

Foundations of Adaptor Signatures

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Thyagarajan



THE UNIVERSITY OF
SYDNEY

Once Upon A Time



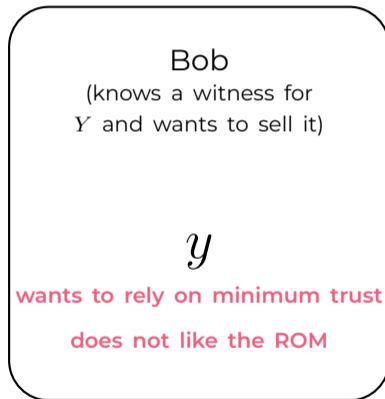
Once Upon A Time



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Adaptor Signatures

Adaptor Signature Interfaces

Signature
Scheme

$$(pk, sk) \leftarrow \text{KGen}(\lambda)$$

$$\sigma \leftarrow \text{Sign}(sk, m)$$

$$b \leftarrow \text{Vrfy}(pk, m, \sigma)$$

Hard
Relation

$$(Y, y) \leftarrow \text{GenR}(\lambda)$$

$$\tilde{\sigma} \leftarrow \text{pSign}(sk, m, Y)$$

$$b \leftarrow \text{pVrfy}(pk, m, \tilde{\sigma}, Y)$$

$$\sigma \leftarrow \text{Adapt}(pk, \tilde{\sigma}, y)$$

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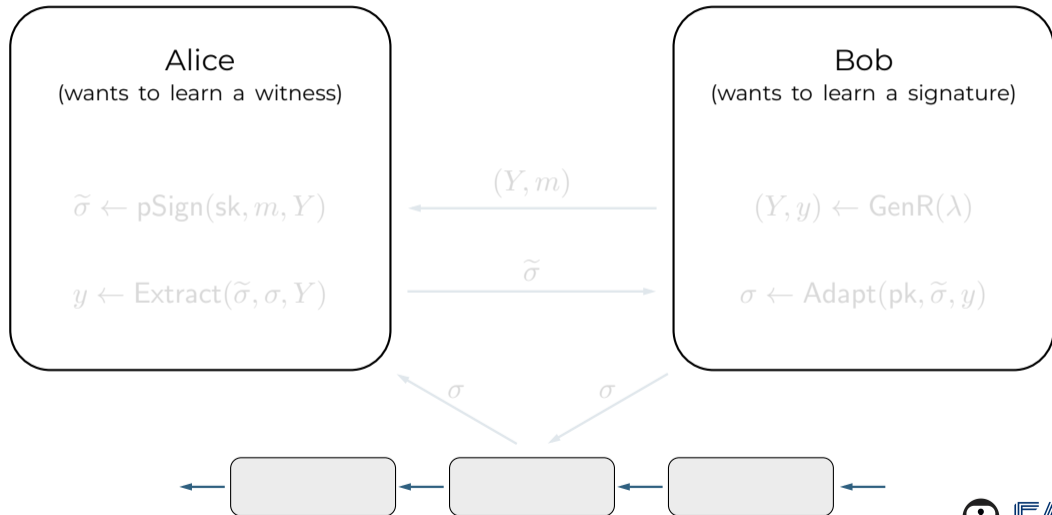
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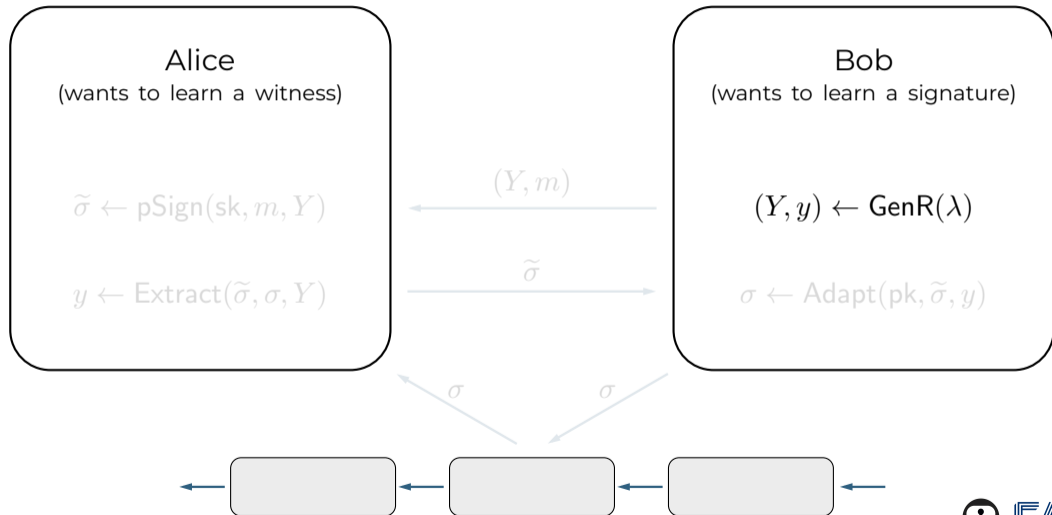
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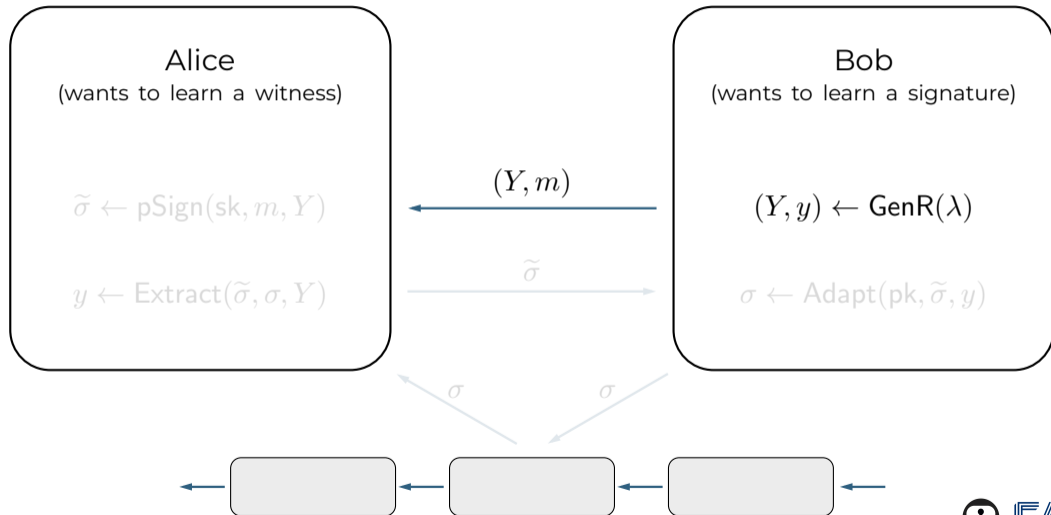
Fair Exchange using Adaptor Signatures



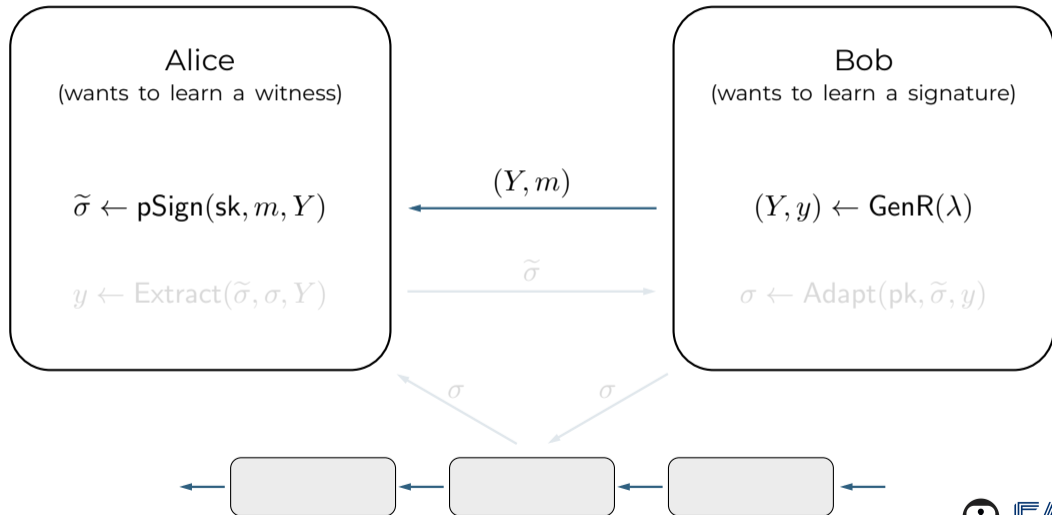
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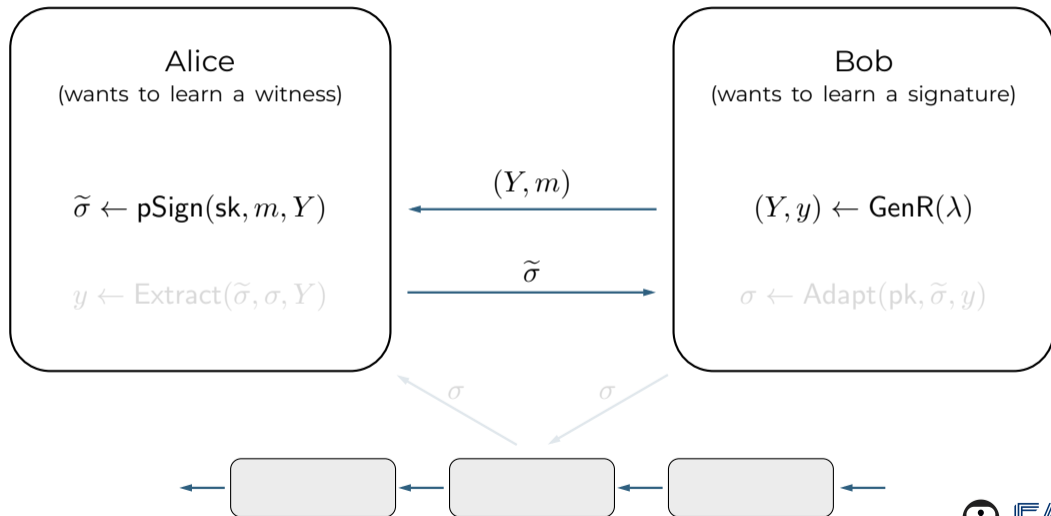
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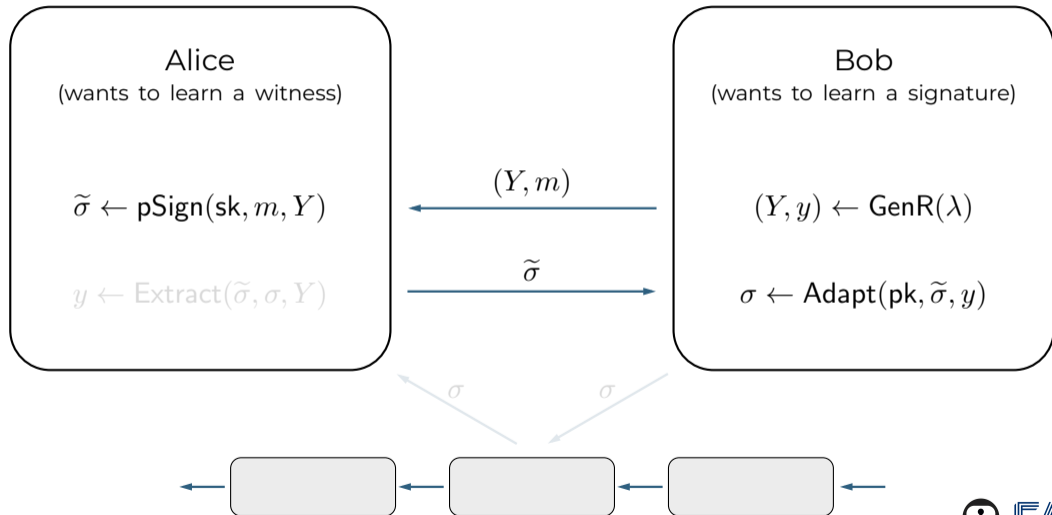
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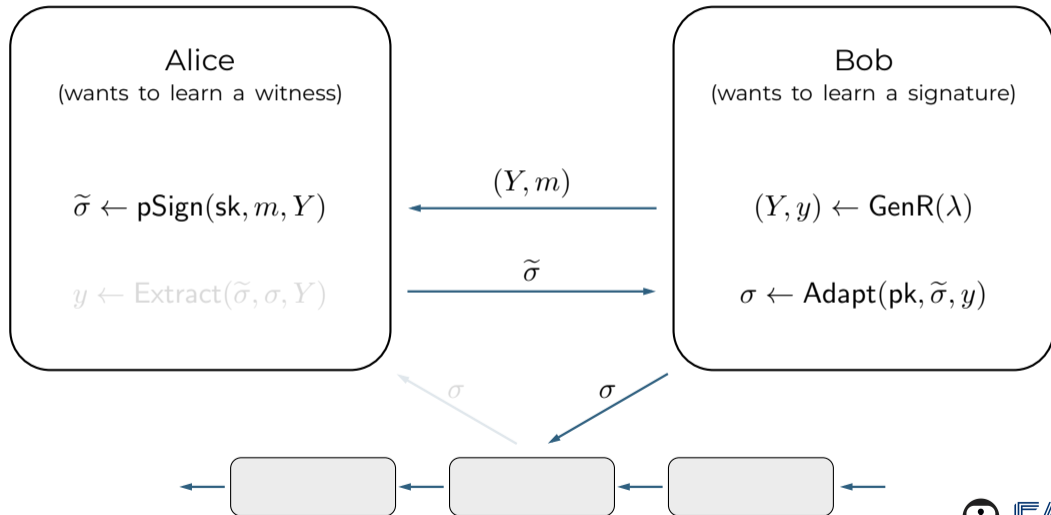
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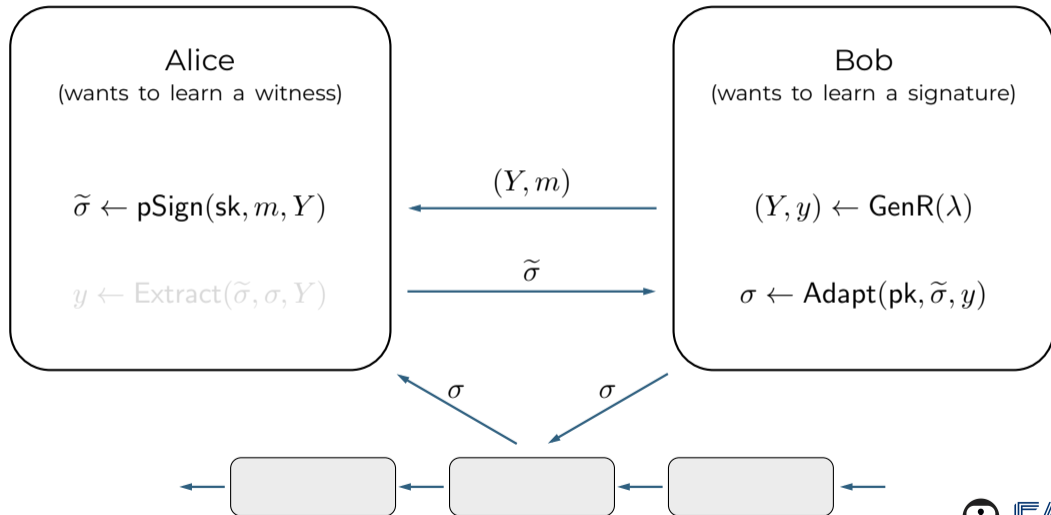
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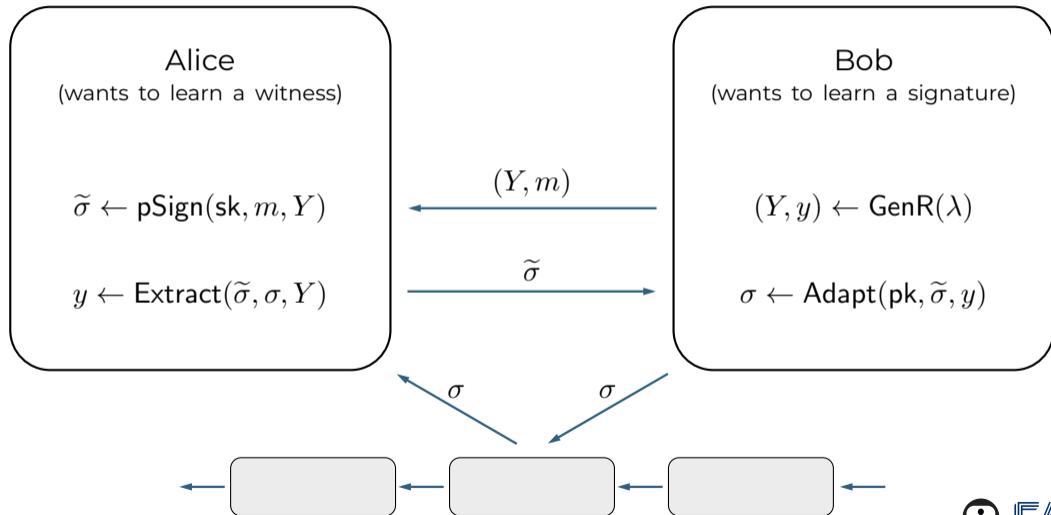
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Fair Exchange using Adaptor Signatures



Adaptor Signatures in the Literature

- Introduced by Andrew Poelstra 2017
- Formally defined by Aumayr et al. [AEEFHMMR'21]
- Applications:
 - (Generalized) Payment Channels [AEEFHMMR'21]
 - (Blind) Coin Mixing [GMMMTT'22, QPMSESELYY'23]
 - Oracle-Based Payments [MTVFMM'23]
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Given a signature scheme, building a secure adaptor signature is hard.

There is no secure adaptor signature in the standard model.

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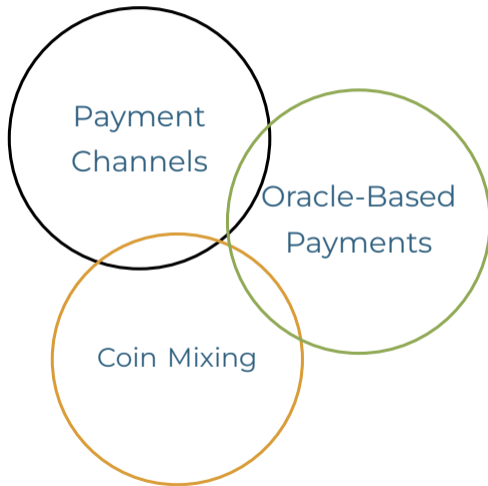
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Adaptor signatures were formalized to build **payment channels**.

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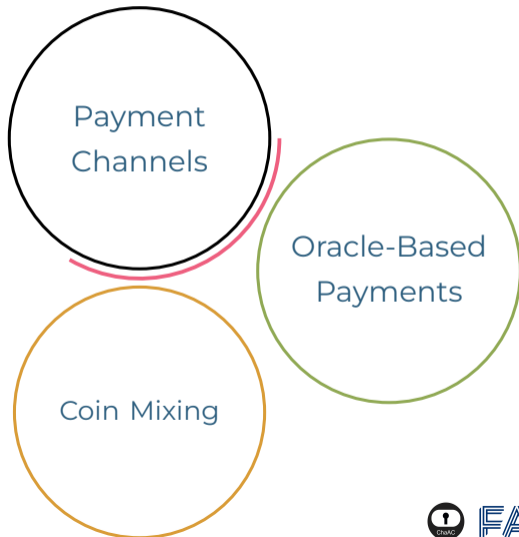
Oracle-Based Payments

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Our Contribution



Gaps



Definitions



Constructions



**Transparent
Reductions**

Our Contribution



Gaps



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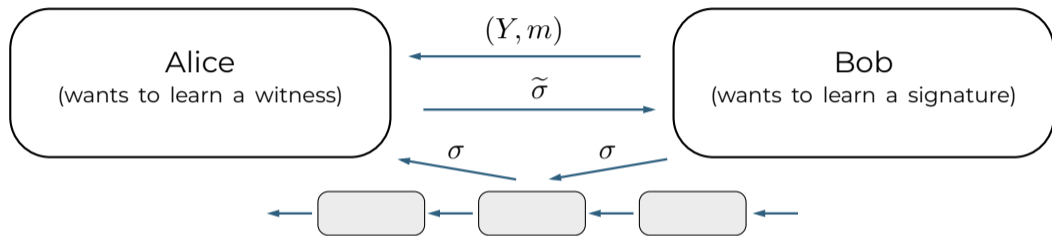


Constructions



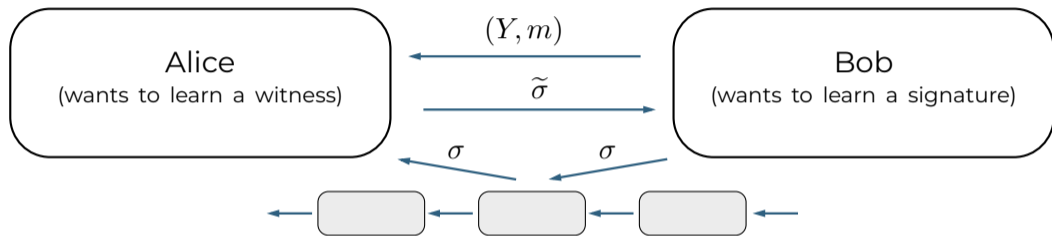
Transparent
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Adaptor Signature Formalization



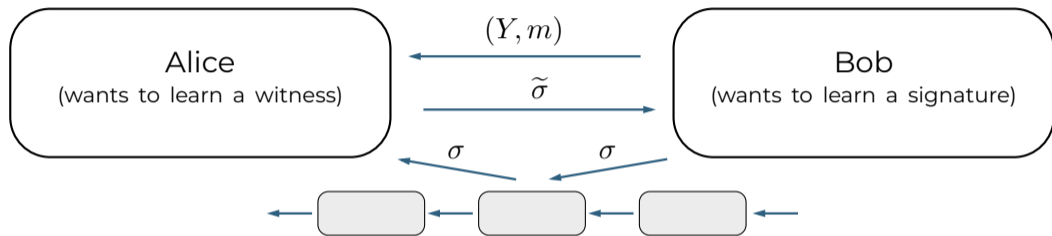
- The definition is a **one-shot experiment**
 - The adversary can only learn a single challenge pre-signature
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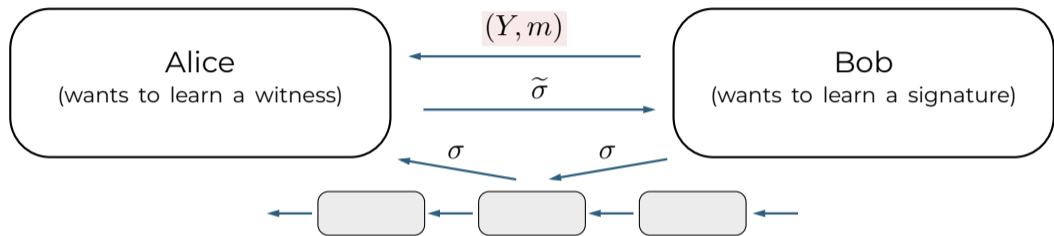
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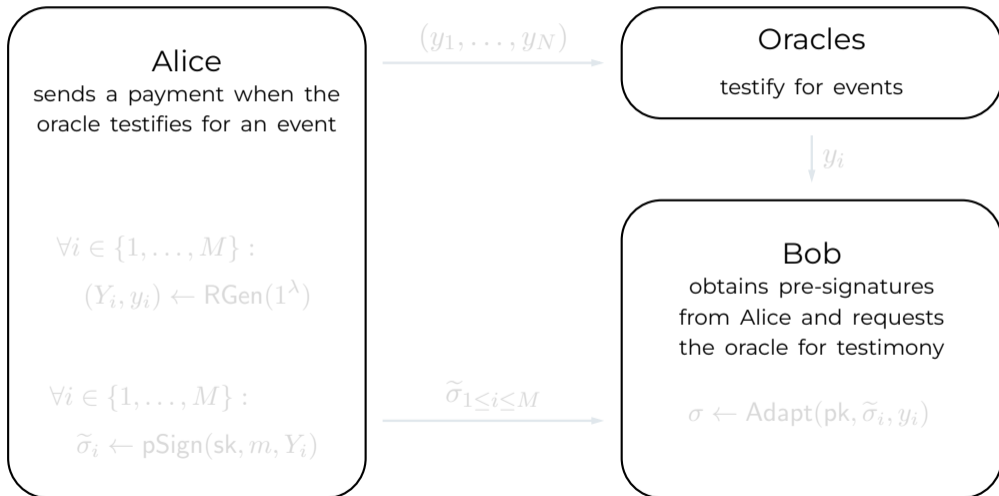
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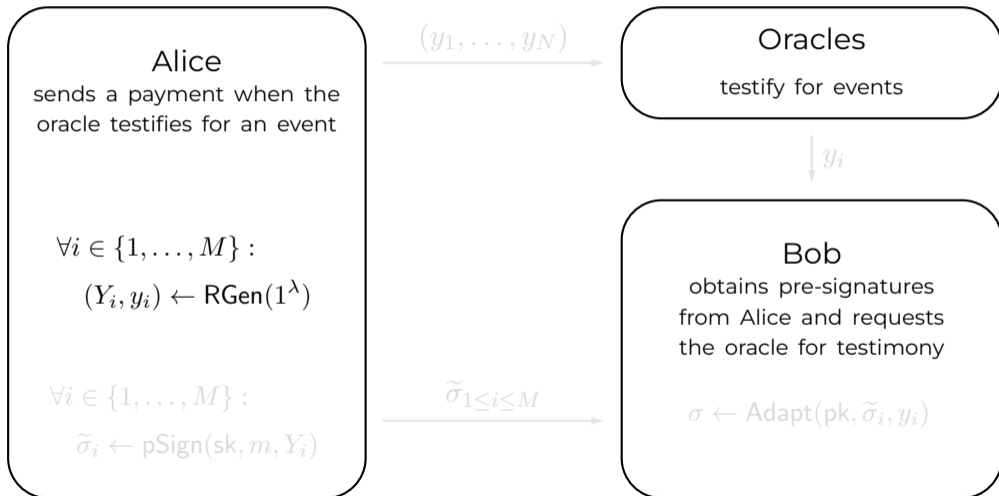


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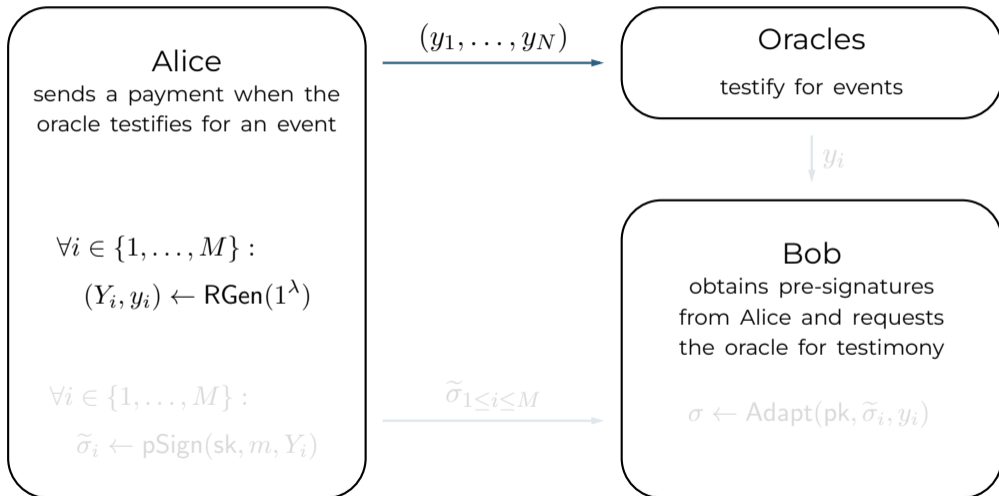
Oracle-Based Conditional Payments [MTVFMS'22]



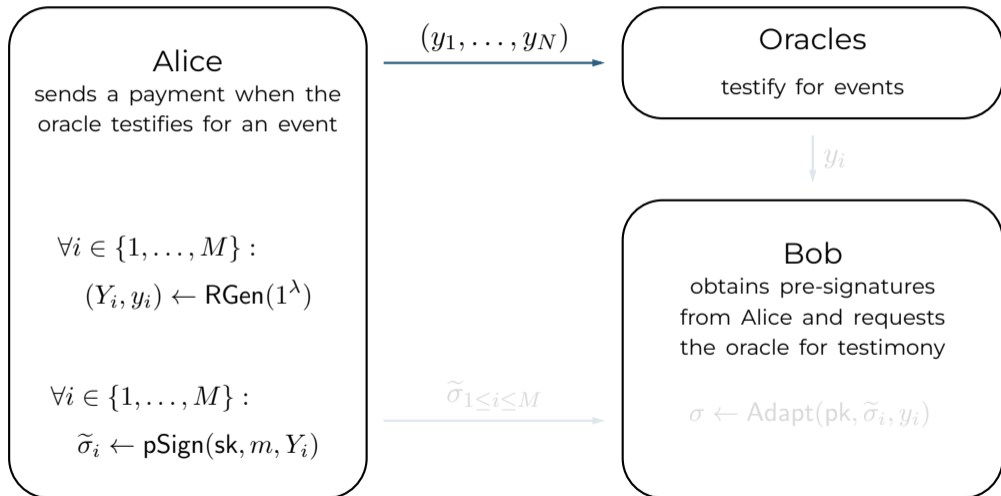
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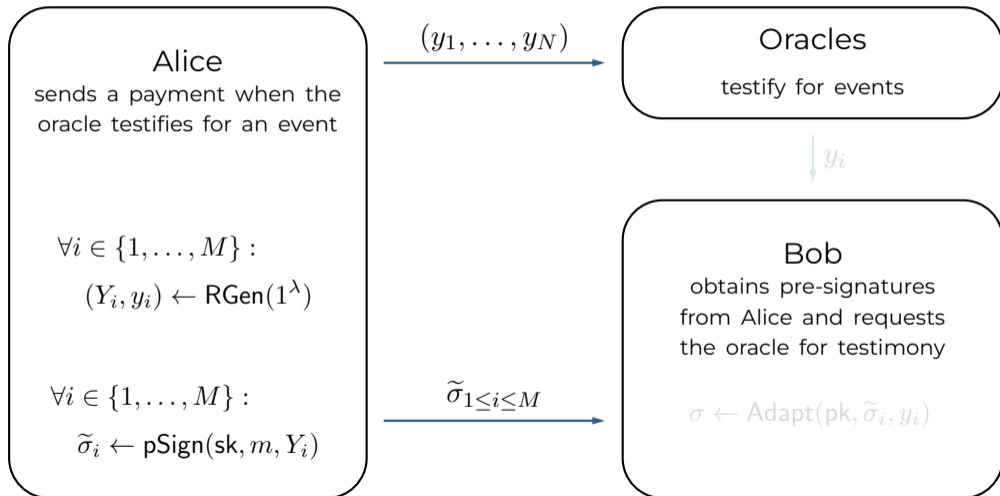
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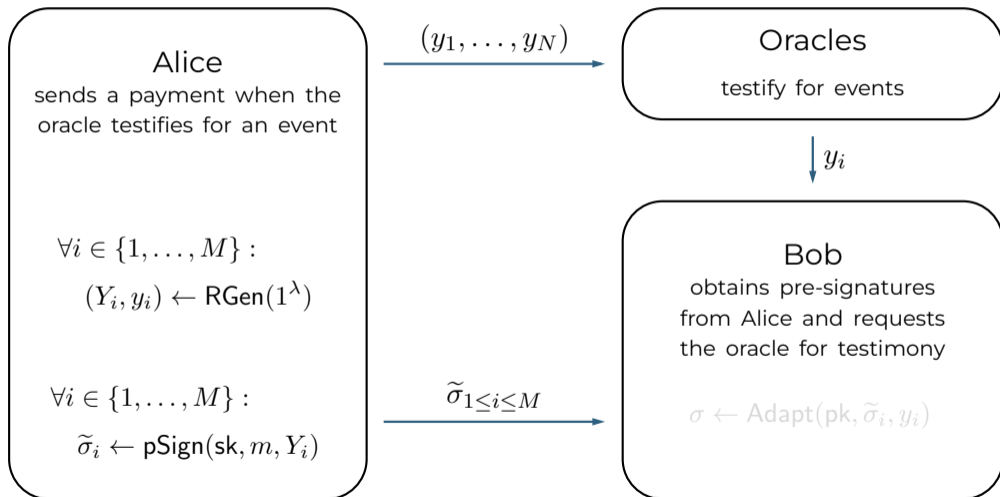
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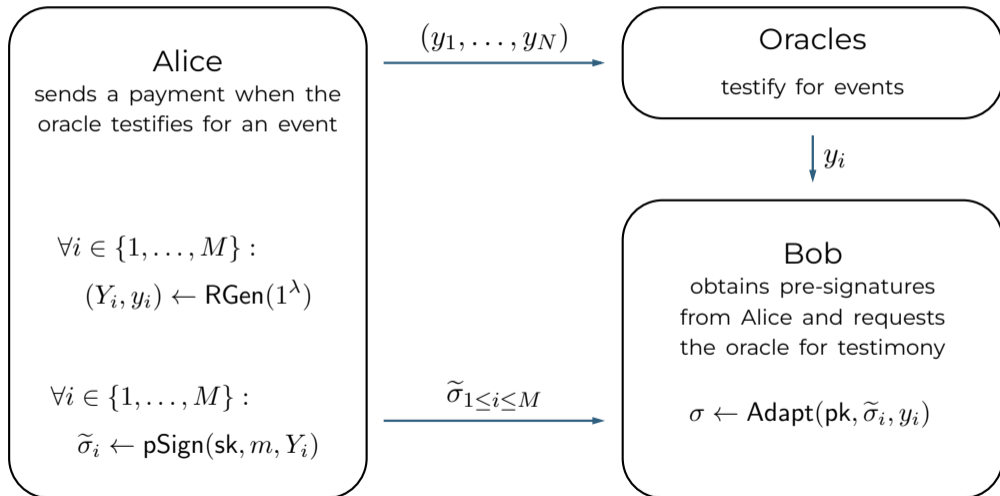
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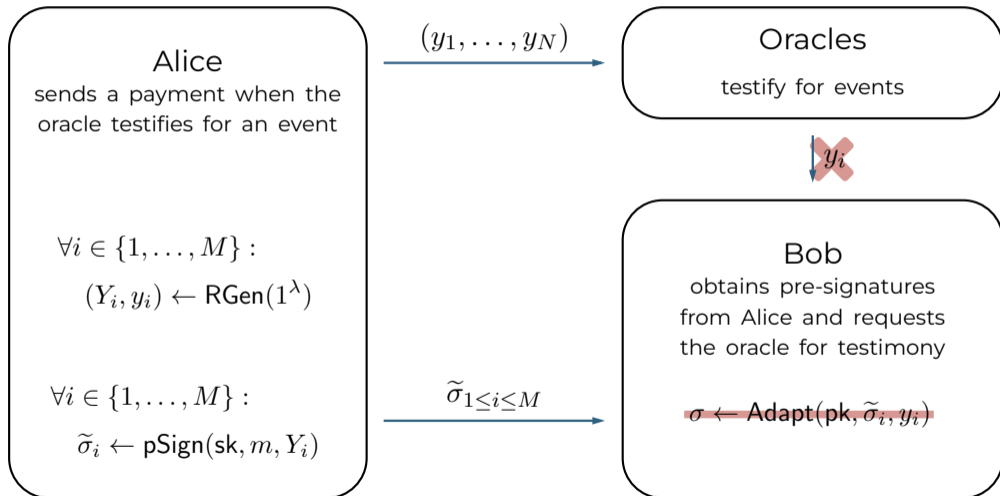
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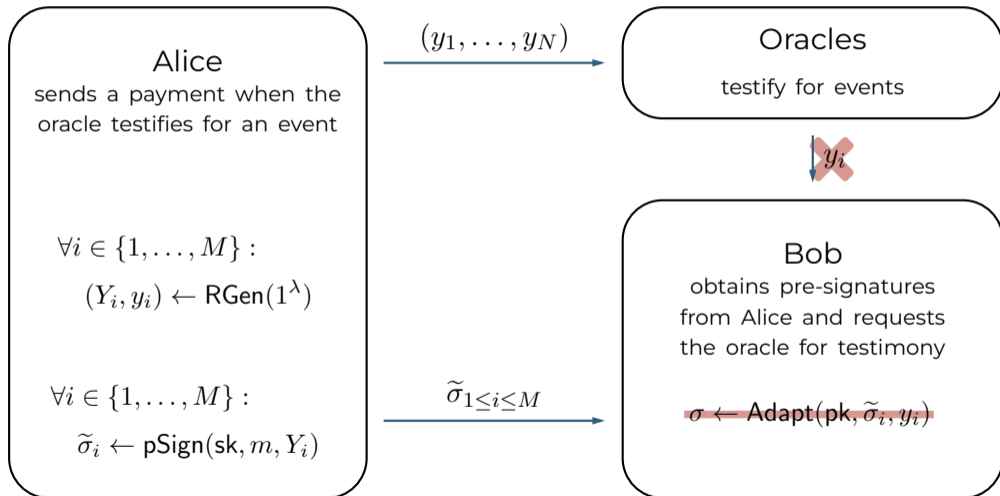
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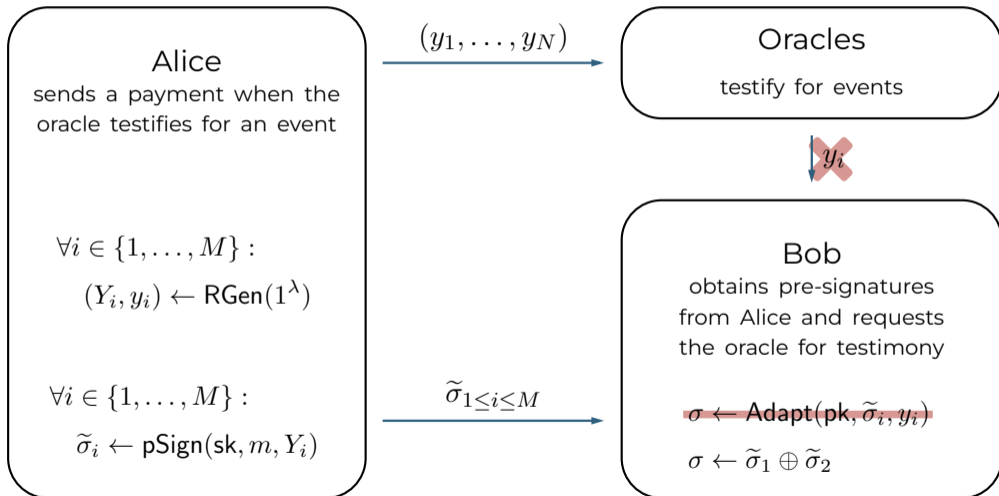
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Overview



Gaps



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Constructions



Transparent
Reductions

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Can we generically transform signatures into adaptor signatures?

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Dichotomic Signatures: Pre-Signing

- The signature consists of two parts

$$\sigma = (\sigma_1, \sigma_2)$$

- The signature uses a homomorphic one-way function

$$R = \text{OWF}(r); Y = \text{OWF}(y); r, y \in \mathbb{Z}_p$$

- One part can be computed using

$$\sigma_1 = \Sigma_1(\text{sk}, m; \text{OWF}(r) \cdot \text{OWF}(y))$$

- The other part can be computed using

$$\sigma_2 = \Sigma_2(\text{sk}, m; r)$$

pSign(sk, m, Y)

1 : $r \leftarrow \$ \mathbb{Z}_p; R \leftarrow g^r$

2 : $h \leftarrow \mathcal{H}(\text{pk}, R \cdot Y, m)$

3 : **return** $(R \cdot Y, \text{sk} \cdot h + r)$

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Dichotomic Signatures: Adapt/Extract

Adapt(pk, $\tilde{\sigma}$, y)

1 : parse $\tilde{\sigma}$ as $(\tilde{\sigma}_1, \tilde{\sigma}_2)$

2 : **return** $(\tilde{\sigma}_1, \tilde{\sigma}_2 + y)$

- The second part of the signature is homomorphic in the randomness

Extract(Y, $\tilde{\sigma}$, σ)

1 : parse $\tilde{\sigma}$ as $(\tilde{\sigma}_1, \tilde{\sigma}_2)$

2 : parse σ as (σ_1, σ_2)

3 : **return** $\sigma_2 - \tilde{\sigma}_2$

$$\Sigma_2(\text{sk}, m; r) + y = \Sigma_2(\text{sk}, m; r + y)$$

Dichotomic Signatures: A Definition

A signature scheme w.r.t. a homomorphic one-way function OWF is dichotomic; if

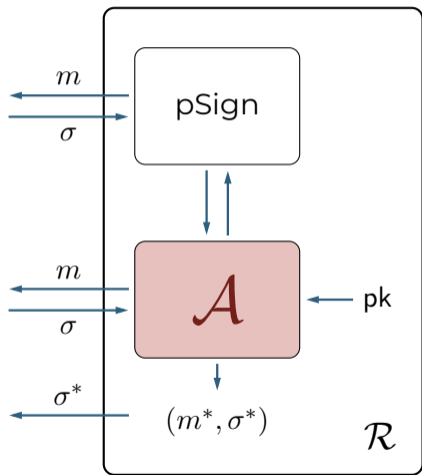
- It is decomposable

$$\sigma = (\sigma_1, \sigma_2) = (\Sigma_1(\text{sk}, m; \text{OWF}(r)), \Sigma_2(\text{sk}, m; r))$$

- It is homomorphic in the randomness

$$\Sigma_2(\text{sk}, m; r) + y = \Sigma_2(\text{sk}, m; r + y)$$

Proving Security

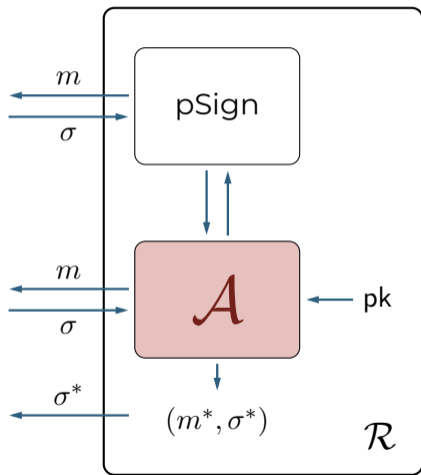


- We need to simulate pre-signatures to the adversary
- We cannot use the random oracle

Converting a signature into a pre-signature seems impossible

- We **cannot** reduce to the strong unforgeability directly

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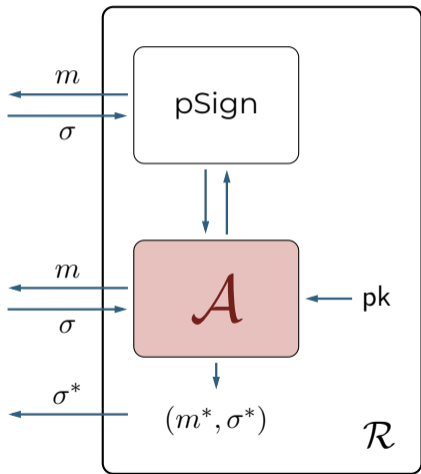


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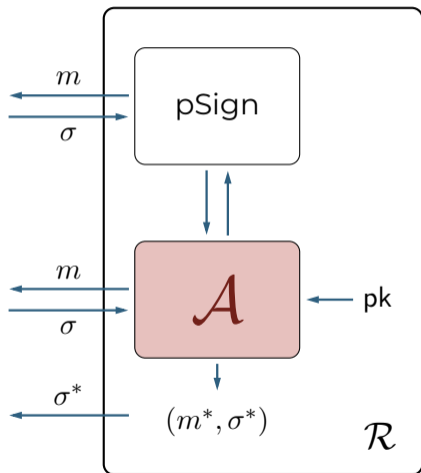


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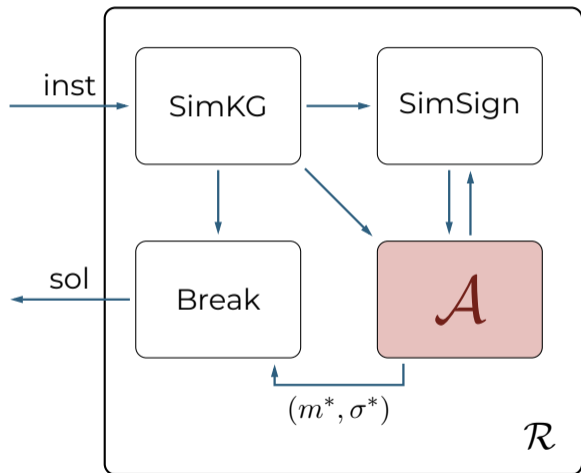


Constructions



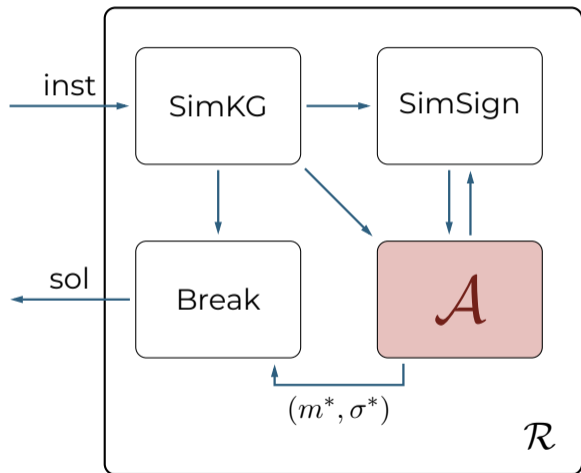
Transparent
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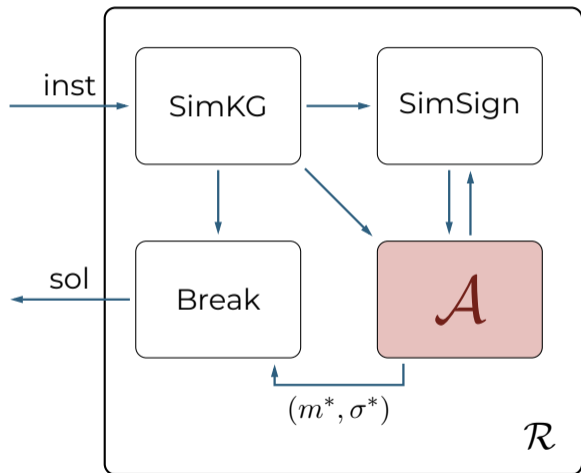
- **SimKG:**
 - Simulates keys ($\text{simSK}, \text{simPK}$)
- **SimSign:**
 - Simulates signatures using simSK
- **Break:**
 - Solve problem instance using valid forgery

Transparent Reductions



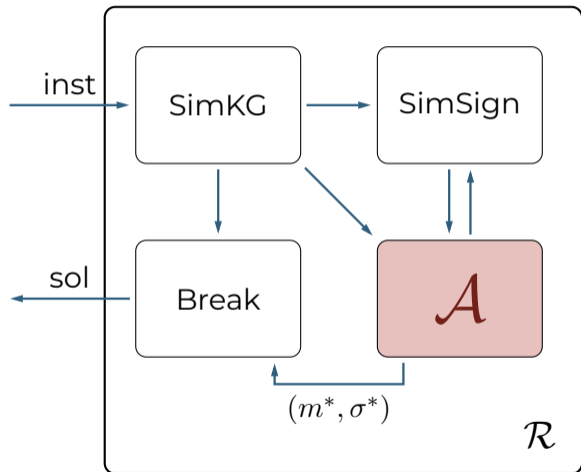
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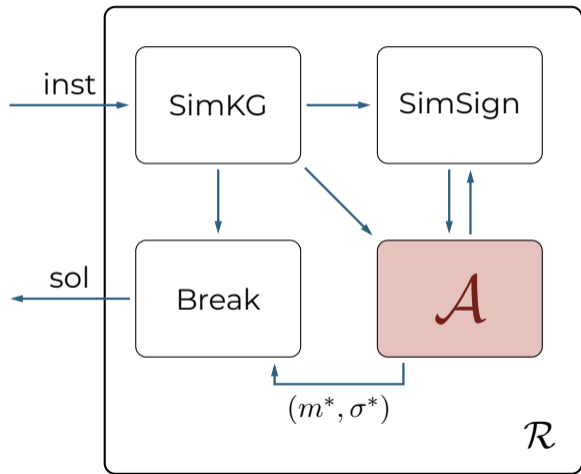
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Simulating Pre Signatures



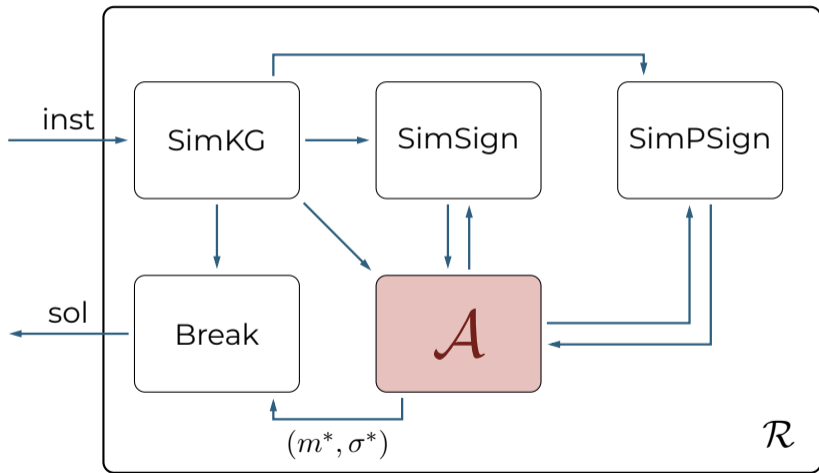
- So far, we **can**:
 - Simulate keys
 - Provide a signature oracle
 - Break the problem instance using a forgery
- So far, we **cannot**:
 - Provide a pre-signature oracle

Simulating Pre Signatures



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Simulatable Transparent Reductions



A Framework For Adaptor Signatures

A secure adaptor signature scheme requires the following three checks:

- The signature scheme is **dichotomic**
- There is a **transparent reduction** from the strong unforgeability to an underlying hard problem
- We can simulate a pre-signature oracle (**simulatability**)

Conclusion



Gaps



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**Transparent
Reductions**