Toward Malicious Constant-Rate 2PC via Arithmetic Garbling

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Toward Malicious Constant-Rate 2PC via Arithmetic **Garbling**

Garbling Scheme [BHR12]

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- A garbling scheme consists of four procedures:
 - A garbling procedure: $Gb(1^{\kappa}, C) \rightarrow \hat{C}, e, d$
 - An encoding procedure: $\operatorname{En}(e, \overrightarrow{x}) \to \overrightarrow{X}$
 - An evaluation procedure: $\operatorname{Ev}(\hat{C}, \overrightarrow{X}) \to \overrightarrow{Y}$
 - A decoding procedure: $De(d, \vec{Y}) \rightarrow \vec{y}$

of four procedures $b(1^{\kappa}, C) \rightarrow \hat{C}, e, d$ $En(e, \overrightarrow{x}) \rightarrow \overrightarrow{X}$ $e: Ev(\hat{C}, \overrightarrow{X}) \rightarrow \overrightarrow{Y}$ $De(d, \overrightarrow{Y}) \rightarrow \overrightarrow{y}$

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Correctness Obliviousness

of four procedures $b(1^{\kappa}, C) \rightarrow \hat{C}, e, d$ $En(e, \overrightarrow{x}) \rightarrow \overrightarrow{X}$ $e: Ev(\hat{C}, \overrightarrow{X}) \rightarrow \overrightarrow{Y}$ $De(d, \overrightarrow{Y}) \rightarrow \overrightarrow{y}$

ess Privacy Authenticity

Toward Malicious Constant-Rate **2PC** via Arithmetic Garbling

2PC

 $C(\overrightarrow{x_0}, \overrightarrow{x_1})$



2PC





Semi-Honest 2PC



 $C(\overrightarrow{x_0}, \overrightarrow{x_1})$



Semi-Honest 2PC





 $C(\overrightarrow{x_0}, \overrightarrow{x_1})$



Semi-Honest 2PC $C(\overrightarrow{x_0}, \overrightarrow{x_1})$















constant-round





constant-round



$\tilde{O}(|C|)$ -communication



Garbled truth tables gate by gate



 $Enc_{X_1,Y_0}(Z_0)$ $Enc_{X_1,Y_1}(Z_1)$

3



Garbled truth tables gate by gate



 $Enc_{X_1,Y_0}(Z_0)$ $Enc_{X_{1},Y_{1}}(Z_{1})$











Garbled truth tables gate by gate



 $-Z_{0}, Z_{1} \qquad \text{Enc}_{X_{0}, Y_{0}}(Z_{0}) \\ -Z_{0}, Z_{1} \qquad \text{Enc}_{X_{0}, Y_{1}}(Z_{0})$ $\operatorname{Enc}_{X_1,Y_0}(Z_0)$ $Enc_{X_{1},Y_{1}}(Z_{1})$

		\bigcirc
[Yao86, LP09]	4 <i>ĸ</i>	4 <i>ĸ</i>
[NPS99]	Зк	Зк
[KS08]	Зк	0
[ZRE15]	2κ	0
[RR21]	1.5ĸ	0





Garbled truth tables gate by gate



 $Enc_{X_1,Y_0}(Z_0)$ $Enc_{X_1,Y_1}(Z_1)$

Communication Rate (K)

		\bigoplus
[Yao86, LP09]	4κ	4ĸ
[NPS99]	Зк	Зк
[KS08]	Зк	0
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Toward Malicious Constant-Rate 2PC via **Arithmetic Garbling**





[AIK11], [BMR16], [BLLL23], [LL24], [Heath24]



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Bounded Integer Computation (BIC)



[AIK11], [BMR16], [BLLL23], [LL24], [Heath24]

Bounded Integer Computation (BIC)

 $B \in \mathbb{Z}^+$



[AIK11], [BMR16], [BLLL23], [LL24], [Heath24]





Garbled <u>Arithmetic</u> Circuits

Bounded Integer Computation (BIC)

 $B \in \mathbb{Z}^+$



[AIK11], [BMR16], [BLLL23], [LL24], [Heath24]



O(1)





Every wire is *B*-bounded.

Bounded Integer Computation (BIC)



[AIK11], [BMR16], [BLLL23], [LL24], [Heath24]

Assume DCR, there exists a constant-rate arithmetic garbling in the bounded integer model



Toward Malicious Constant-Rate 2PC via Arithmetic Garbling



Toward Malicious Constant-Rate 2PC via Arithmetic Garbling



semi-honest

Our focus





Semi-honest \Rightarrow Malicious











Semi-honest \Rightarrow Malicious





(& malicious secure OT/OLE)





Ensure this is correctly garbled





Semi-honest \Rightarrow Malicious

For the Boolean garblings, this is possible:





Semi-honest \Rightarrow Malicious

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XCut-and-choose $O(\lambda)$ -rate [LP07, …]





Semi-honest \Rightarrow Malicious

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 - **X**Cut-and-choose $O(\lambda)$ -rate [LP07, …]
 - Authenticated garblings O(1)-rate [IKO+11, WRK17, ...]





Semi-honest \Rightarrow Malicious

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 - **X**Cut-and-choose $O(\lambda)$ -rate [LP07, …]
 - Authenticated garblings O(1)-rate [IKO+11, WRK17, …]
 - Dual execution O(1)-rate **1-bit leakage** [MF06, …]








(& malicious secure OT/OLE)

Semi-honest \Rightarrow Malicious

- For the Boolean garblings, this is possible:
 - **X**Cut-and-choose $O(\lambda)$ -rate [LP07, …]
 - **X**Authenticated garblings O(1)-rate [IKO+11, WRK17, …]
 - **X**Dual execution O(1)-rate **1-bit leakage** [MF06, …]







• Constant-rate BLLL GC is *not* well-defined w.r.t. the malicious security.

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• There exists an "overflow attack" that even works for a fully correct GC.

- The best achievable security is against:
 - malicious G with 1-bit leakage
 - semi-honest E

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There exists an "overflow attack" that even works for a fully correct GC.

- The best achievable security is against:
 - malicious G with 1-bit leakage
 - semi-honest E

• We can achieve the best achievable security efficiently.

Our Results

Constant-rate BLLL GC is not well-defined w.r.t. the malicious security.

There exists an "overflow attack" that even works for a fully correct GC.

Roadmap

Constant-rate BLLL GC

- The overflow attack
- Our protocol based-on BLLL GC



Constant-rate BLLL GC

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Review of Constant-Rate BLLL GC







 $w k_0^w + k_1^w \in \mathcal{R}^n$ **Evaluator**





Communication-free information-theoretic ADD/MUL



Communication-free information-theoretic ADD/MUL

Boolean







Communication-free information-theoretic ADD/MUL

Boolean









Communication-free information-theoretic ADD/MUL

Boolean









Communication-free information-theoretic ADD/MUL

Boolean



garbled truth table







Communication-free information-theoretic ADD/MUL

Boolean



garbled truth table









Communication-free information-theoretic ADD/MUL

Boolean



garbled truth table

GC labels





AIK



Communication-free information-theoretic ADD/MUL

Boolean



garbled truth table

GC labels



no communication



Communication-free information-theoretic ADD/MUL





garbled truth table







GC labels

Communication-free information-theoretic ADD/MUL

KE gadgets





GC labels

Communication-free information-theoretic ADD/MUL

KE gadgets

Communication-free information-theoretic ADD/MUL













Communication-free information-theoretic ADD/MUL









Communication-free information-theoretic ADD/MUL



linear key/msg homomorphism









Communication-free information-theoretic ADD/MUL



linear key/msg homomorphism



GC labels







Communication-free information-theoretic ADD/MUL



linear key/msg homomorphism



GC labels





Communication-free information-theoretic ADD/MUL



linear key/msg homomorphism



GC labels





Communication-free information-theoretic ADD/MUL



linear key/msg homomorphism



GC labels











BLLL constant-rate KE gadgets from DCR

GC labels

Communication-free information-theoretic ADD/MUL

KE gadgets

Review of Constant-Rate BLLL GC AIK GC paradigm over a ring GC labels Communication-free information-theoretic ADD/MUL **KE** gadgets **BLLL constant-rate KE gadgets from DCR** $\operatorname{Enc}_{a}(\overline{c})$ $g_1^a(N+1)^{2c_1}, \dots, g_n^a(N+1)^{2c_n} \mod N^{\zeta+1}$

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Review of Constant-Rate BLLL GC AIK GC paradigm over a ring GC labels Communication-free information-theoretic ADD/MUL **KE** gadgets **BLLL constant-rate KE gadgets from DCR** Why BIC? $\operatorname{Enc}_{a}(\overline{c})$ $g_1^a(N+1)^{2c_1}, \dots, g_n^a(N+1)^{2c_n} \mod N^{\zeta+1}$ The value can only be [-B,B]Group of unknown order **Group of order** N^{ζ} **—— Use as key of next** Enc.(\cdot) $x \in \mathbb{Z}_{N^{\zeta}} \Rightarrow x \in \mathbb{Z}$




Constant-rate BLLL GC

The overflow attack

Our protocol based-on BLLL GC

Roadmap

The Overflow Attack Which part looks attackable?







- **AIK GC paradigm over a ring**
 - GC labels
- Communication-free information-theoretic ADD/MUL
 - KE gadgets
 - **BLLL constant-rate KE gadgets from DCR**

The Overflow Attack Maliciously change inputs

The Overflow Attack **Maliciously change inputs**







Bounded-integer computation:

Every wire is *B*-bounded.



The Overflow Attack Maliciously change inputs





Bounded-integer computation:

Some wire value may **not** be *B*-bounded.





 $x \in \mathbb{Z}_{N^{\zeta}}$ \longrightarrow $x \in \mathbb{Z}$

$x \in [-B, B]$





$x \notin [-B, B]$



 $x \in \mathbb{Z}_{N\zeta}$ \longrightarrow $\tilde{x} \in \mathbb{Z}$

E could no longer decrypt the ciphertext

$x \notin [-B, B]$











A fully correct GC

 $B \in \mathbb{Z}^+$ $\overrightarrow{x_1} \in \mathbb{Z}^*$

Dec $\rightarrow \bot$





A fully correct GC









 $Dec \rightarrow \bot$

 $B \in \mathbb{Z}^+$ $\overrightarrow{x_1} \in \mathbb{Z}^*$

























The Overflow Attack Remark: Boolean GC is secure against malicious E







Where do overflows happen







malicious G with 1-bit leakage and

semi-honest E

The <u>Best</u> Achievable Security

Constant-rate BLLL GC

- The overflow attack
- Our protocol based-on BLLL GC













- **X**Cut-and-choose $O(\lambda)$ -rate [LP07, …]
- **Authenticated garblings** O(1)-rate [IKO+11, WRK17, ...]
- **X**Dual execution O(1)-rate **1-bit leakage** [MF06, ...]









Feasibility: GMW compiler with zkSNARK

- **X**Cut-and-choose $O(\lambda)$ -rate [LP07, …]
- Authenticated garblings O(1)-rate [IKO+11, WRK17, ...]
- **X**Dual execution O(1)-rate [MF06, ...] **1-bit leakage**













A fully An almost correct GC





A fully An almost correct GC



We design custom ZK to:

- **1. Authenticate randomness**
 - (Offline) VOLE over $\mathbb{Z}_{N^{\zeta}}$
 - (Online) ADD/MUL VOLEs
- 2. Get almost correct KE
 - (Online) Σ -protocol

Key Insight: If E does not abort, E will learn $C(x'_0, \vec{x_1})$



Conclusion

Conclusion

security than 1-bit leakage.

model assuming DCR and LPN.

 Our overflow attack works for any arithmetic garbling in the BIC model, and there is no hope to get better

 Our protocol achieves the first constant-rate constantround 2PC with the best achievable security in the BIC





- A fully correct GC
- Constant-rate GCs beyond BIC model
- Characterize the leakage functions learnt by E's overflow attacks







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