CISPA
HELMHOLTZ CENTER FOR INFORMATION SECURITY

Early Stopping for Any Number of Corruptions

Julian Loss, CISPA Helmholtz Center for Information Security
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- Validity: all receivers output m if S is honest
- Consistency: all receivers output the same message
- Termination: All parties terminate

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- Can we terminate earlier when $f \ll t$ parties are corrupted?

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- **This work:** $O(\min\{f^2, t\})$ for $t < n$ and signatures

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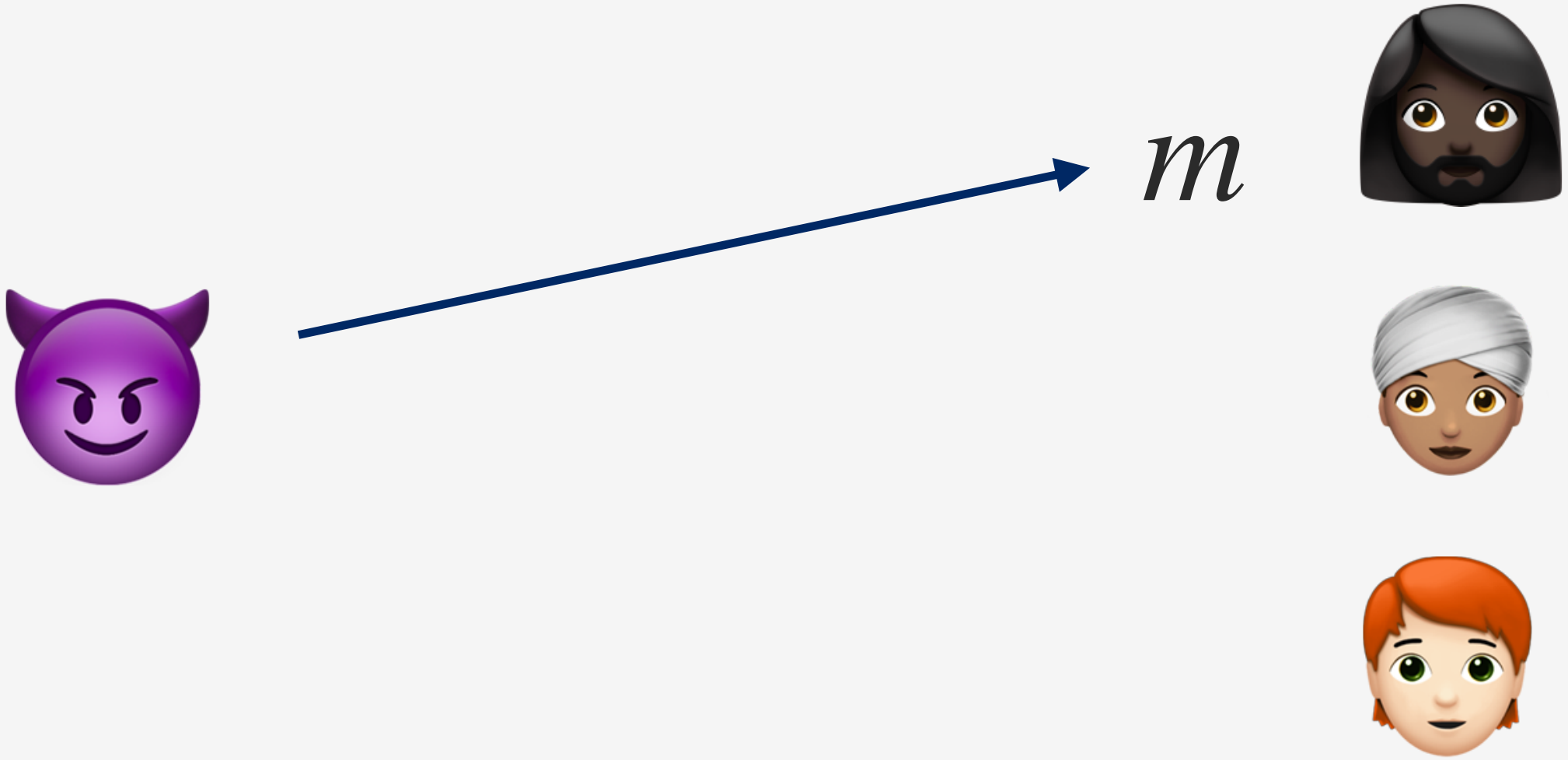

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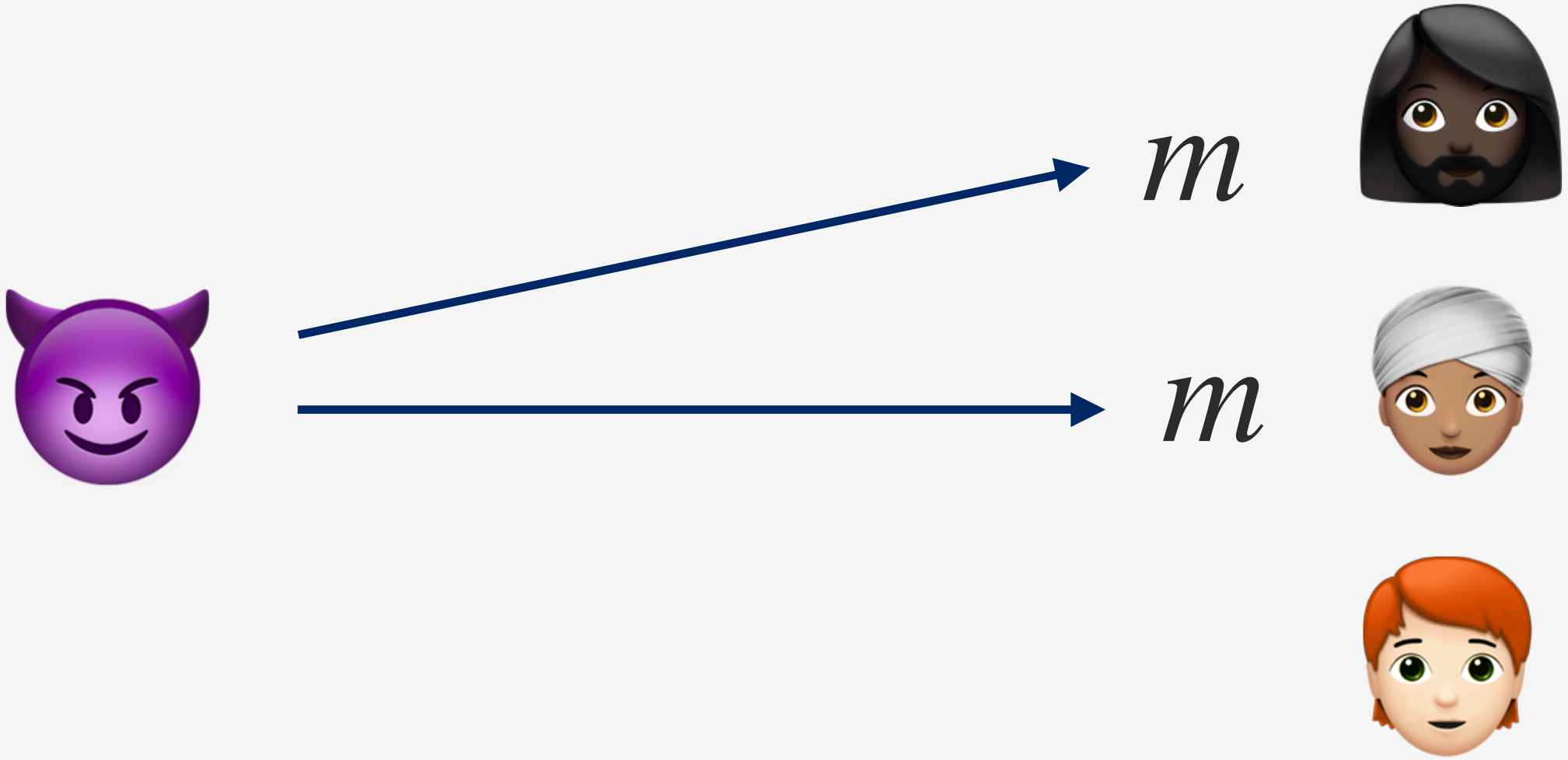

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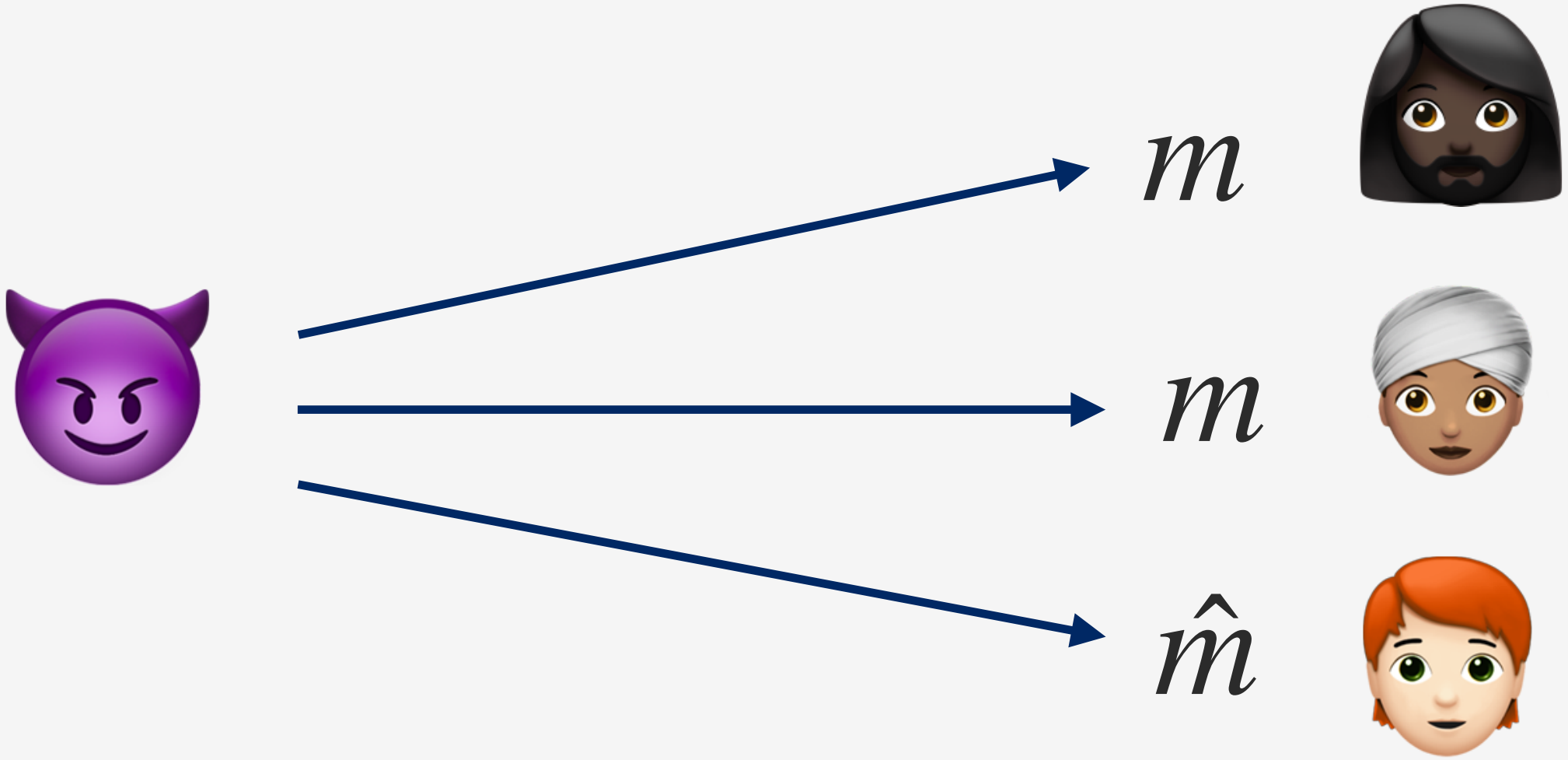

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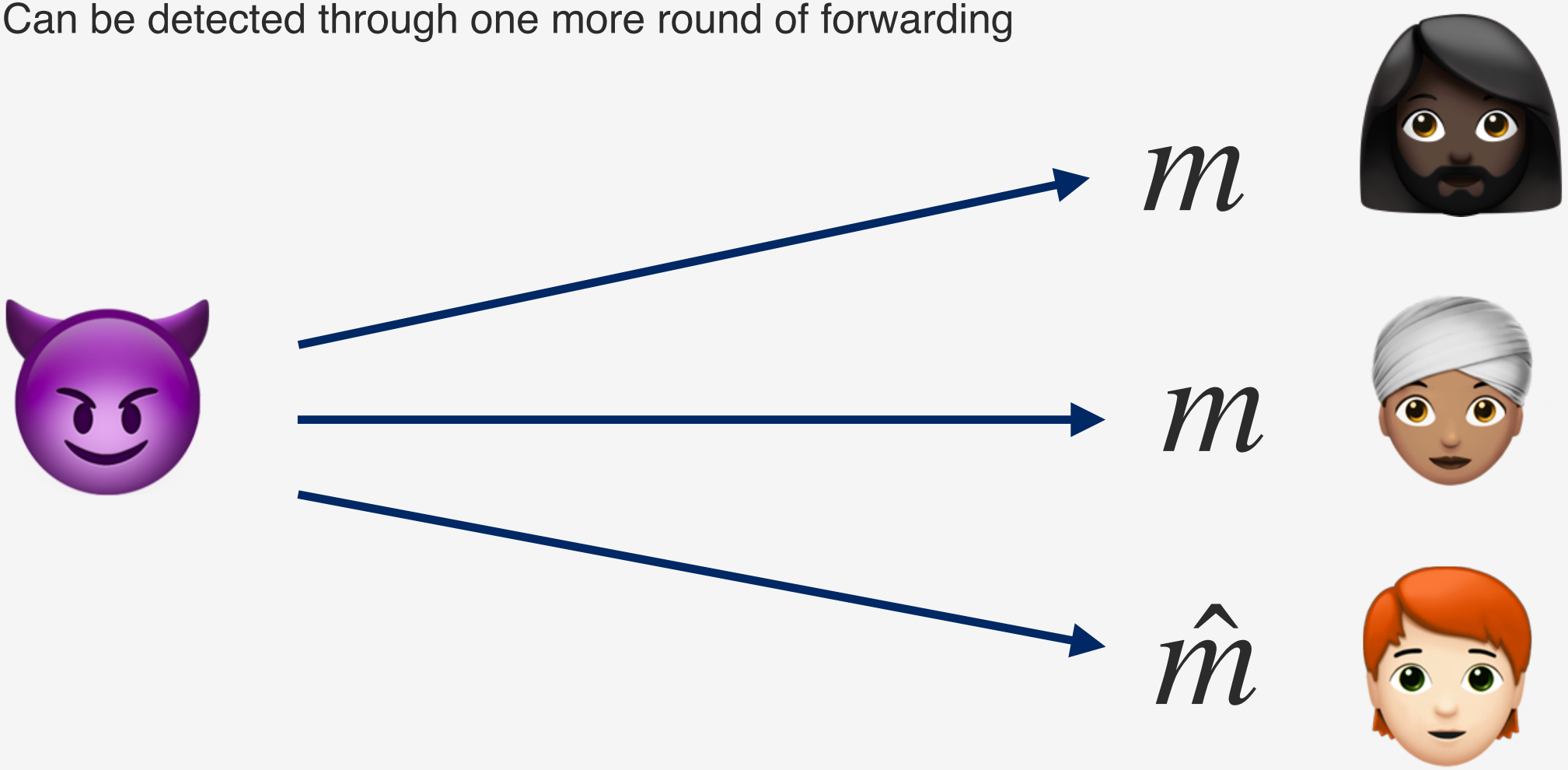

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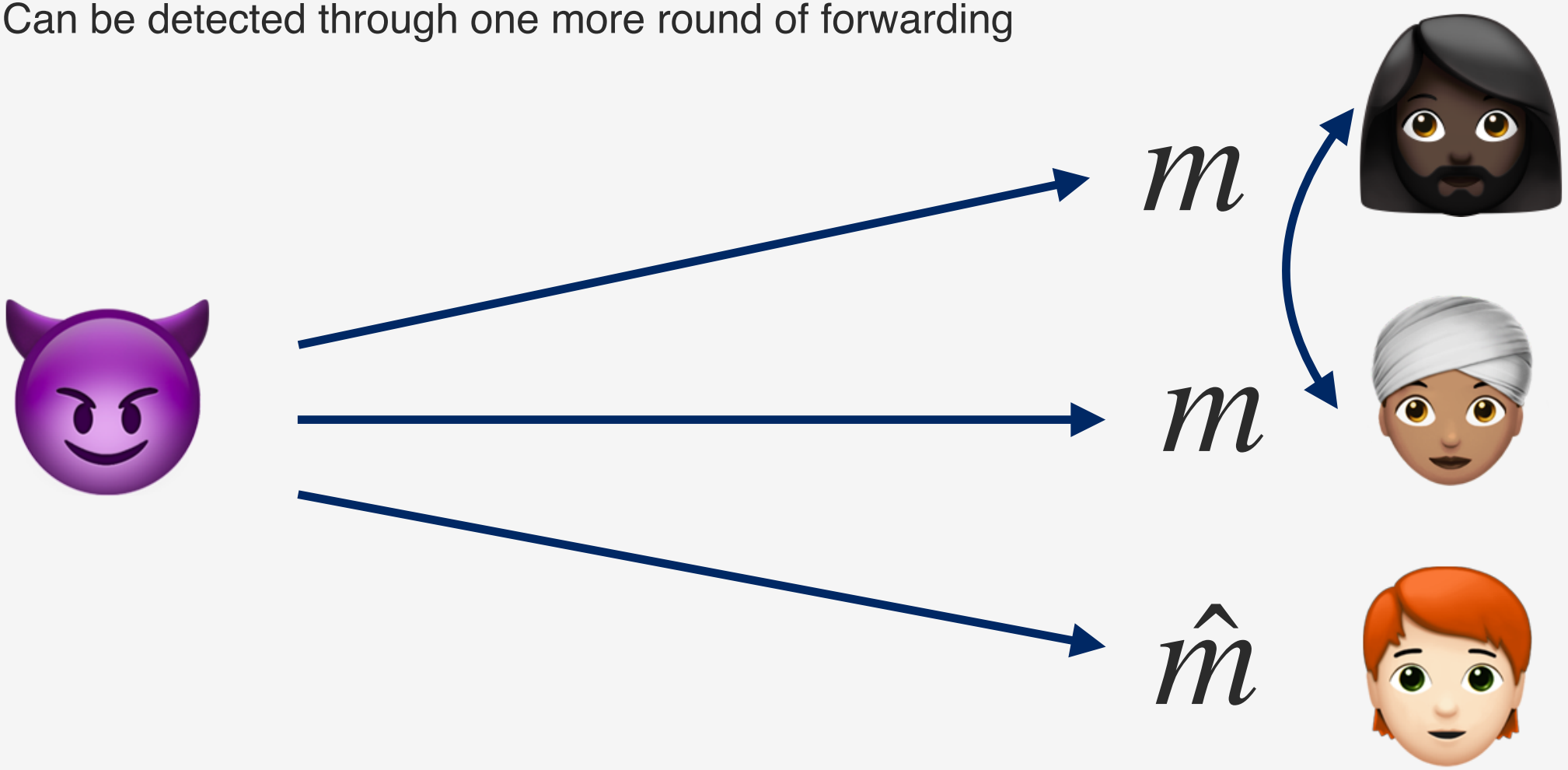

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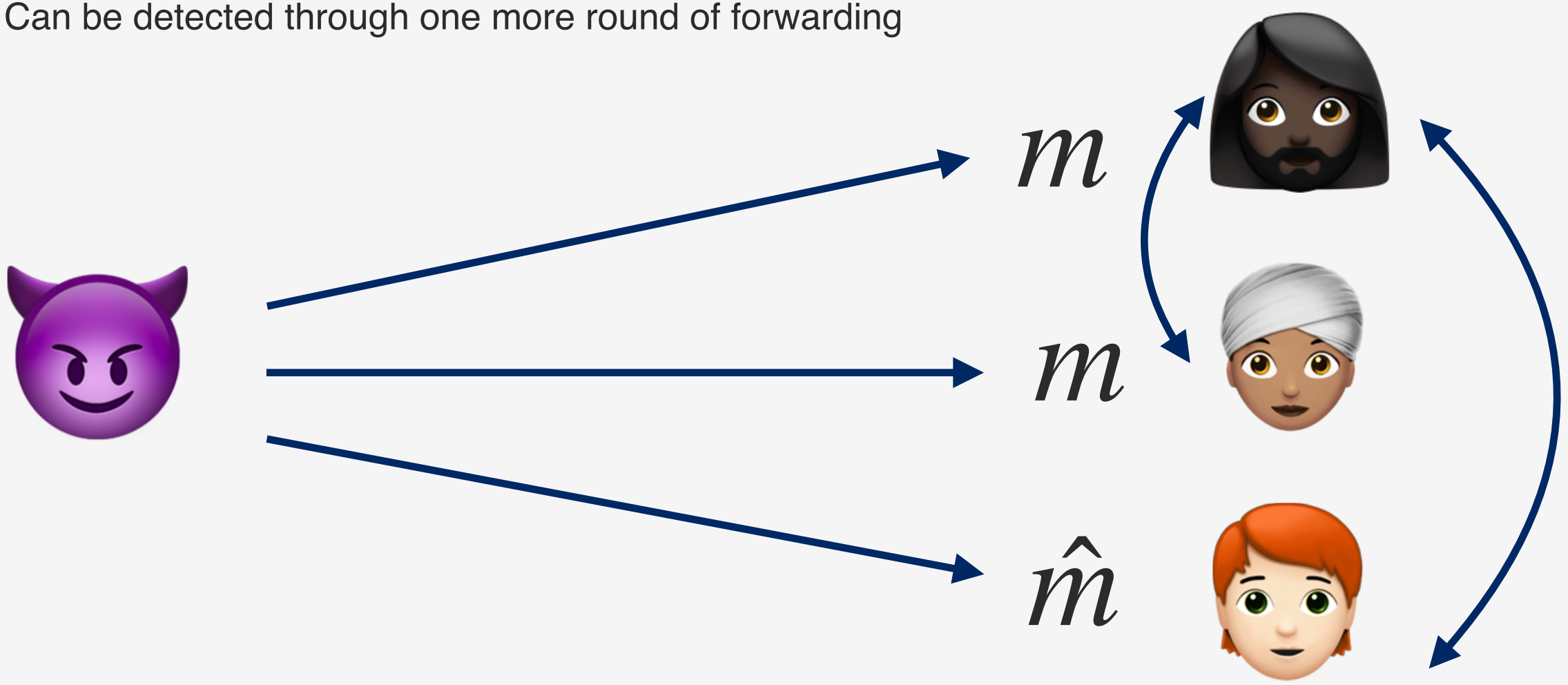

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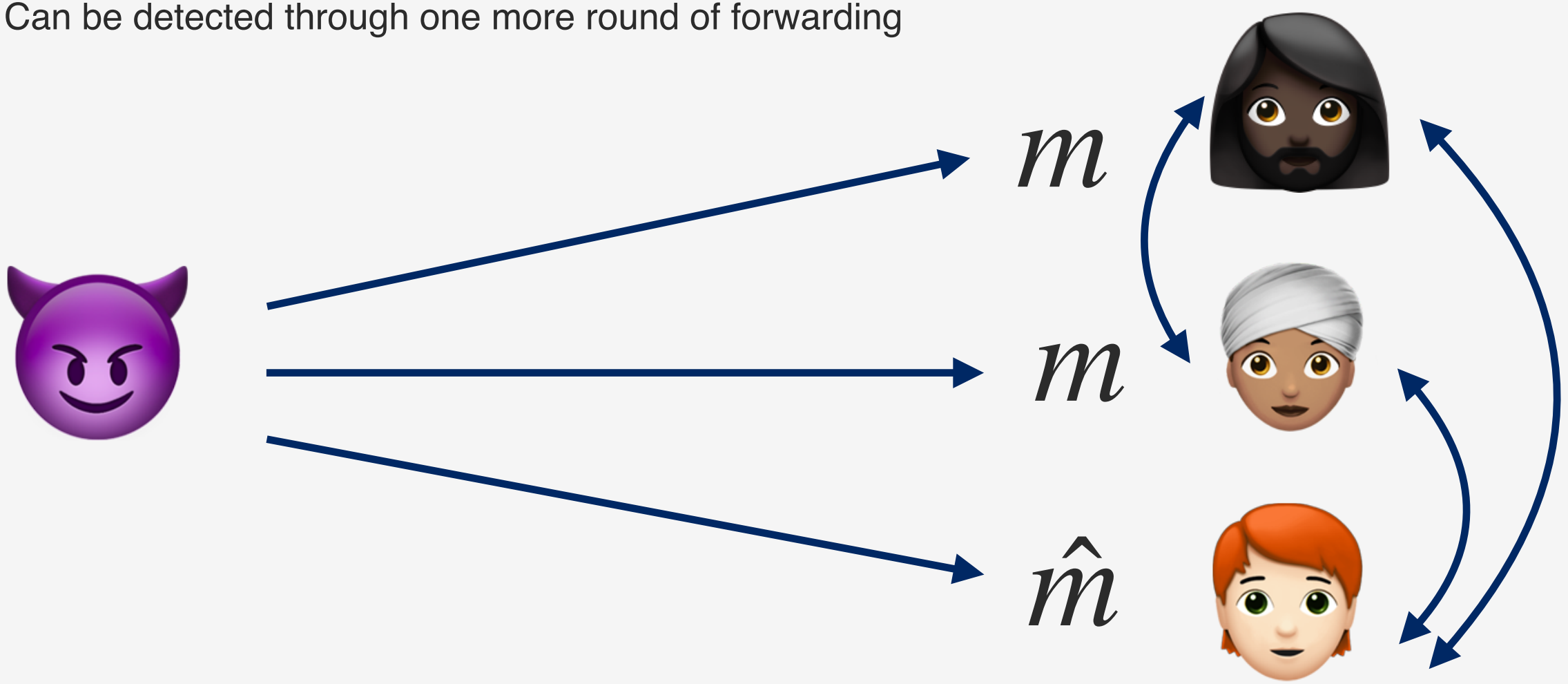

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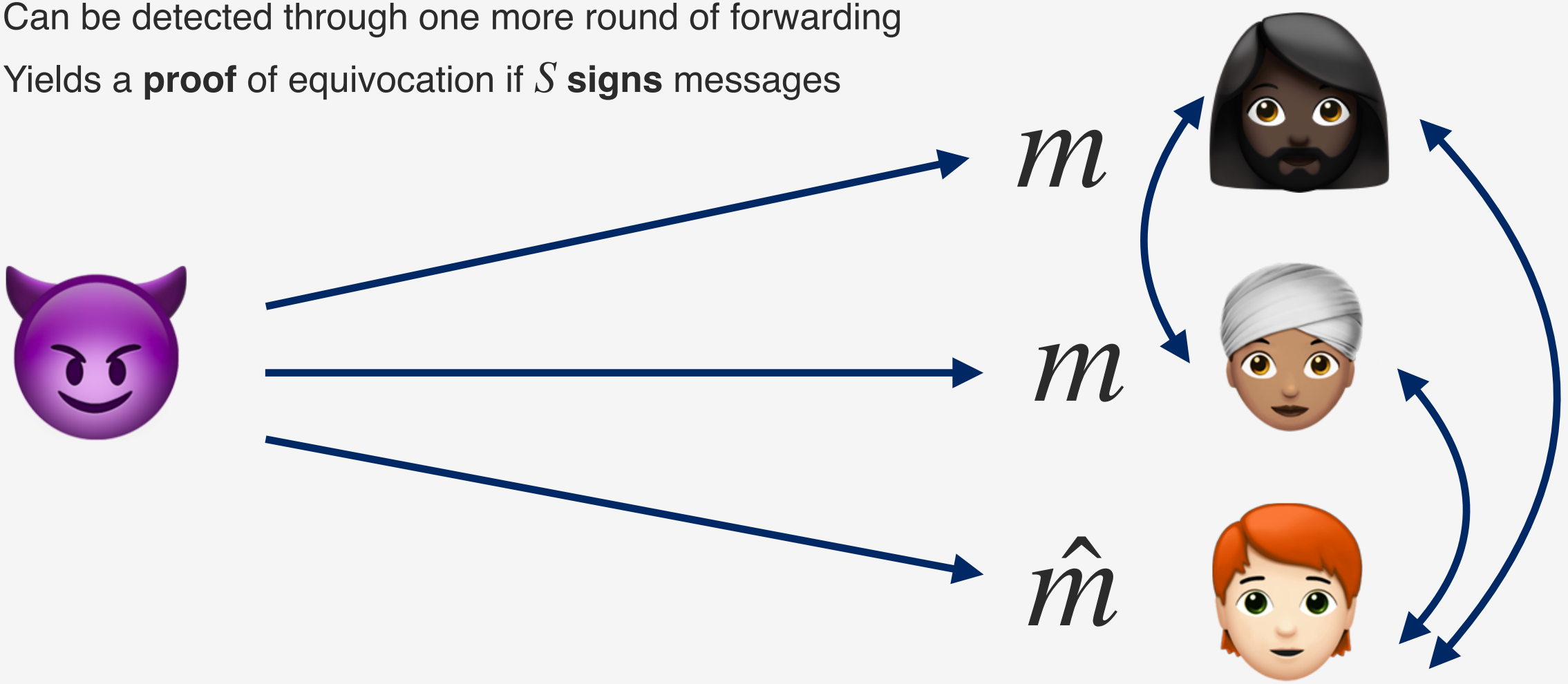
- Can be detected through one more round of forwarding
- Yields a **proof** of equivocation if S **signs** messages



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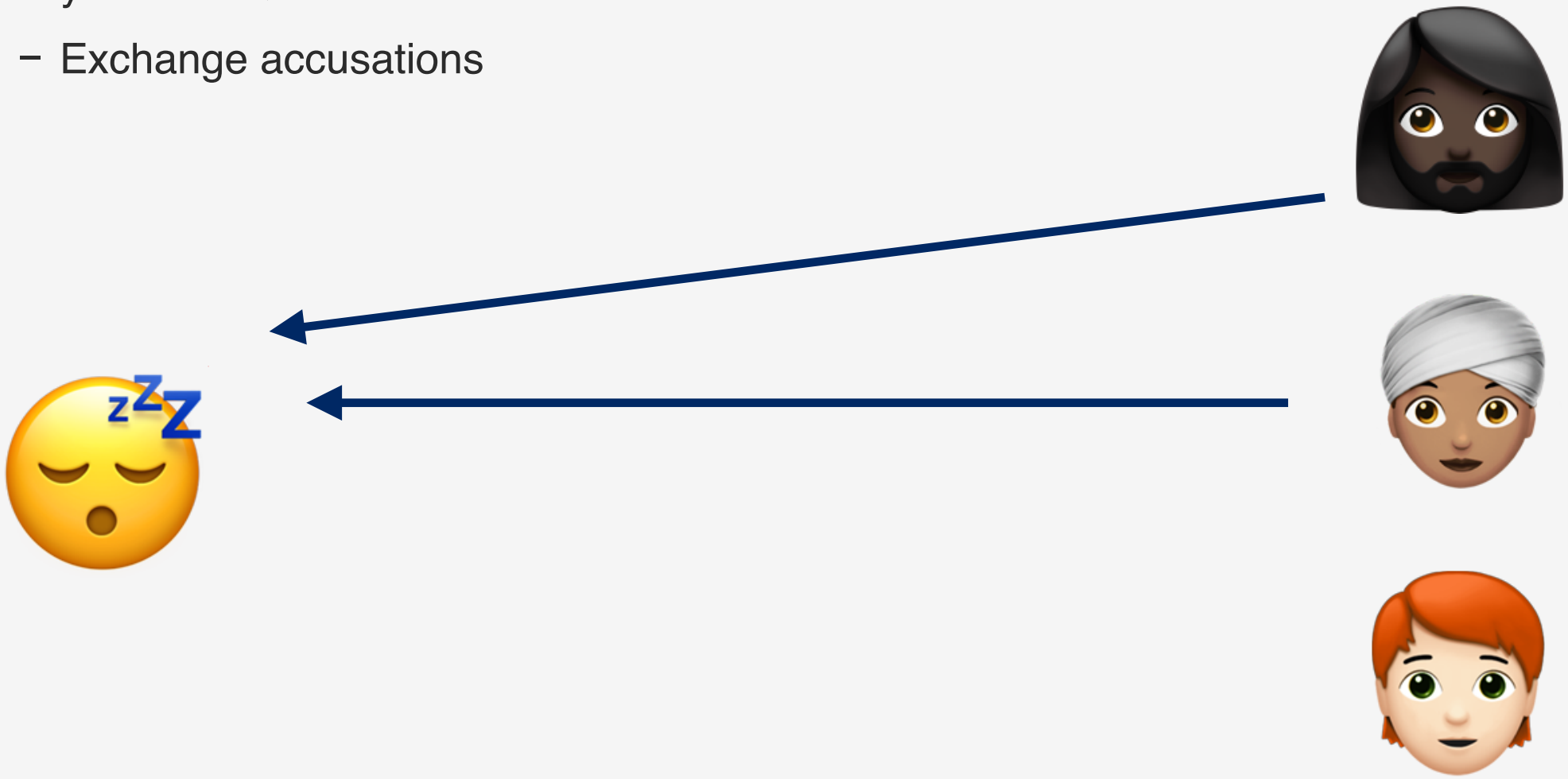

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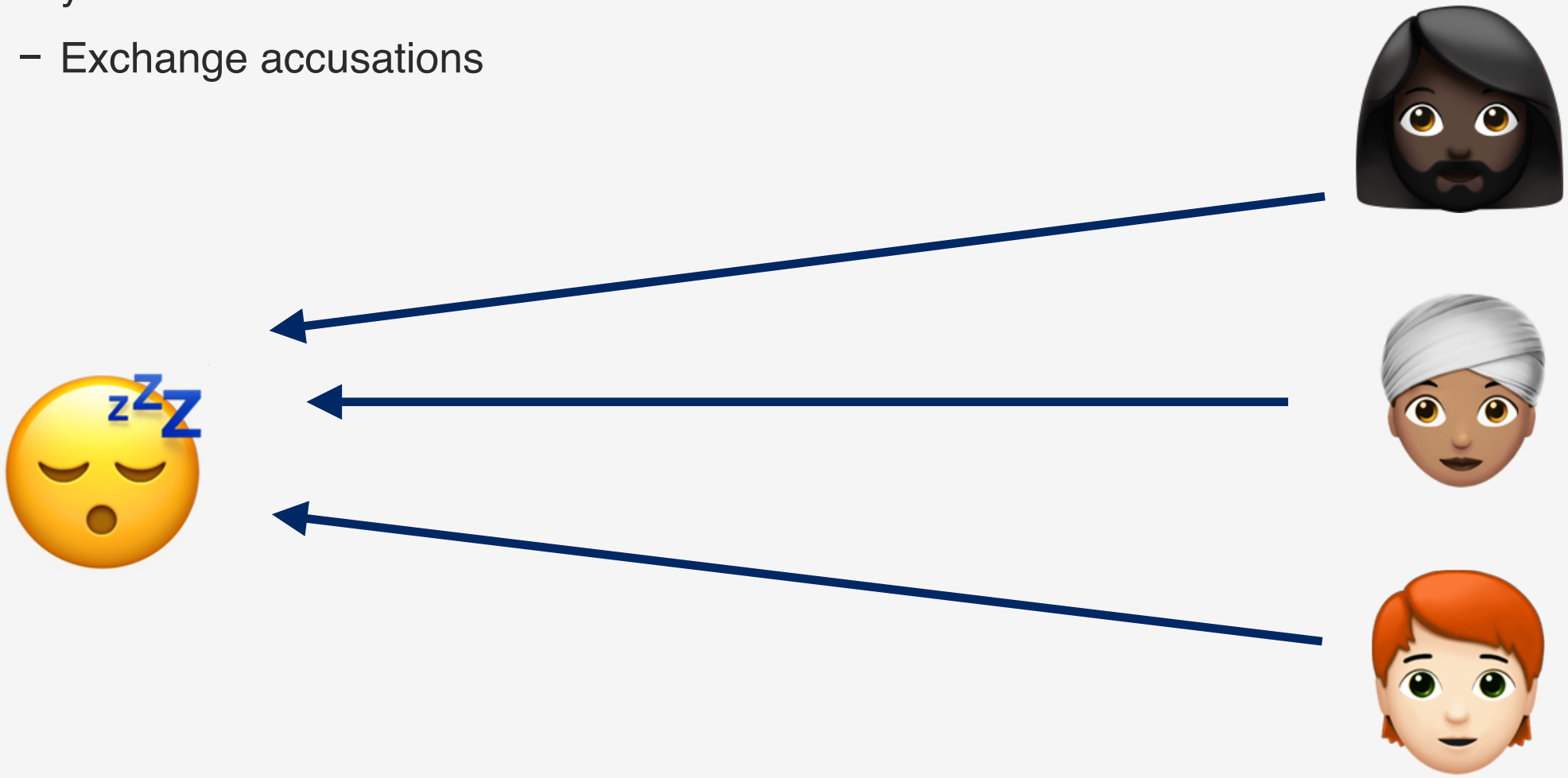

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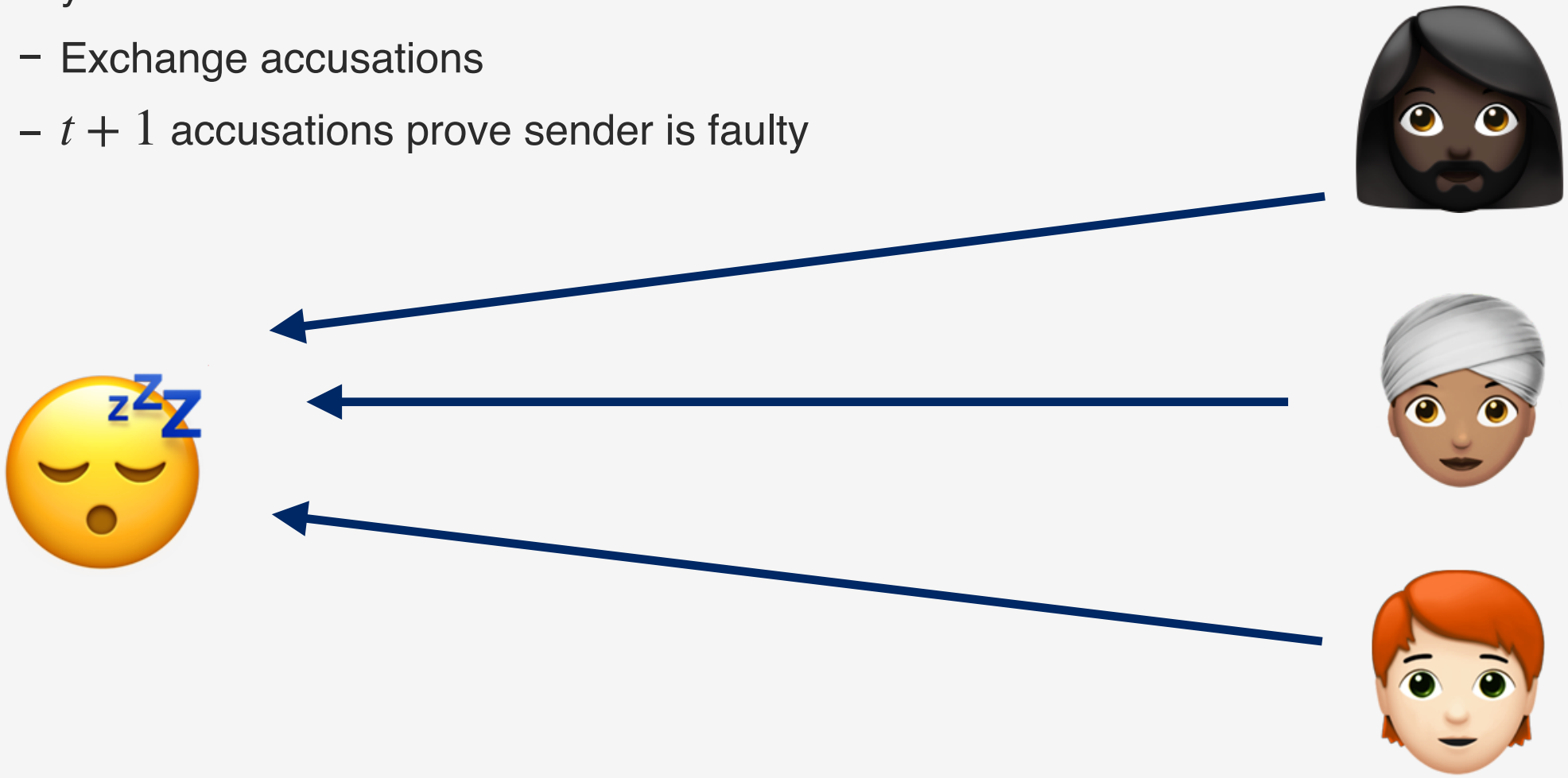

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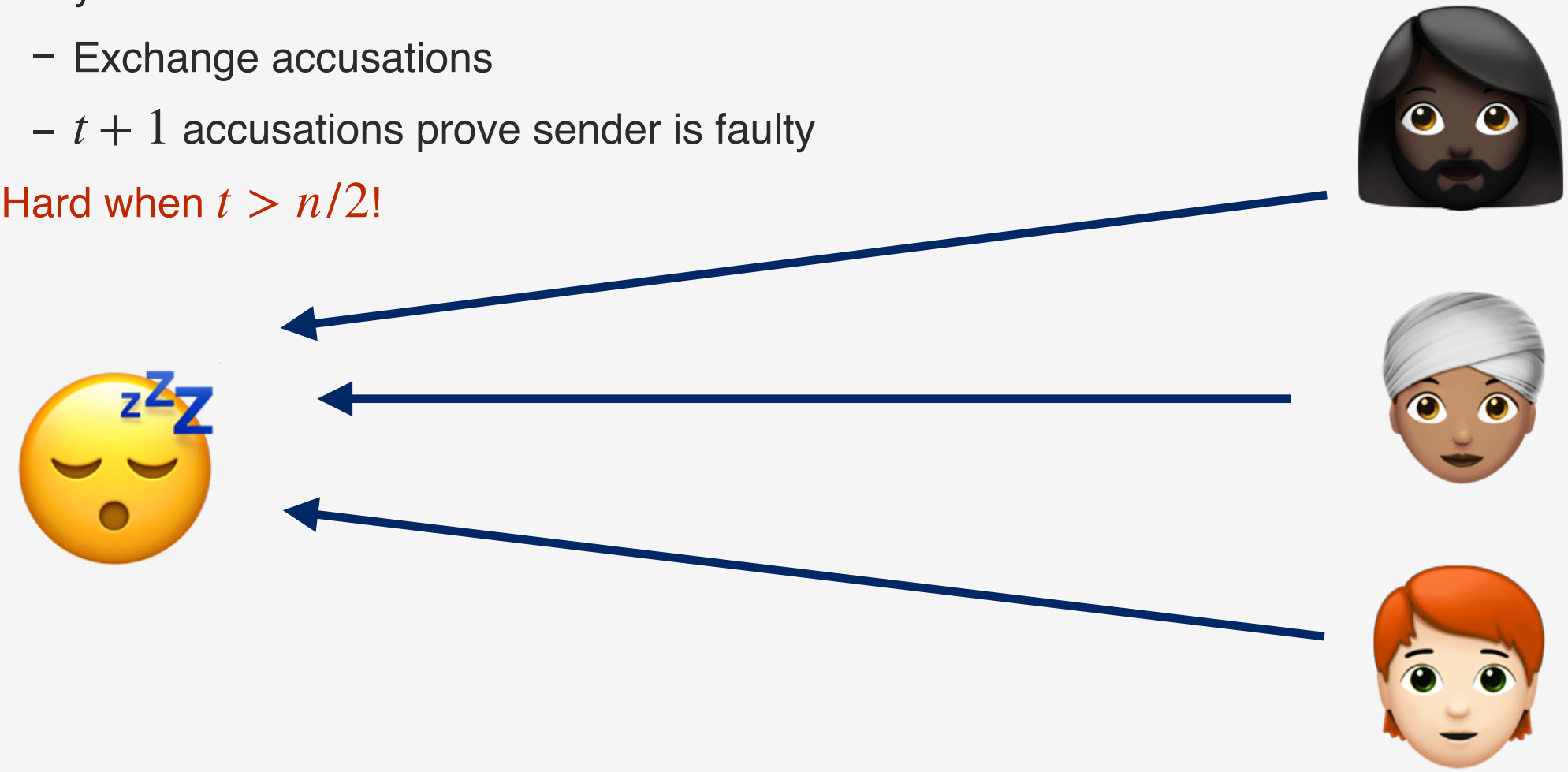

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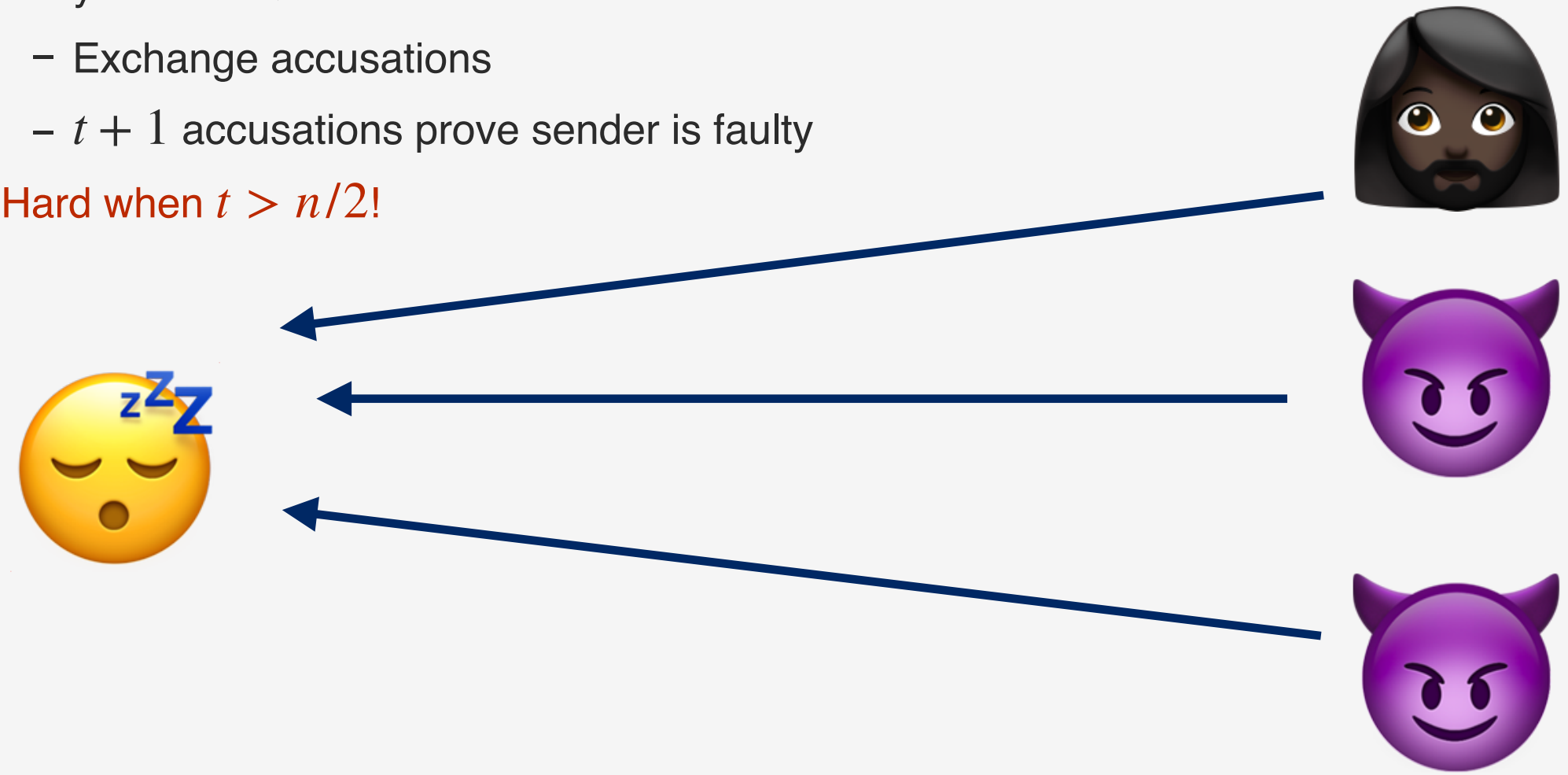

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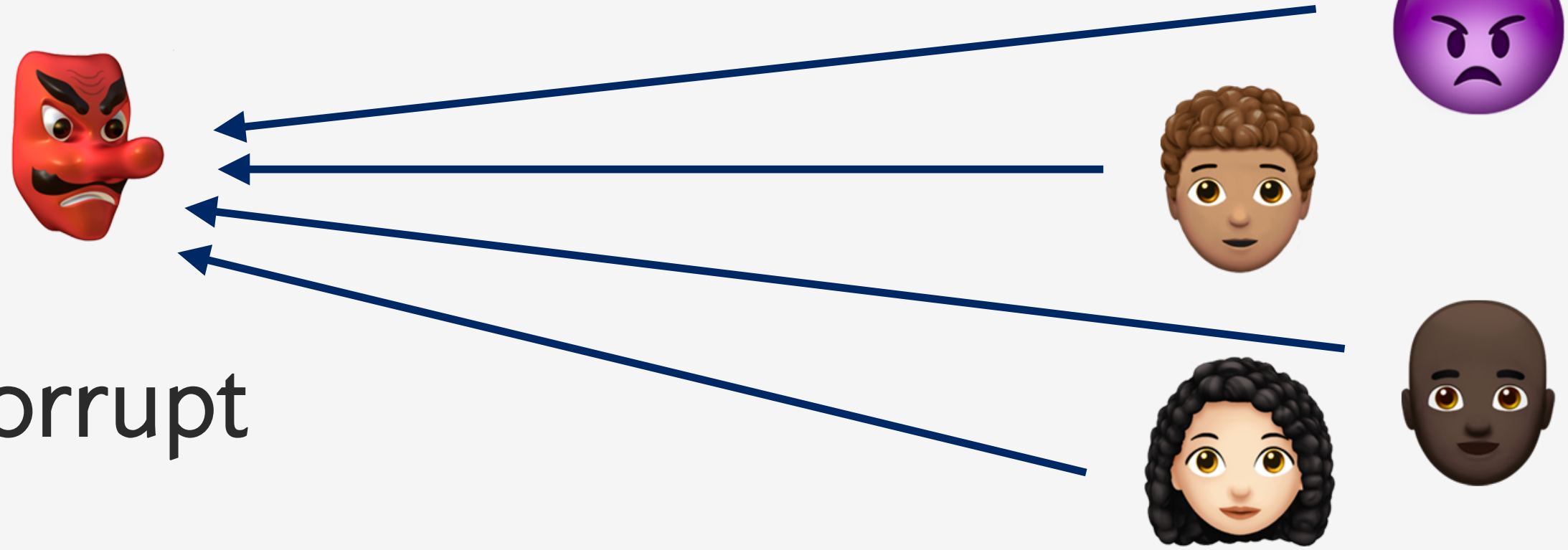

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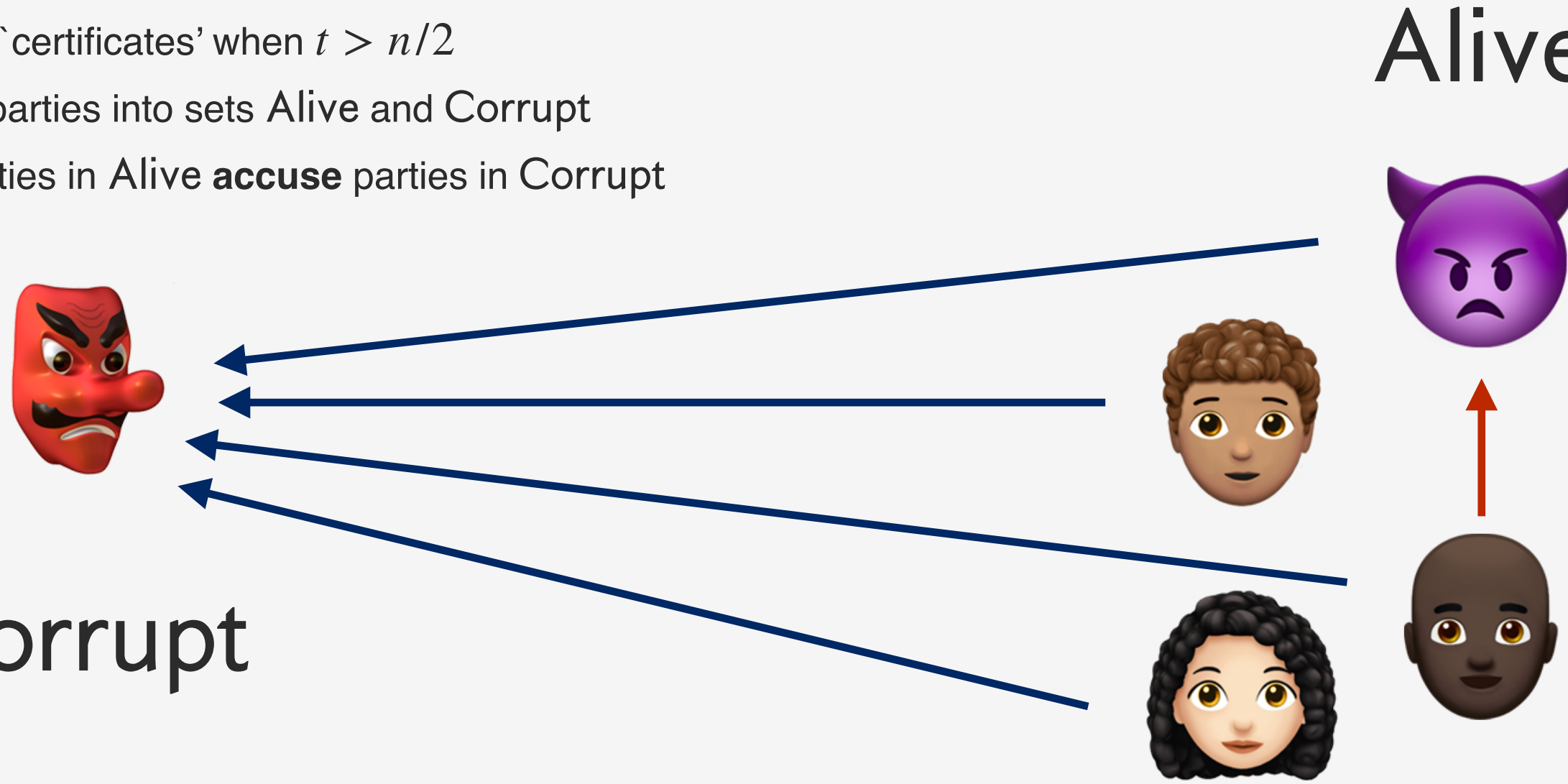

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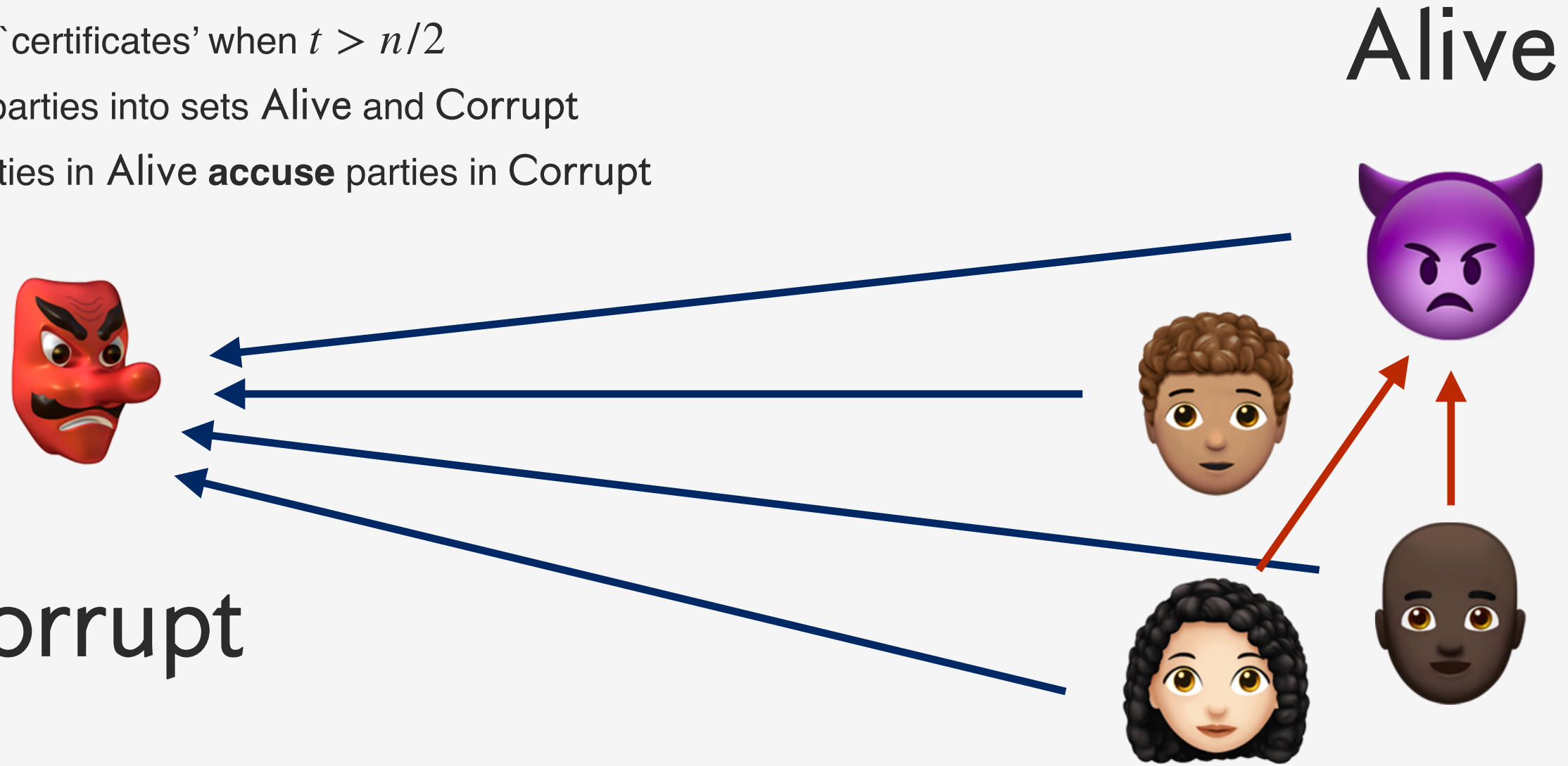

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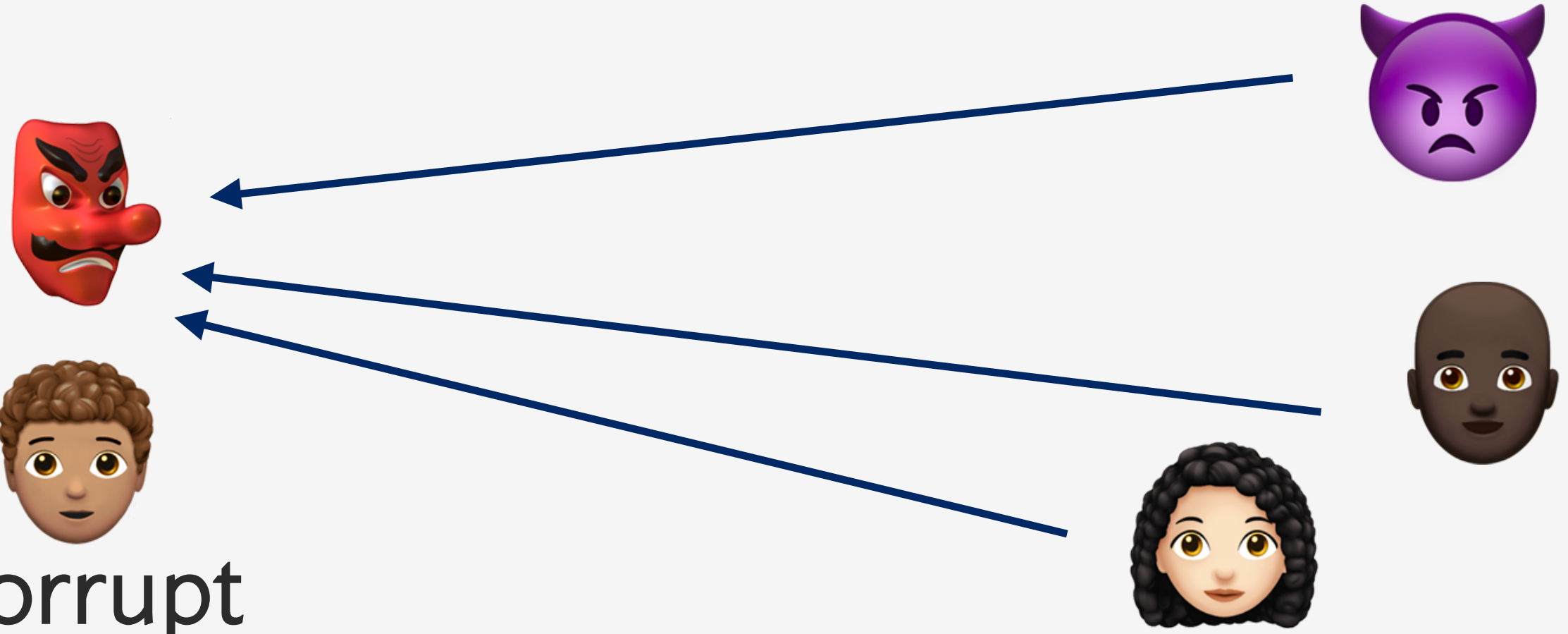

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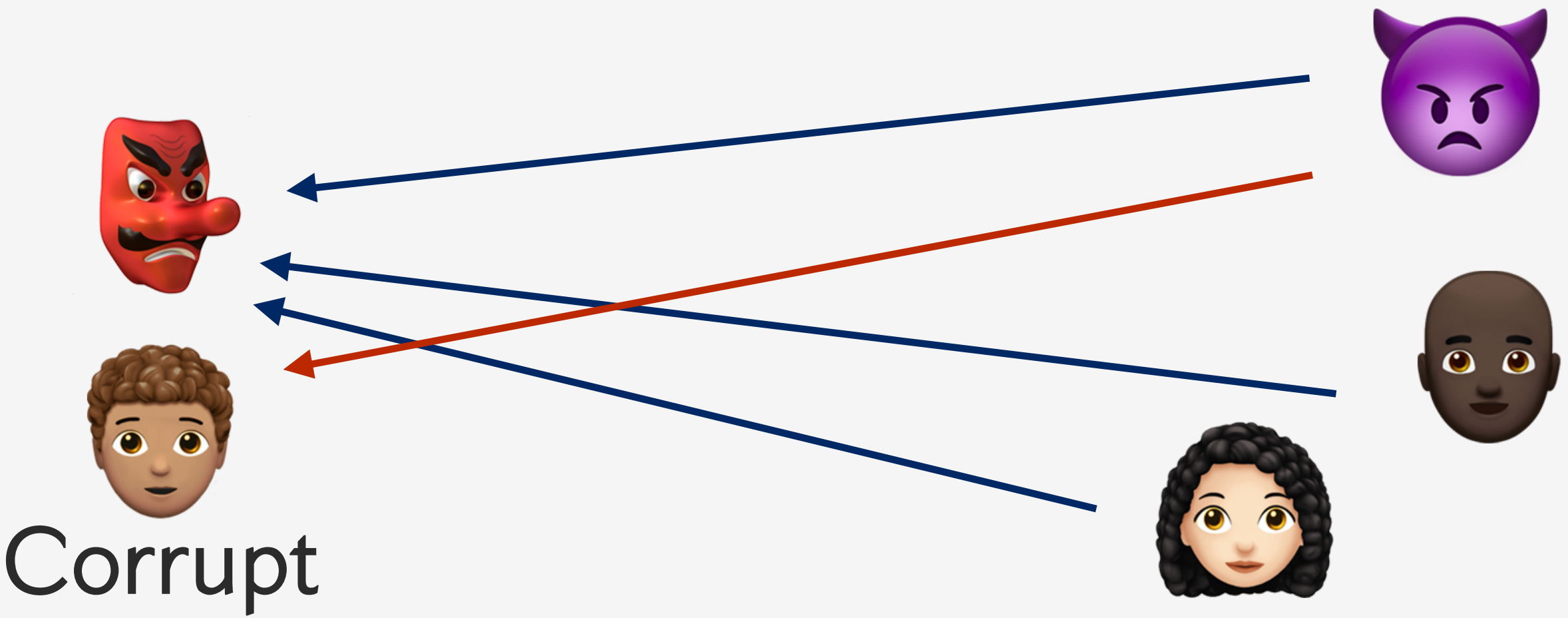

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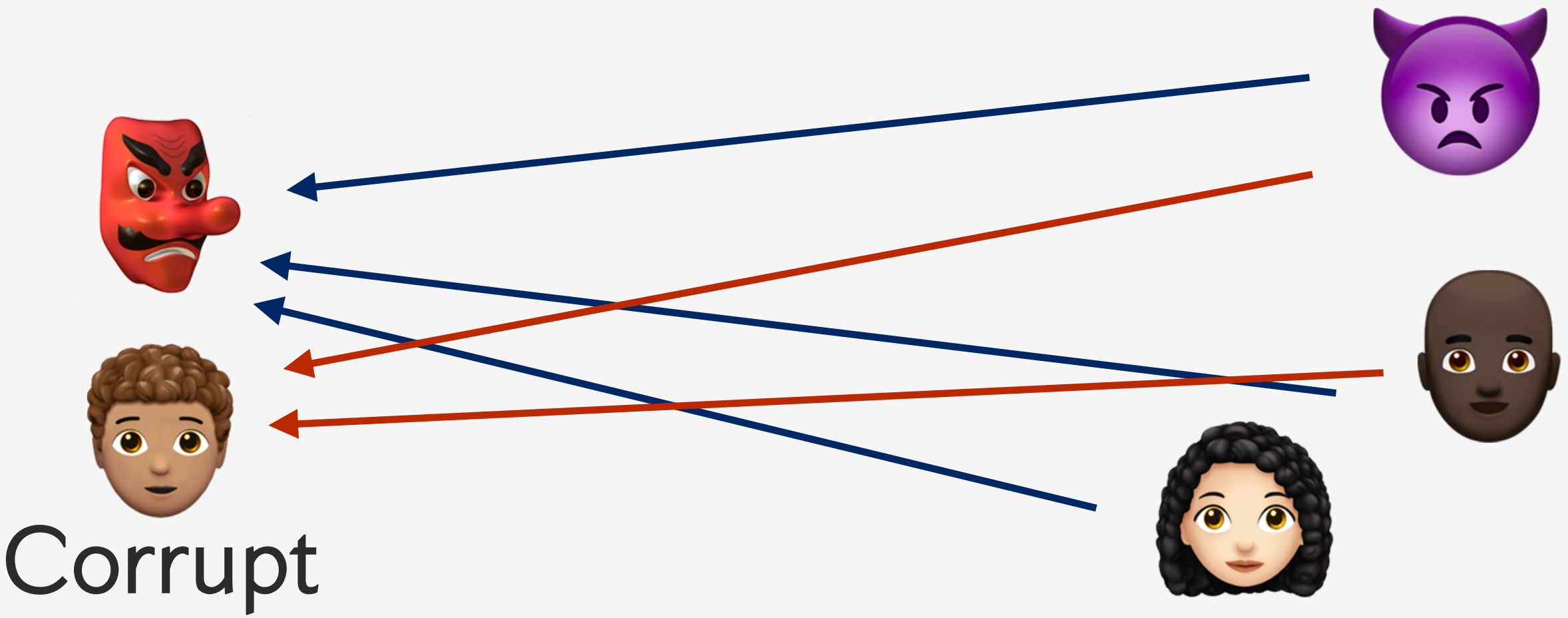

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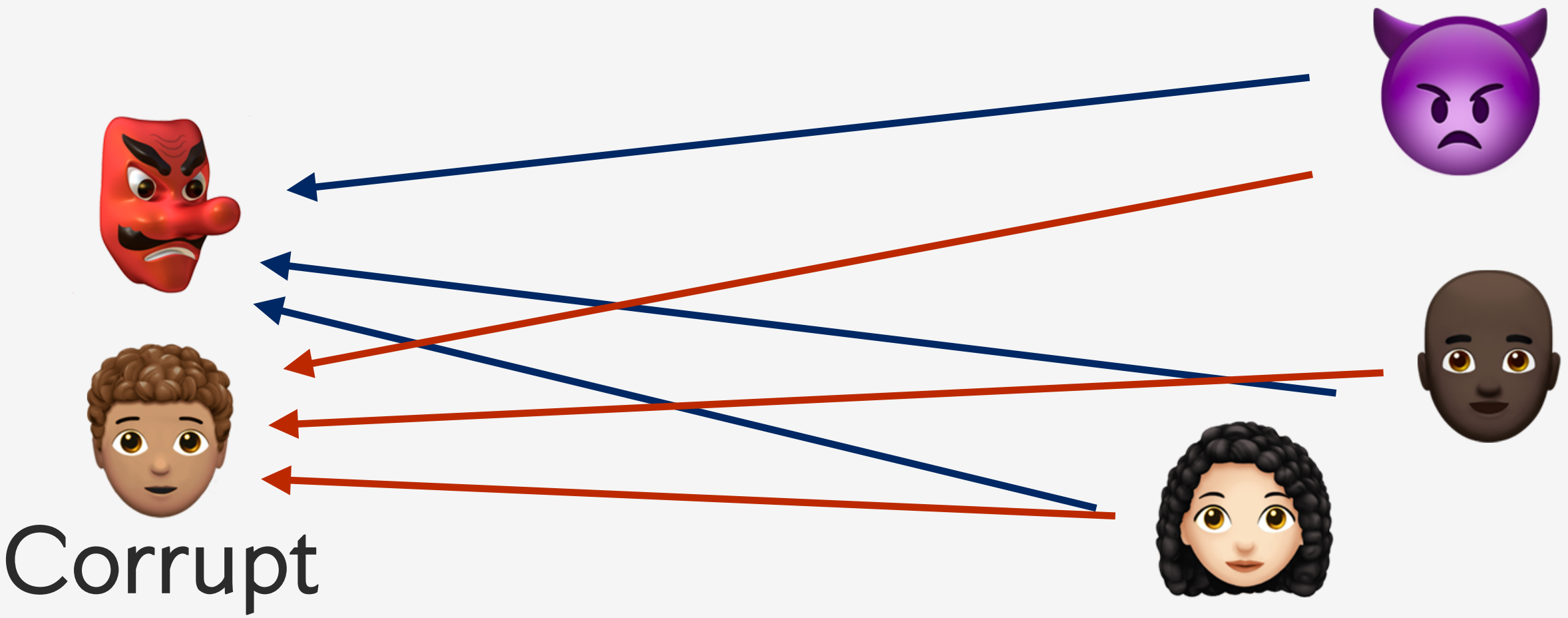

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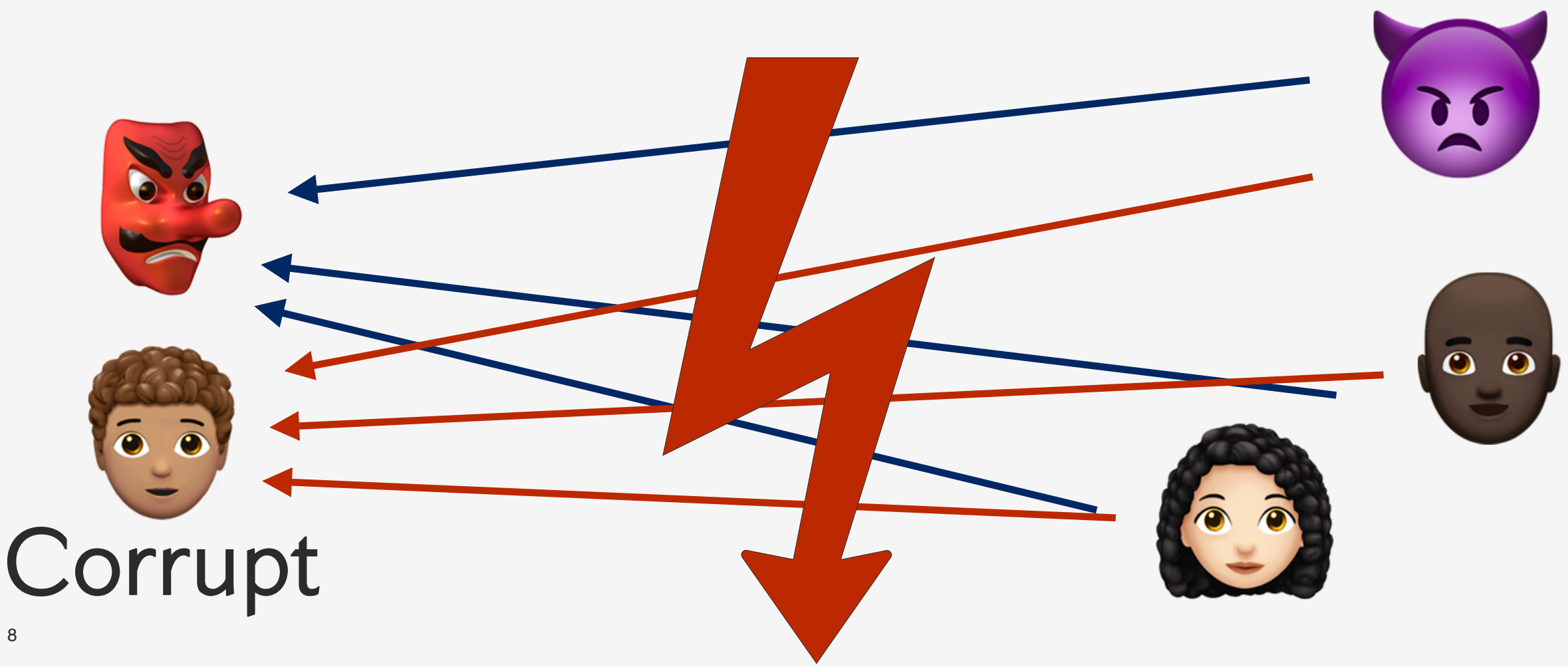

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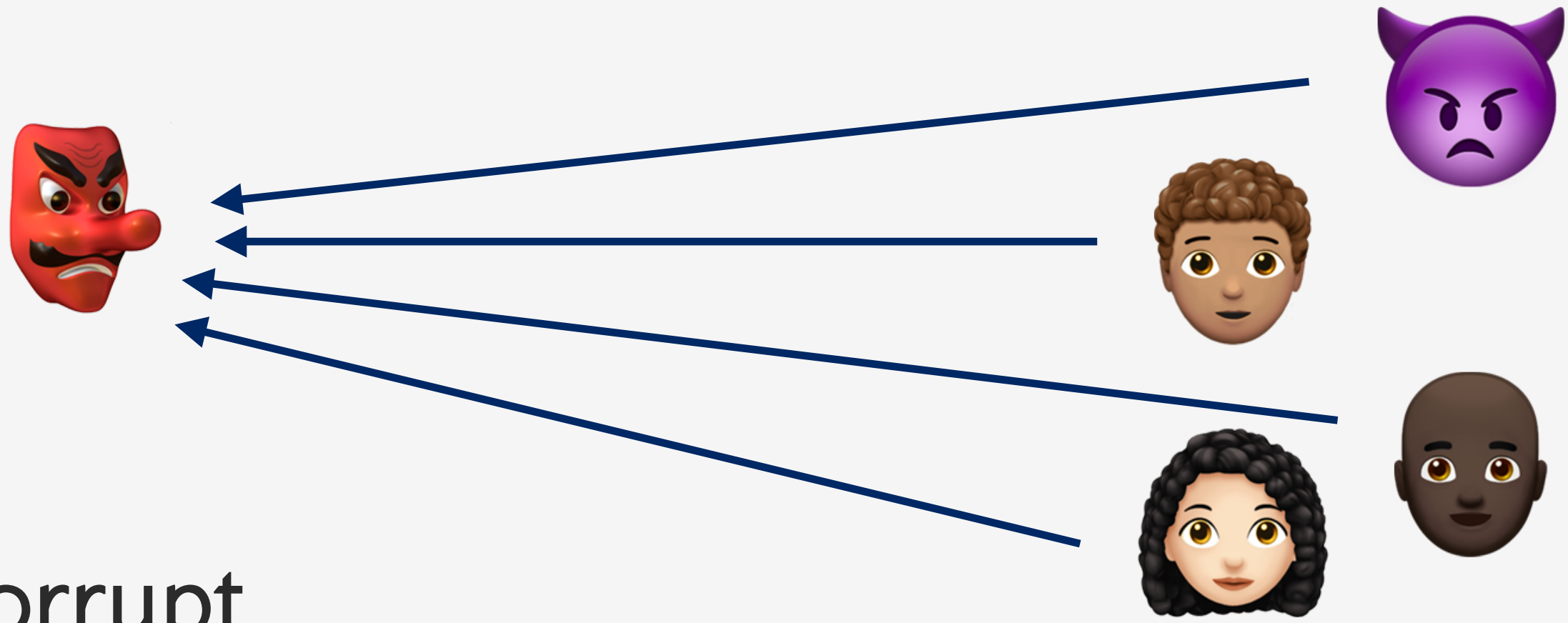

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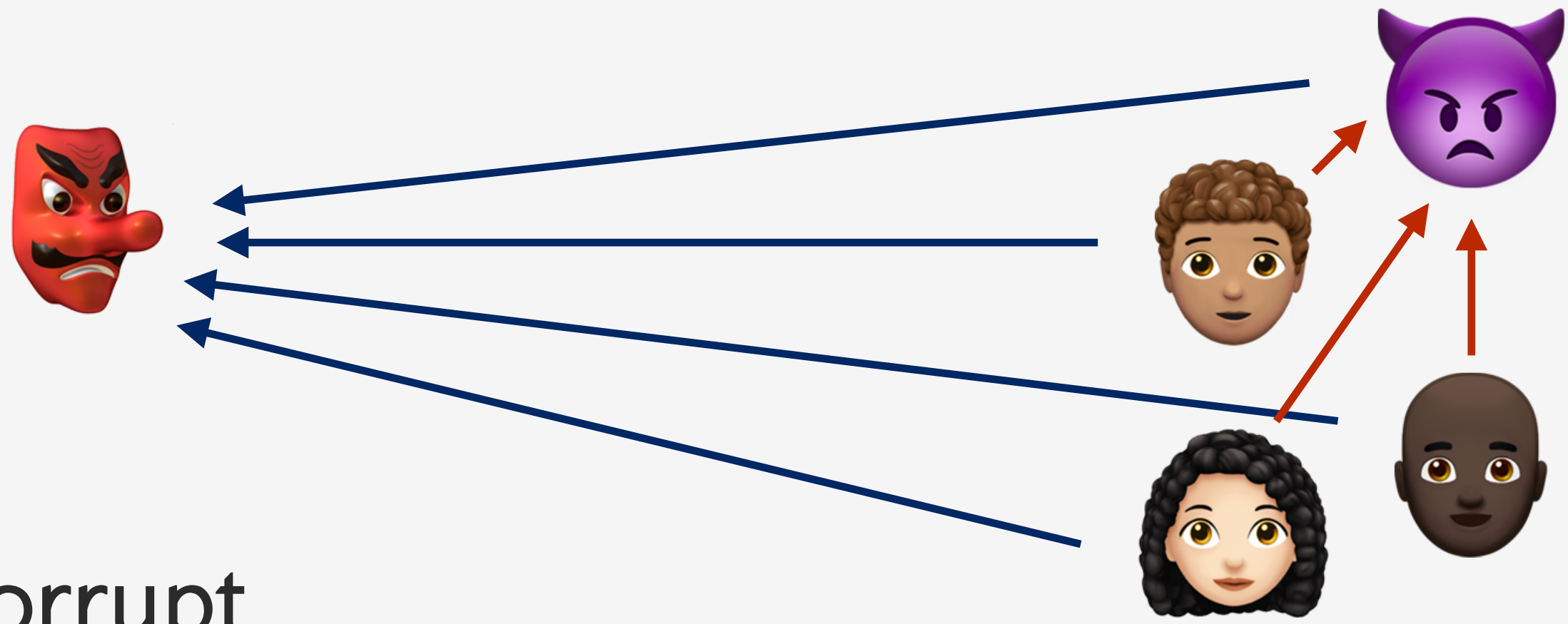

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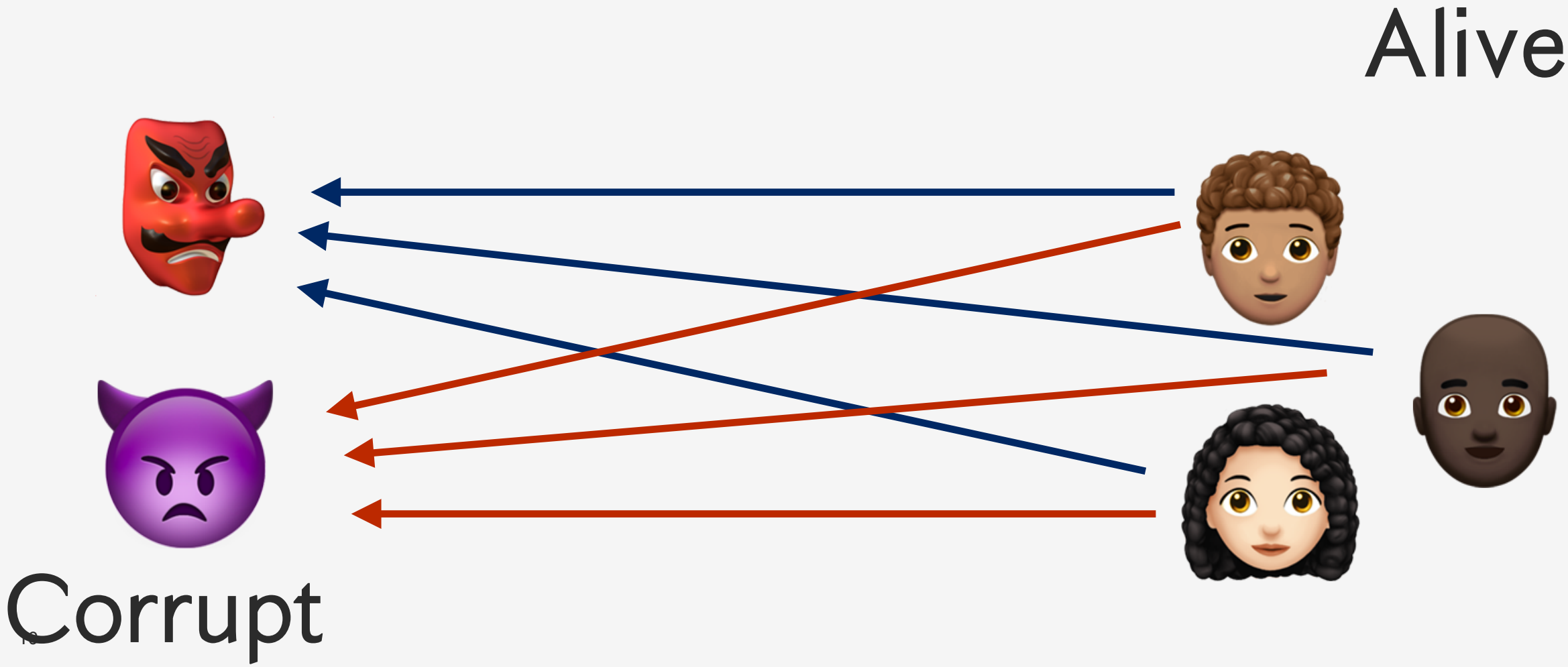

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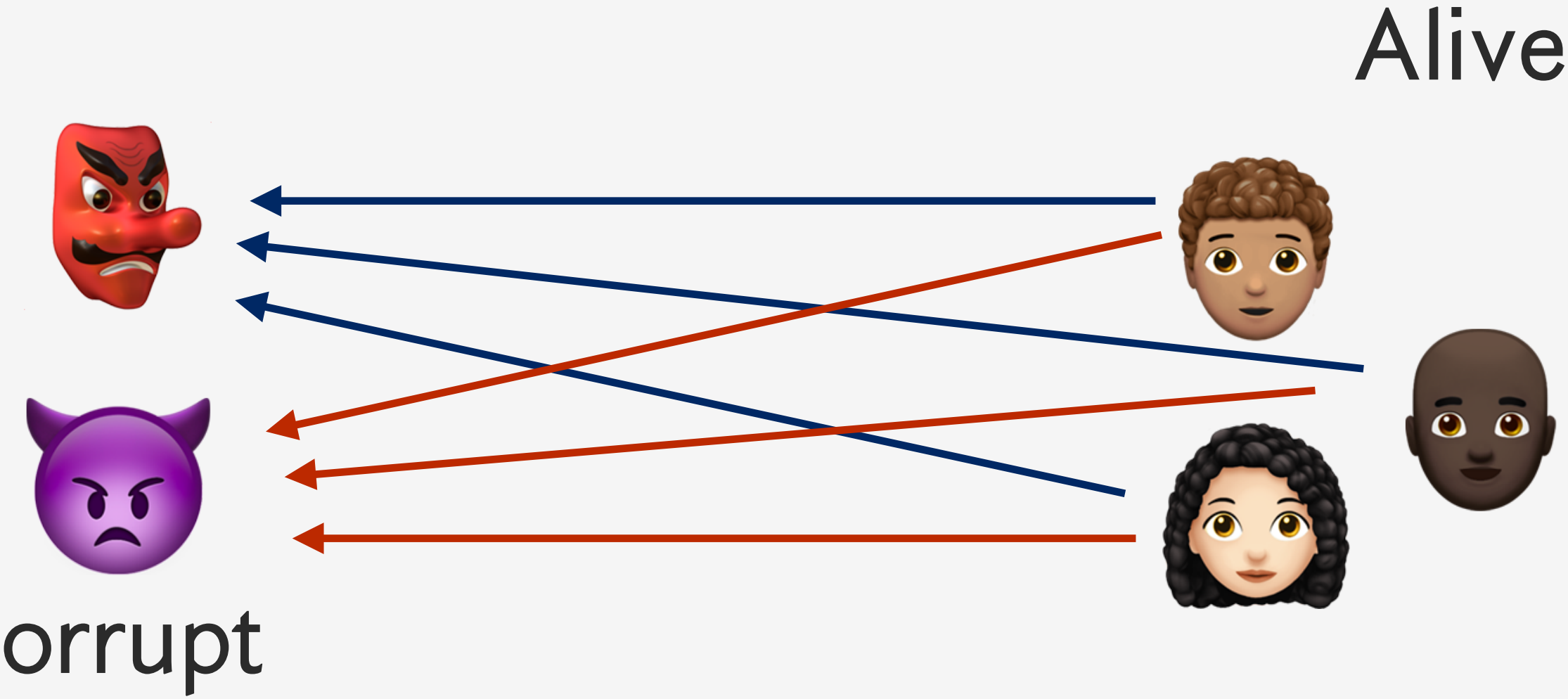

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- Update polariser with new accusations
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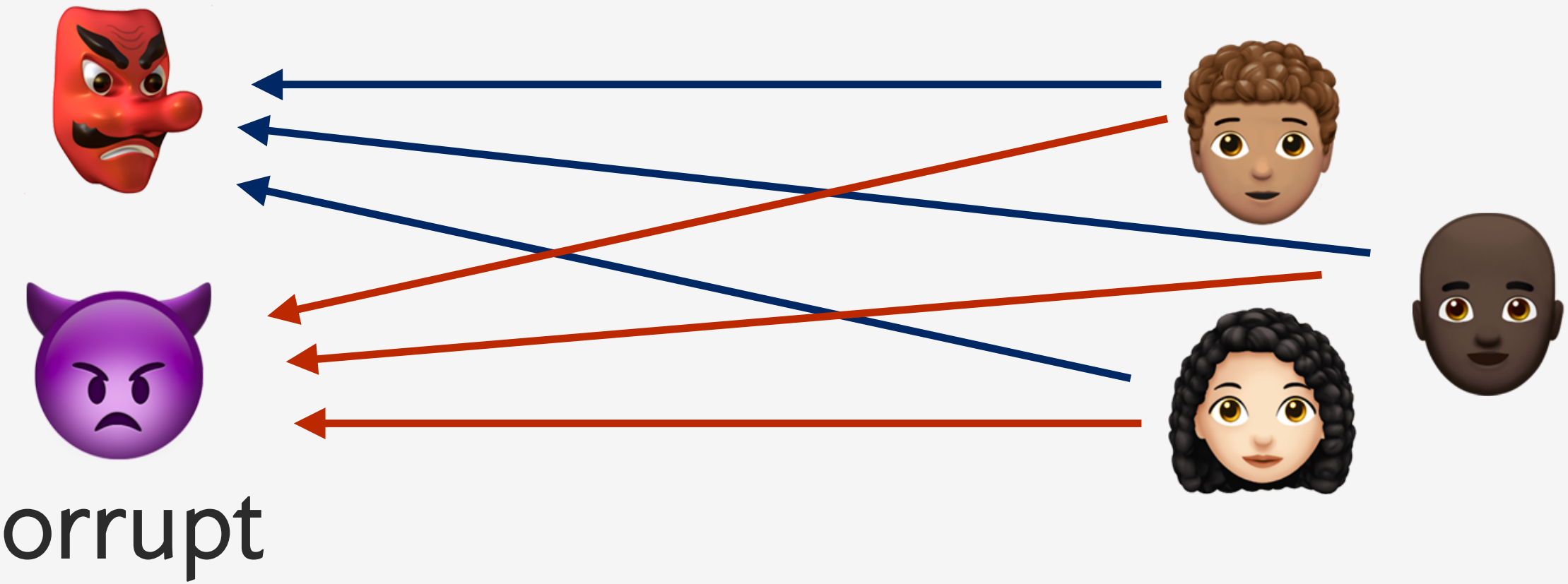
- $\Longrightarrow$ Polarisers can be **forwarded/updated** safely between honest parties!



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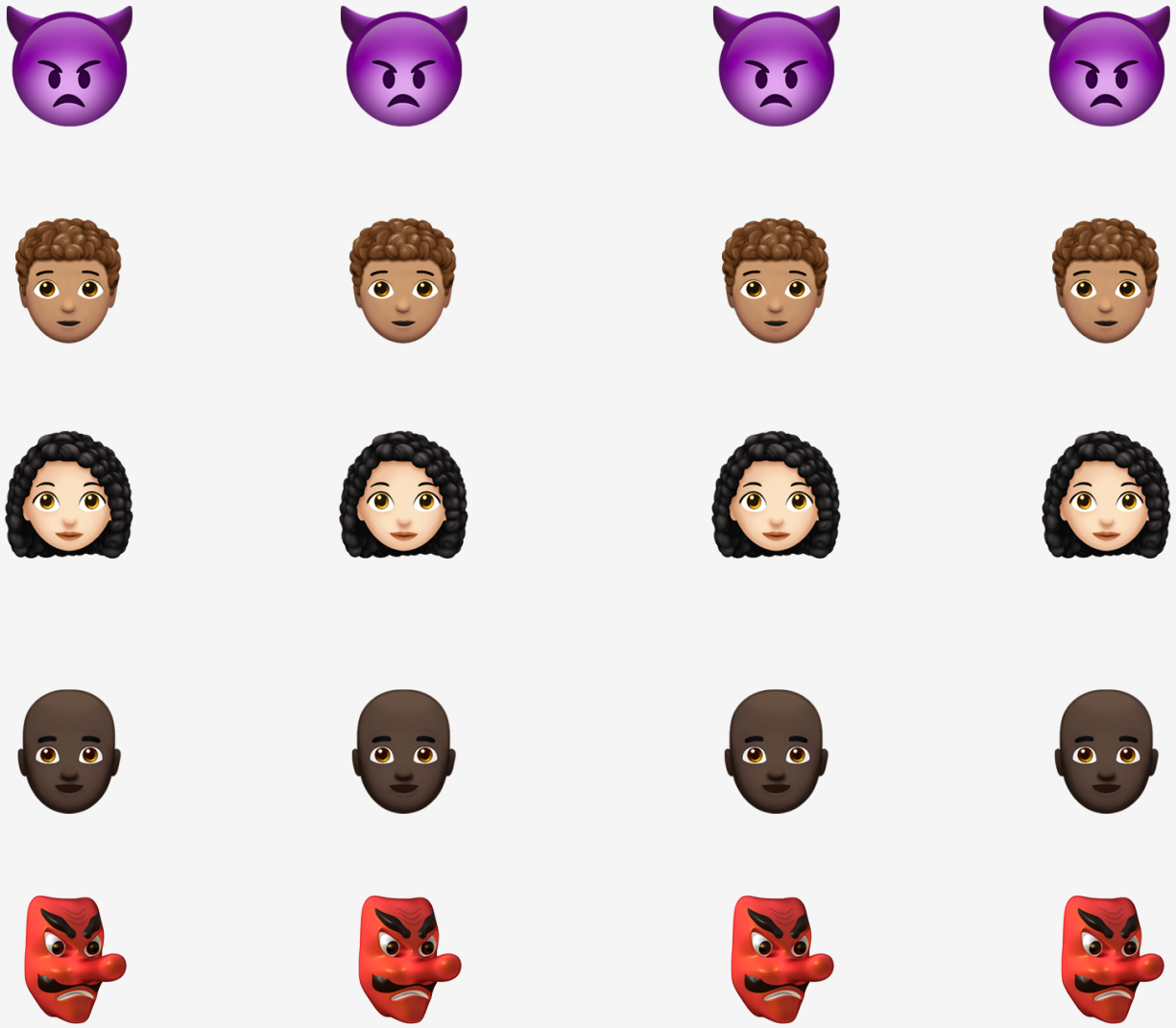

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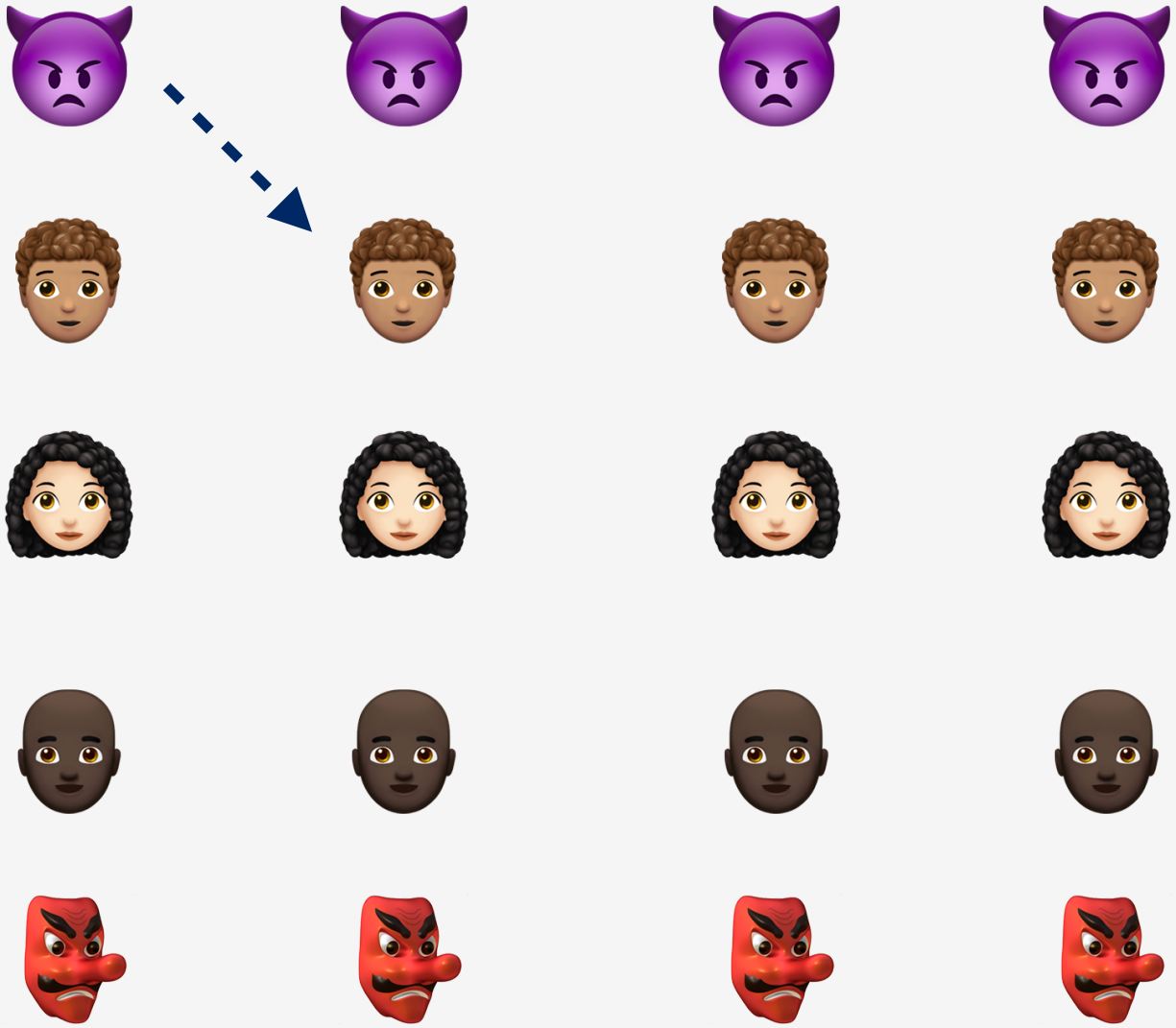

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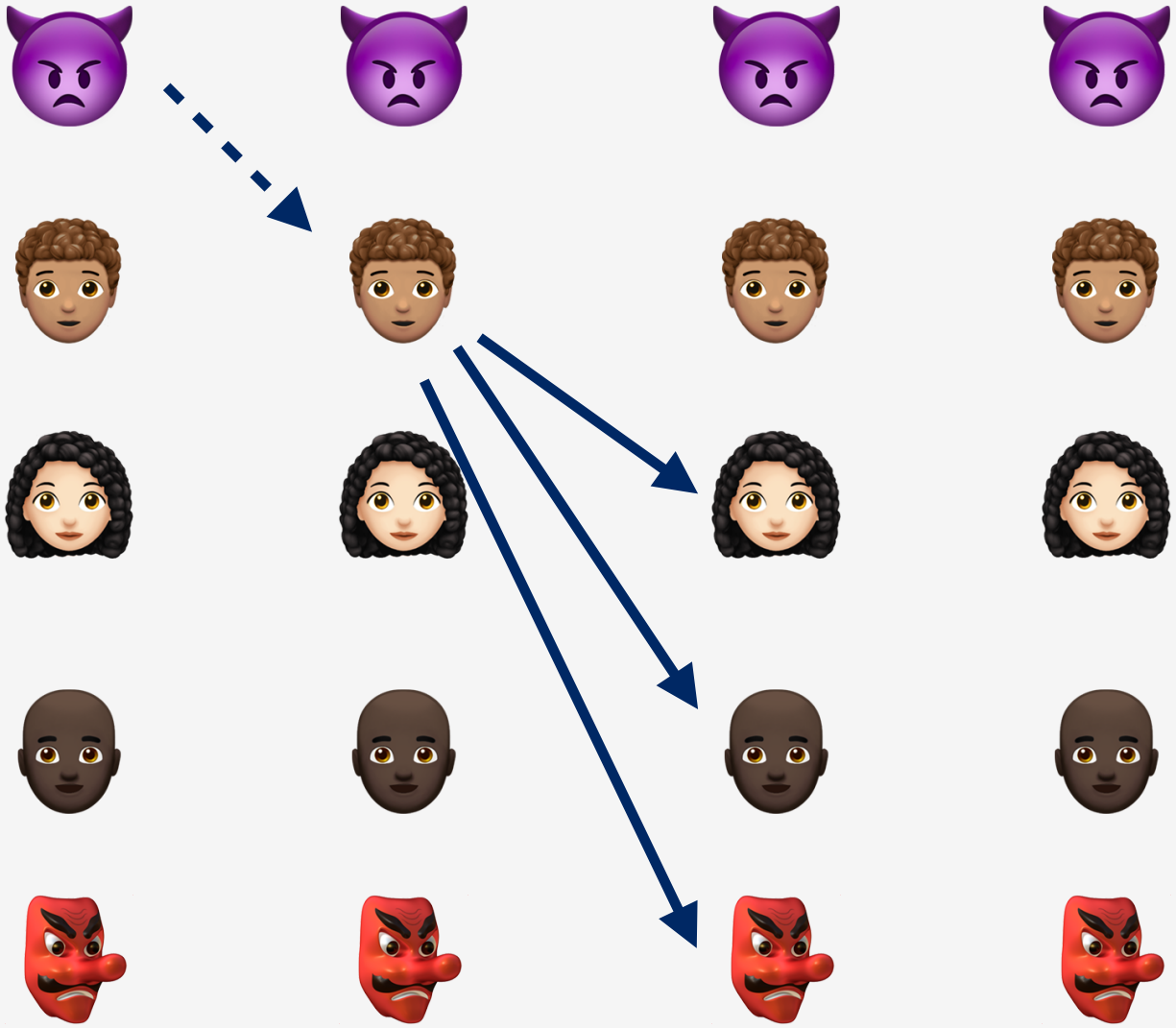

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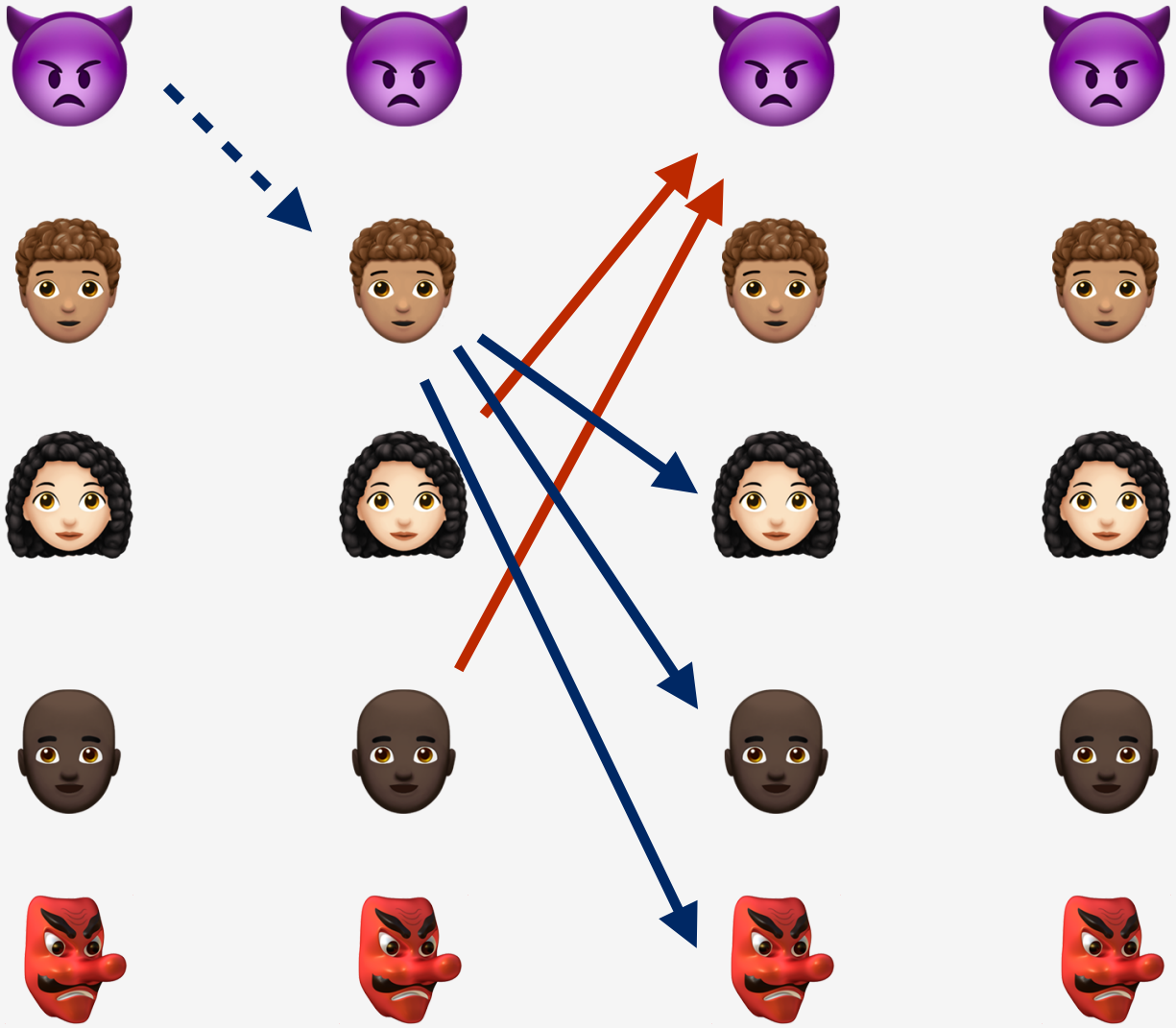

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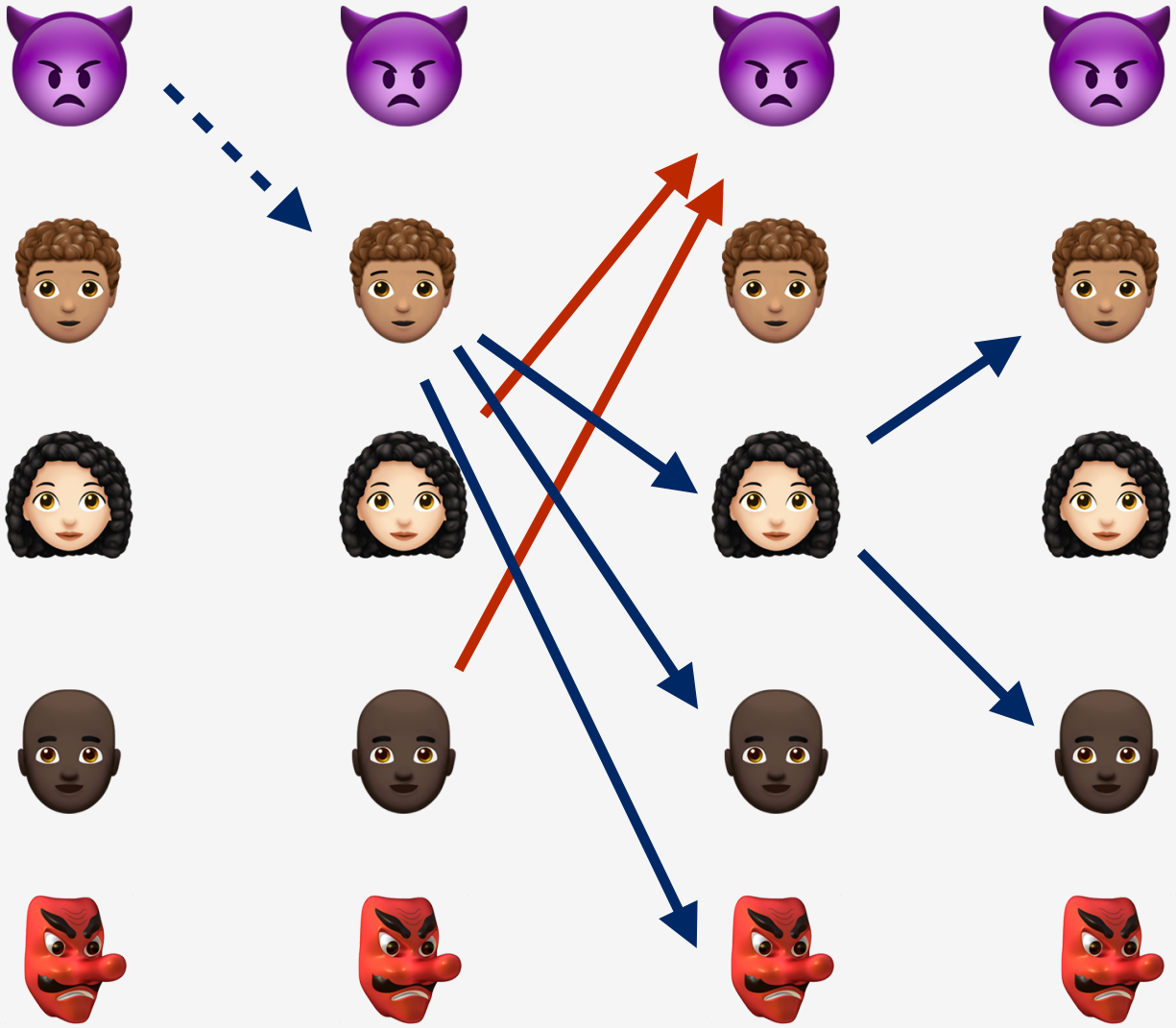

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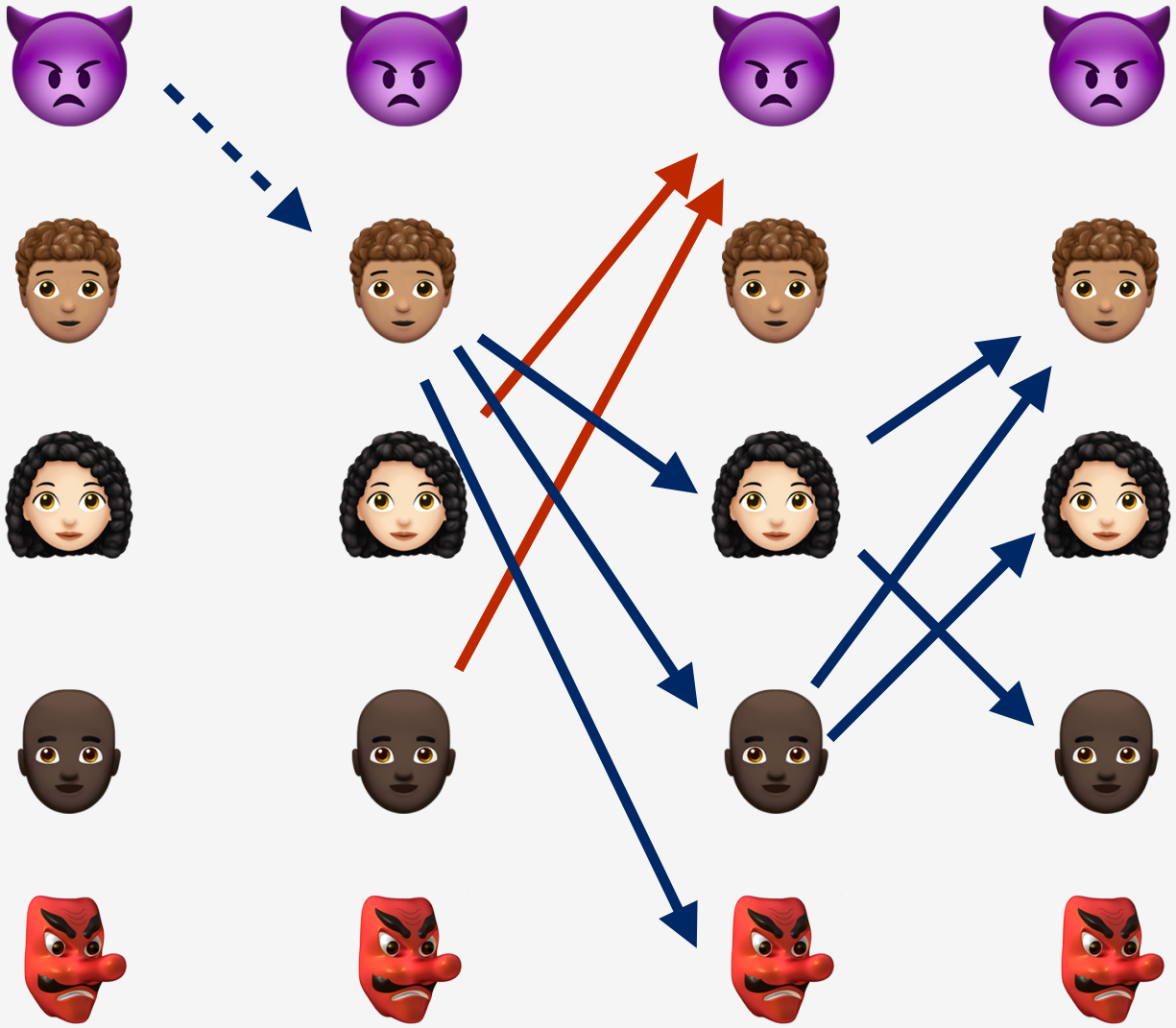

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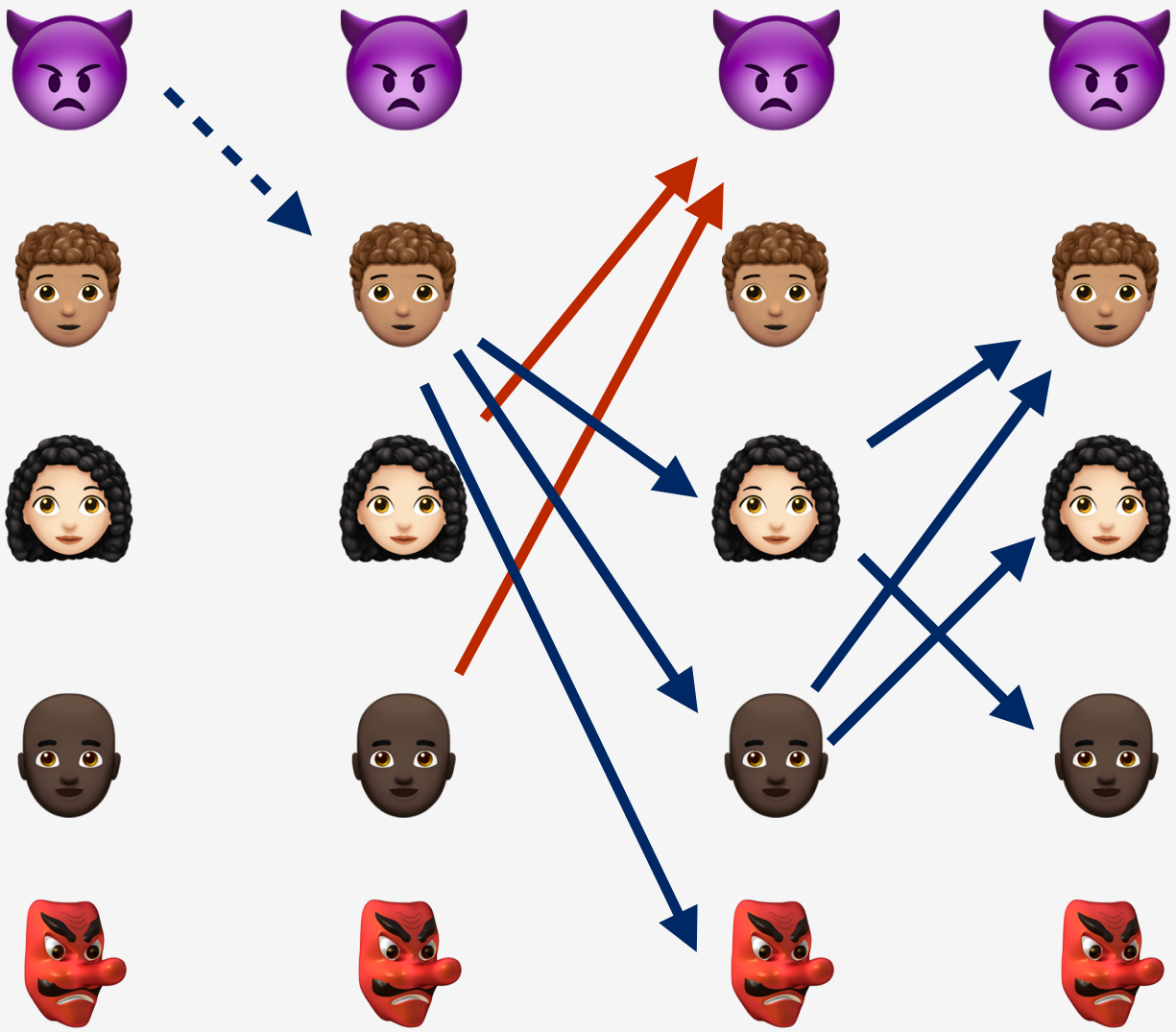

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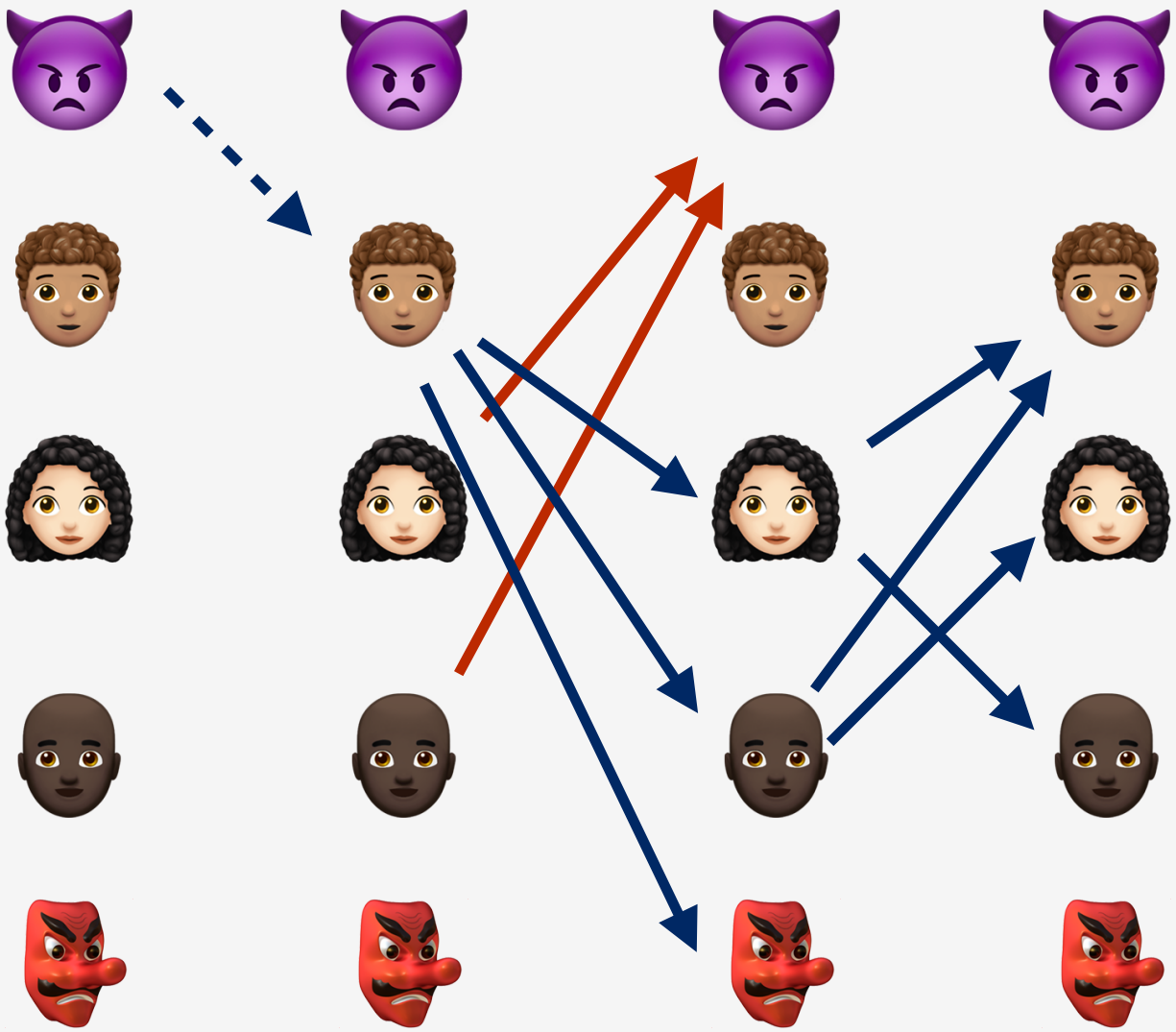

- All parties output P_1's message by Round 4
- No malicious party identified

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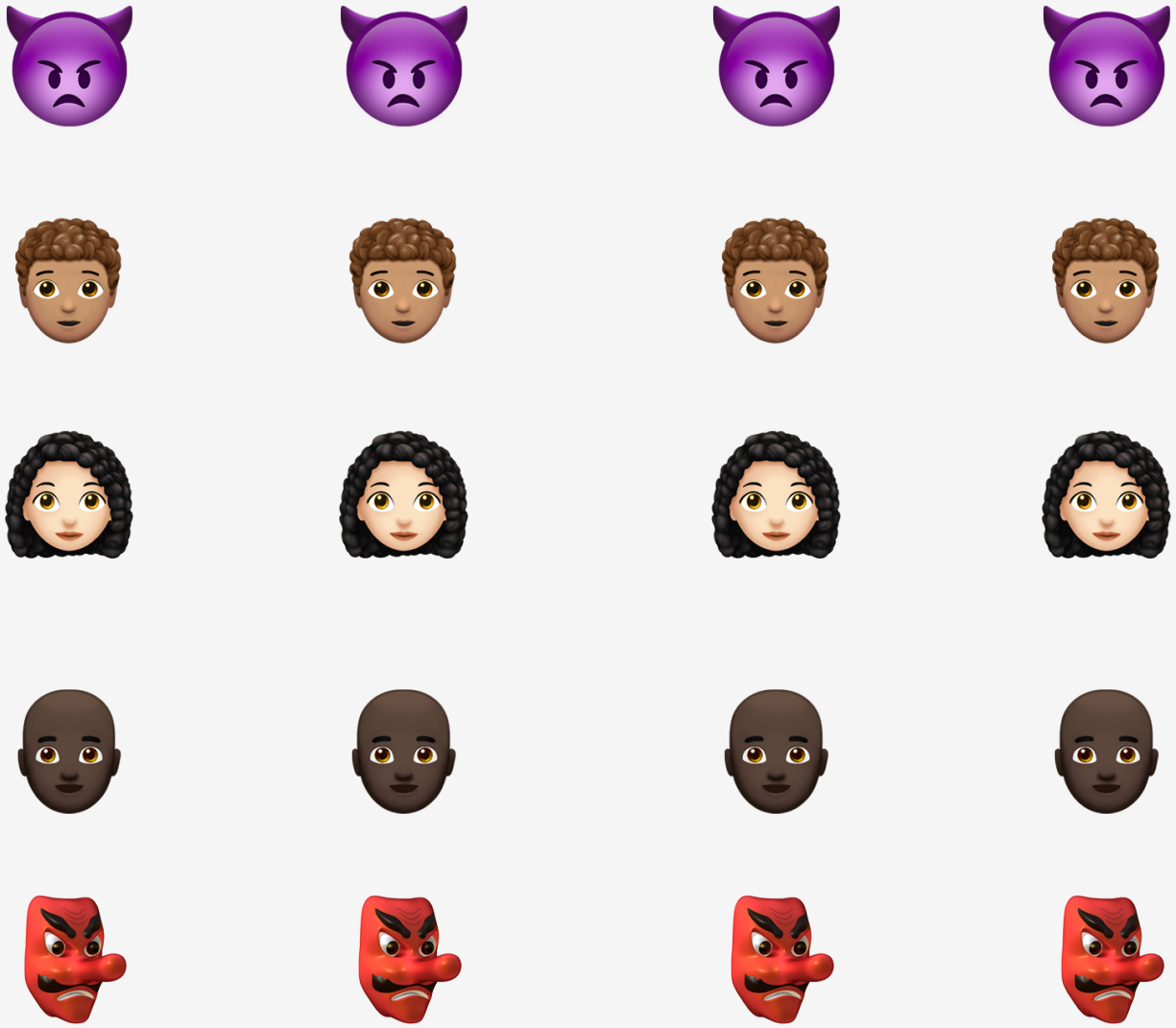

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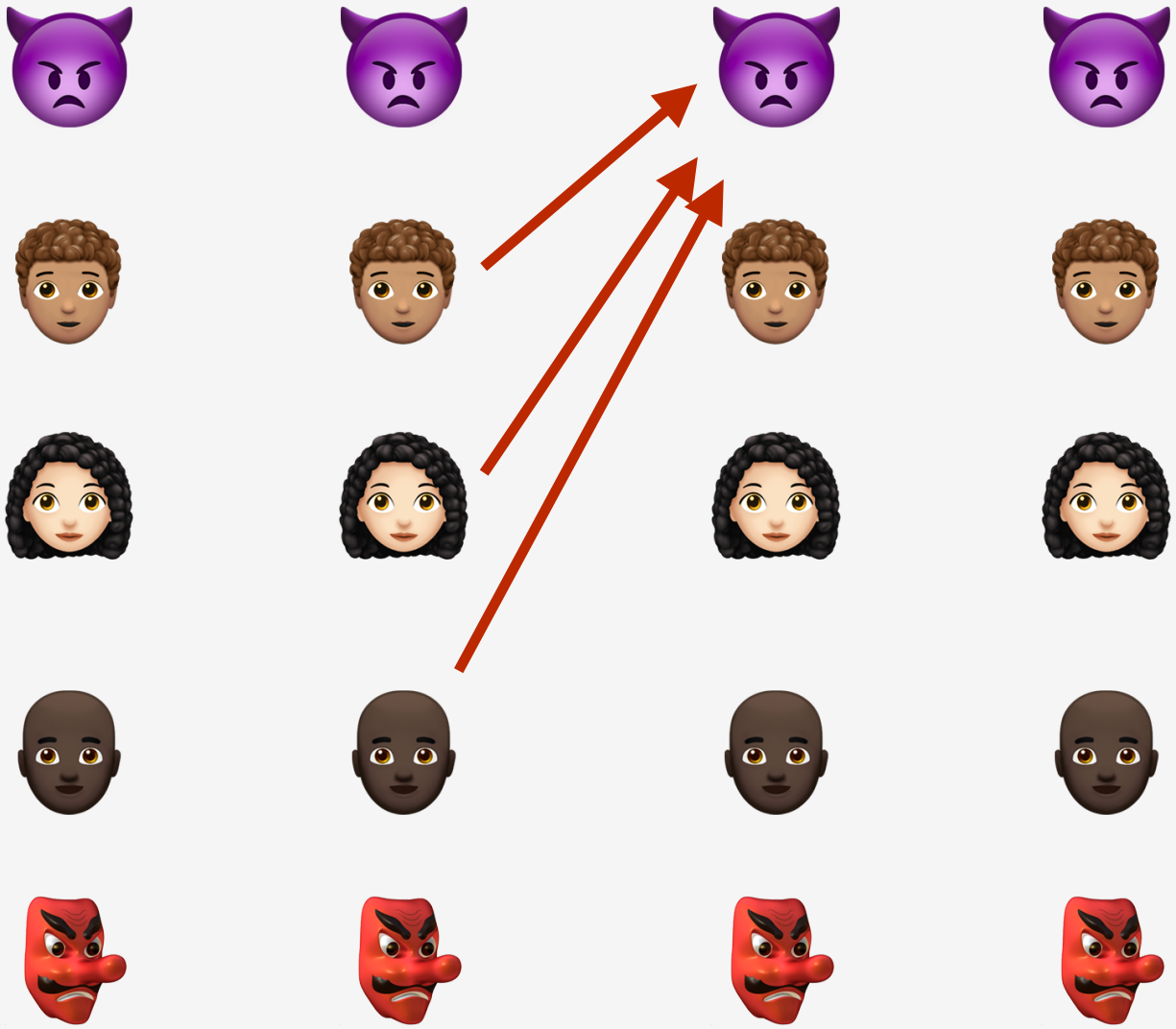

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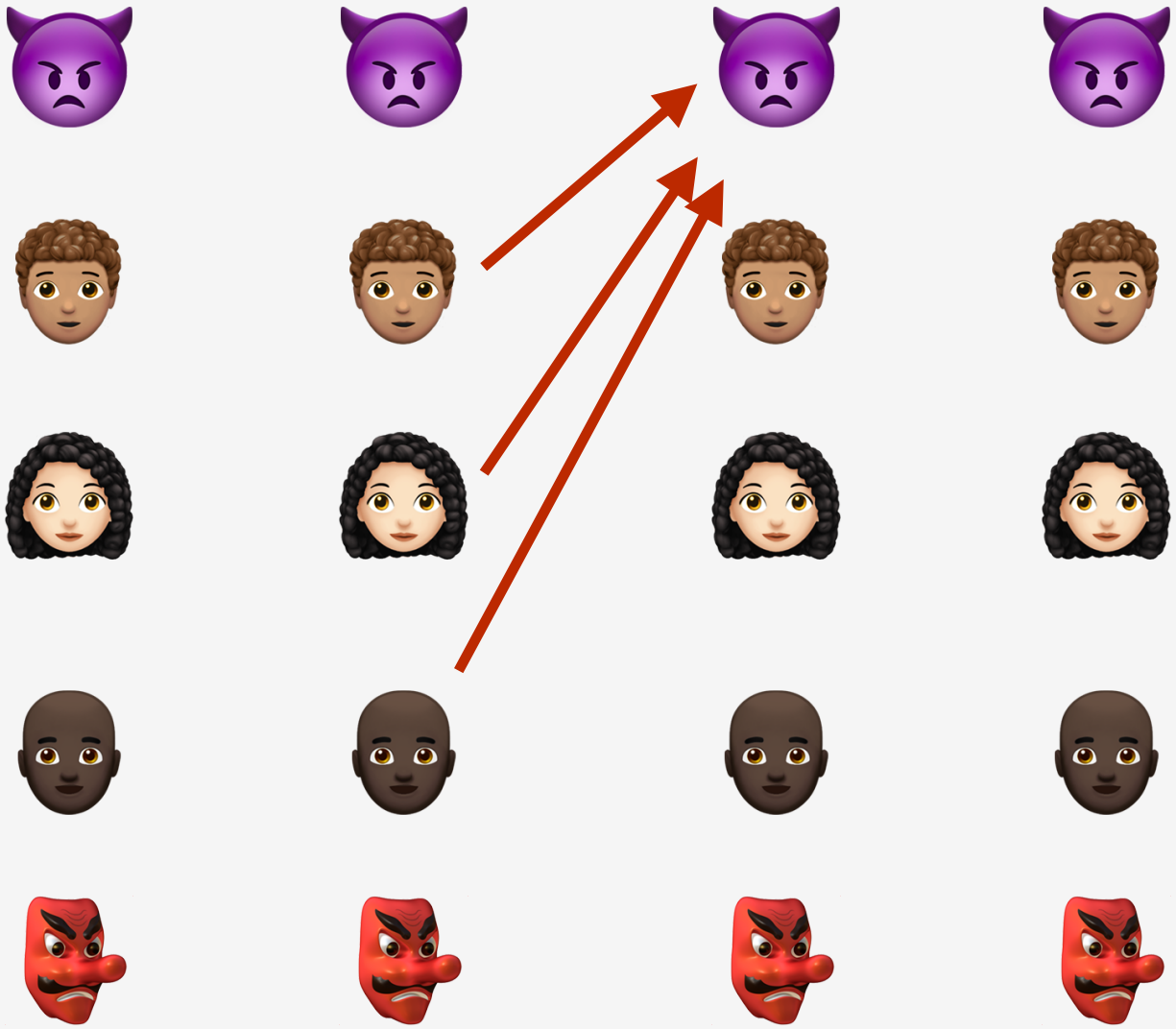

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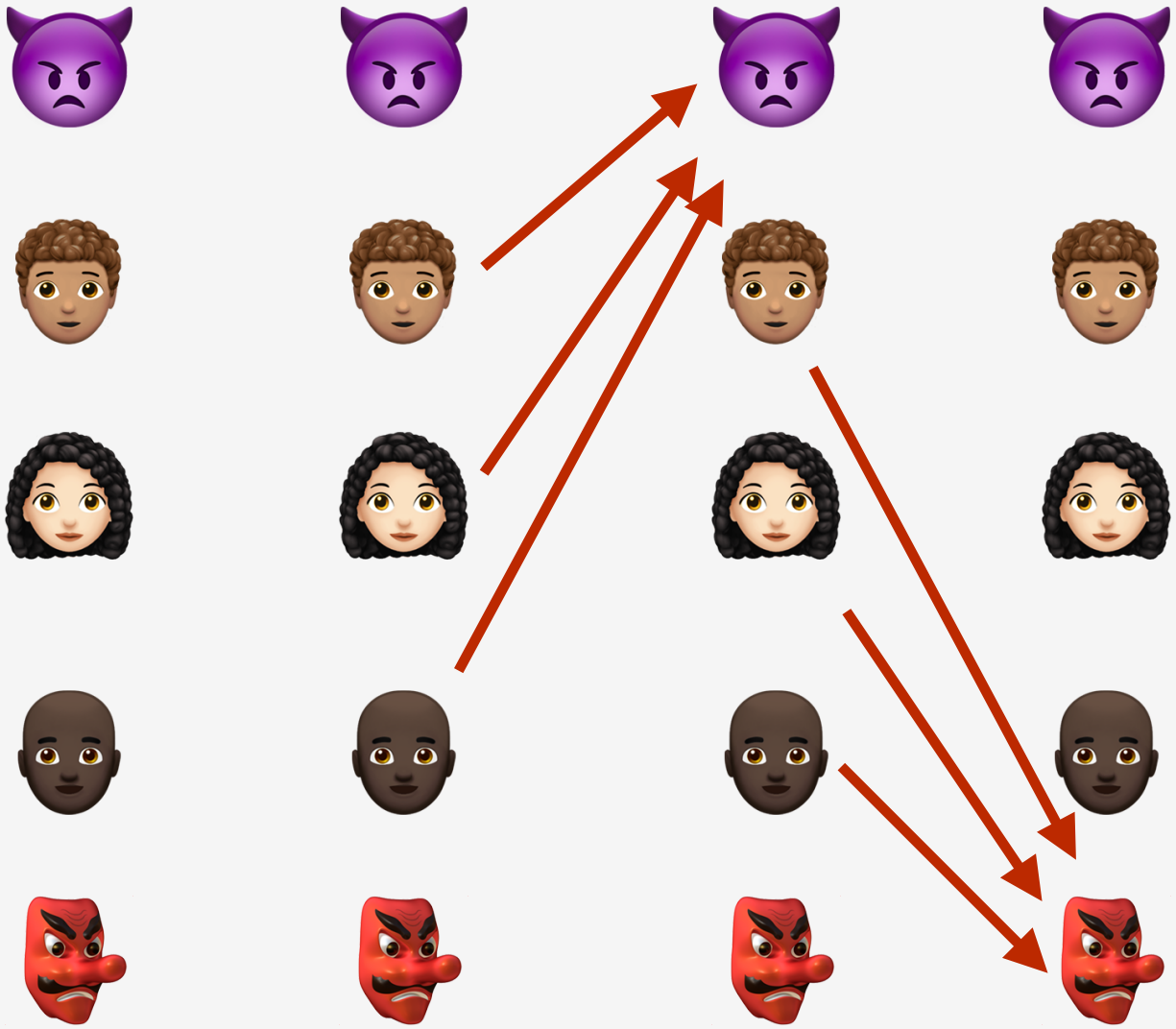

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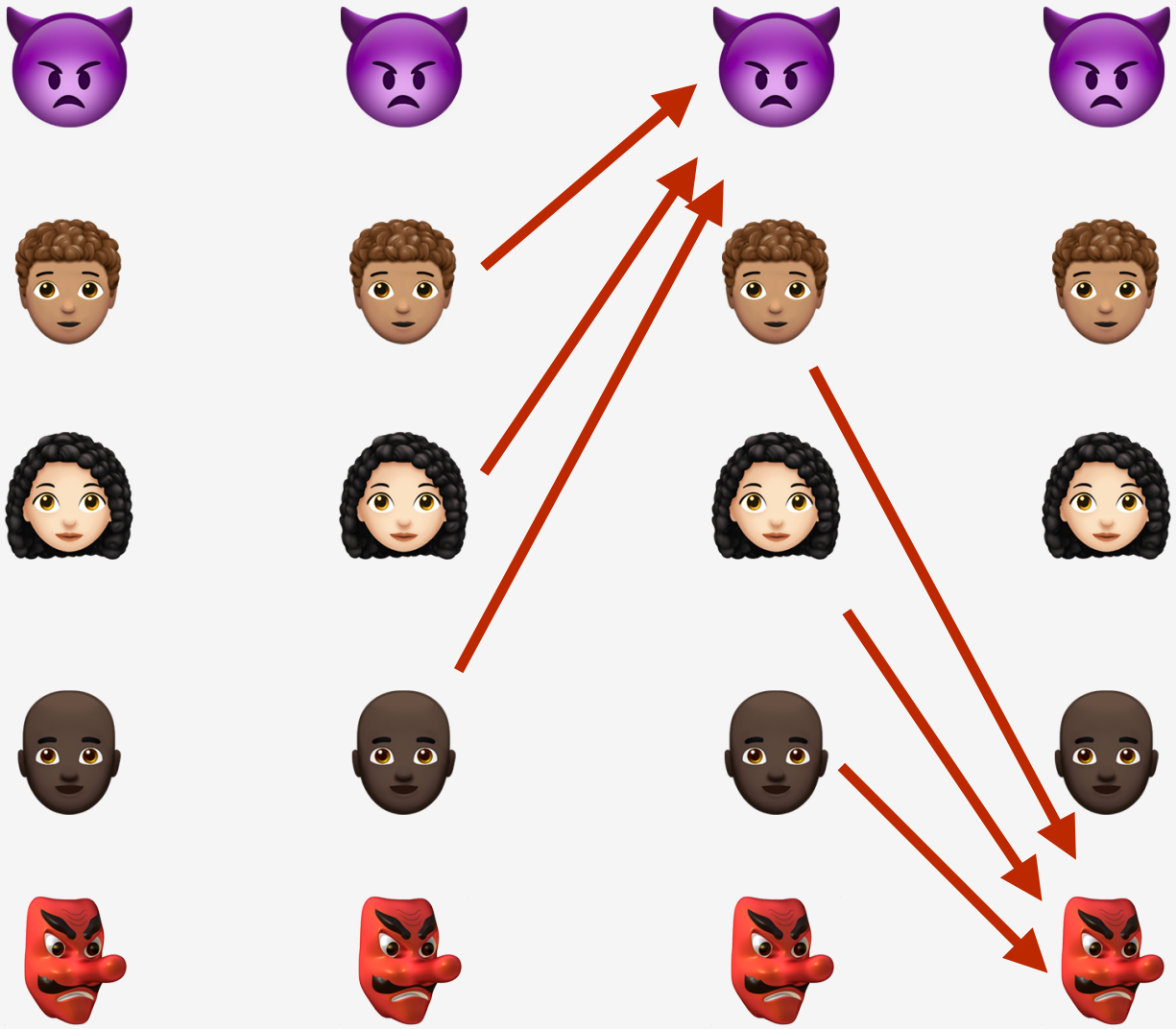

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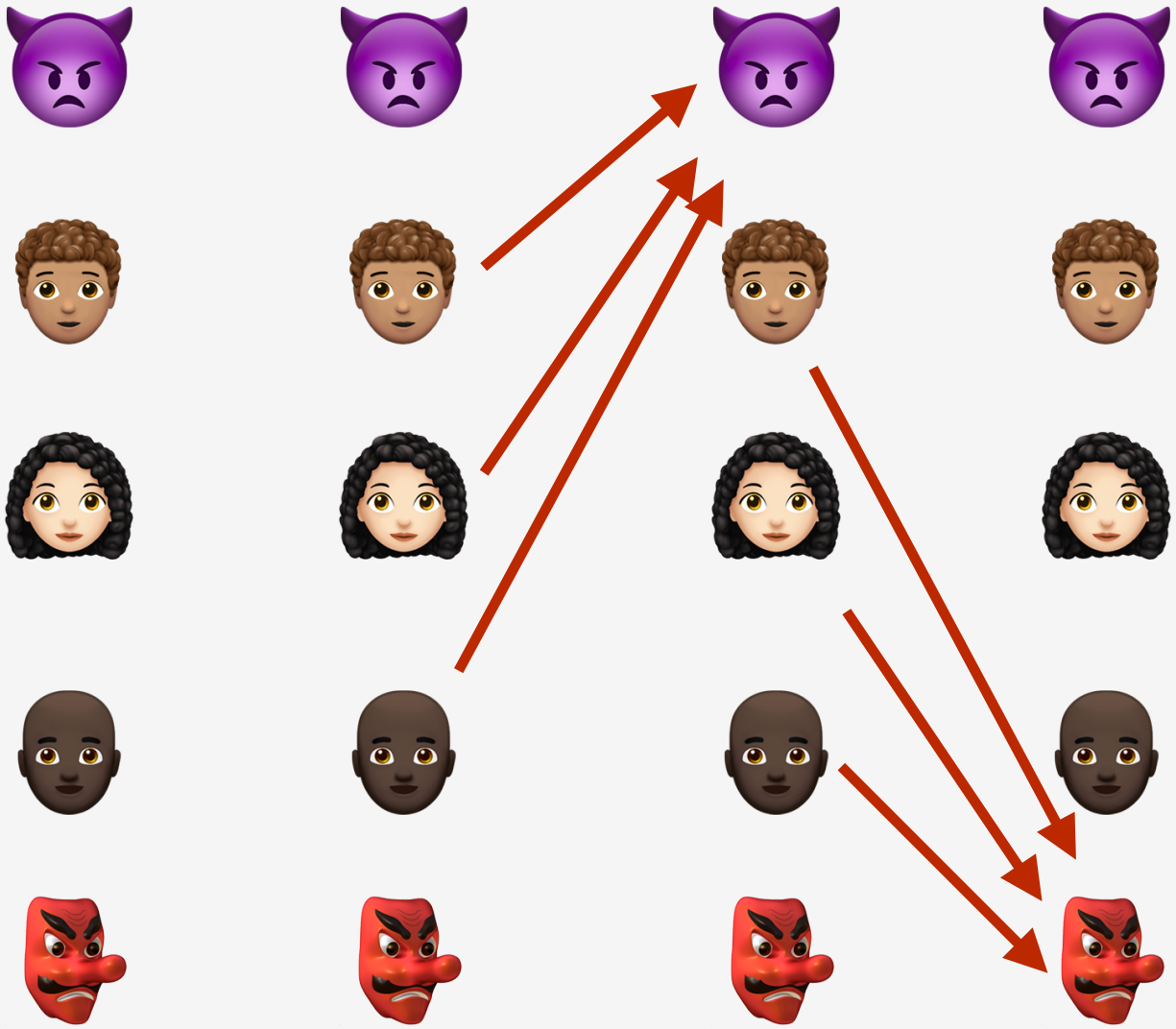

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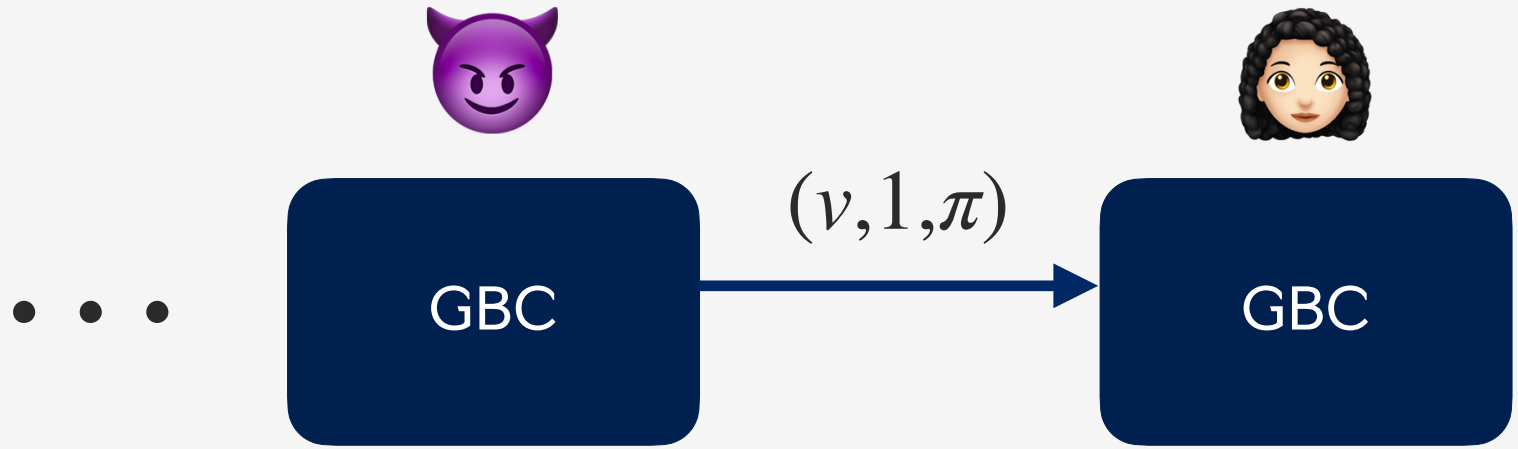

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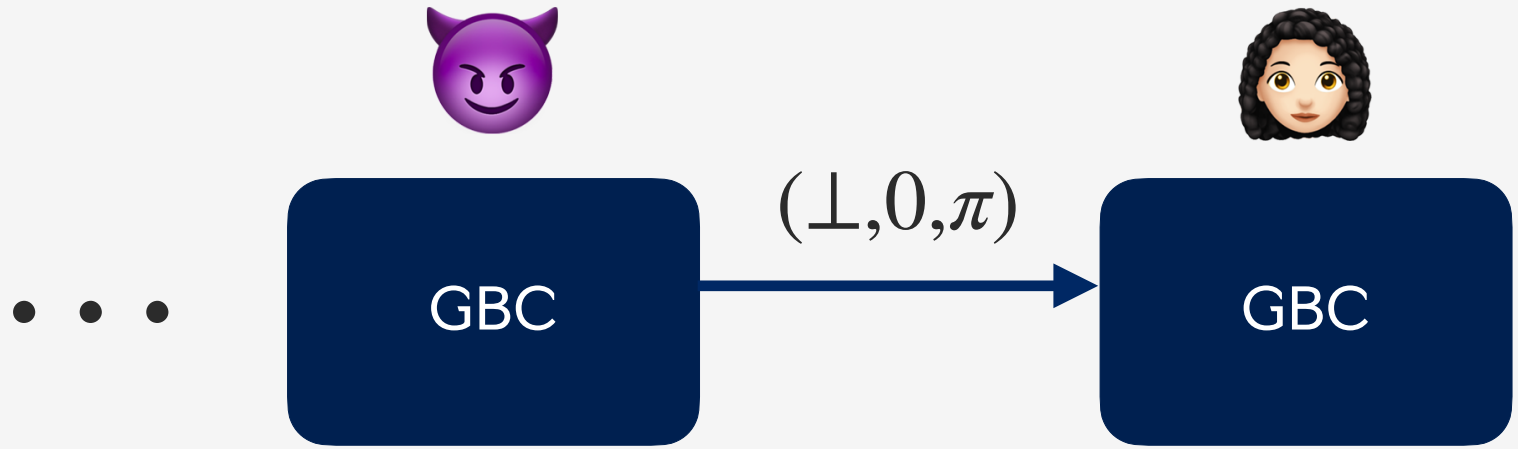

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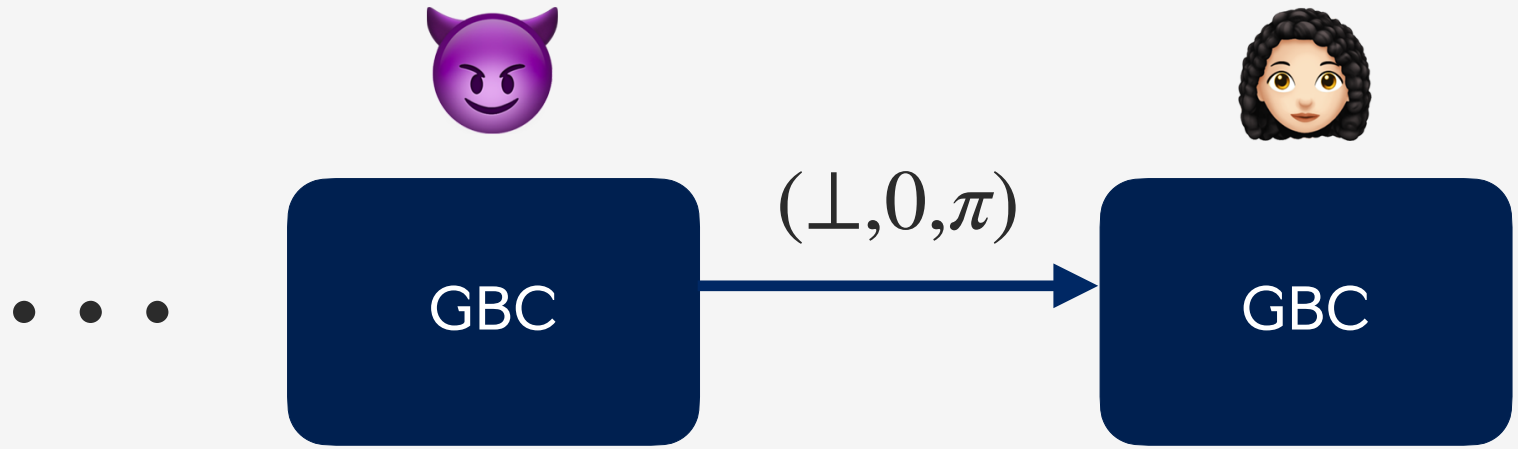

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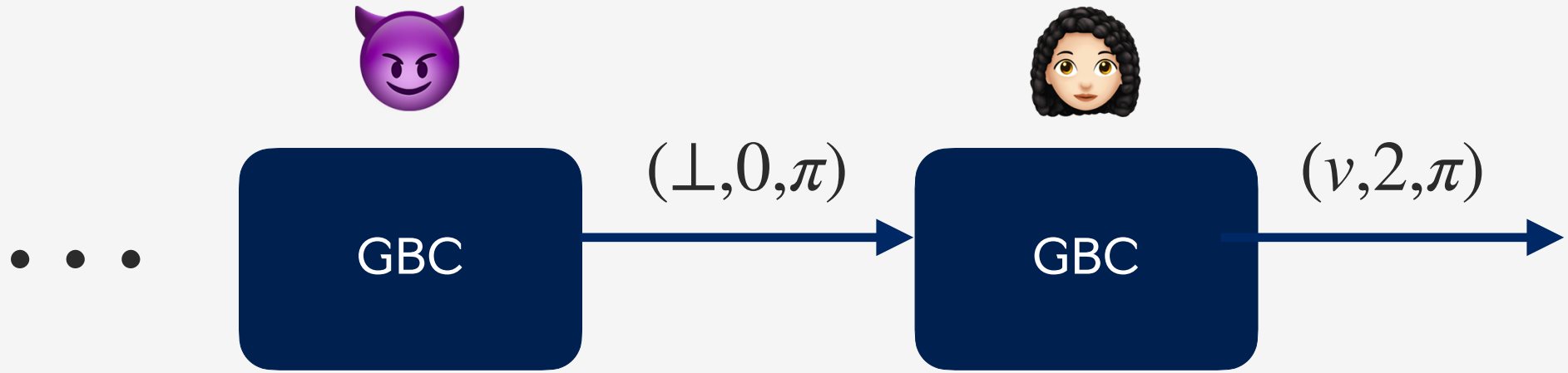

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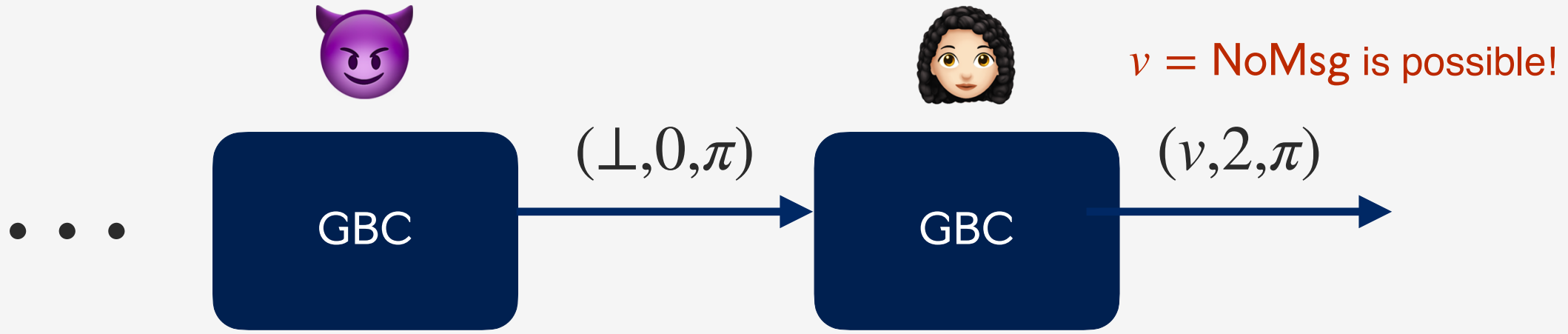

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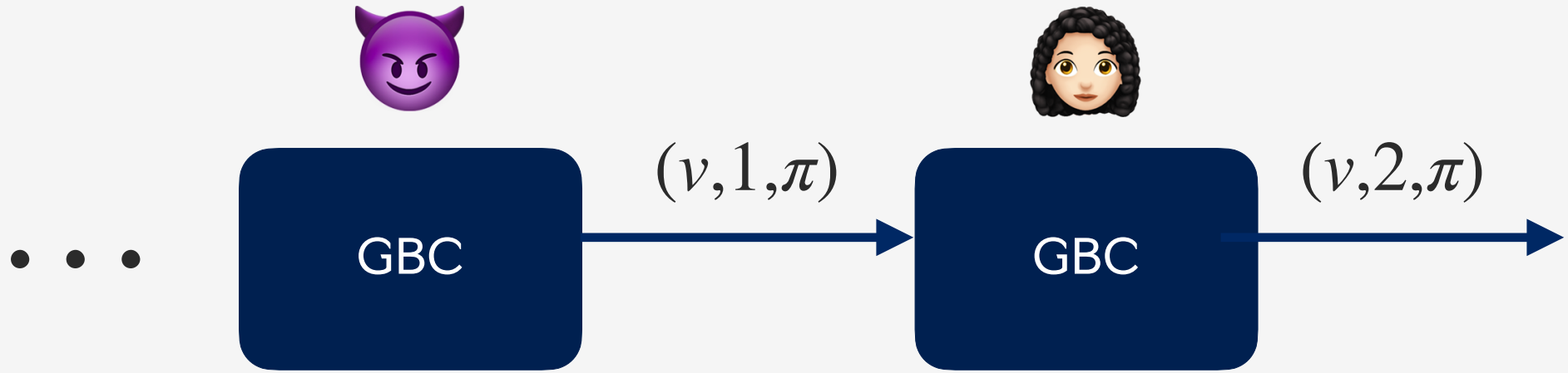

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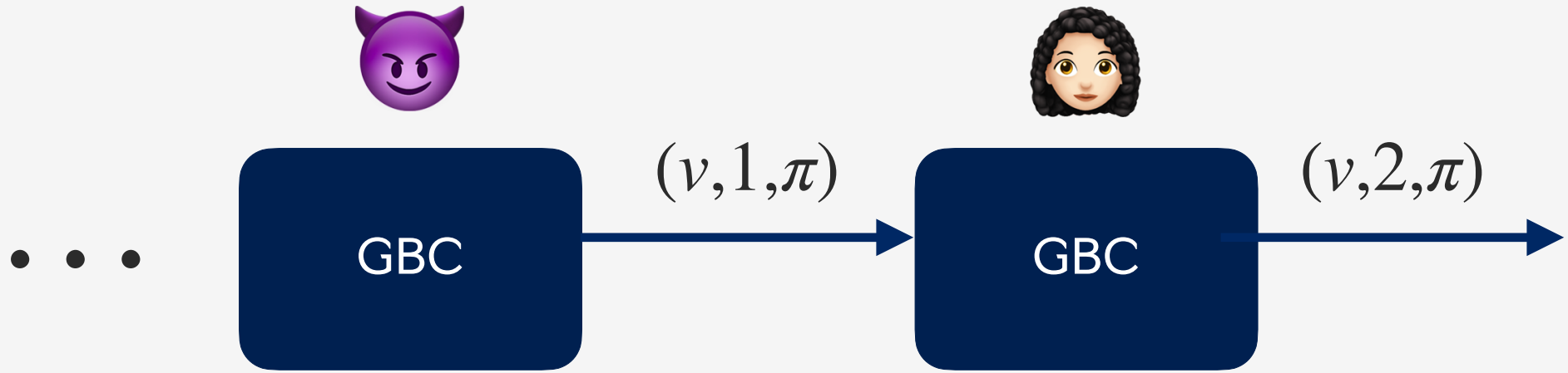

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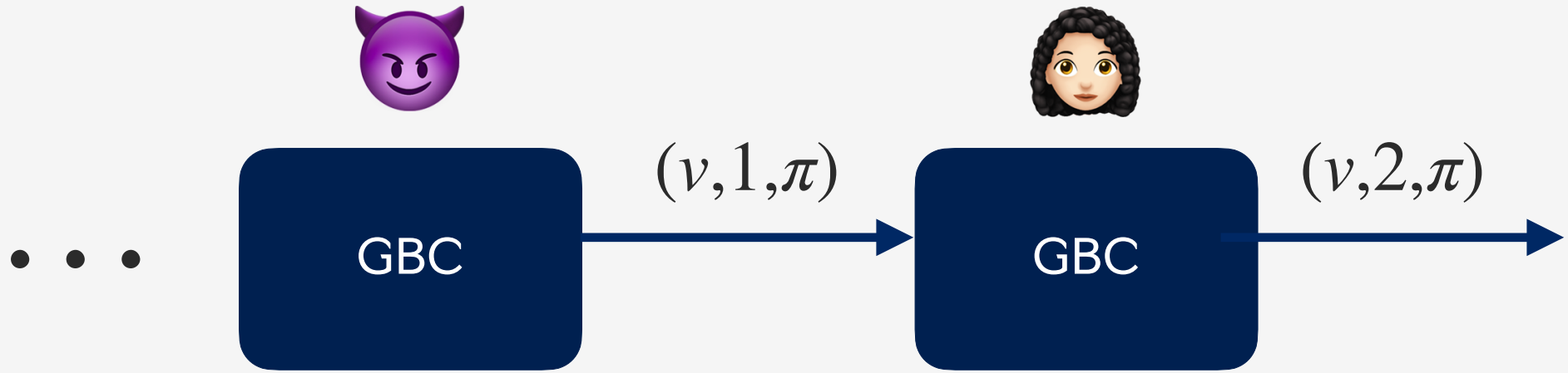

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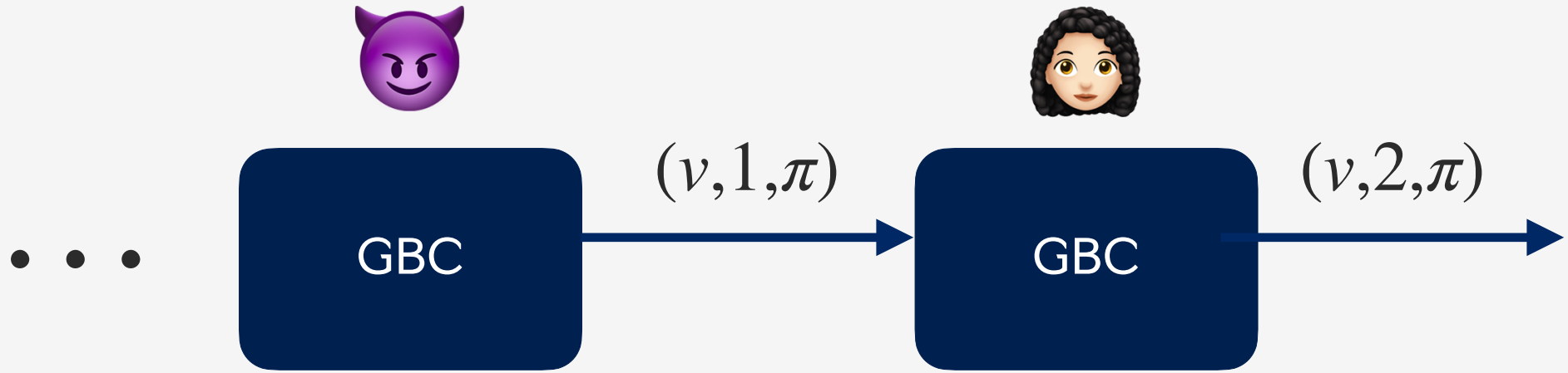

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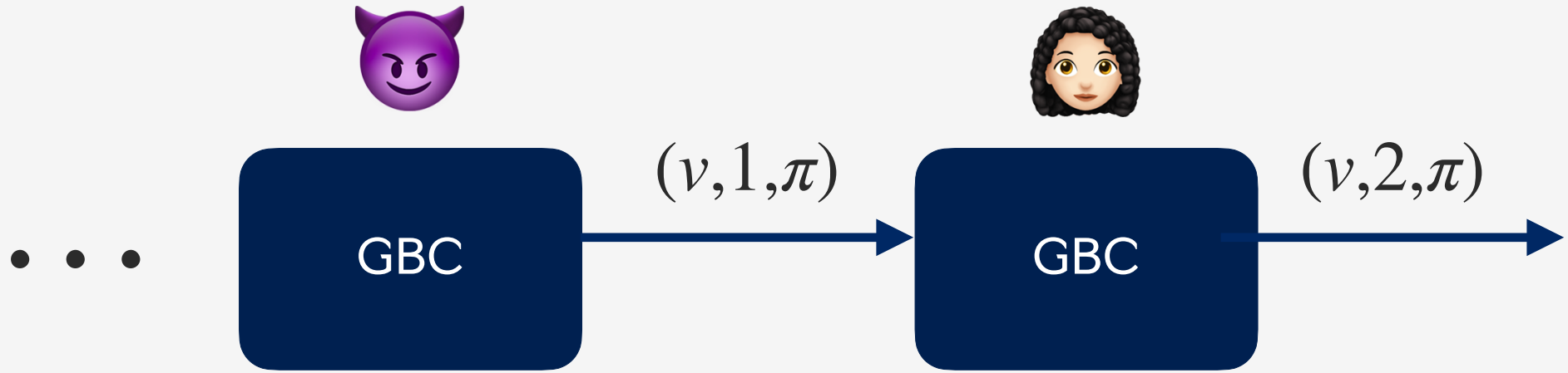

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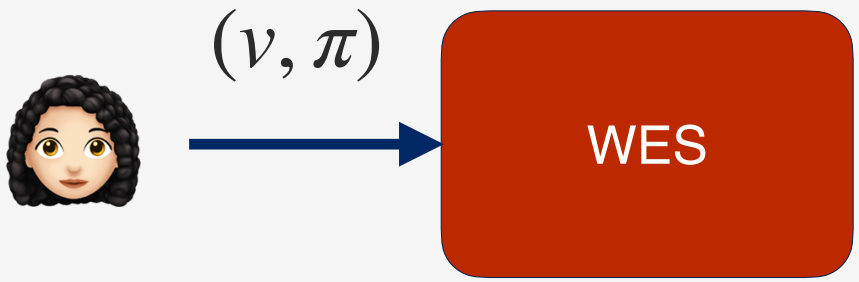

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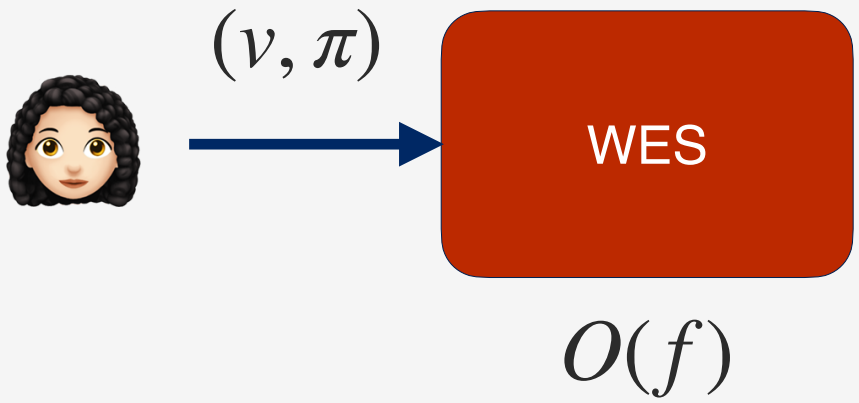

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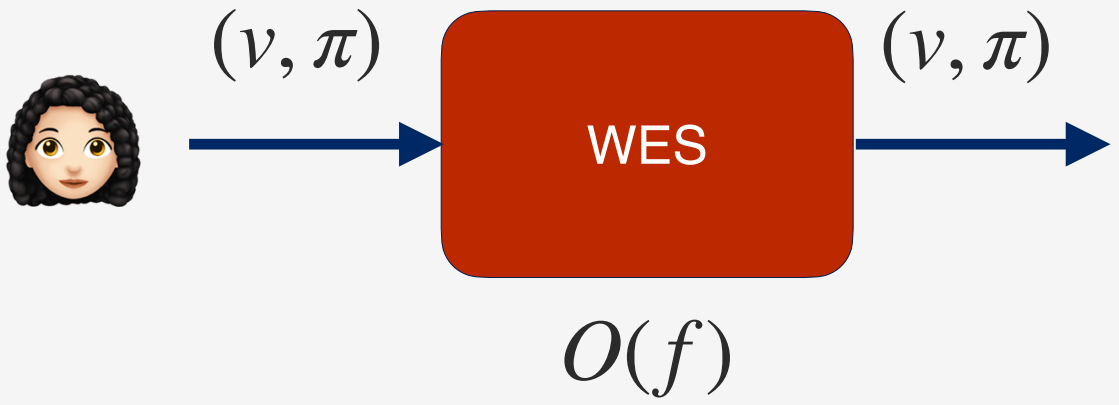

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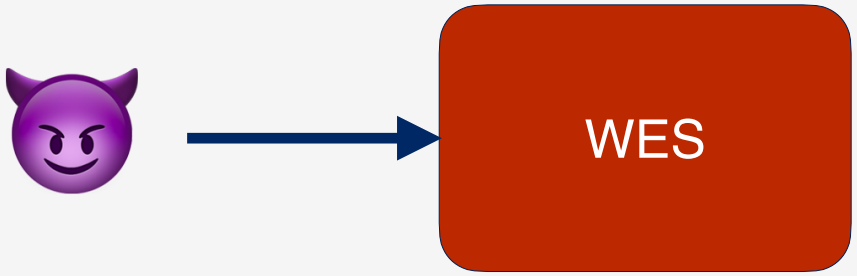

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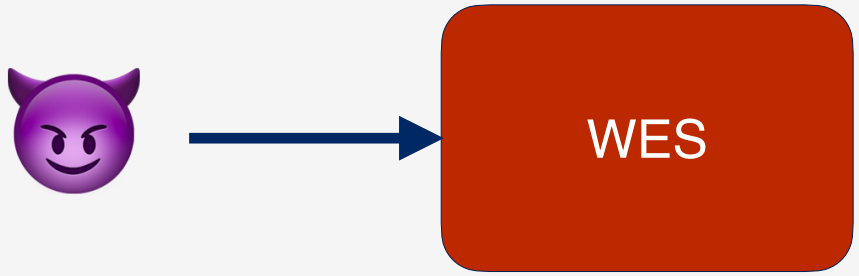

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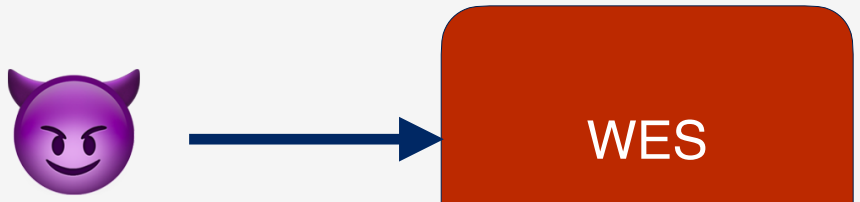

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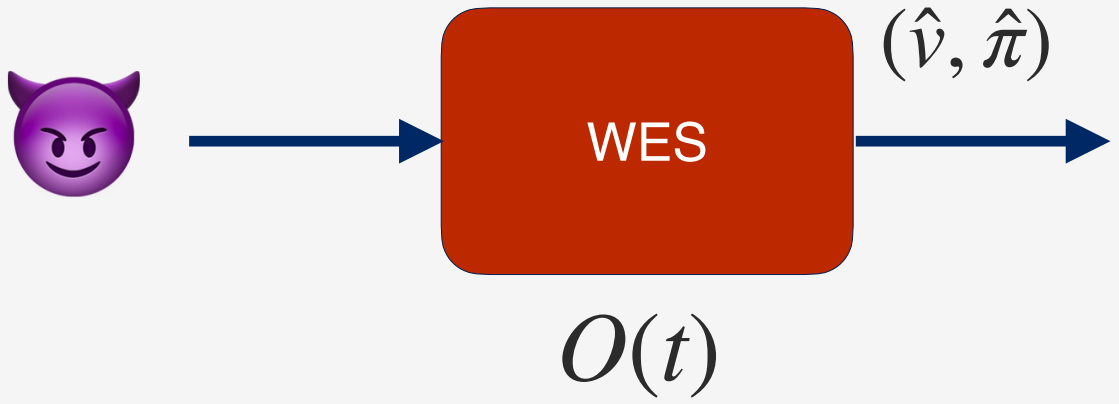

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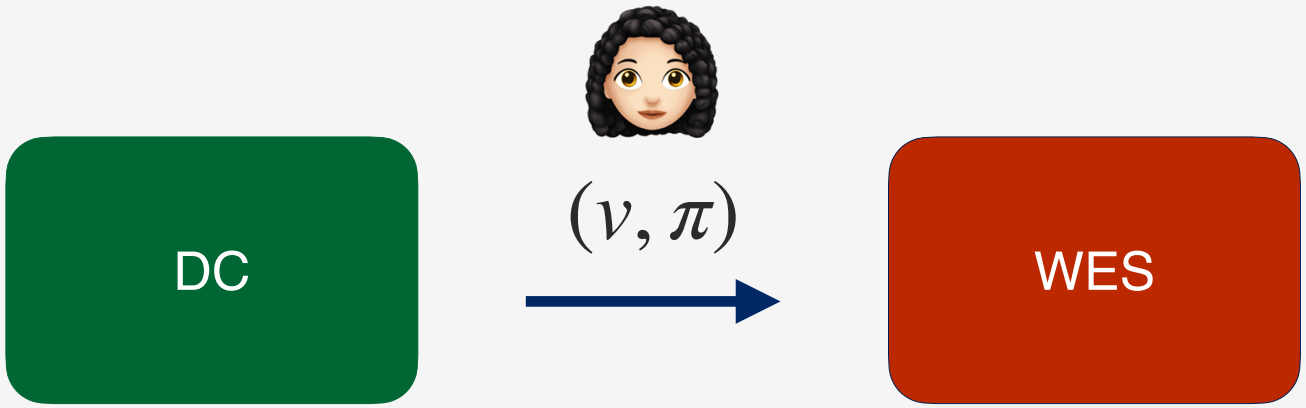

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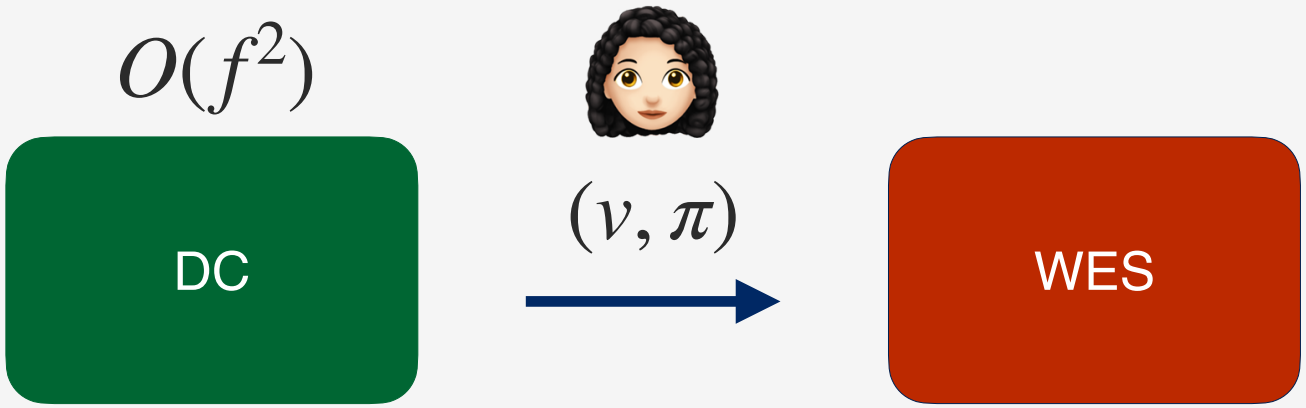

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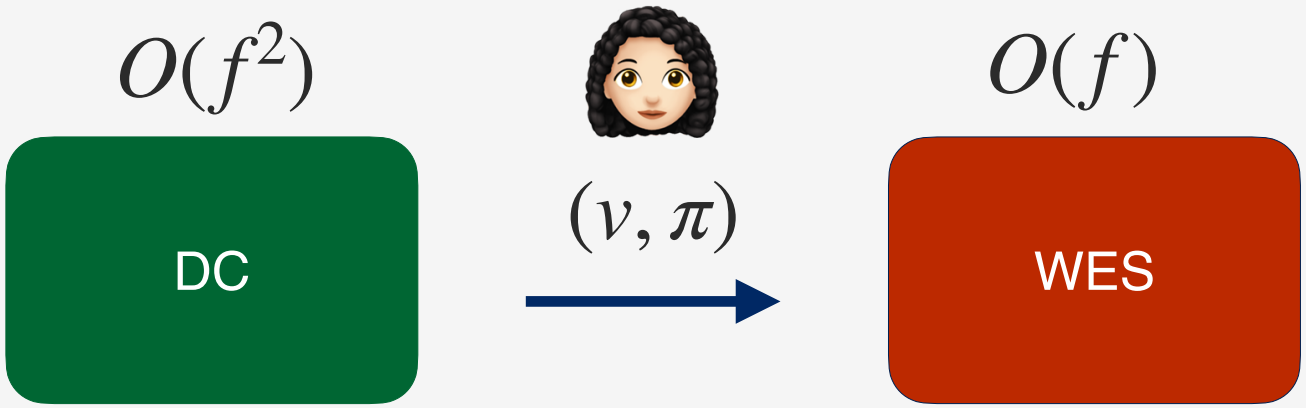

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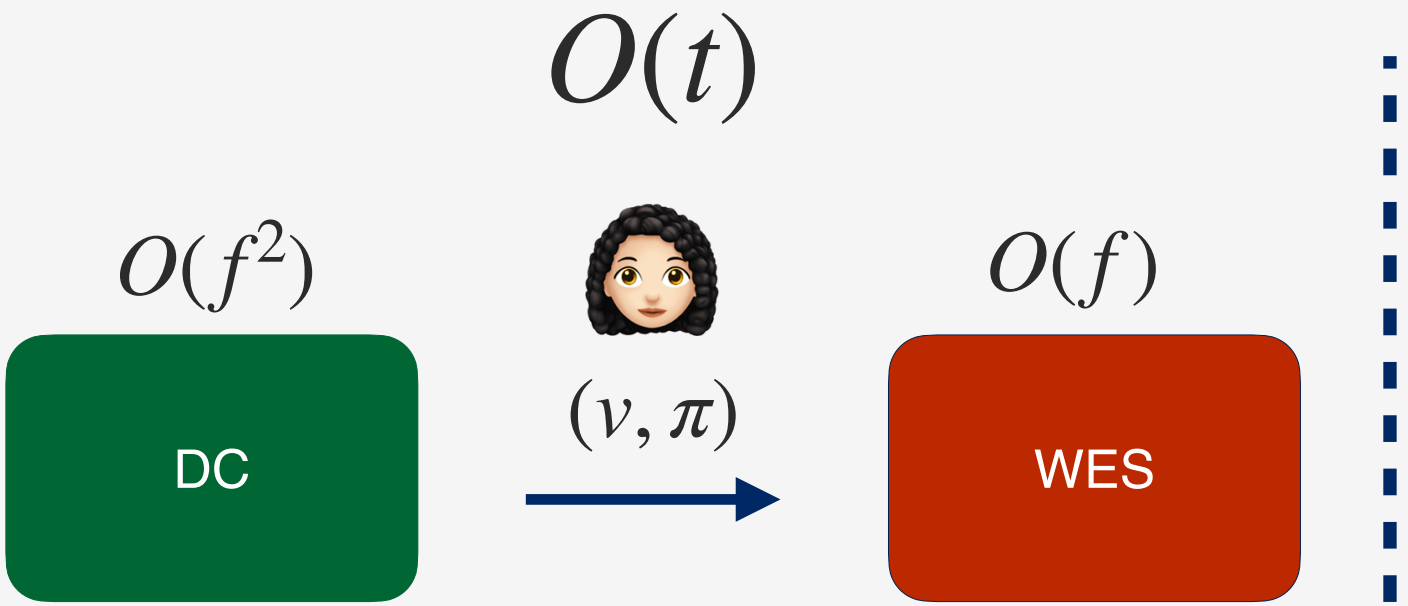

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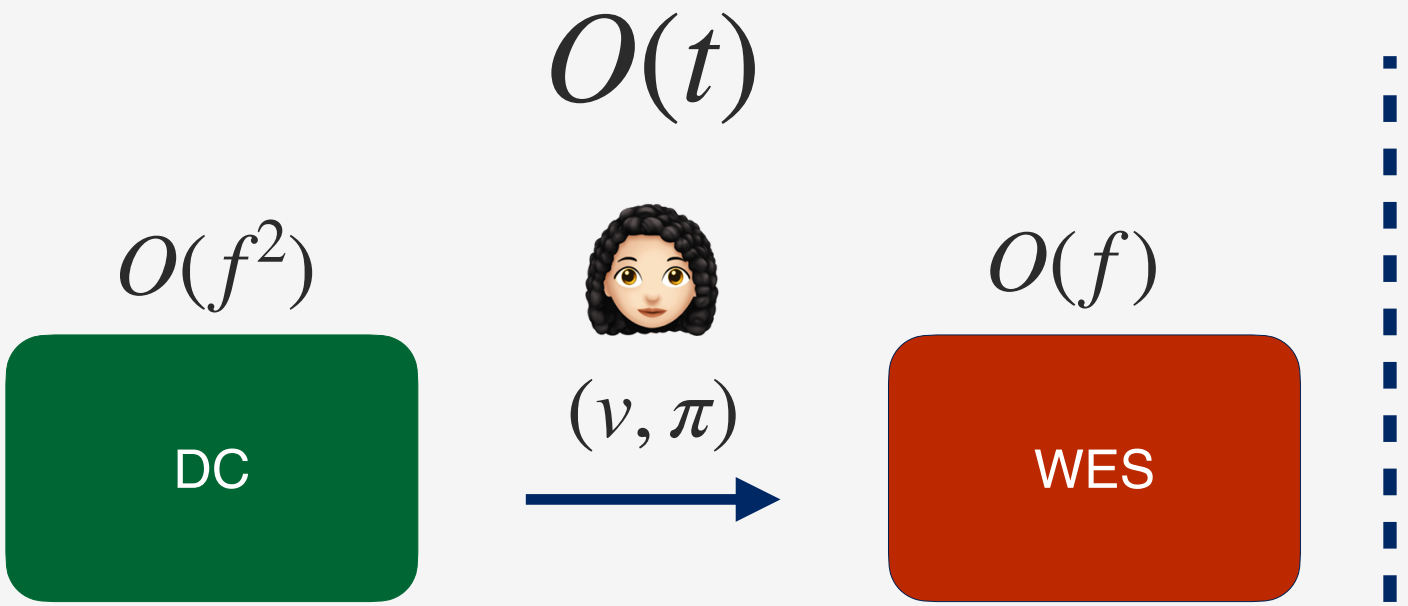

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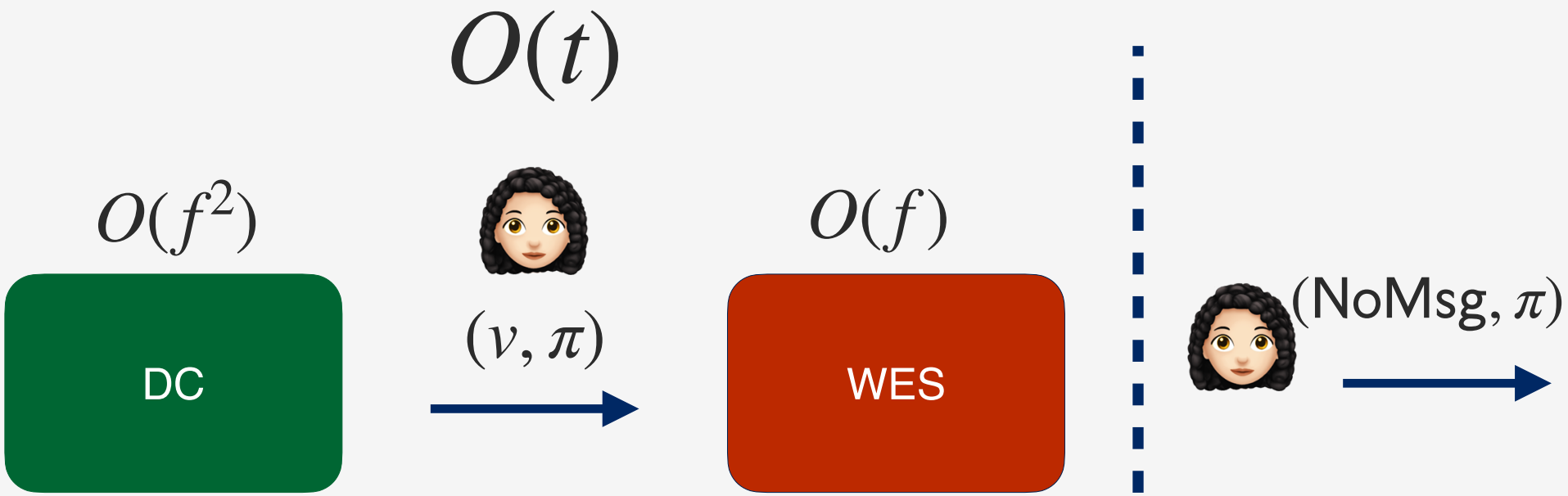

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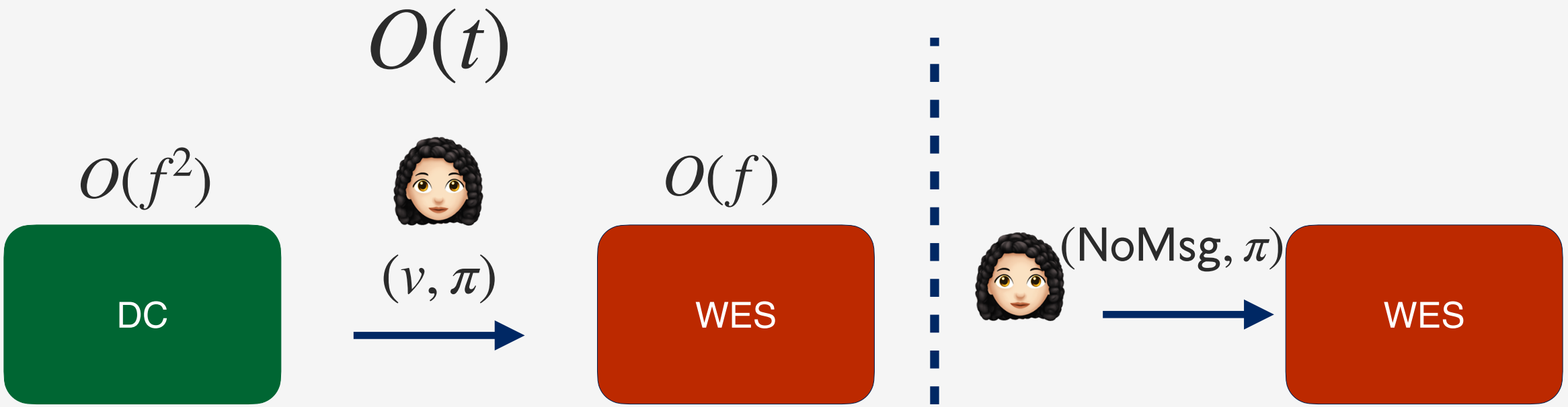

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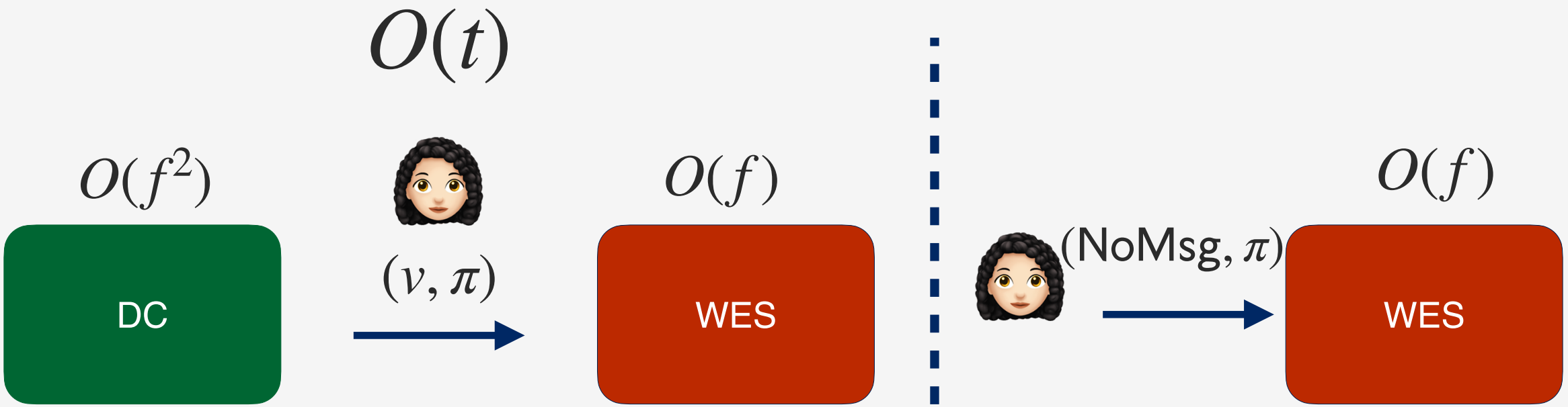

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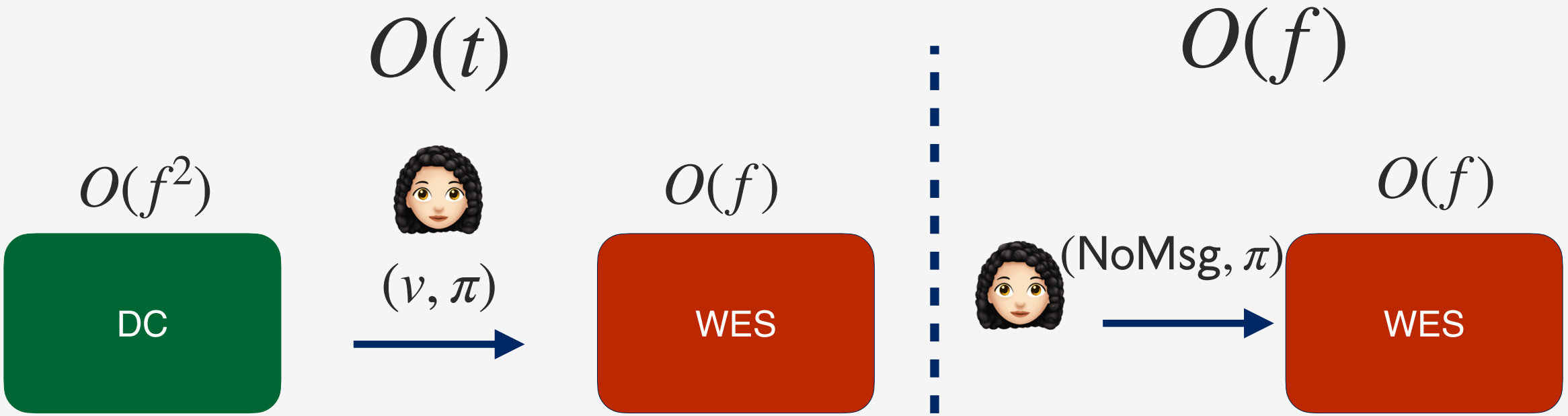

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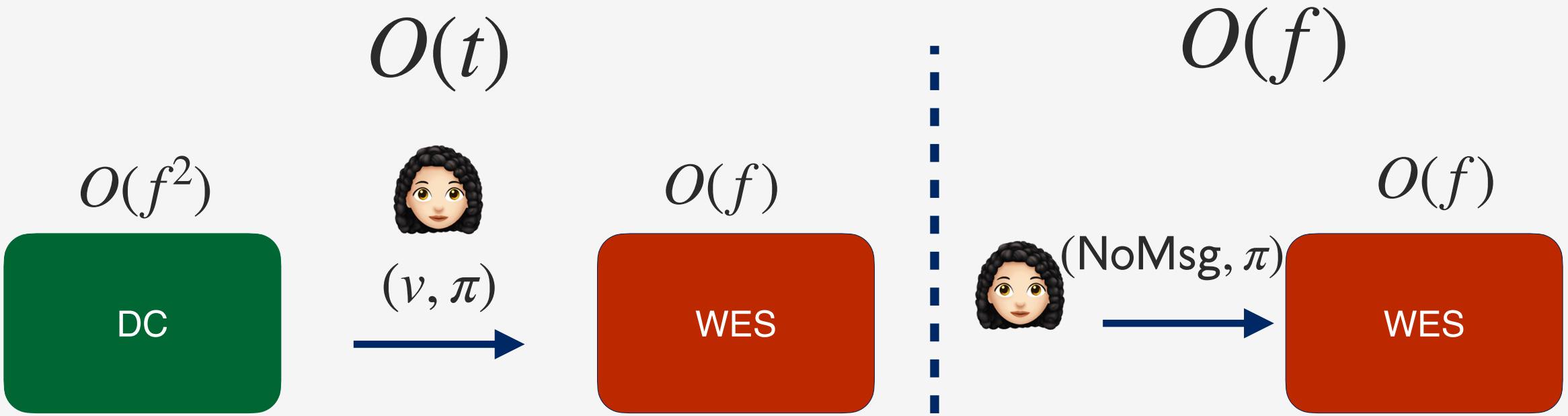

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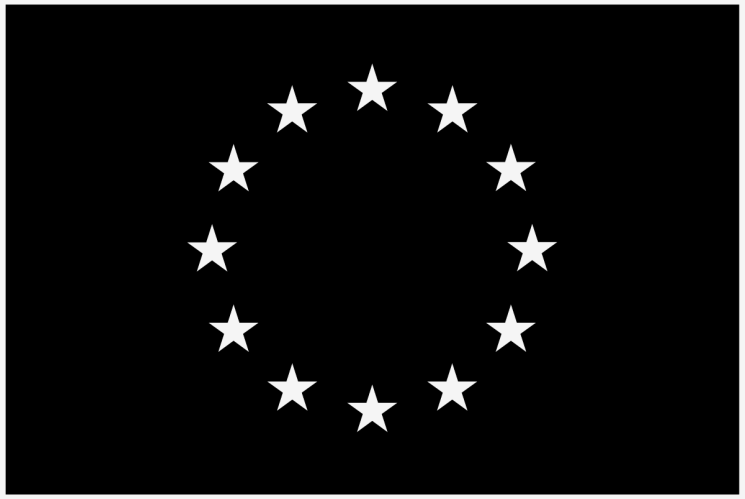
Funding Information



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Thank you!