Closing the Efficiency Gap between Synchronous and Network-Agnostic Consensus

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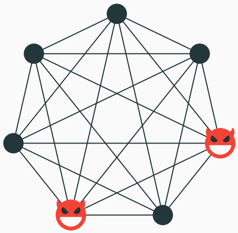
Consensus (Byzantine Agreement)

Parties $P_1, P_2, \ldots, P_n$ with ℓ-bit inputs.

Up to t of the parties are byzantine.



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The parties agree on an output.

Validity

common input $\boldsymbol{m} \implies$ output $\boldsymbol{m}$

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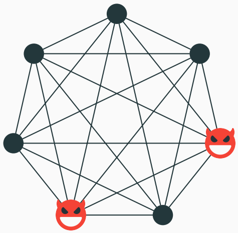

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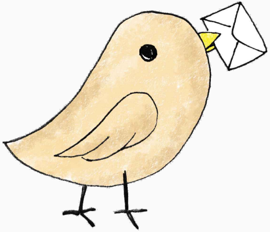
Intrusion Tolerance

The common output is either an honest input, or a special value $\perp$.

Network-Agnostic Setting

Synchronous Setting: Messages arrive after Δ time, clocks synchronized. Security possible with setup when $t < \frac{n}{2}$ [8].

Asynchronous Setting: Messages arrive after arbitrary delays, clocks not synchronized. Security requires $t < \frac{n}{3}$ [15].



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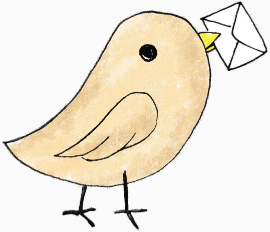


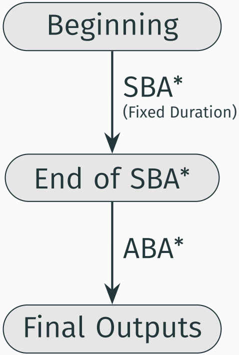
Network-Agnostic Setting (Blum, Katz, Loss [2])

Network synchronous ($\leq t_s$ corruptions), or asynchronous ($\leq t_a$ corruptions). The parties don't know if the network is synchronous or not.

Consensus when $t_a \leq t_s$ is possible iff $2t_s + t_a < n$ [2].

Fallback Compilation

[2] Blum, Katz and Loss compile a synchronous consensus protocol SBA* and an asynchronous consensus protocol ABA*.



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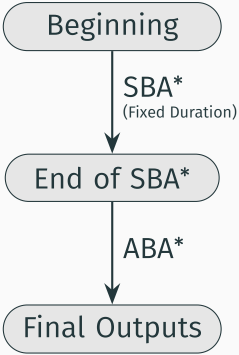

★ SBA* achieves validity against t_a corruptions, even if the network is asynchronous.

★ ABA* achieves validity against t_s corruptions when the network is synchronous.

Previous Works

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2023: Bacho, Collins, Liu-Zhang and Loss designed a new ABA* which works with a bulletin-PKI setup, supports ℓ-bit inputs, and has intrusion tolerance [1].

- **Communication complexity:** $\mathcal{O}(n^3\kappa + \ell n^3)$.

The Generic Approach

2022: Ghinea, Goyal and Liu-Zhang designed a λ-round SBA with a statistical error probability $\lambda^{-\Omega(\lambda)}$ when $2t_s \leq (1-\varepsilon)n$ [12]. How do we get this for SBA*?

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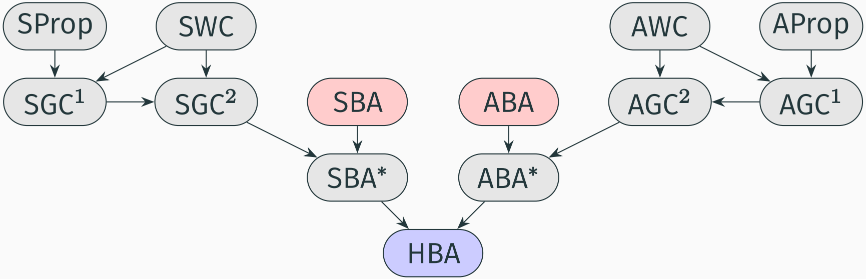

We can compile **any** fixed-duration SBA and **any** ABA.

Overhead: 13 or 16 rounds when the network is synchronous.

Example – Asynchronous 2-Graded Consensus

Assume $t_a \leq t_s$ and $2t_s + t_a < n$.

Asynchronous 2-Graded Consensus

Inputs: $m_i \in \{0,1\}^\ell$ **Outputs:** $(y_i, g_i) \in (\{0,1\}^\ell \cup \{\bot\}) \times \{0,1,2\}$

- **t_s-intrusion tolerance:** If no party has an input m, then no party P_i obtains $y_i = m$.
- **6-round t_s-validity with liveness:** If the parties run forever with a common input m, then they output $(m, 2)$, and do so within 6Δ time if the network is synchronous.

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Complexity: $\mathcal{O}(n^2)$ messages, $\mathcal{O}(\ell n^2)$ bits

Asynchronous Consensus – Termination

Old Way [2, 7, 1]

Sign your ABA* output and multicast it.

A $(t_s + 1)$-certificate on y proves y is the correct output. Upon having one, multicast it, output y and terminate.

Termination against t_s corruptions.

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New Way – Bracha's Broadcast Style [3]

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Solution: In synchrony, everyone outputs by some time T. Don't terminate earlier.

Bonus Efficiency: Don't send ABA messages before the time $T + \Delta$.

Complexity Summary

Adapting techniques by Momose and Ren [13] and using their SBA to obtain SBA*, we achieve (with no $\mathsf{CC}_{\mathsf{ABA}}$ in synchrony):

Resilience	Setup	Communication Complexity
$2t_s + t_a < n$	Bulletin-PKI	$\mathcal{O}(\mathsf{CC}_{\mathsf{ABA}} + n^3\kappa + \ell n^2)$
$2t_s + t_a < n$	Threshold Signatures	$\mathcal{O}(\mathsf{CC}_{\mathsf{ABA}} + n^2\kappa + \ell n^2)$
$2t_s + t_a < n$, $2t_s \leq (1-\varepsilon)n$	Bulletin-PKI	$\mathcal{O}(\mathsf{CC}_{\mathsf{ABA}} + n^2\kappa + \ell n^2)$

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Unique threshold signatures: $\mathsf{CC}_{\mathsf{ABA}} = \mathcal{O}(n^2\kappa)$ [4].

Bulletin-PKI and CRS, or no setup but static security: $\mathsf{CC}_{\mathsf{ABA}} = \mathcal{O}(n^3\kappa)$ [11, 6].

Adaptive security without setup: $\mathsf{CC}_{\mathsf{ABA}} = \mathcal{O}(n^3\kappa \log n)$ [10].

Complexity Summary – Extended

We can reduce the $\mathcal{O}(\ell n^2)$ term to $\mathcal{O}(\ell n)$ with extension protocols.

Thanks to intrusion tolerance, a few rounds suffice after consensus on κ-bit inputs. No need for 2 consensus instances as in [14] by Nayak, Ren, Shi, Vaidya and Xiang.

Resilience	Setup	Complexity Overhead
$2t_s + t_a < n$	Trusted	$\mathcal{O}(n^2\kappa + \ell n)$
$2t_s + t_a < n$	None	$\mathcal{O}(n^2\kappa \mathrm{log} n + \ell n)$
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$2t_s + t_a \leq (1-\delta)n$	None	$\mathcal{O}(n^2\kappa + \ell n)$

When $2t_s + t_a \leq (1-\delta)n$ and the network is synchronous, bulletin-PKI suffices for the complexity $\mathcal{O}(n^2\kappa + \ell n)$ thanks to $(t_s, \delta n)$-intrusion tolerance.

With trusted setup, one can let $\mathrm{CC}_{\mathrm{SBA}} = \mathrm{CC}_{\mathrm{ABA}} = \mathcal{O}(n^2\kappa)$ to achieve network-agnostic consensus with $\mathcal{O}(n^2\kappa + \ell n)$ bits of communication.

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