Public-Coin, Complexity-Preserving, Succinct Arguments for NP from Collision Resistance

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[3]Cornell Tech

Cryptographic Proofs for NP

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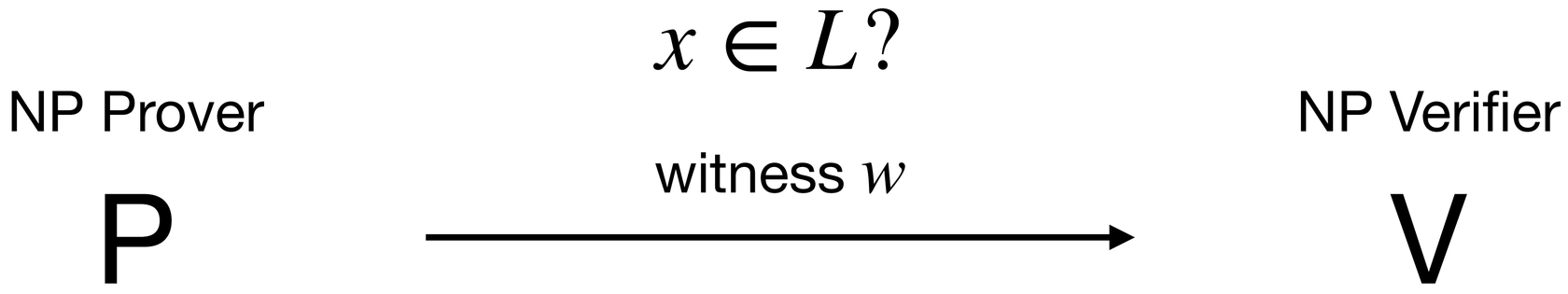

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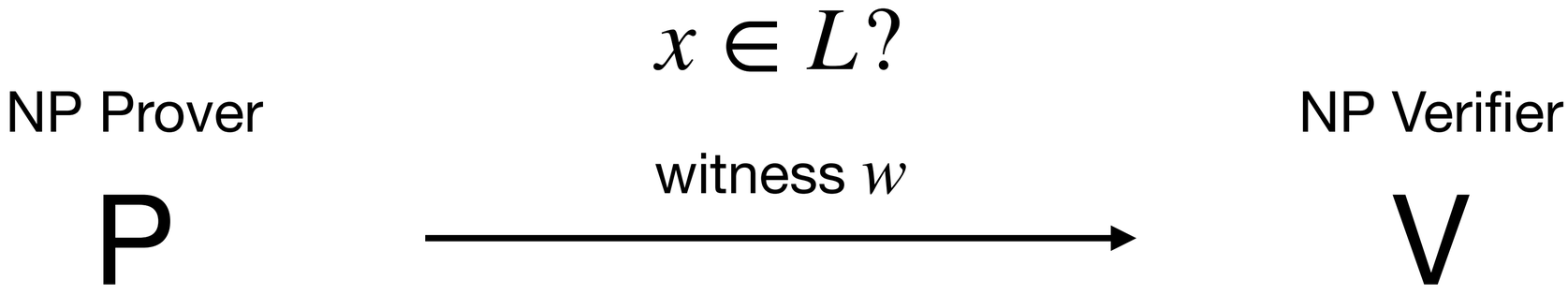

Completeness: If $x \in L$, $\exists w$ s.t. $V(x, w) = 1$.

Soundness: If $x \notin L$, $\forall w\ V(x, w) = 0$.

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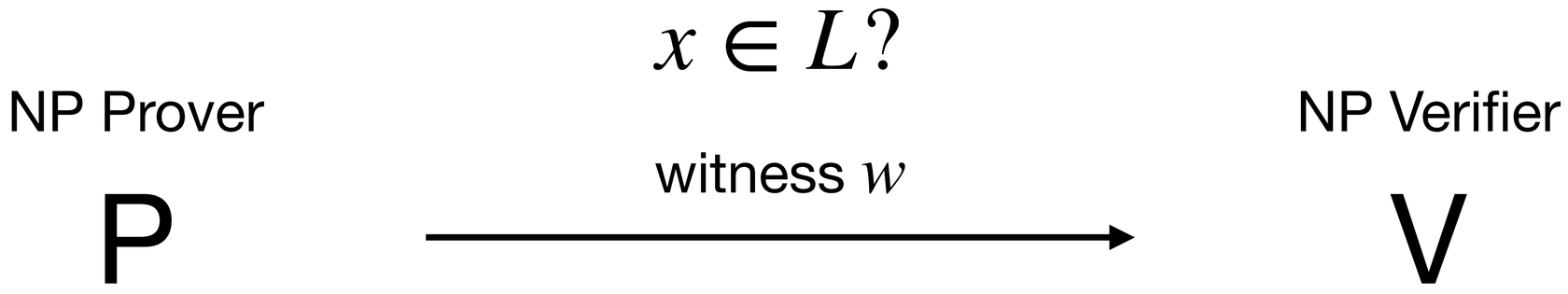

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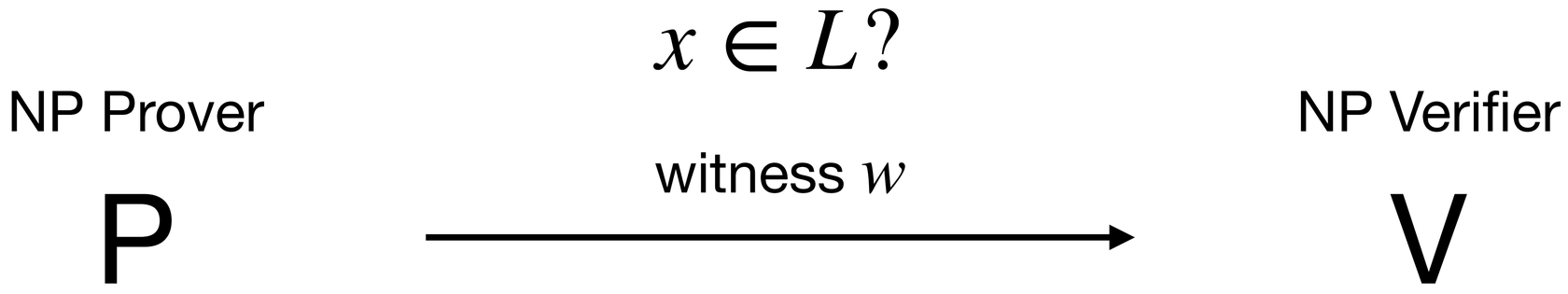

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Equivalent to non-deterministic computation

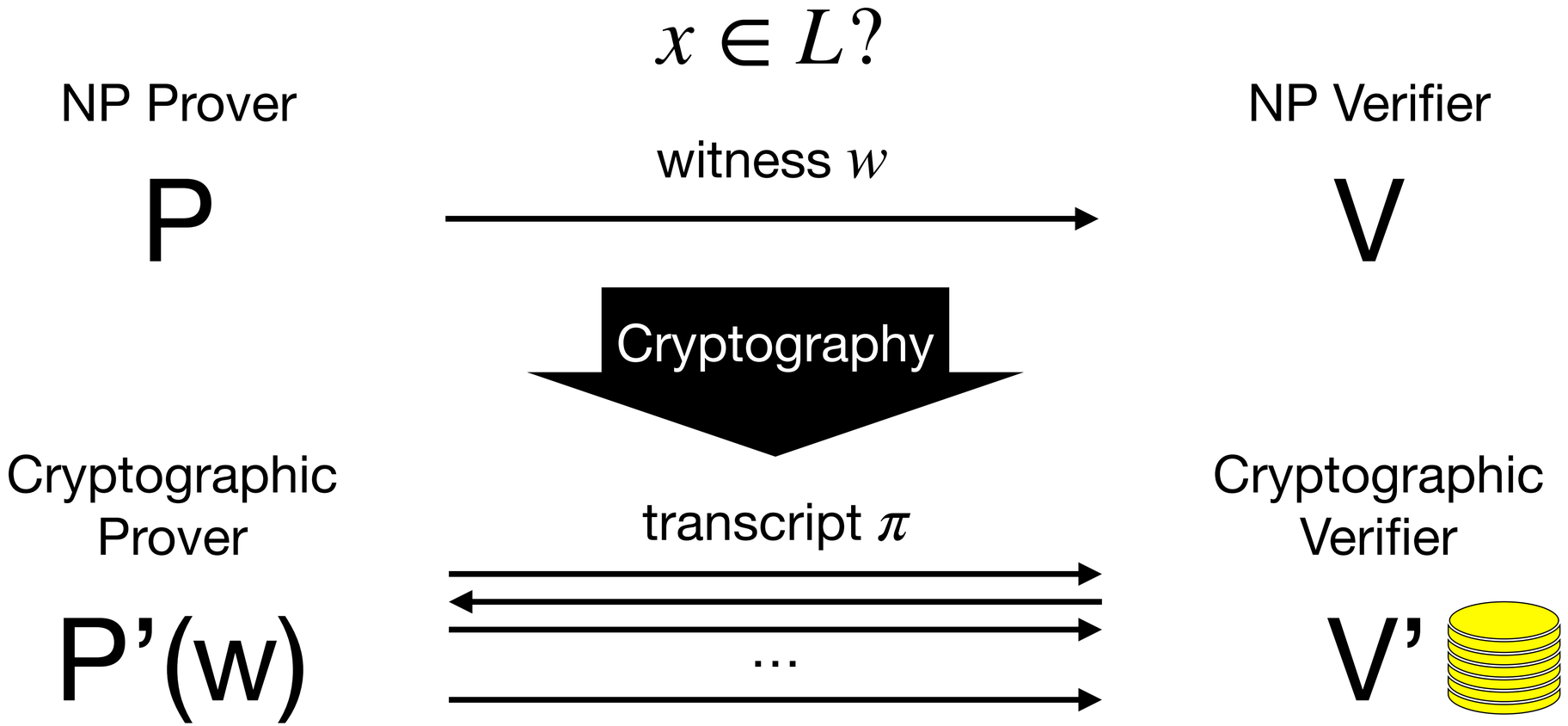
Assumptions:

- $V(x, w)$ runs in **time** T and **space** S as a **RAM program**

Cryptographic Proofs for NP

$x \in L?$

Cryptographic Proofs for NP



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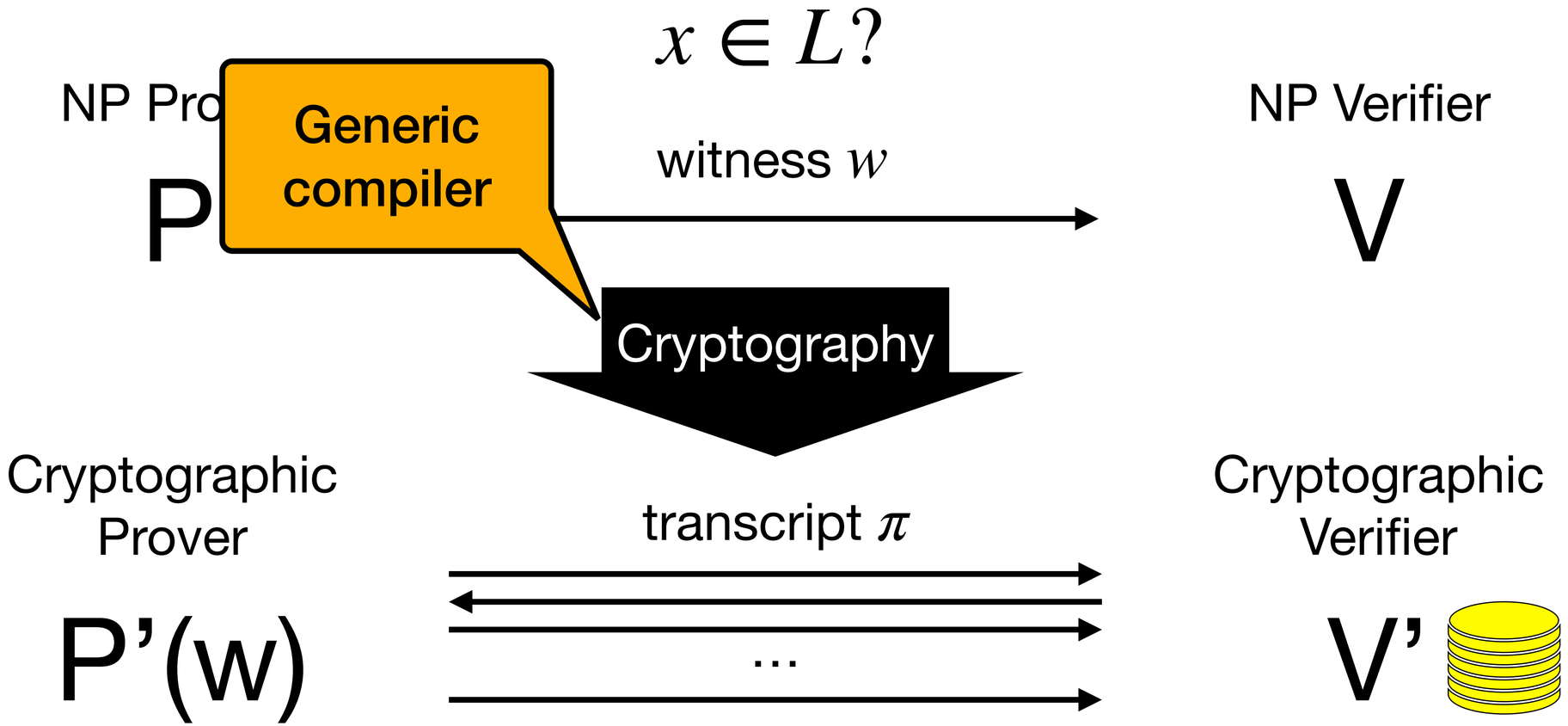

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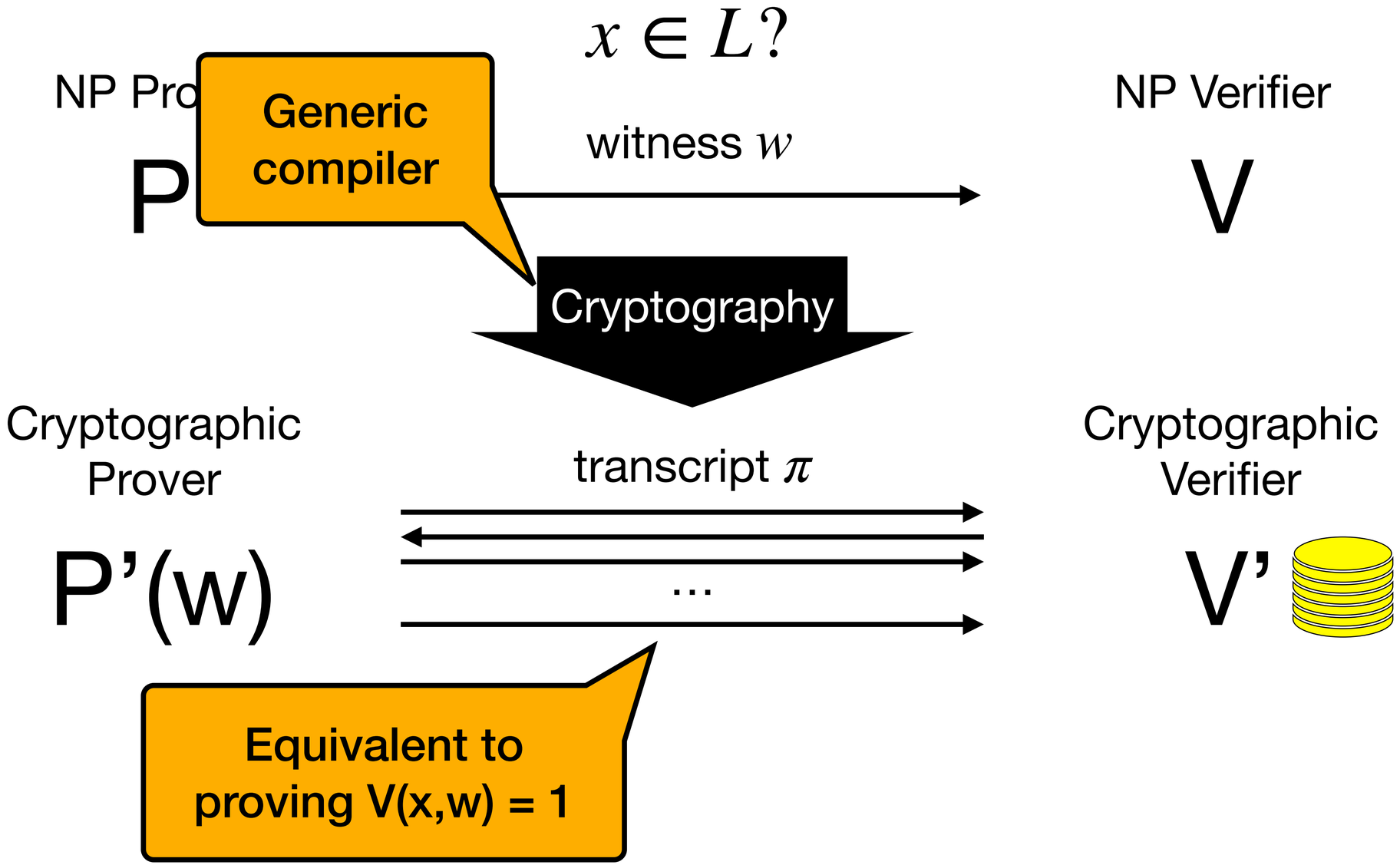


Cryptographic Proofs for NP

Cryptographic Prover

P’(w)

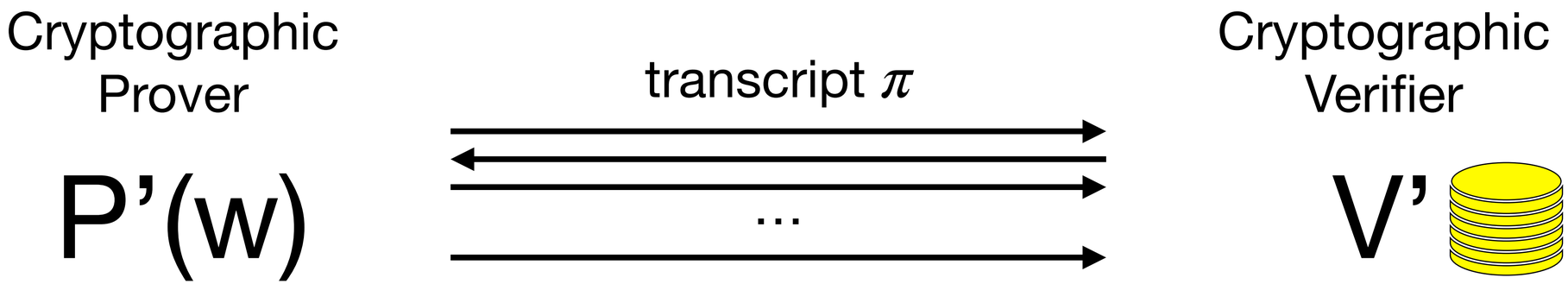
transcript π

…

Cryptographic Verifier

V’

Cryptographic Proofs for NP



Computational Soundness (Argument):

If $x \notin L$, no **poly-time** prover P* can convince V' w.h.p.

Argument of Knowledge:

If poly-time P* convinces V', it **"knows"** a witness.

Public-coin (optional):

V' sends random coins and has **no private state**.

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Goals: **Privacy** **Efficiency**

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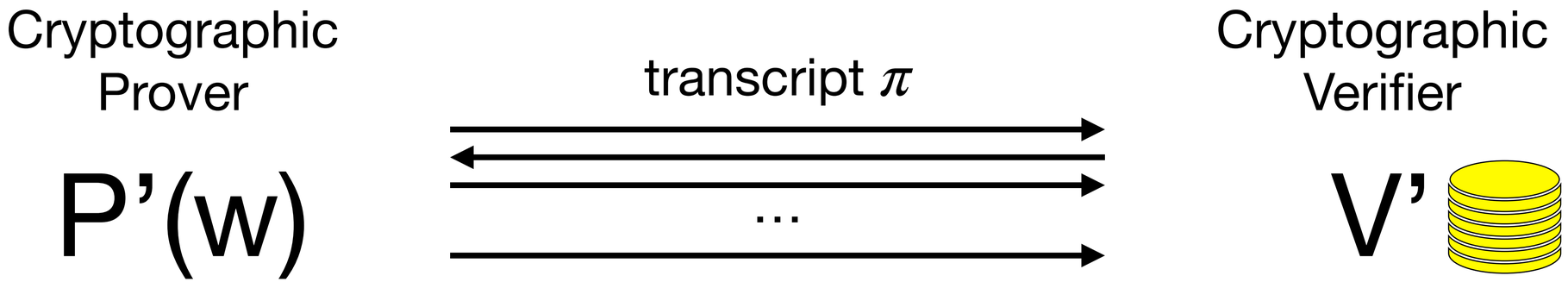
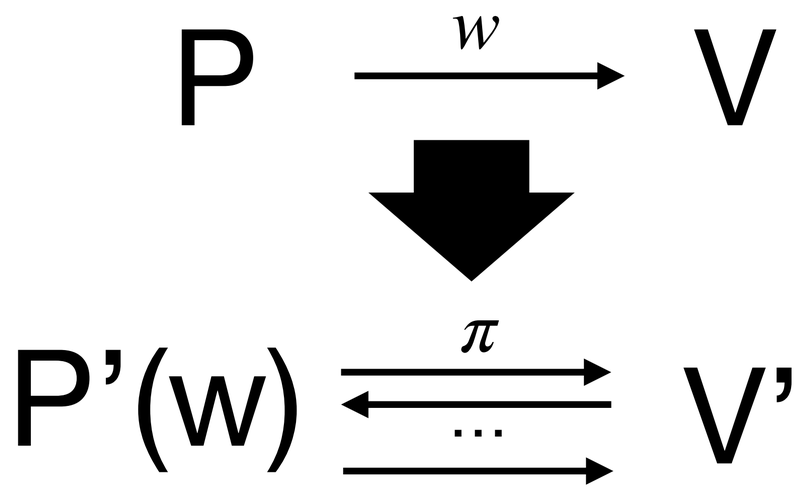


Goals: **Privacy** **Efficiency**

Focus of this talk

Efficiency Goals

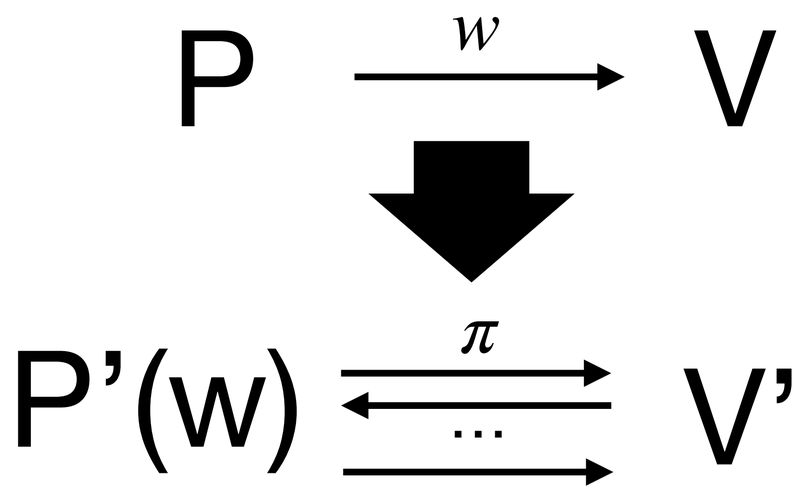


1. Low bandwidth

2. Efficient verifier

3. Scalable prover

Efficiency Goals



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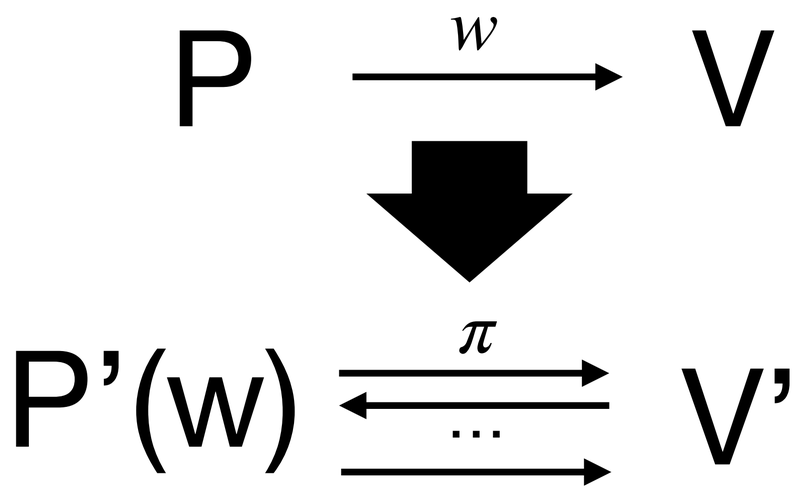
Succinct Argument:

- $|\pi| = \text{polylog}(T)$
- $\text{Time}(V') = \text{poly}(|x|, \log T)$

Efficiency Goals



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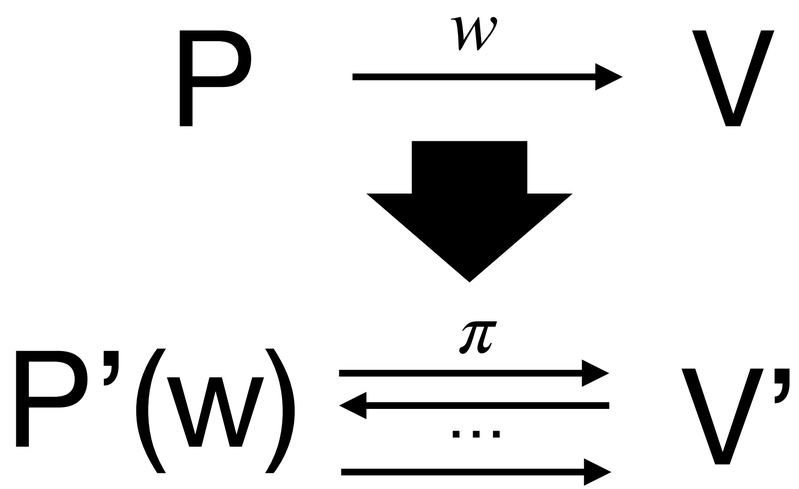

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Theorem [Kilian '92]:

There exists a 4-message **succinct argument** for NP from **collision-resistant hash functions**

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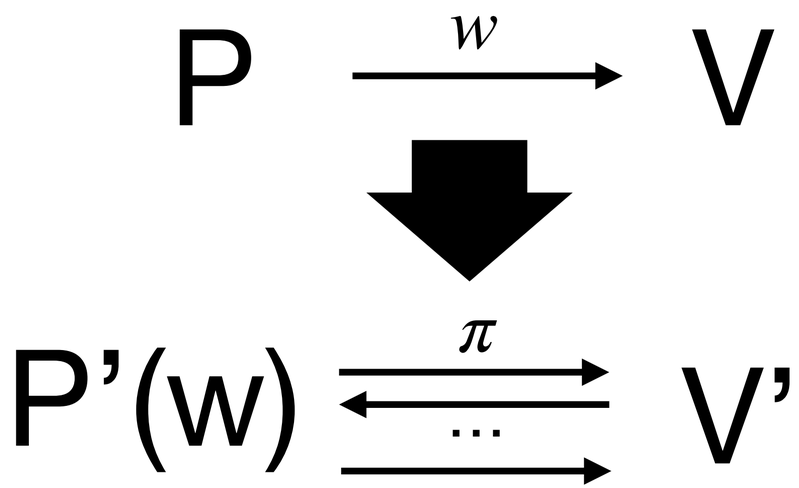

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Minicrypt, unstructured

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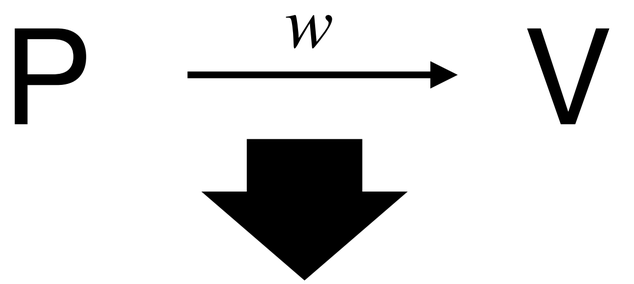

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Probabilistically Checkable Proofs (PCPs) + Hash Trees

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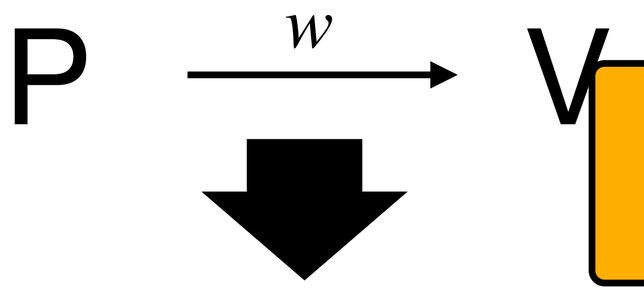

V'

width

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Poly-time in T is not scalable!

3. Scalable prover

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The Quest for a Scalable Prover

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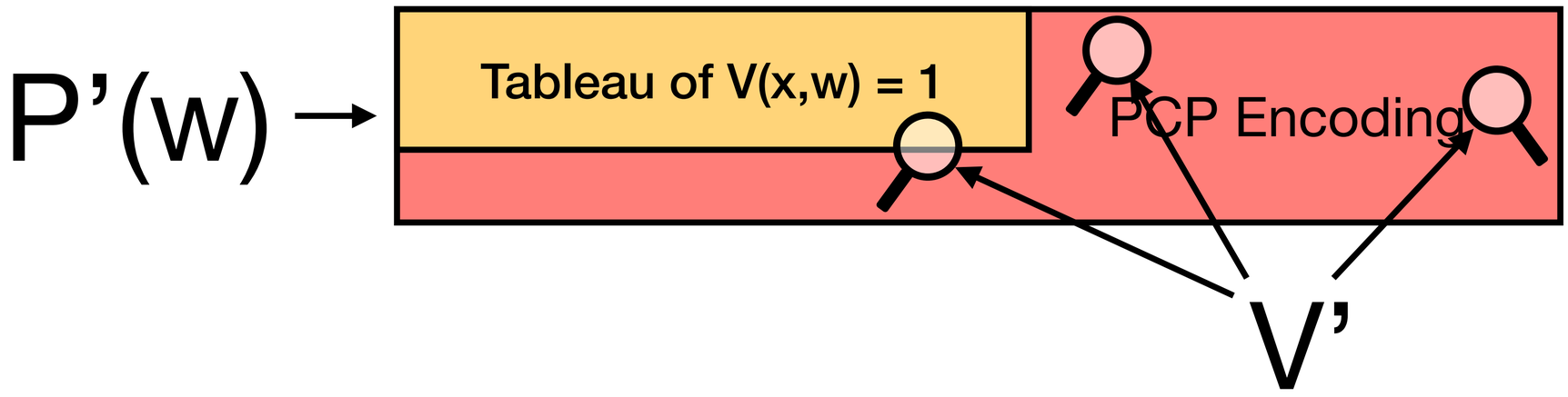
More Efficient PCPs

[BFLS91],[BGHSV05], [D07],[BS08],[M09], [BCGT13], [BKKMS16],…



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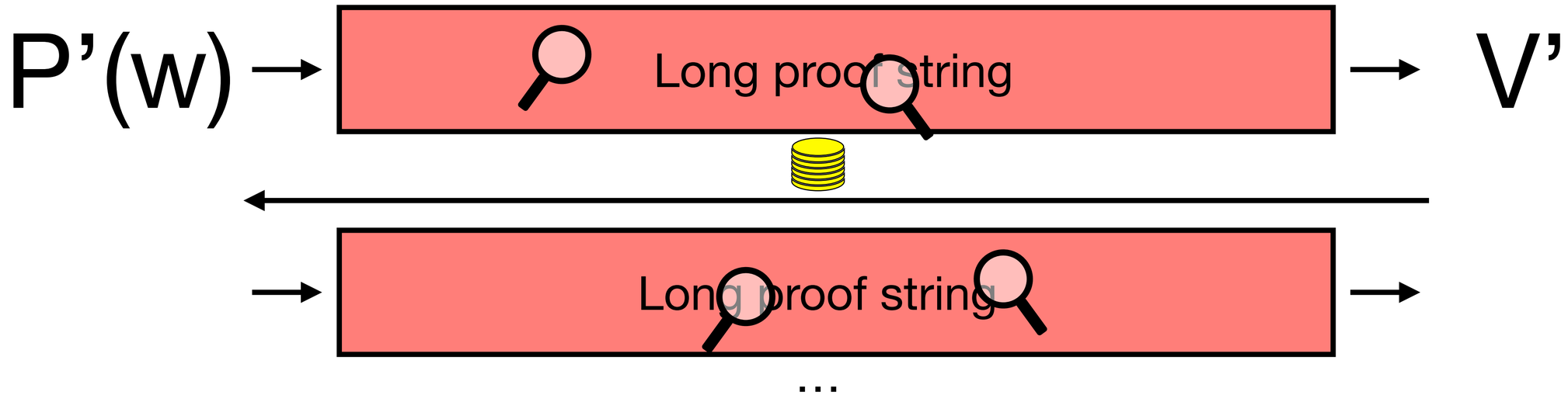

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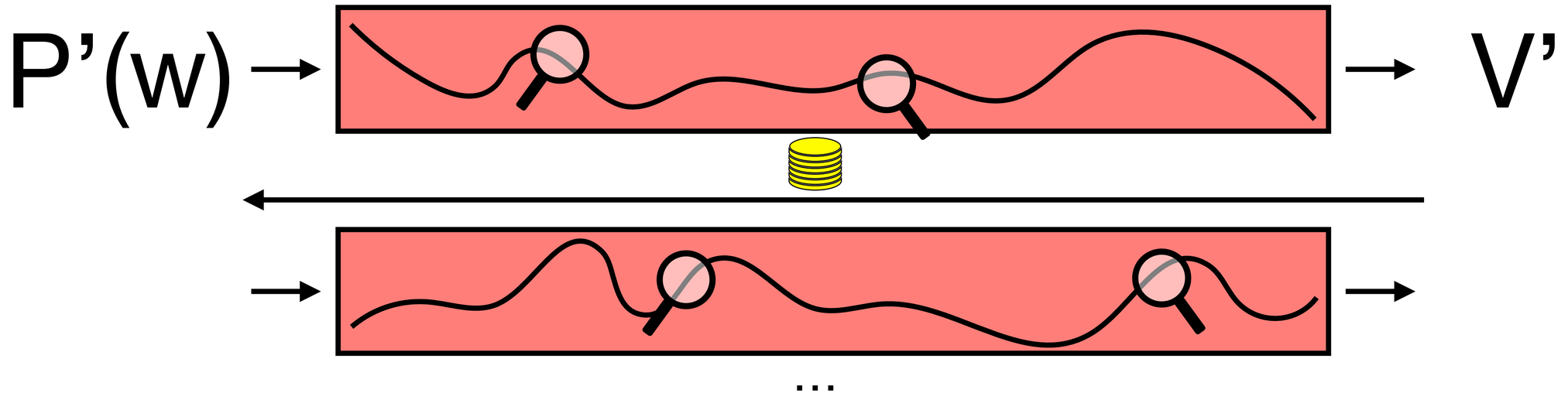

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Recursive Composition
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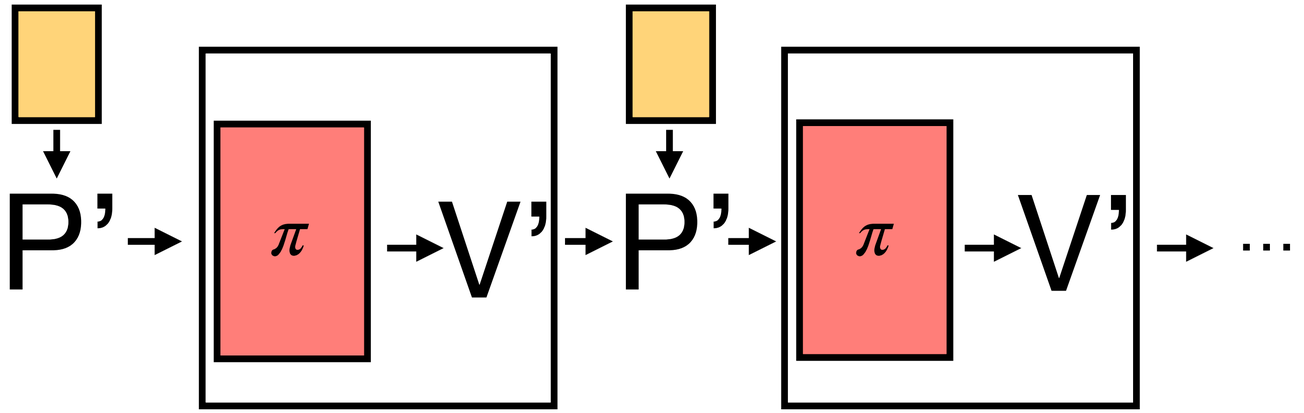

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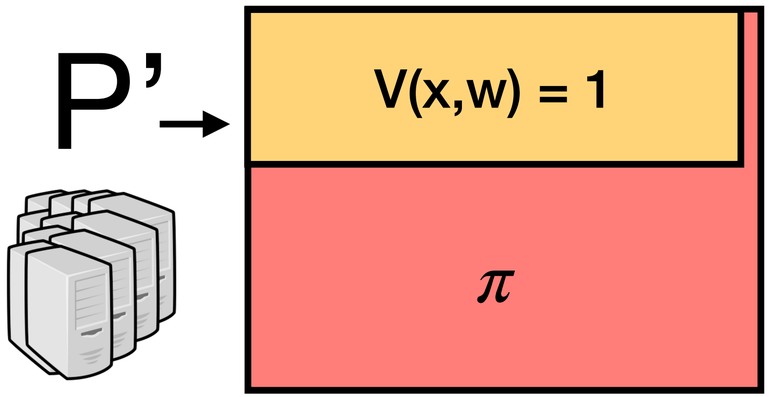

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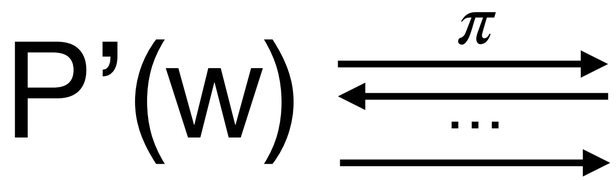
Space-Efficient/ Complexity-Preserving
[BC12],[BCCT12],[BHRRS20], [BHRRS21],[BBHV22]

Blueprint for Kilian-Based Arguments



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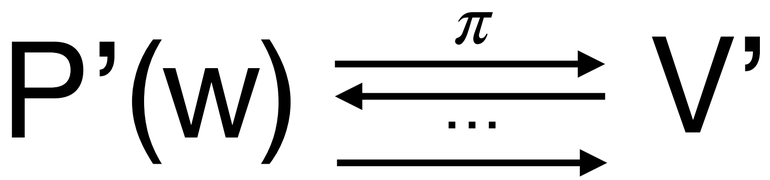

1. Write down computation tableau

Tableau of V(x,w) = 1

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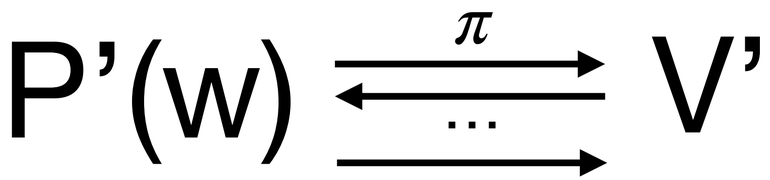

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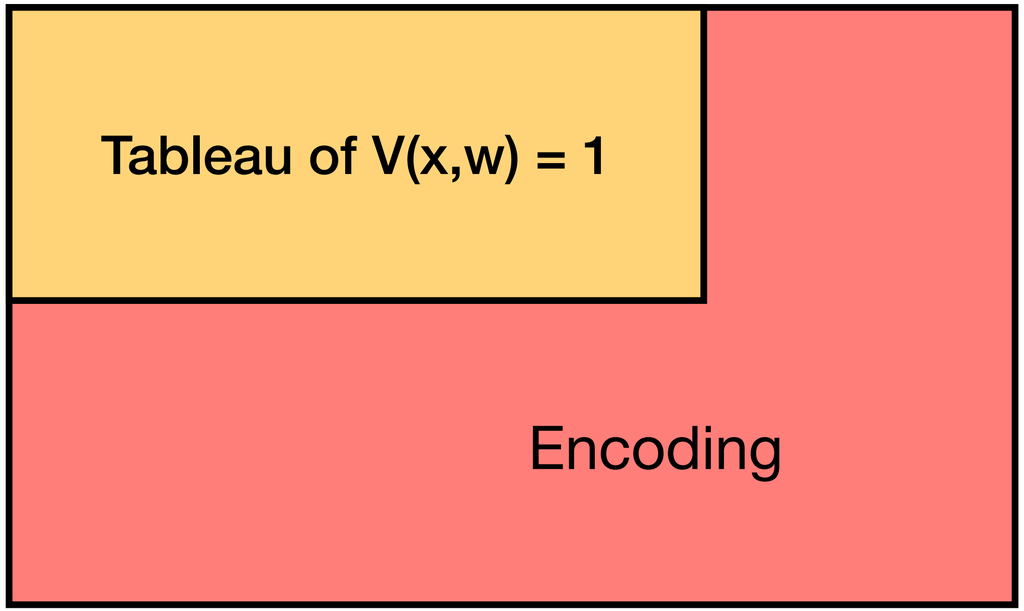
2. Encode tableau



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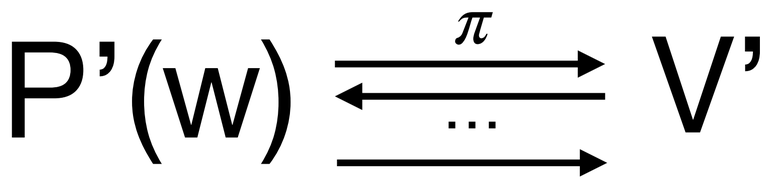

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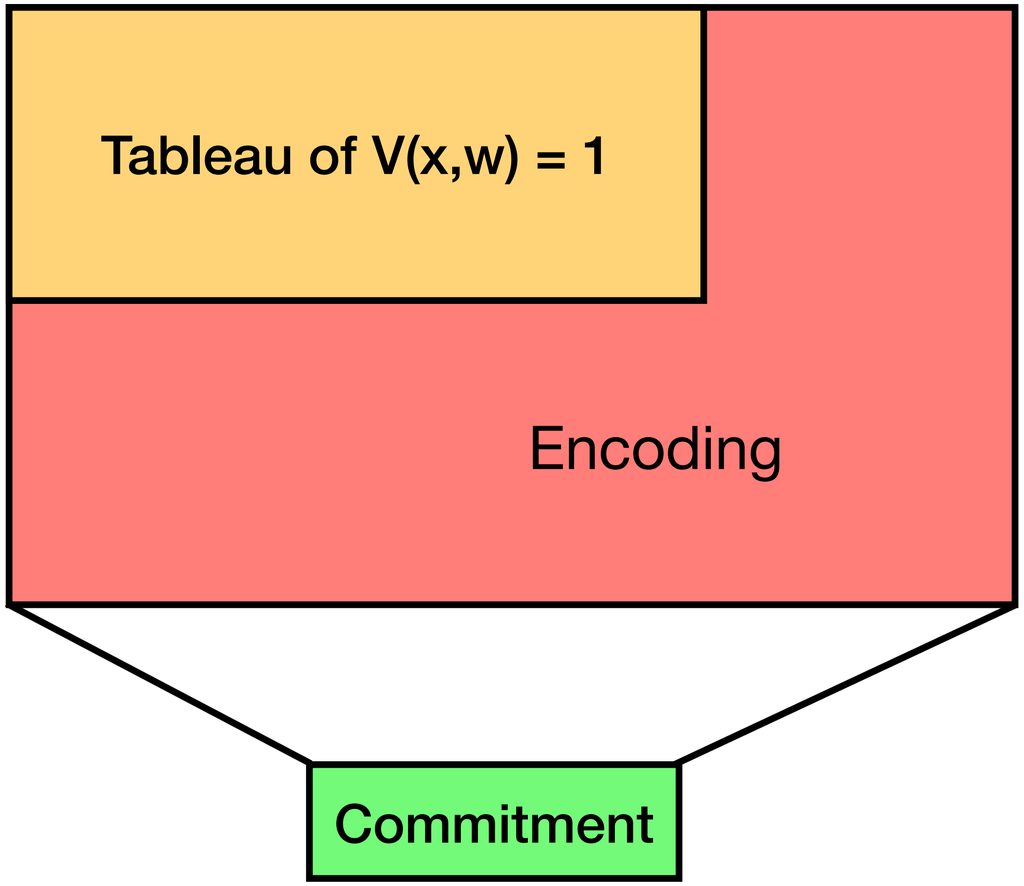
3. Cryptographically commit to encoding



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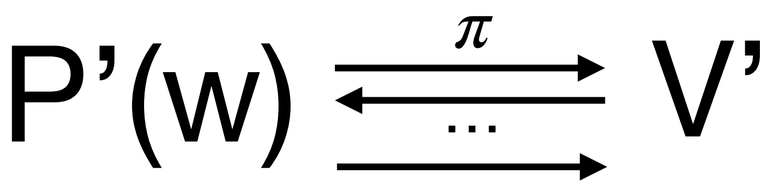

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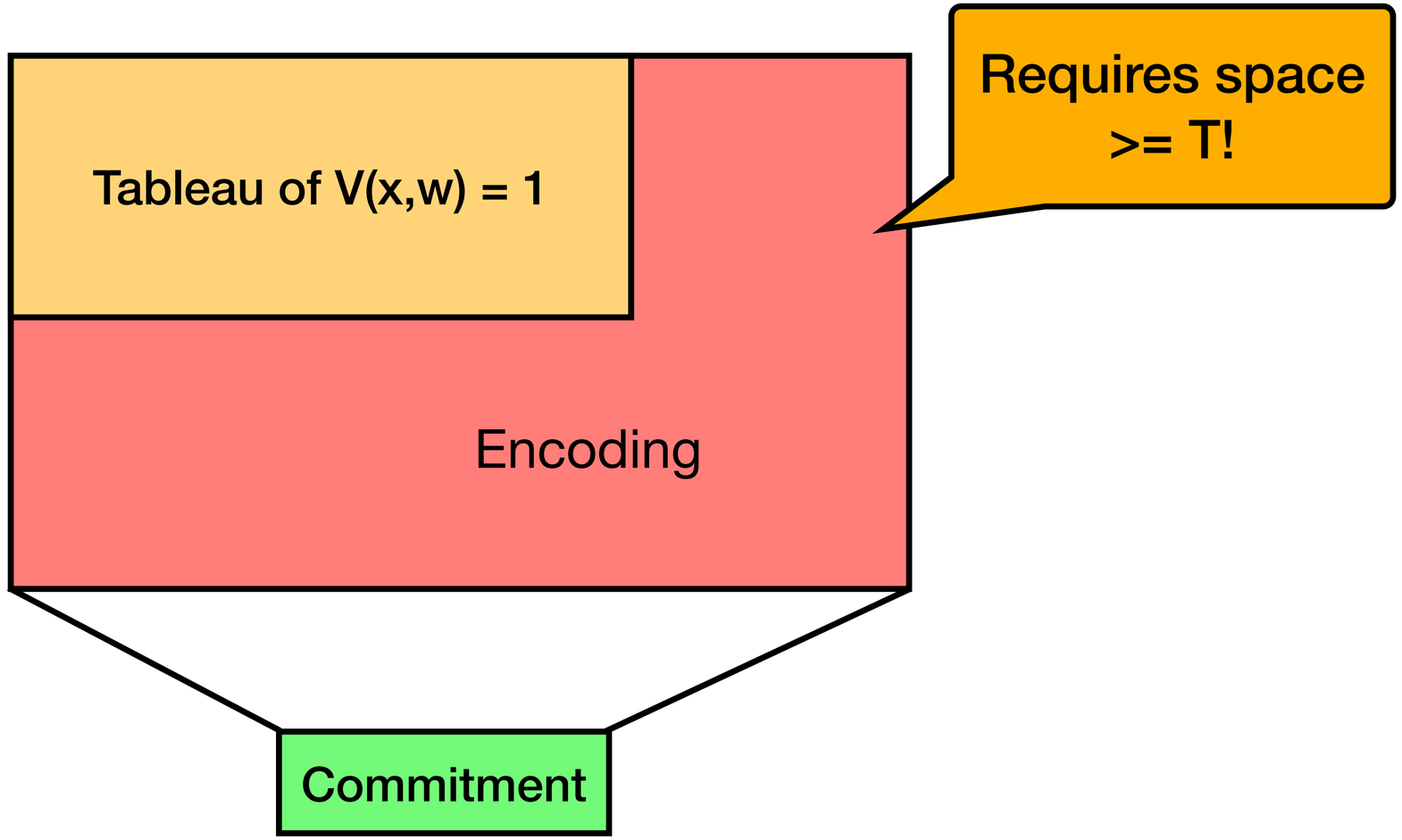

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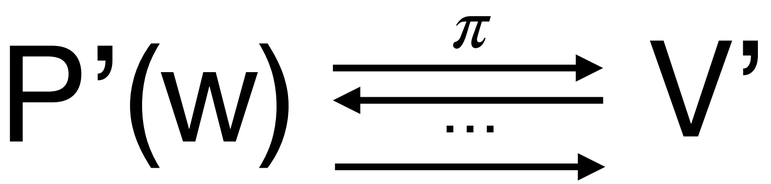

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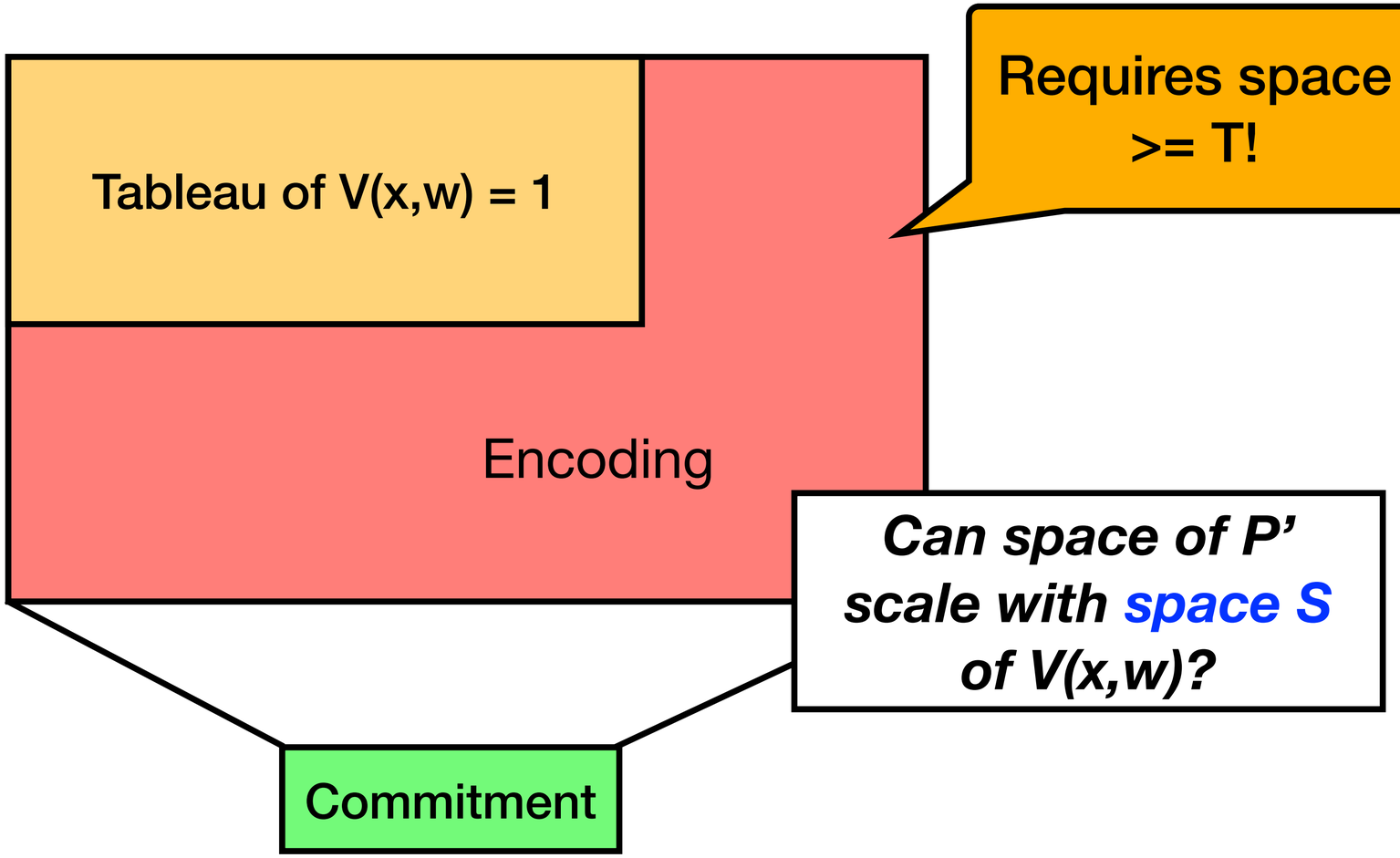
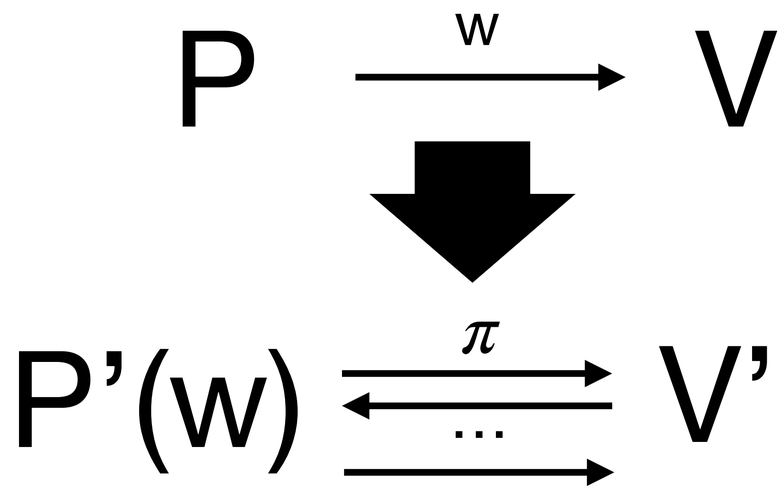


Complexity-Preserving Arguments



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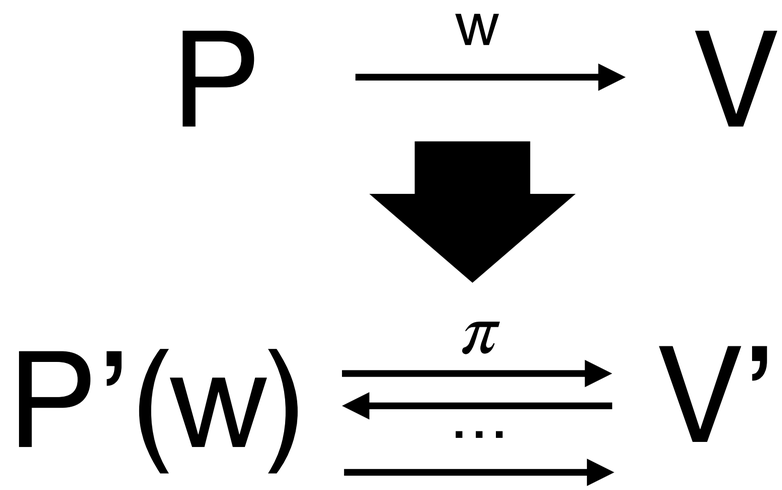
and space $\tilde{O}(S)$

Complexity-Preserving Ar



Best possible up to polylog factors!

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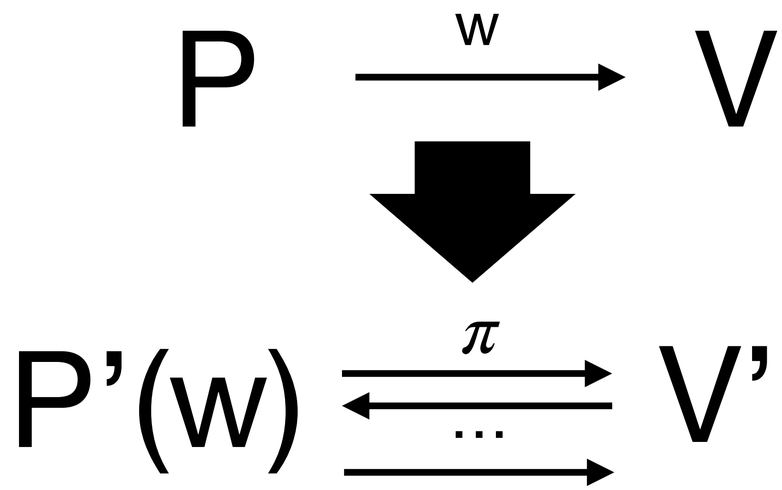

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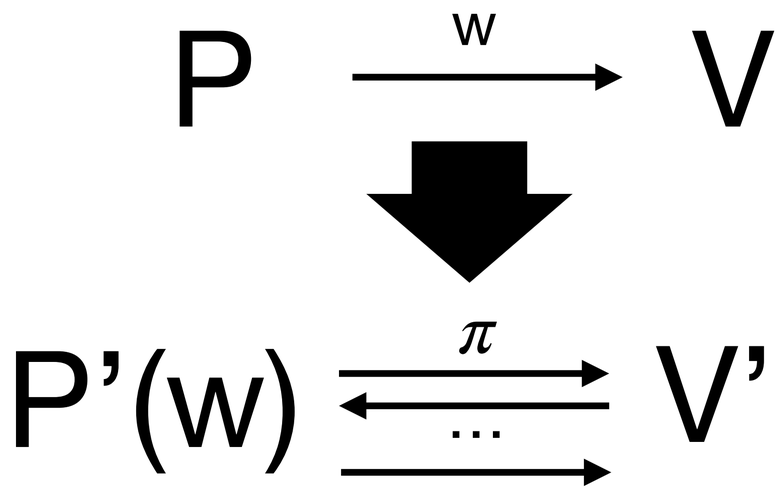

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There exists a 4-message **complexity-preserving** succinct argument for NP from **fully homomorphic encryption**

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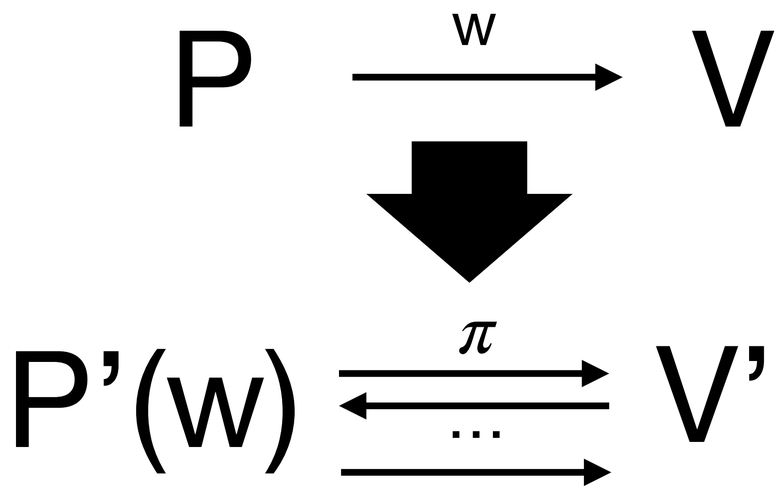

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Open Questions:

Public verifiability?

Minicrypt?

Towards resolving these questions

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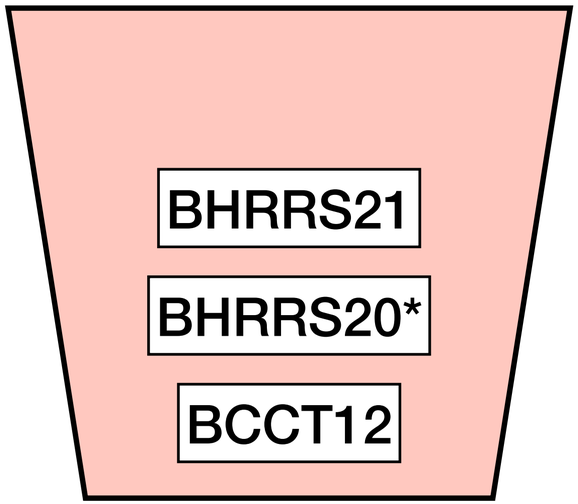
Publicly-verifiable, Complexity-Preserving, Succinct Arguments

*not fully succinct
**only privately verifiable

Towards resolving these questions

Publicly-verifiable, Complexity-Preserving, Succinct Arguments

Non-standard assumptions



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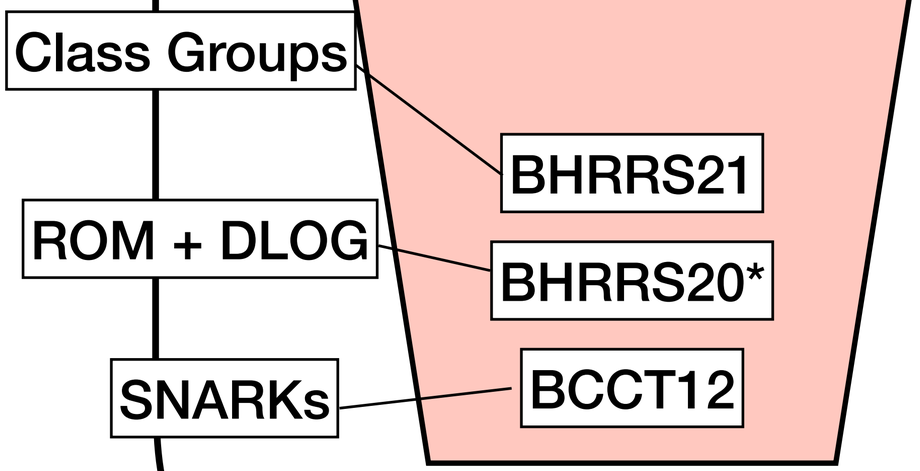

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Non-standard assumptions

BHRRS21

BHRRS20*

BCCT12

Non-interactive for P

KLVW23

DGKV22

PP22

HR18**

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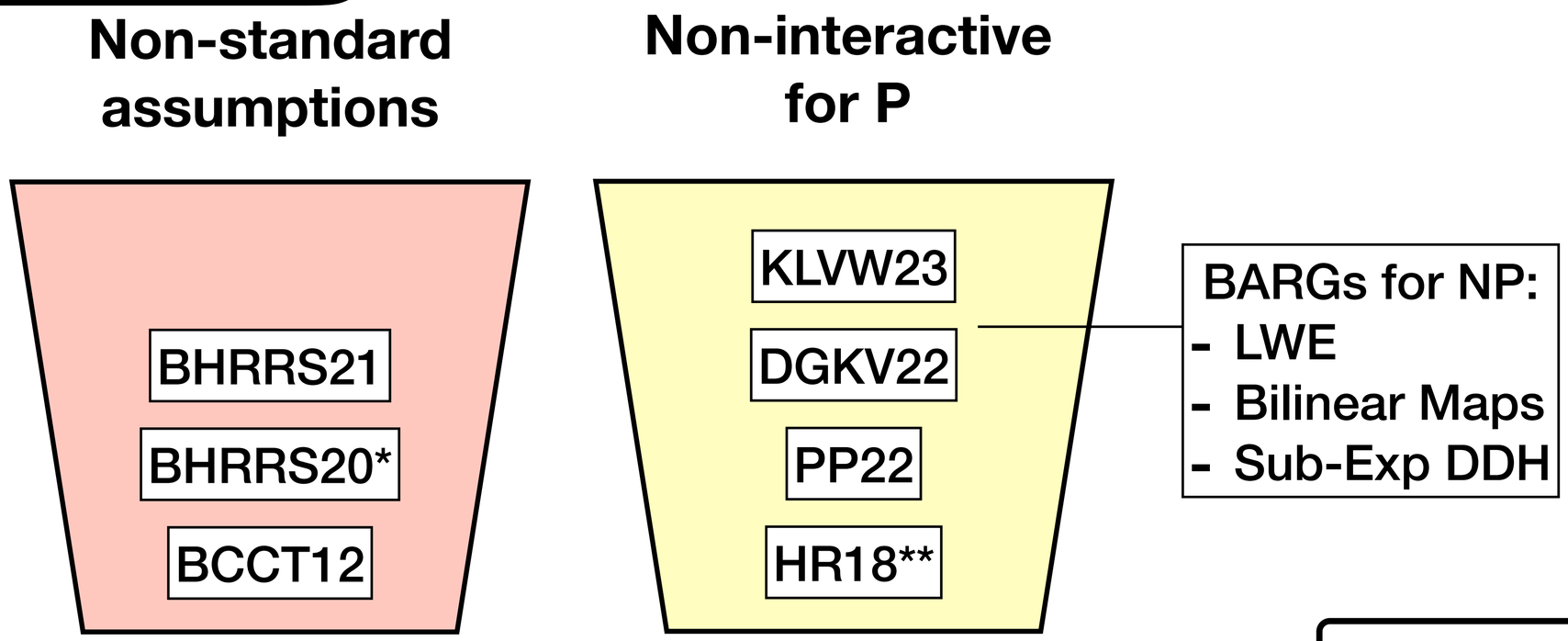

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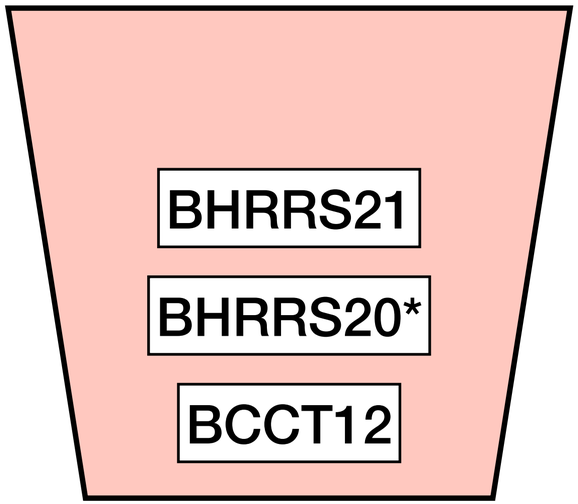

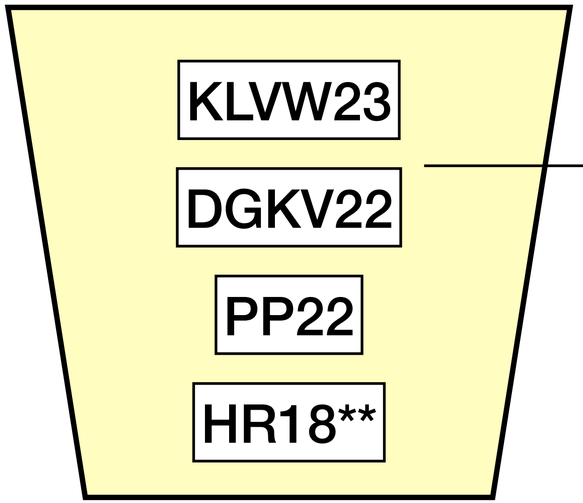

Still all Cryptomania assumptions!

BARGs for NP:
- LWE
- Bilinear Maps
- Sub-Exp DDH

*not fully succinct
**only privately verifiable

Towards resolving these questions

Publicly-verifiable, Complexity-Preserving, Succinct Arguments

Non-standard assumptions

- BHRRS21
- BHRRS20*
- BCCT12

Non-interactive for P

- KLVW23
- DGKV22
- PP22
- HR18**

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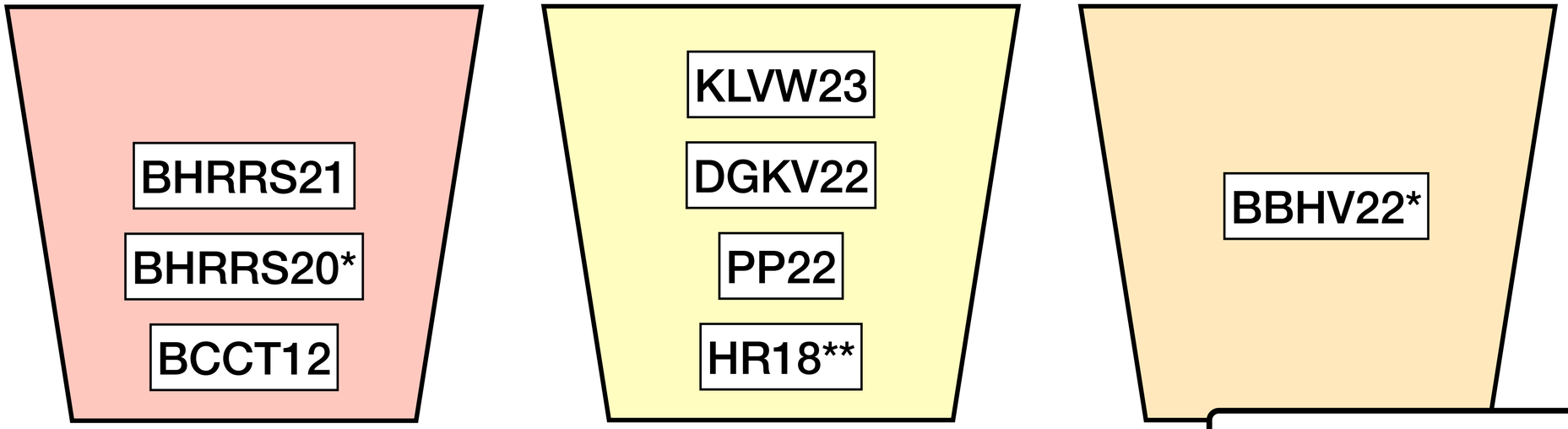
Publicly-verifiable, Complexity-Preserving, Succinct Arguments



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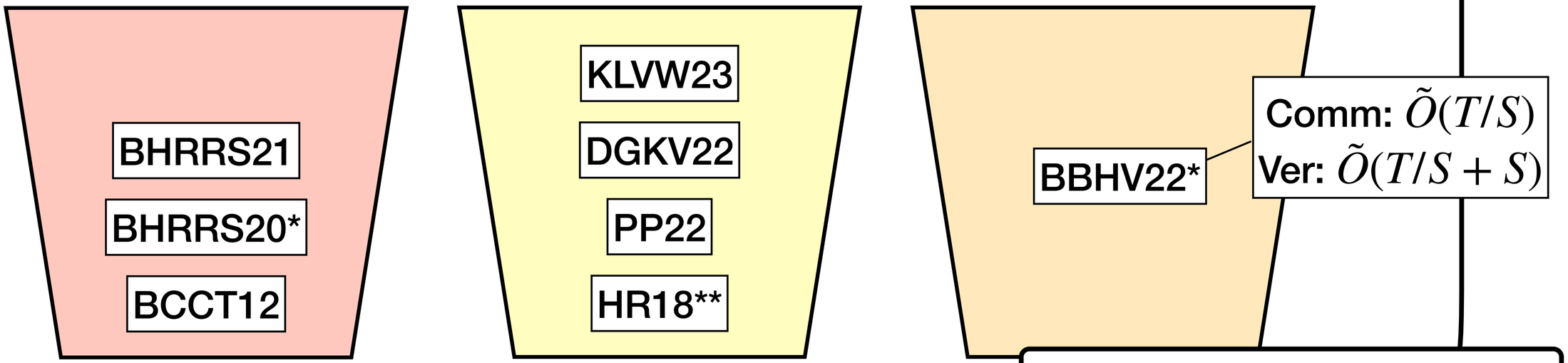

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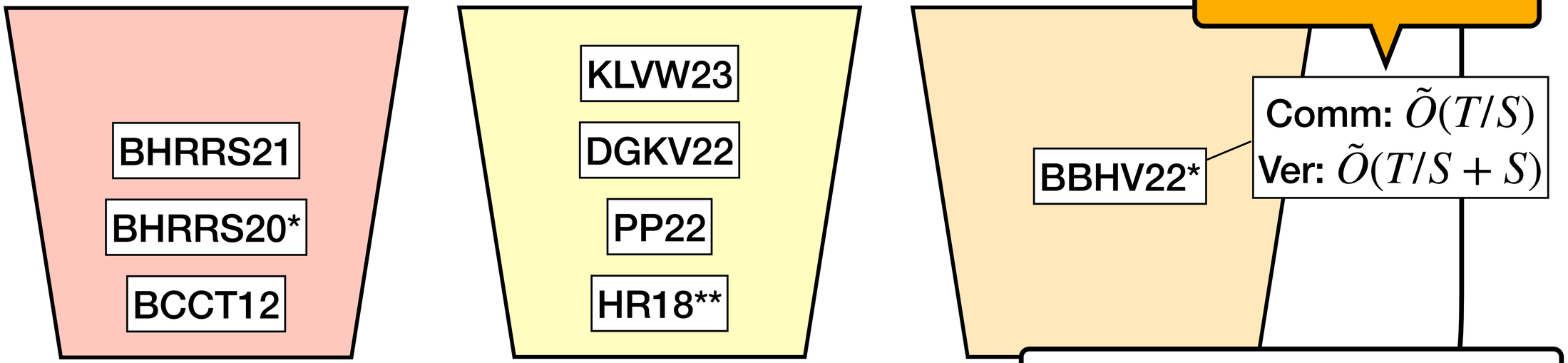

"Tight" wrt black-box constructions

*not fully succinct
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Our Main Result

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Theorem:
There exists a **public-coin** $O(\log_\lambda T)$**-message complexity-preserving** succinct argument for NP from **collision-resistant hashing**

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[BC12] Questions:

Publicly verifiable ✓

Minicrypt ✓

Our Main Result

Theorem:
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[BC12] Questions:

- Publicly verifiable ✓
- Minicrypt ✓

Remaining Open Questions:

- Fixed constant rounds?
- Universal argument?
- Turing machines?

Corollaries to Our Main Result

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Zero-Knowledge

"Commit-and-prove" via [BGGHKMR88]

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Generic transformation of [EFKP20]

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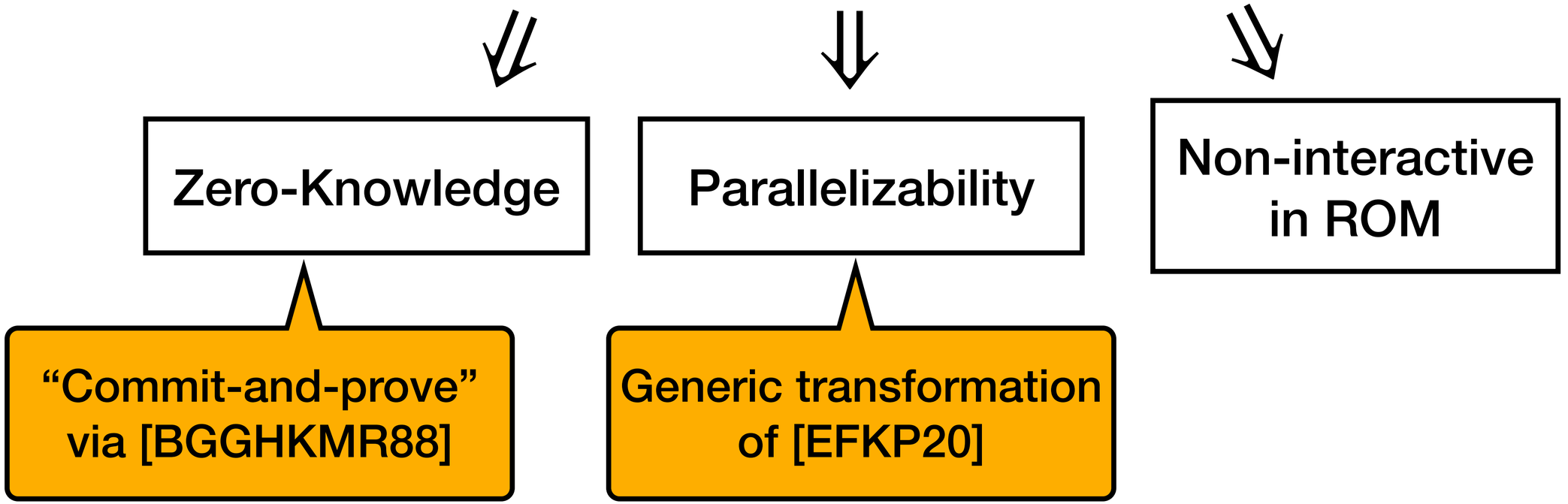

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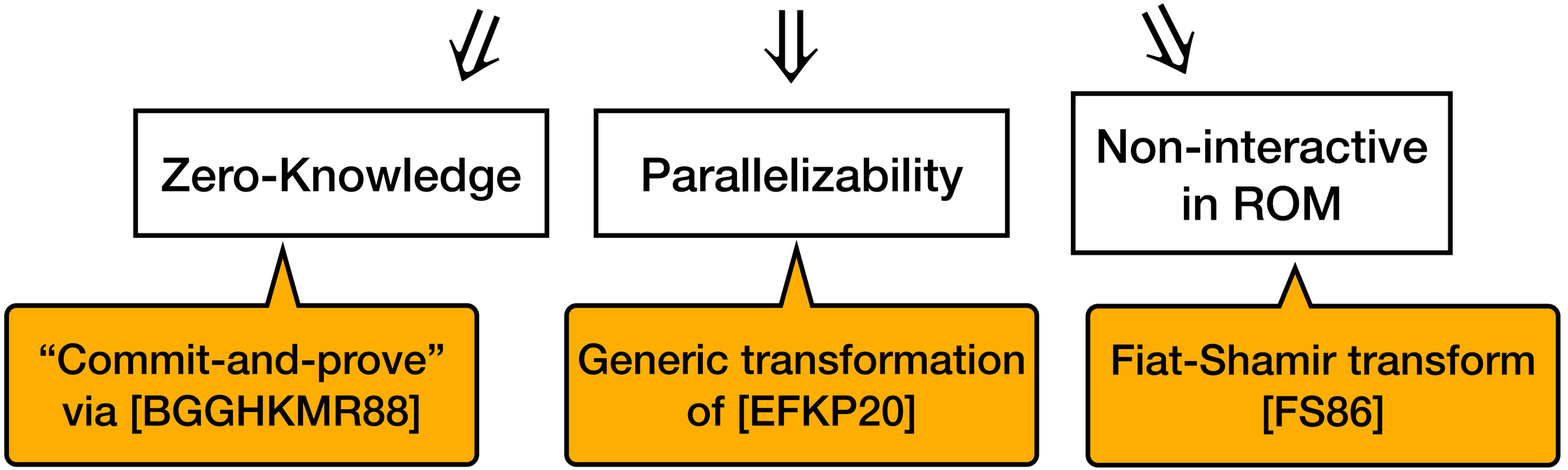

Roadmap for Our Construction

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Reduction via hash trees [KP16]

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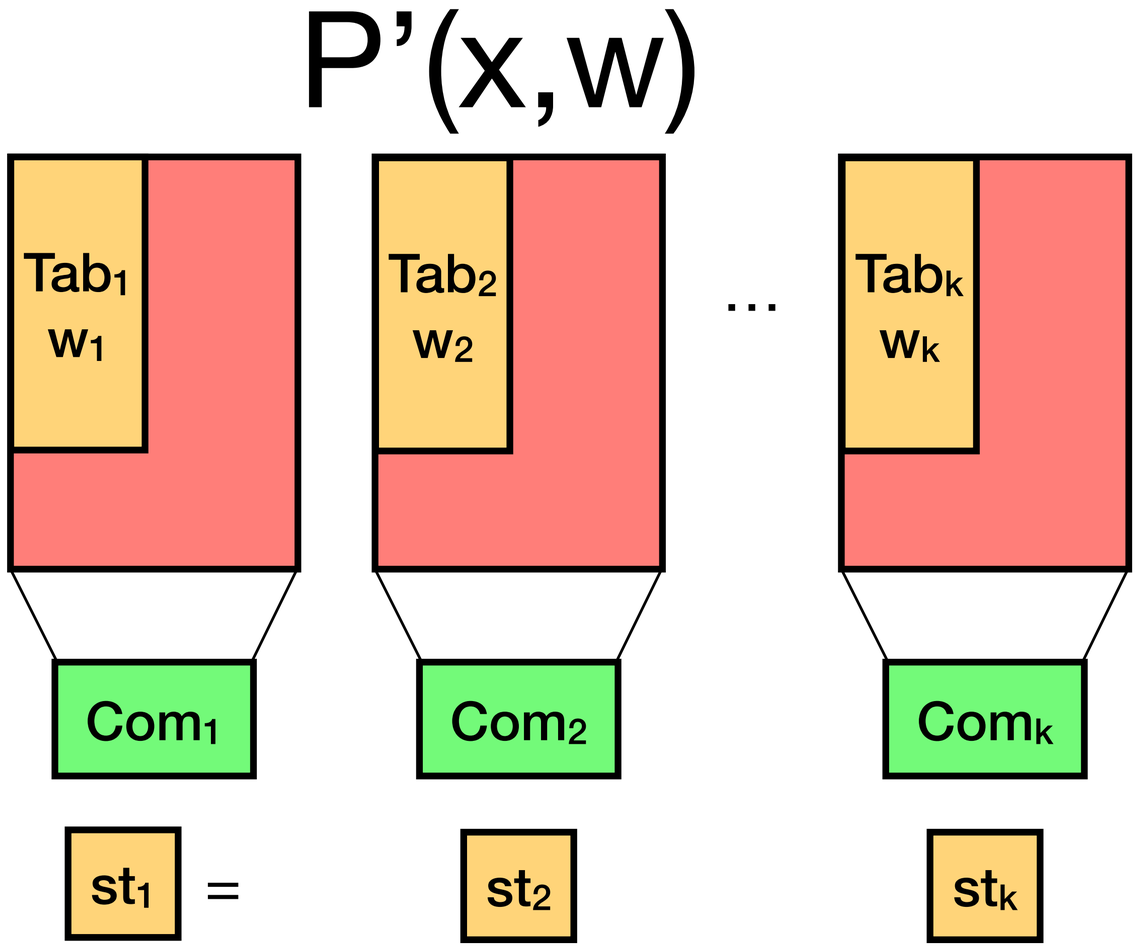
A $\tilde{O}(T^{1/2}S)$-space protocol: Kilian in Parallel



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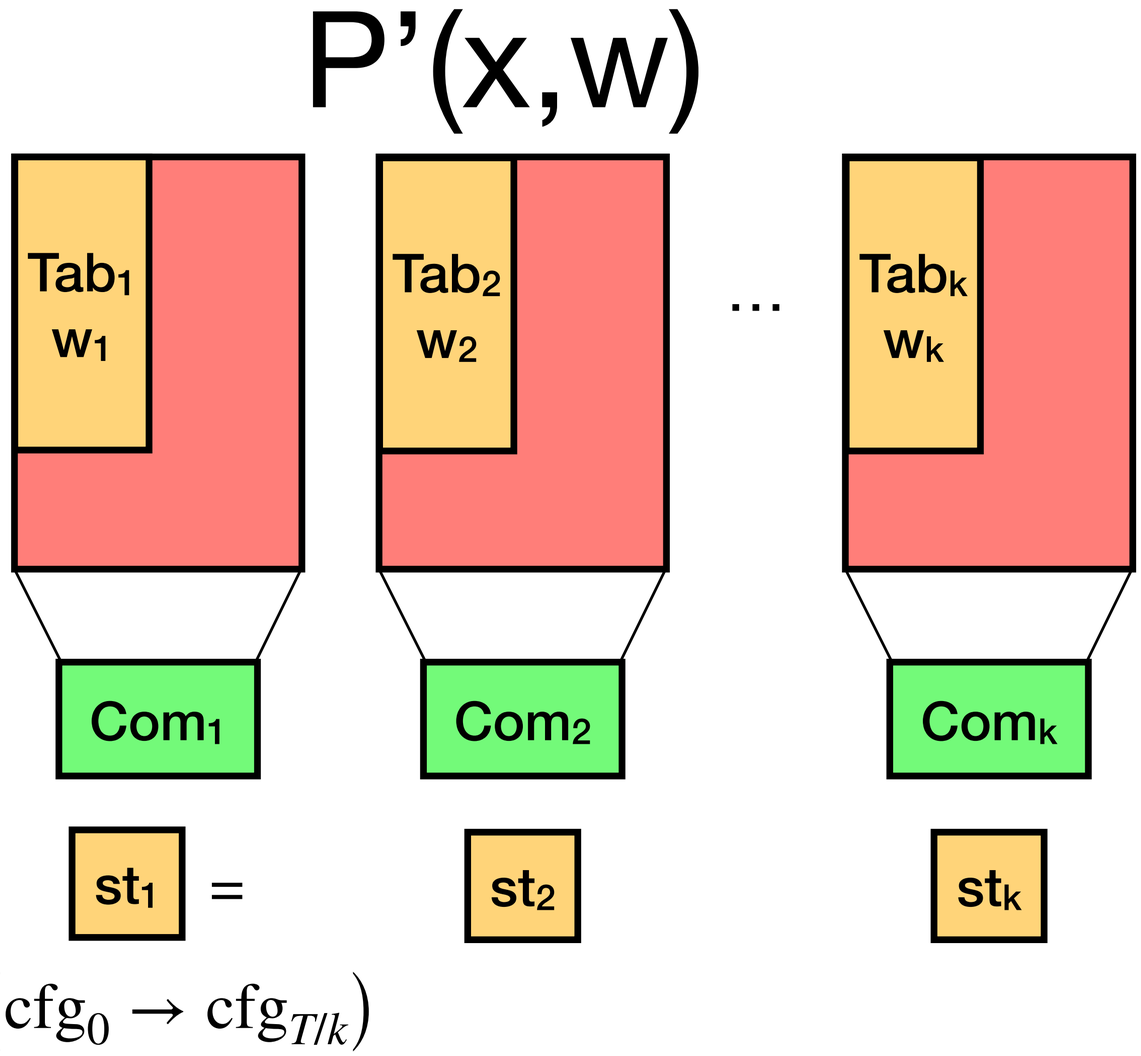


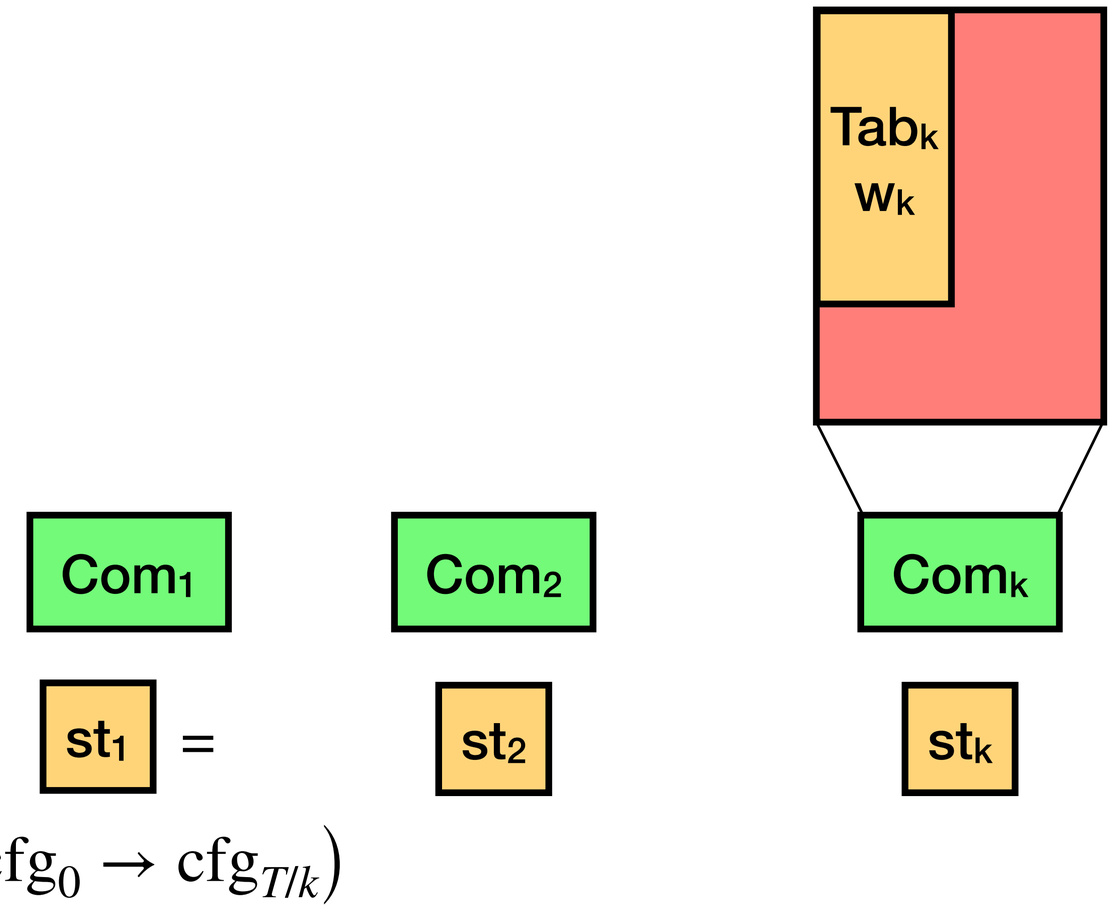
Prover efficiency:

- Time $k \cdot \tilde{O}(ST/k) = \tilde{O}(ST)$
- Space $\tilde{O}(ST/k + Sk)$

  $k = T^{1/2} \Rightarrow \tilde{O}(T^{1/2}S)$

A $\tilde{O}(T^{1/2}S)$-space protocol: Kilian in Parallel

P’(x,w)



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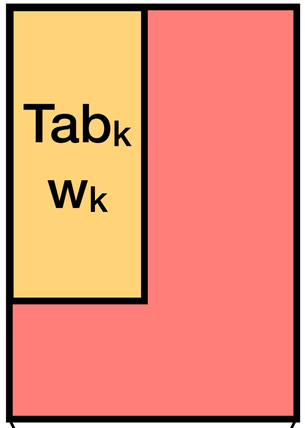

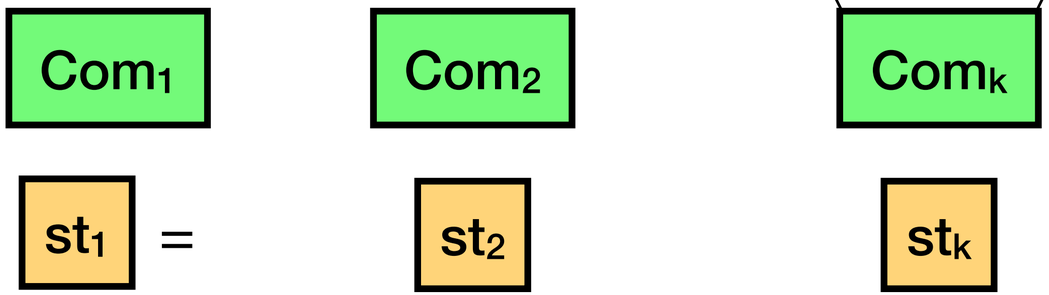

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Communication/ Verification:

- $\tilde{O}(kS) = \tilde{O}(T^{1/2}S)$

A $\tilde{O}(T^{1/2}S)$-space protocol

Intermediate Result:

There exists a public-coin 4-message argument for NP from CRH with:

- prover **time** $\tilde{O}(TS)$ and **space** $\tilde{O}(T^{1/2}S)$
- **comm/ verification** $\tilde{O}(T^{1/2}S)$.

1. A $\tilde{O}(T^{1/2}S)$-space protocol ✓

2. A solution for small space

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Need to address:

1. comm/ verification
2. prover space

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Commit-and-prove!

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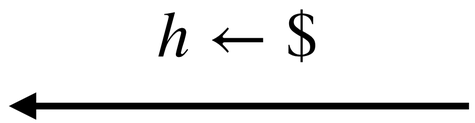
Recursion!

Recursive Protocol

$P_r(w)$ V_r

Recursive Protocol

$P_r(w)$ V_r



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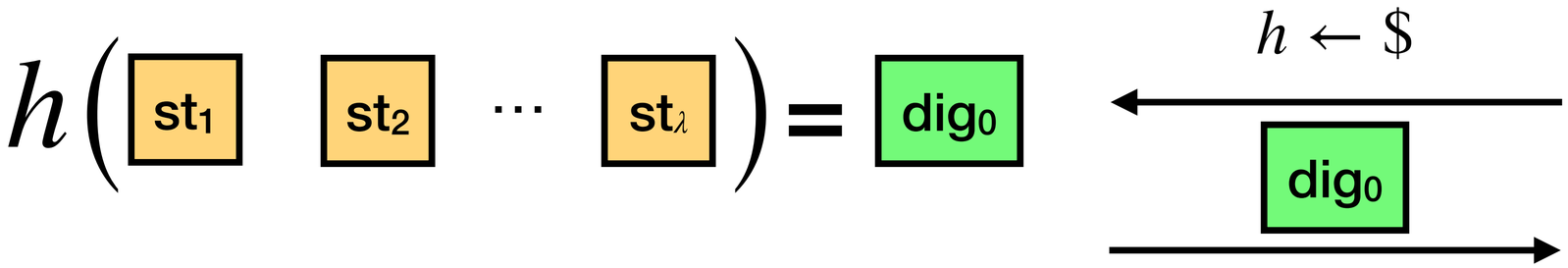

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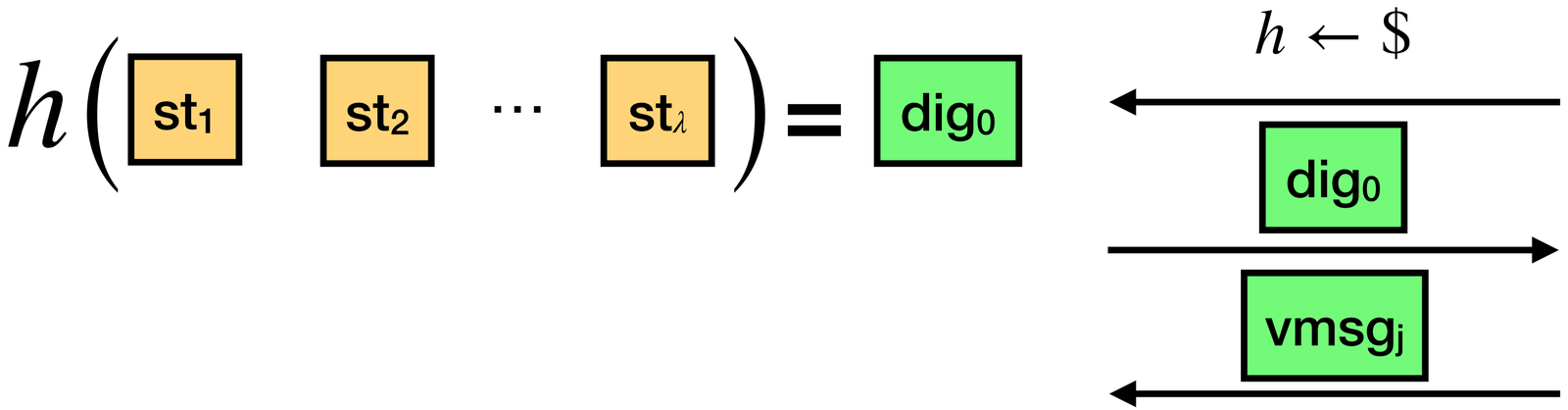

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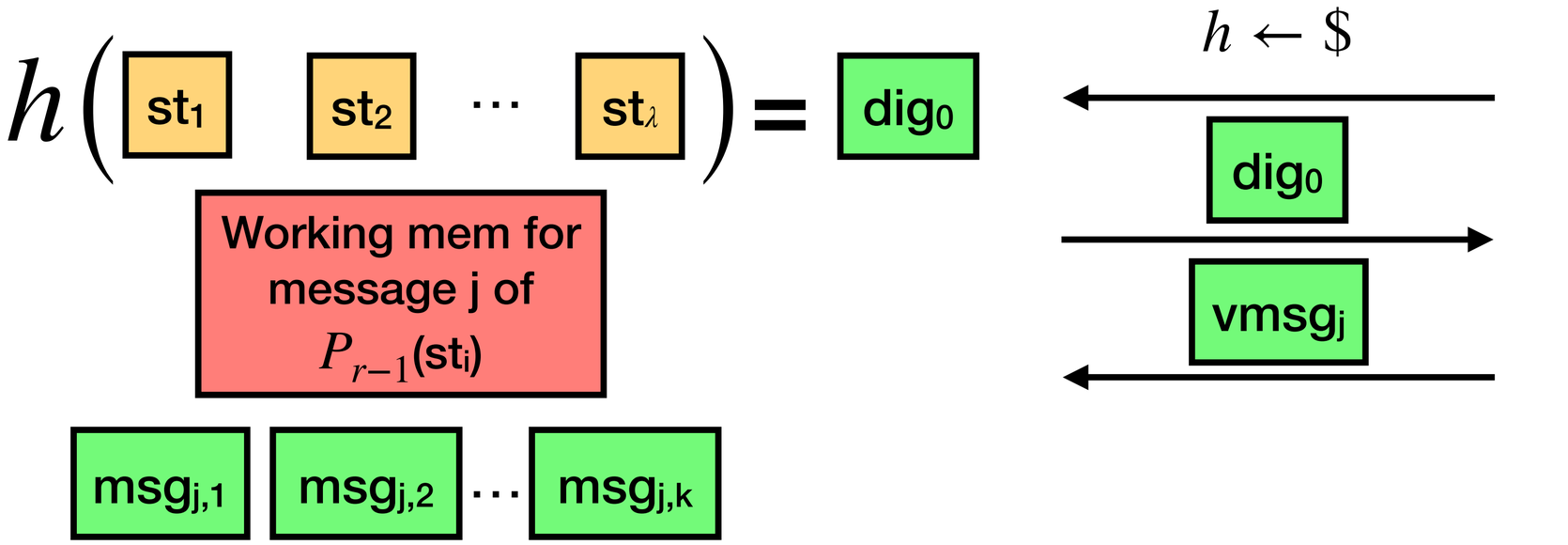

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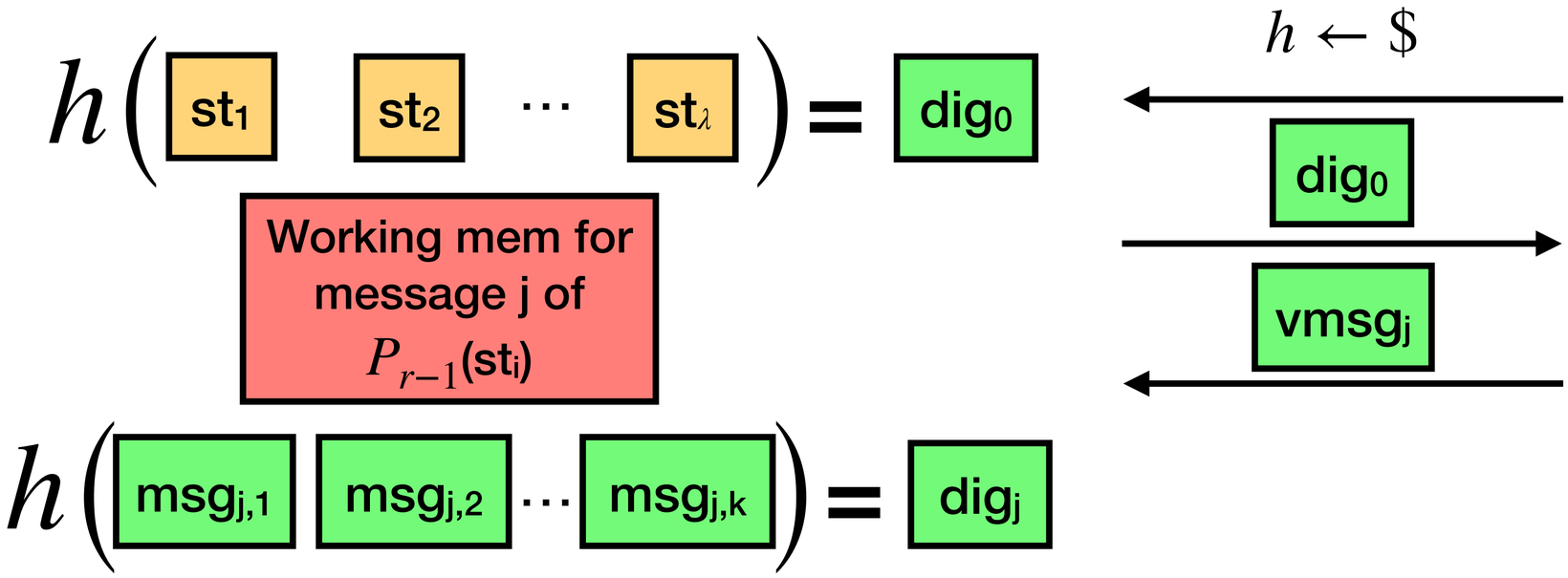

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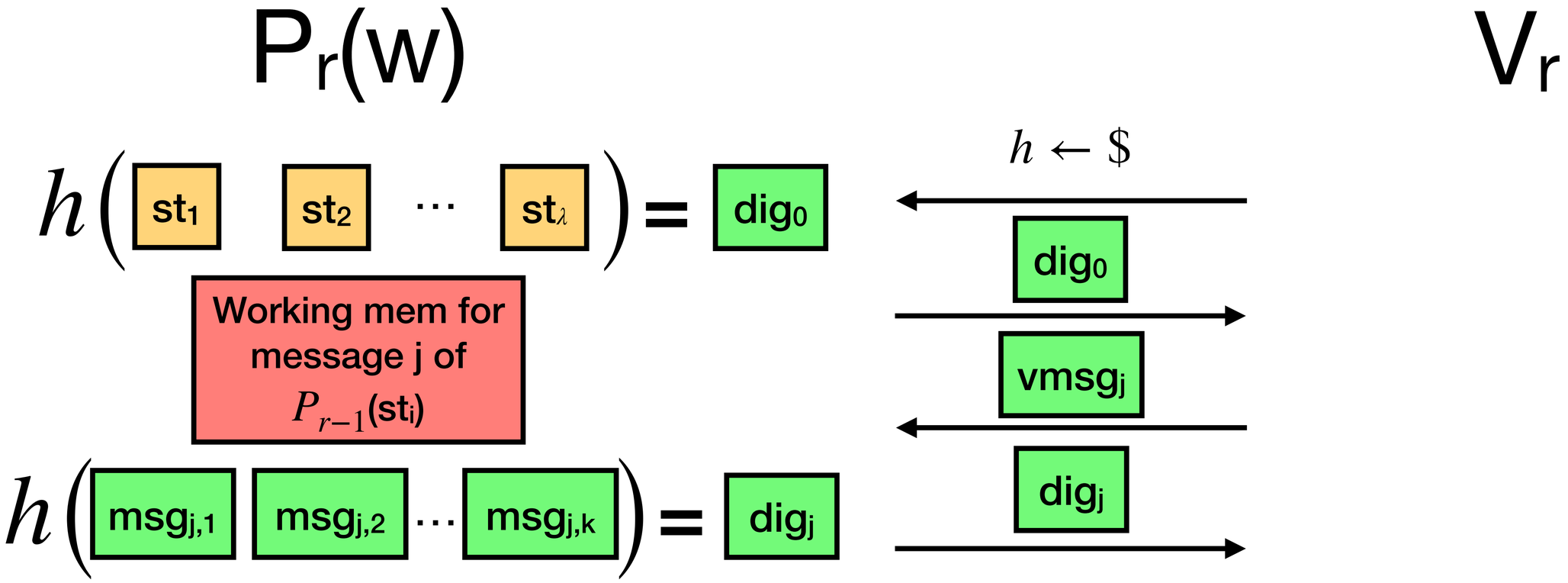

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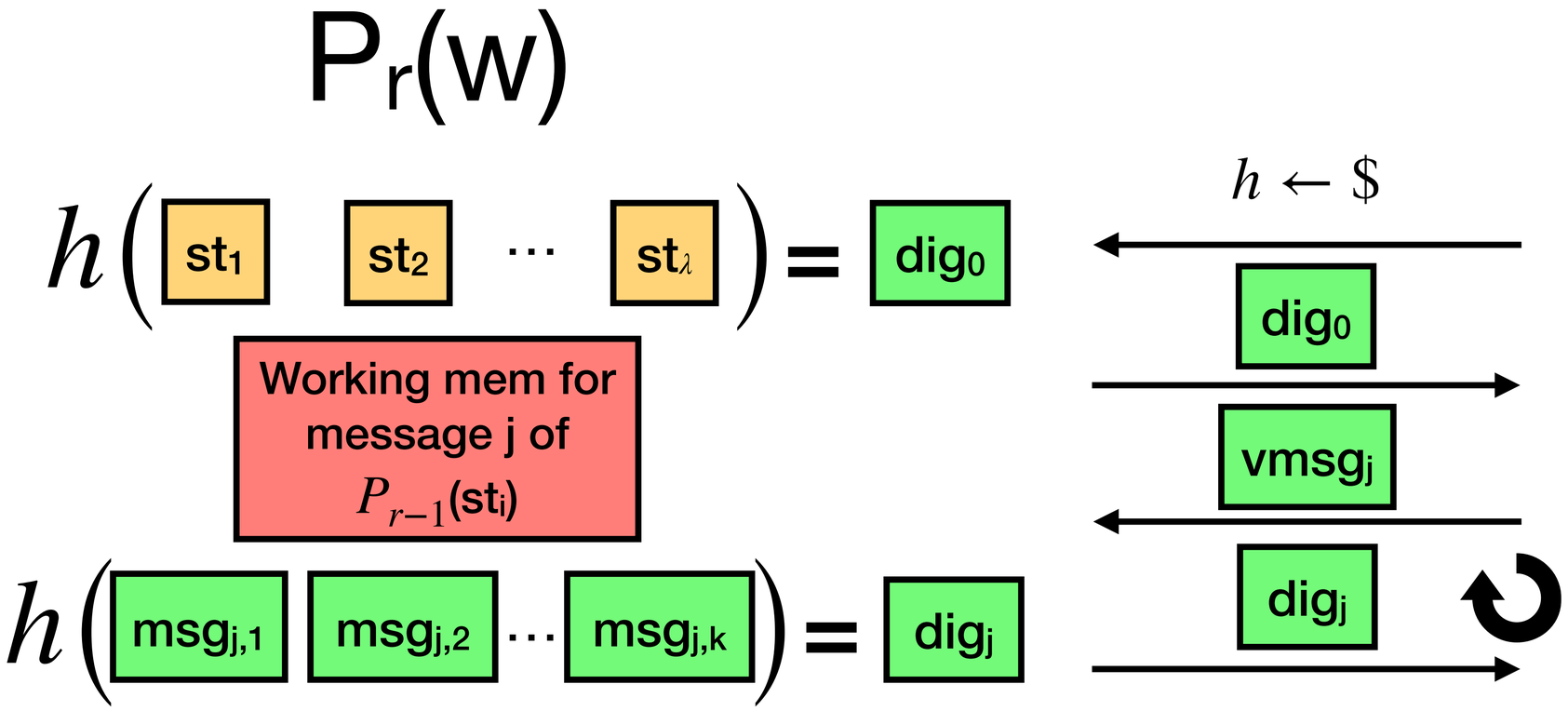

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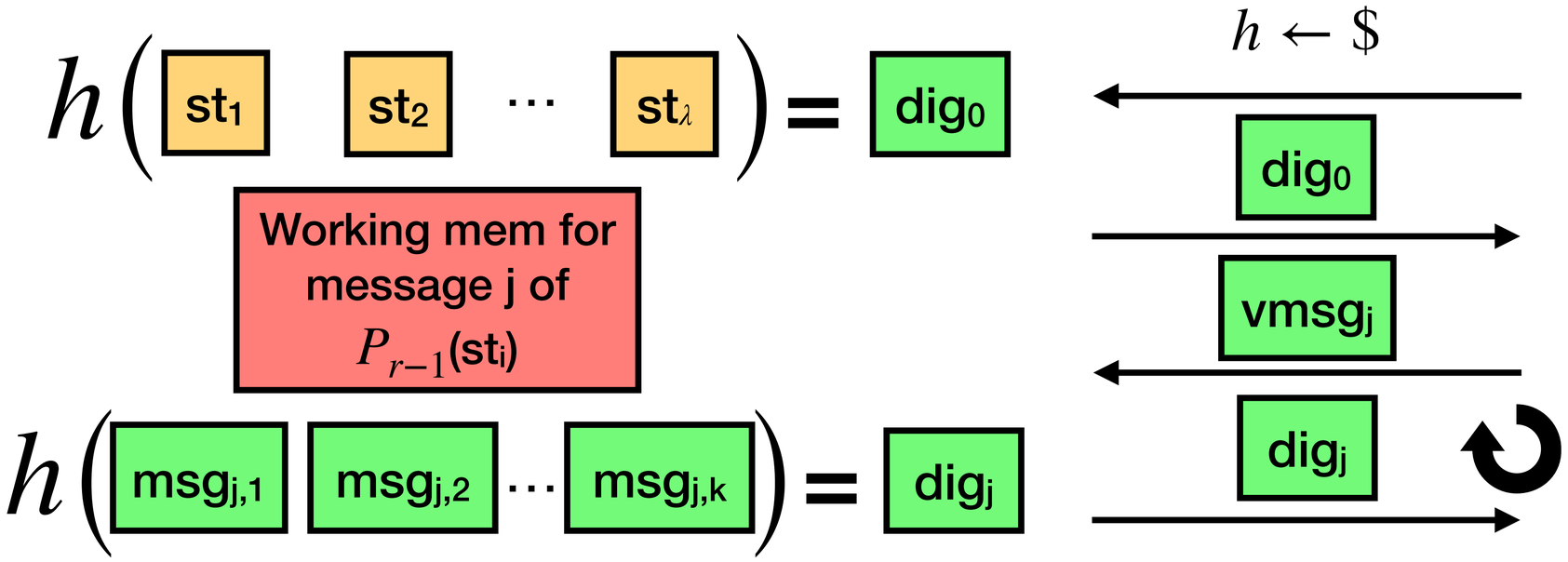
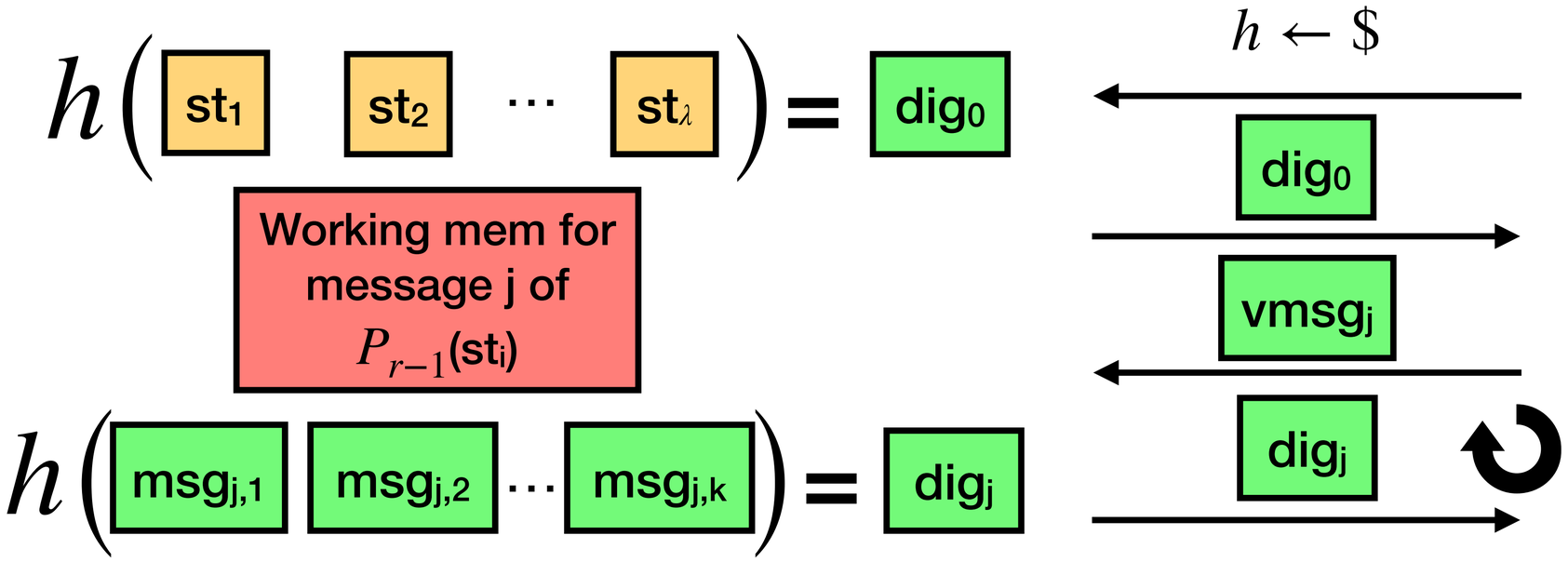


$$R_{\text{merge}} = \left\{ \begin{array}{l} ((\text{cfg}, \text{cfg}', \vec{\text{dig}}), (\vec{\text{st}}, \text{Msg})) : \\ \quad 1.\ \text{dig}_0 = h(\vec{\text{st}}) \\ \quad 2.\ \forall j \geq 1, \text{dig}_j = h(\text{Msg}_{j,*}) \\ \quad 3.\ \forall i \in [k], V_{r-1}(\text{st}_i, \text{Msg}_{*,i}) = 1 \end{array} \right\}$$

Succinct argument for R_{merge}

Recursive Protocol

$P_r(w)$ V_r



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1. Verify succinct argument

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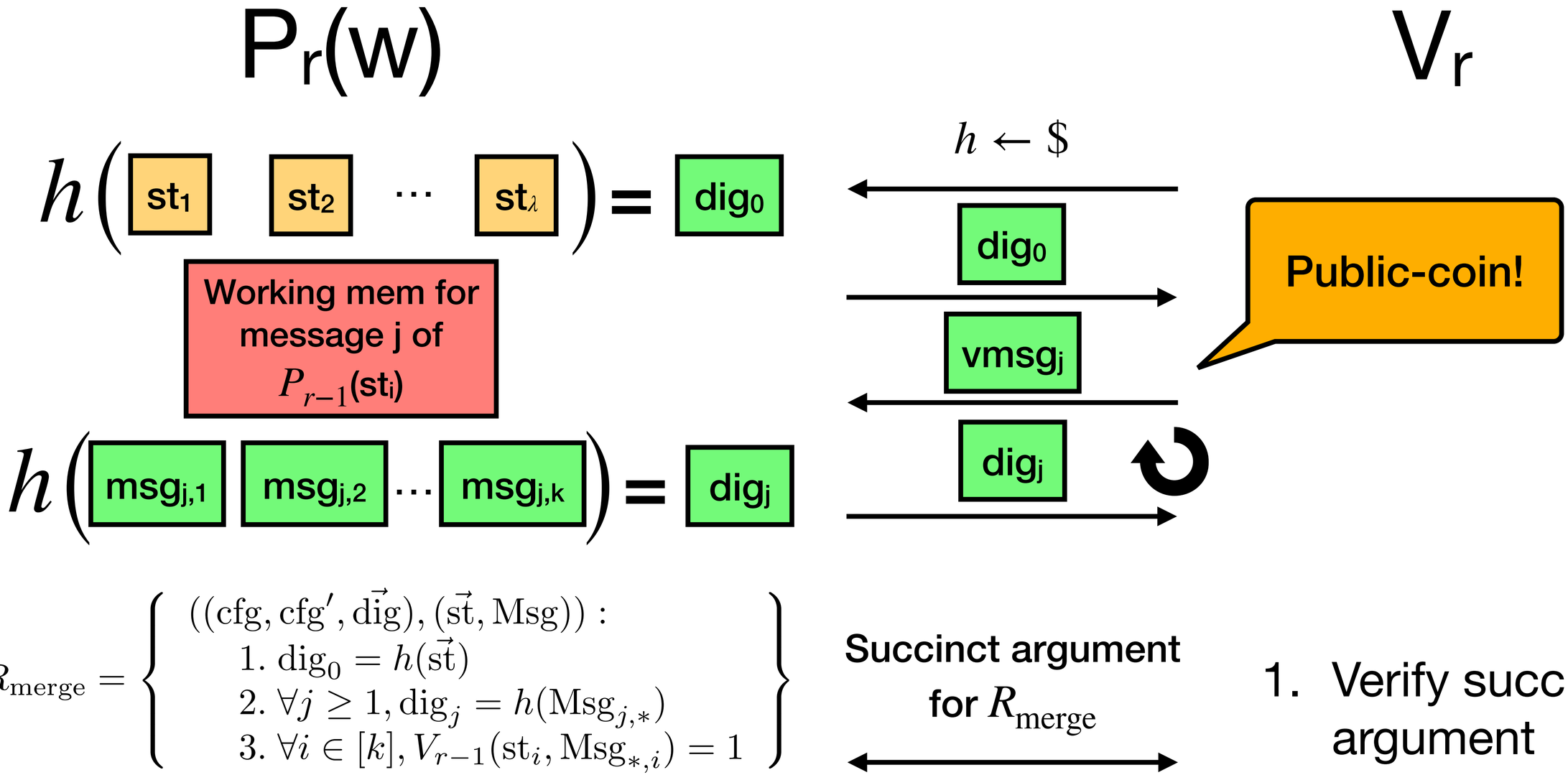

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