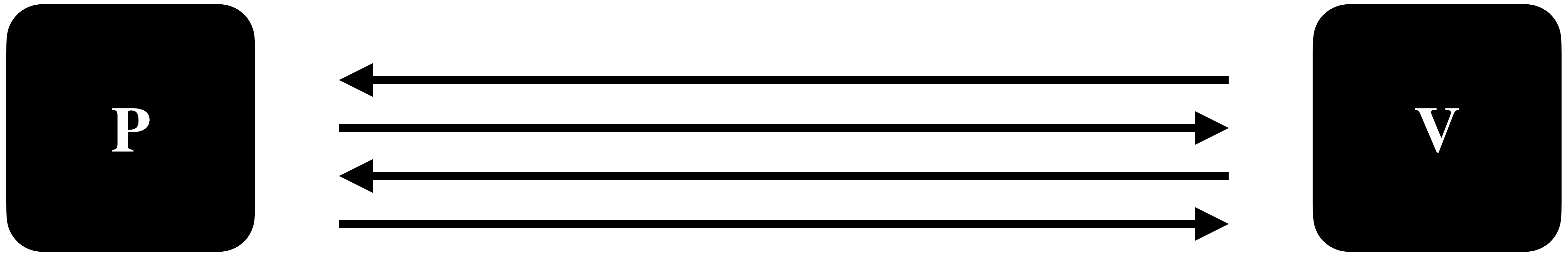

Prover now knows:

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Verifier now knows:

- Common reference string crs′
- Commitment $\mathbf{t}'$
- New claim: $g(u') = v'$ and Open(crs′, $\mathbf{t}', g, (\mathbf{z_b})_\mathbf{b}) = 1$

Evaluation Protocol II

Split and fold (Evaluations)

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$$f \in \mathscr{R}_q^{<d}[X]$$

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Fast Reed-Solomon Interactive Oracle Proofs of Proximity

Eli Ben-Sasson* Iddo Bentov† Ynon Horesh* Michael Riabzev*

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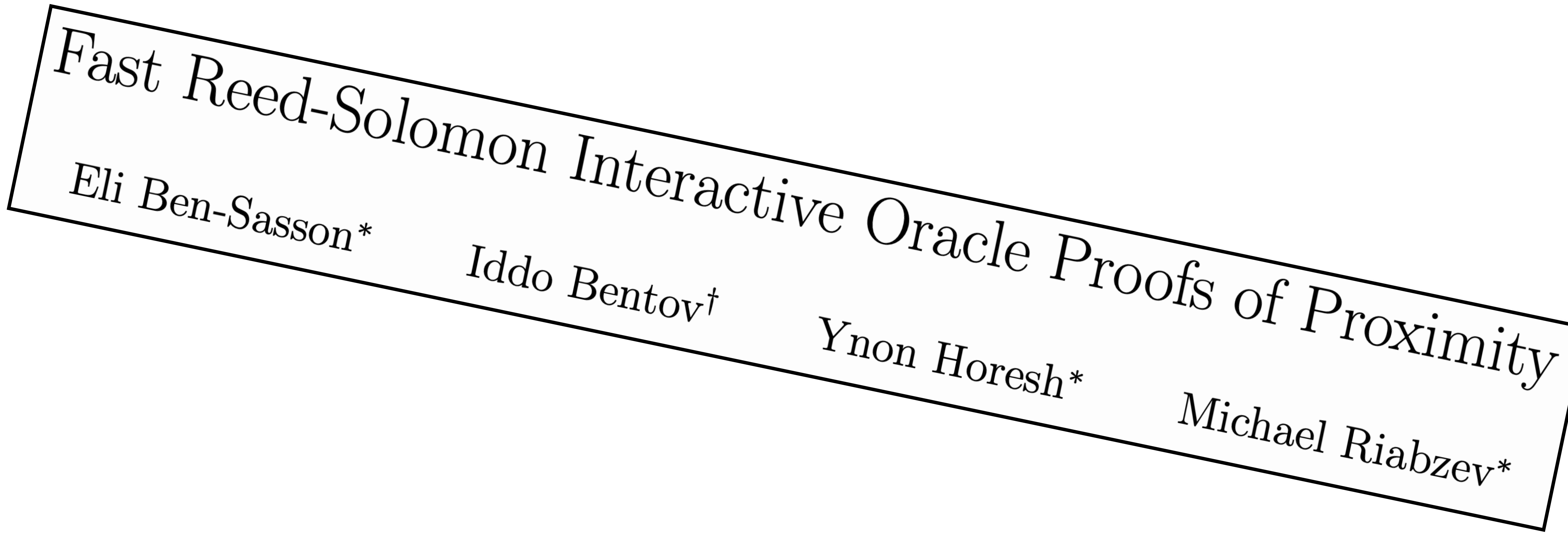
$f \in \mathscr{R}_q^{<d}[X]$

$\downarrow$ Split

$$f(X) = f_L(X^2) + X \cdot f_R(X^2)$$

Evaluation Protocol II

Split and fold (Evaluations)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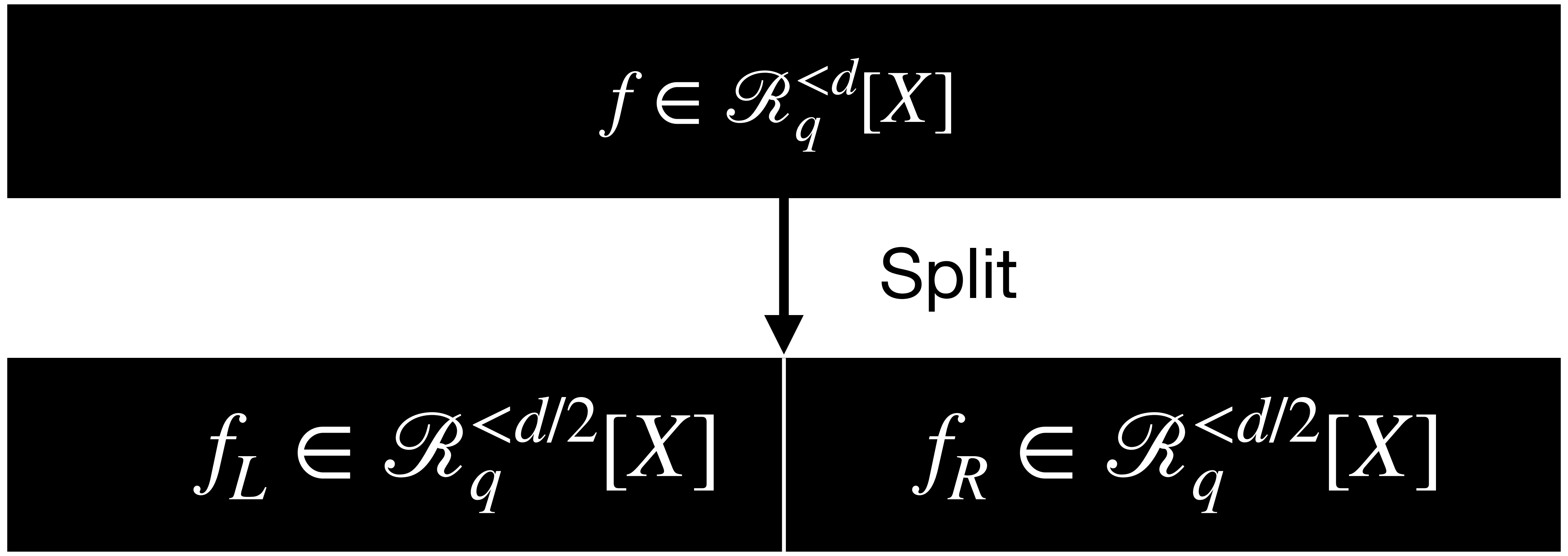

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Split

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α_0, α_1

V

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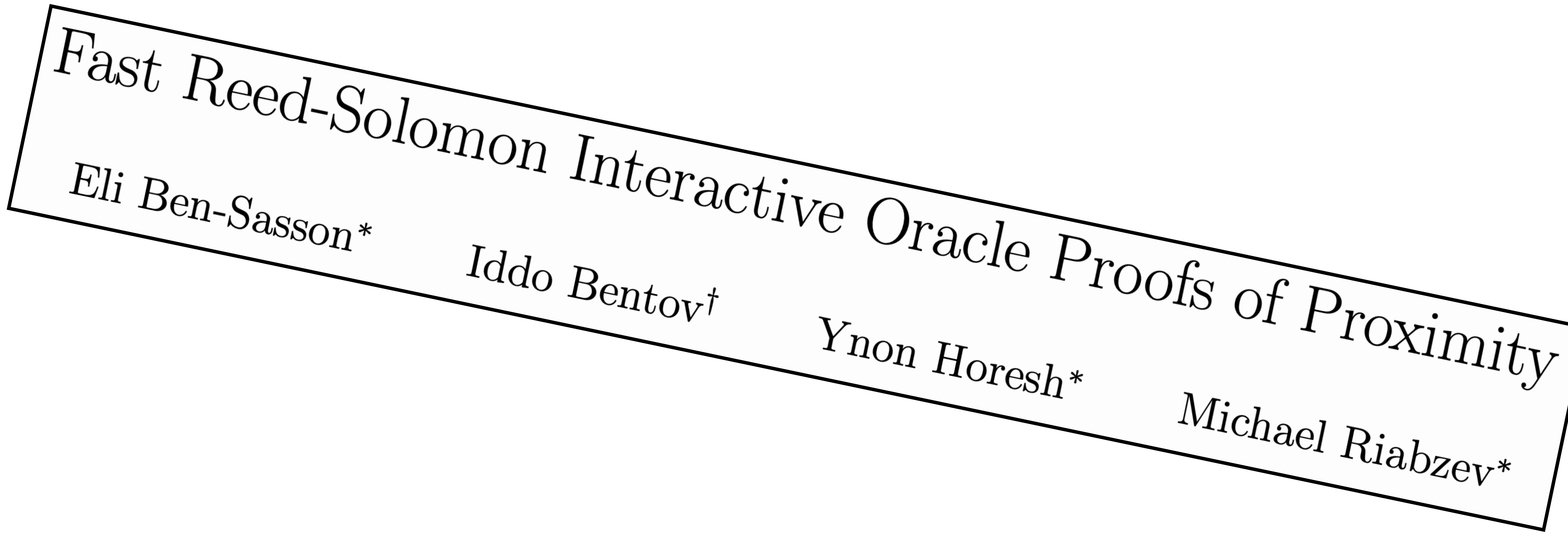
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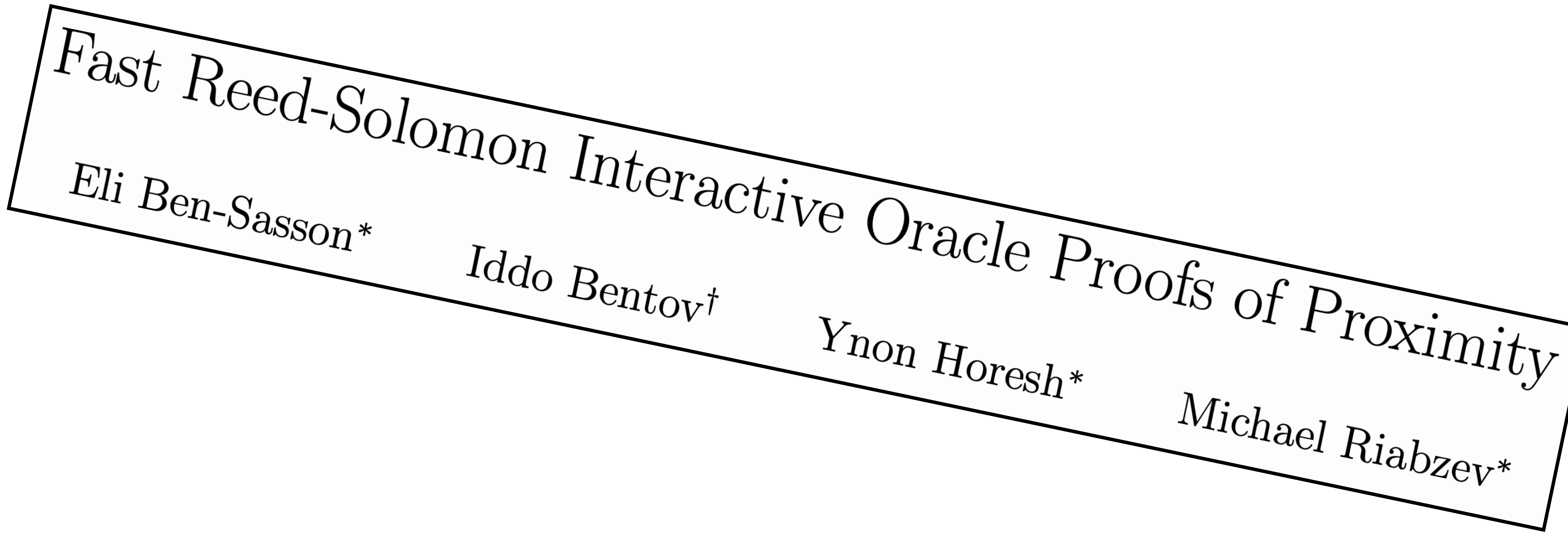
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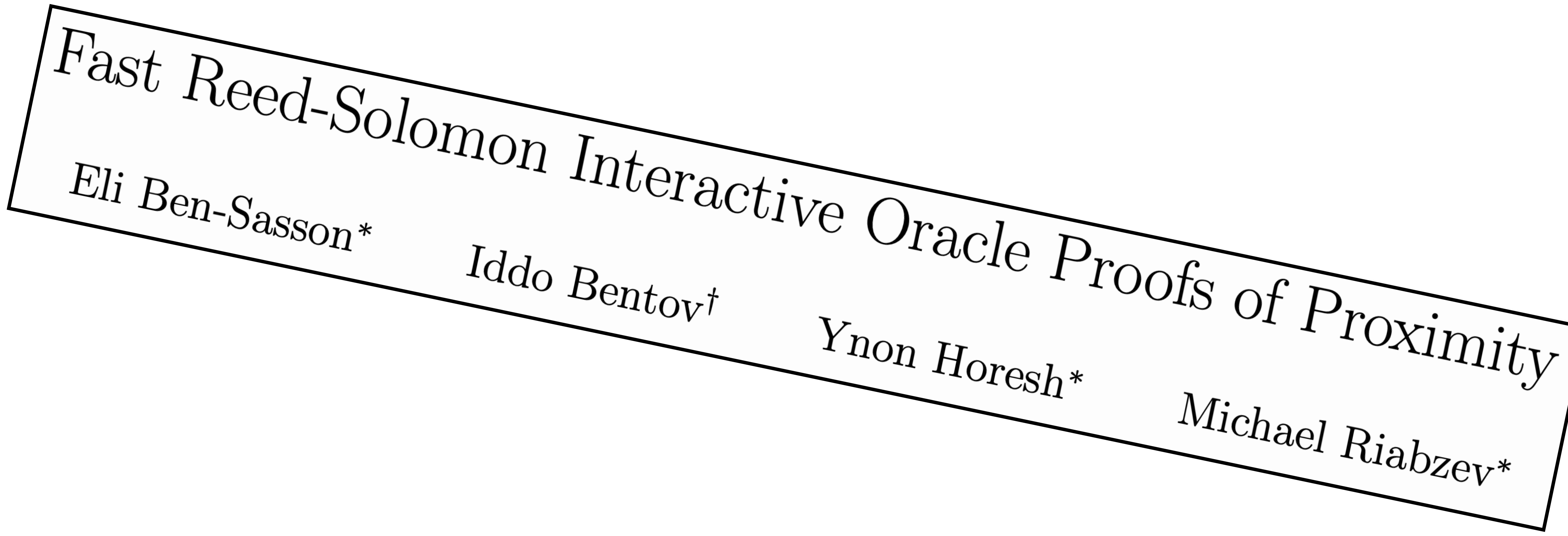

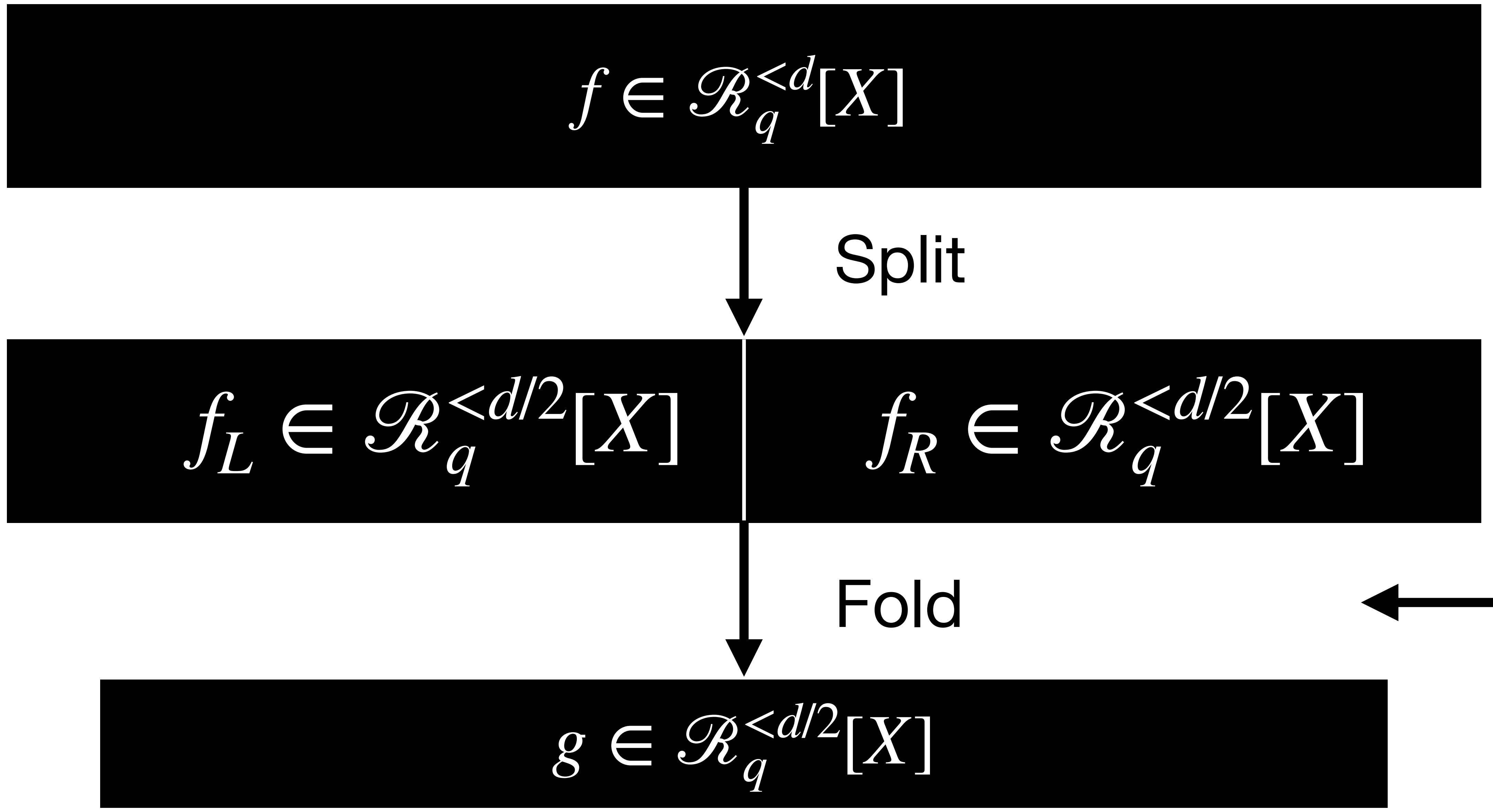

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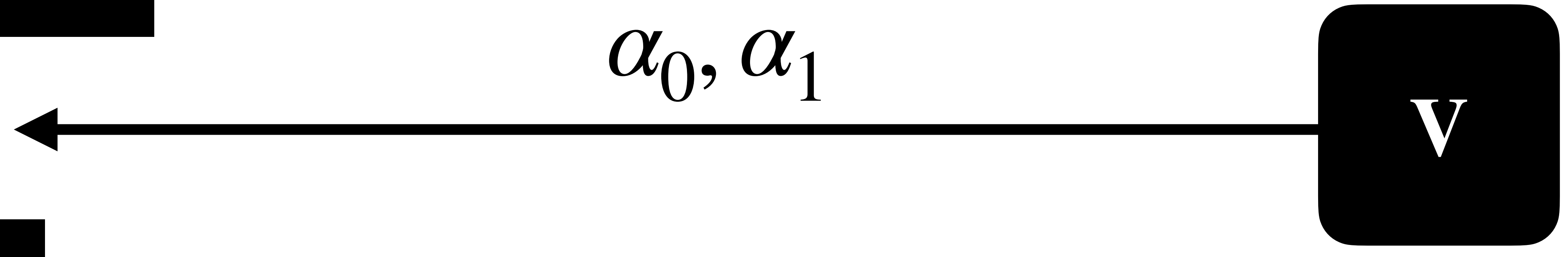

$$g(X) = \alpha_0 f_L(X) + \alpha_1 f_R(X)$$

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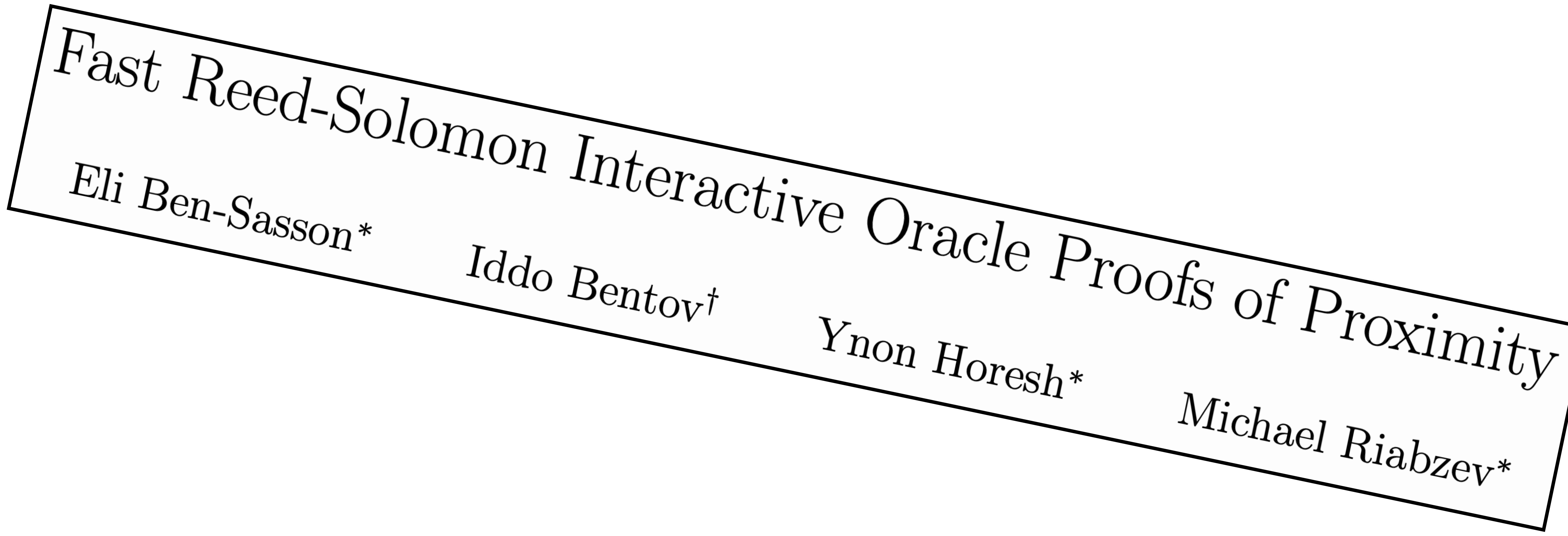
Evaluation Protocol II

Split and fold (Evaluations)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Fold

α_0, α_1

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$g \in \mathcal{R}_q^{<d/2}[X]$

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If $f(u) = v$, then $g(u^2) = \alpha_0 z_0 + \alpha_1 z_1$.

Evaluation Protocol III

Split and fold (Openings)

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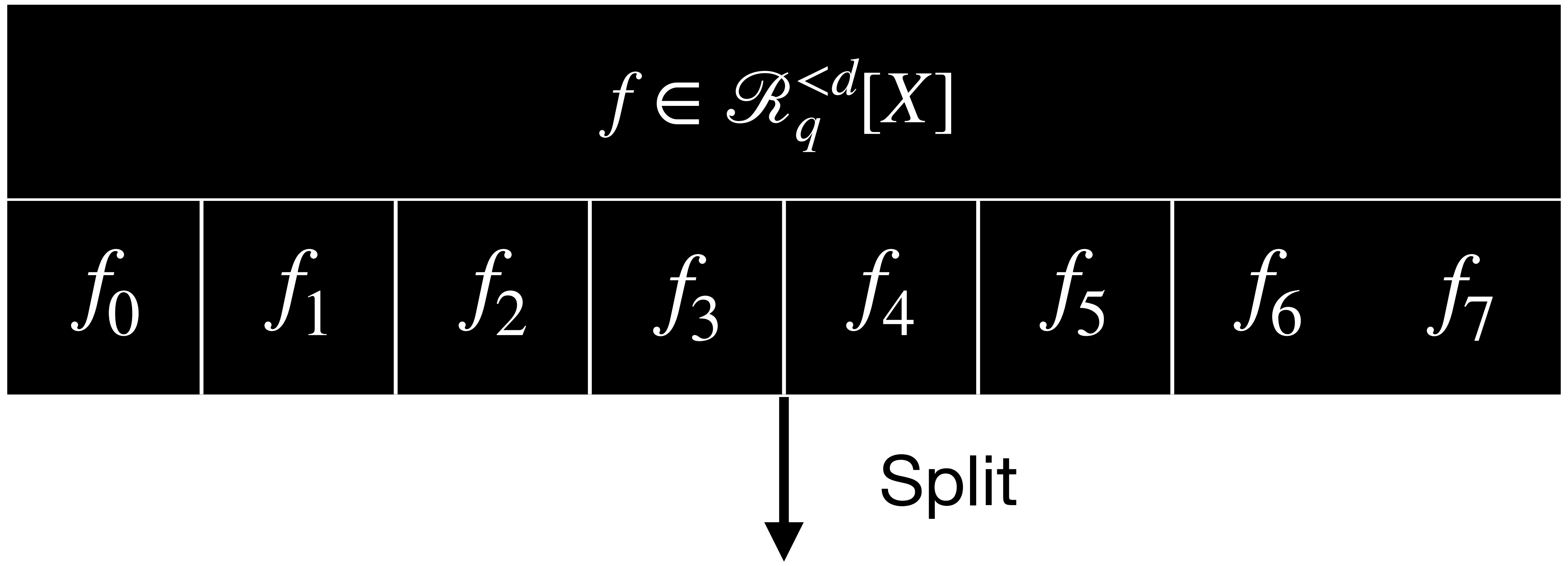
Split and fold (Openings)

$f \in \mathscr{R}_q^{<d}[X]$							
f_0	f_1	f_2	f_3	f_4	f_5	f_6	f_7

Evaluation Protocol III

Split and fold (Openings)



Evaluation Protocol III

Split and fold (Openings)

$f \in \mathcal{R}_q^{<d}[X]$							
f_0	f_1	f_2	f_3	f_4	f_5	f_6	f_7

$\downarrow$ Split

$f_L \in \mathcal{R}_q^{<d/2}[X]$				$f_R \in \mathcal{R}_q^{<d/2}[X]$			
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Fold

Evaluation Protocol III

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f_0	f_2	f_4	f_6	f_1	f_3	f_5	f_7

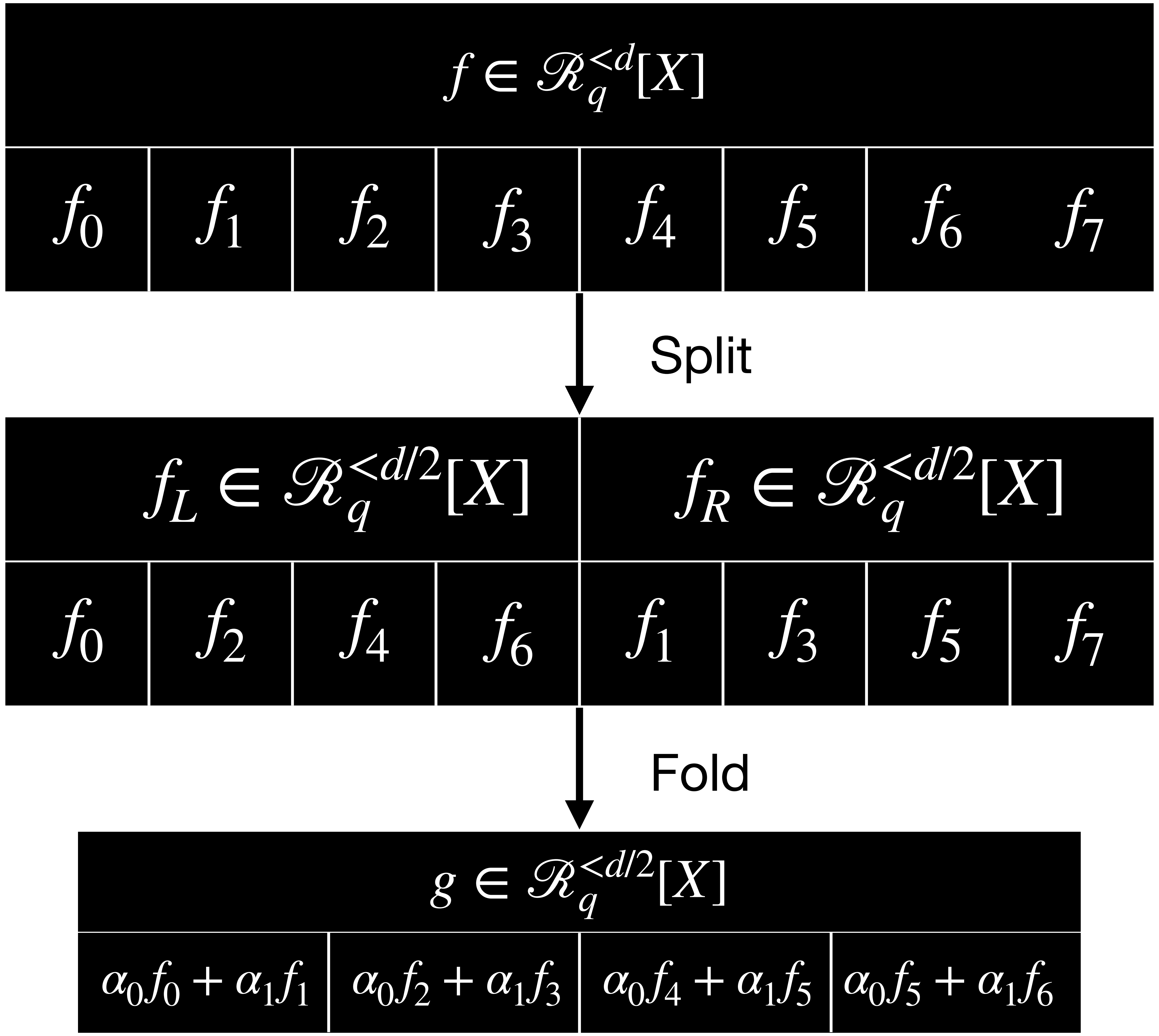
$\downarrow$ Fold

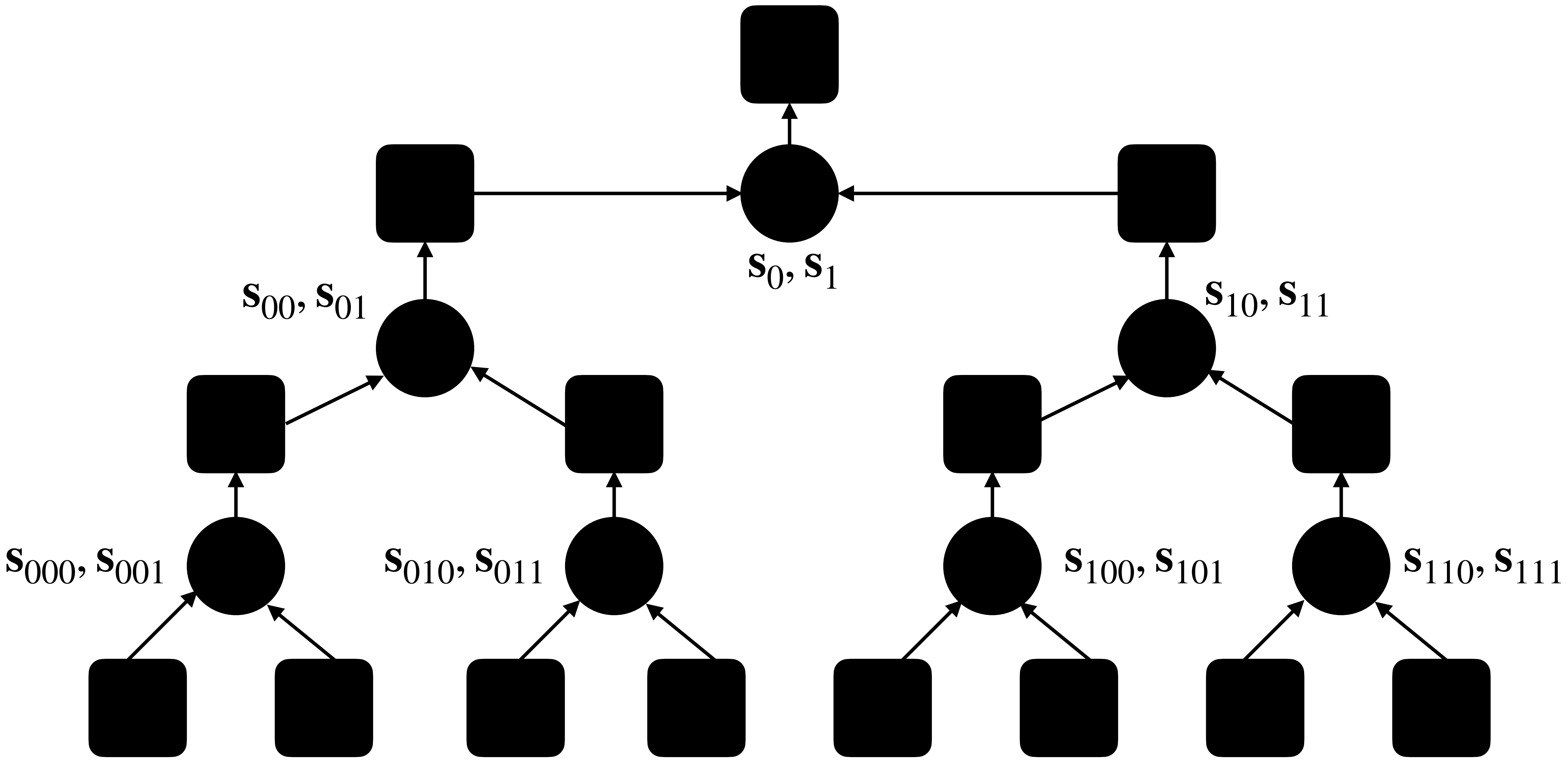
$g \in \mathscr{R}_q^{<d/2}[X]$

$\alpha_0 f_0 + \alpha_1 f_1$	$\alpha_0 f_2 + \alpha_1 f_3$	$\alpha_0 f_4 + \alpha_1 f_5$	$\alpha_0 f_5 + \alpha_1 f_6$

Evaluation Protocol III

Split and fold (Openings)





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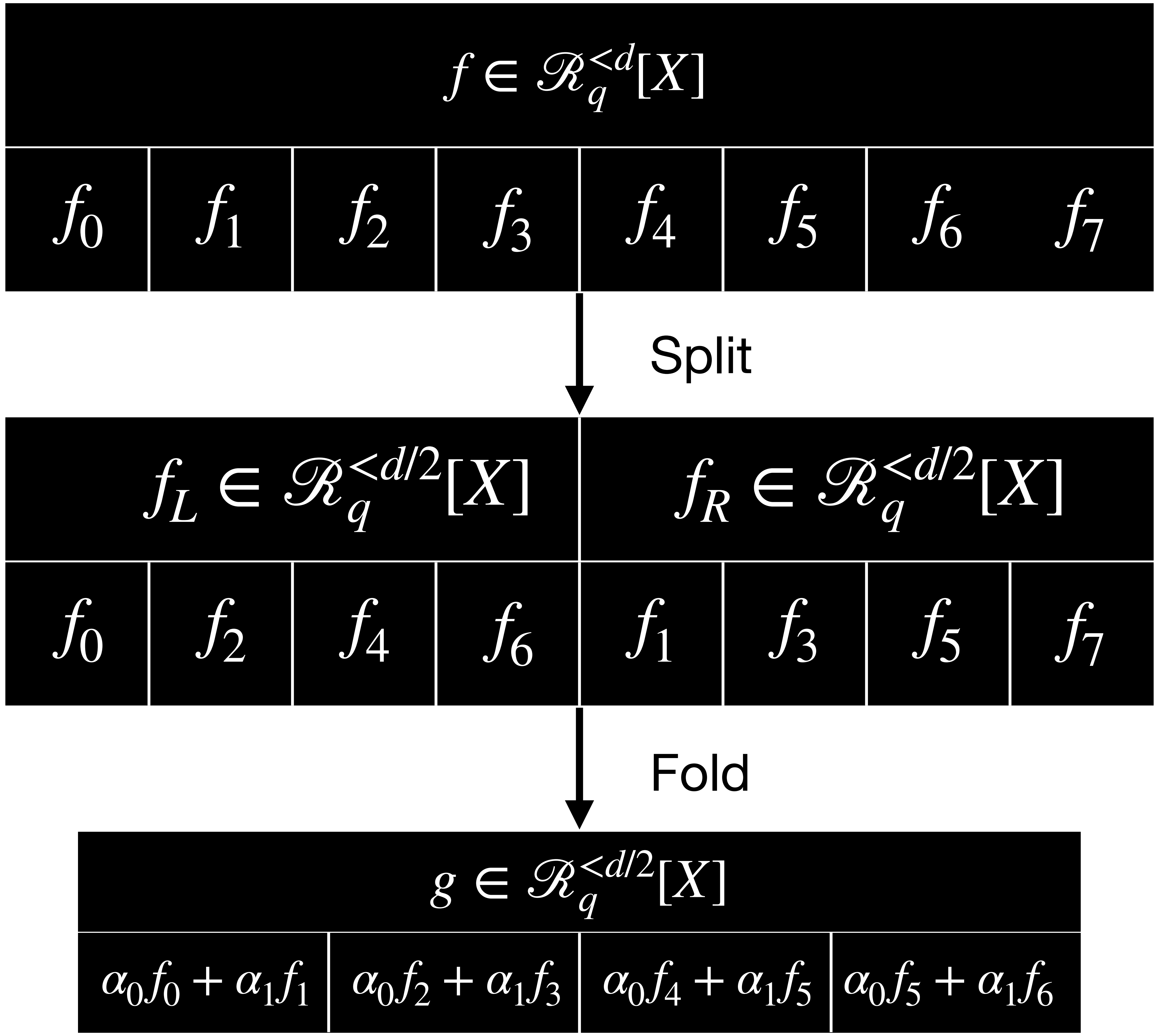

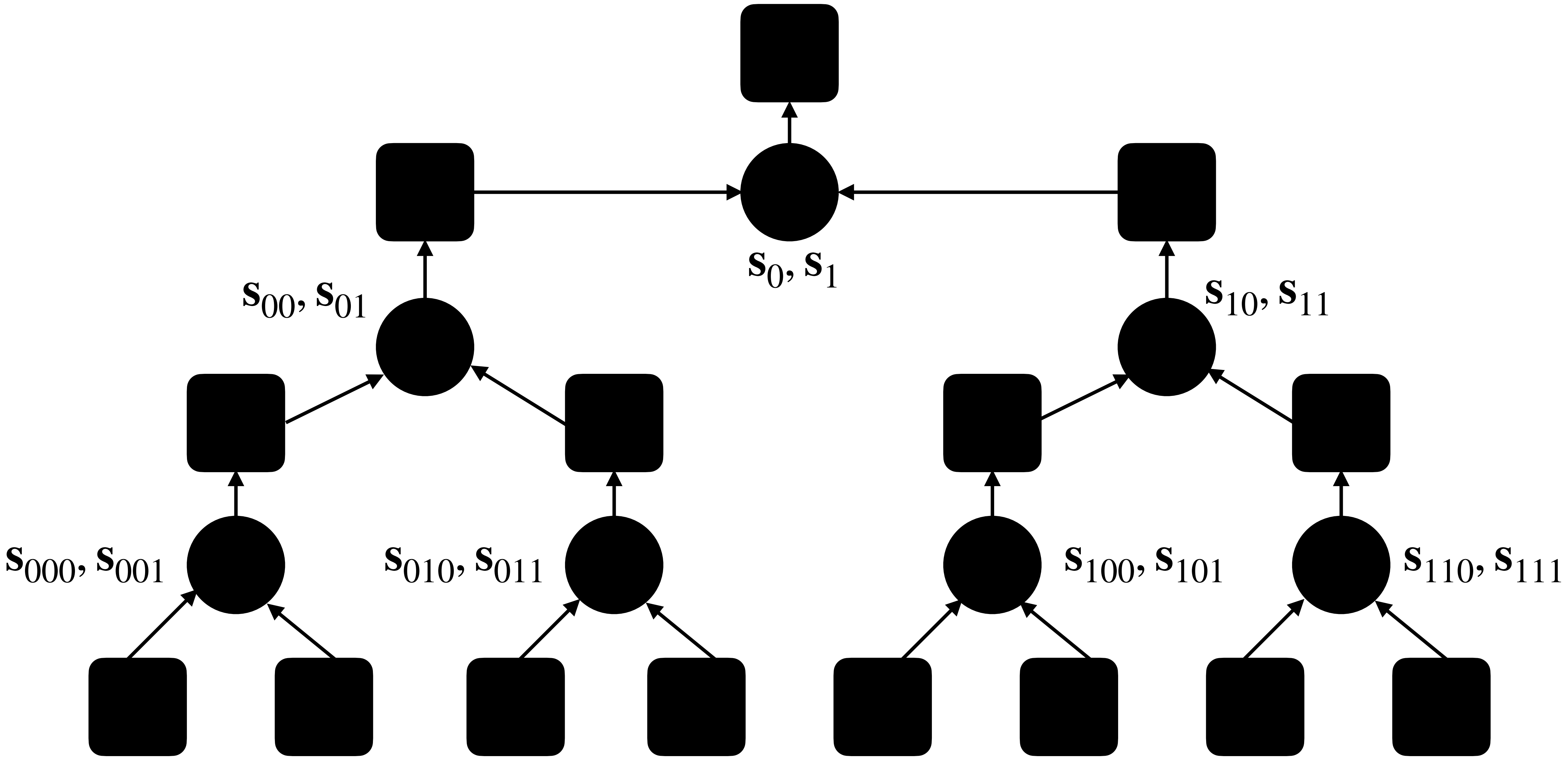

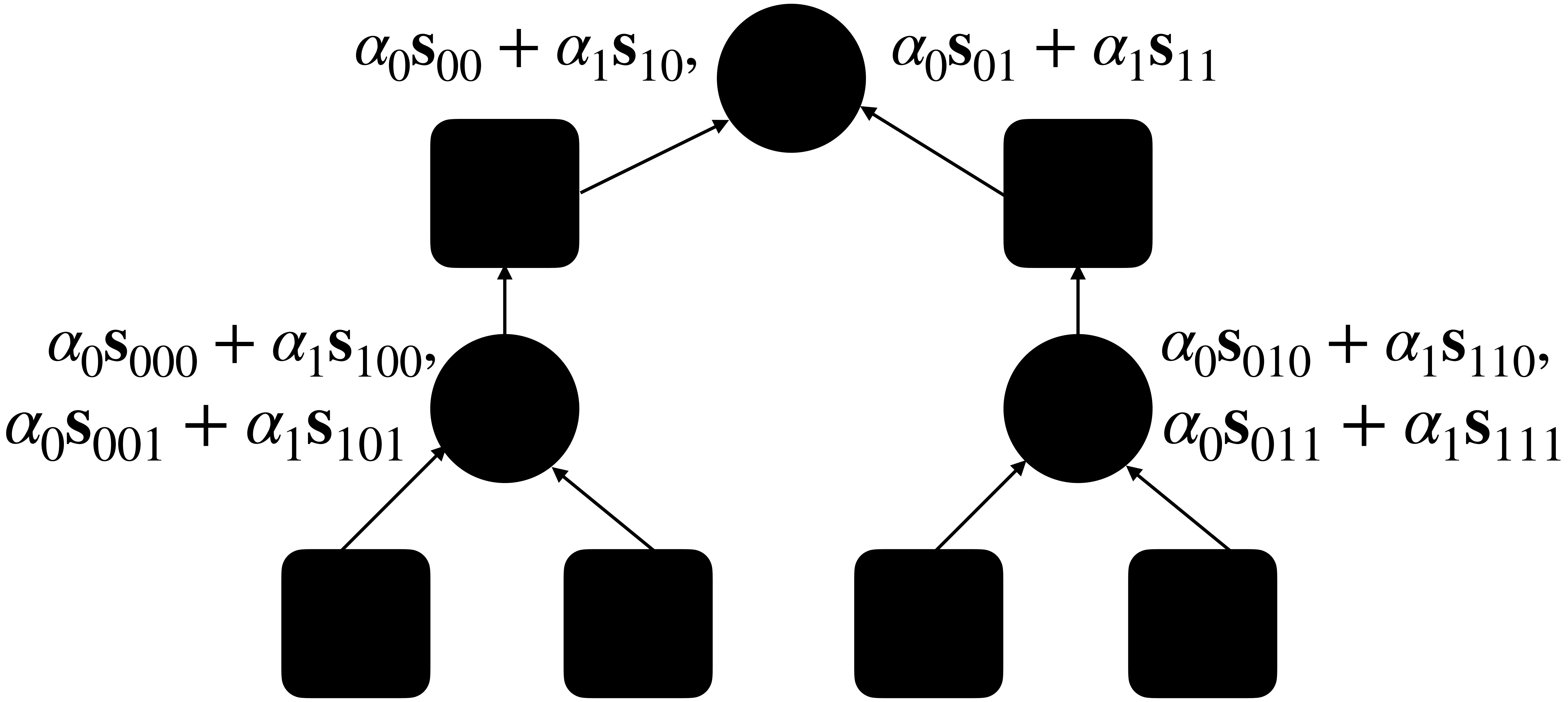

Evaluation Protocol IV

Split and fold (Commitment)

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$$\sum_{j \in [h-1]} w_{1+j}^{b_{1+j}} \mathbf{A}_{1+j} \mathbf{s}_{\mathbf{b}:1+j} + g_{\mathbf{b}} \mathbf{e} = \alpha_0 \cdot (\mathbf{t} - w_1^0 \mathbf{A}_1 \mathbf{s}_0) + \alpha_1 \cdot (\mathbf{t} - w_1^1 \mathbf{A}_1 \mathbf{s}_1)$$

Evaluation Protocol IV

Split and fold (Commitment)

- We have shown how to compute new evaluations and openings
- If α_i are short, the new openings also are.
- How does the verifier compute new commitment? With some magic:

$$\sum_{j \in [h-1]} w_{1+j}^{b_{1+j}} \mathbf{A}_{1+j} \mathbf{s}_{\mathbf{b}:1+j} + g_{\mathbf{b}} \mathbf{e} = \alpha_0 \cdot (\mathbf{t} - w_1^0 \mathbf{A}_1 \mathbf{s}_0) + \alpha_1 \cdot (\mathbf{t} - w_1^1 \mathbf{A}_1 \mathbf{s}_1)$$

- Prover reveals $\mathbf{s}_0, \mathbf{s}_1$. Verifier sets RHS as new updated commitment.

BASIS-👨‍👩‍👧‍👧 [WW23]

BASIS-👨‍👩‍👧‍👧 [WW23]

BASIS Game

BASIS-👨‍👩‍👧‍👧 [WW23]

BASIS Game

$$\mathbf{A}^{\star} \leftarrow \mathcal{R}_q^{m \times n}$$

BASIS-👨‍👩‍👧‍👧 [WW23]

BASIS Game

$$\mathbf{A}^\star \leftarrow \mathcal{R}_q^{m\times n}$$

$$\mathsf{aux} \leftarrow \mathsf{Samp}(\mathbf{A}^\star)$$

BASIS-👨‍👩‍👧‍👧 [WW23]

BASIS Game

$\mathbf{A}^\star \leftarrow \mathcal{R}_q^{m \times n}$

aux $\leftarrow$ Samp$(\mathbf{A}^\star)$

return $(\mathbf{A}^\star$, aux$)$ to $\mathscr{A}$

BASIS-👨‍👩‍👧‍👧 [WW23]

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Recap:

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- PRISIS and Merkle-PRISIS commitments
- Multi-instance PRISIS assumptions
- h-PRISIS$_2$ reduces to MSIS
- Succinct evaluation protocol for Merkle-PRISIS
- Boosting soundness via claim bundling

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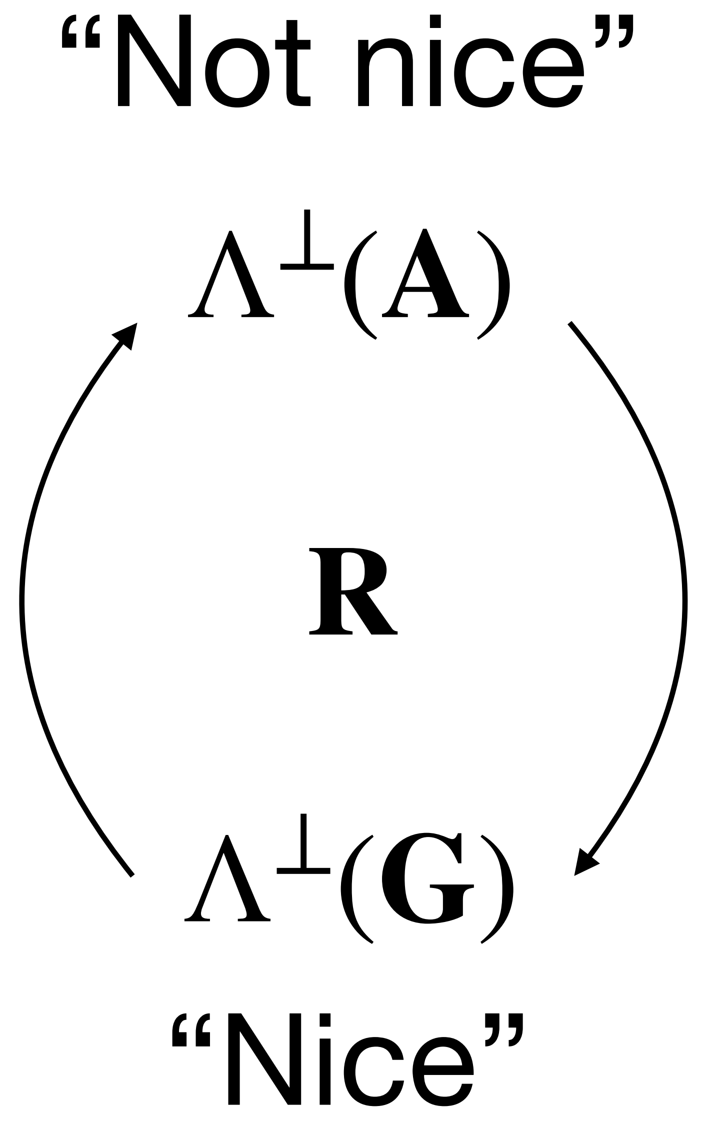

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Trapdoor Resampling [WW23]

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- Given $(\mathbf{A}, \mathbf{R})$, can sample new trapdoor $\mathbf{T}$ for some matrix $\mathbf{B}$ “related” to $\mathbf{A}$

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$$\begin{tikzcd} \Lambda^{\perp}(\mathbf{A}) \arrow[d, bend left=60, ""{name=U}] \\ \Lambda^{\perp}(\mathbf{G}) \arrow[u, bend left=60, ""{name=D}] \arrow[from=U, to=D, phantom, "\mathbf{R}"] \end{tikzcd}$$

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- BASIS style assumption say:

“Given $\mathbf{A}, \mathbf{B}, \mathbf{T}$, hard to find short $\mathbf{x}$ for $\mathbf{A}\mathbf{x} = \mathbf{0}$”