A generic algorithm for efficient key recovery in differential attacks – and its associated tool

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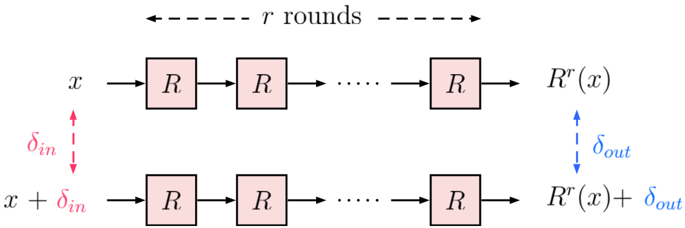
Differential cryptanalysis

- Cryptanalysis technique introduced by Biham and Shamir in **1990**.
- Based on the existence of a high-probability **differential** $(\delta_{in}, \delta_{out})$.



- If the probability of $(\delta_{in}, \delta_{out})$ is (much) higher than $\max(2^{-n}, 2^{-\kappa})$, where n is the block size, κ the key length, then we have a differential distinguisher.

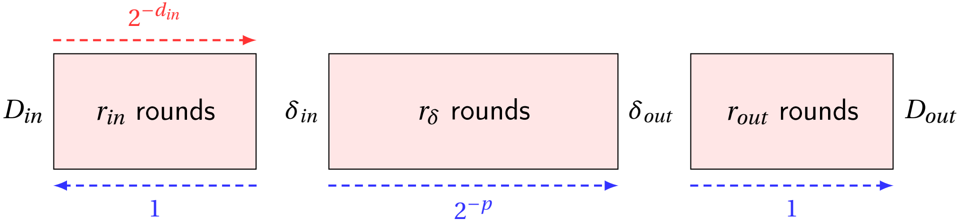
Key recovery attack

A differential distinguisher can be used to mount a key recovery attack.

- This technique broke many block ciphers of the 70s-80s, e.g. `DES`, `FEAL`, etc.
- New primitives should come with arguments of resistance by design against this technique.
- Most of the arguments used rely on showing that differential distinguishers of high probability do not exist after a certain number of rounds.
- Not always enough: A deep understanding of how the key recovery works is necessary to claim resistance against these attacks.

The key recovery problem

Overview of the key recovery procedure



First step: Construct $2^{p+d_{in}}$ pairs $((P,C),(P',C'))$ s.t. $P+P' \in D_{in}$.

- Use of structures of size $2^{d_{in}}$ → Data complexity: $\approx 2^{p+1}$, Memory complexity: $2^{d_{in}}$

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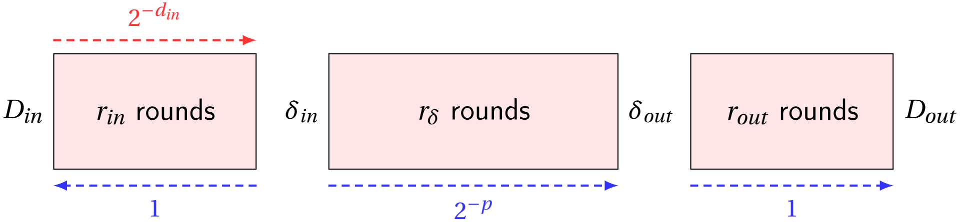

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- Number of pairs for the attack: $N = 2^{p+d_{in}-(n-d_{out})}$

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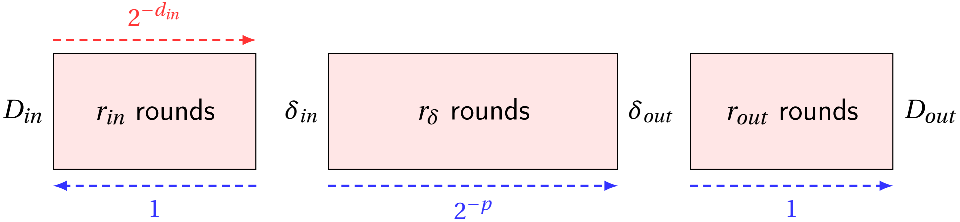

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Third step: Core key recovery

Core key recovery

Goal

Determine the pairs for which there exists an associated key that leads to the differential.

A candidate is a triplet $((P, C), (P', C'), k)$ such that the (partial) key candidate k encrypts (resp. decrypts) (P, P') (resp. (C, C')) to the input (resp. output) of the differential.

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What is the complexity of this procedure?

- Upper bound: $\min(2^{\kappa}, N \cdot 2^{|\mathcal{K}|})$,
- Lower bound: $N + N \cdot 2^{|\mathcal{K}|-d_{in}-d_{out}}$,
  where $N \cdot 2^{|\mathcal{K}|-d_{in}-d_{out}}$ is the number of expected candidates.

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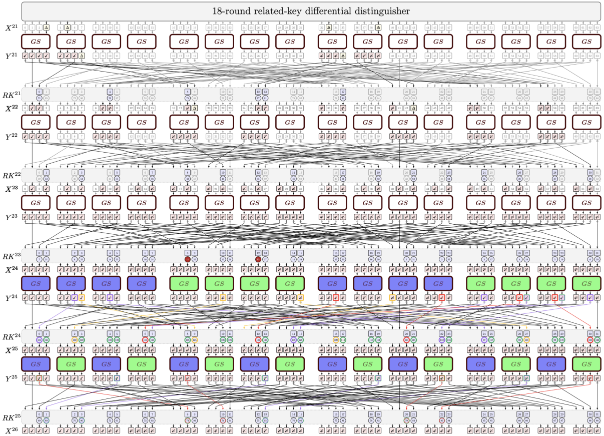
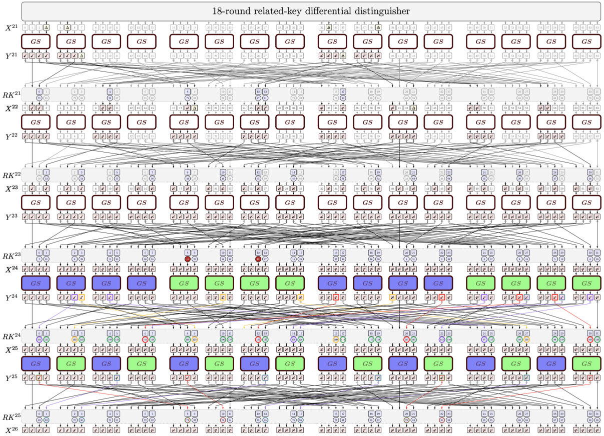
A key recovery is efficient, if its complexity is as close as possible to the lower bound.

The key recovery problem



Potentially too many active S-boxes and key guesses.

The key recovery problem

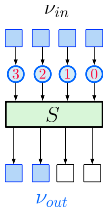


Our goal : Automatise the key recovery for SPN block ciphers with a bit-permutation as linear layer and an (almost) linear key schedule.

Efficient key recovery

Solving an active S-box S

Determine the triplets (x, x', k) s. t. $x + x' \in \nu_{in}$ and $S(x+k) + S(x'+k) \in \nu_{out}$.
Discard the other triplets.



Example: this active S-box has $2^{8+4-2} = 2^{10}$ solutions.

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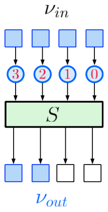

Can be generalised to any subset of active S-boxes!

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Efficient key recovery

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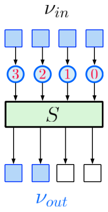

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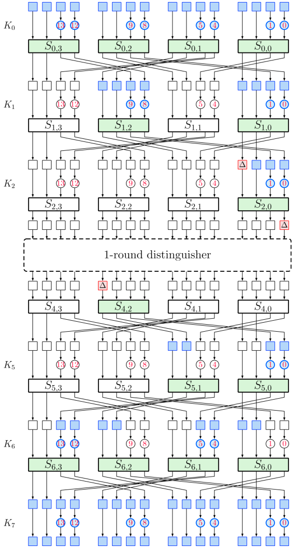
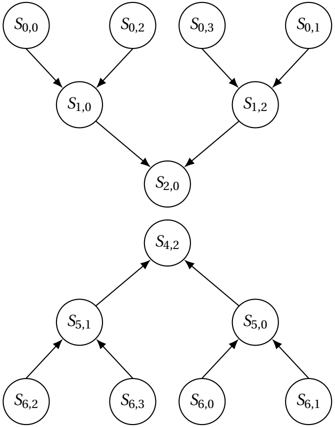
Example: this active S-box has $2^{8+4-2} = 2^{10}$ solutions.

Goal: Reduce the number of triplets as early as possible whilst maximizing the number of determined key bits in the involved key material $\mathcal{K}$.

An algorithm for efficient key recovery

Modeling the key recovery as a graph





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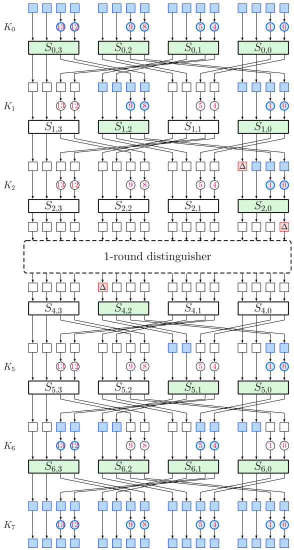
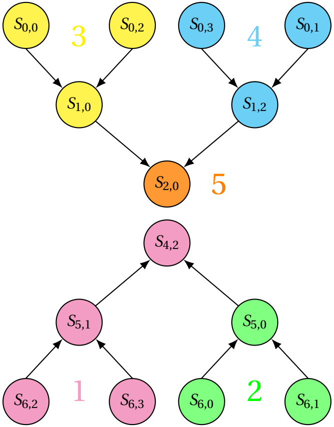




Key recovery:
partition of the nodes + associated order

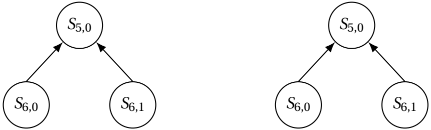
Considering strategies

Strategy $\mathscr{S}_X$ for a subgraph X

Procedure that allows to enumerate all the possible values that the S-boxes of X can take under the differential constraints imposed by the distinguisher.

Parameters of a strategy $\mathscr{S}_X$:

- number of solutions $\mathscr{N}$;
- online time complexity $\mathscr{T}$.



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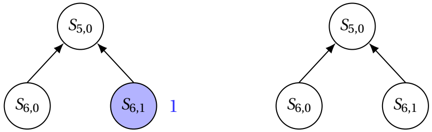

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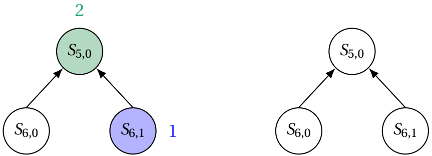

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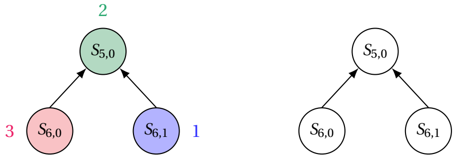

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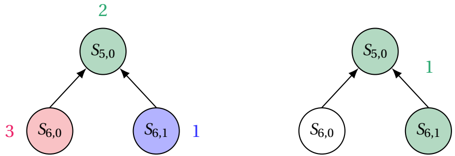

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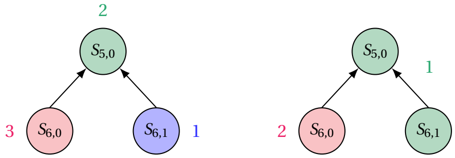

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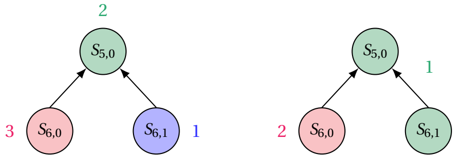

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Objective: Build an efficient strategy for the whole graph.

→ Based on basic strategies, i.e. strategies for a single S-box.

Comparing two strategies

Compare two strategies $\mathscr{S}_X^1$ and $\mathscr{S}_X^2$ for the same subgraph X

1. Choose the one with the best time complexity.
2. If same time complexity, choose the one with the best memory complexity.

Compare $\mathscr{S}_X^1$ and $\mathscr{S}_Y^2$ when $Y \subset X$

If the number of solutions and time complexity of $\mathscr{S}_X^1$ are not higher than those of $\mathscr{S}_Y^2$, then we can freely replace $\mathscr{S}_Y^2$ by $\mathscr{S}_X^1$.

Merging two strategies

Let $\mathscr{S}_X$ and $\mathscr{S}_Y$ two strategies for the graphs X and Y respectively.

- The number of solutions of $\mathscr{S}' = merge(\mathscr{S}_X, \mathscr{S}_Y)$ **only depends** on $X \cup Y$:

Number of solutions of $\mathscr{S}'$

$Sol(X \cup Y) = Sol(X) + Sol(Y) - \#$ bit-relations between the nodes of X and Y ⚠ log scale

Time and memory associated to $\mathscr{S}'$

- $T(\mathscr{S}') \approx \max(T(\mathscr{S}_X), T(\mathscr{S}_Y), Sol(X \cup Y))$
- $M(\mathscr{S}') \approx \max(M(\mathscr{S}_X), M(\mathscr{S}_Y), \min(Sol(\mathscr{S}_X), Sol(\mathscr{S}_Y)))$

A dynamic programming approach

- The online time complexity of $merge(\mathscr{S}_X, \mathscr{S}_Y)$ **only depends** on the time complexities of $\mathscr{S}_X$ and $\mathscr{S}_Y$.
- An optimal strategy for $X \cup Y$ **can always** be obtained by merging two optimal strategies for X and Y.
- Use a bottom-up approach, merging first the strategies with the smallest time complexity to reach a graph strategy with a minimal time complexity.

Dynamic programming approach

Ensure that, for any subgraph X, we only keep one optimal strategy to enumerate it.

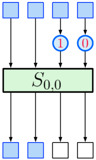
Sieving

Idea: Use the differential constraints to filter out pairs that cannot follow the differential, regardless of the value of the key.

- Example:

$$(x_3, x_3', x_2, x_2', x_1 \oplus x_1', x_0 \oplus x_0')$$

Filter: $36/2^6 = 2^{-0.83}$.



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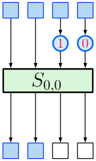


Pre-sieving

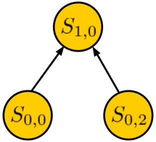
Apply a sieve on all S-boxes of the external rounds.

Advantage : The key recovery is performed on $N' \leq N$ pairs.

Precomputing partial solutions

Idea

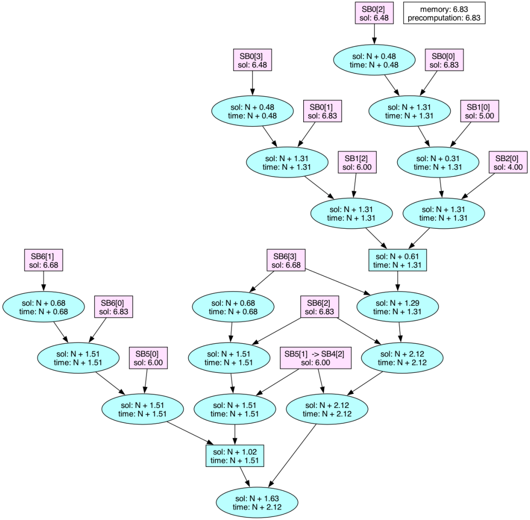
Precompute the partial solutions to some subgraph.



- Impact on the memory complexity and the offline time of the attack.
- The optimal key recovery strategy depends on how much memory and offline time are allowed.

Applications of our tool: KYRYDI

Application to the toy cipher



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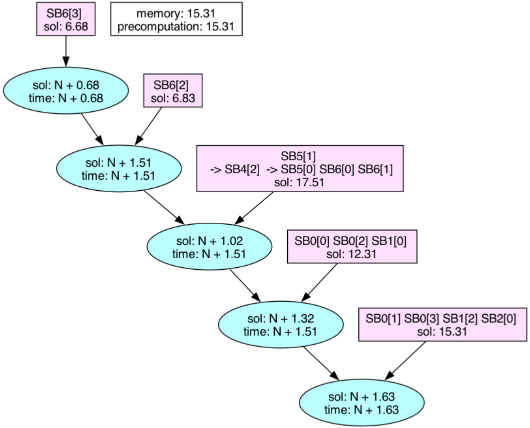

Application to other ciphers

Start from an existing distinguisher that led to the best key recovery attack against the target cipher.

- RECTANGLE: Extended by one round the previous best attack.
- PRESENT-80: Extended by two rounds the previous best differential attack.
- GIFT-64 and SPEEDY-7-192: Best key recovery strategy without additional techniques.

Extensions and improvements

- Handle ciphers with more complex linear layers.
- Handle ciphers with non-linear key schedules.
- Incorporate tree-based key recovery techniques by exploiting the structure of the involved S-boxes.

The best distinguisher **does not always** lead to the best key recovery!

Ultimate goal

Combine the tool with a distinguisher-search algorithm to find the best possible attacks.

Other open problems

- Prove optimality.
- The tool works for (impossible) differential attacks:

  → Apply a similar approach to other attacks.

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Thanks for your attention!

Link to **KYRYDI**:

`https://gitlab.inria.fr/capsule/kyrydi`