The NISQ Complexity of Collision Finding

Yassine Hamoudi, Qipeng Liu, Makrand Sinha

CNRS, LaBRI UC San Diego U. of Illinois

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Noisy Intermediate-Scale Quantum

Limitations of short-term quantum computers:

- limited error correction
- small coherence time
- few logical qubits
- …

NISQ complexity: understand what cannot be done with NISQ computers

Collision finding

4	3	0	6	3	2	1

Find x, y *with* $H(x) = H(y)$ *in a function* $H : [N] \to [N]$

Collision finding

4	3	0	6	3	2	1

Find x, y *with* $H(x) = H(y)$ *in a function* $H : [N] \to [N]$

- Subroutines of many quantum algorithms and crypto. attacks
- Current speedups (BHT, Ambainis' quantum walk…) are not NISQ

Can we get quantum speedups for Collision finding in NISQ era?

How to model NISQ complexity?

Model 1

Shallow quantum circuits

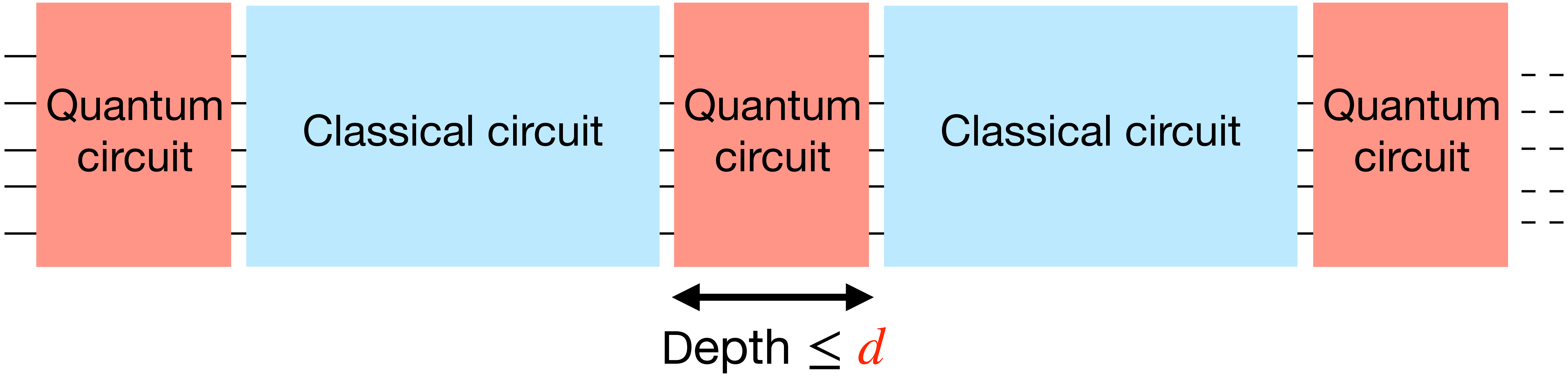
Model 2

Costly gates

Model 3

Noisy gates

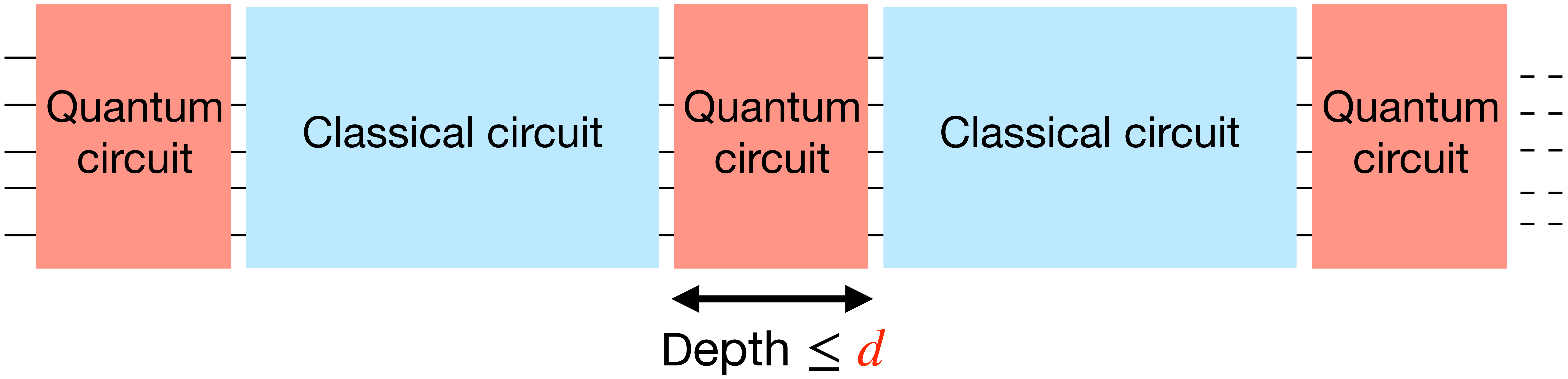
Model 1
Shallow quantum circuits



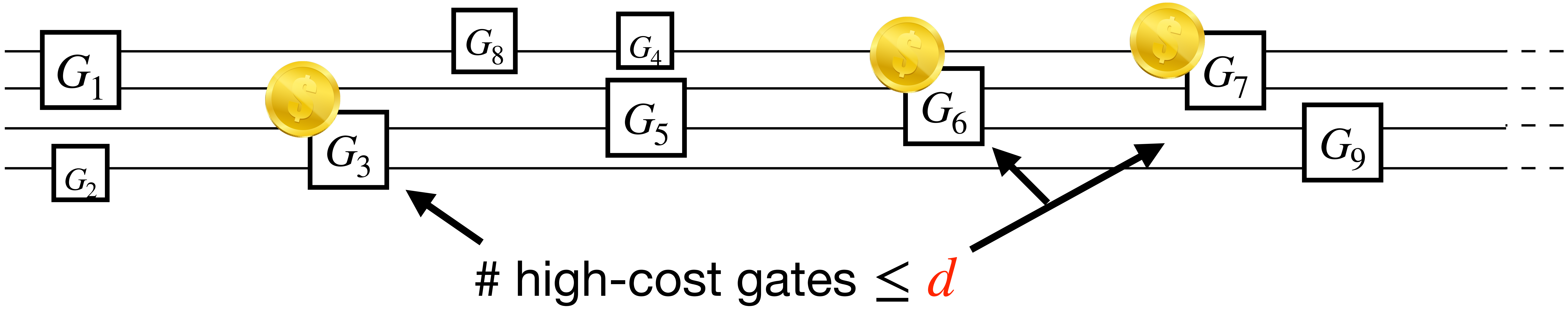
Model 2
Costly gates

Model 3
Noisy gates

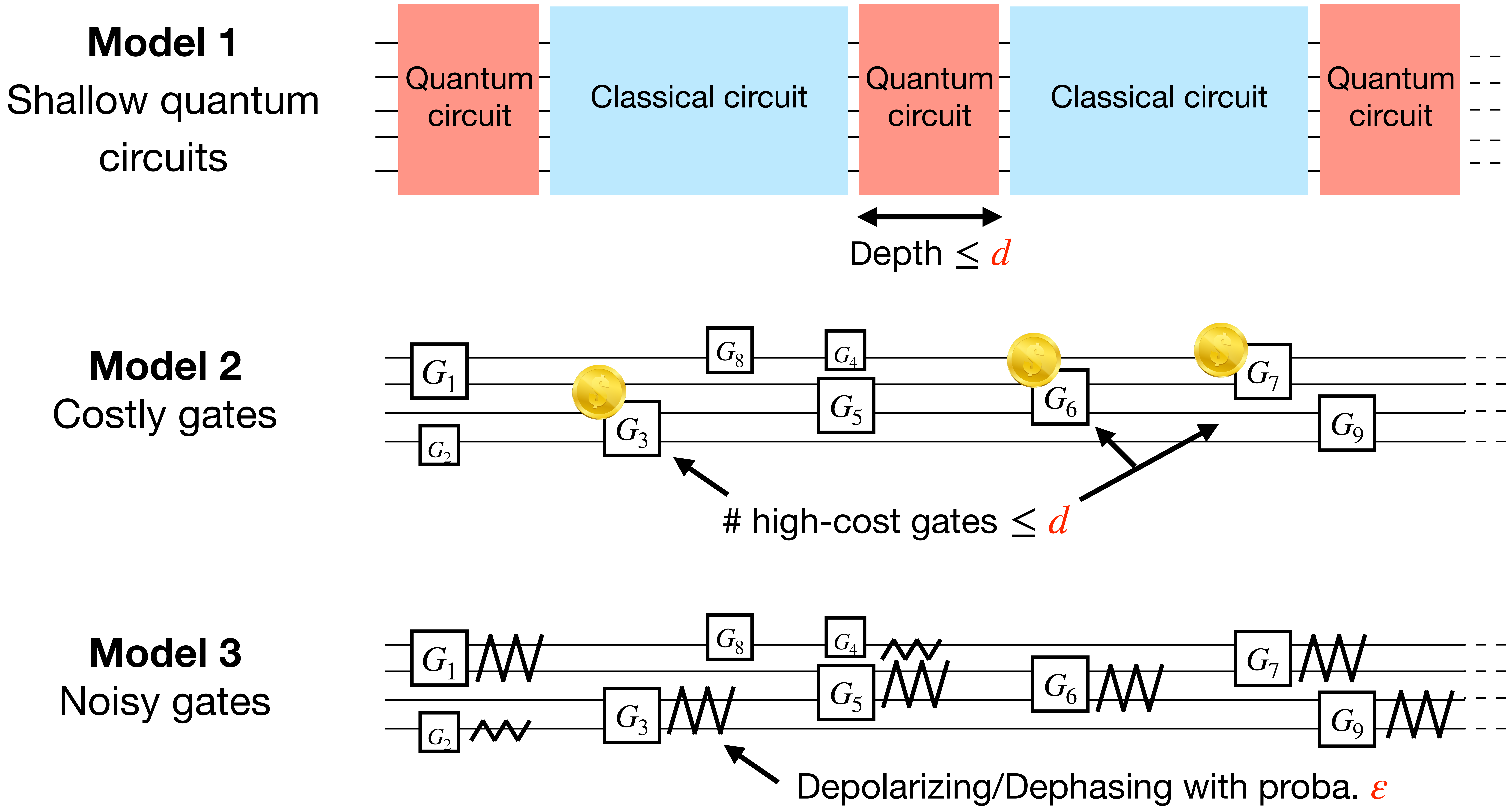
Model 1
Shallow quantum circuits



Model 2
Costly gates



Model 3
Noisy gates



Model 1
Shallow quantum circuits

Quantum circuit

Classical circuit

Quantum circuit

Classical circuit

Quantum circuit

Depth ≤ d

Model 2
Costly gates

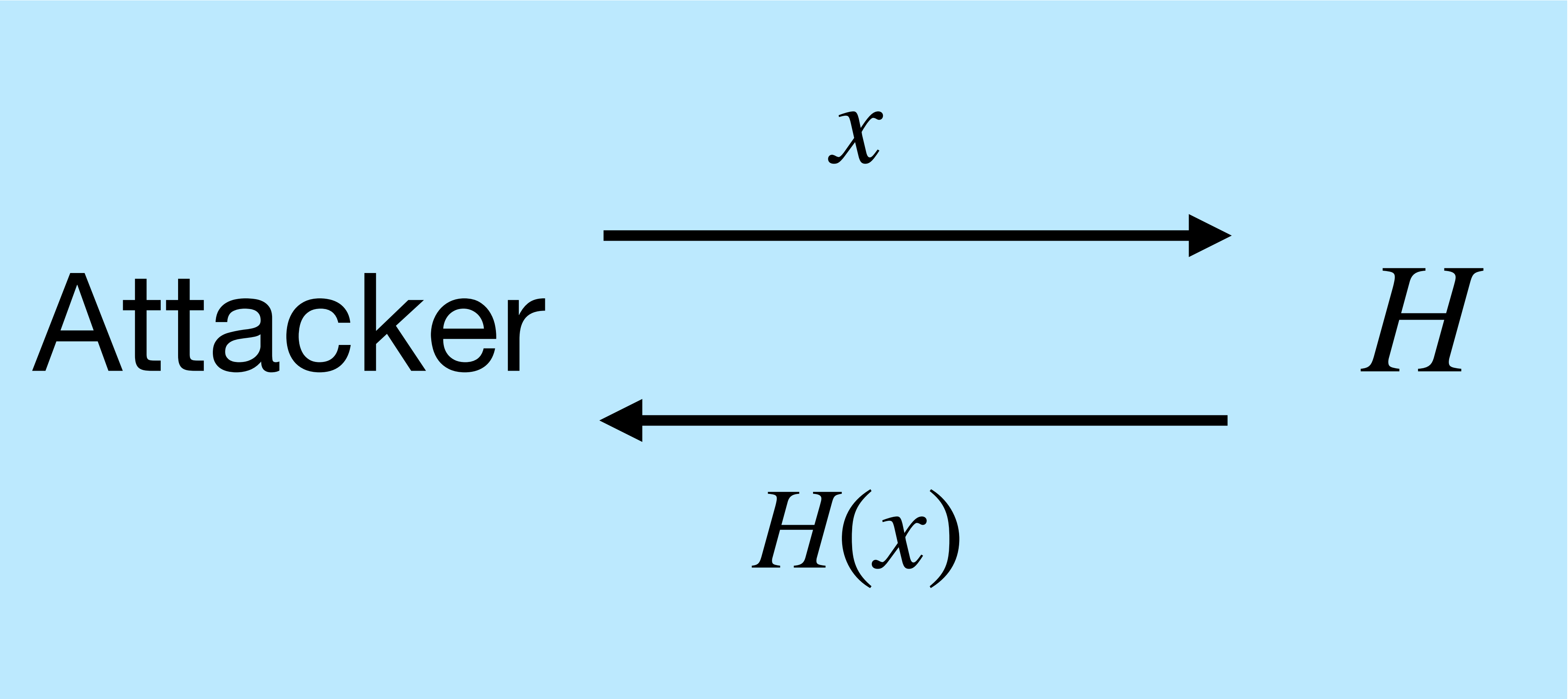
# high-cost gates ≤ d

Model 3
Noisy gates

Depolarizing/Dephasing with proba. ε

Random oracle model

Classical query



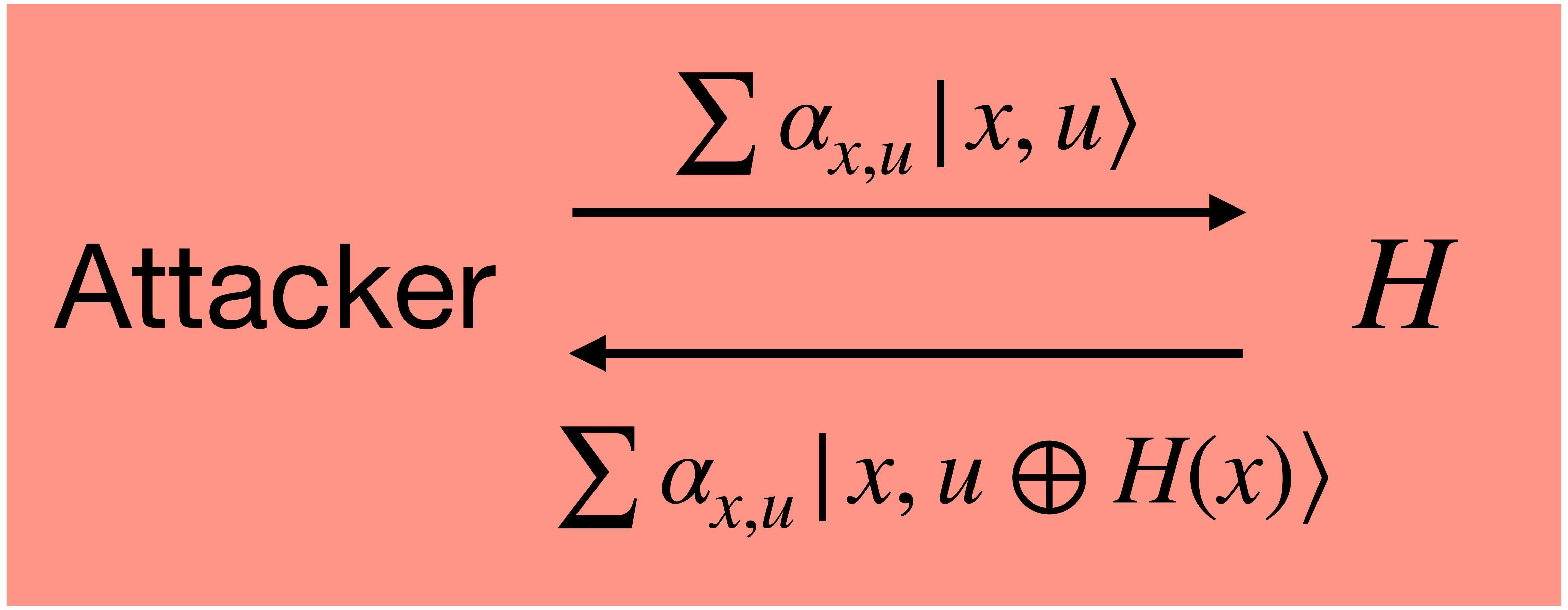
Attacker

x

$H(x)$

H

Quantum query



Attacker

$\sum \alpha_{x,u} | x, u \rangle$

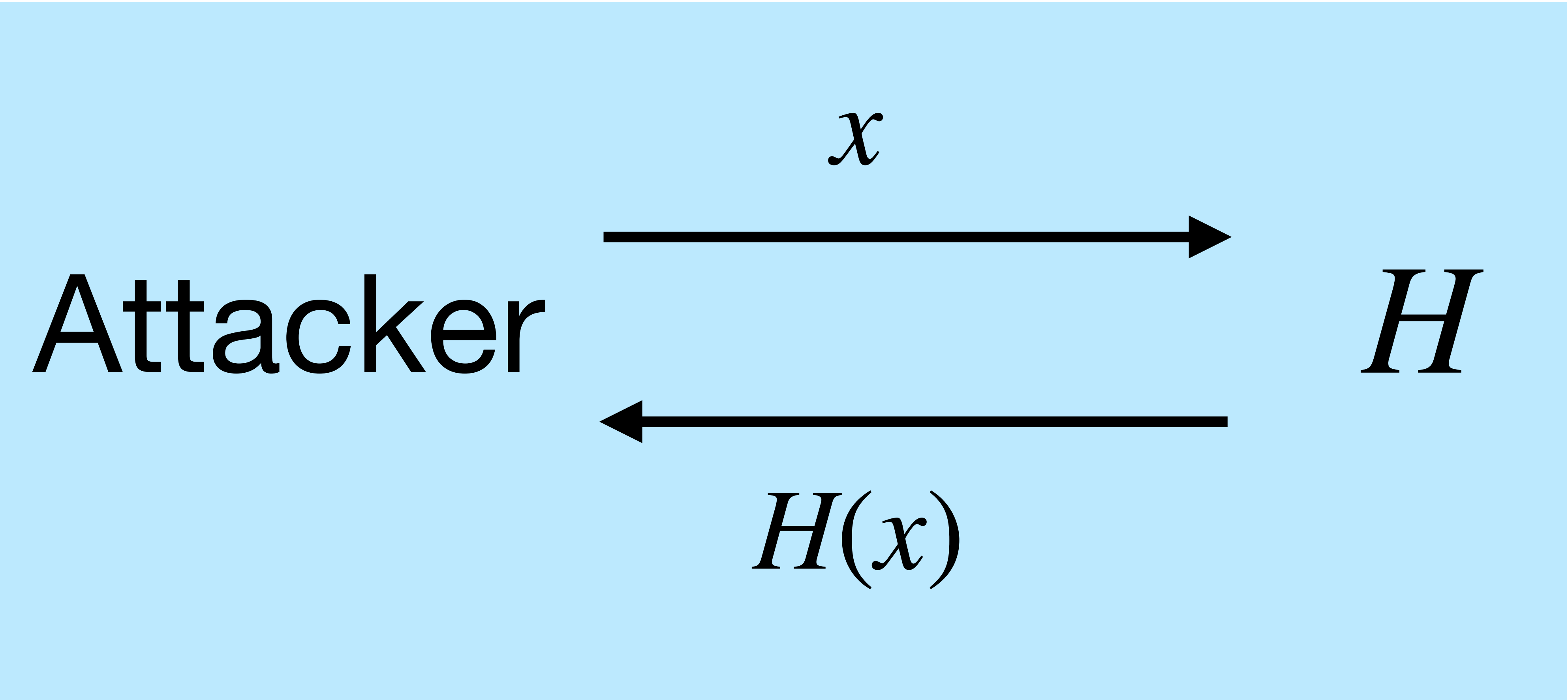
$\sum \alpha_{x,u} | x, u \oplus H(x) \rangle$

H

Black-box interface to an “ideal” hash function

Random oracle model

Classical query


Attacker

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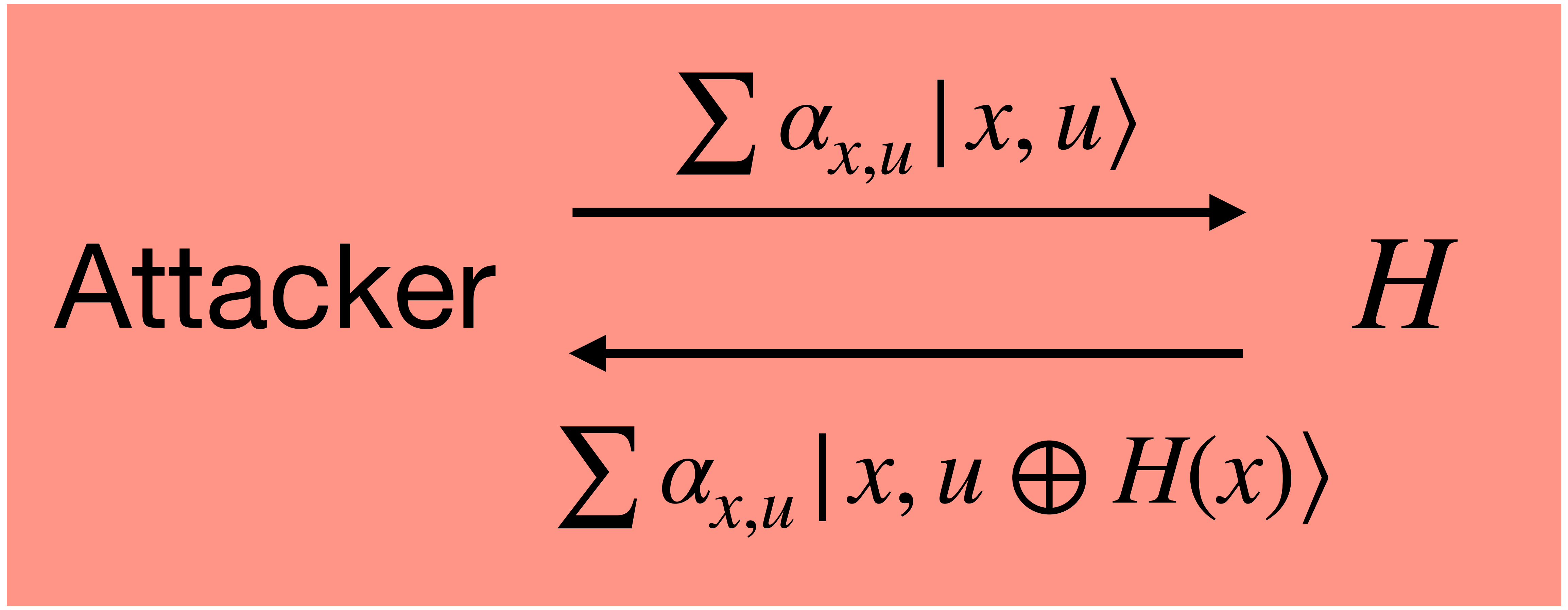
H

$H(x)$

Quantum query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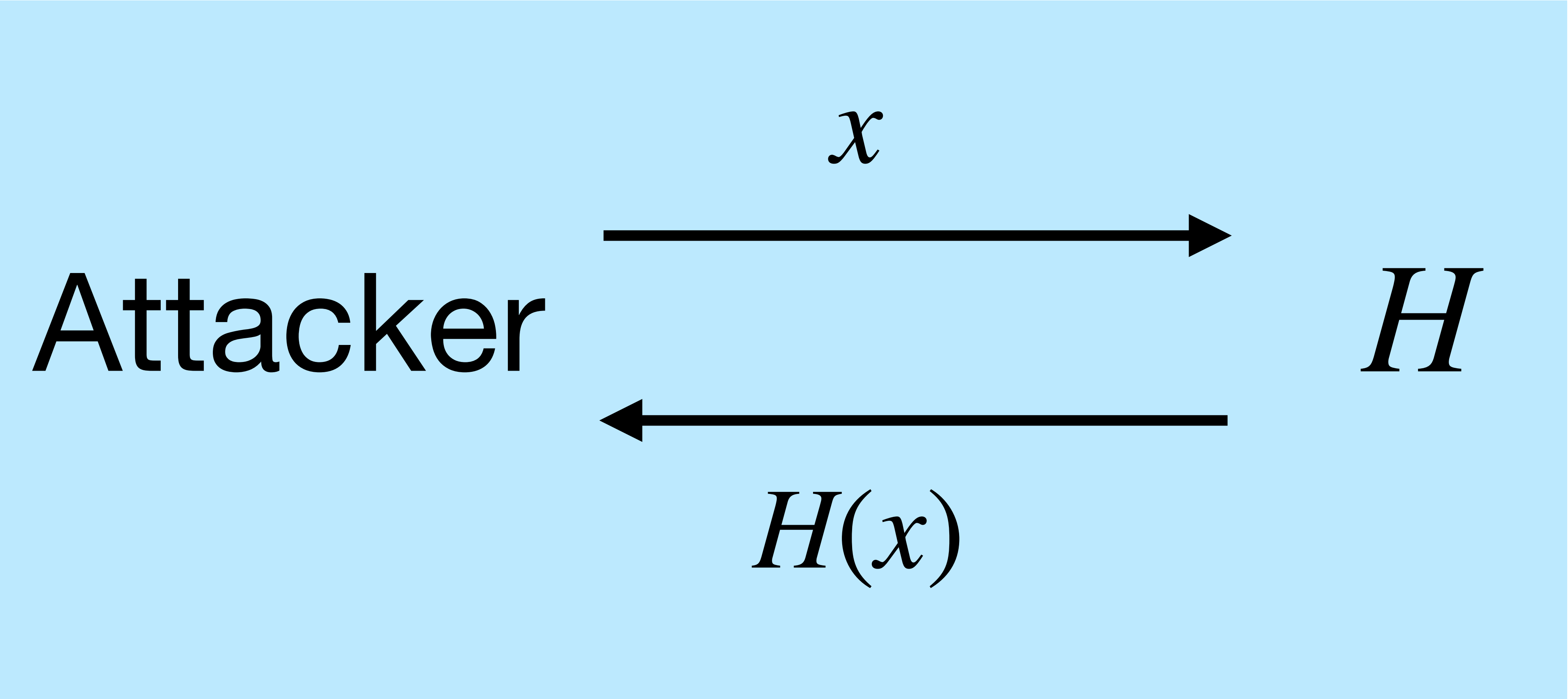
Black-box interface to an “ideal” hash function

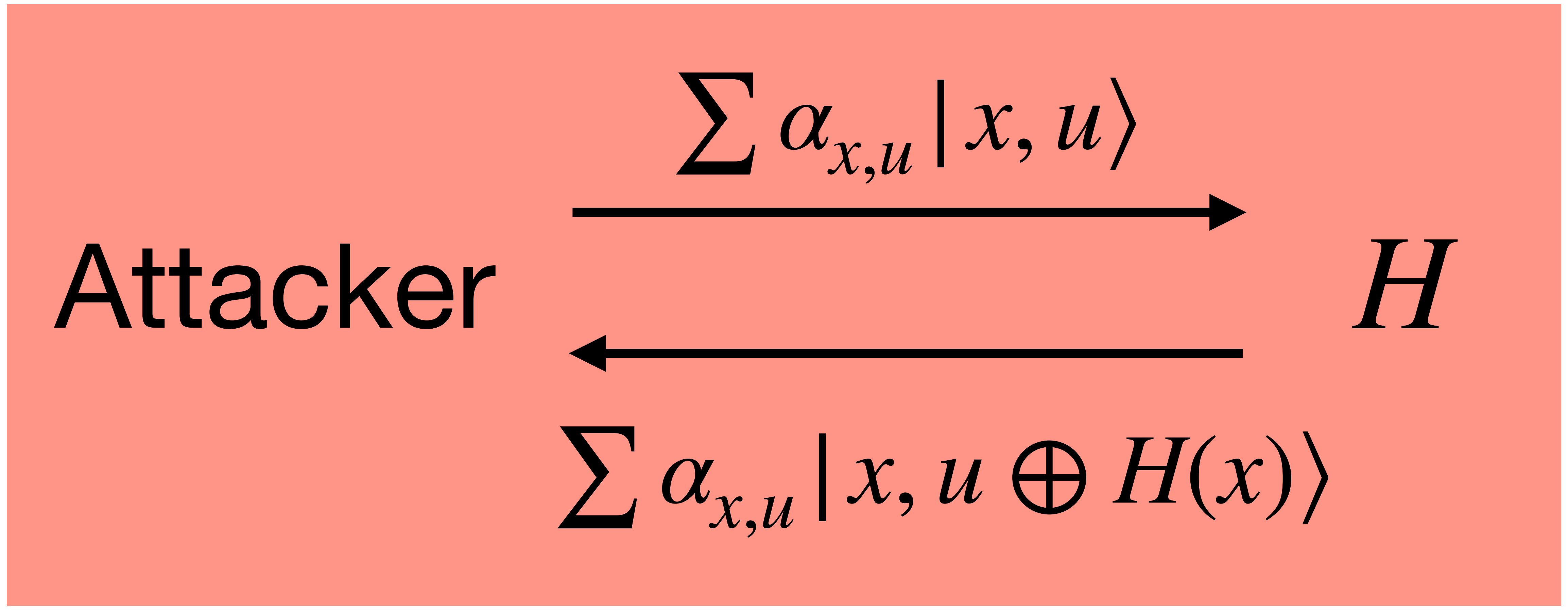
- Existing quantum attacks are designed in this model

Random oracle model

Classical query



Quantum query


$\sum \alpha_{x,u} | x, u \rangle$
Attacker
H
$\sum \alpha_{x,u} | x, u \oplus H(x) \rangle$

Black-box interface to an “ideal” hash function

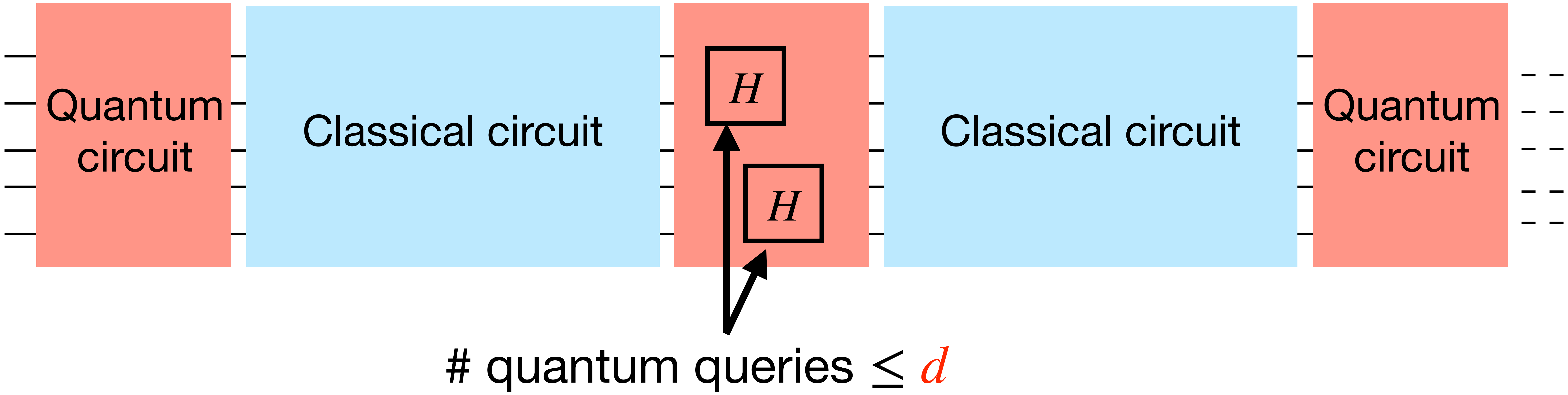
- Existing quantum attacks are designed in this model
- Quantum queries are often the most time-consuming part

Model 1
Shallow quantum circuits

Model 2
Costly gates

Model 3
Noisy gates

Model 1
Shallow quantum circuits



Model 2
Costly gates

Model 3
Noisy gates

Model 1
Shallow quantum circuits

Model 2
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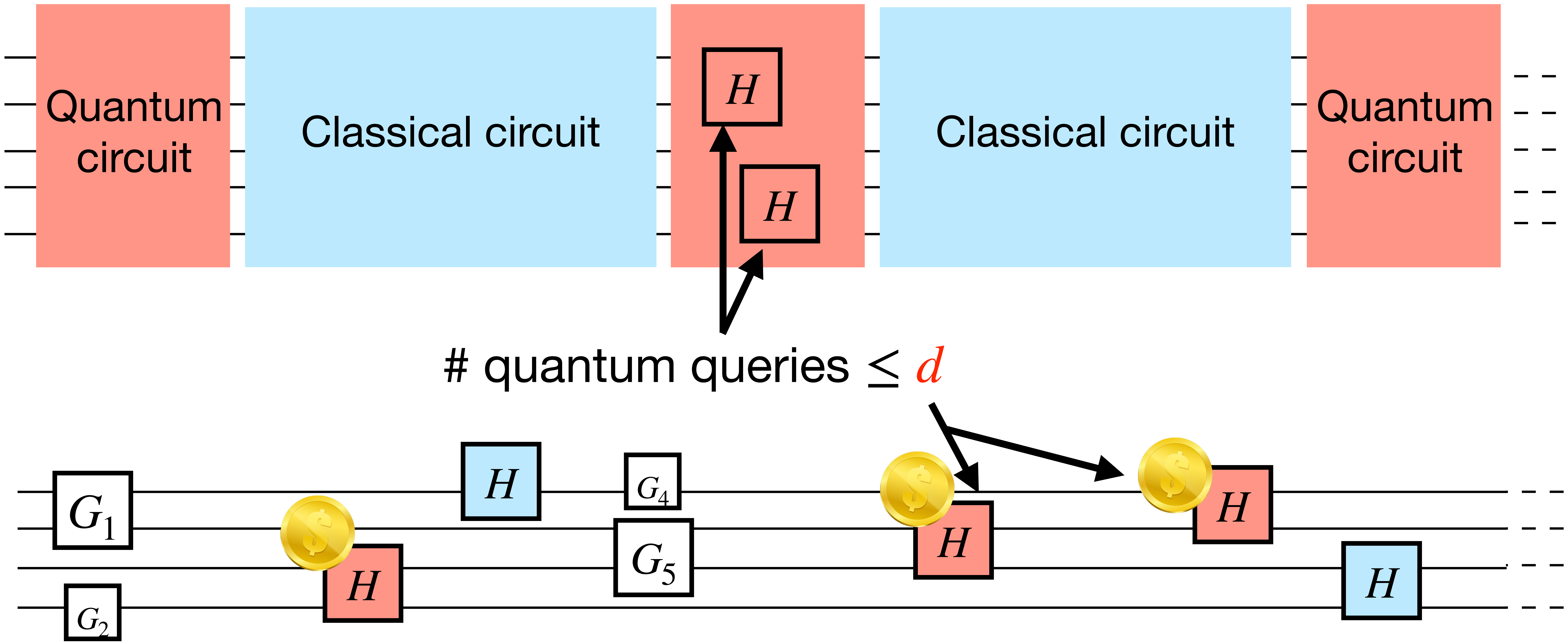
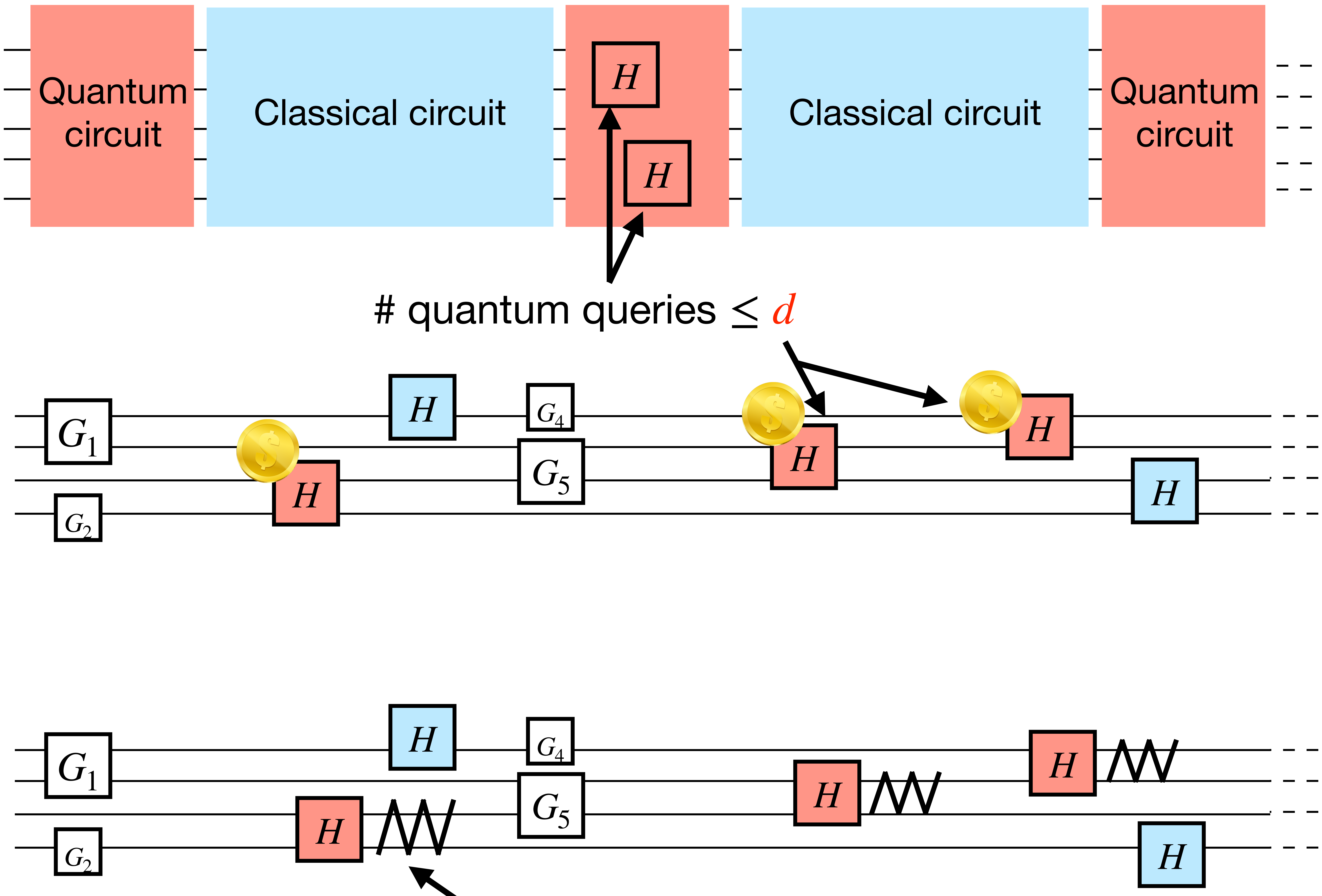


Model 3
Noisy gates

Model 1
Shallow quantum circuits

Model 2
Costly gates

Model 3
Noisy gates



# quantum queries ≤ d

Depolarizing/Dephasing with proba. ε

Main results

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1/ No significant speedup for Collision finding in NISQ models

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2/ Tight characterization of optimal speedups in “super-NISQ” models

3/ New framework and techniques for analyzing NISQ complexity

4/ Similar results for Preimage search

Extends to QROM: [Sun, Zheng’19], [Chen, Cotler, Huang, Li’22], [Rosmanis’22’23]

Depth vs Quantum queries

(Model 1)

Number of queries
$\sqrt{N}$
Birthday paradox
$N^{1/3}$
BHT algorithm
0
$N^{1/3}$
Circuit depth
(coherence time)

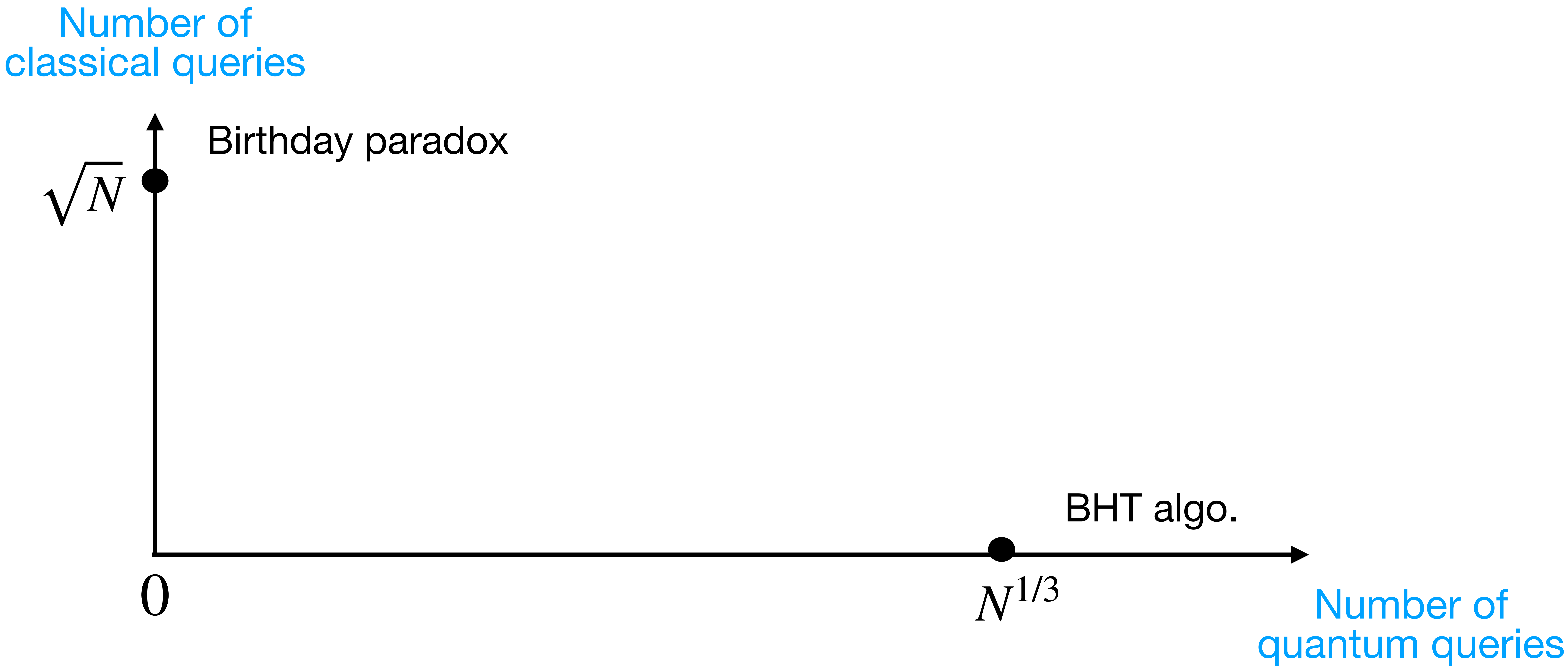
Depth vs Quantum queries

(Model 1)

Number of queries
Birthday paradox
$\sqrt{N}$
NISQ
$x \mapsto \sqrt{N/x}$
BHT algorithm
$N^{1/3}$
0
$N^{1/3}$
Circuit depth
(coherence time)

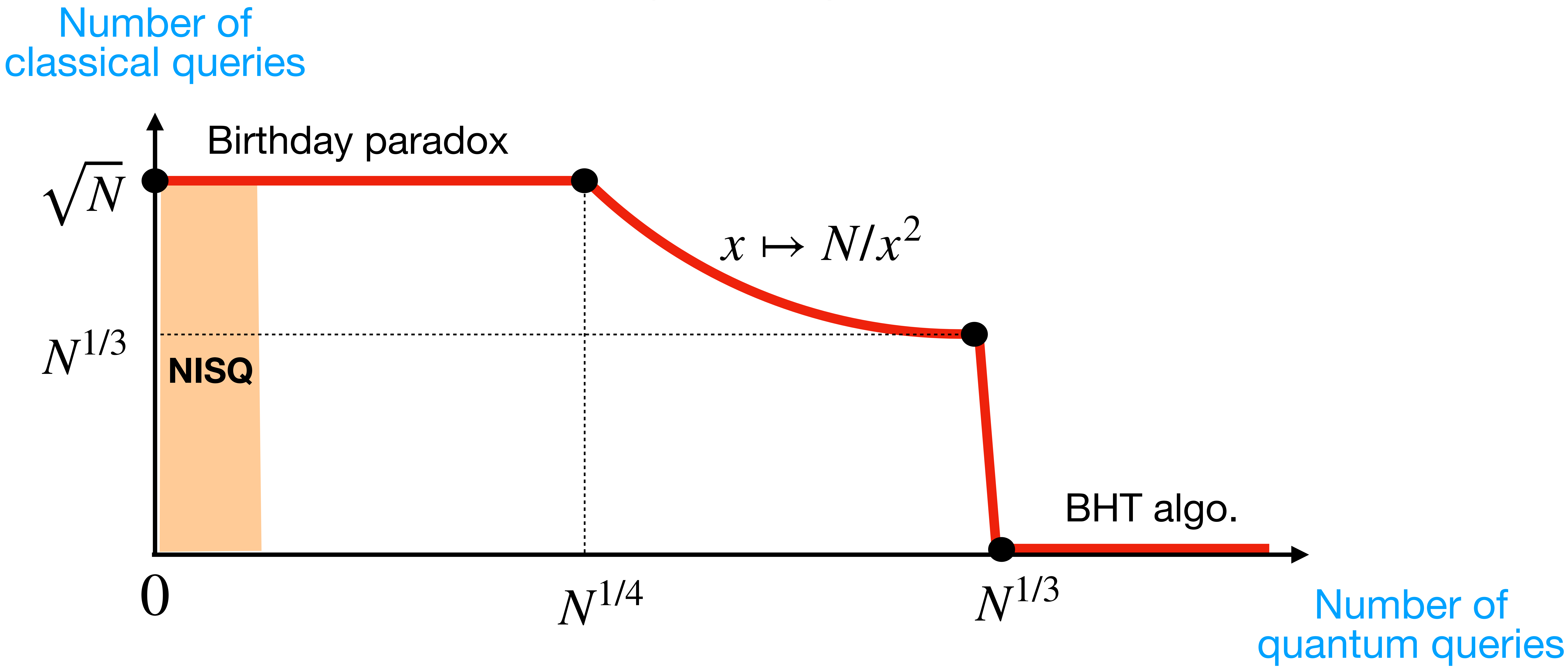
Classical queries vs Quantum queries

(Model 2)



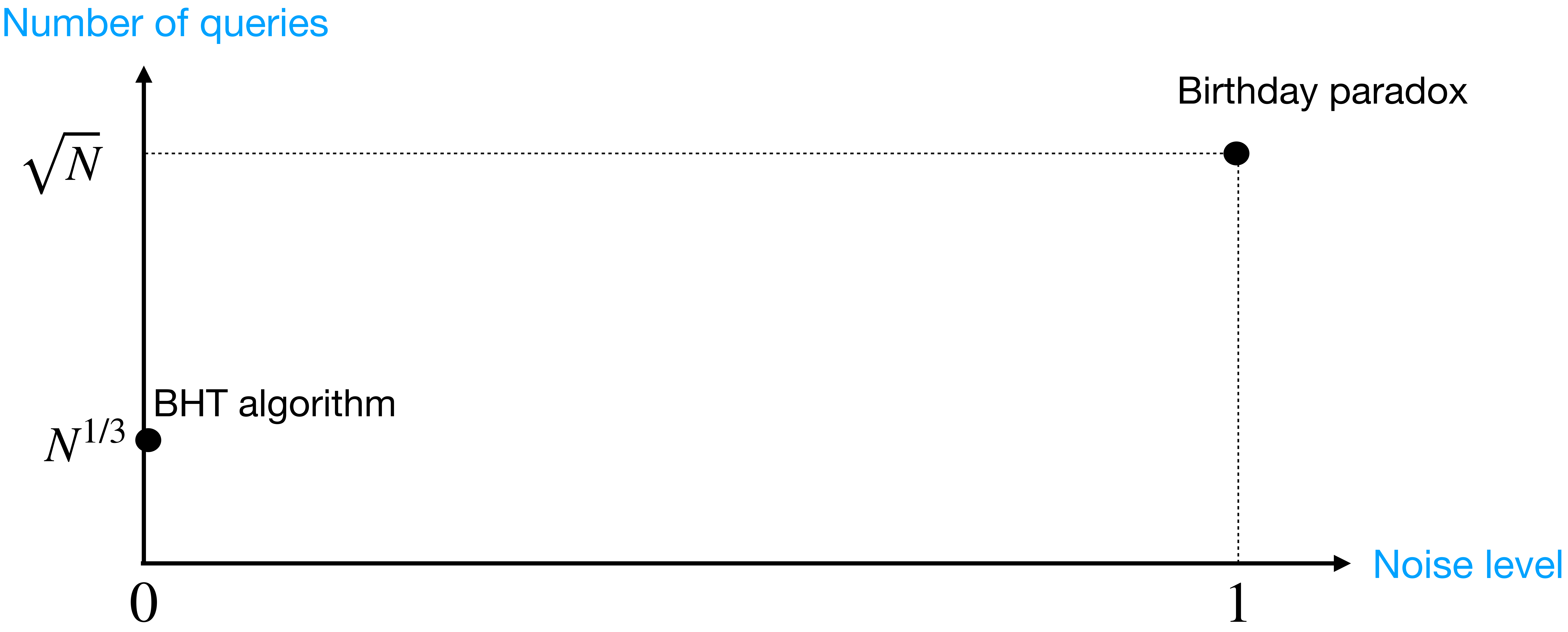
Classical queries vs Quantum queries

(Model 2)



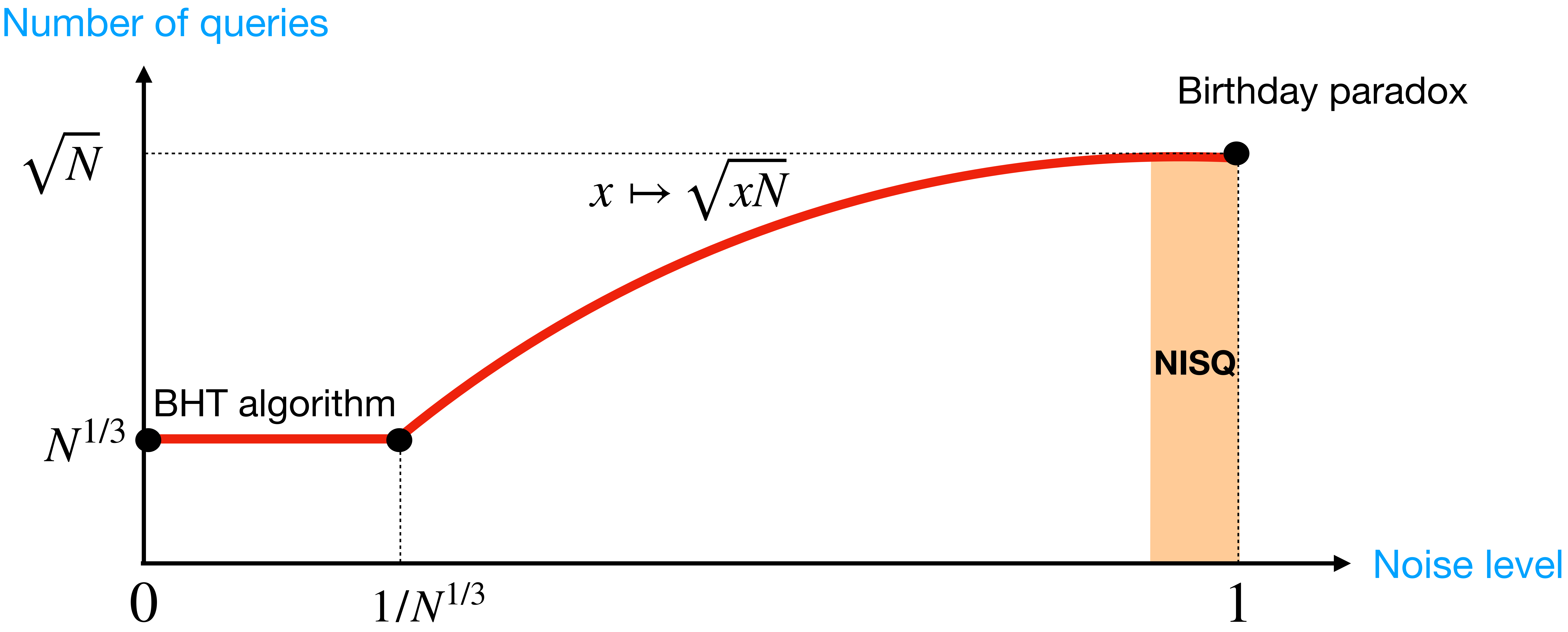
Noise vs Quantum queries

(Model 3)



Noise vs Quantum queries

(Model 3)

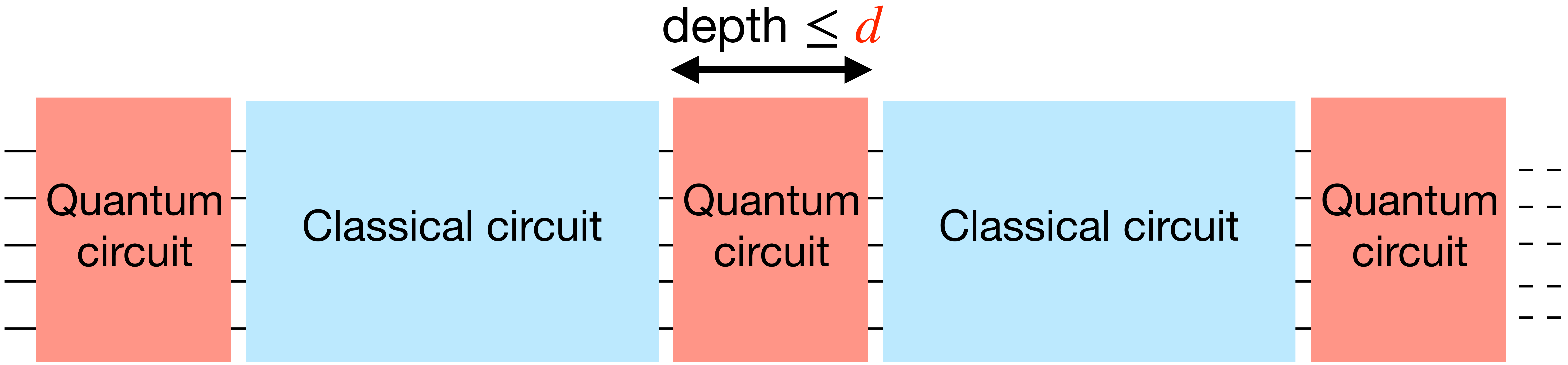


Proof methods

Idea 1: Dropping the depth constraint

Model 1

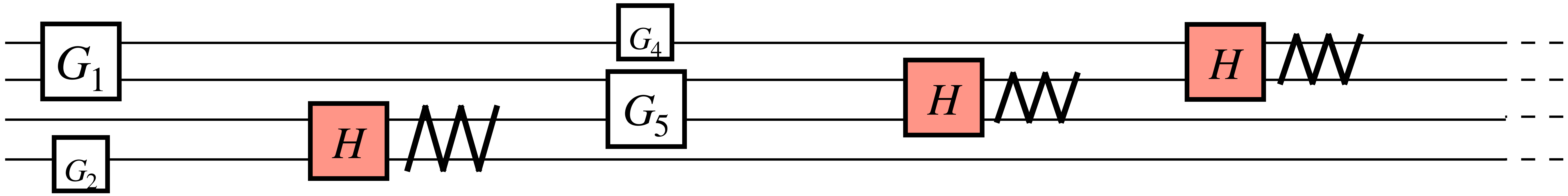
Shallow quantum circuits



Shallow circuits can be simulated if noise $\varepsilon \leq 1/d$

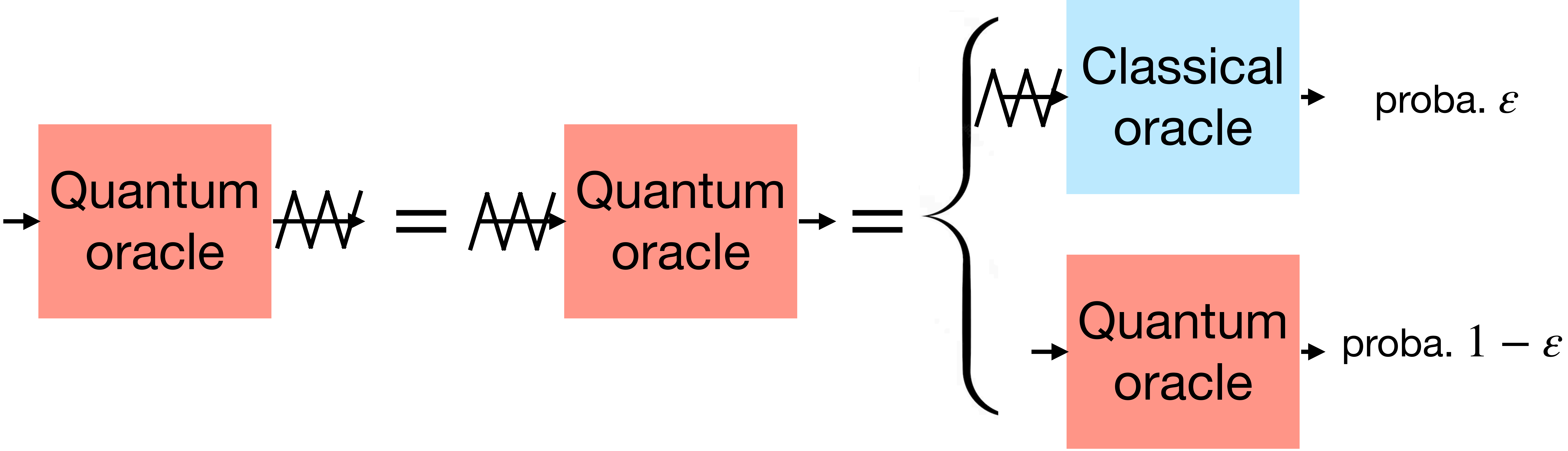
Model 3

Dephasing noise

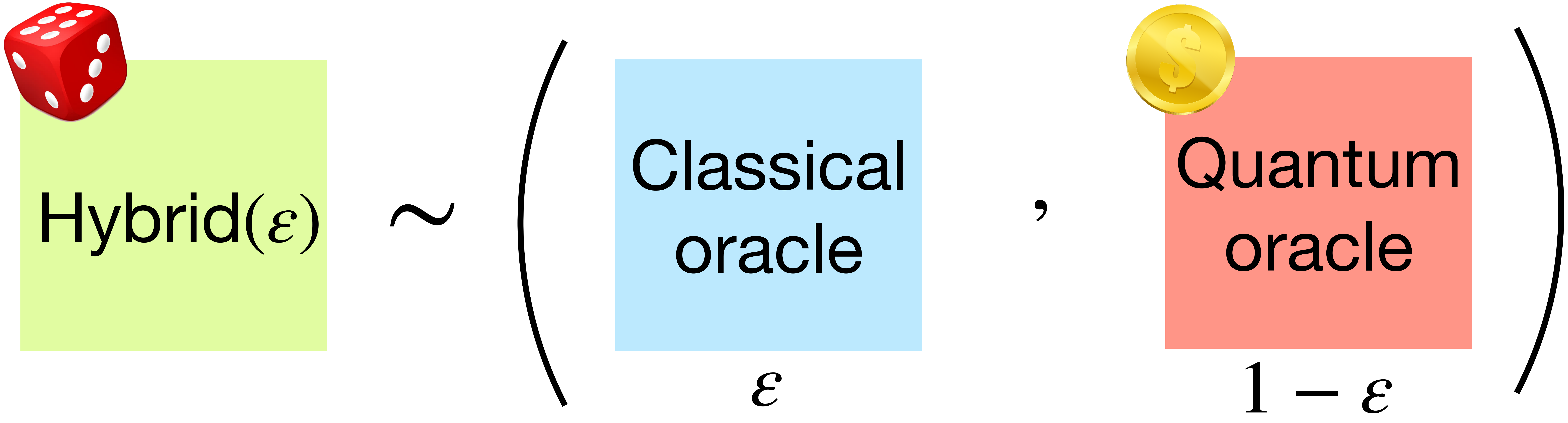


Idea 2: Hybrid oracles

Observation: (dephasing) noise commutes with quantum oracle



Idea 2: Hybrid oracles



Hybrid(ε) $\sim$ (Classical oracle , Quantum oracle)

ε $\quad$ $1-\varepsilon$

Equivalently: quantum oracle collapses into classical oracle with proba. ε

Idea 3: Hybrid *compressed* oracles

Extend the oracle purification technique of [Zhandry, CRYPTO'19] to hybrid oracles

1/ We devise a way of simultaneously recording classical and quantum queries into a classical-quantum database

2/ We relate the probability of finding a collision to some progress measure on this database