Perfect (Parallel) Broadcast in Constant Expected Time via Statistical VSS

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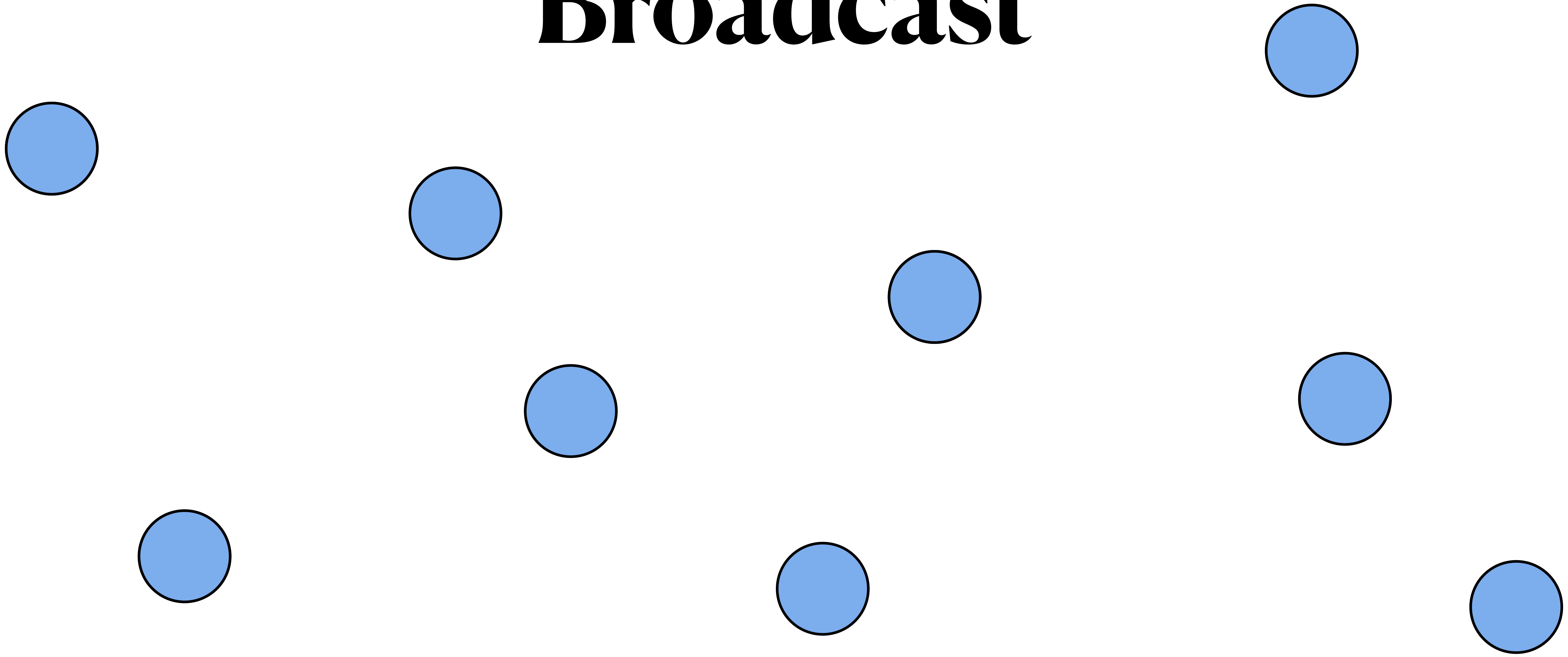
Broadcast



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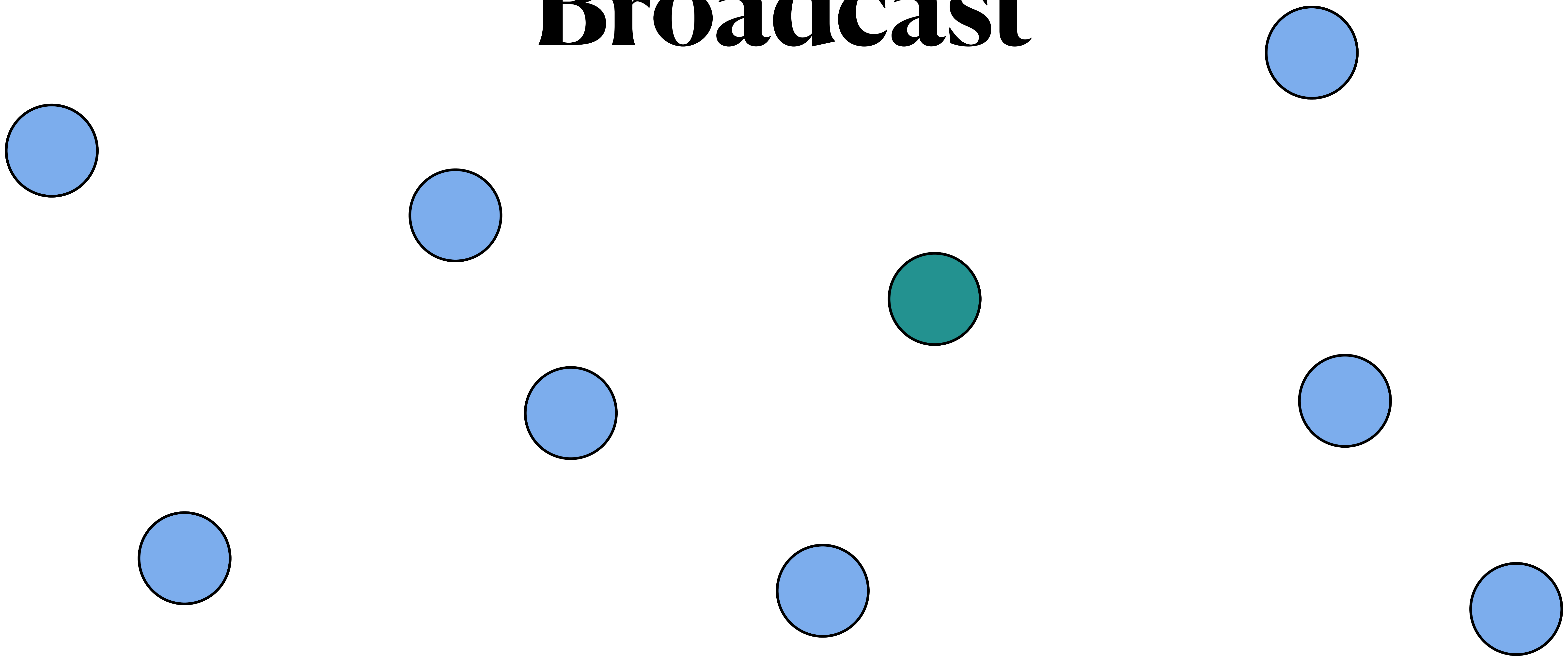

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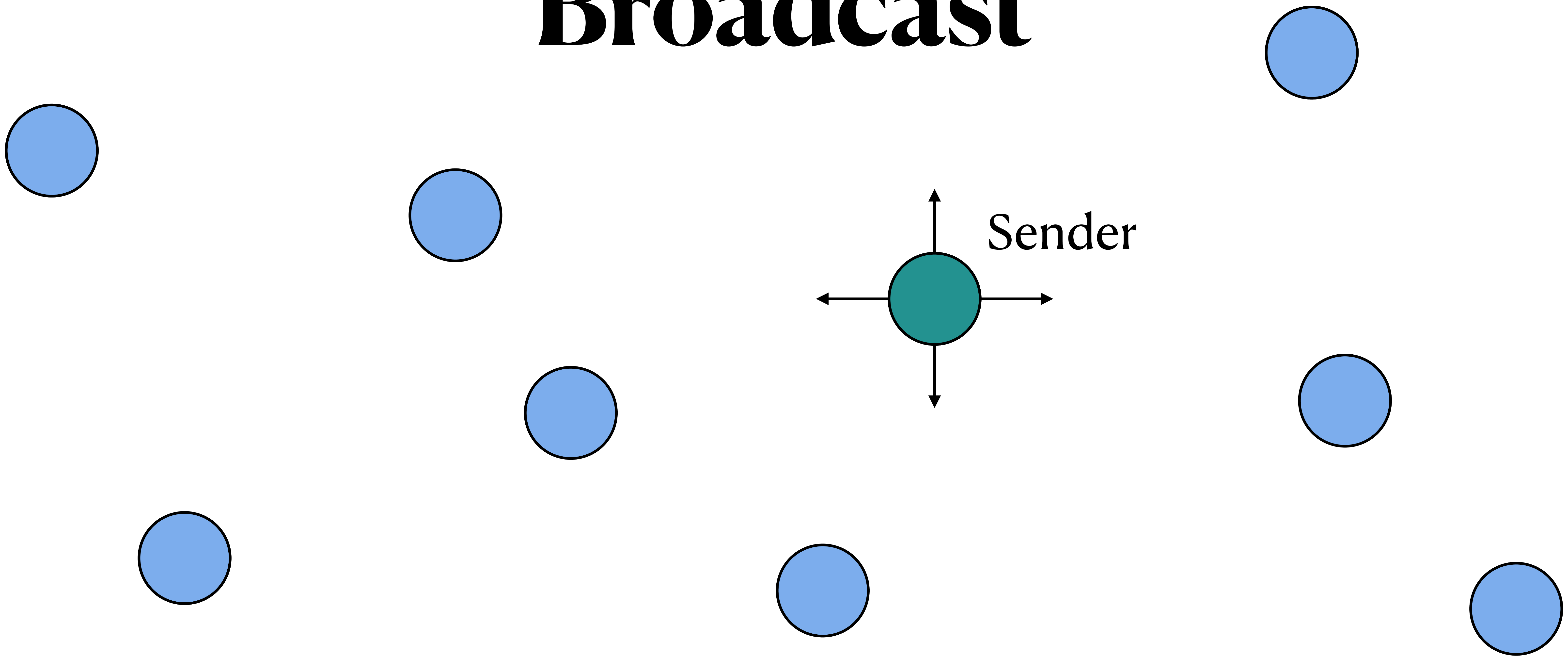

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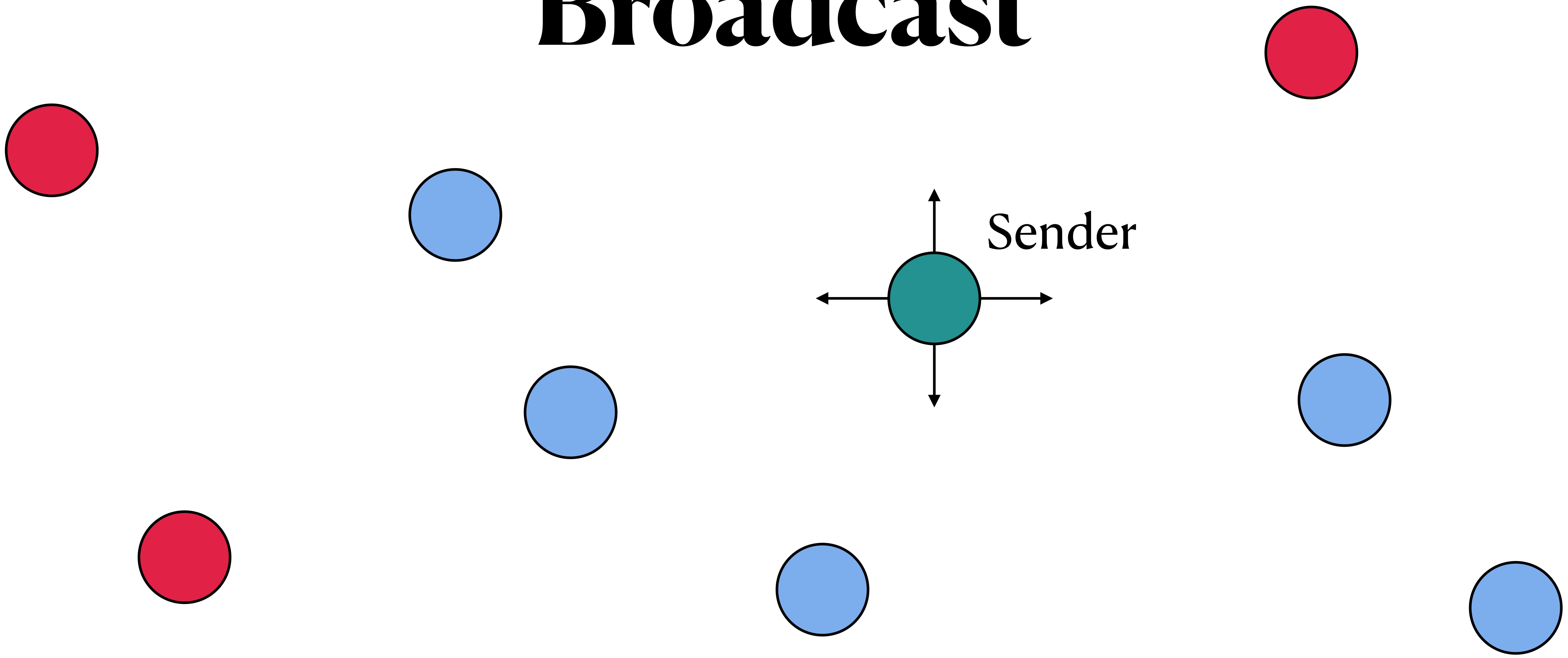

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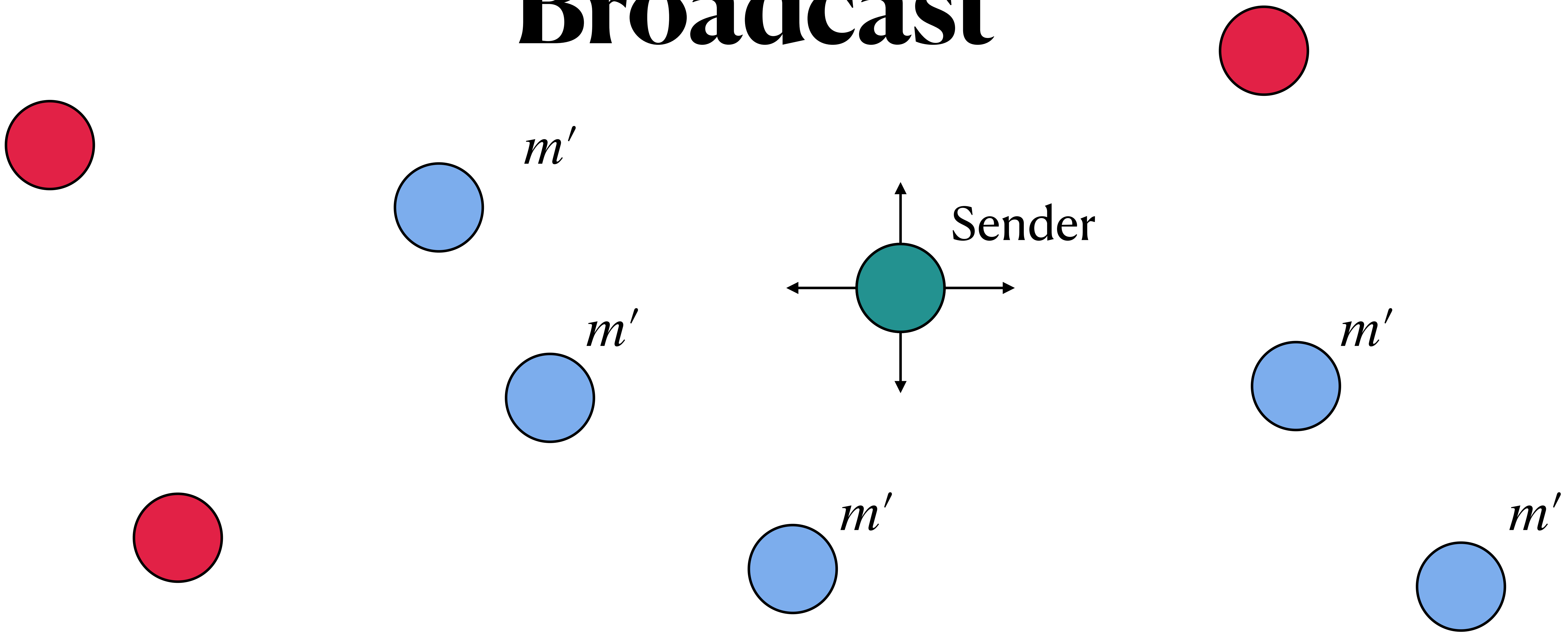

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- **Agreement:** Everyone outputs the same message *m’*

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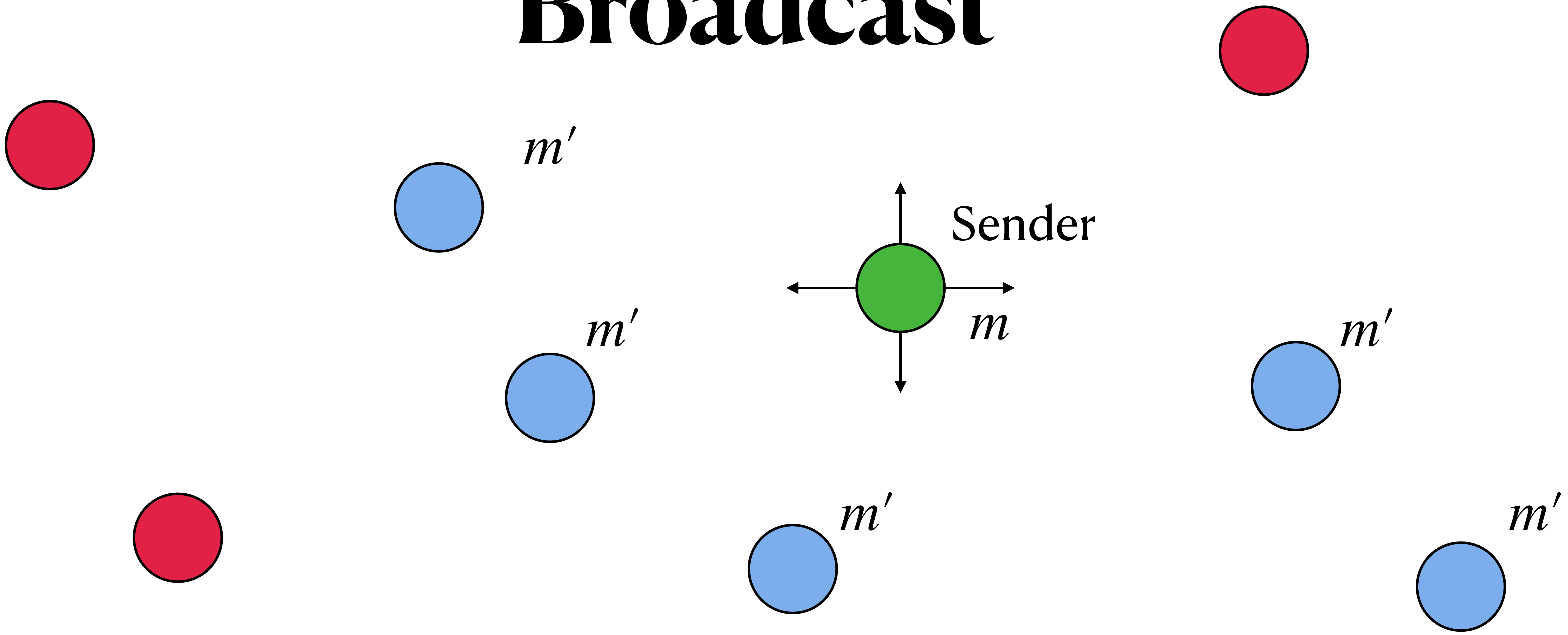

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- **Validity:** For honest sender *m'=m*

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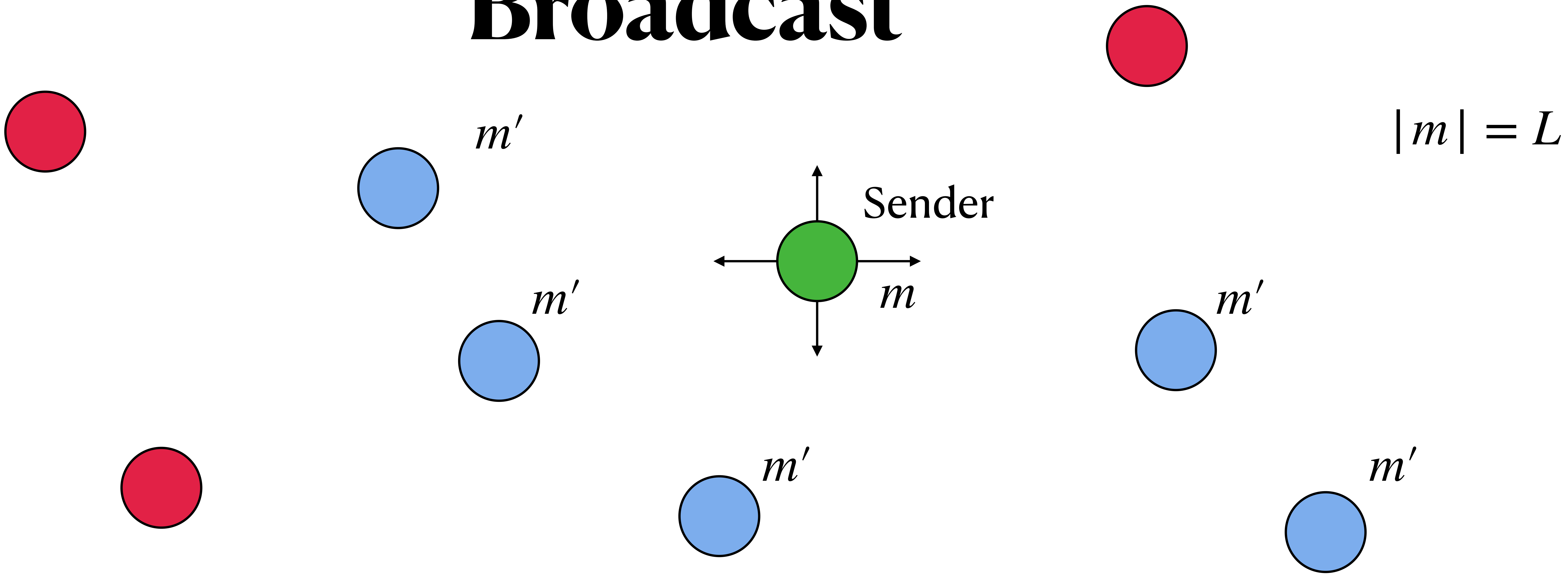

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- No computational hardness assumptions
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- **Resilience:** $t < n/3$ is necessary [PSL80,LSP82]
- **Rounds:** Deterministic $\Omega(n)$ [FL82]
- **Communication:** $\Omega(n^2)$ messages [DR82] (also [ACD+23])

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$O(nL + n^2)$

State of the Art: Broadcast

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Succinct with High Latency

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More Comm. but with expected Low Latency

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Communication

$O(nL + n^2 \log n)$

Rounds

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Communication

[CW89], [BGP92] + [Che21]

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State of the Art: Broadcast

	Succinct with High Latency	More Comm. but with expected Low Latency
	[CW89], [BGP92] + [Che21]	[FM88]
Communication	$O(nL + n^2 \log n)$	$O(n^2L) + \mathrm{E}(\mathrm{poly}(n))$
Rounds	$\Omega(n)$	$\mathrm{E}(O(1))$

State of the Art: Broadcast

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		$O(n^2 L) + \mathsf{E}(\text{poly}(n))$	[FM88]
[CW89], [BGP92] + [Che21]	$O(nL + n^2 \log n)$	$O(n^2 L) + \mathsf{E}(O(n^6 \log n))$	[KK06]
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Rounds	
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$L = 1$
$n = 300$

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$O(nL + n^2 \log n)$	$O(n^2 L) + \mathrm{E}(O(n^6 \log n$
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$n = 300$
$n^2 \approx 11\mathrm{KB}$

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Rounds

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Rounds

$\Omega(n)$

$\mathsf{E}(O(1))$

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$n = 300$
$n^2 \approx 11\text{KB}$

$L = 1$
$n = 300$
$n^4 \approx 1\text{GB}$

10 round protocol
$O(3000)$ rounds

10 round protocol
Expected $O(10)$ rounds

Succinct Broadcast with Expected Low Latency?

Our Results #1: Broadcast

	Succinct with High Latency	**More Comm. but with expected Low Latency**	
Communication	[CW89], [BGP92] + [Che21]	[AAPP22]	
	$O(nL + n^2 \log n)$	$O(nL) + \mathsf{E}(O(n^4 \log n))$	
		$O(nL + n^2)$	Best we can hope for
Rounds	$\Omega(n)$	$\mathsf{E}(O(1))$	

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		$O(nL + n^2)$	Best we can hope for
		$O(nL) + \mathsf{E}(O(n^3 \log^2 n))$	This work
Rounds	$\Omega(n)$	$\mathsf{E}(O(1))$	

Broadcast for MPC

- Secure computation protocols assume broadcast
- [BGW88] Verifiable Secret Sharing:
  - Complain about the dealer
  - Vote on the dealer

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$1 \times \mathsf{BC}(L)$

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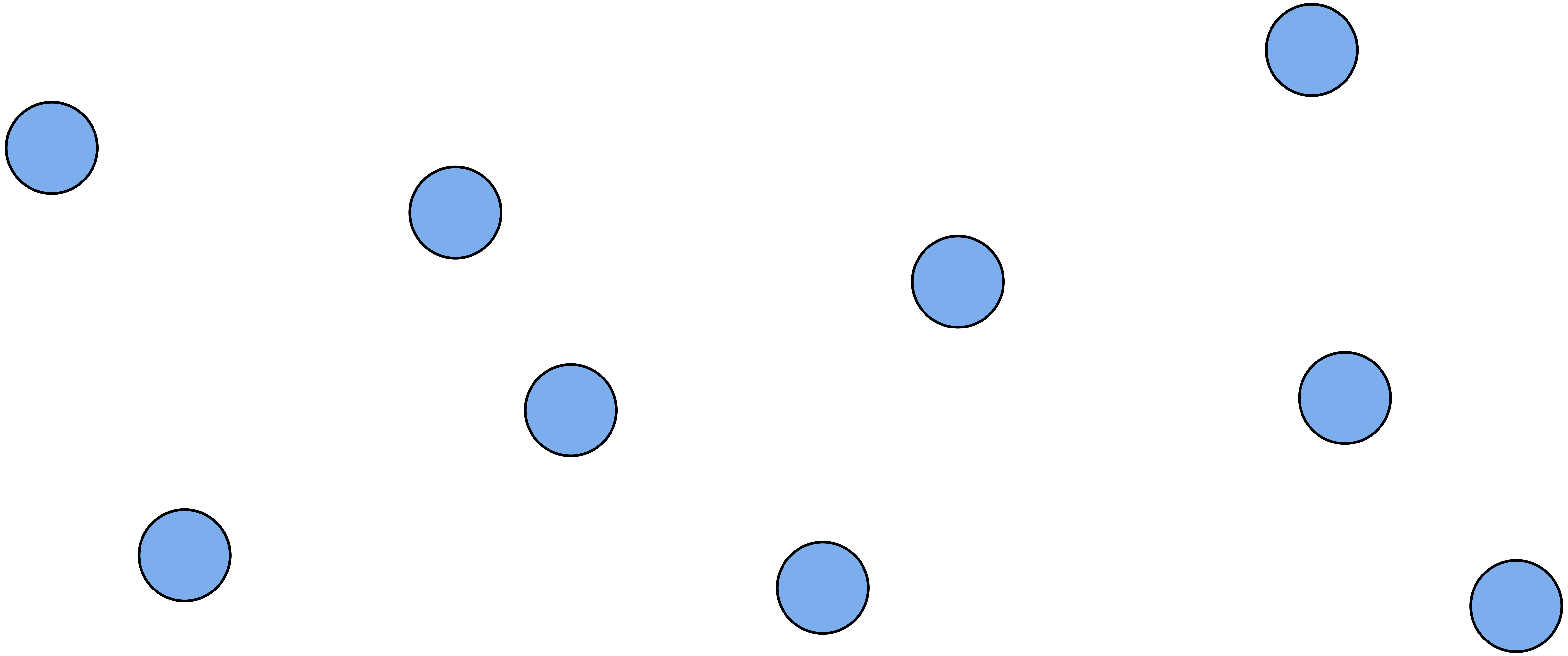
$1 \times \mathsf{BC}(L)$ $\quad$ $n \times \mathsf{BC}(L)$

Parallel Broadcast



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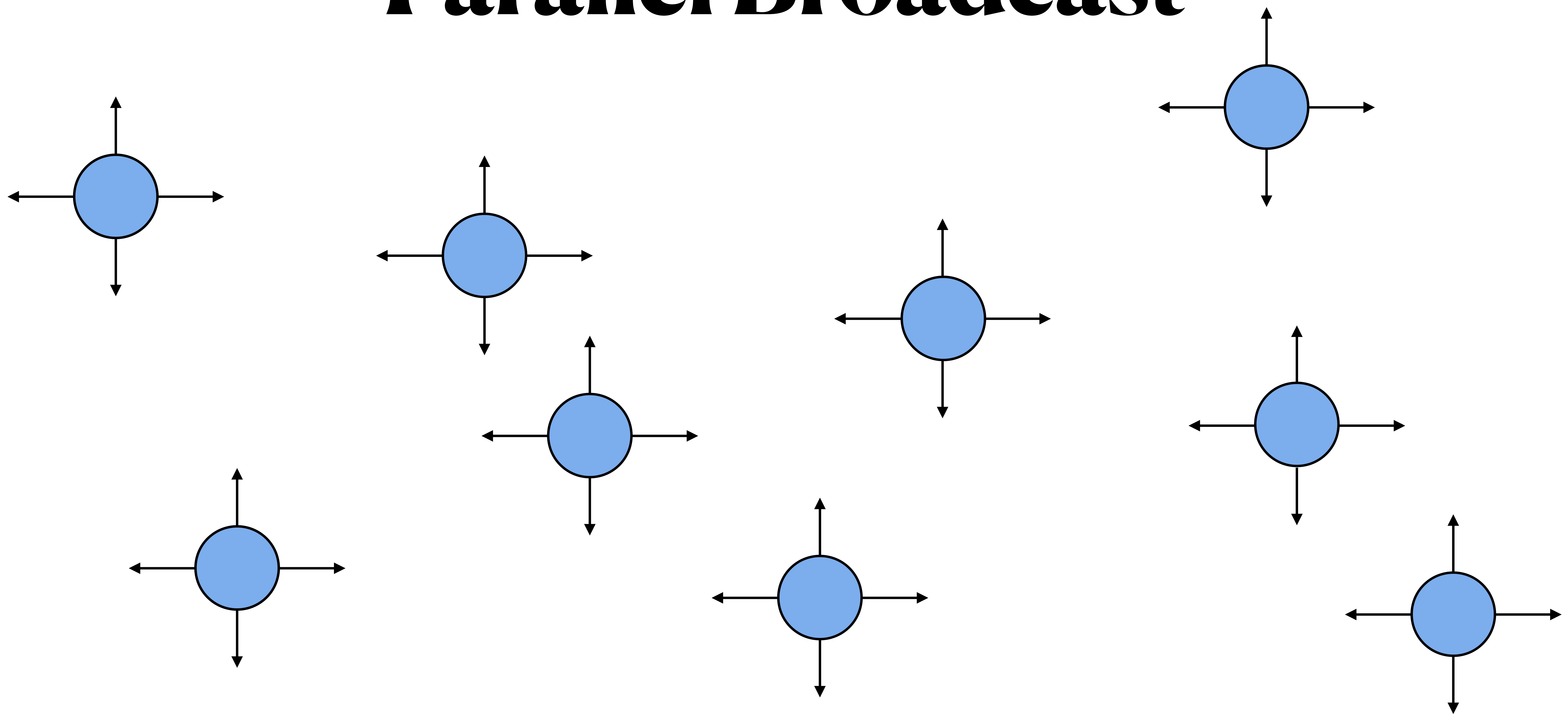

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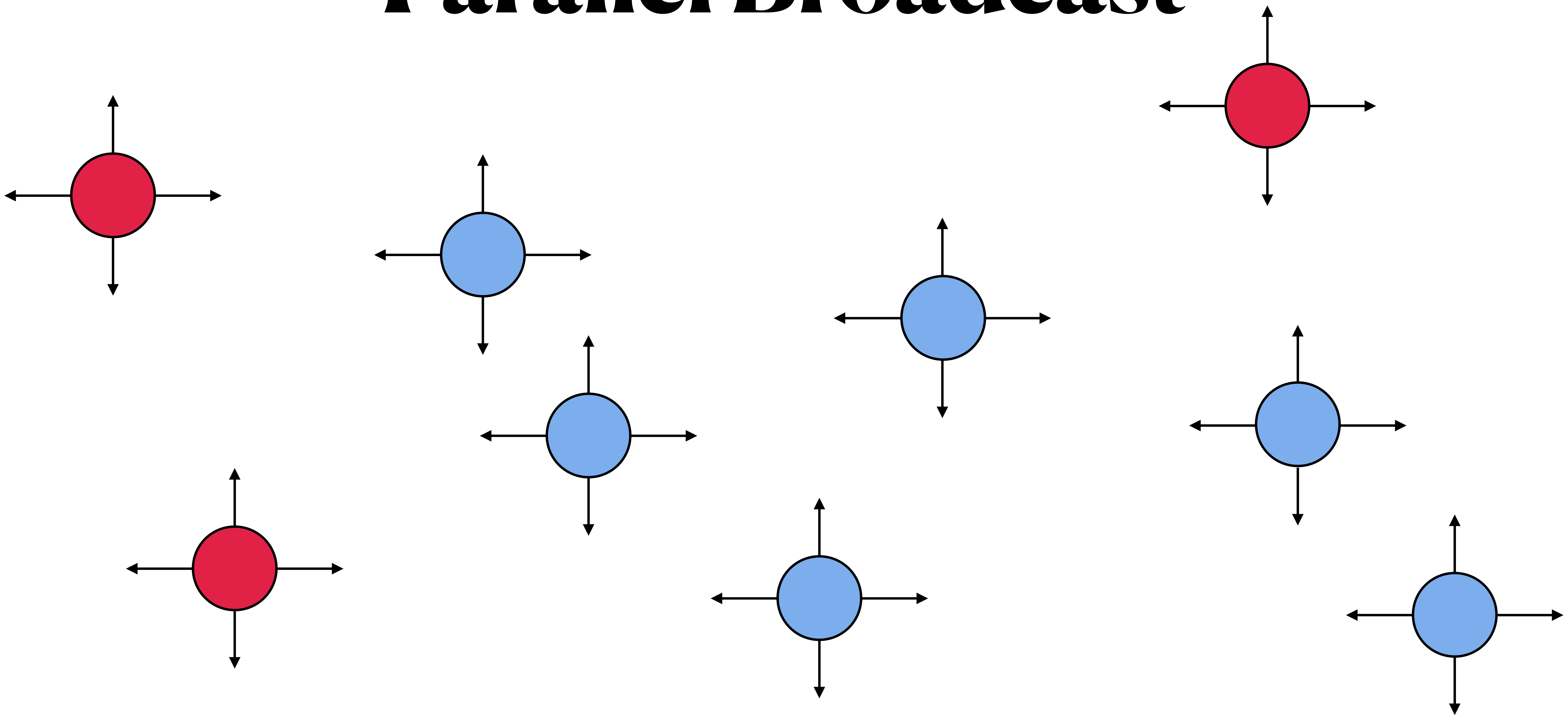

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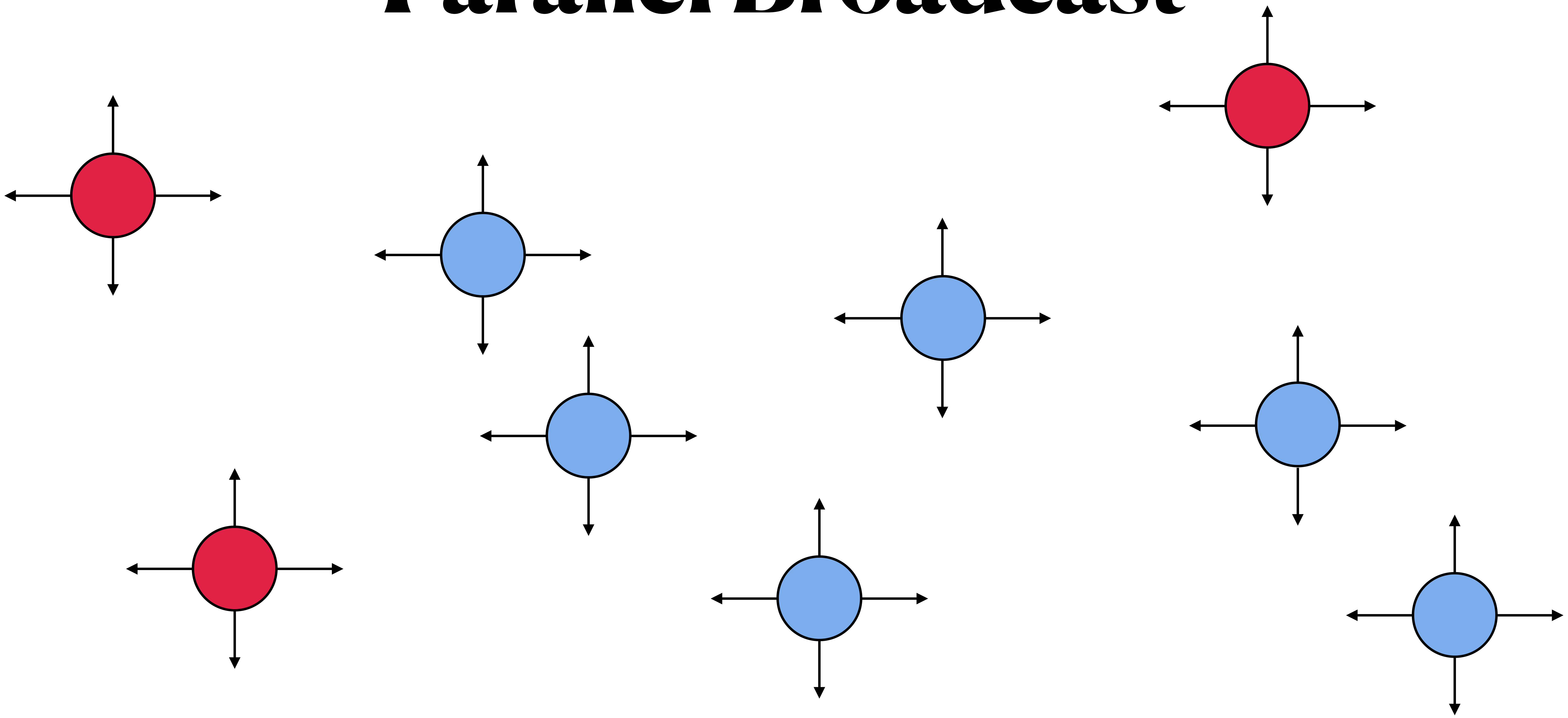

- **Agreement:** On the messages of all senders
- **Validity:** Output each honest sender's message

State of the Art: Parallel Broadcast

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$O(n^2L + n^3 \log n)$

Rounds

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$O(n^2 L) + \mathrm{E}(\mathrm{poly}(n))$

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Rounds: $\Omega(n)$	**Rounds:** $\mathrm{E}(O(1))$

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Succinct (Parallel) Broadcast with Expected Low Latency?

Our Results #2: Parallel Broadcast

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[CW89], [BGP92] + [Che21]			
Communication	$O(n^2L + n^3 \log n)$	$O(n^2L) + \mathrm{E}(O(n^4 \log n))$	[AAPP22]
		$O(n^2L)$	Best we can hope for
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We are optimal for $L \geq n \log^2 n$

Broadcast

[KK06] Framework

Broadcast

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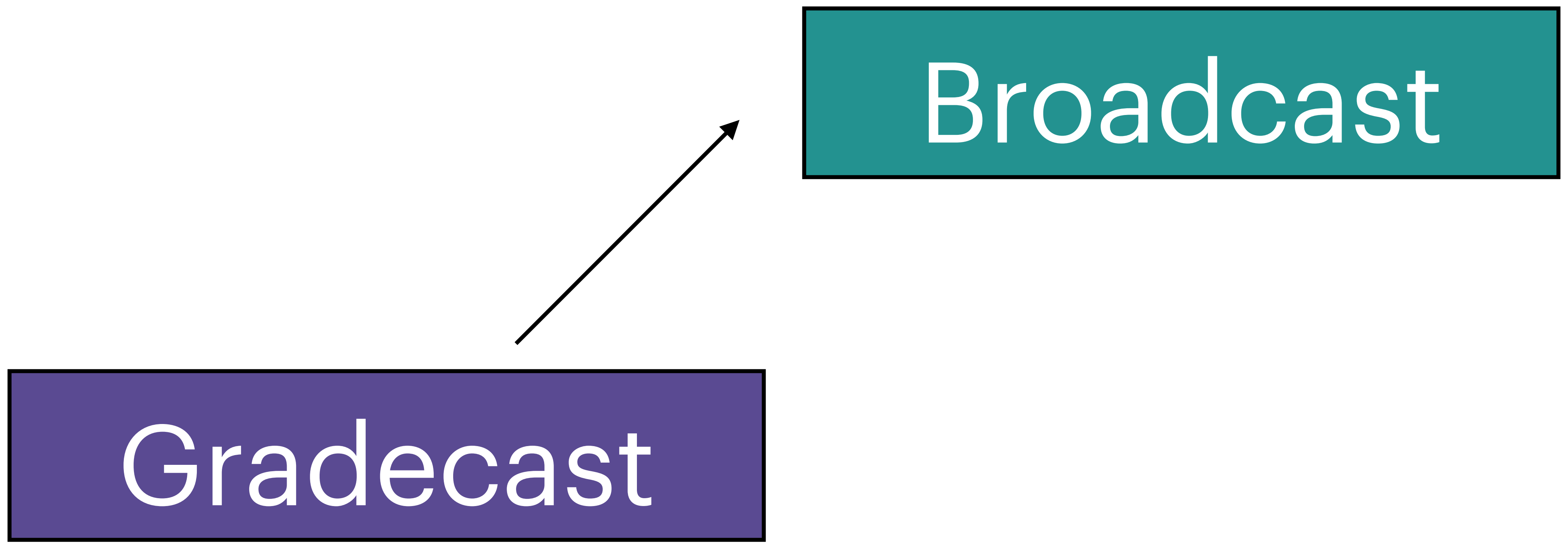

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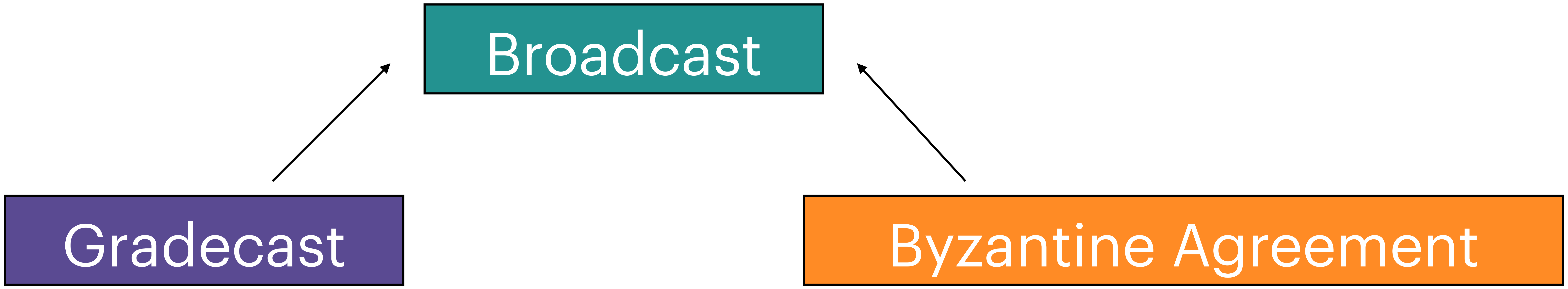

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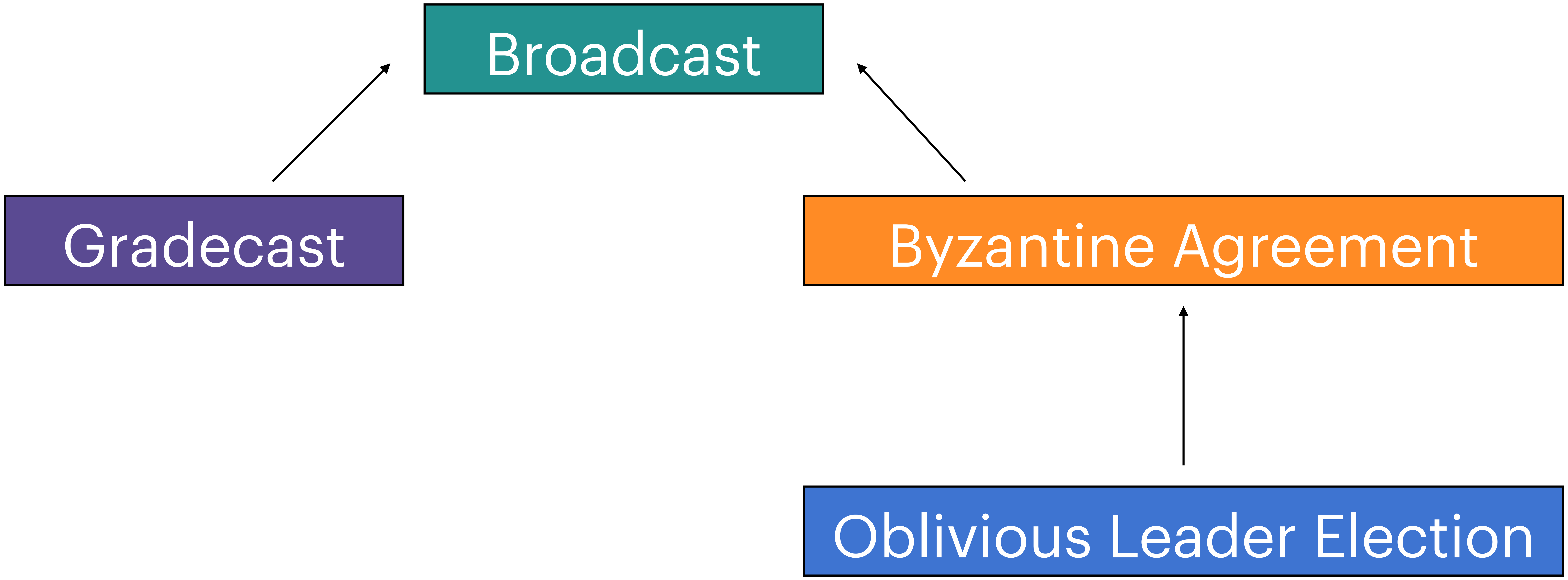

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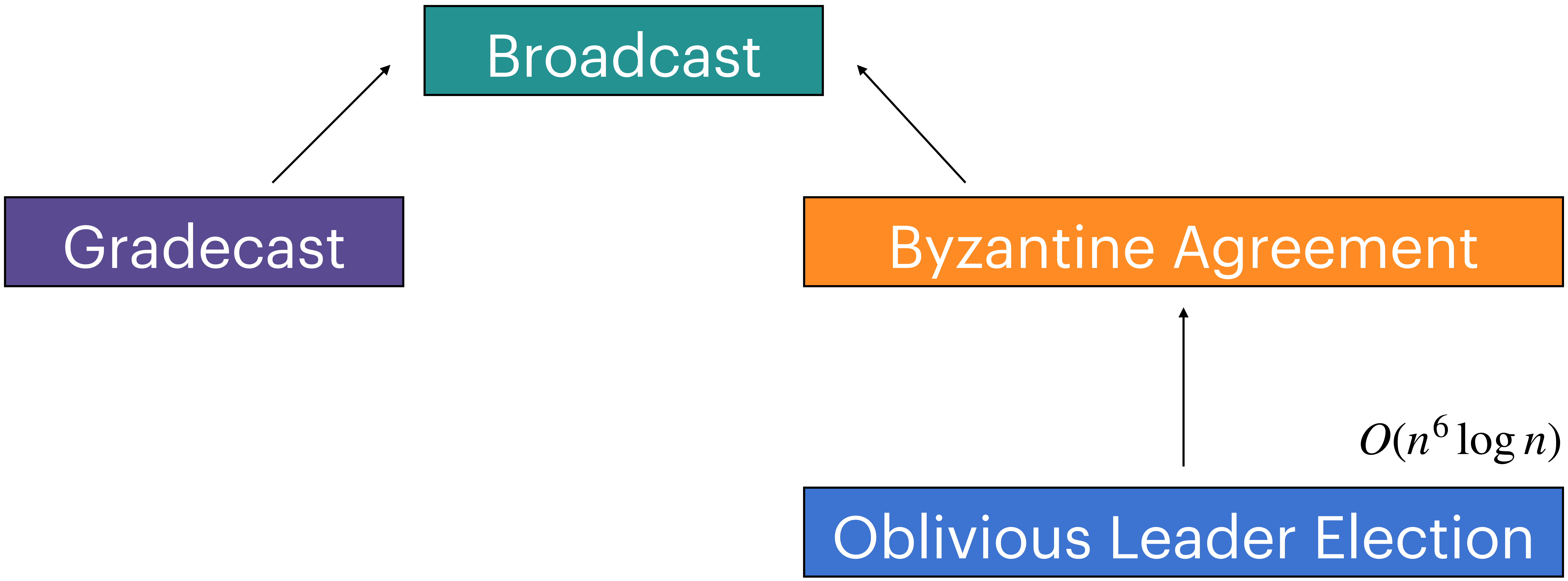

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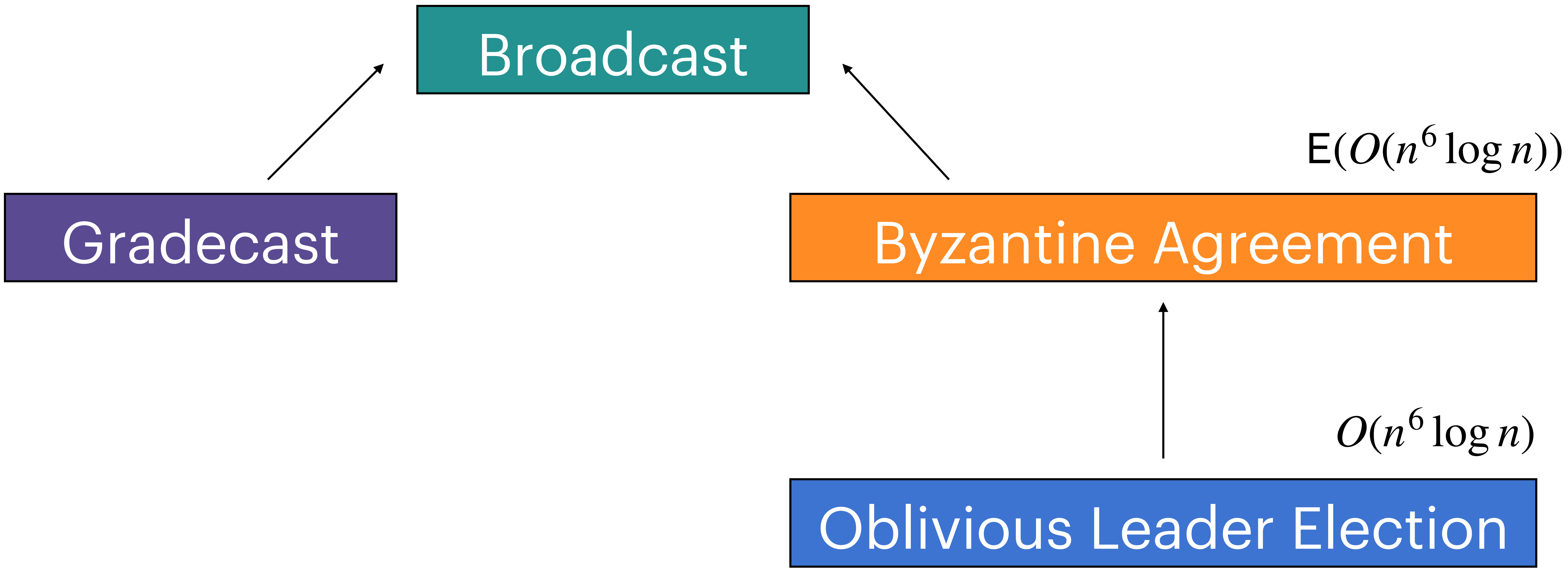

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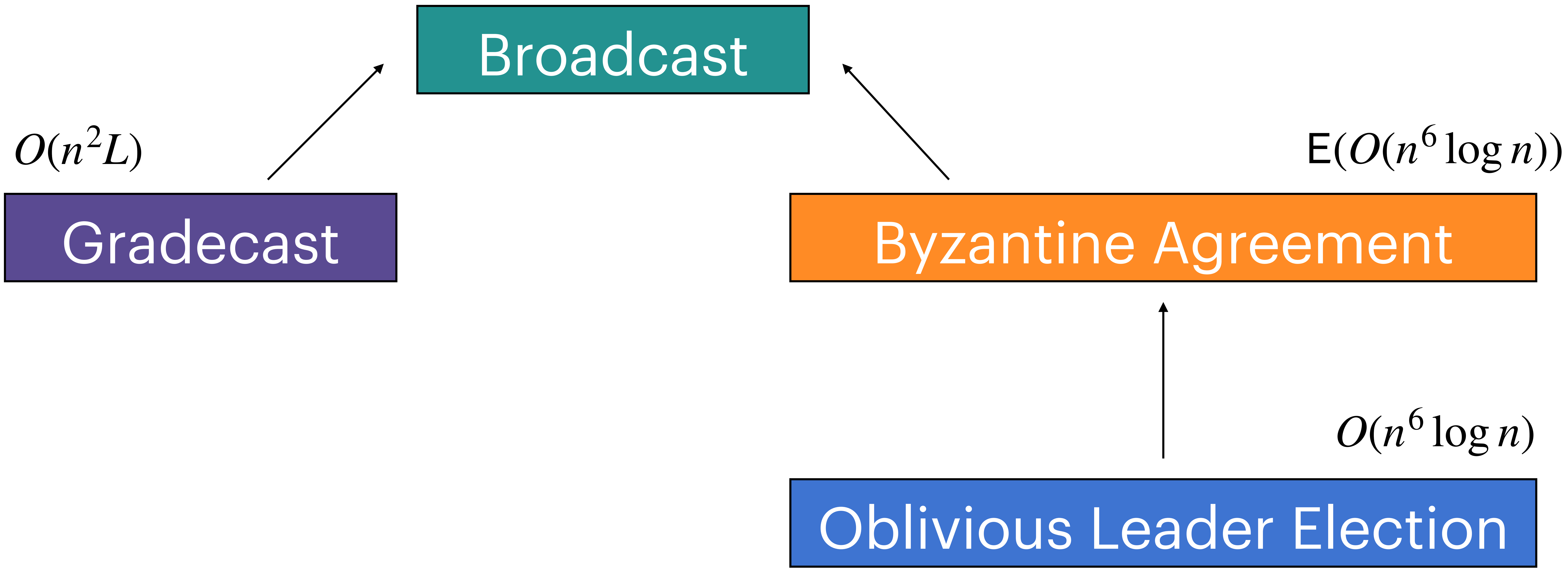

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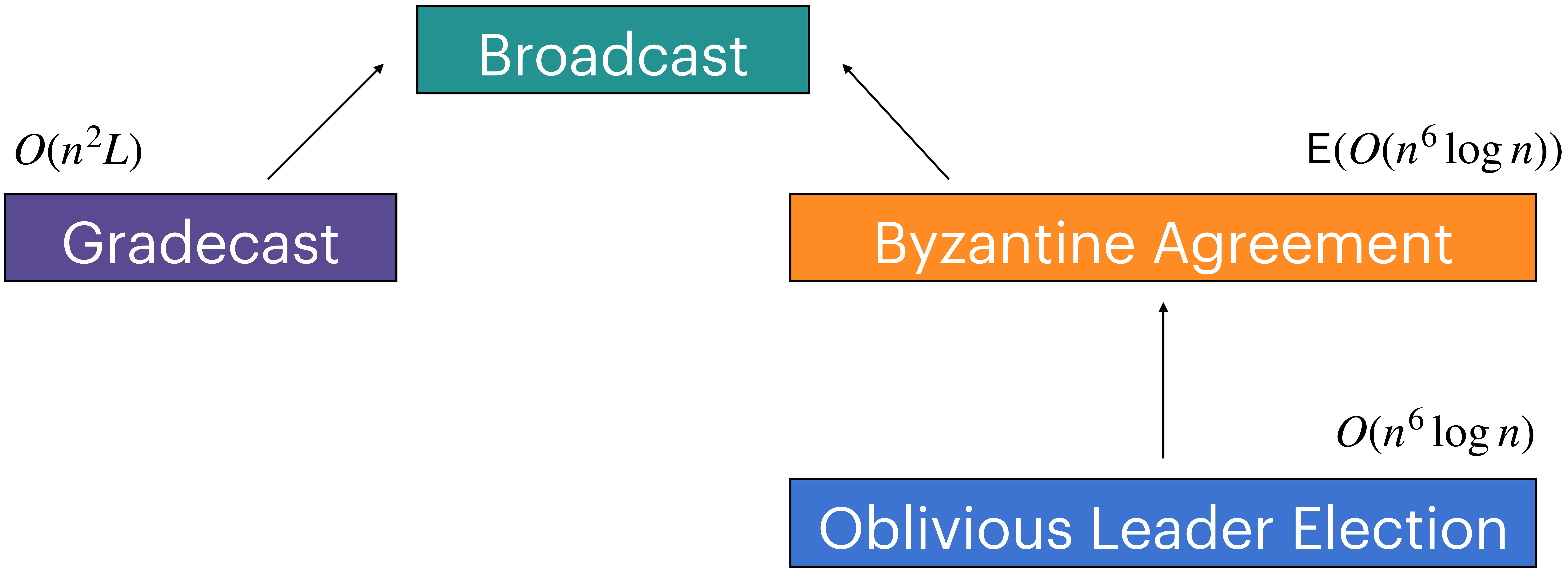

[Abraham, Asharov, Patil, Patra; 22] Improvements

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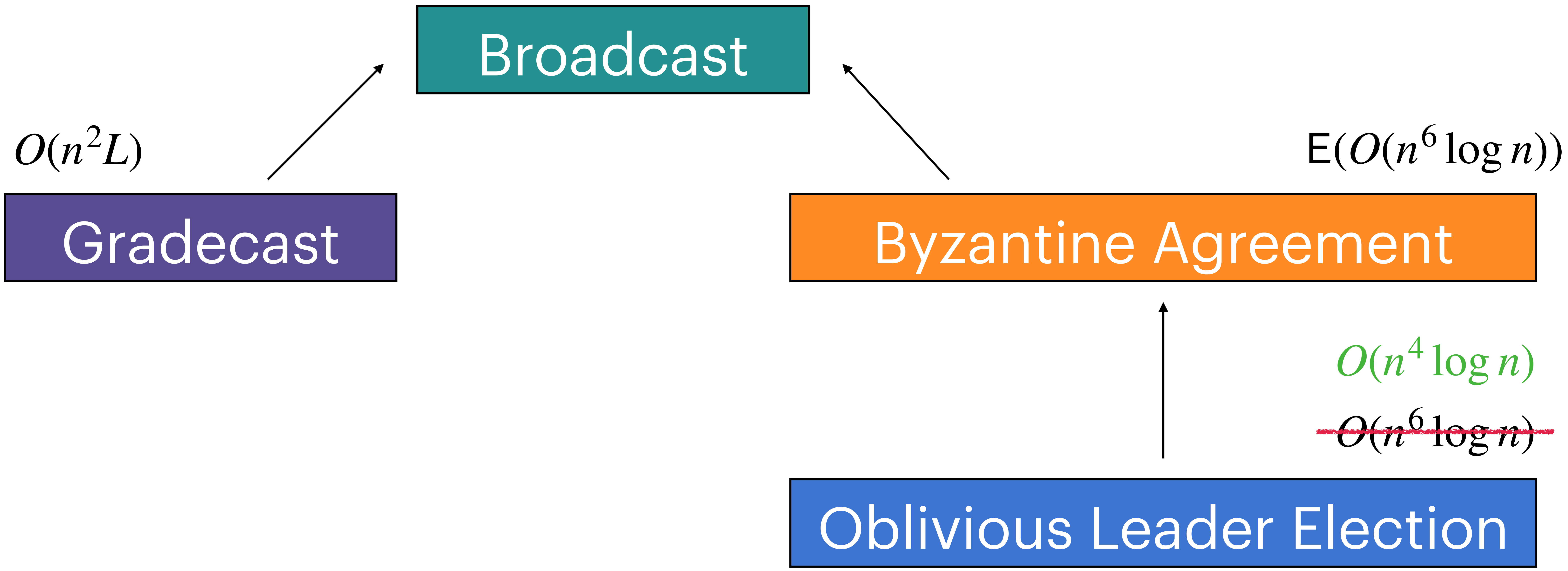

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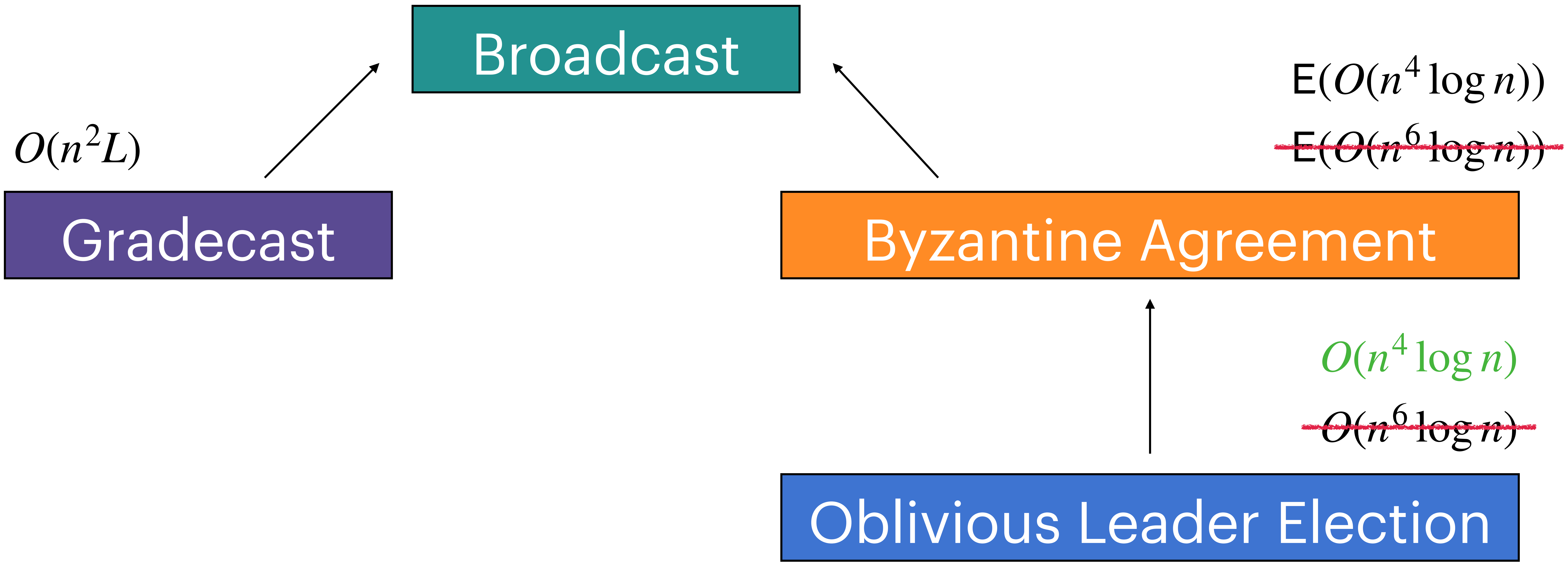

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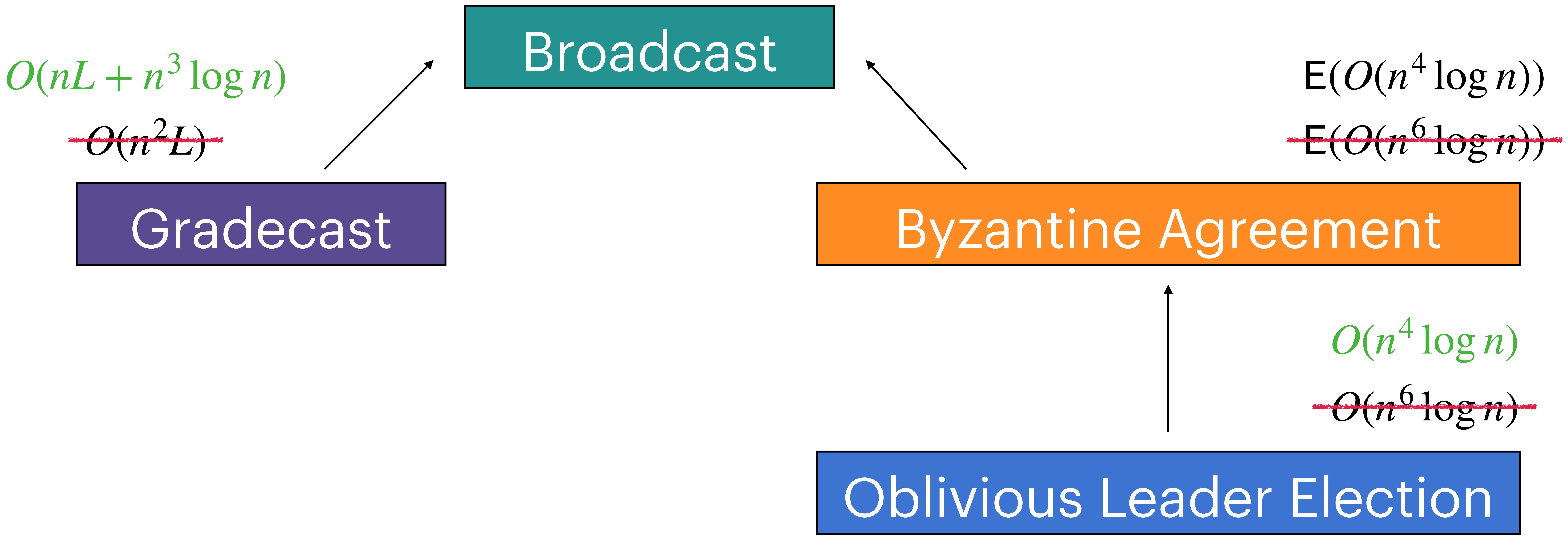

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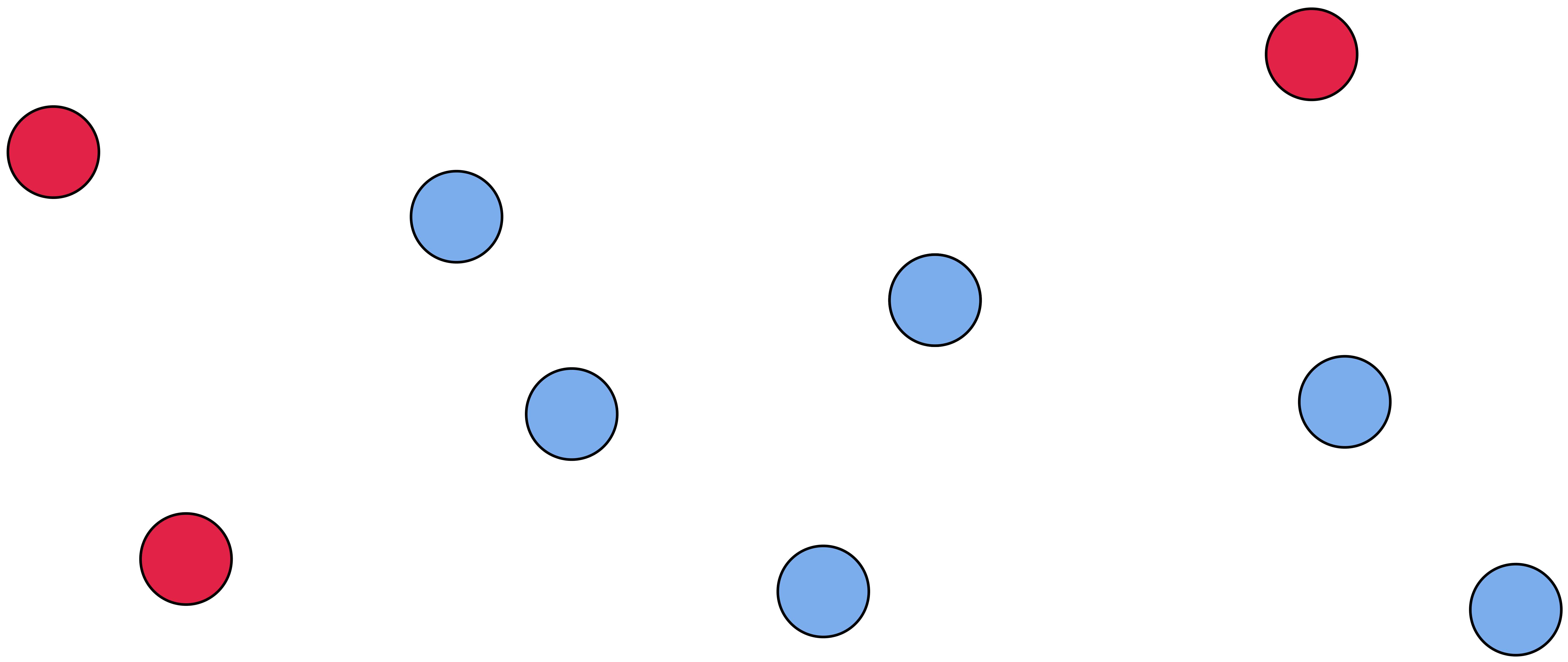
Oblivious Leader Election



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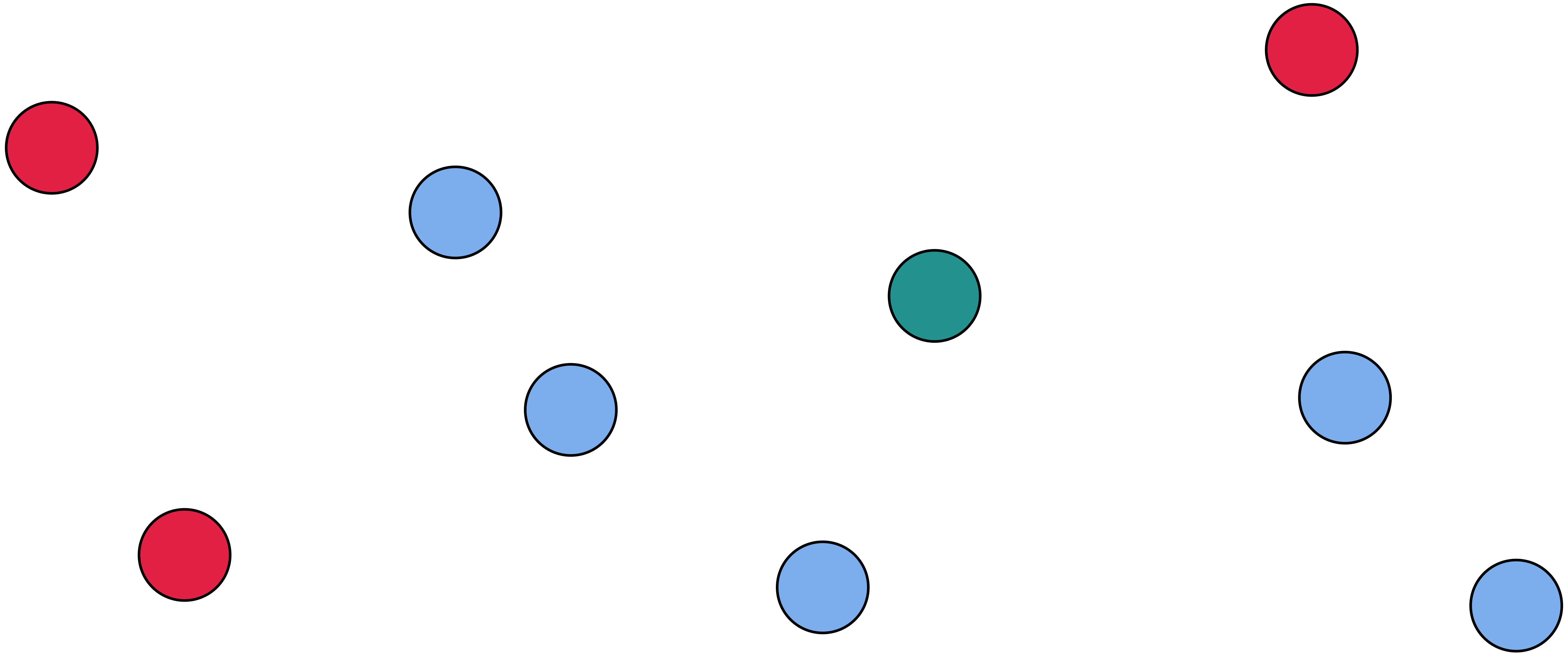

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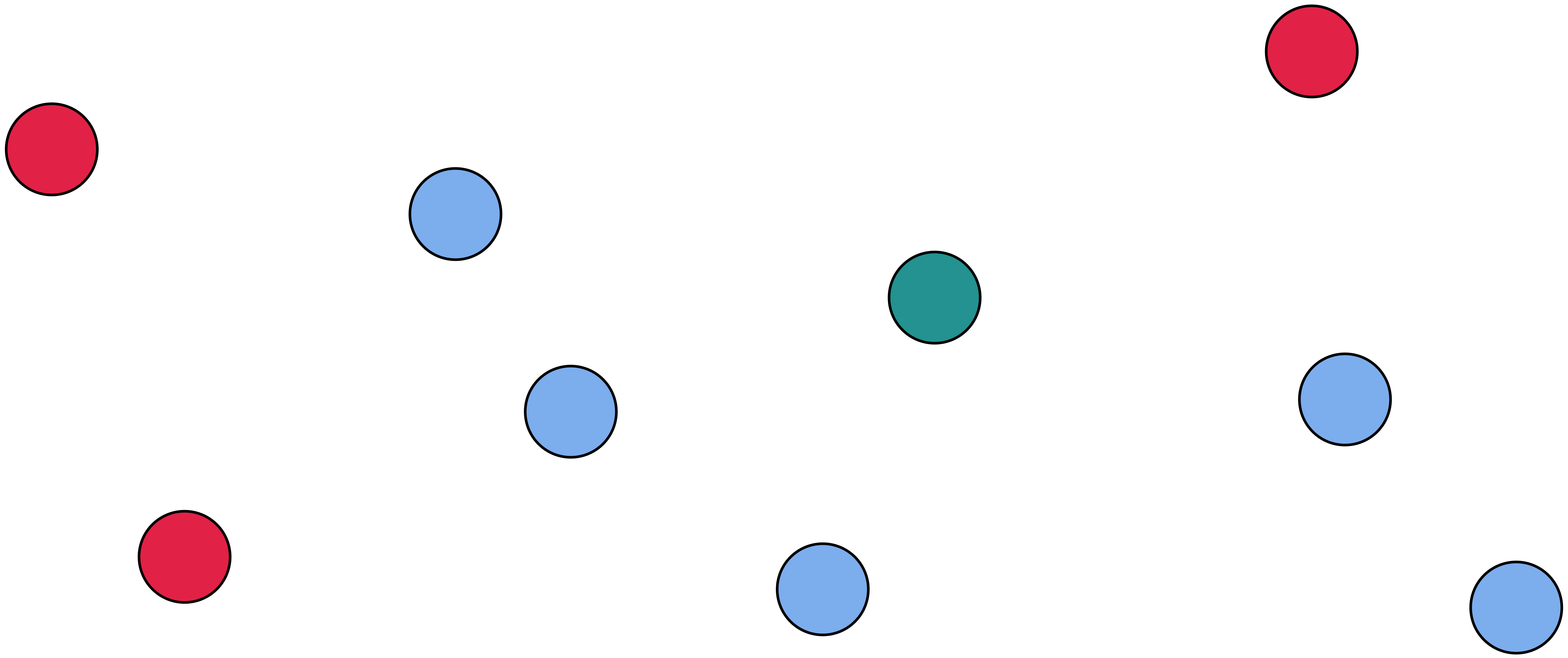
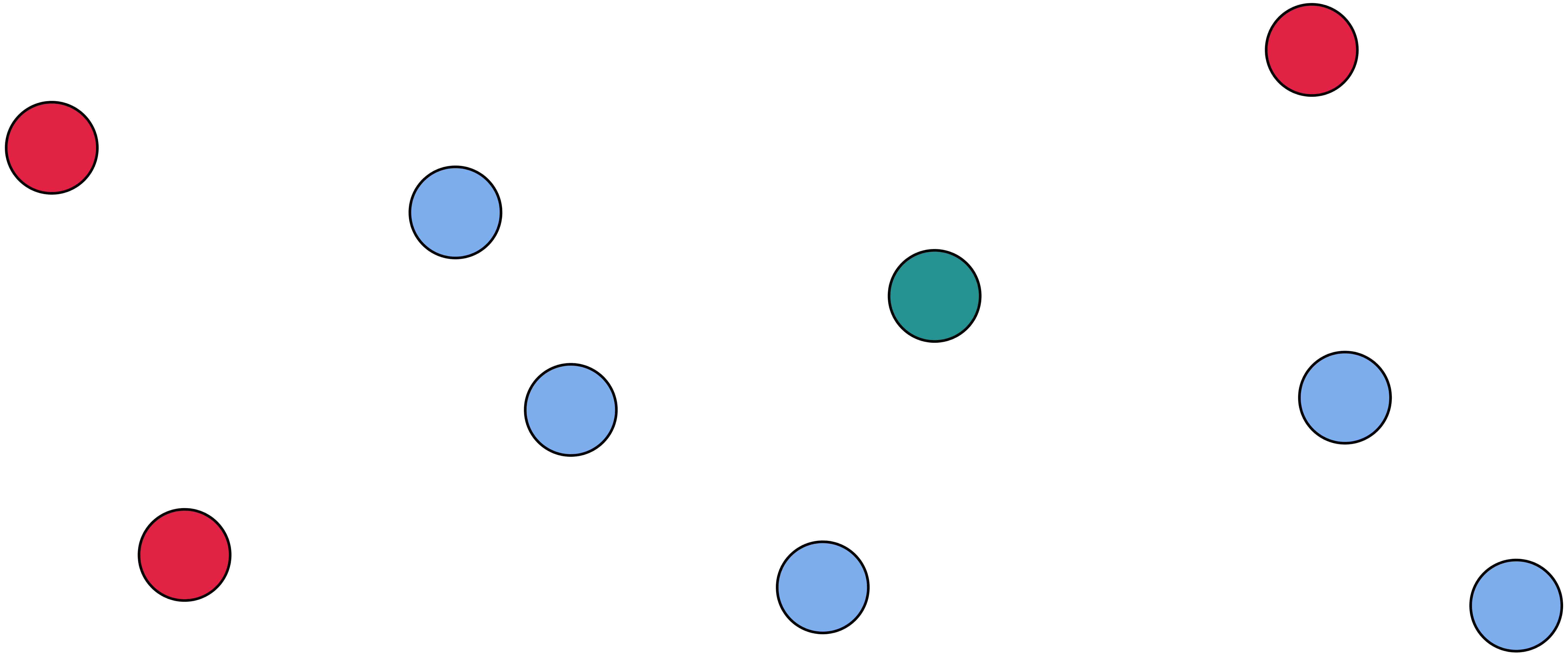


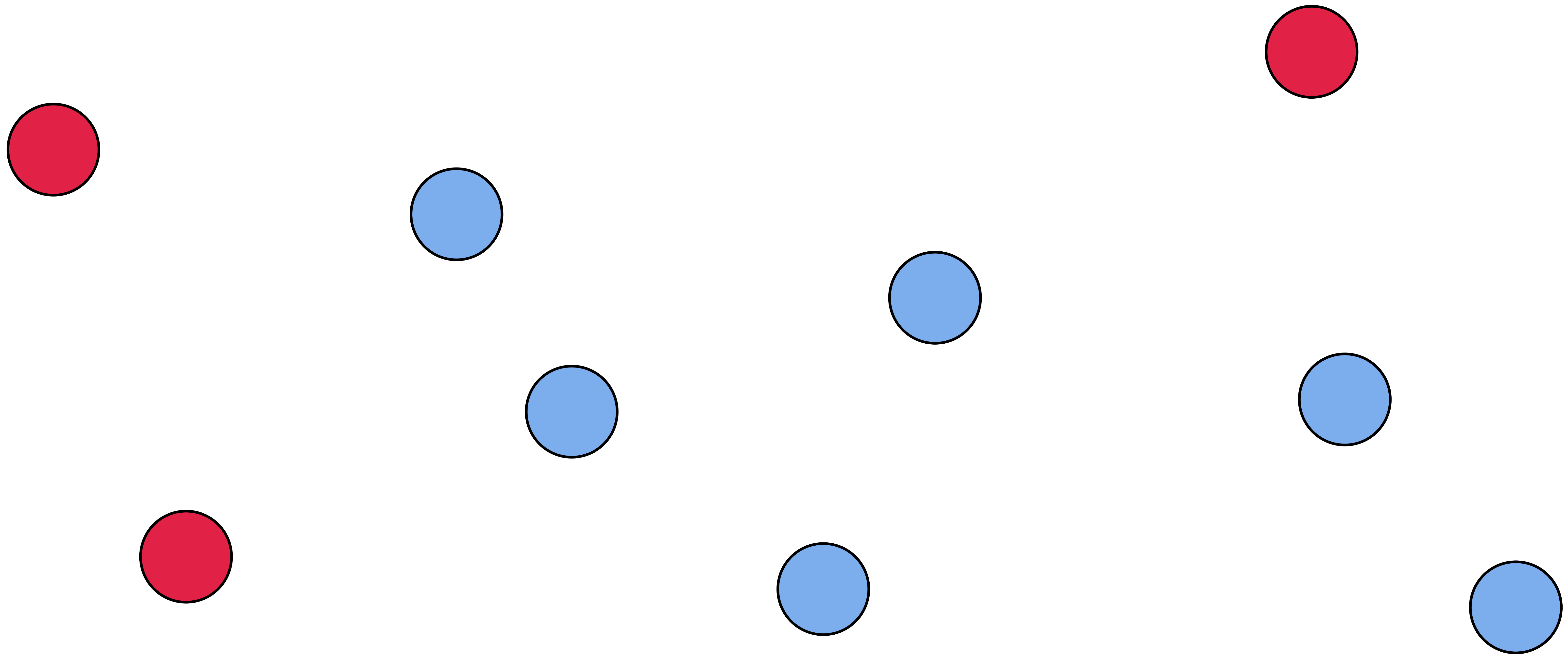
$$\Pr[\text{Everyone agrees on an honest leader}] \geq \frac{1}{2}$$

Oblivious Leader Election



$\Pr[\text{Everyone agrees on an honest leader}] \geq \frac{1}{2}$ Can be any constant!

Oblivious Leader Election



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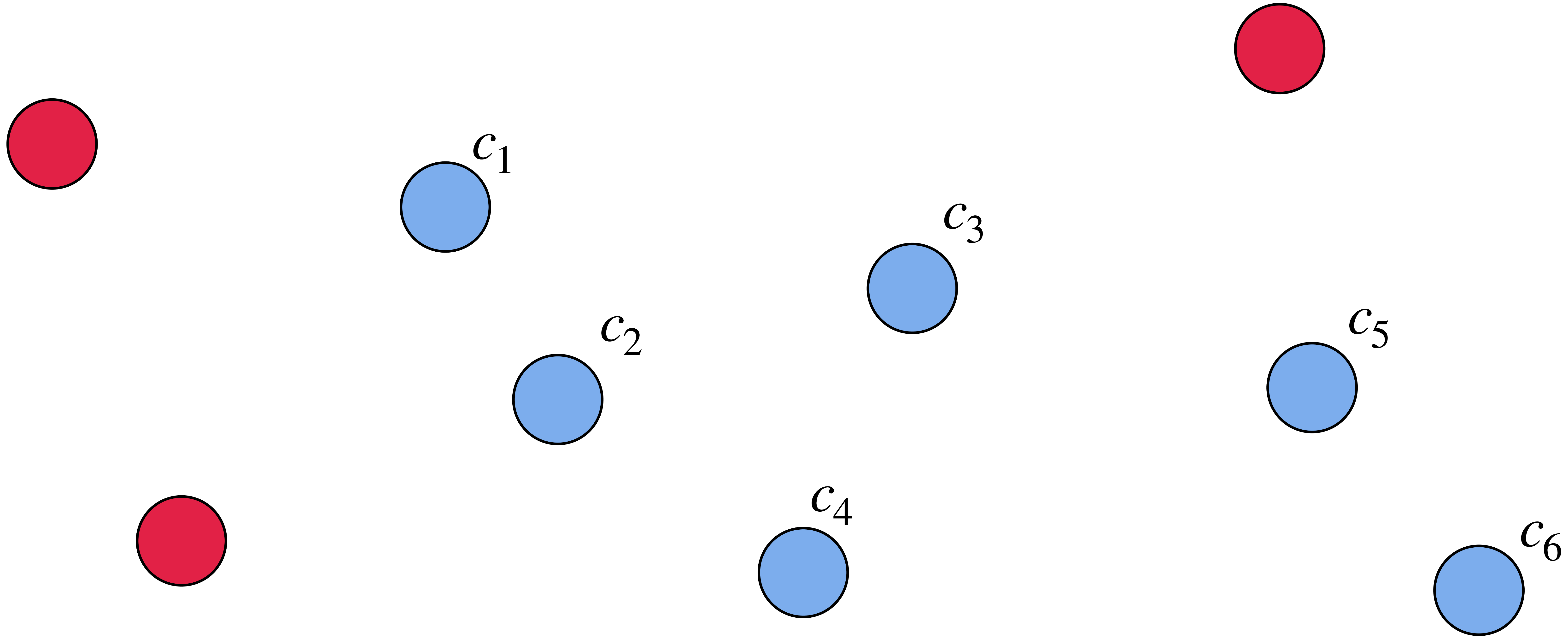

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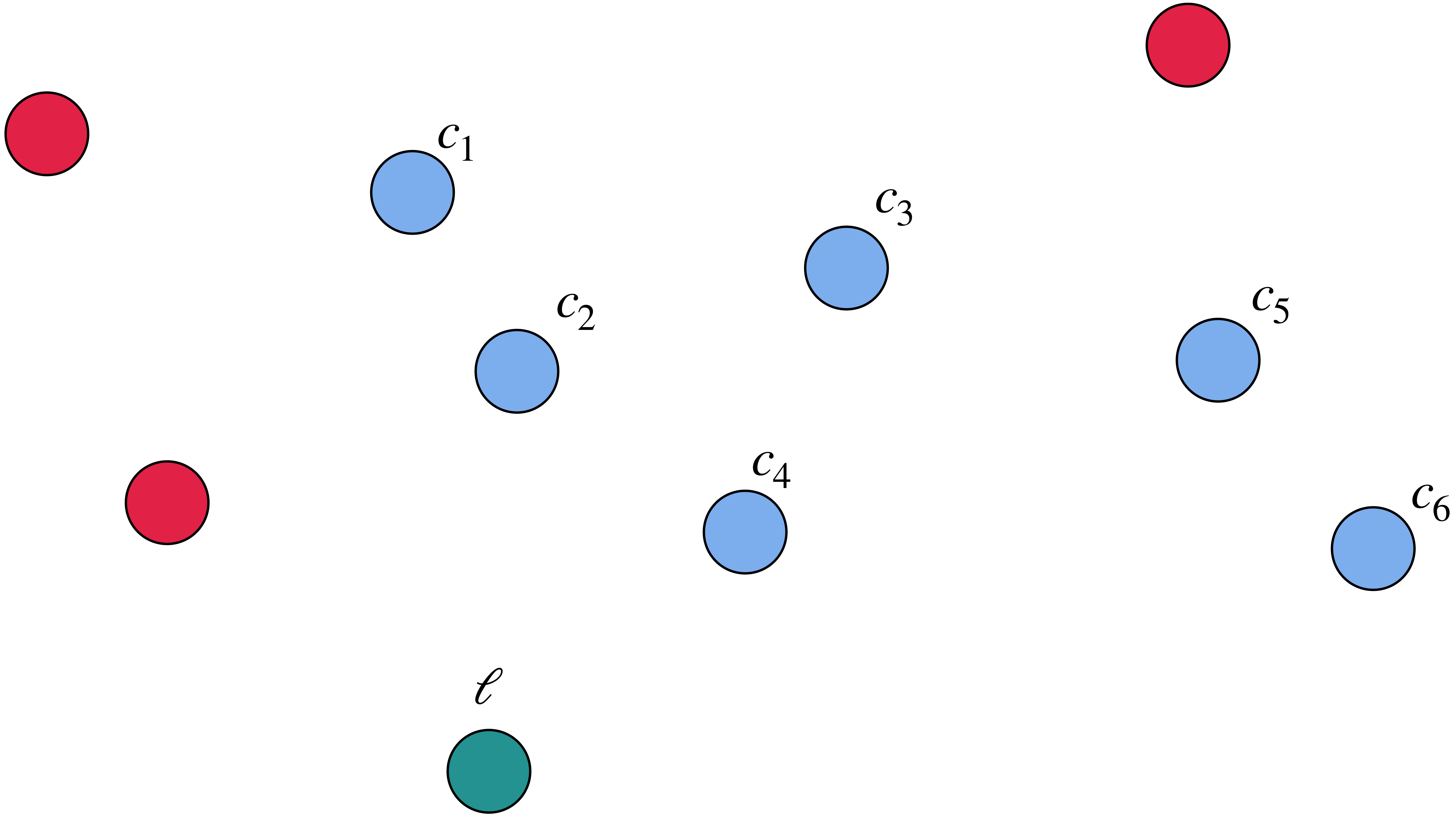

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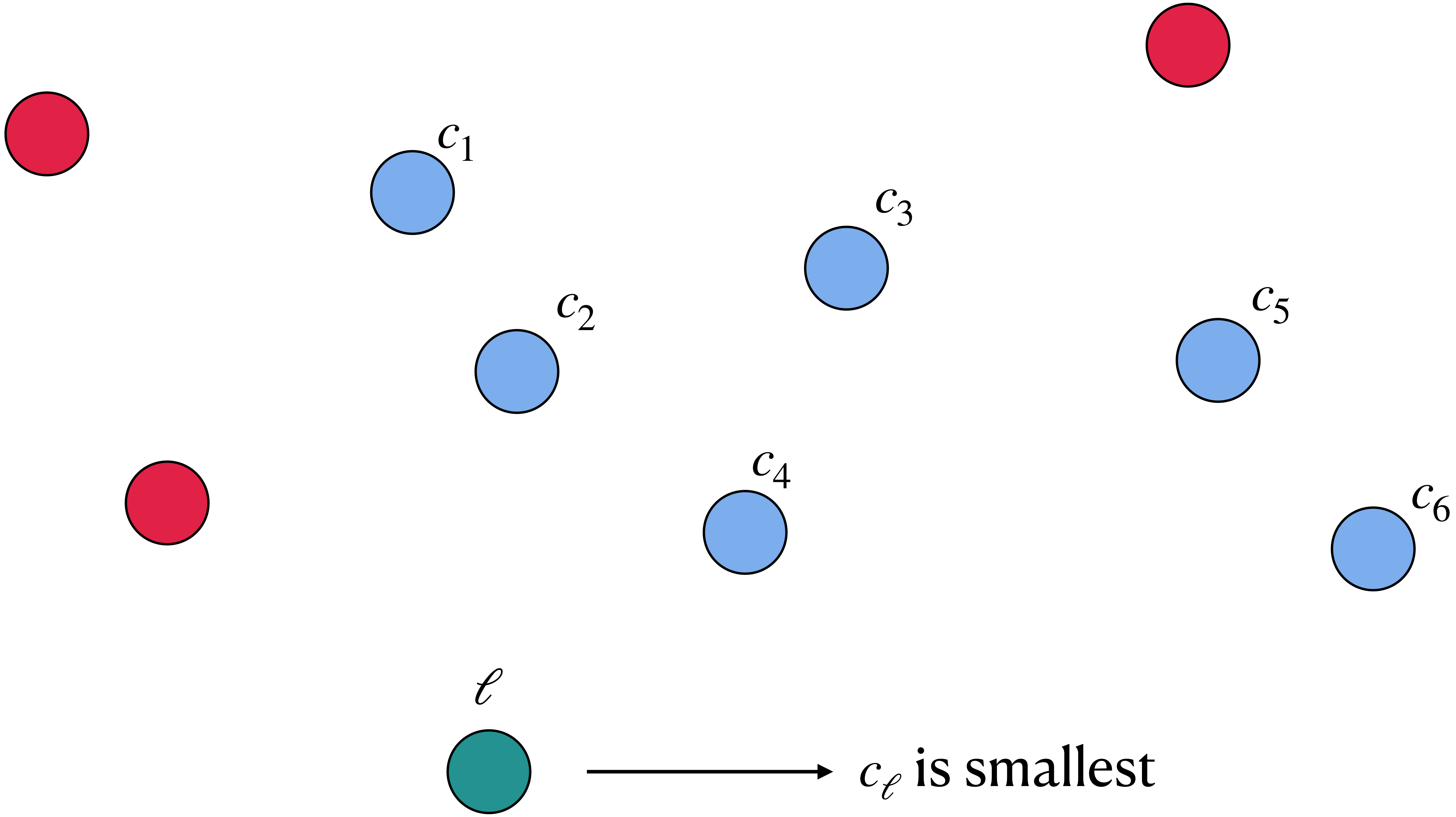

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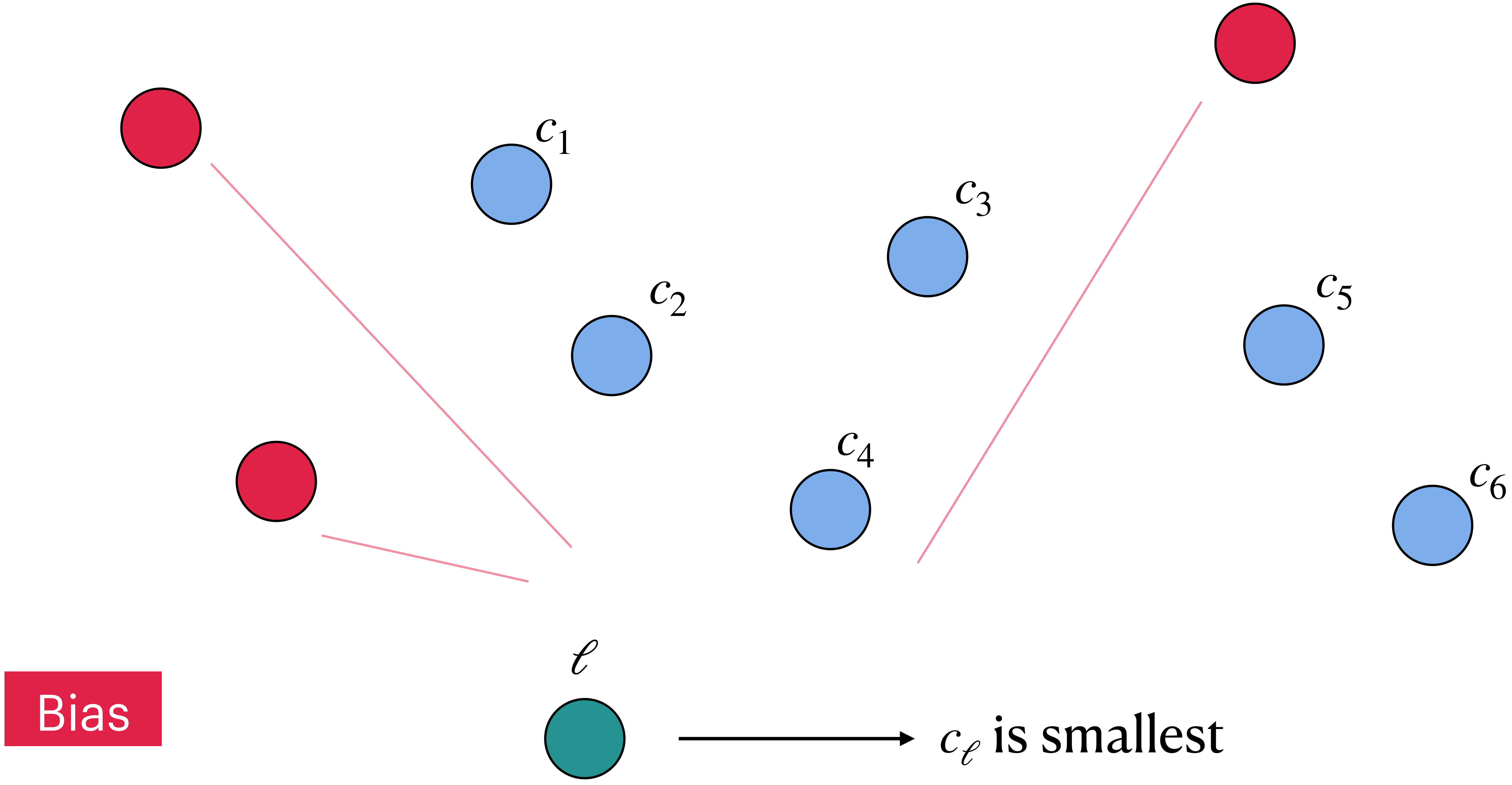
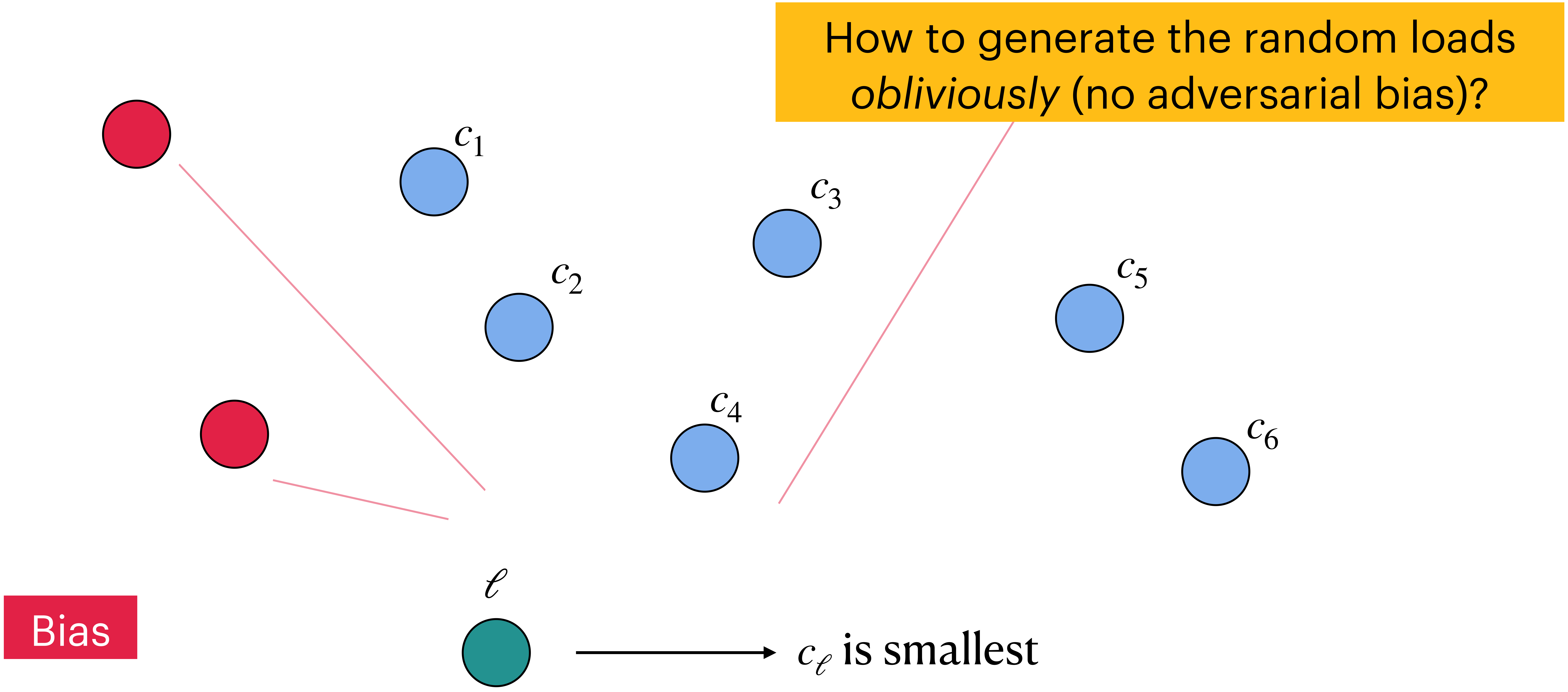


Oblivious Leader Election

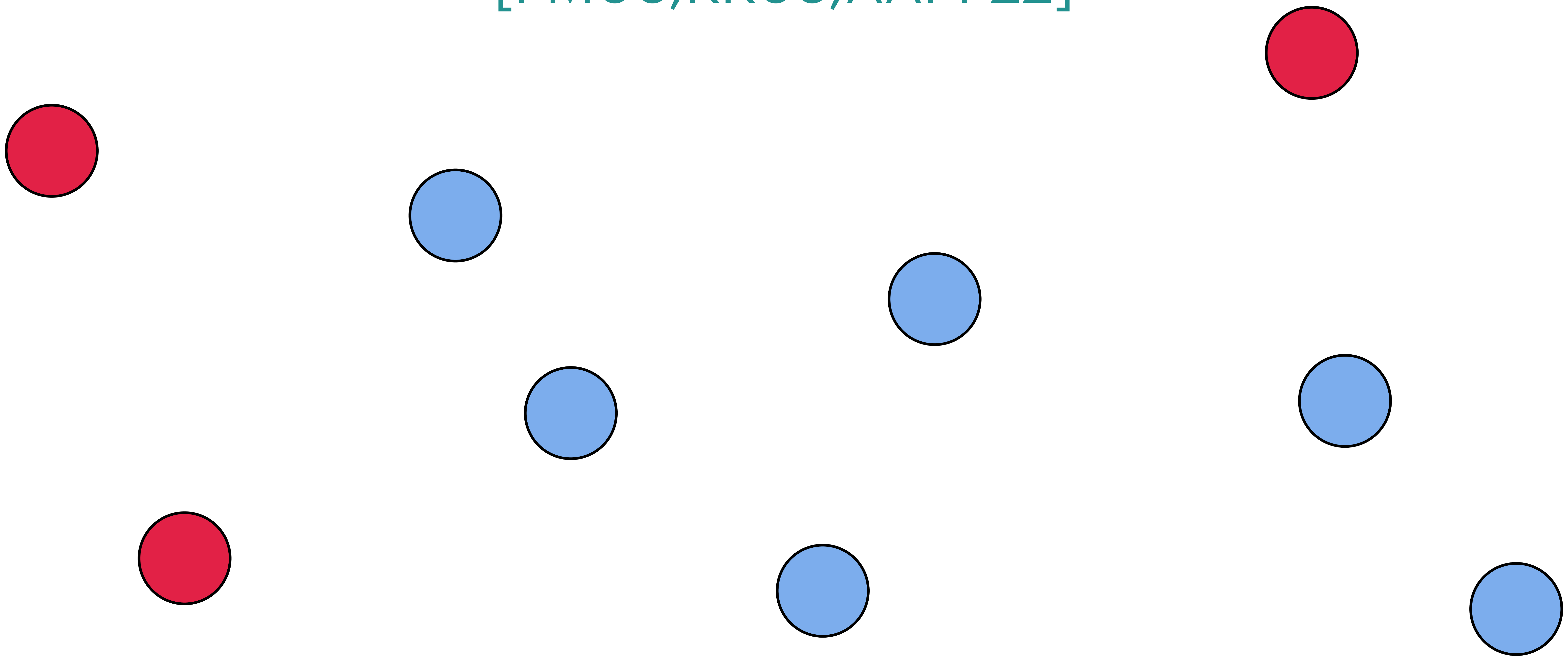


Leader Election from Commitments

[FM06,KK08,AAPP22]

Leader Election from Commitments

[FM06,KK08,AAPP22]



Leader Election from Commitments

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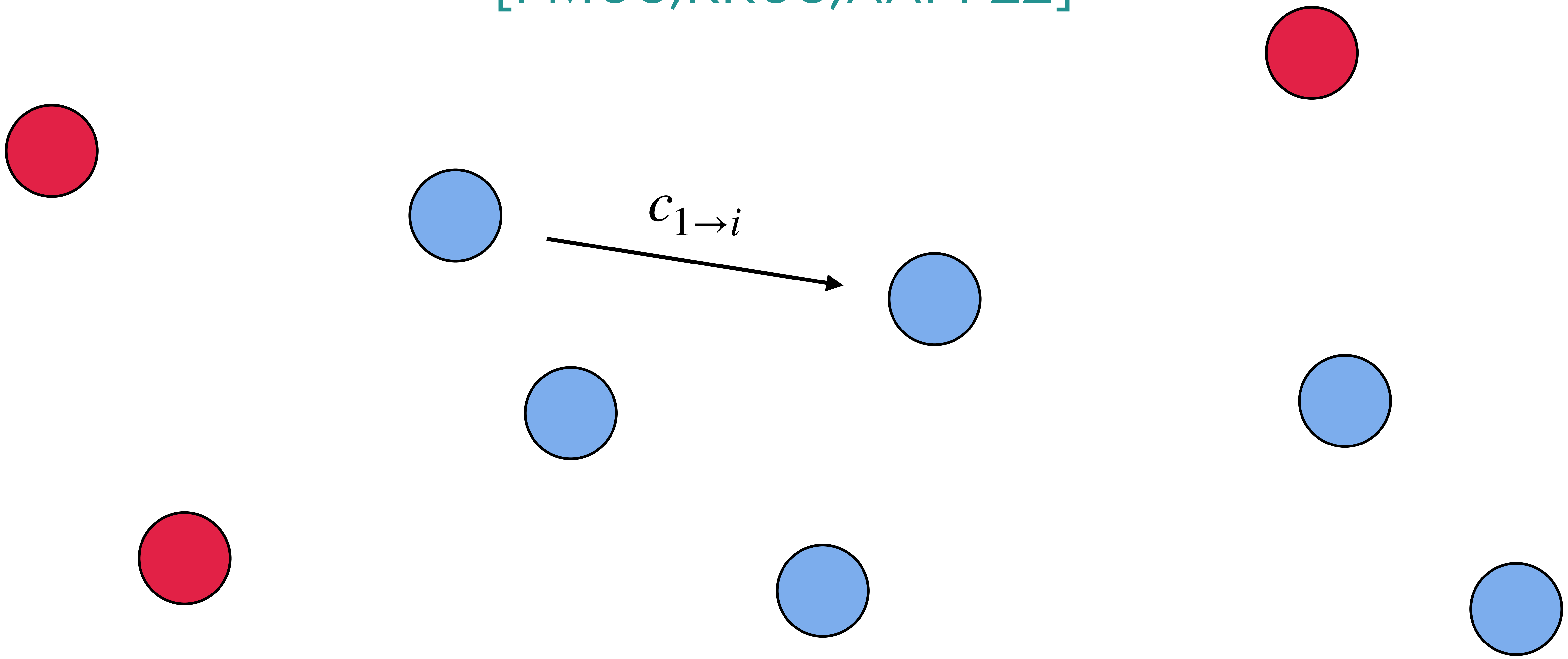

Leader Election from Commitments

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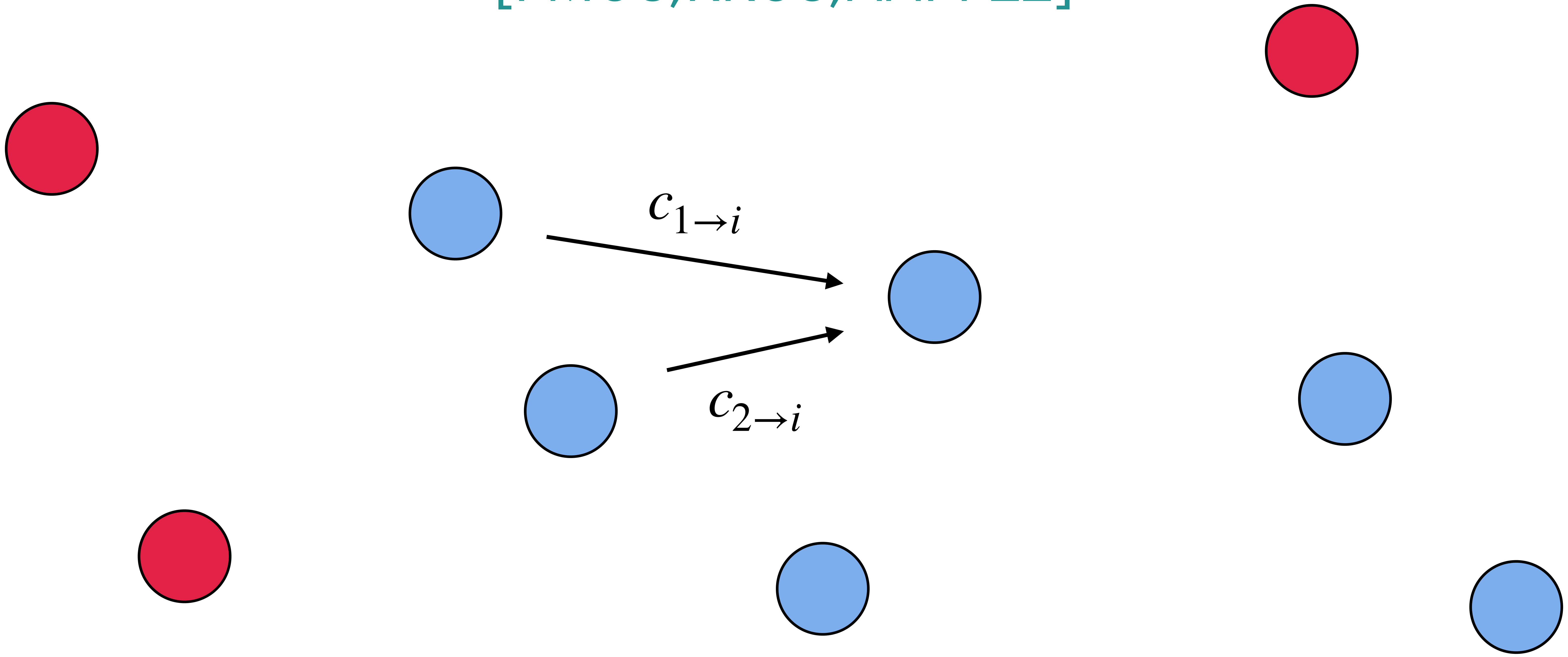
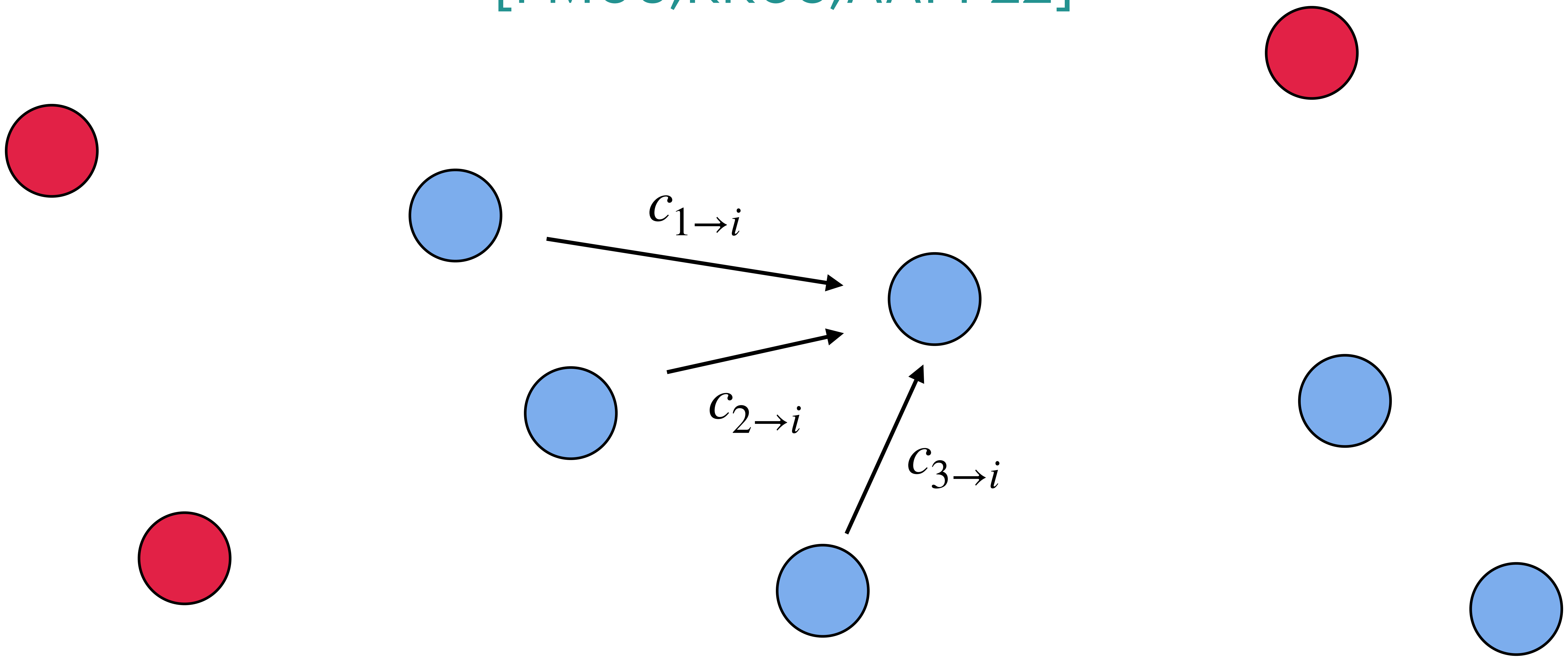


Leader Election from Commitments

[FM06,KK08,AAPP22]



Leader Election from Commitments

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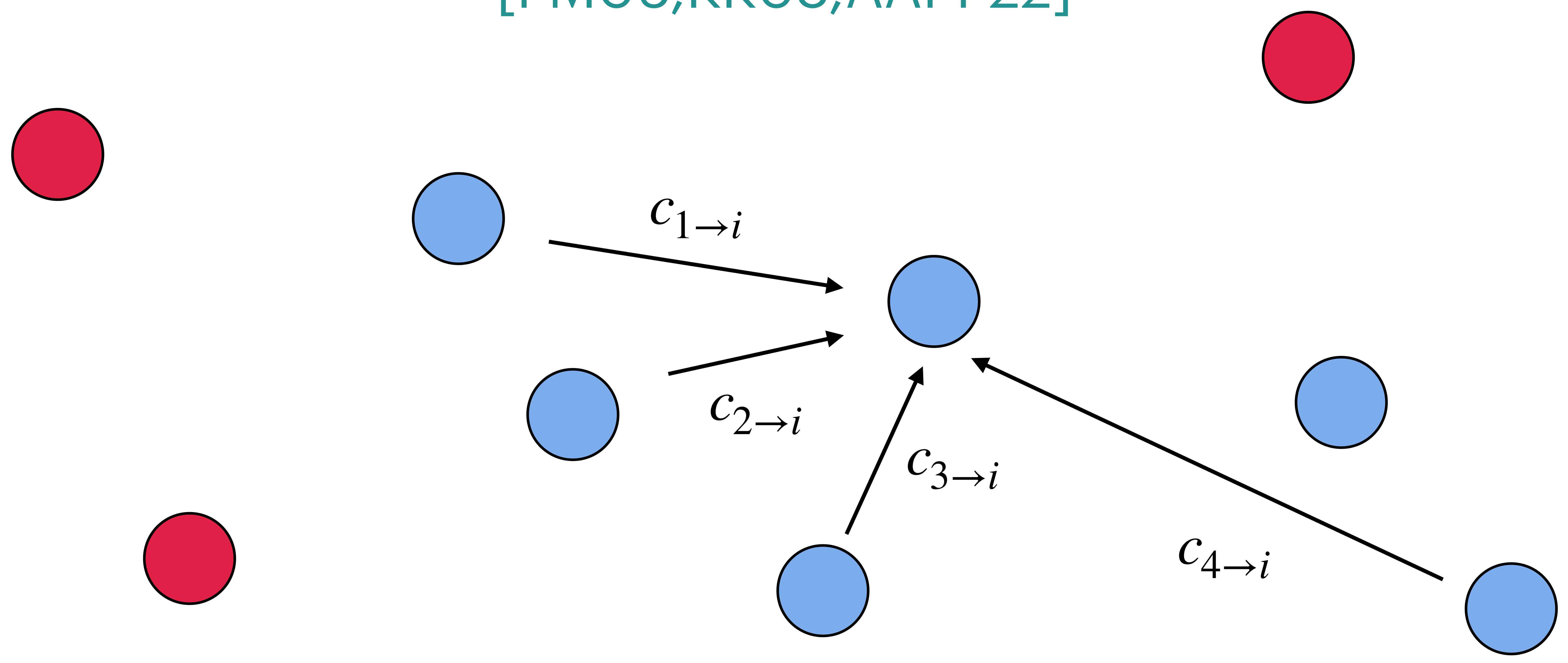

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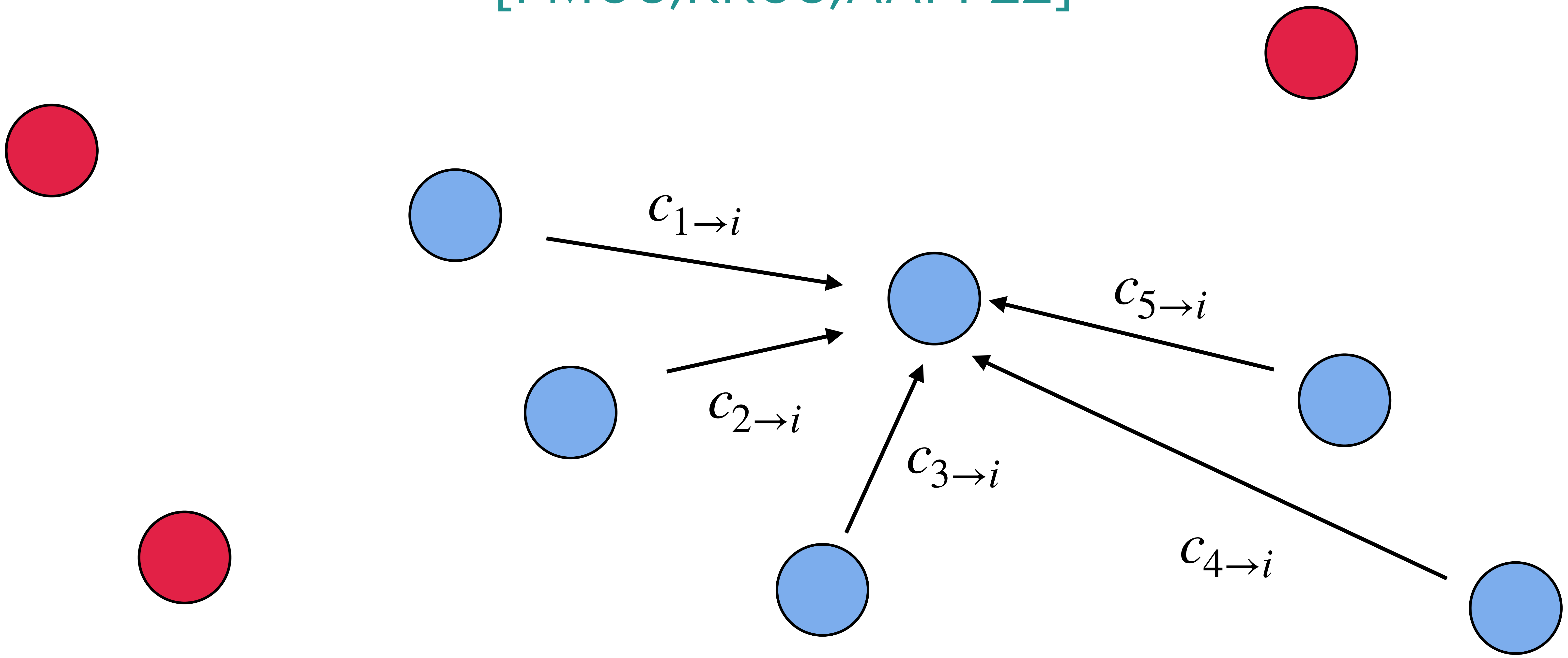
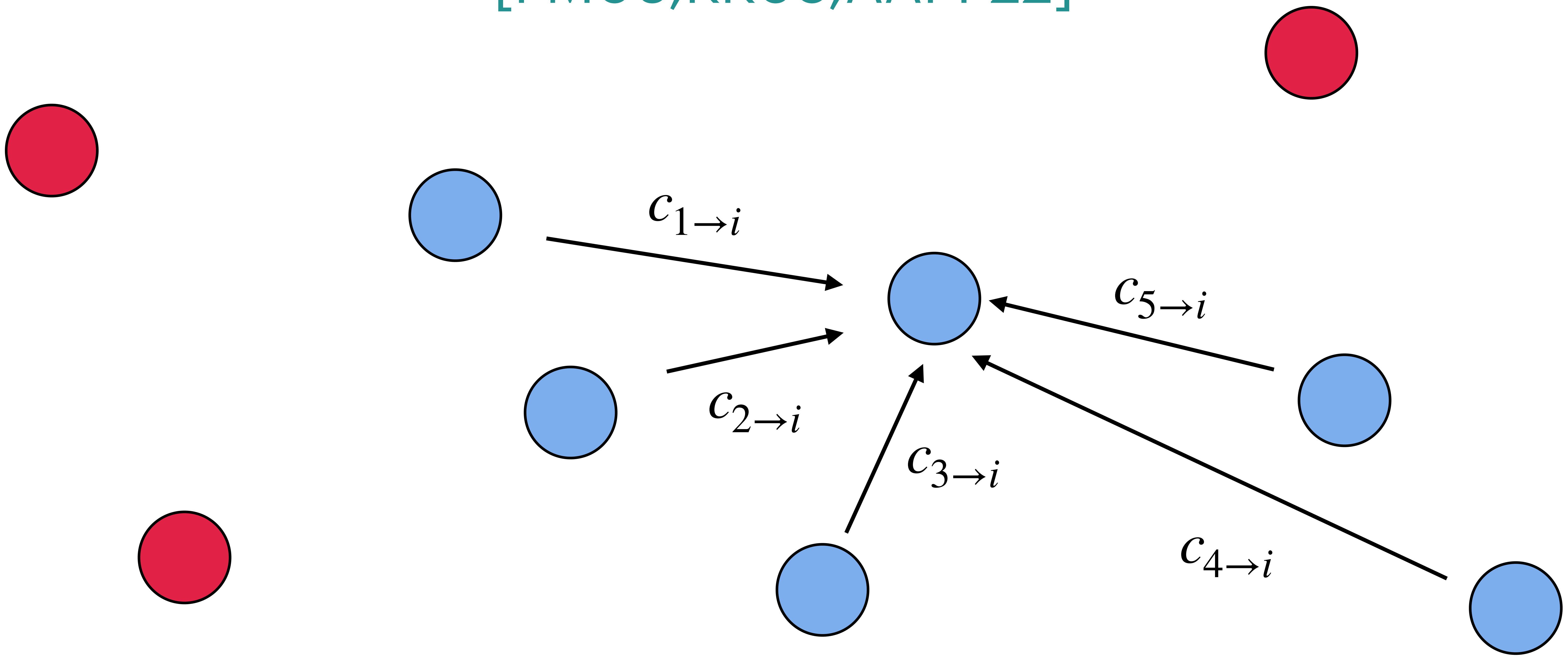


Contribution via
commit + reveal

…lection from Commitments

[FM06,KK08,AAPP22]



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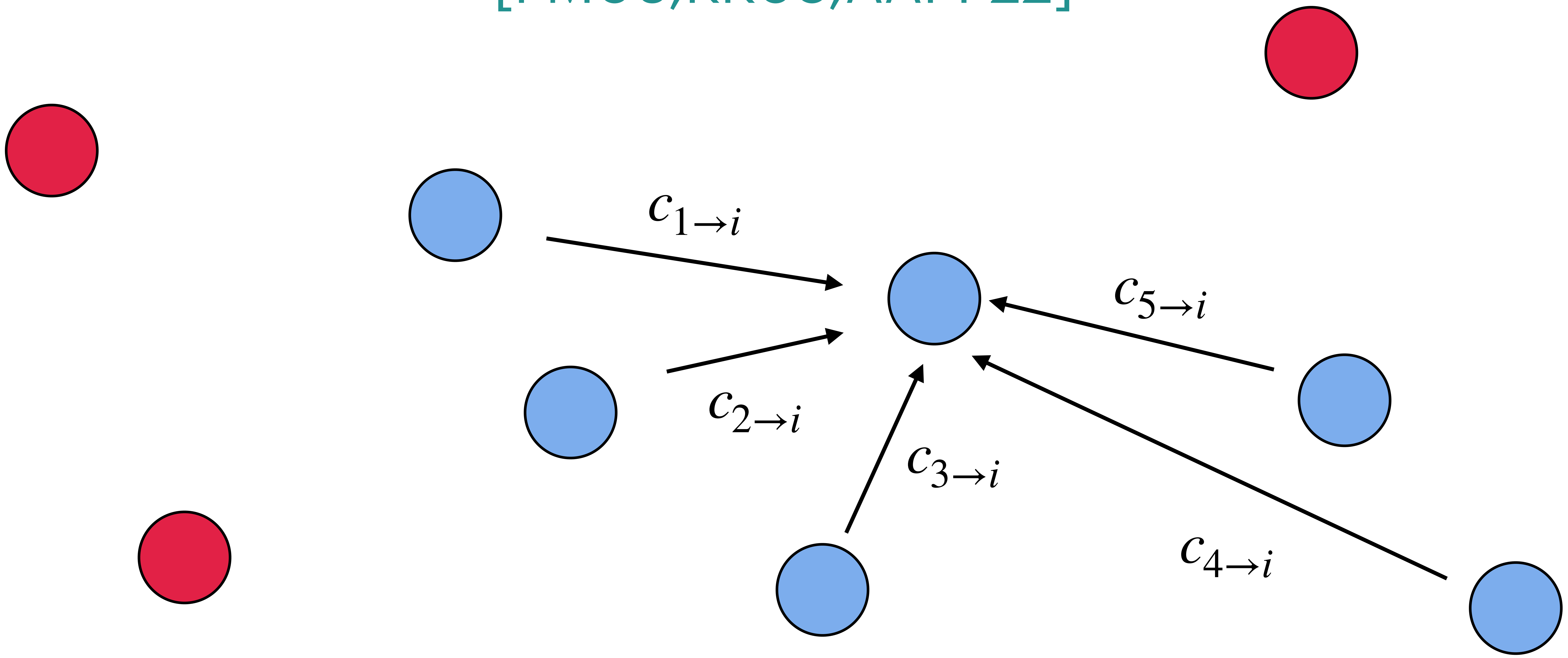
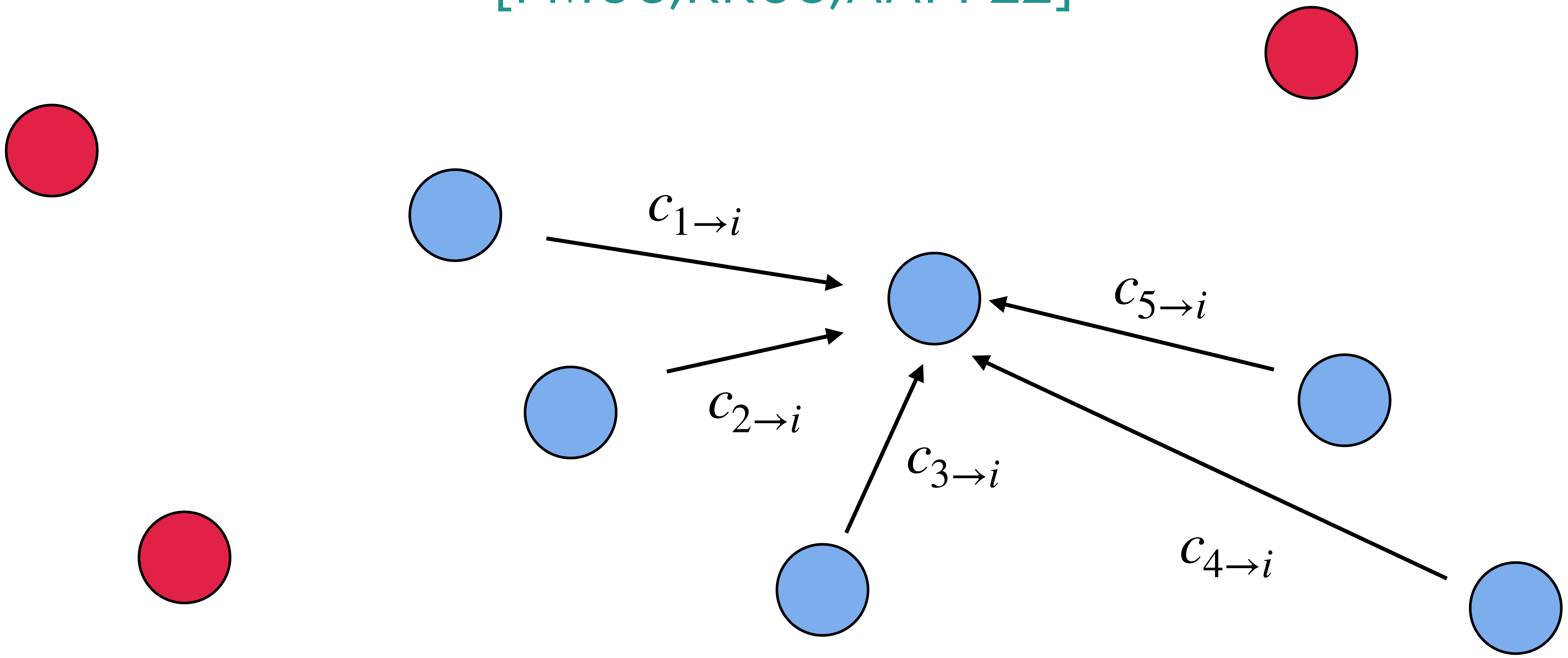


$$c_i = \sum_{j=1}^{n} c_{j\to i}$$

lection from Commitments

Contribution via
commit + reveal

[FM06,KK08,AAPP22]



Adversary cannot bias!

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Contribution via
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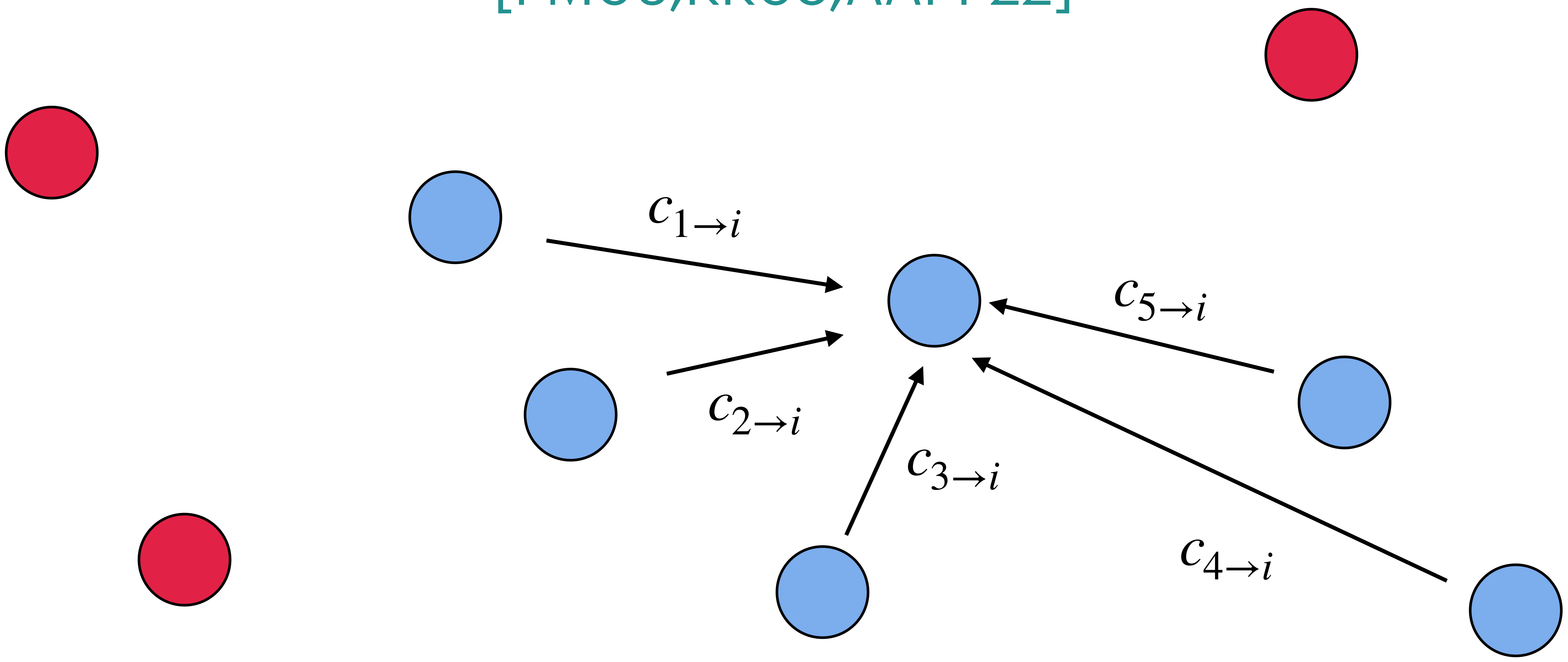
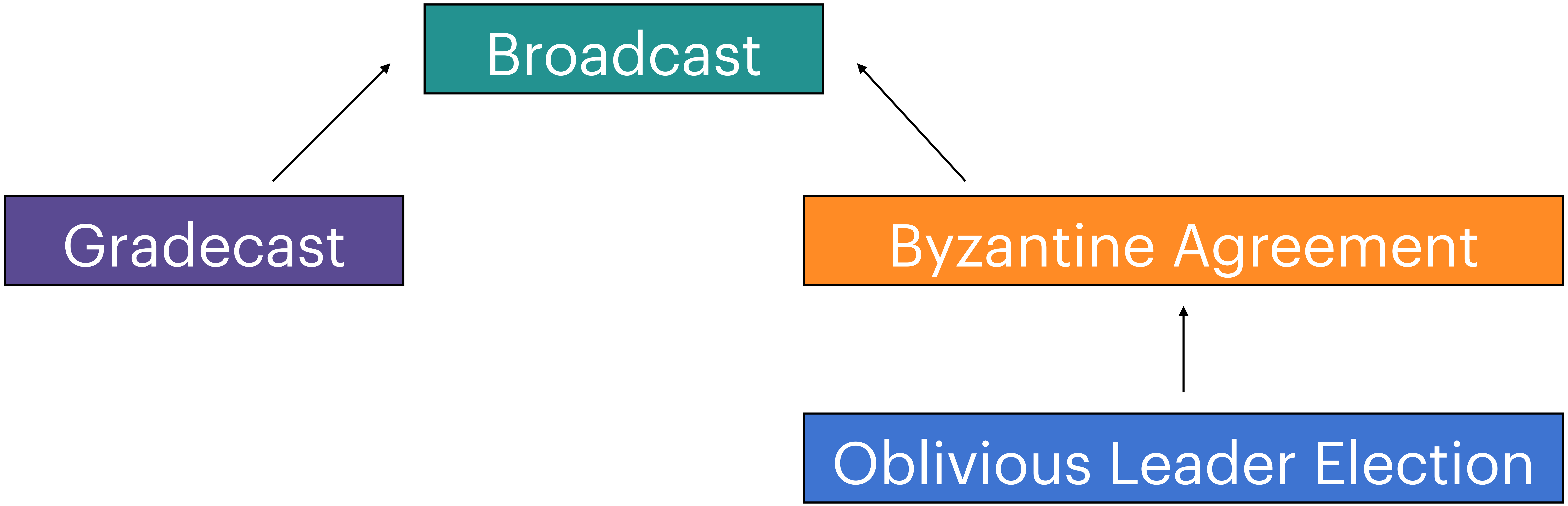


Adversary cannot bias!

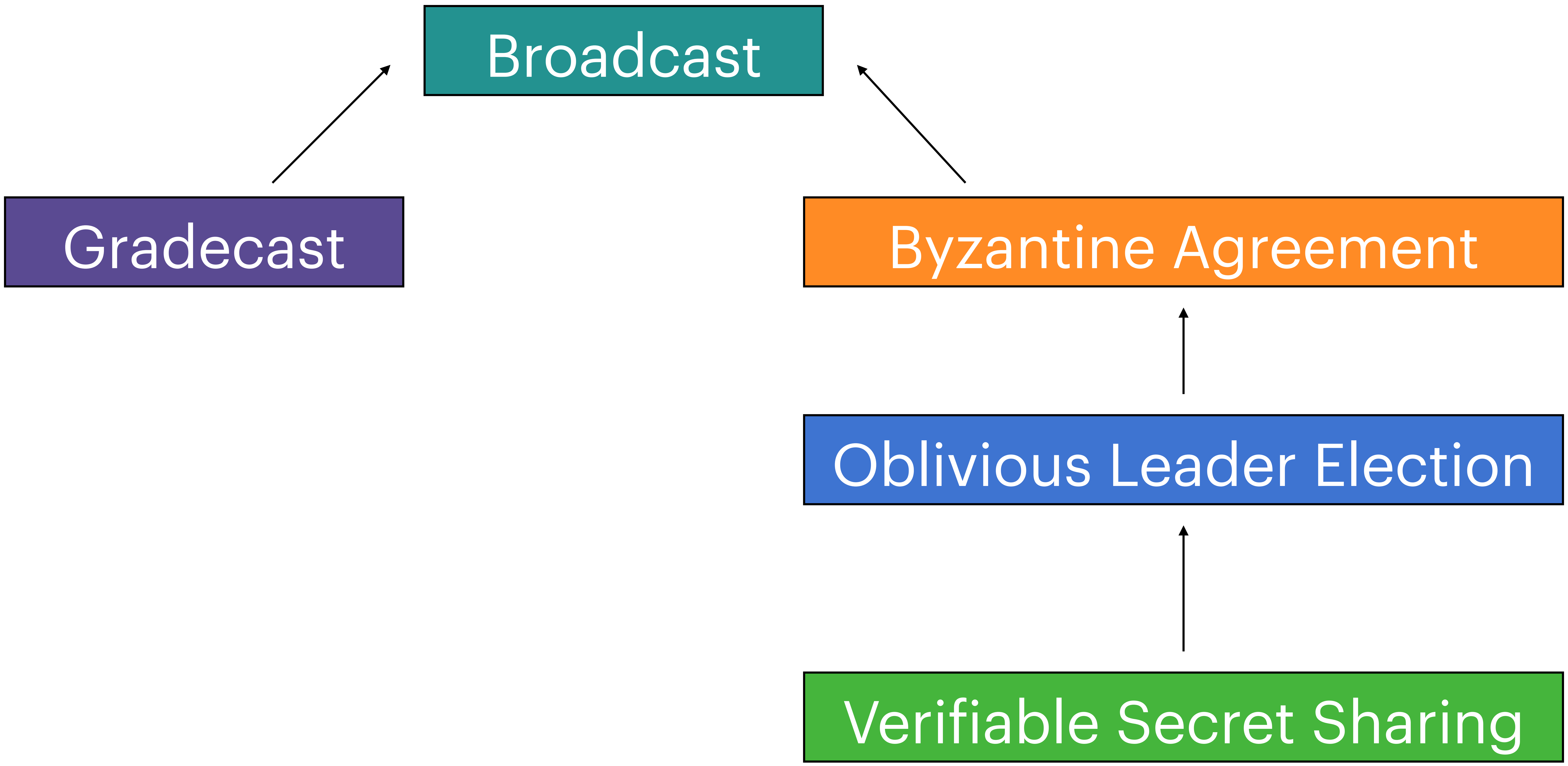
$$c_i = \sum_{j=1}^{n} c_{j\to i}$$

Each party receives at least one uniformly random contribution from an honest party

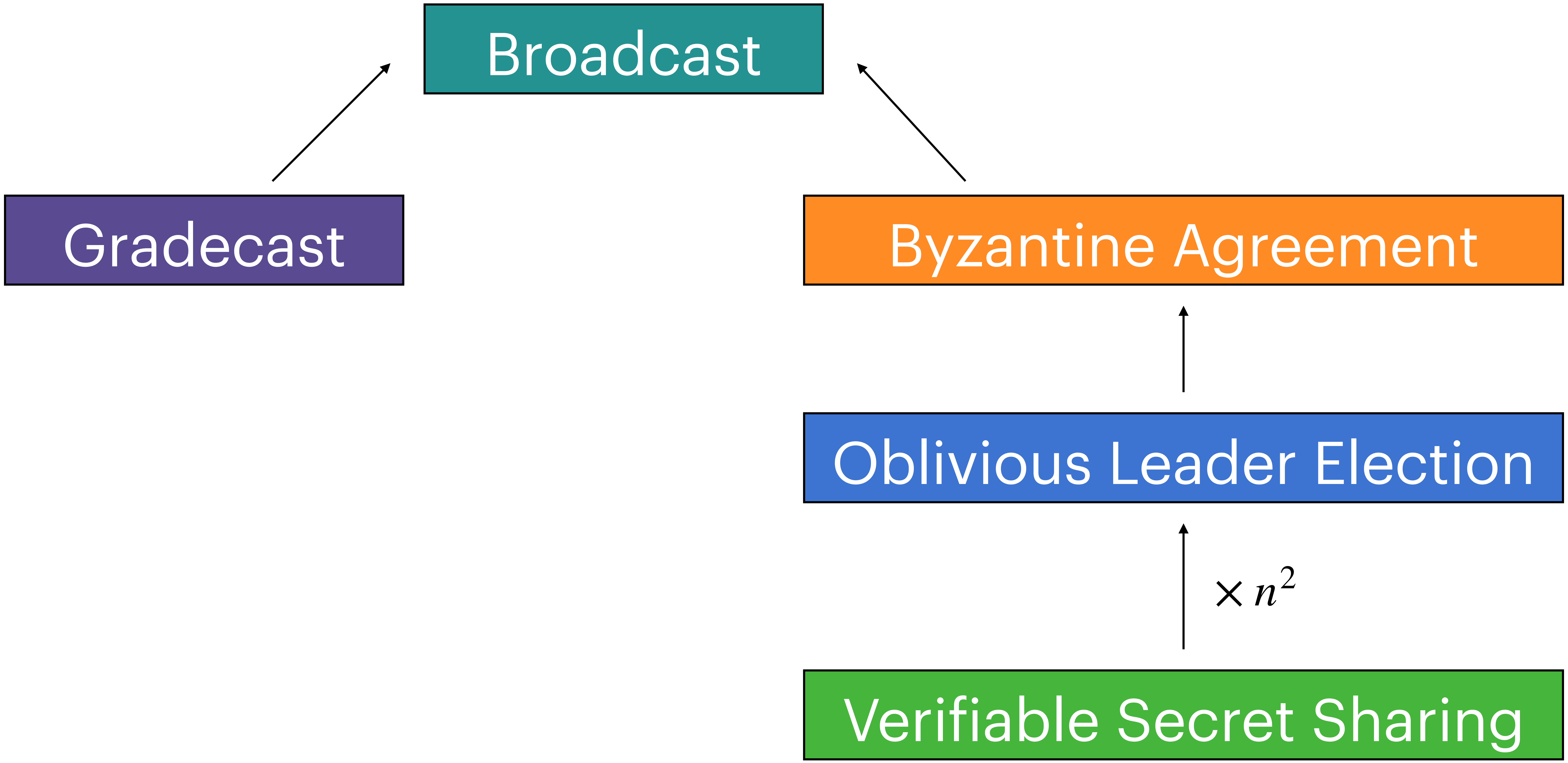
[KK06] Framework



[KK06] Framework



[KK06] Framework



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← Probability that OLE fails!

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✅ $\Pr[\text{Everyone agrees on honest leader}] \geq \frac{1}{2}$

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Statistical security suffices!

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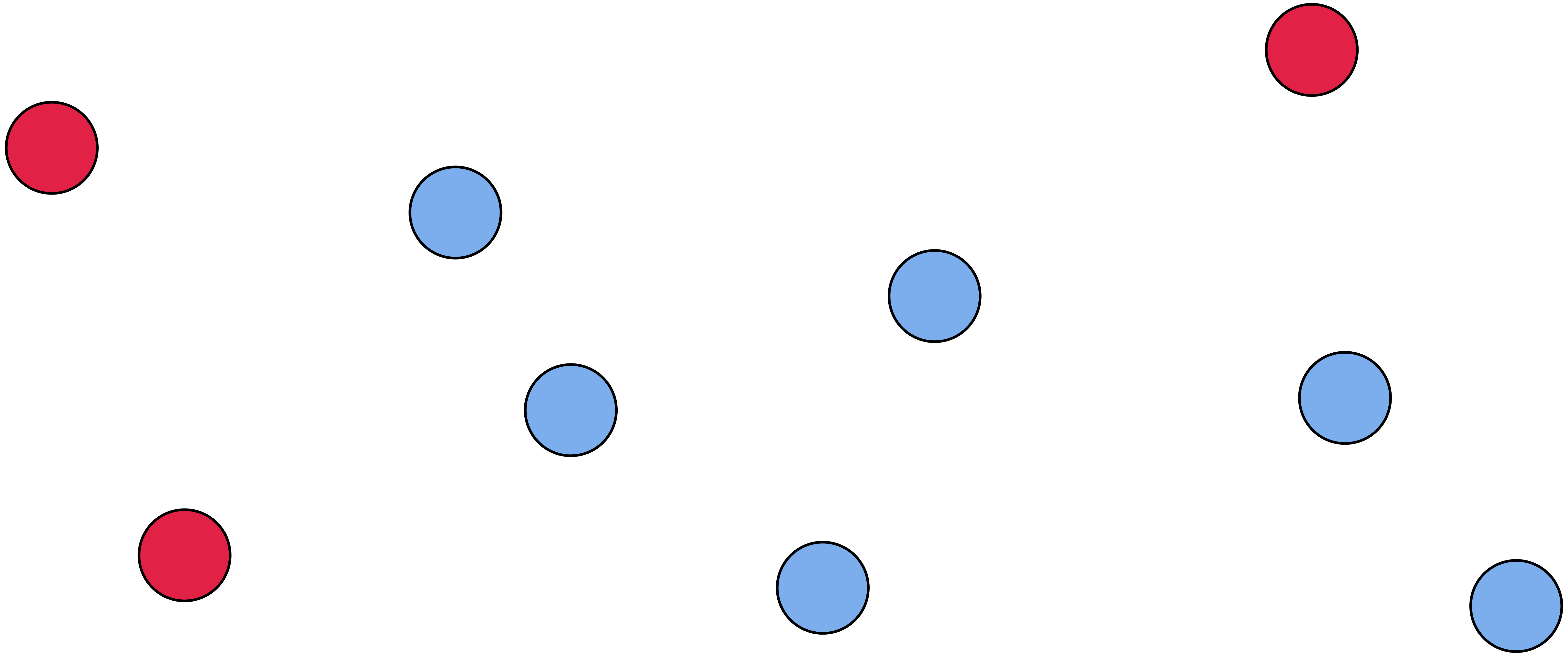
Statistical error

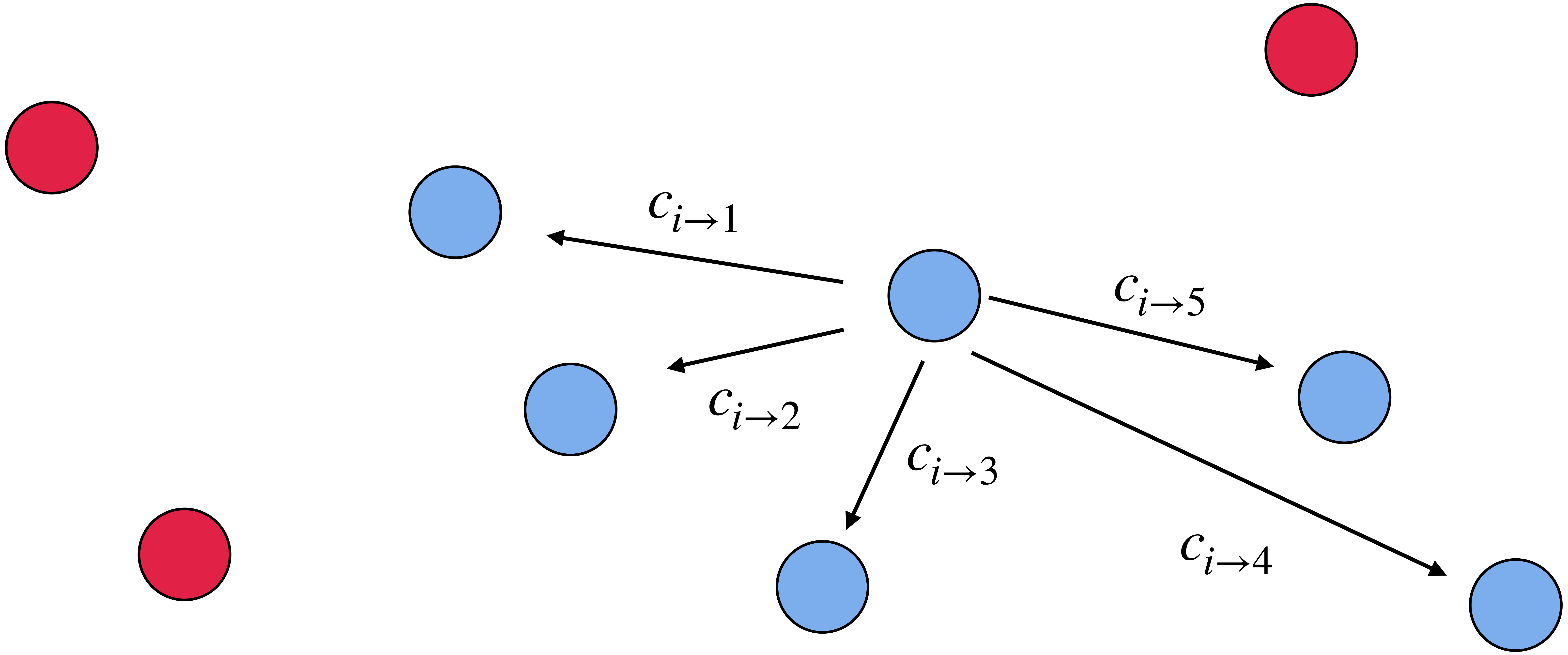
Statistical security suffices!

Leads to fewer secrets!

Reducing the # of Secrets

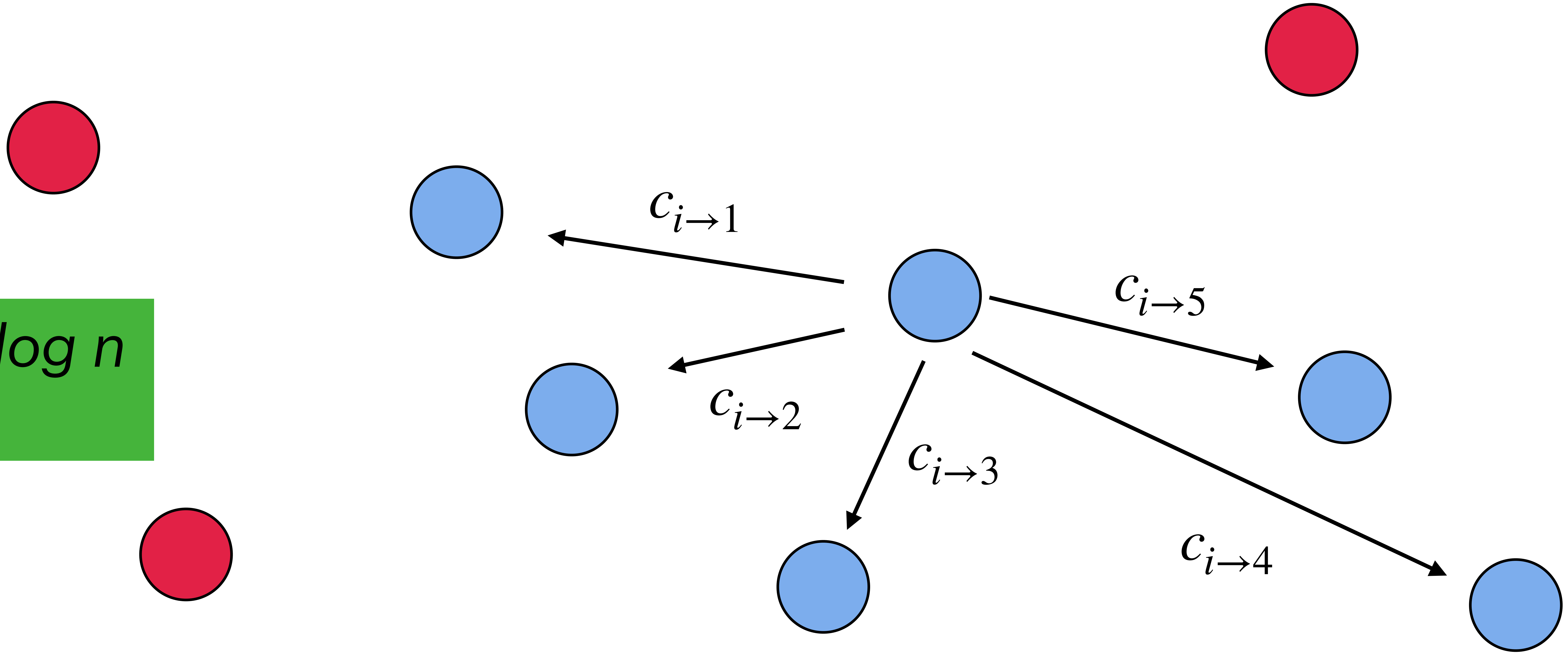


Reducing the # of Secrets



Reducing the # of Secrets

Contribute to *log n* parties!



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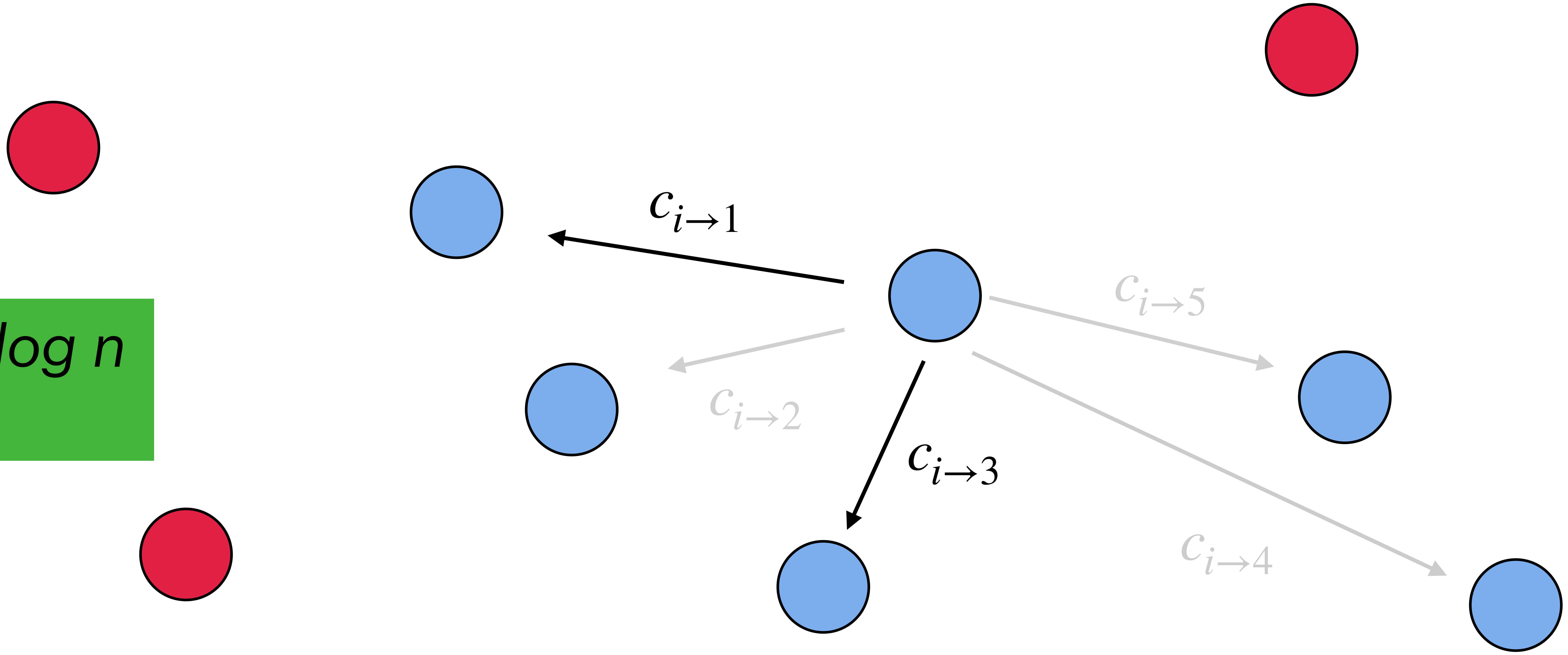
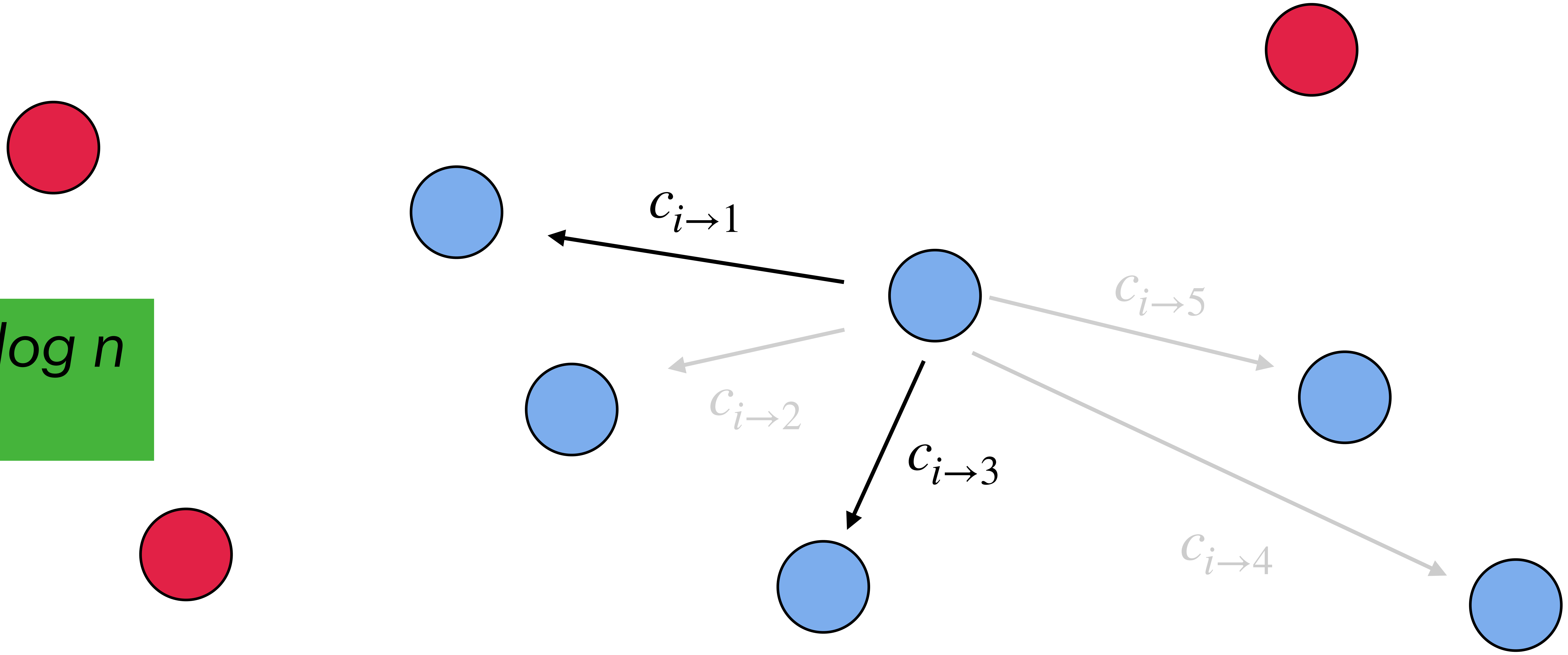


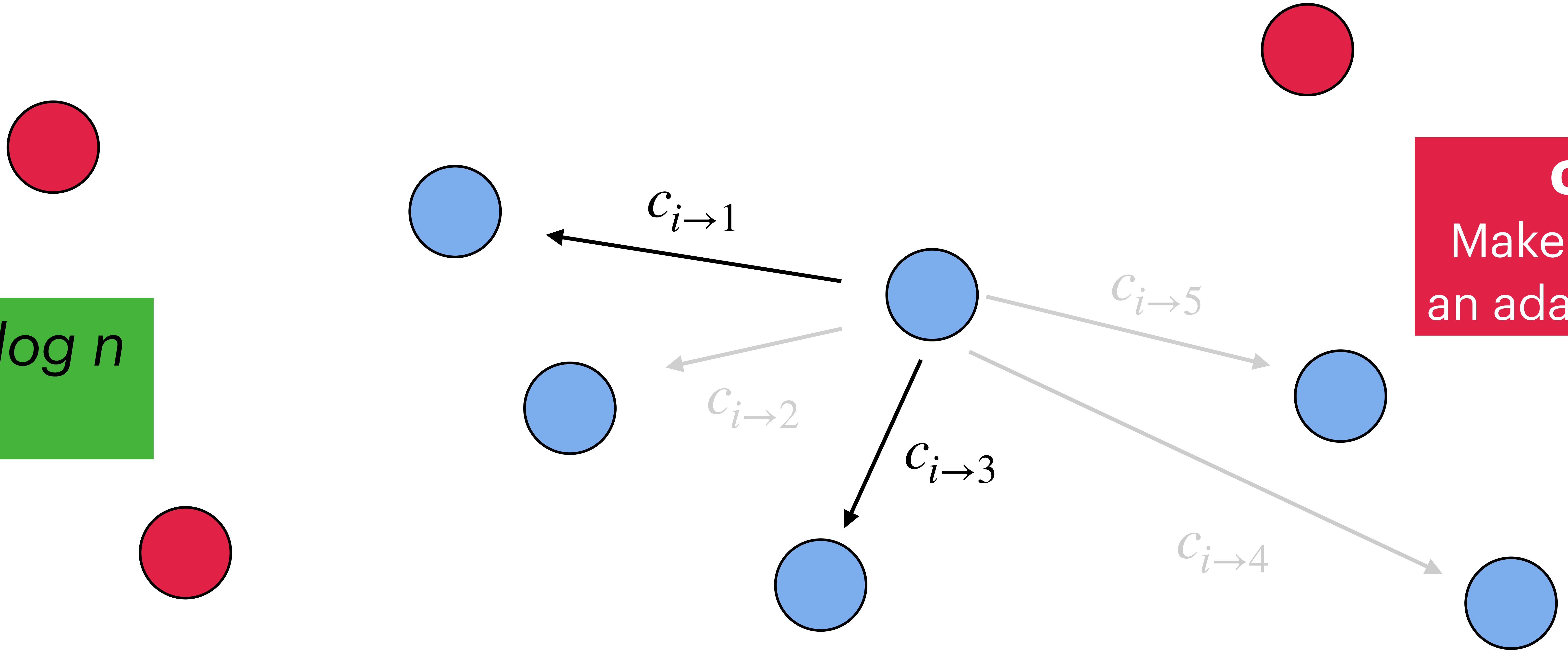
Reducing the # of Secrets



Contribute to *log n* parties!

We need: With high probability each party receives at least one honest contribution

Reducing the # of Secrets



Contribute to *log n* parties!

Challenge
Make it work against an adaptive adversary

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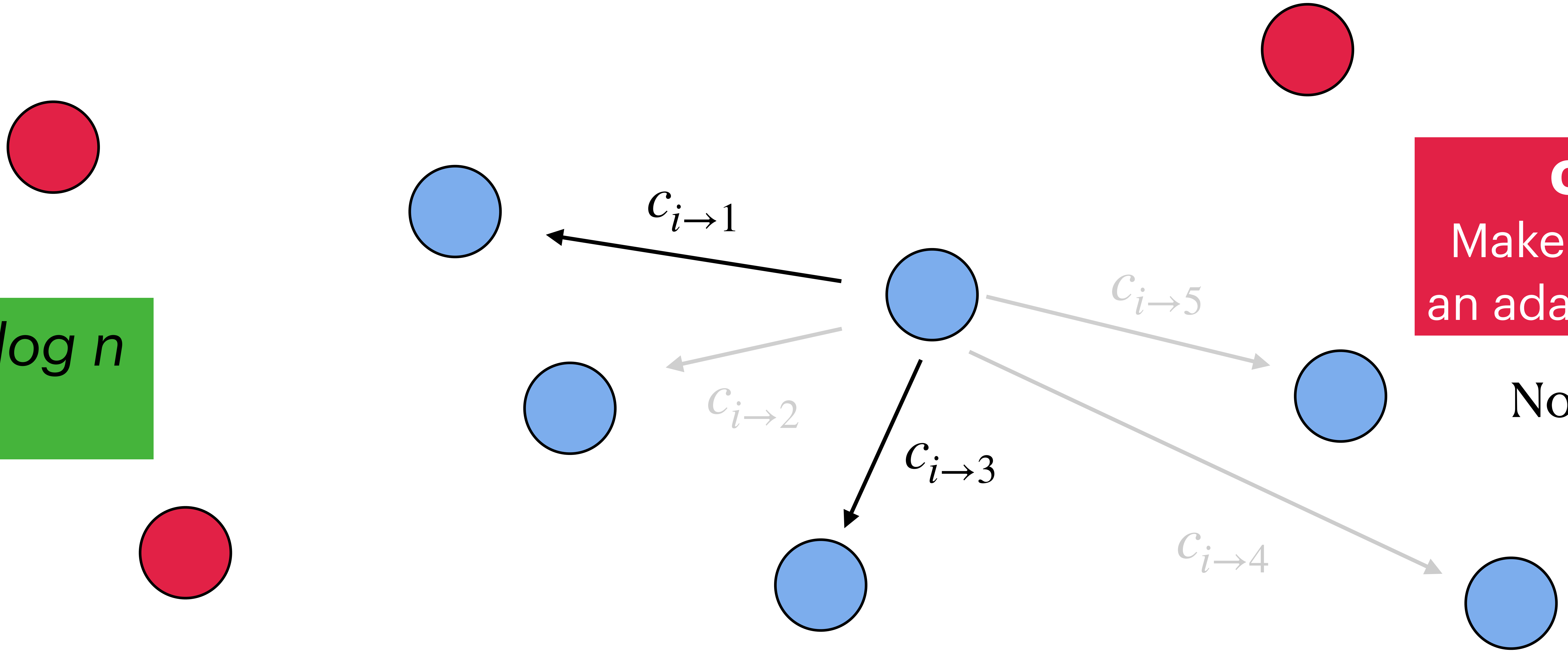
Reducing the # of Secrets



Contribute to *log n* parties!

Challenge
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Not in this talk

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Probability Analysis

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Each party contributes to *log n* parties chosen uniformly at random

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Probability that *j* does not pick *i* once

Probability Analysis

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For *log n* independent samples

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Probability that *j* does not pick *i* once

$\leq n \cdot e^{-\log n}$

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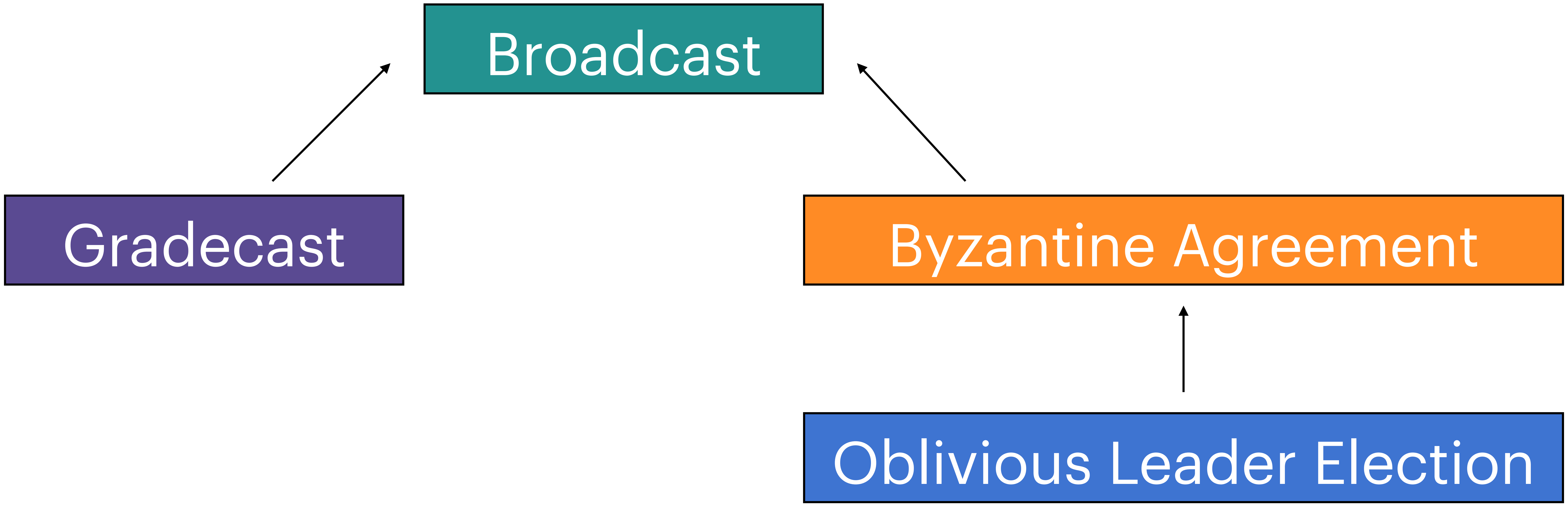
[KK06] Framework



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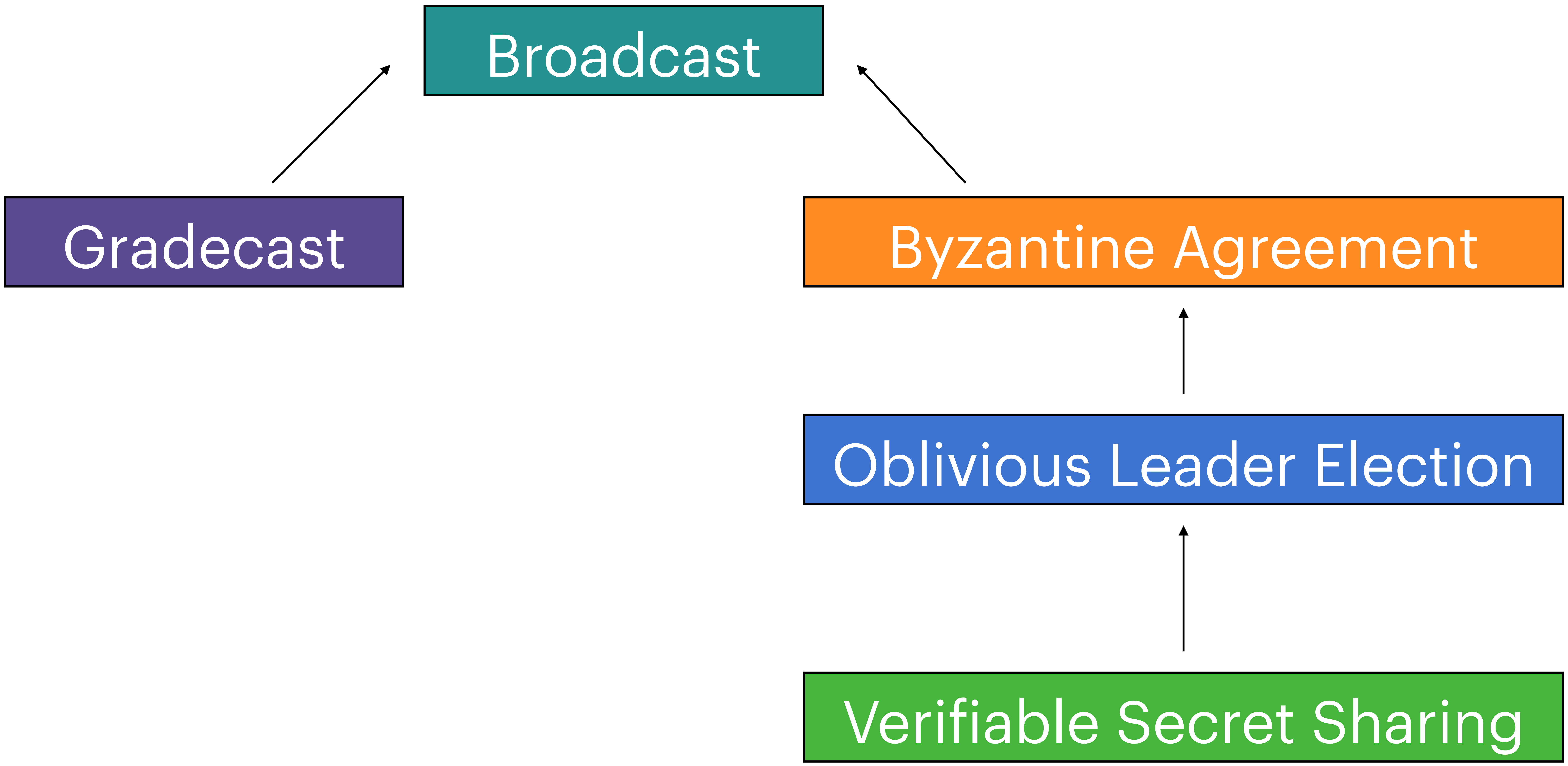

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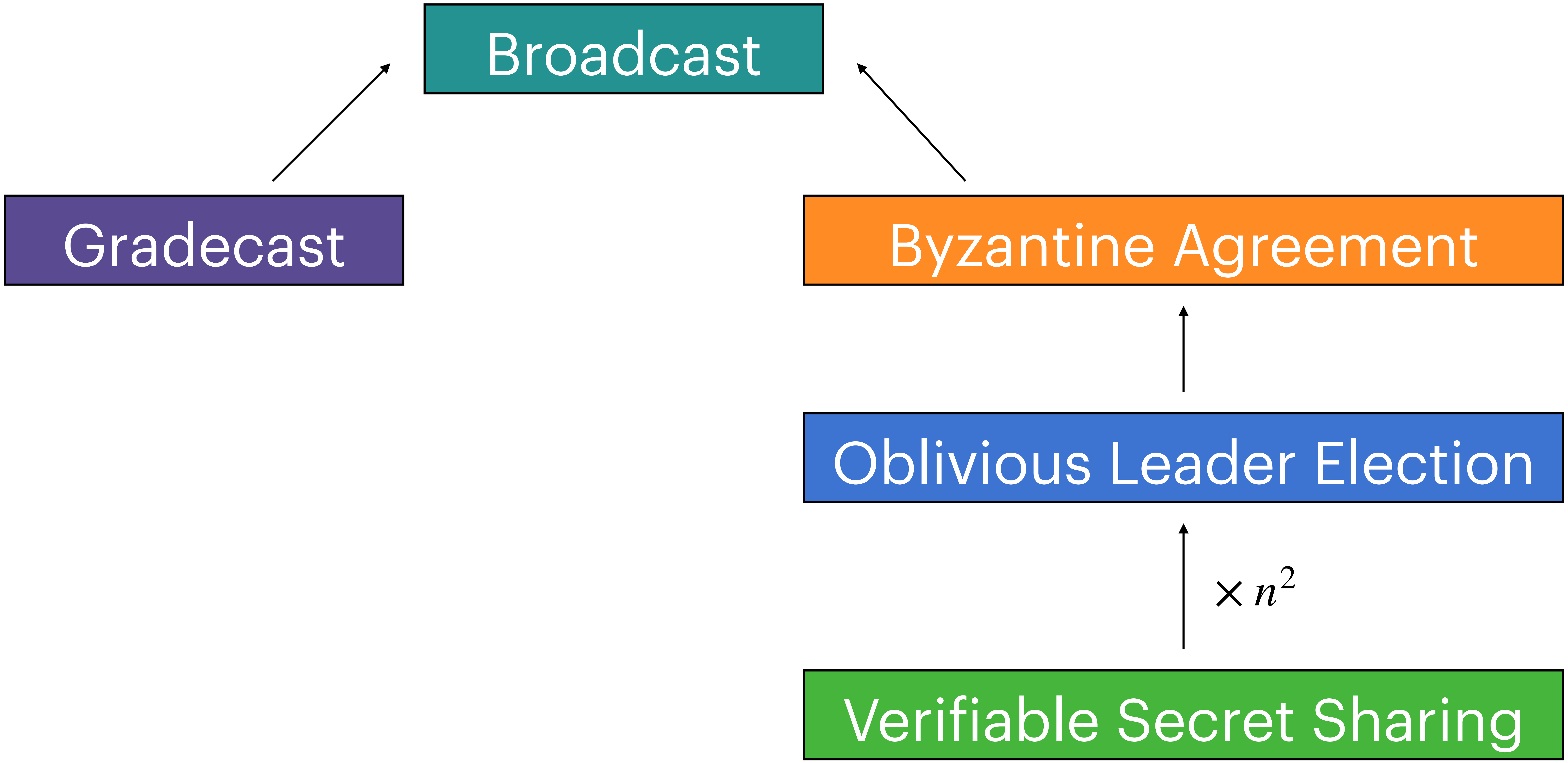

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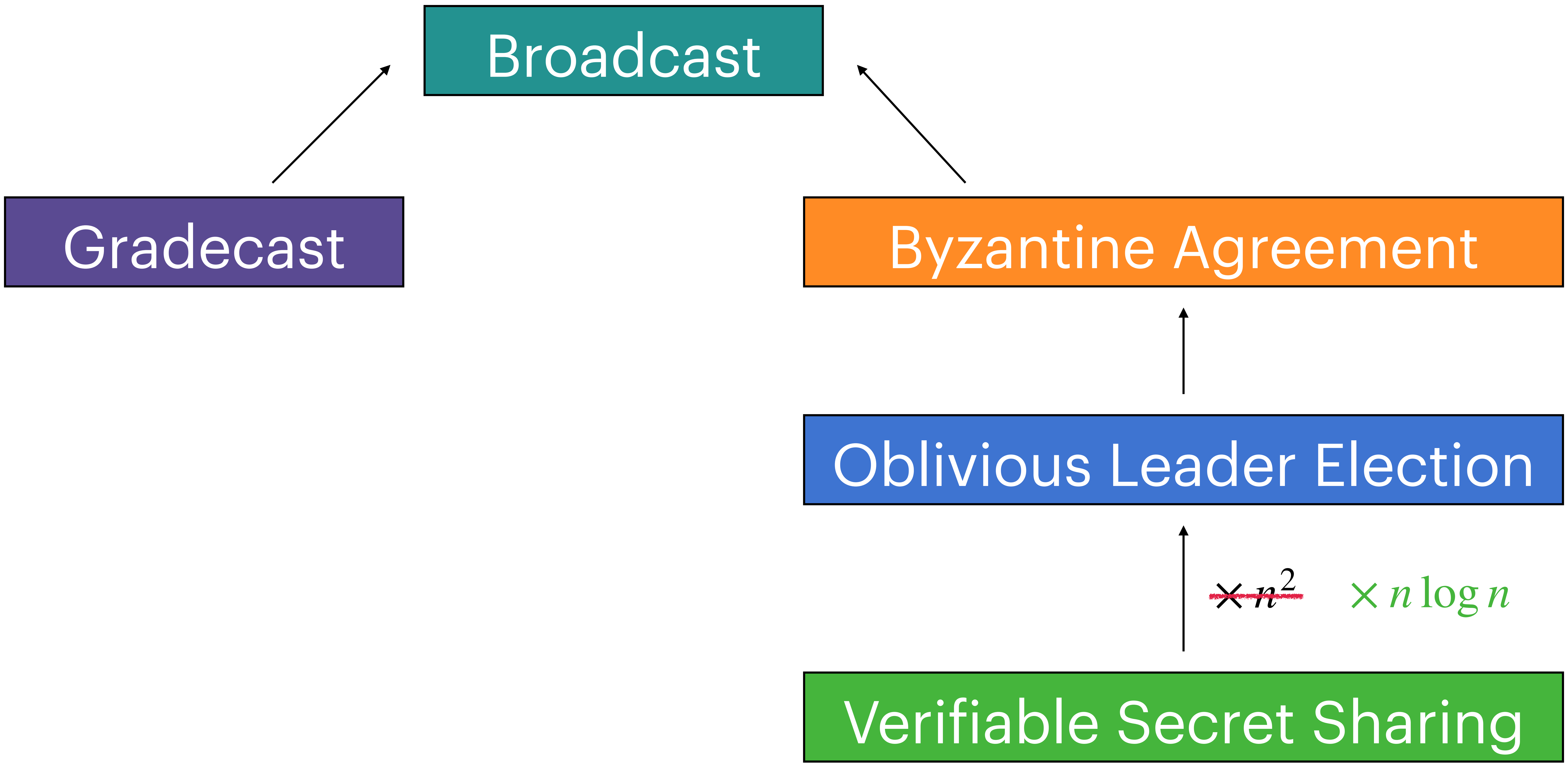

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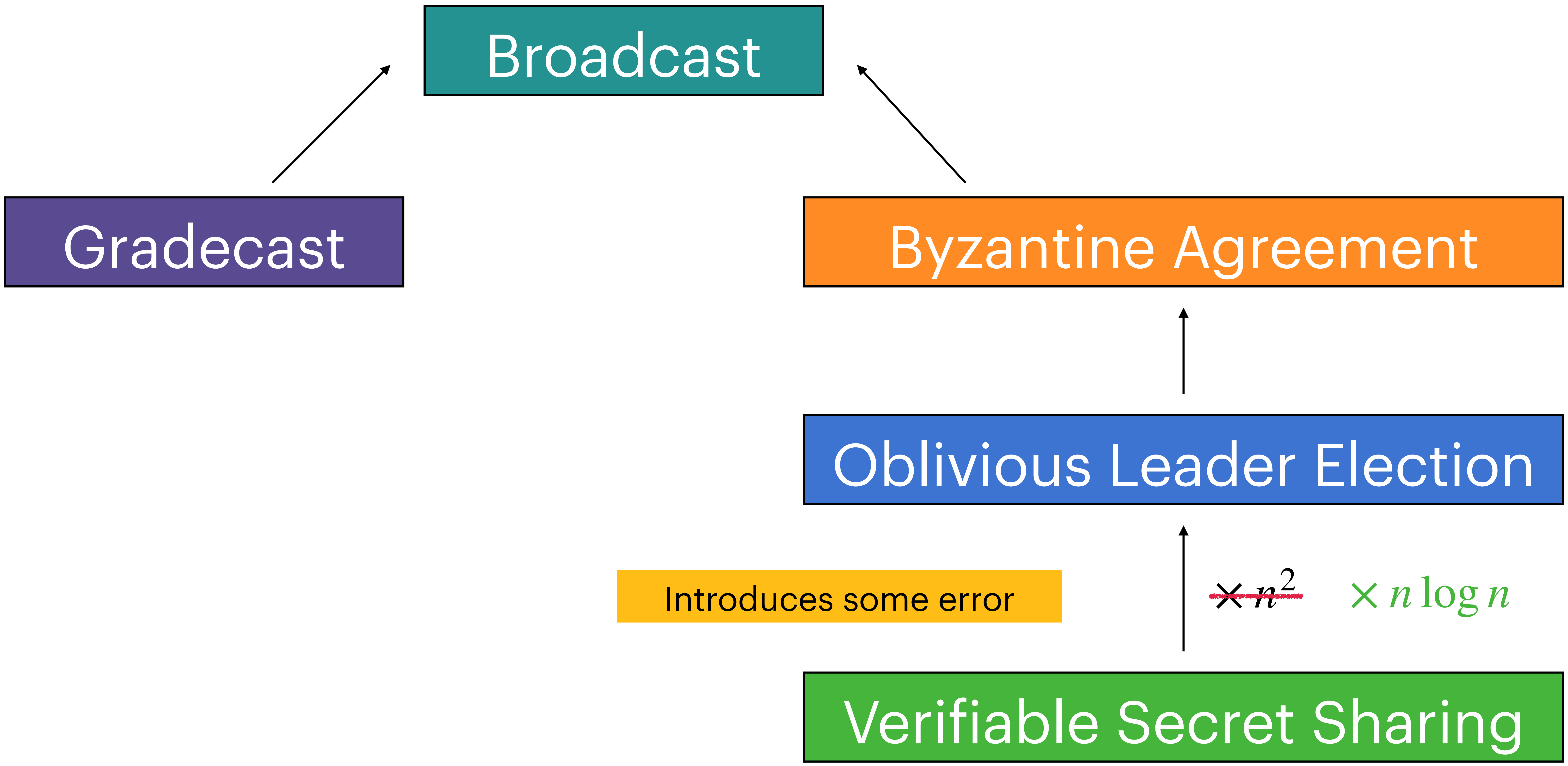


Contributions

- **Conceptual contributions:**
  - Statistical OLE suffices
- **Technical contributions:**
  - Statistical OLE with lesser secrets

Verifiable Secret Sharing

Verifiable Secret Sharing

Information Theoretic Commitments!

Verifiable Secret Sharing

Information Theoretic Commitments!

Dealer

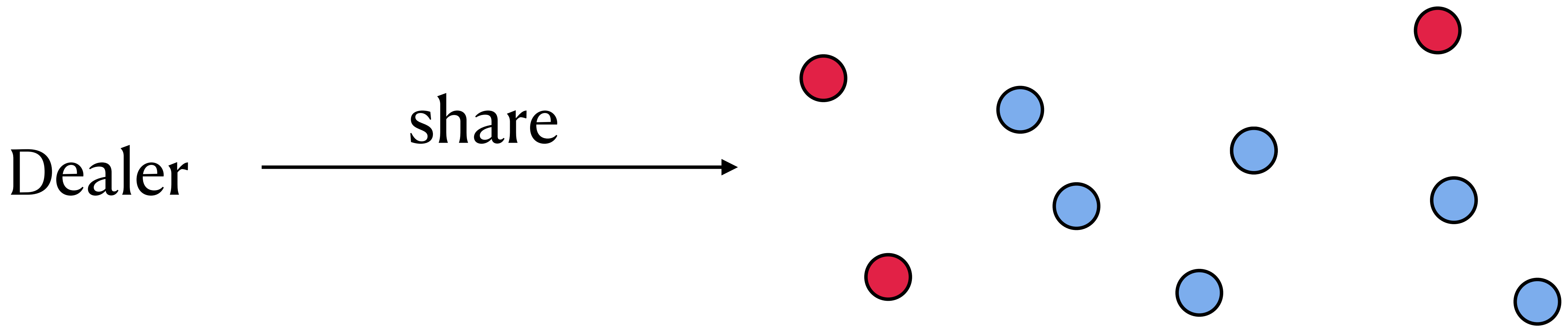
Verifiable Secret Sharing

Information Theoretic Commitments!

Dealer → share

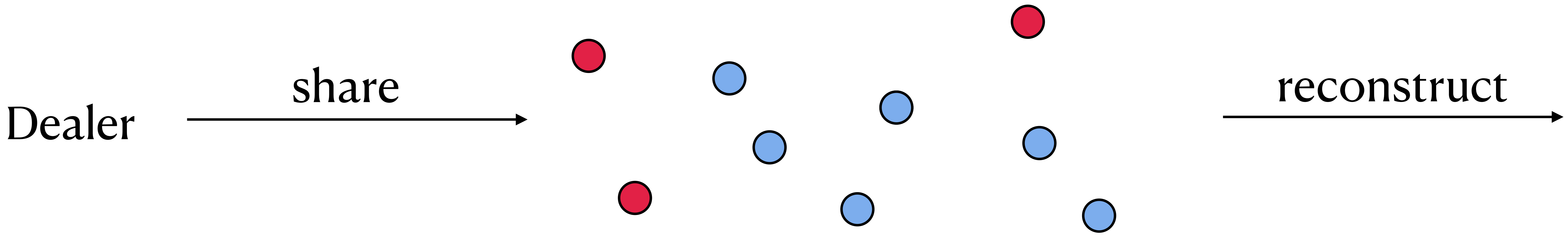
Verifiable Secret Sharing

Information Theoretic Commitments!



Verifiable Secret Sharing

Information Theoretic Commitments!



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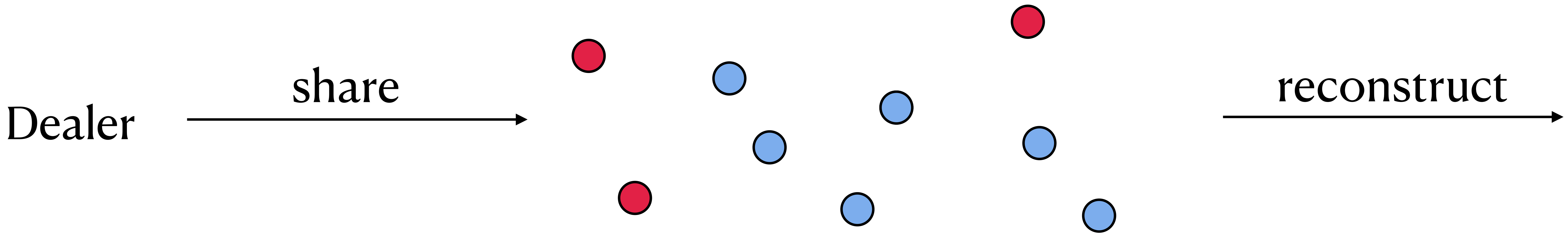
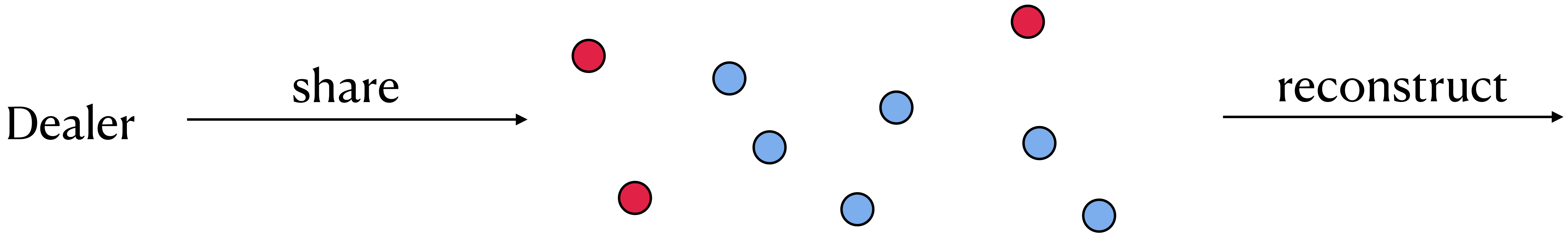


- Designated dealer can "*share*" a secret *s* among *n* parties
- **Honest dealer:** *s* is private and reconstruction succeeds
- **Corrupt dealer:** Some *s'* is defined and reconstruction succeeds

Verifiable Secret Sharing

Information Theoretic Commitments!



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Verifiable Secret Sharing

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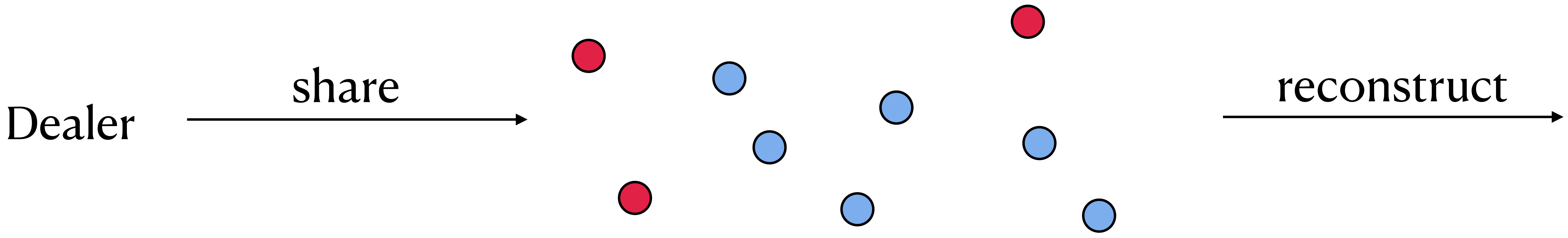

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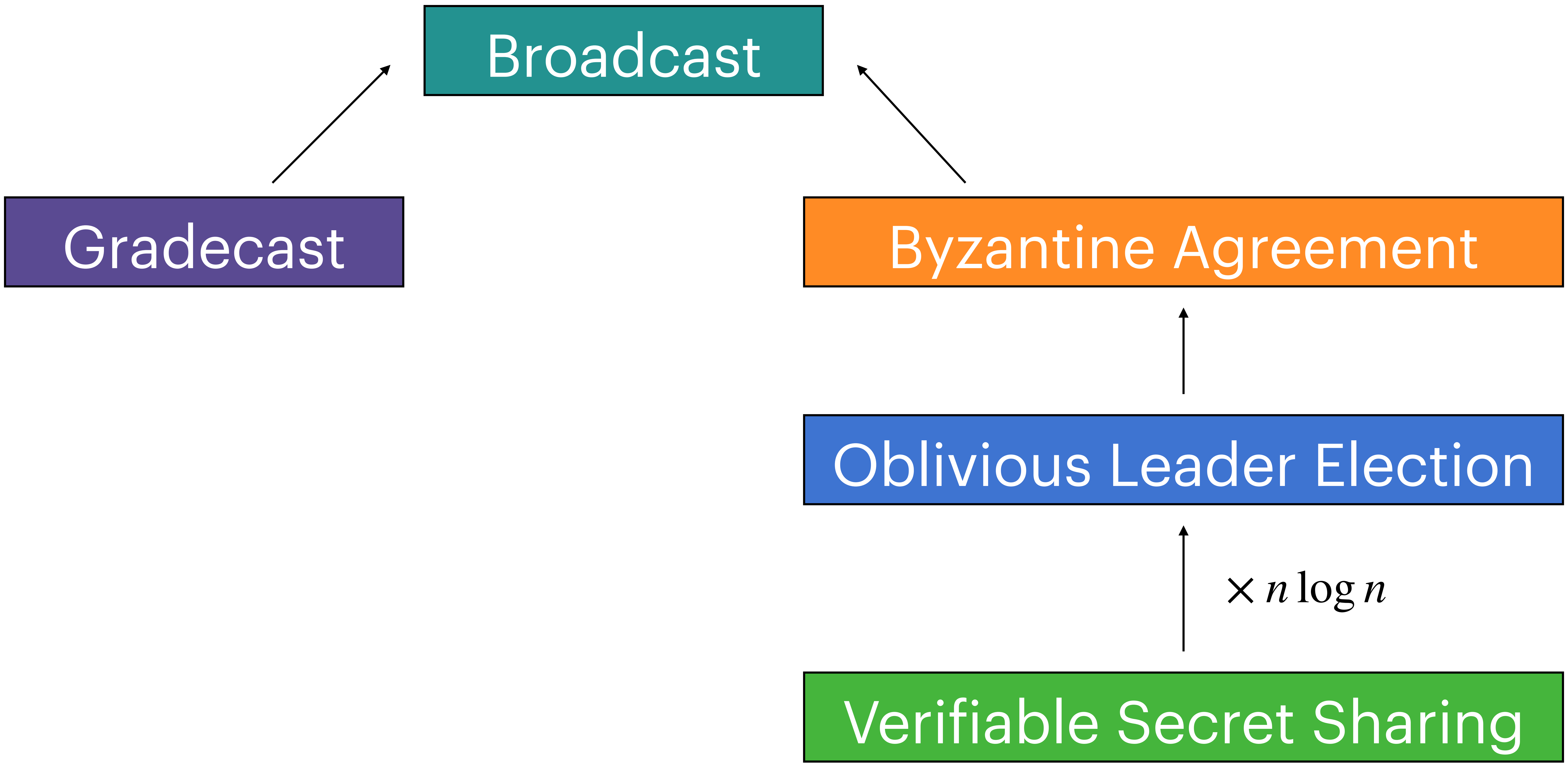
[KK06] Framework



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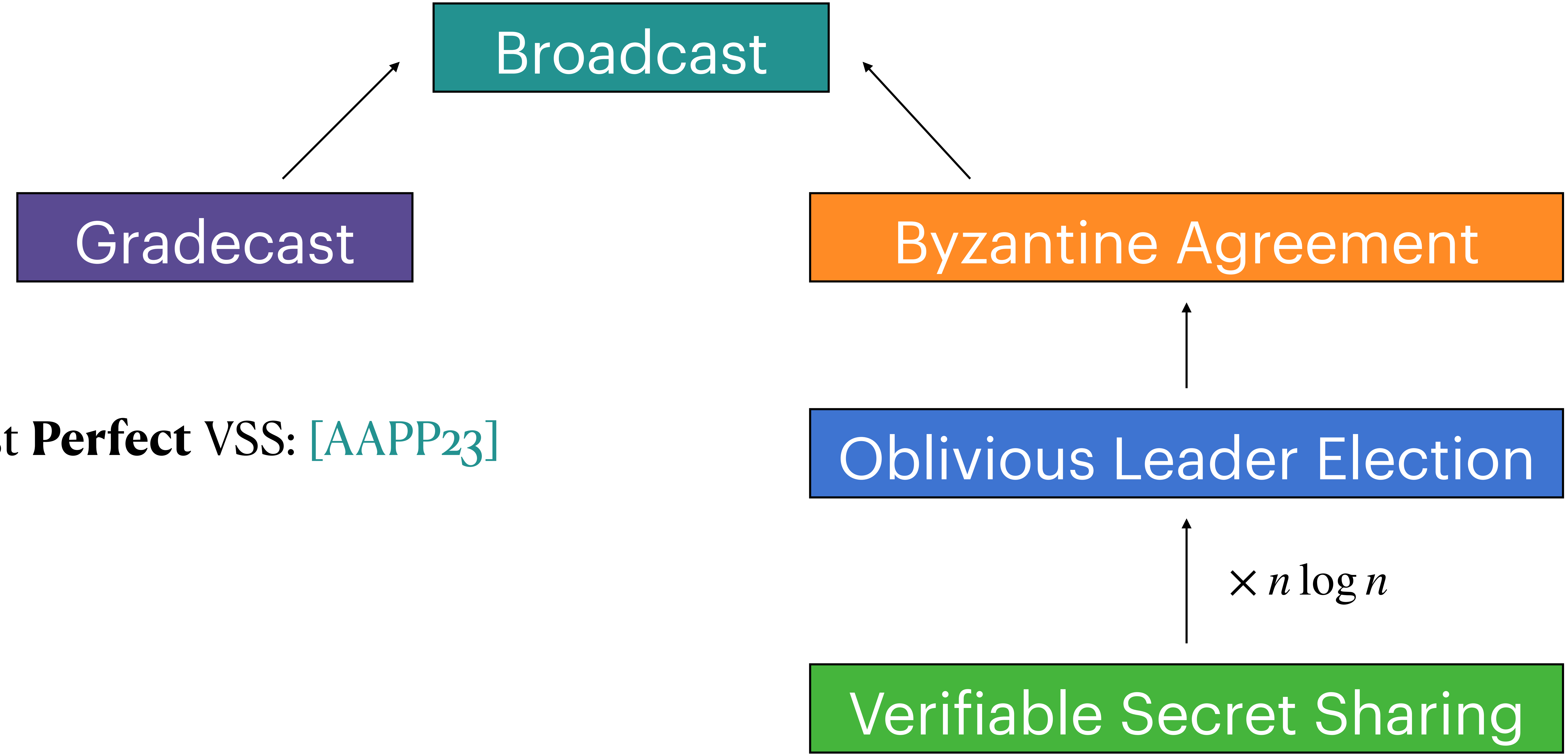
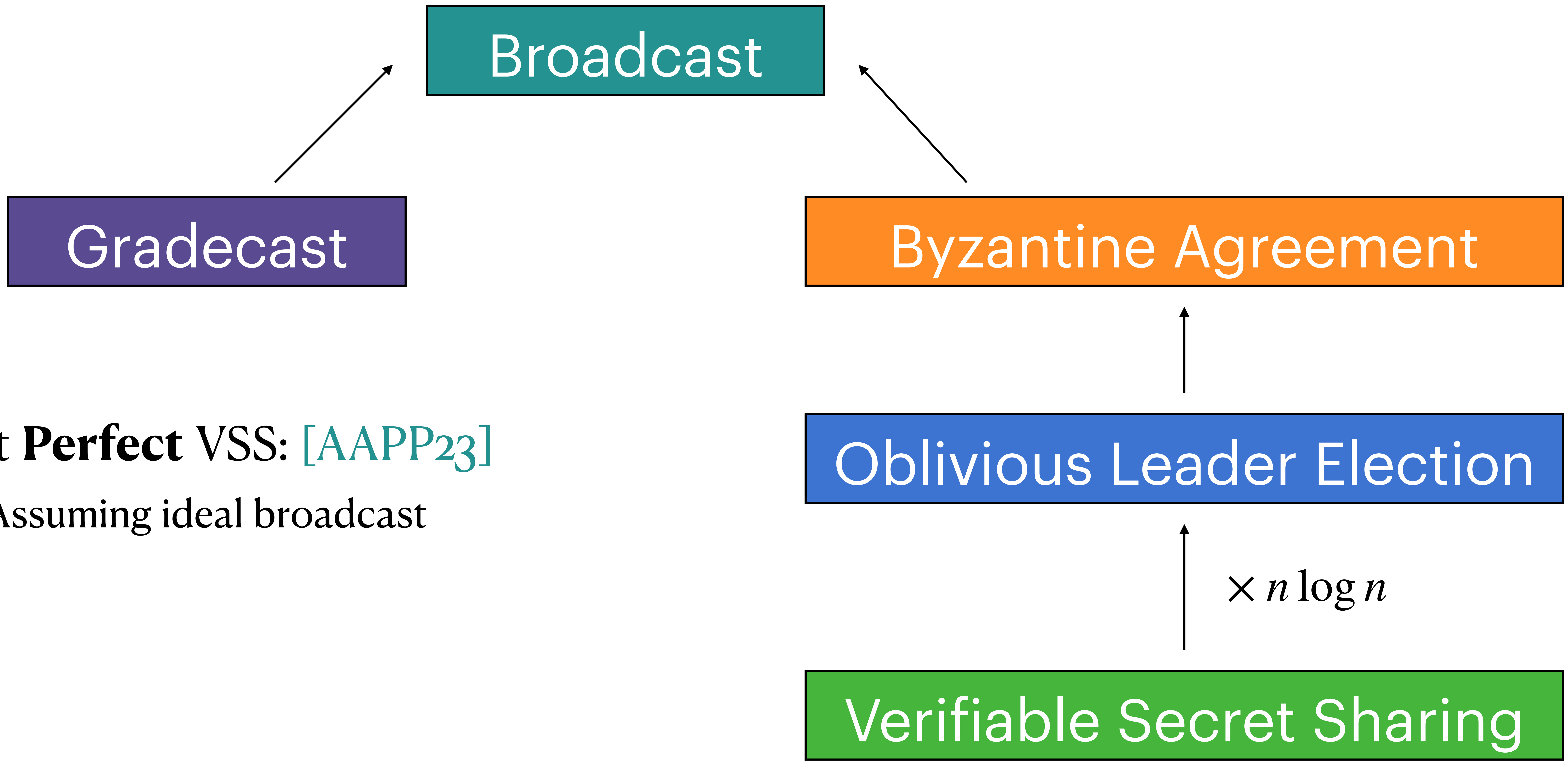


Best **Perfect** VSS: [AAPP23]

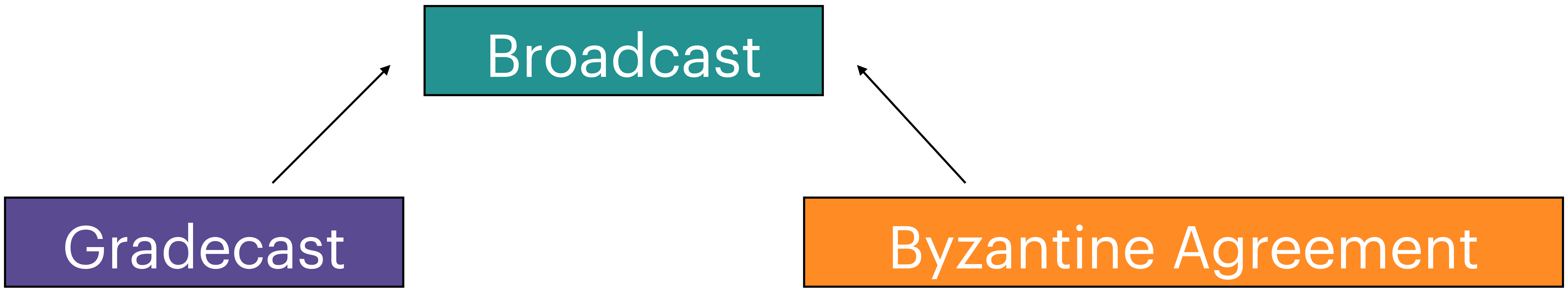
[KK06] Framework



Best **Perfect** VSS: [AAPP23]

Assuming ideal broadcast

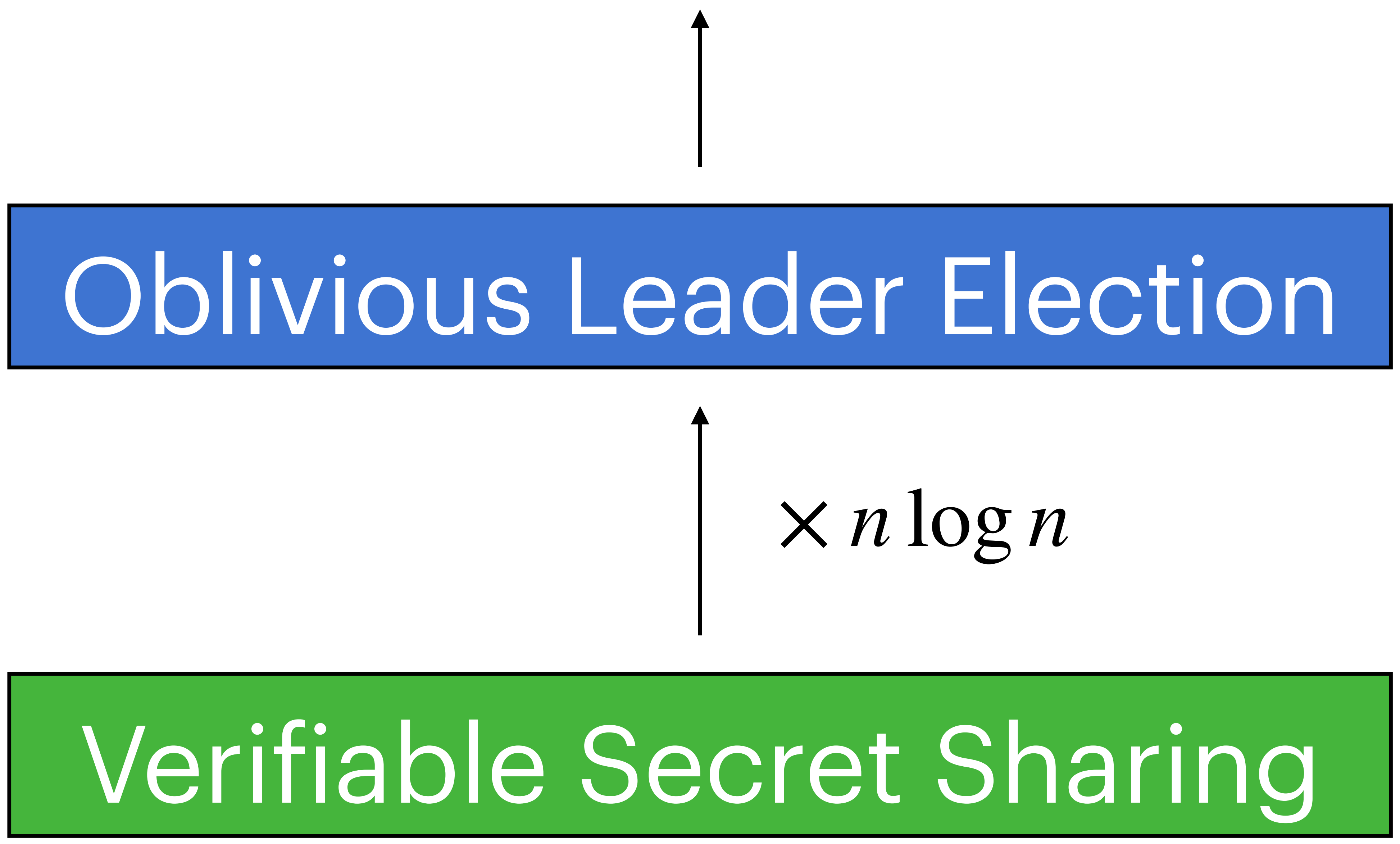
[KK06] Framework



Best **Perfect** VSS: [AAPP23]

Assuming ideal broadcast

$\tilde{O}(mn + n^3)$ for m secrets



[KK06] Framework

Broadcast

Gradecast

Byzantine Agreement

Oblivious Leader Election

$\times n \log n$

Verifiable Secret Sharing

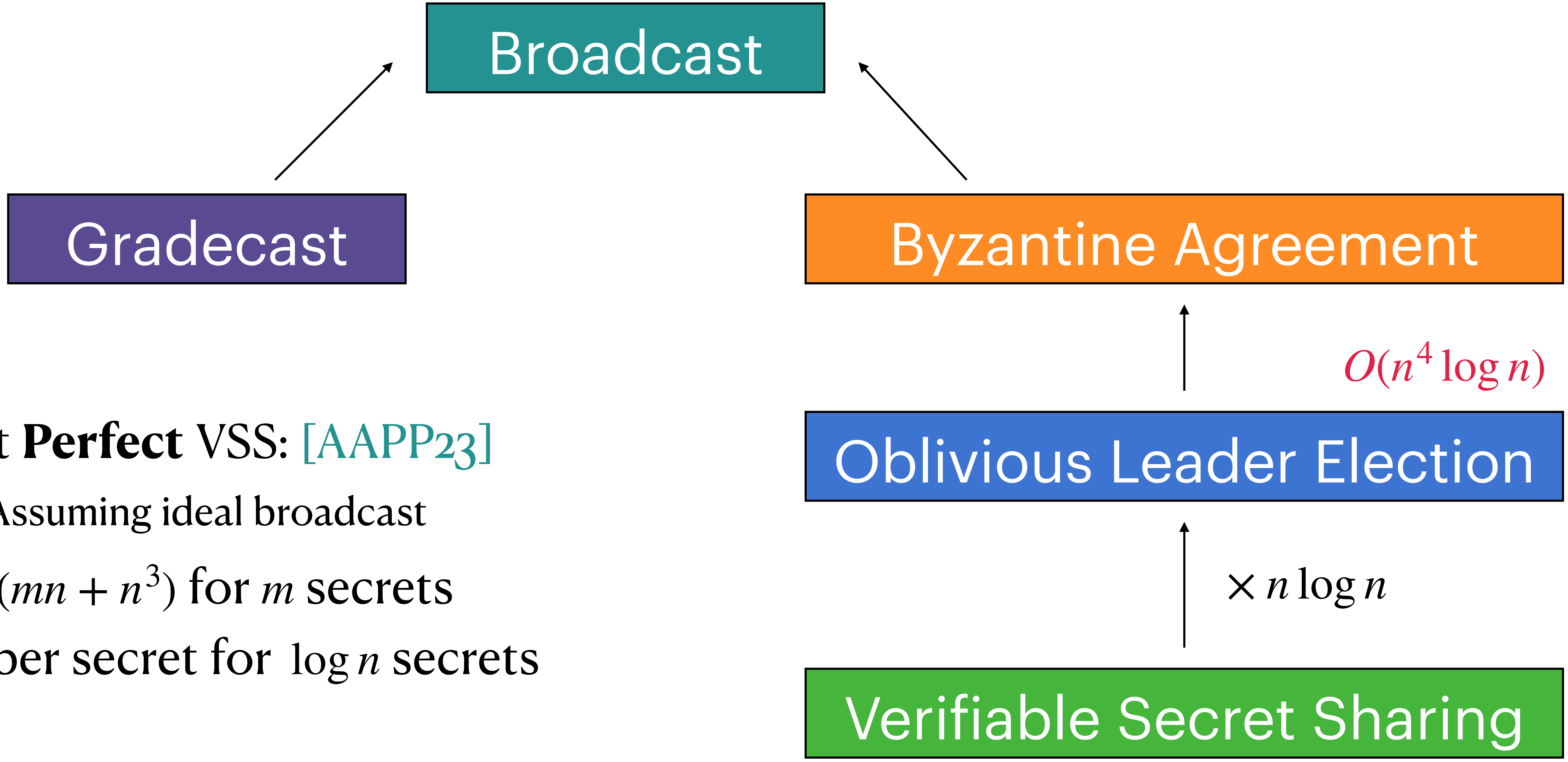
Best **Perfect** VSS: [AAPP23]

Assuming ideal broadcast

$\tilde{O}(mn + n^3)$ for m secrets

$O(n^3)$ per secret for $\log n$ secrets

[KK06] Framework



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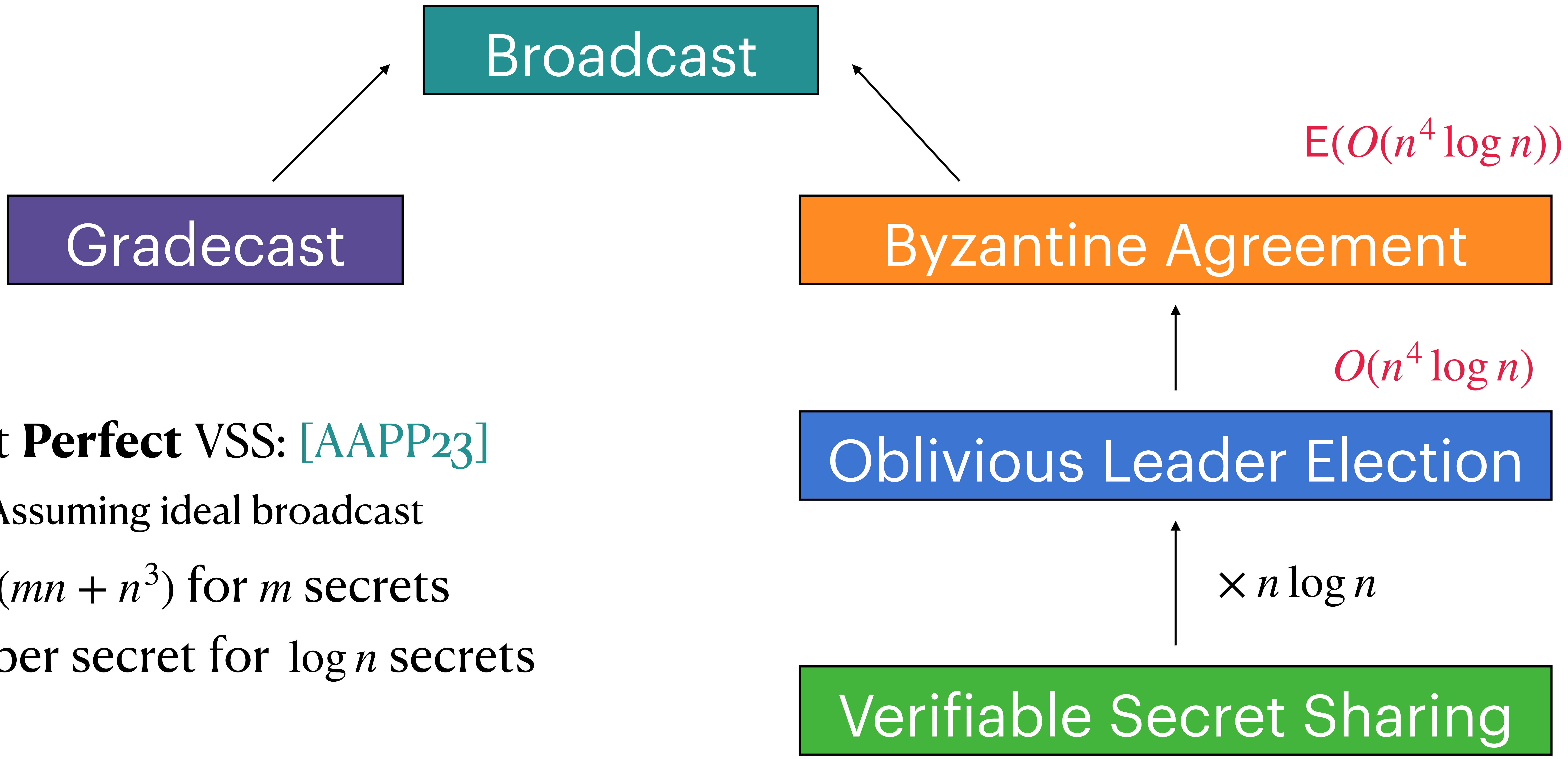

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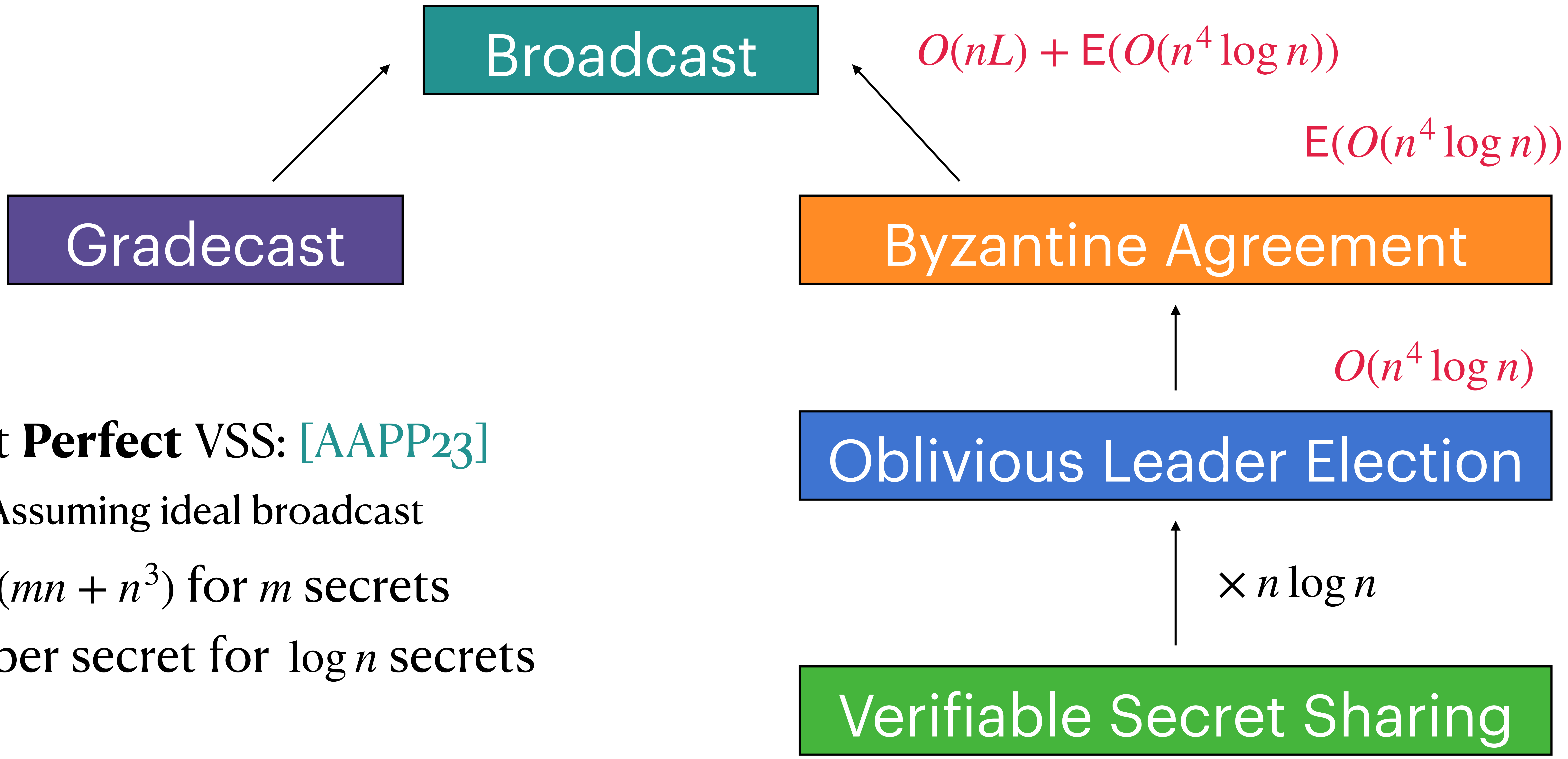

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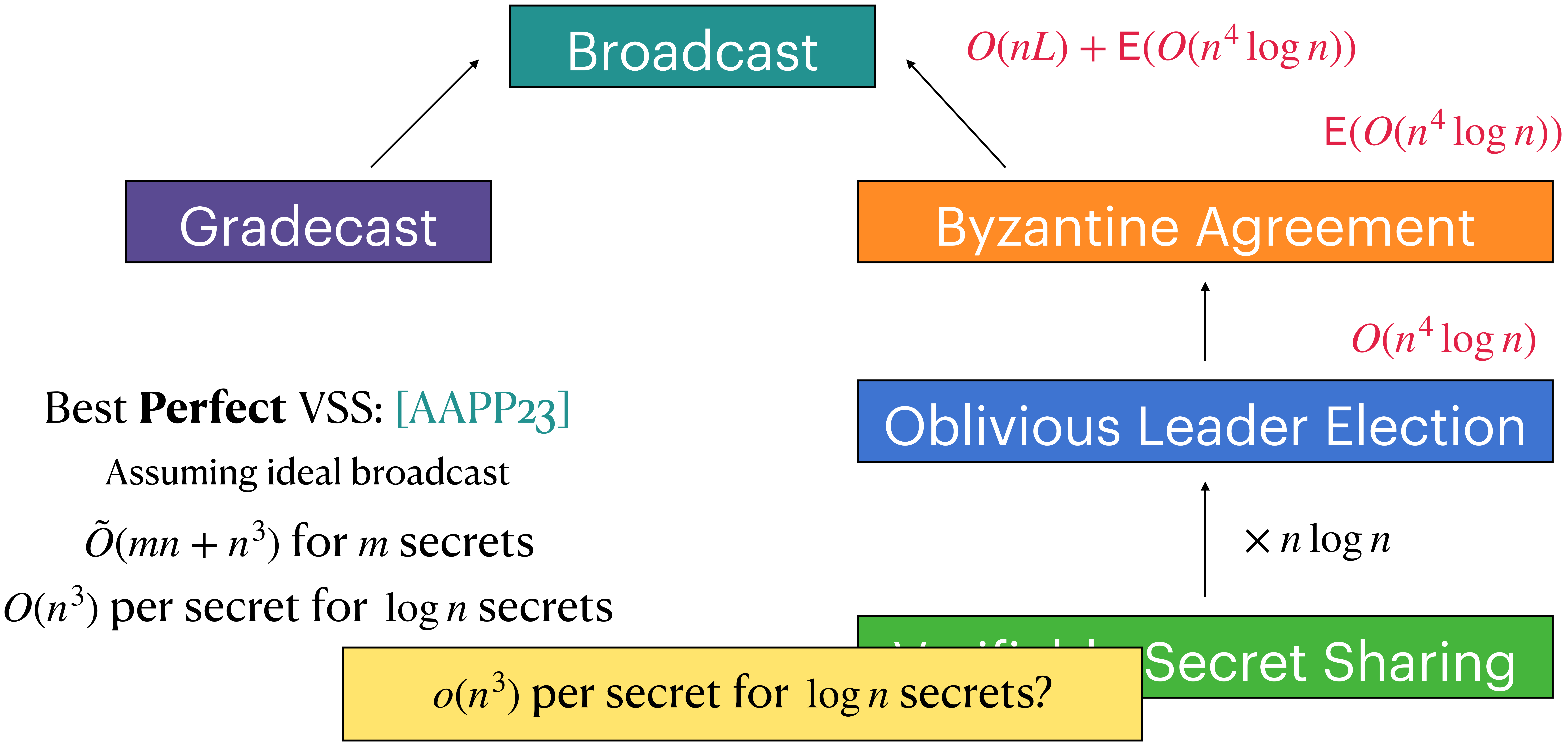

We're not done yet!

Why Perfect?

Why Perfect?

$$\Pr[\text{No agreement } \textbf{OR} \text{ corrupted leader } \textbf{OR} \text{ the VSS fails}] \leq \frac{1}{2}$$

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Best Perfect VSS for m secrets: [AAPP23]

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Best Perfect VSS for m secrets: [AAPP23]

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Our statistical VSS for m secrets with error ϵ

$\tilde{O}(mn^2 + n^2 \log(n/\epsilon))$

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$\tilde{O}(mn + n^3)$

Our statistical VSS for m secrets with error ϵ

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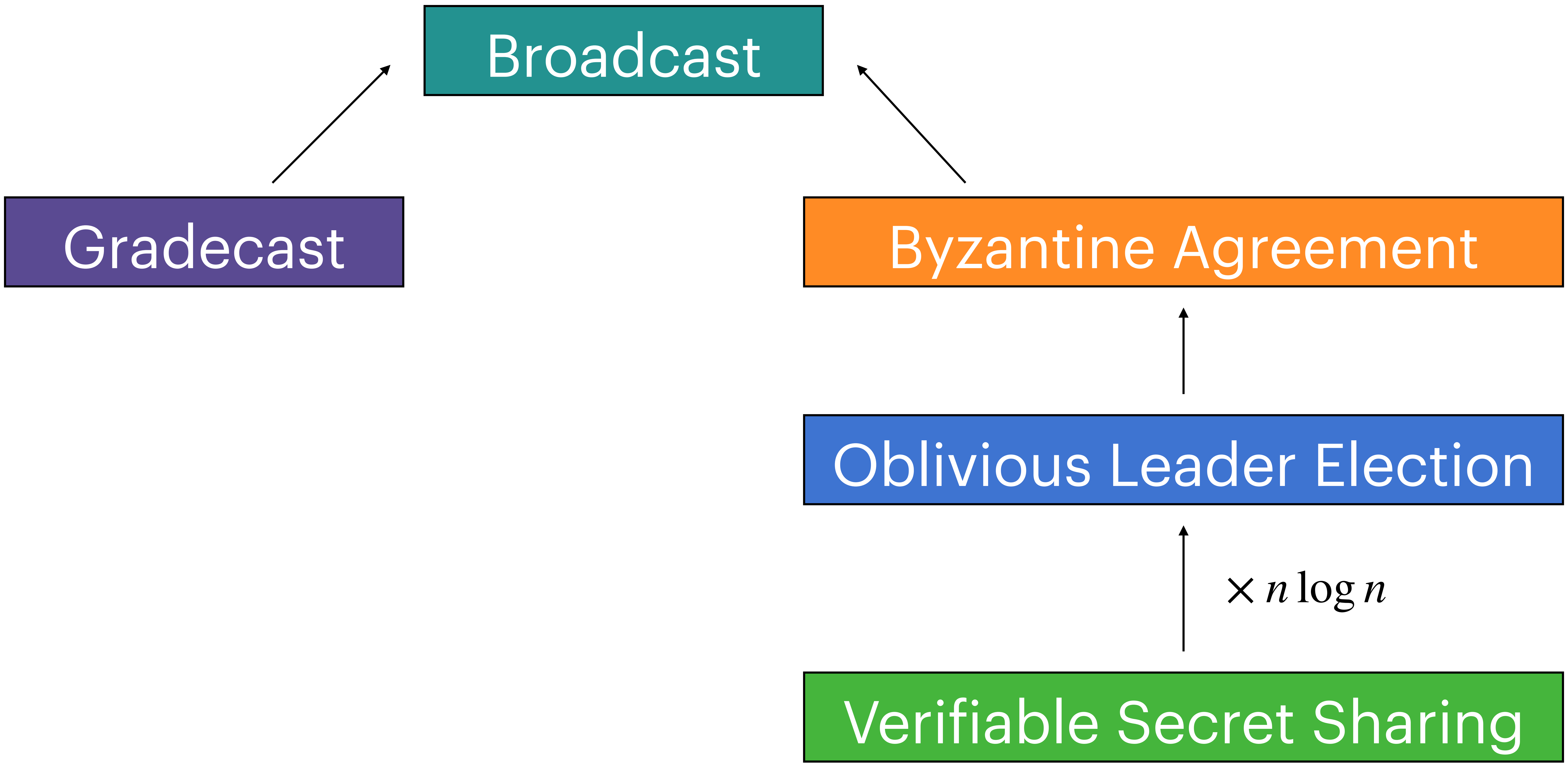
$$\epsilon = \frac{1}{\text{poly } n} \text{ suffices!}$$

[KK06] Framework



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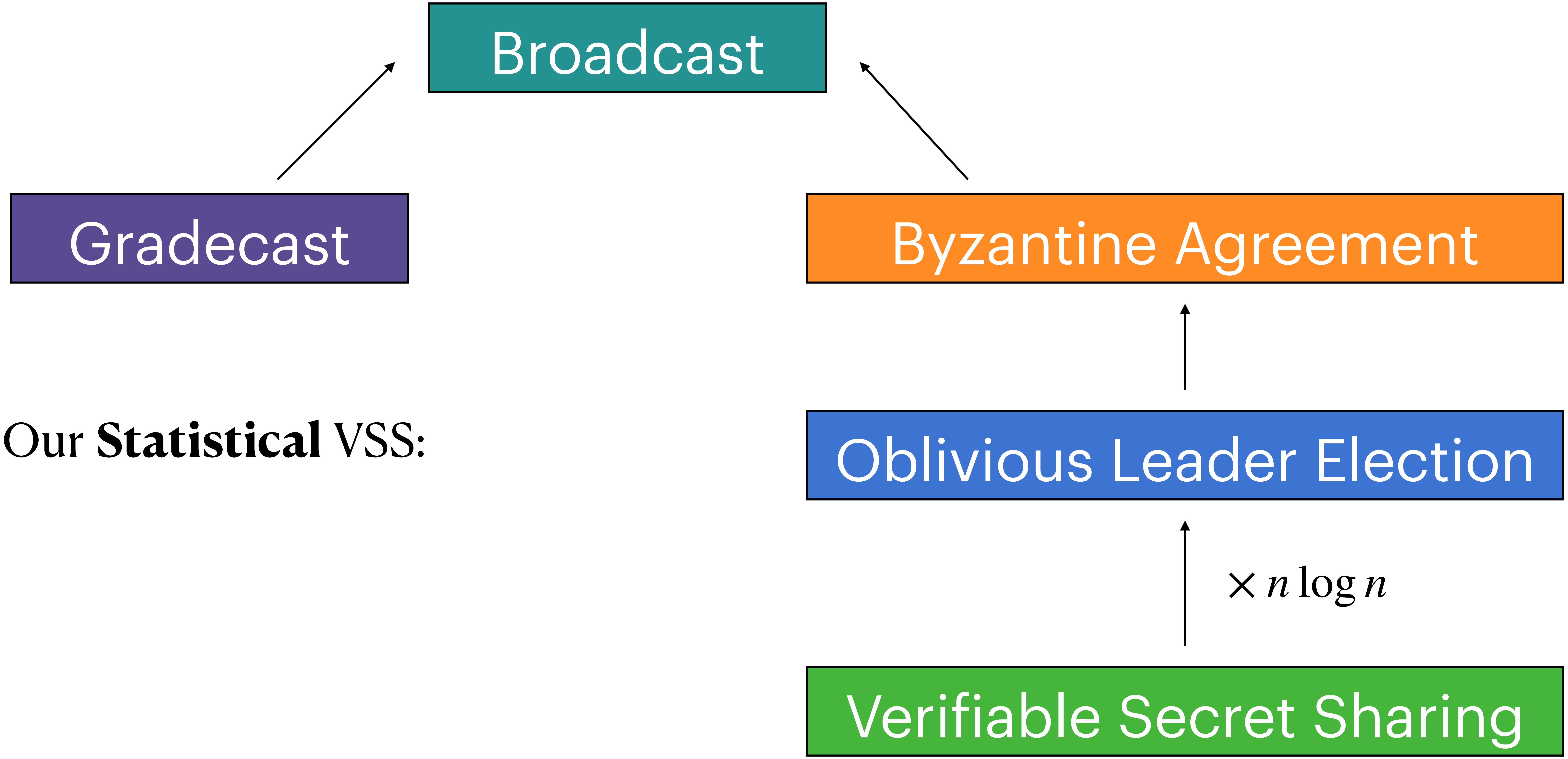
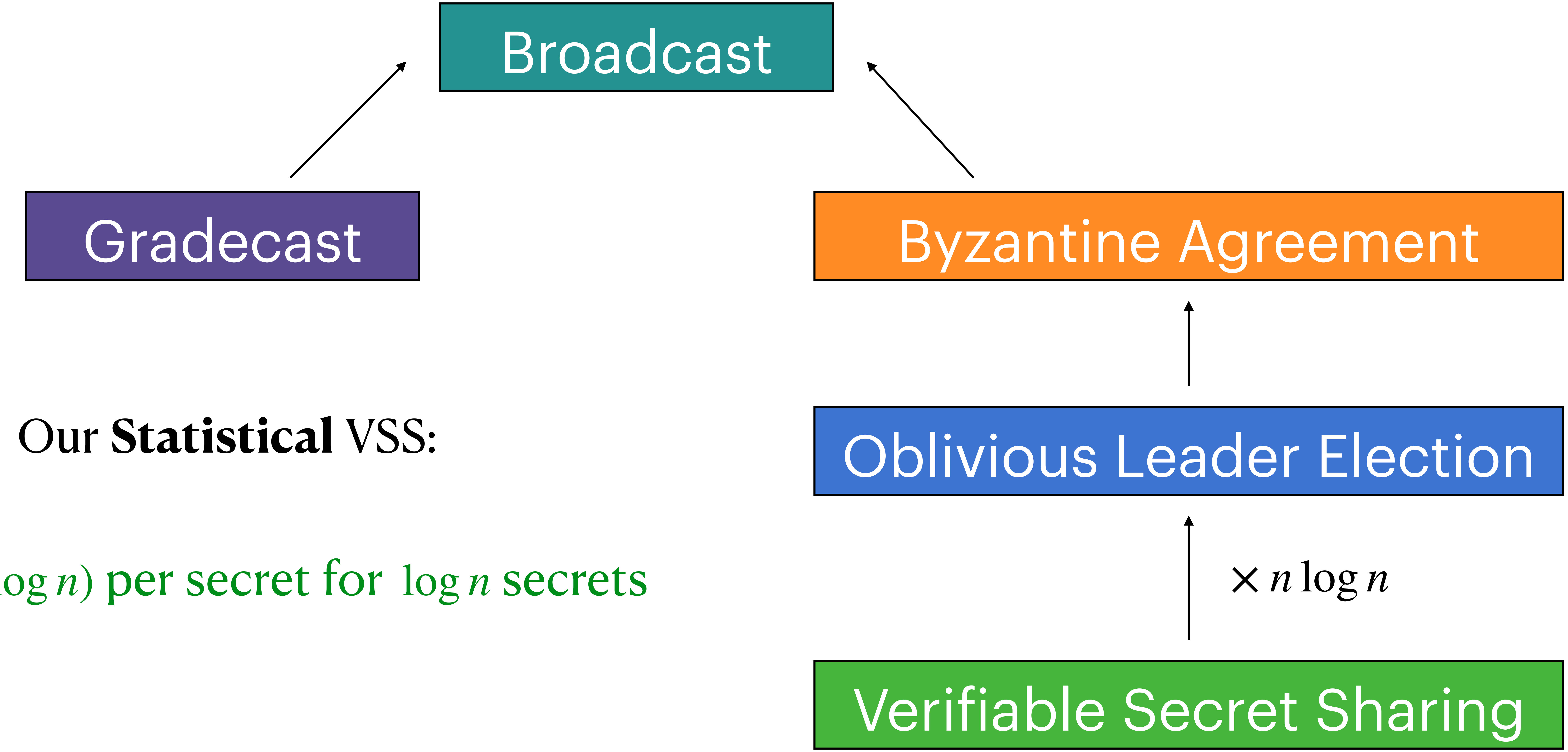


Our **Statistical** VSS:

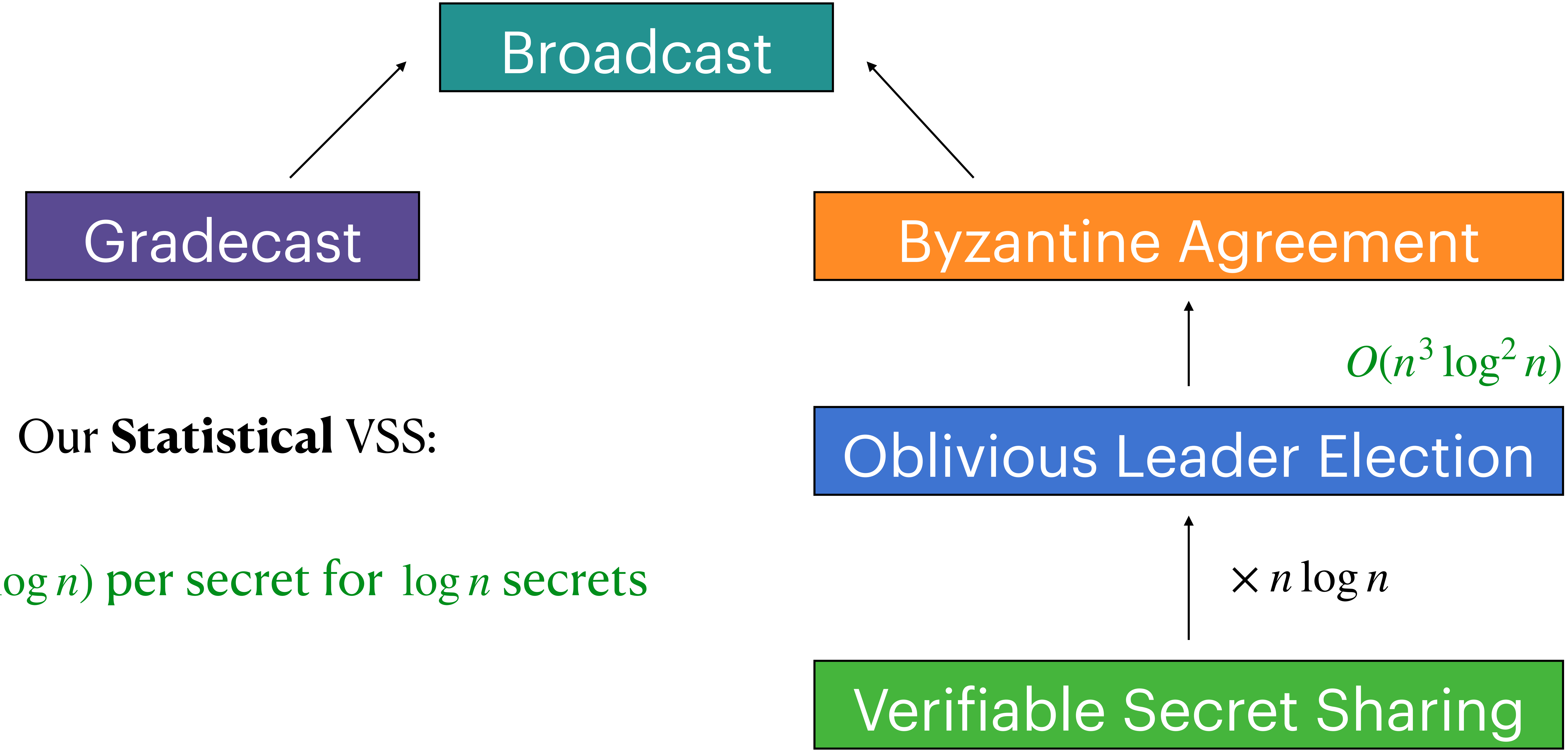
[KK06] Framework



Our **Statistical** VSS:

$O(n^2 \log n)$ per secret for $\log n$ secrets

[KK06] Framework



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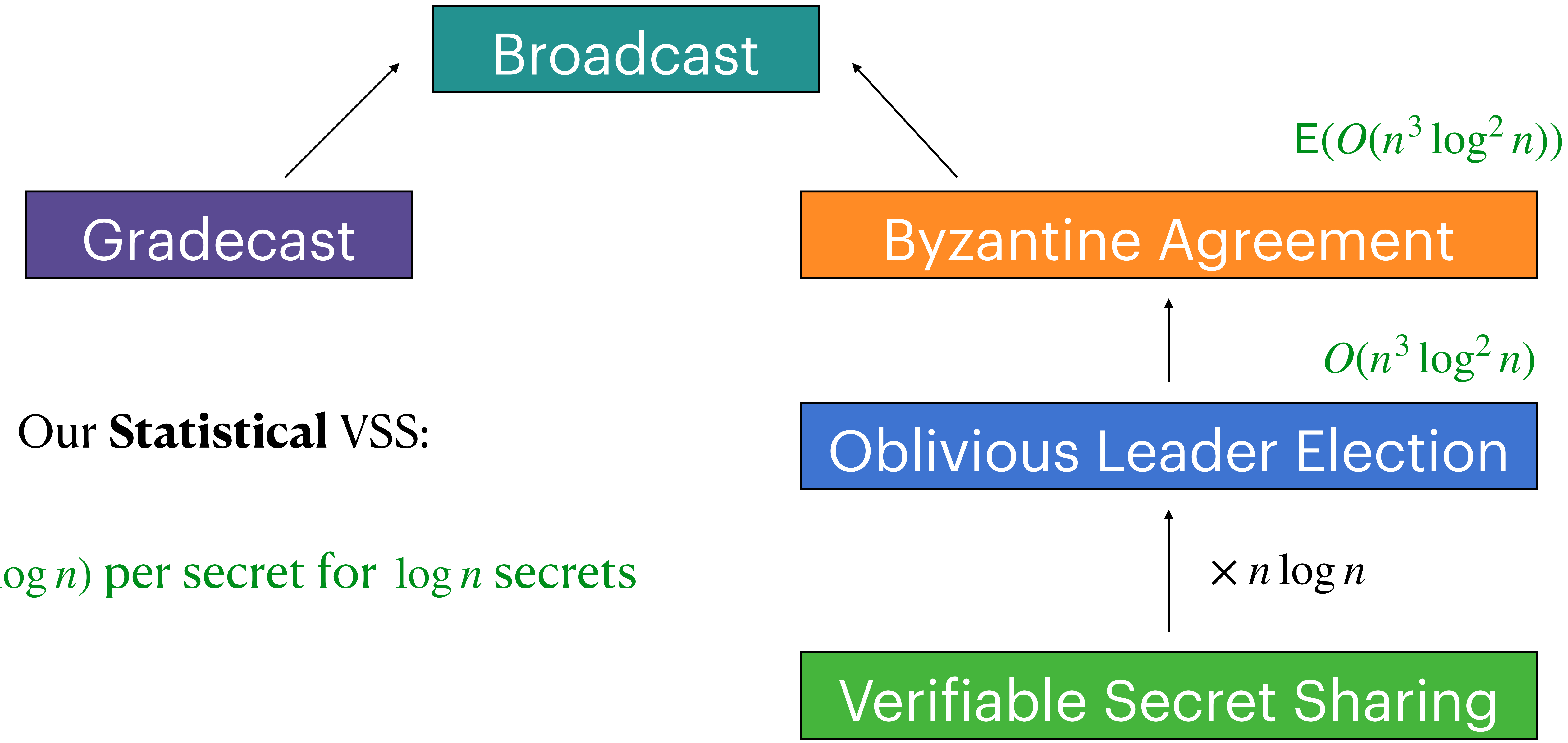
[KK06] Framework



Our **Statistical** VSS:

$O(n^2 \log n)$ per secret for $\log n$ secrets

[KK06] Framework

Broadcast

$O(nL) + \mathsf{E}(O(n^3 \log^2 n))$

Gradecast

$\mathsf{E}(O(n^3 \log^2 n))$

Byzantine Agreement

$O(n^3 \log^2 n)$

Oblivious Leader Election

Our **Statistical** VSS:

$\times\, n \log n$

$O(n^2 \log n)$ per secret for $\log n$ secrets

Verifiable Secret Sharing

Contributions

- **Conceptual contributions:**
  - Statistical OLE suffices
  - OLE from statistical VSS
- **Technical contributions:**
  - Statistical OLE with lesser secrets
  - Amortized Statistical VSS for lesser secrets

Conclusions

	Communication	Rounds	
Perfect Broadcast	$O(nL) + \mathsf{E}(O(n^3 \log^2 n))$	$\mathsf{E}(O(1))$	Optimal for $L \geq n^2 \log^2 n$
Perfect (Parallel) Broadcast	$O(n^2L) + \mathsf{E}(O(n^3 \log^2 n))$	$\mathsf{E}(O(1))$	Optimal for $L \geq n \log^2 n$
Statistical VSS	$O(n^2 \log n)$ per secret for $O(\log n)$ secrets	$O(1)$	

Statistical OLE $\Longrightarrow$ Perfect broadcast in constant expected time

Conclusions

	Communication	Rounds	
Perfect Broadcast	$O(nL) + \mathsf{E}(O(n^3 \log^2 n))$	$\mathsf{E}(O(1))$	Optimal for $L \geq n^2 \log^2 n$
Perfect (Parallel) Broadcast	$O(n^2 L) + \mathsf{E}(O(n^3 \log^2 n))$	$\mathsf{E}(O(1))$	Optimal for $L \geq n \log^2 n$
Statistical VSS	$O(n^2 \log n)$ per secret for $O(\log n)$ secrets	$O(1)$	

Statistical OLE $\Longrightarrow$ Perfect broadcast in constant expected time

Thank you!