Laconic Function Evaluation, Functional Encryption and Obfuscation for RAMs with Sublinear Computation

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Daniel Wichs

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NTT Research

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Laconic Function Evaluation (LFE)

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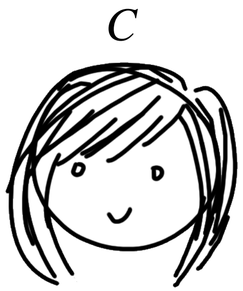
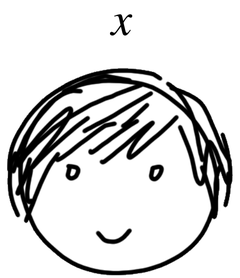
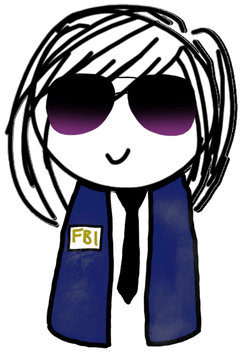


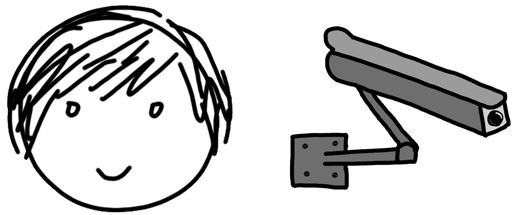


Laconic Function Evaluation (LFE)

C



x



Laconic Function Evaluation (LFE)

* in CRS model, CRS hidden

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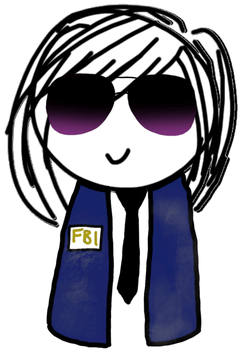

$\text{dig} = \text{Hash}(C)$

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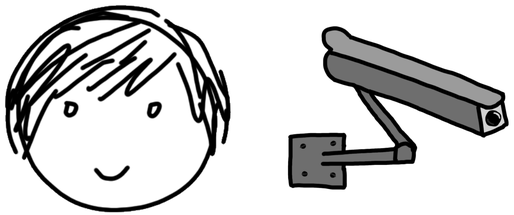

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digest very short compared to $|C|$

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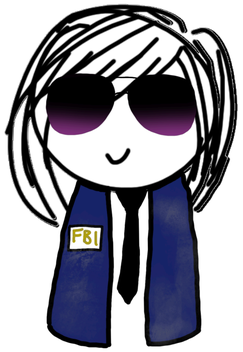

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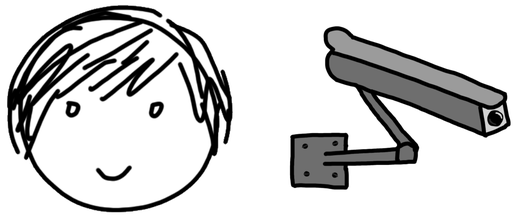

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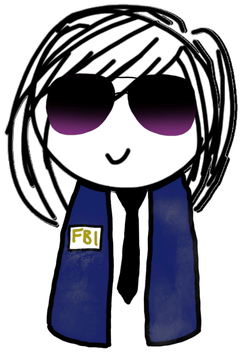

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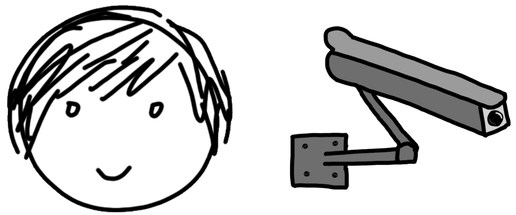

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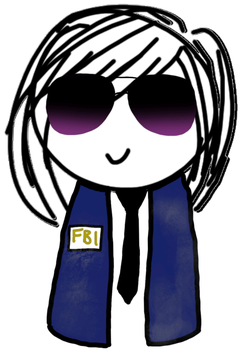

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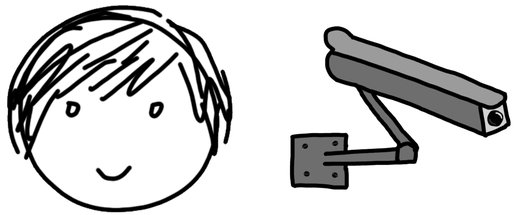

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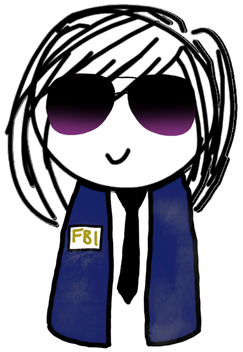

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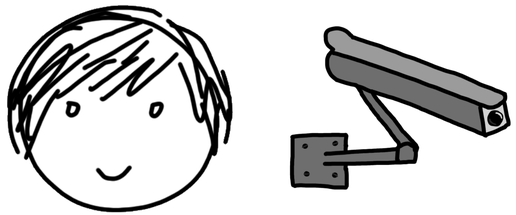

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$\mathsf{Dec}(C, \mathsf{ct}) = C(x)$

Security: Server learns nothing more than $C(x)$

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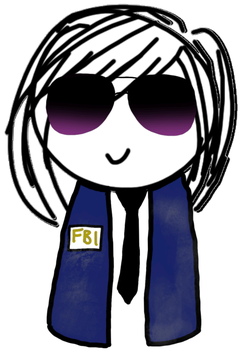

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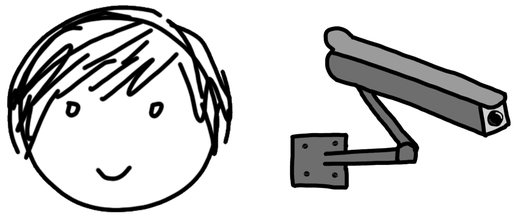

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Like FHE: 2-round 2PC where Server does the computational work

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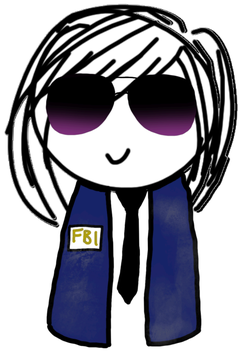

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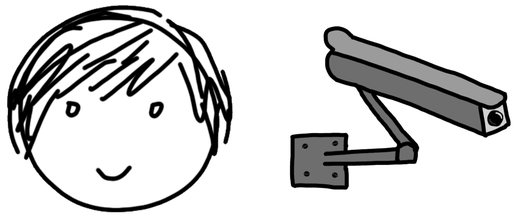

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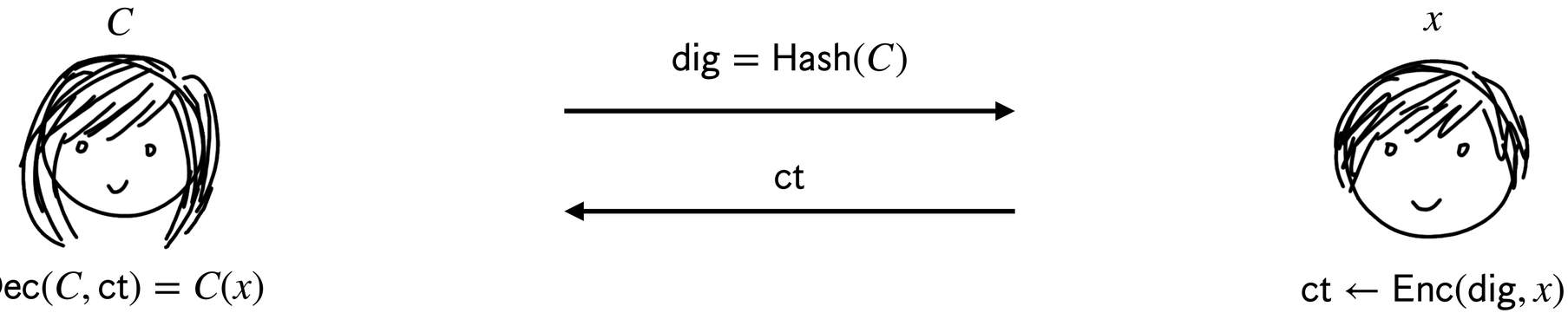
Like FHE: 2-round 2PC where Server does the computational work
But "flipped": Server learns the output (instead of Client)

Laconic Function Evaluation (LFE)



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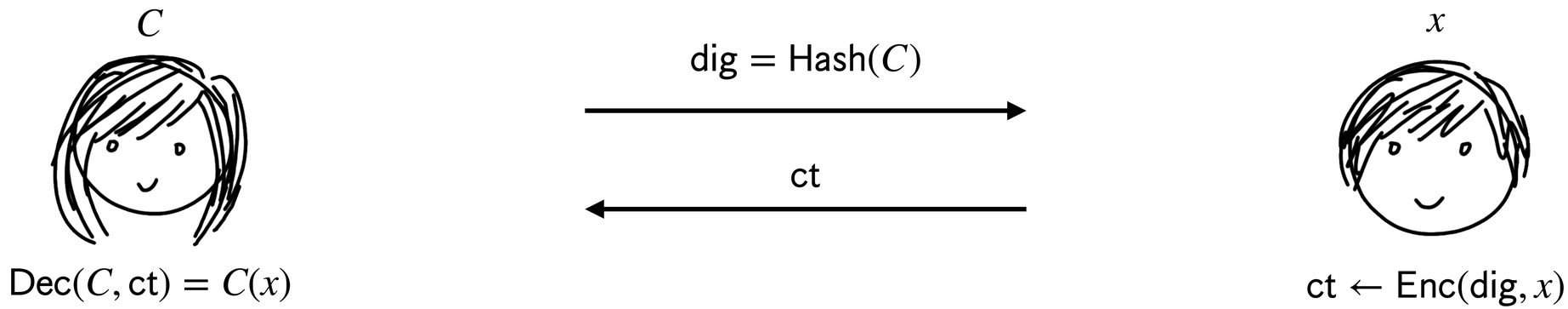

Prior work:

- [Quach-Wee-Wichs'17]: LFE for circuits from LWE

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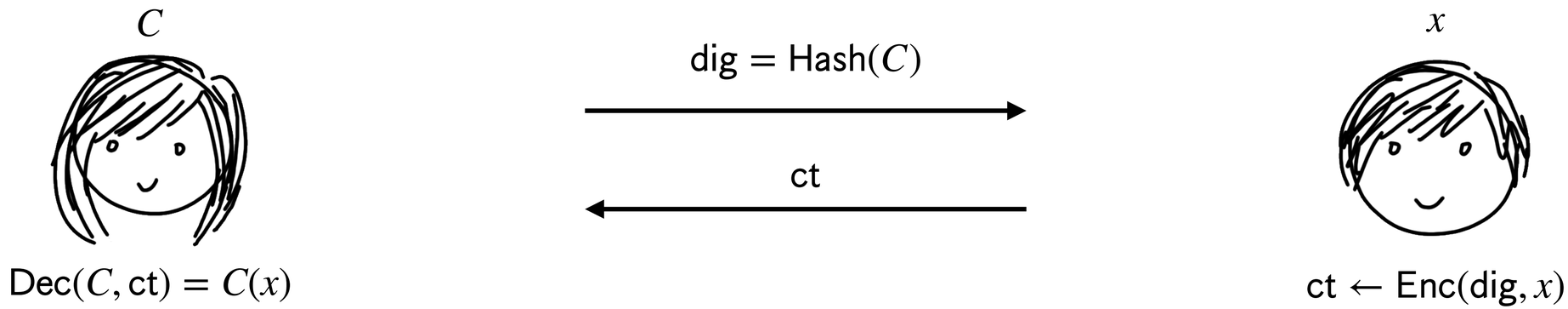

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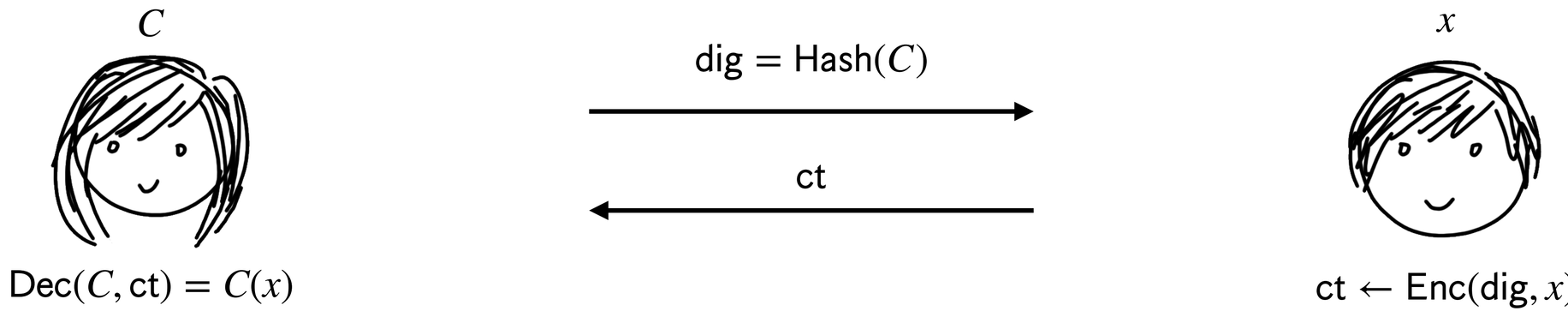

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Problem: Server computation is at least linear in inputs!

LFE for RAMs

x

Main Result: We build LFE for RAMs assuming RingLWE

LFE for RAMs

Goal: output RAM computation $P(x, y)$

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Some fixed RAM program (e.g. universal)

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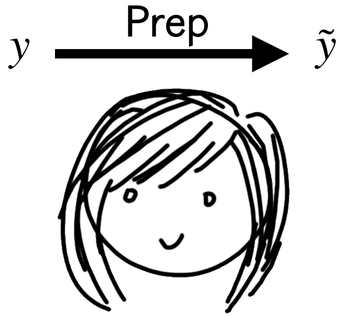

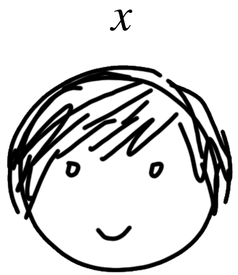

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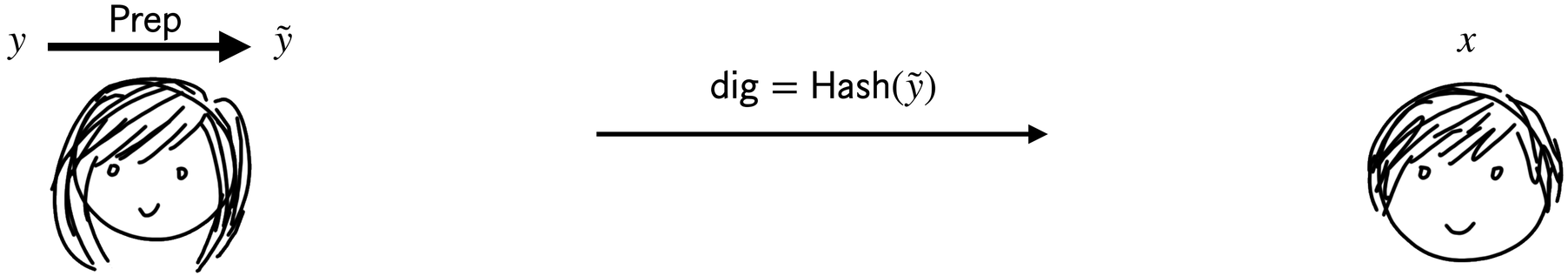

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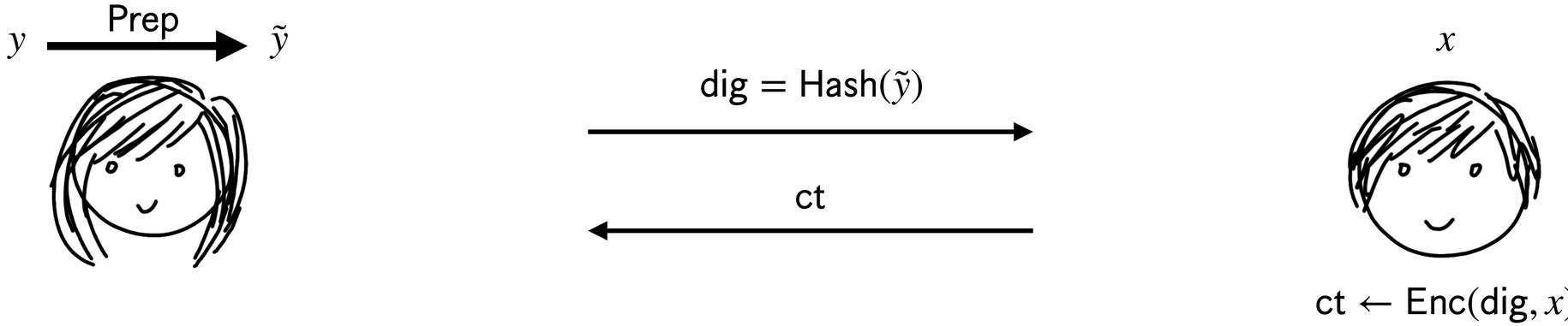

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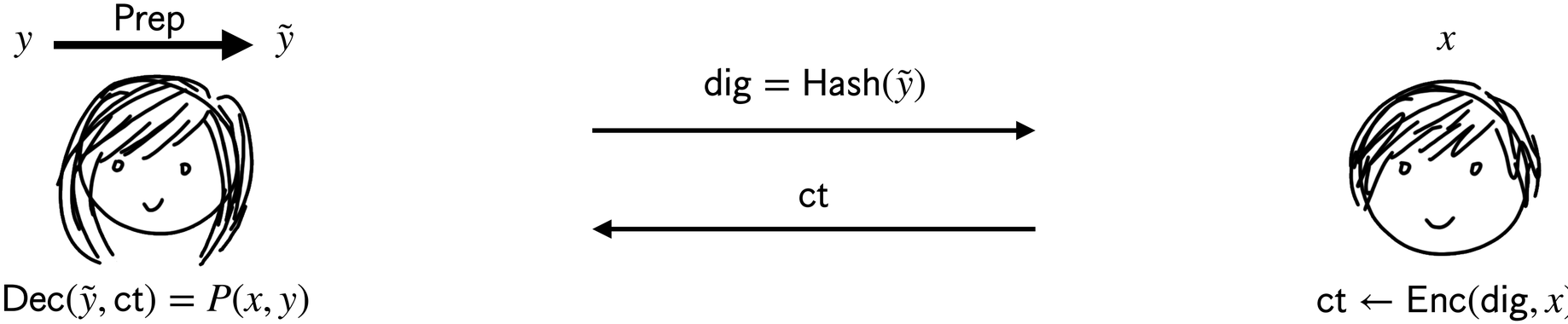

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*suppressing poly(λ) and polylog factors

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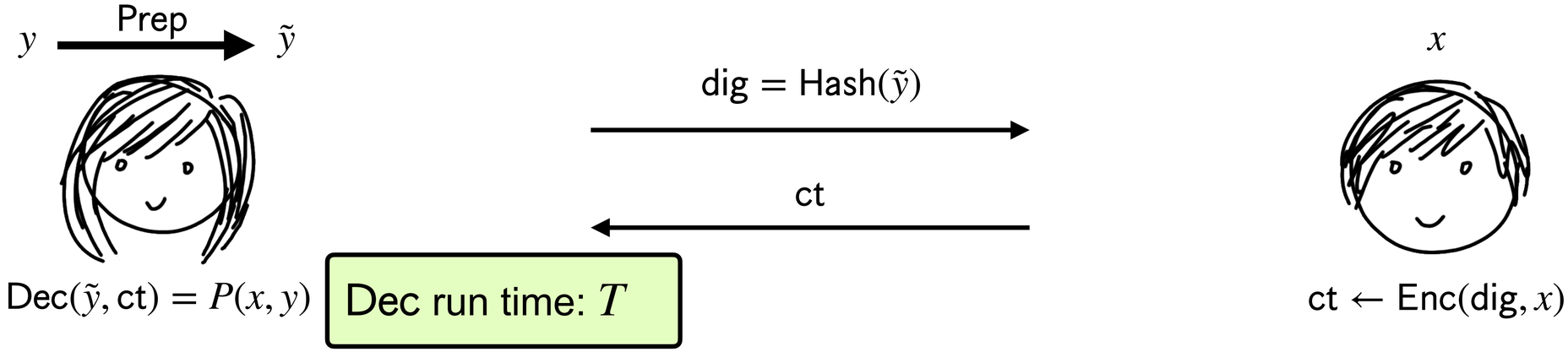

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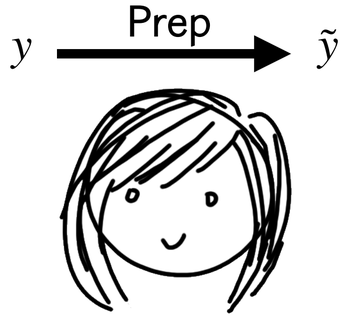
Prep run time: $|y|^{1+\varepsilon}$

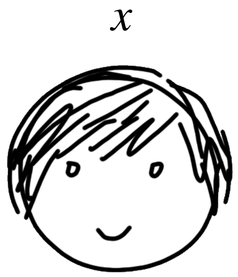


$\text{Dec}(\tilde{y}, \text{ct}) = P(x, y)$

Dec run time: T

$\text{dig} = \text{Hash}(\tilde{y})$

ct



$\text{ct} \leftarrow \text{Enc}(\text{dig}, x)$

Main Result: We build LFE for RAMs assuming RingLWE

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$P(x, y)$ has RAM runtime T

Prep run time: $|y|^{1+\varepsilon}$

y → Prep → $\tilde{y}$

$\text{Dec}(\tilde{y}, \text{ct}) = P(x, y)$

Dec run time: T

$\text{dig} = \text{Hash}(\tilde{y})$

ct

Enc run time: $|x| + T$

x

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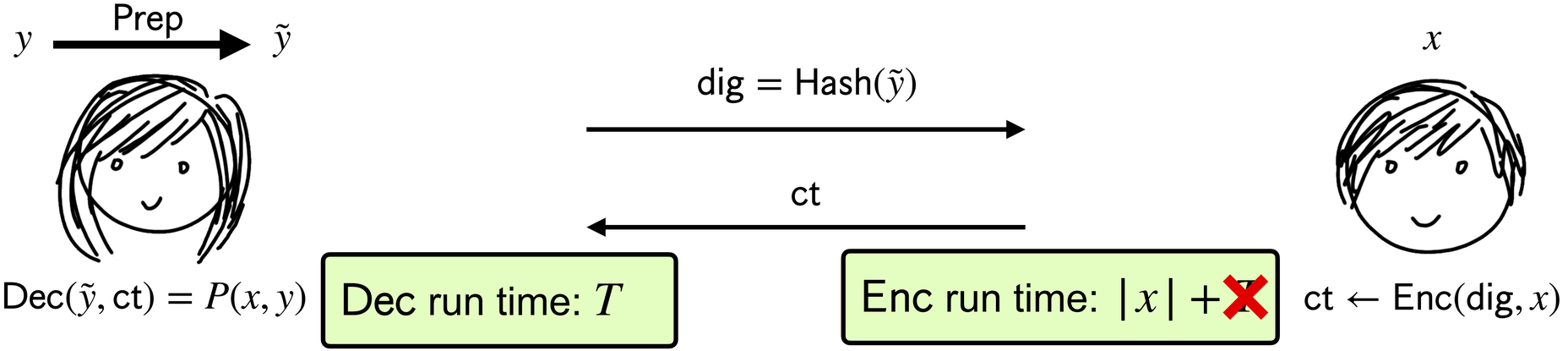

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Additionally assuming iO, get Enc run time just $|x|$

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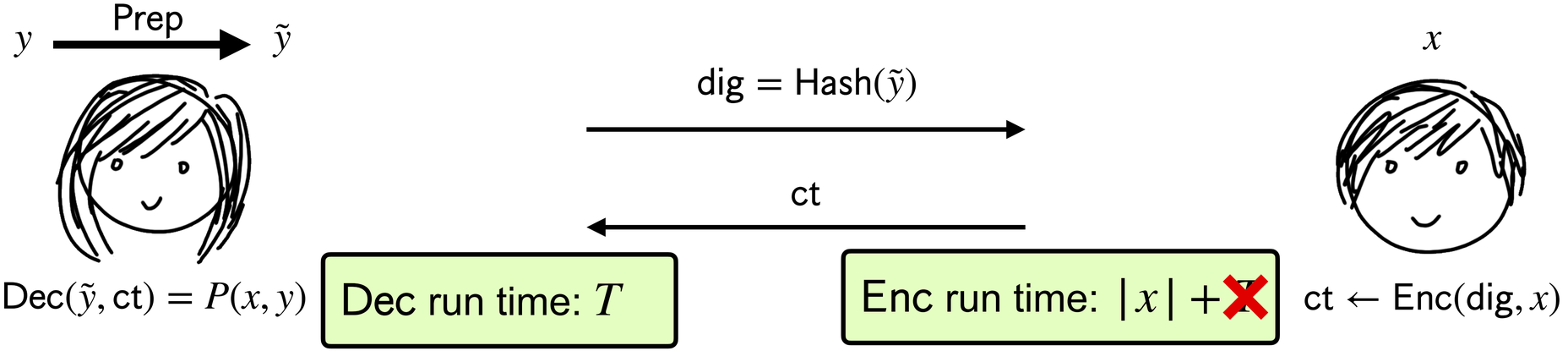

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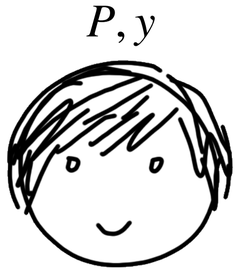
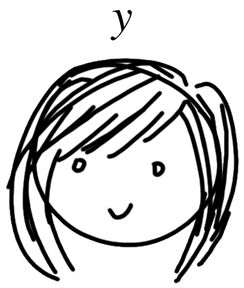
Main challenge: Privately accessing the public database y

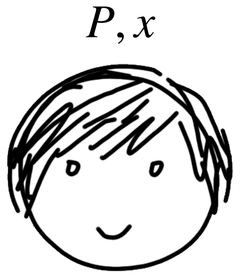
RAM-LFE vs Garbled RAM



Garbled RAM

RAM-LFE





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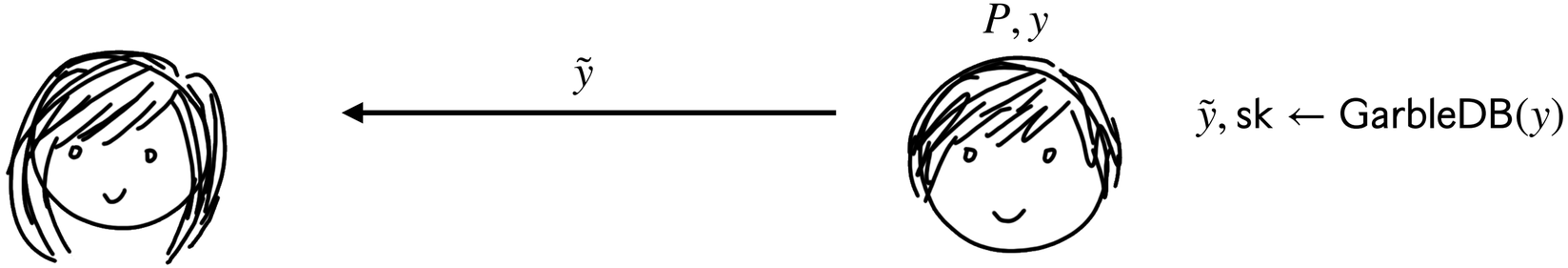


Garbled RAM

RAM-LFE

RAM-LFE vs Garbled RAM

$\tilde{y}$

P, y

$\tilde{y}, \mathsf{sk} \leftarrow \mathsf{GarbleDB}(y)$

y belongs to client and is garbled with respect to their secret key

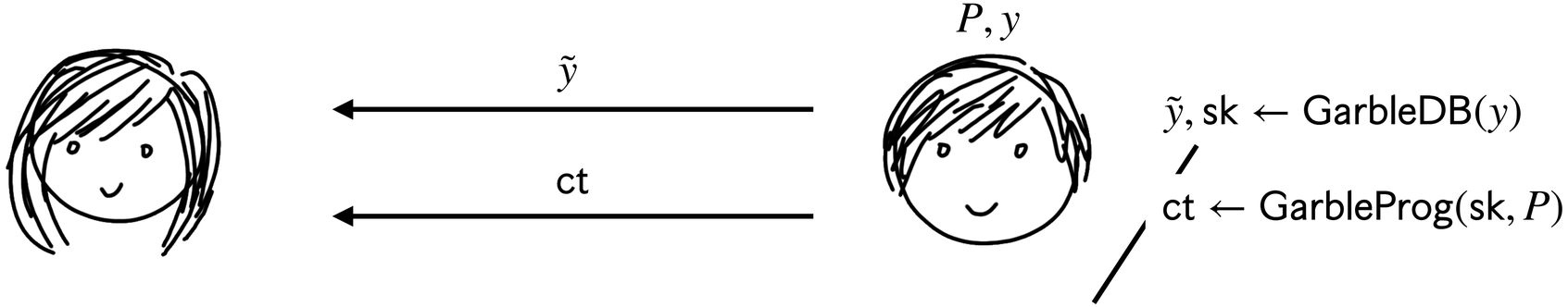
Garbled RAM

RAM-LFE

y

P, x

RAM-LFE vs Garbled RAM



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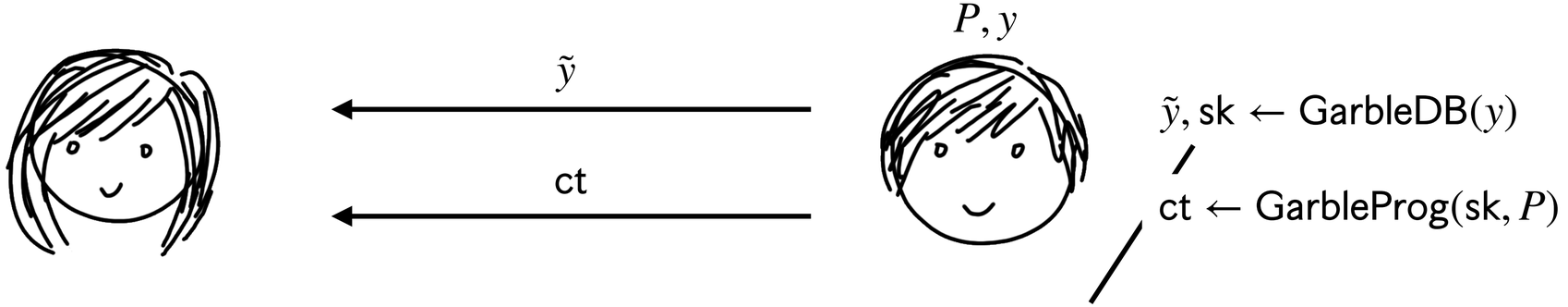

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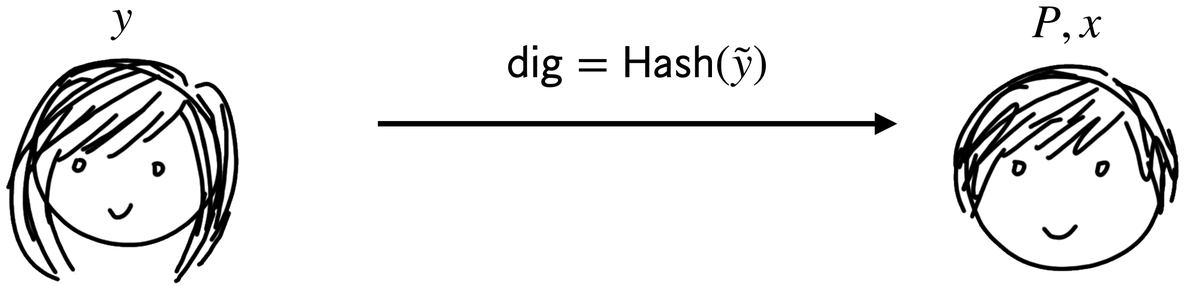

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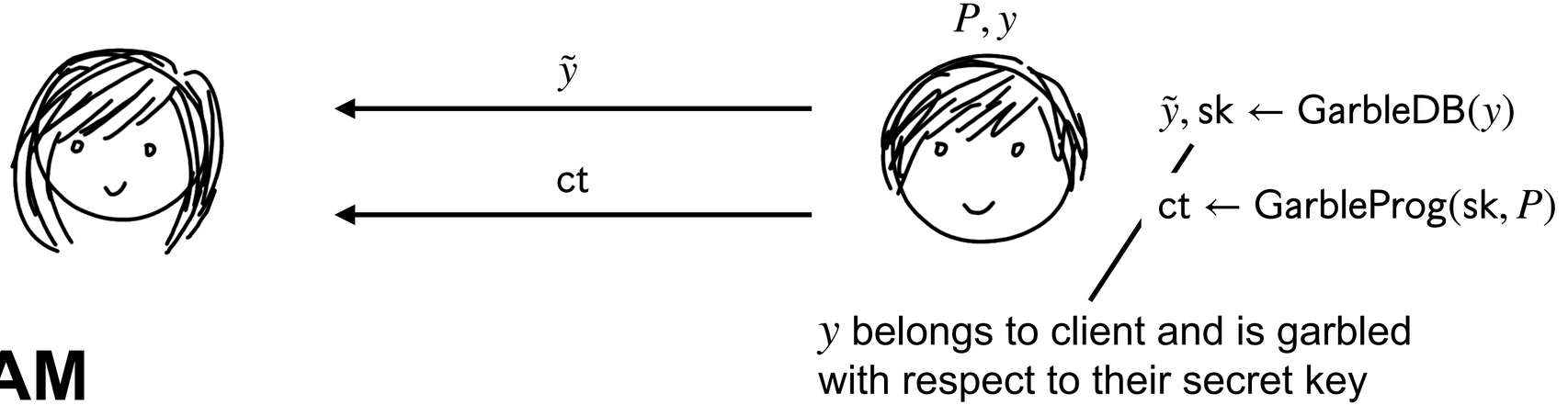

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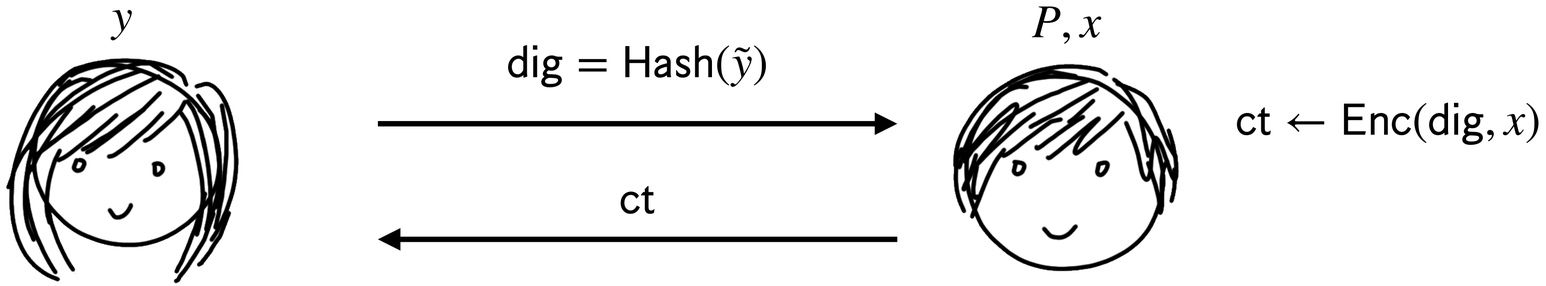

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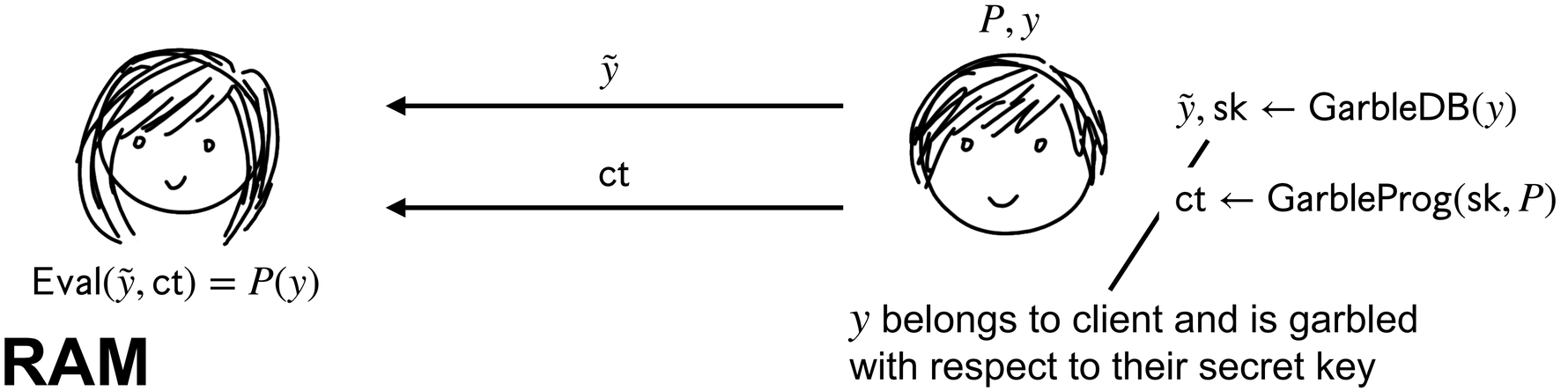

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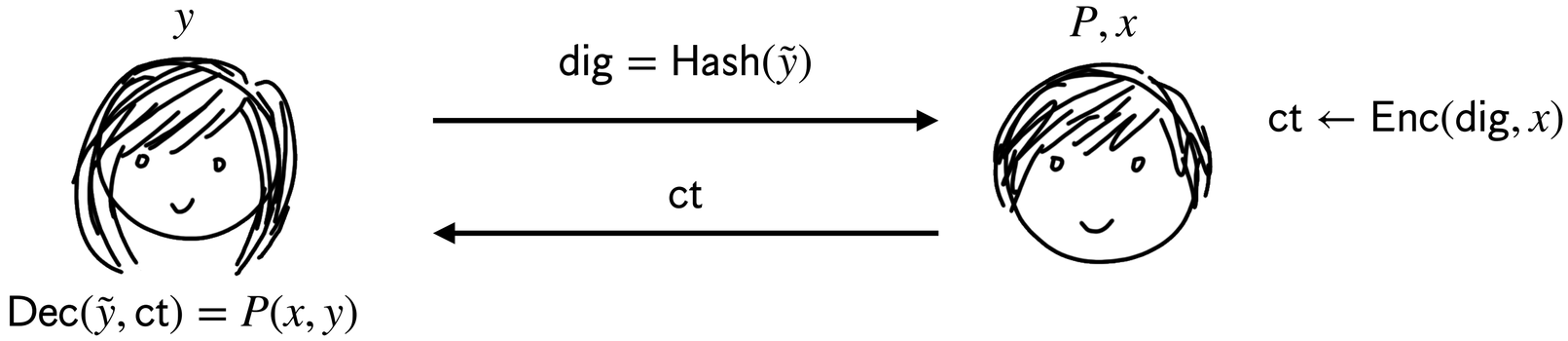

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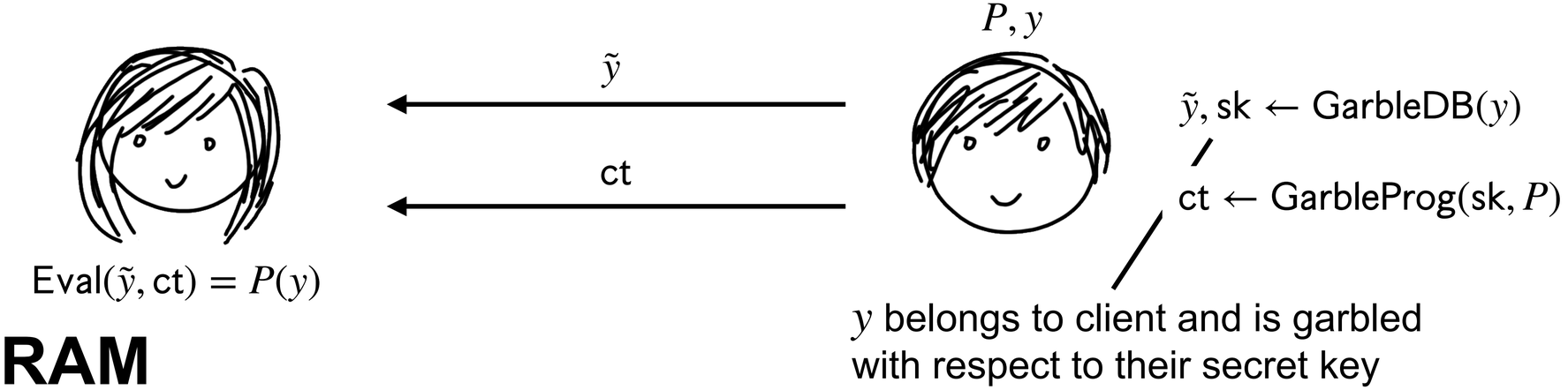

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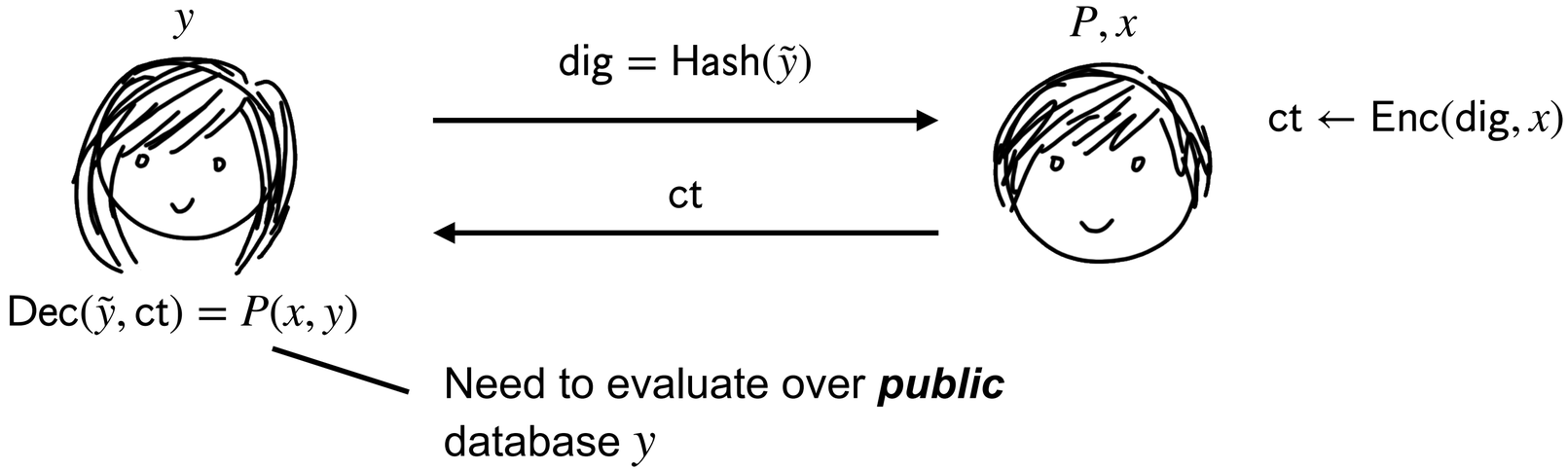
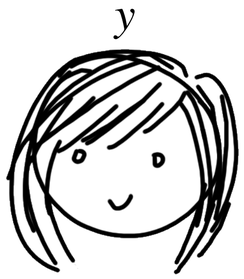
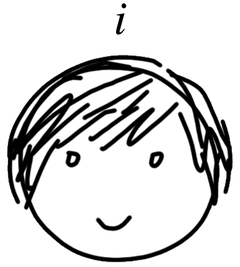


DEPIR vs ORAM

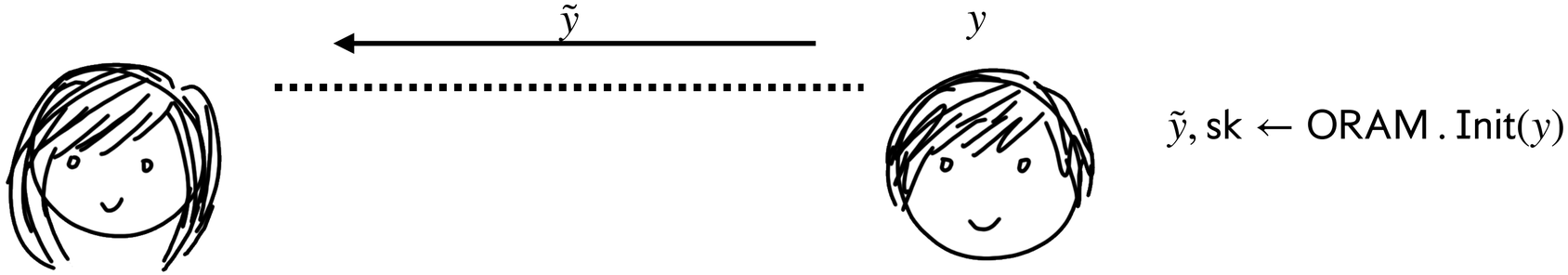
ORAM

DEPIR





DEPIR vs ORAM



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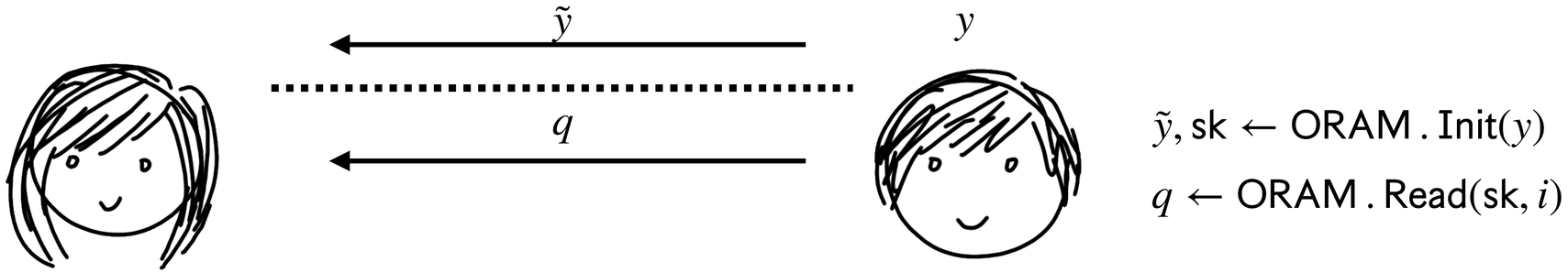

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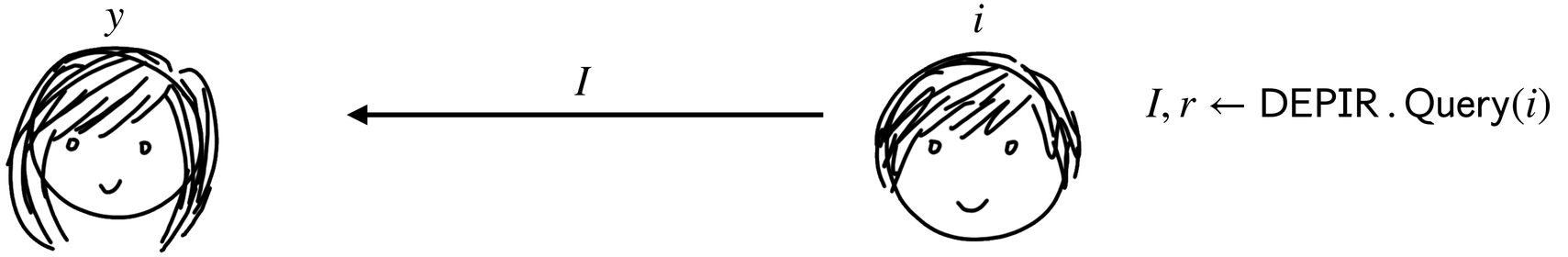

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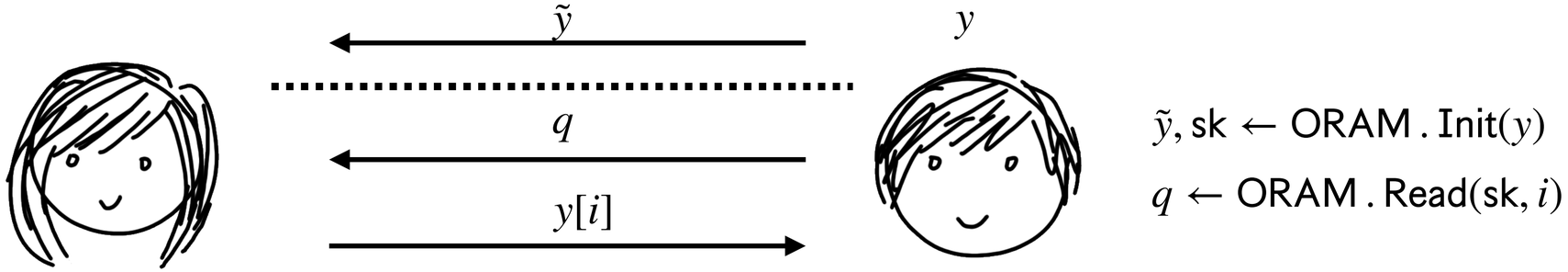


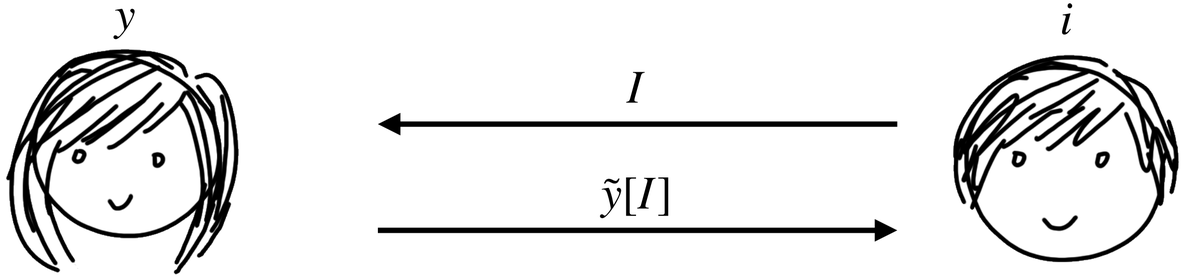
$\tilde{y}, \mathsf{sk} \leftarrow \mathsf{ORAM}.\mathsf{Init}(y)$

$q \leftarrow \mathsf{ORAM}.\mathsf{Read}(\mathsf{sk}, i)$

ORAM

DEPIR

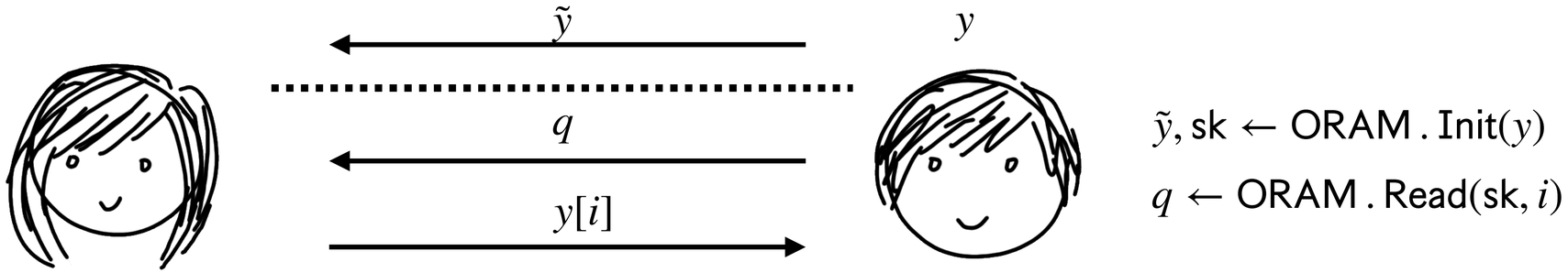
$\tilde{y} := \mathsf{DEPIR}.\mathsf{Prep}(y)$



$I, r \leftarrow \mathsf{DEPIR}.\mathsf{Query}(i)$

$y[i] = \mathsf{DEPIR}.\mathsf{Dec}(\tilde{y}[I], r)$

DEPIR vs ORAM



ORAM — Private database, requires client secret key

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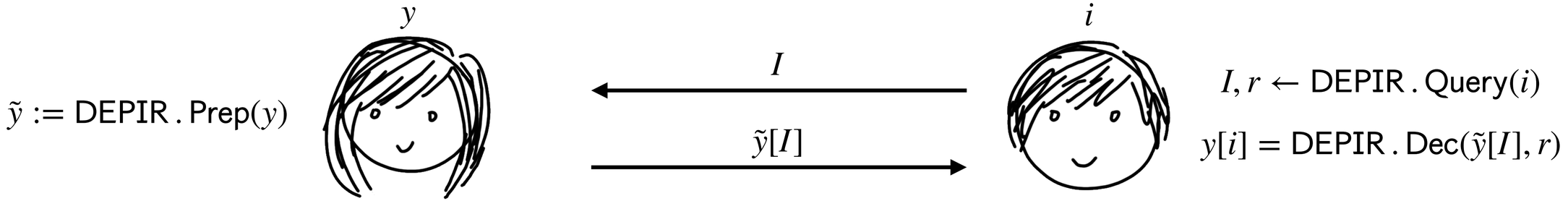

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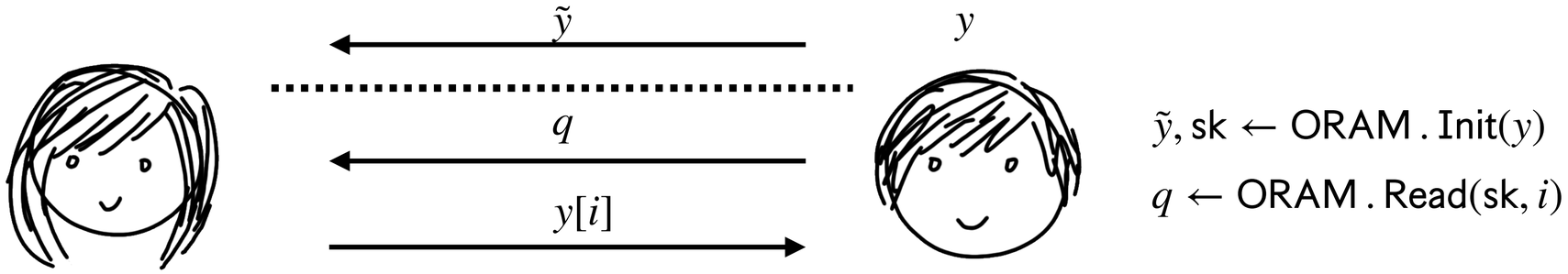

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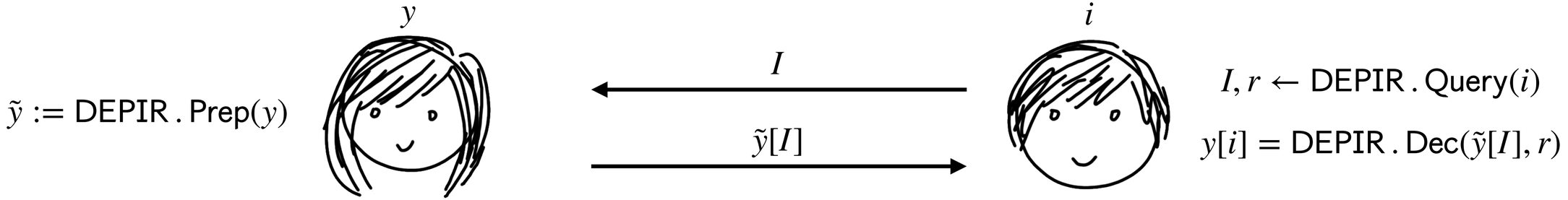

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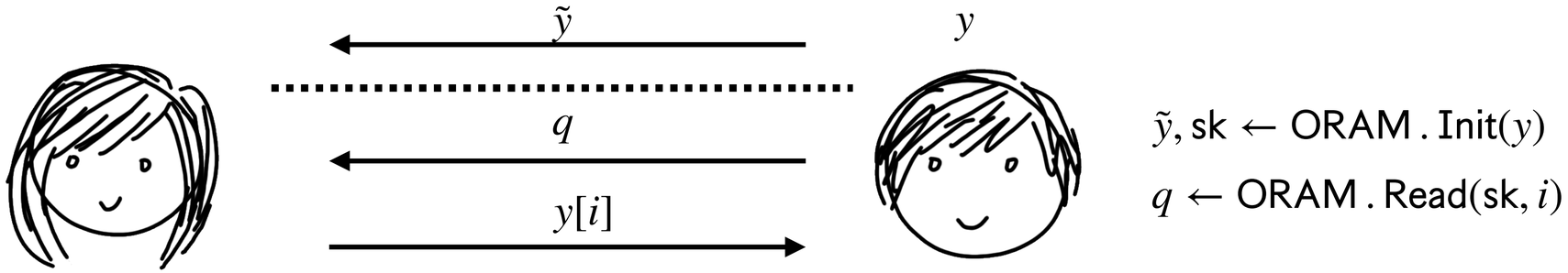

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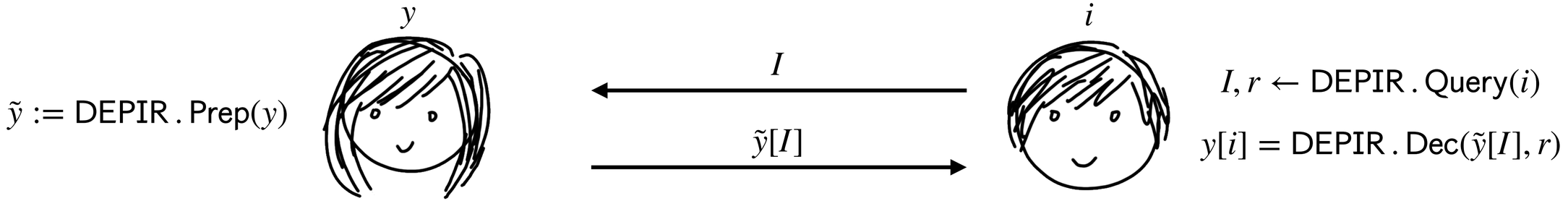

Prior Work: [Lin-**M**-Wichs'23] build DEPIR from RingLWE

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We follow the general template for constructing Garbled RAM

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Requires careful argument and special ORAM construction

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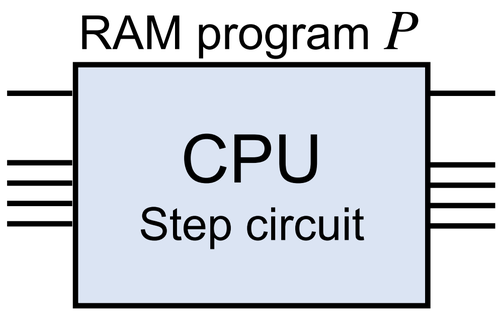

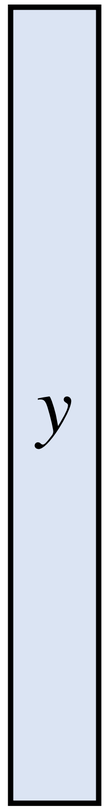

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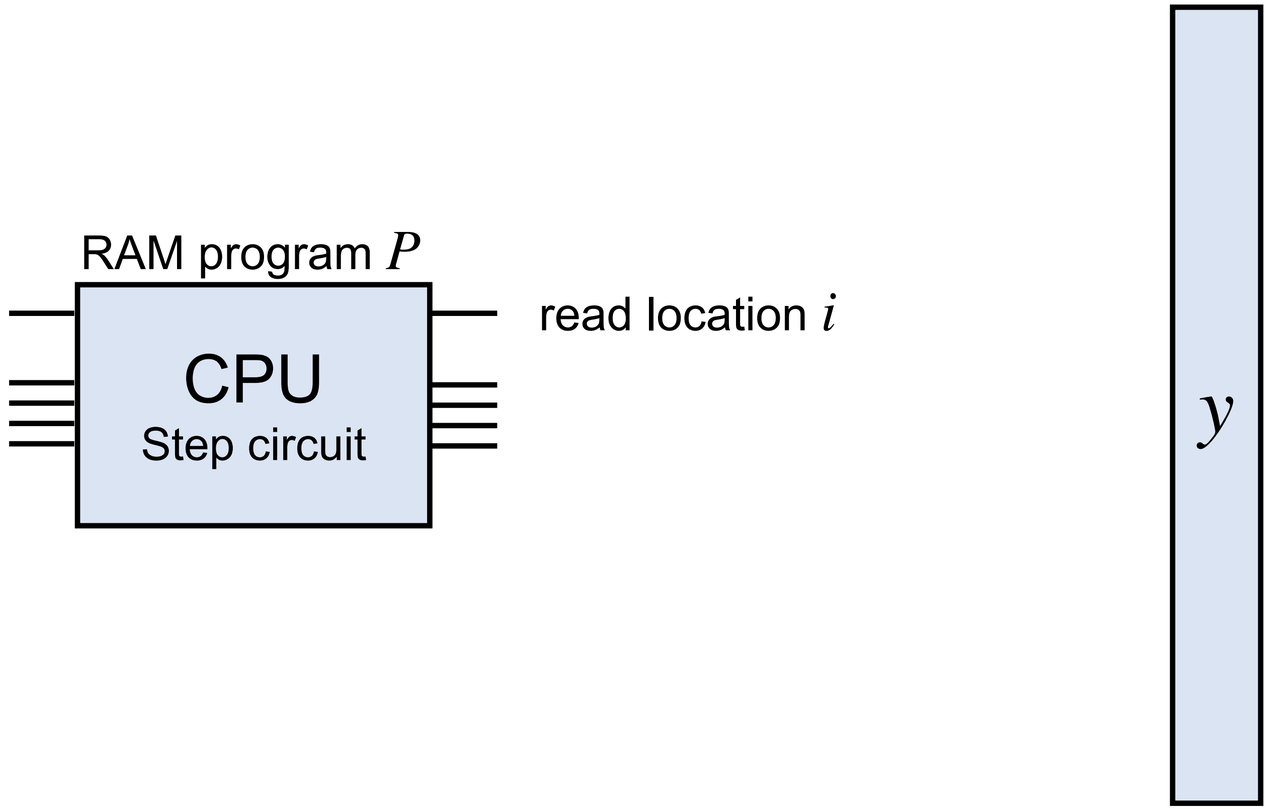

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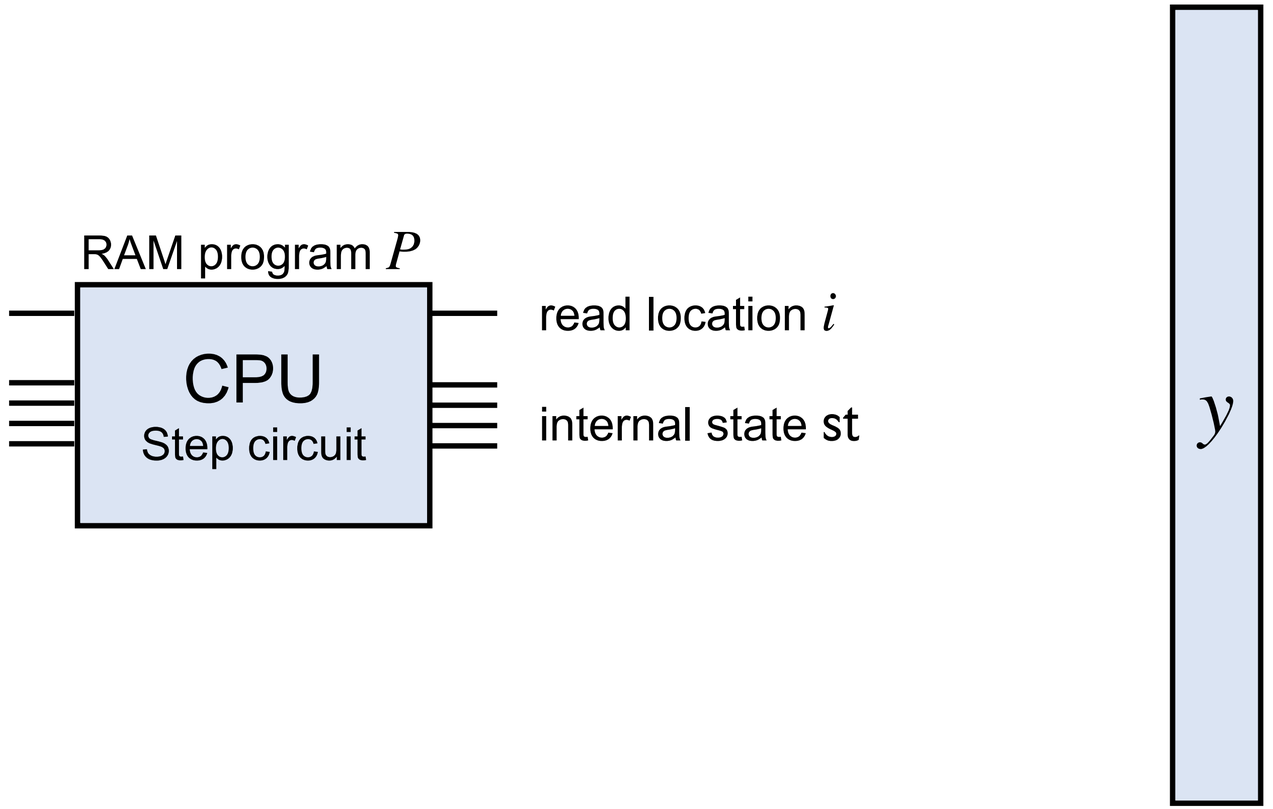

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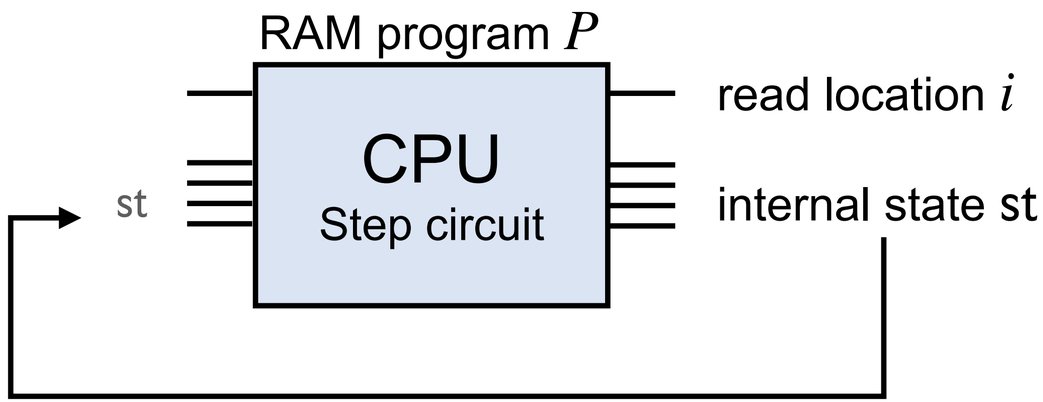

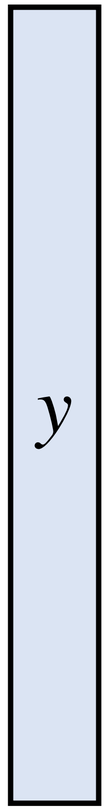

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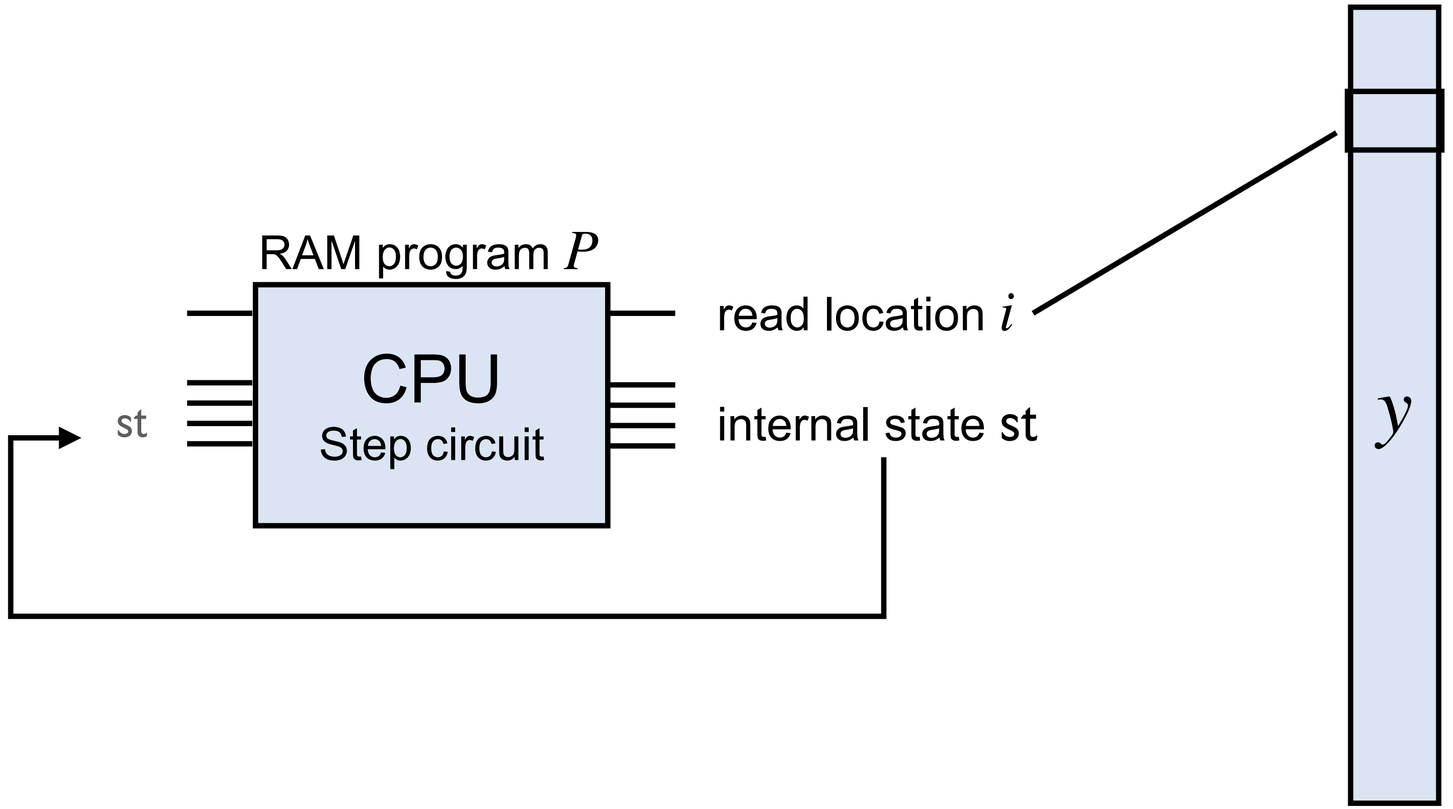

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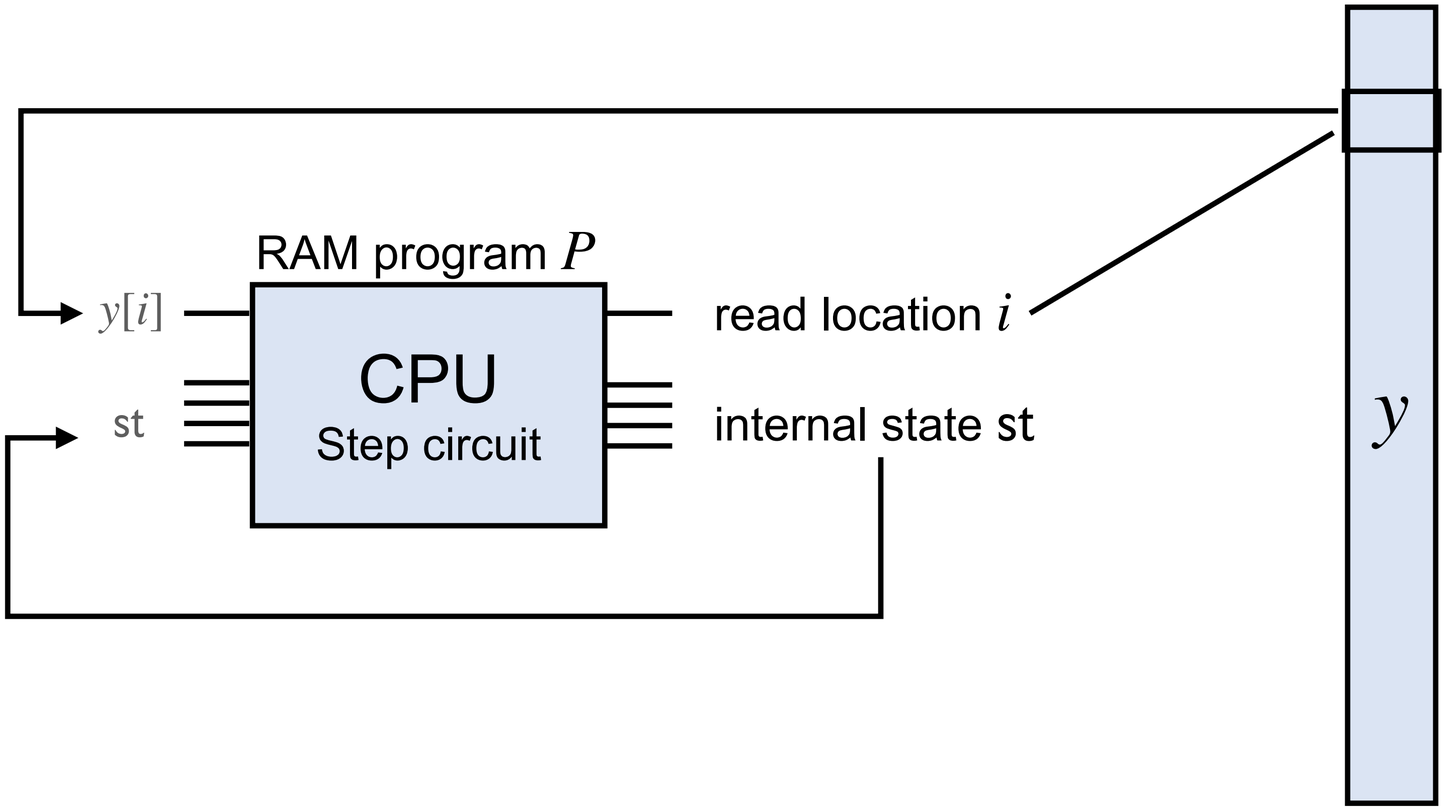

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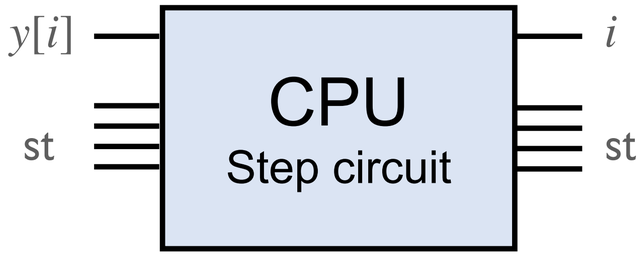

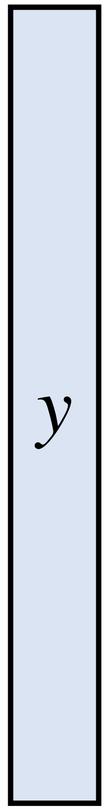

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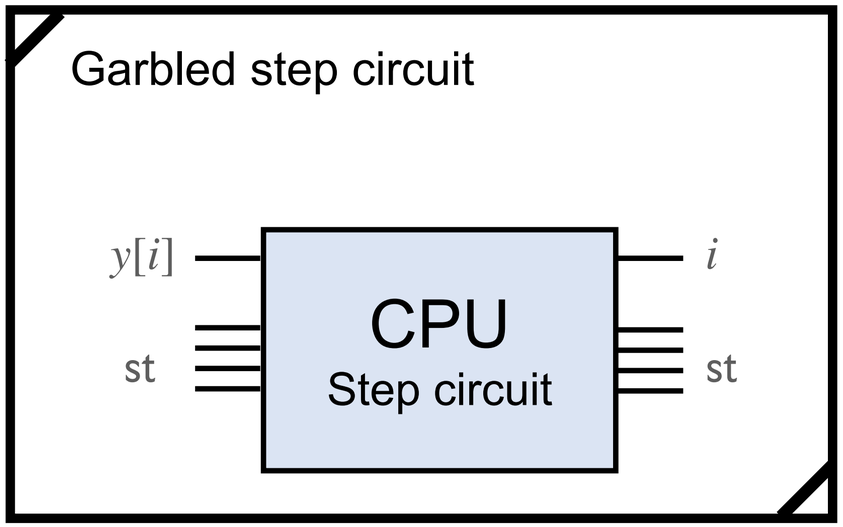

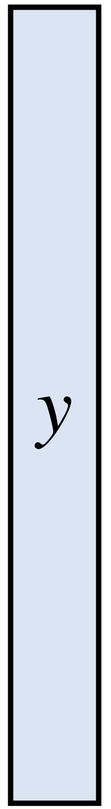

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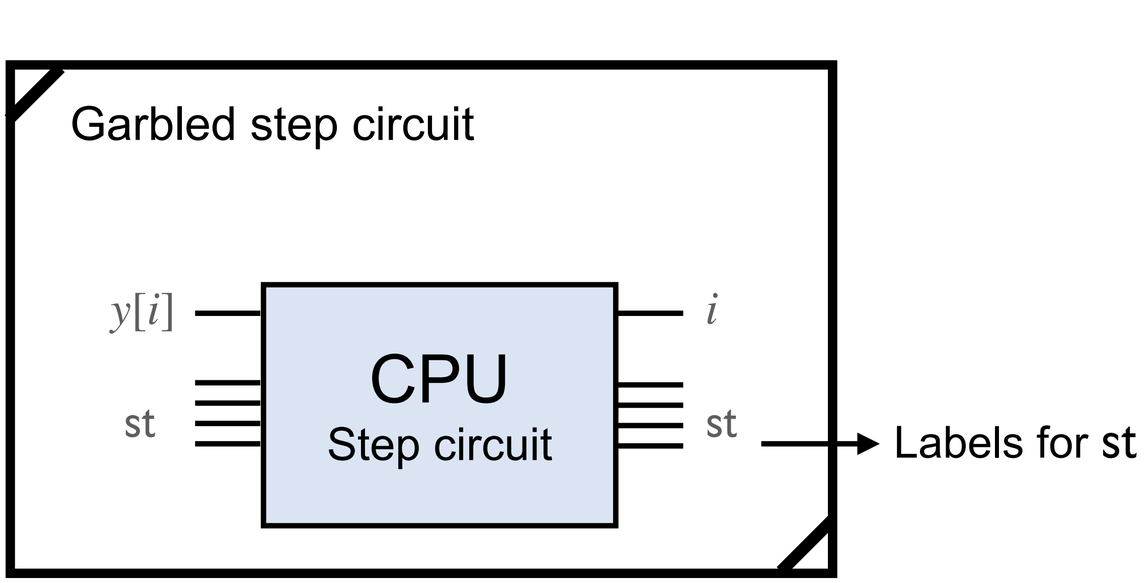

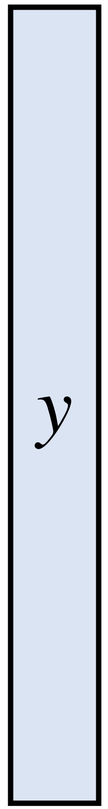

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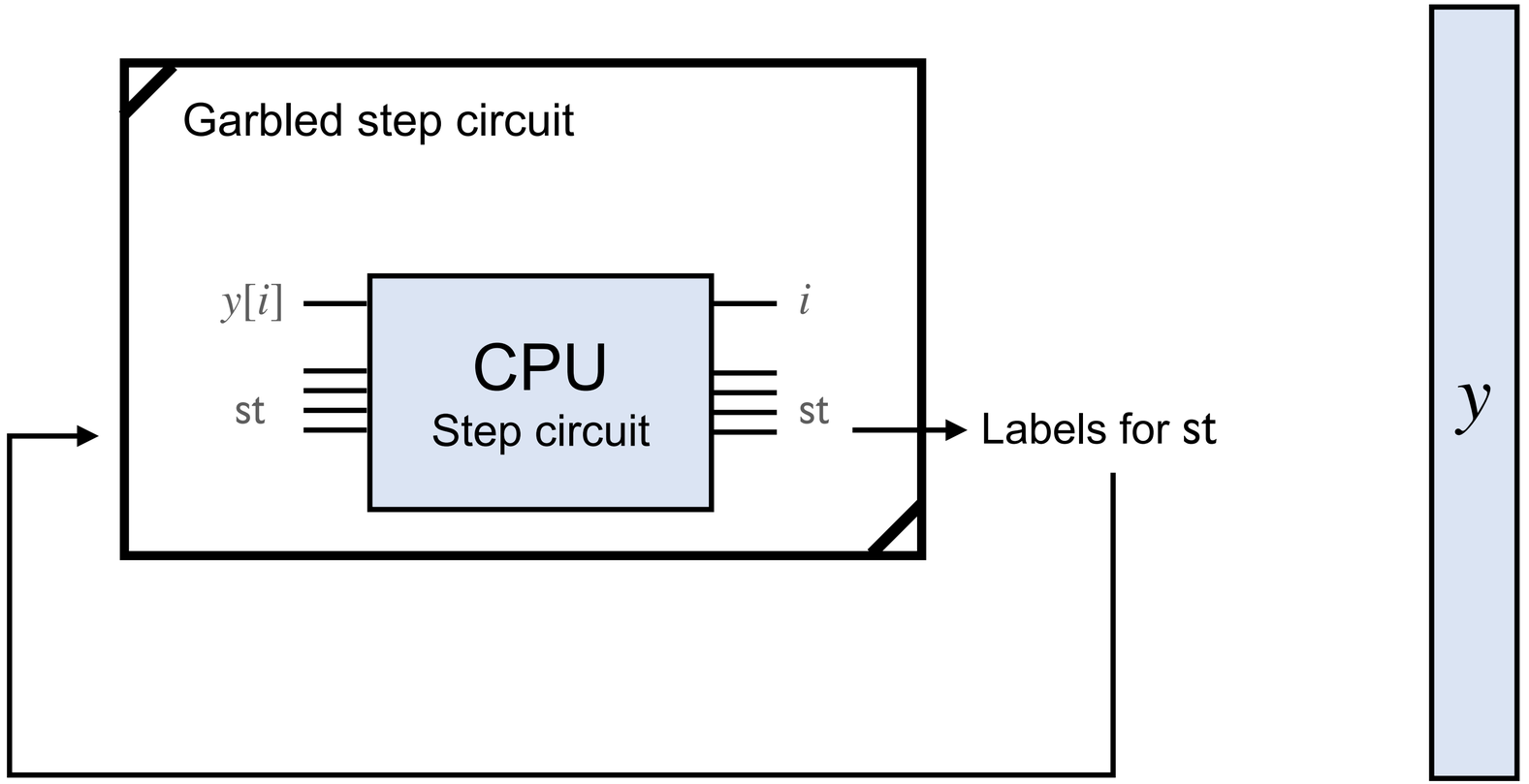

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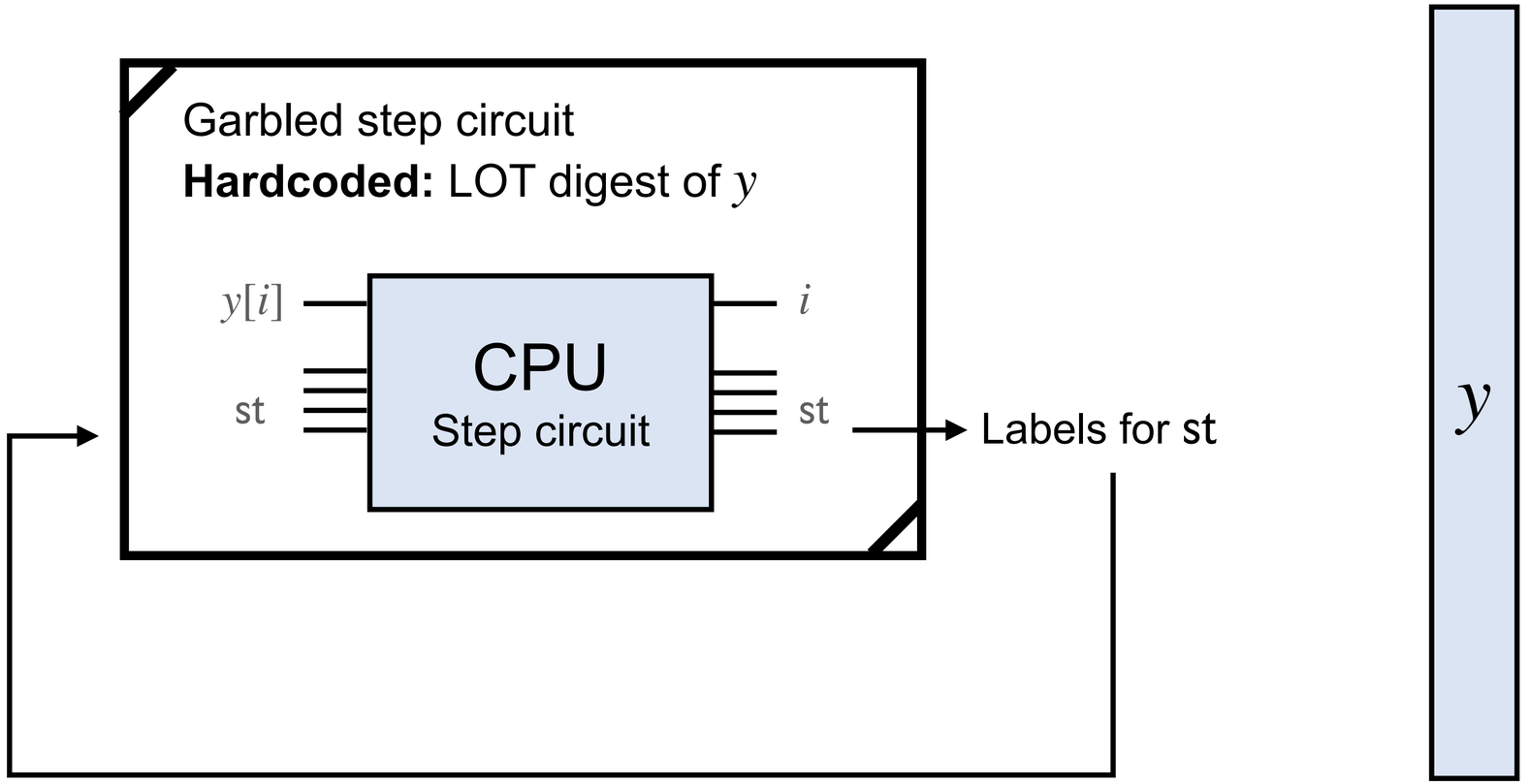

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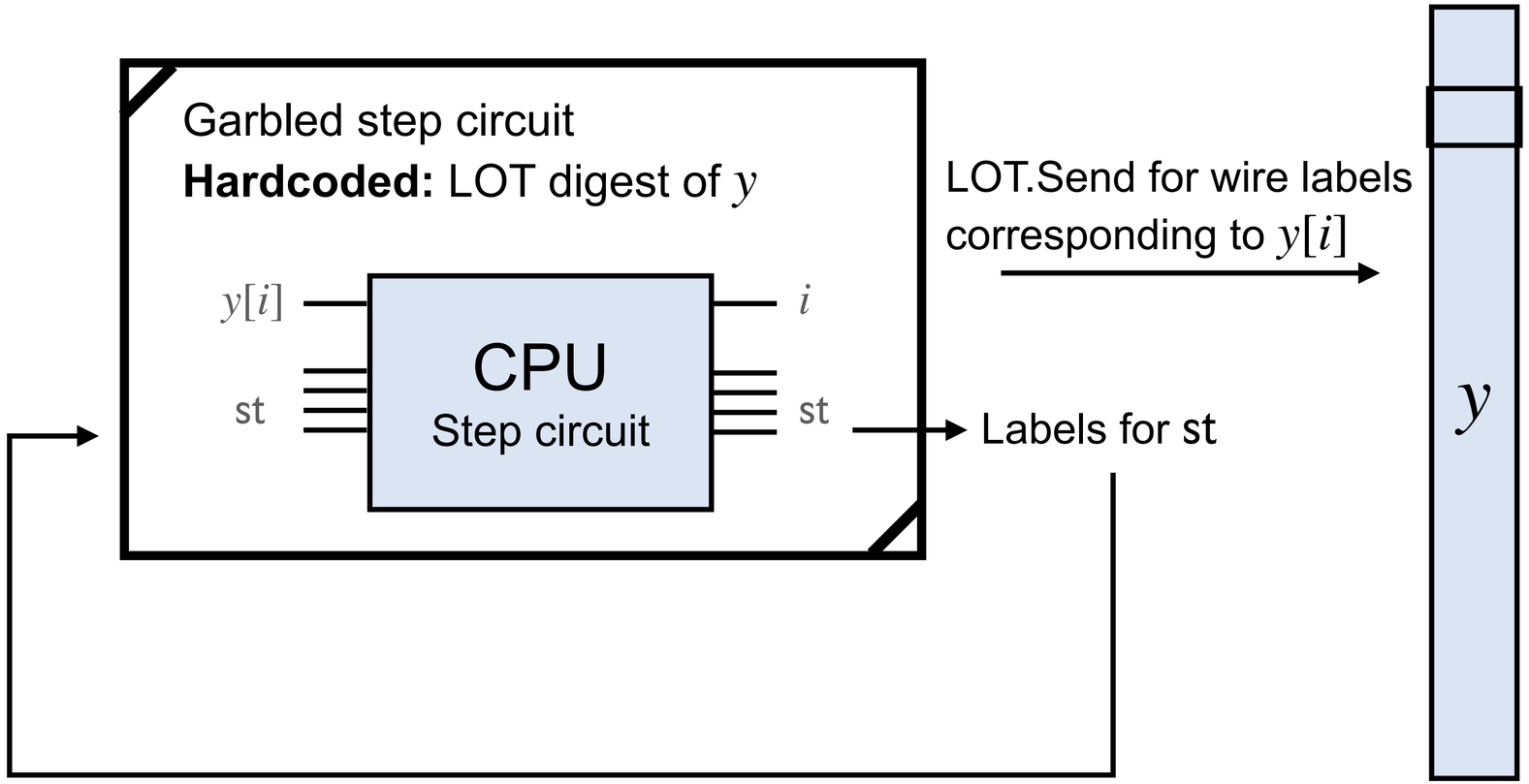

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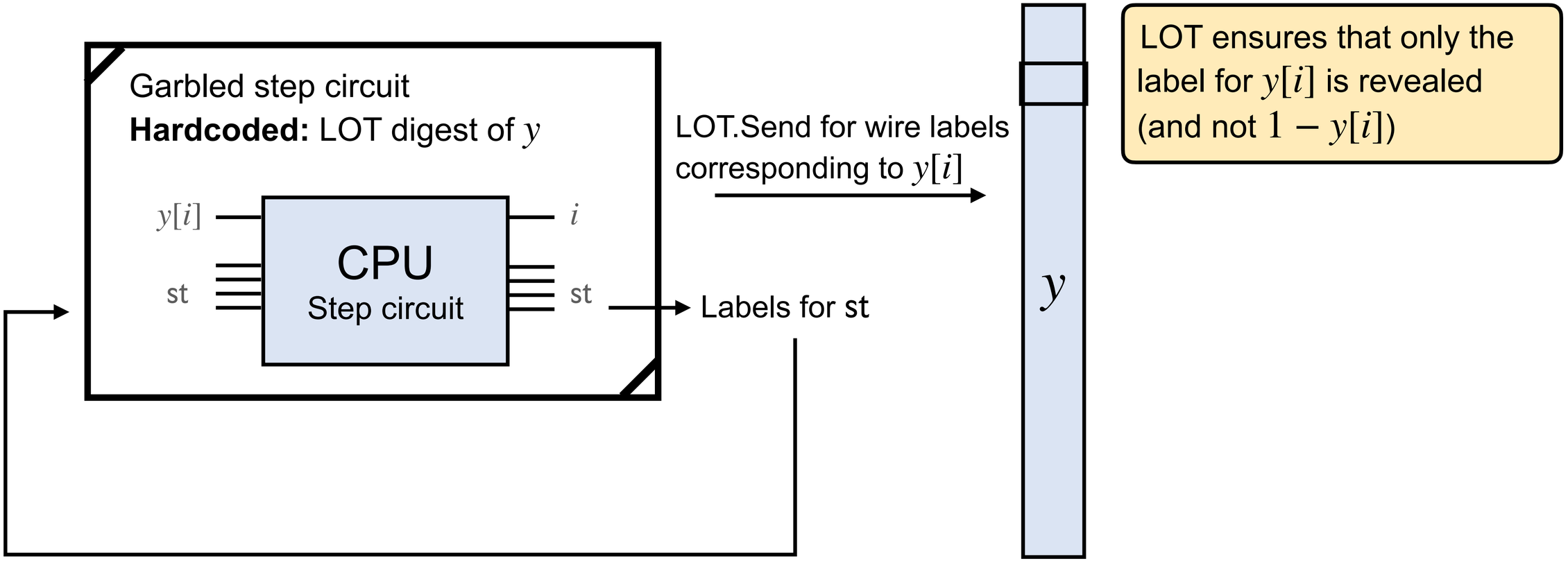

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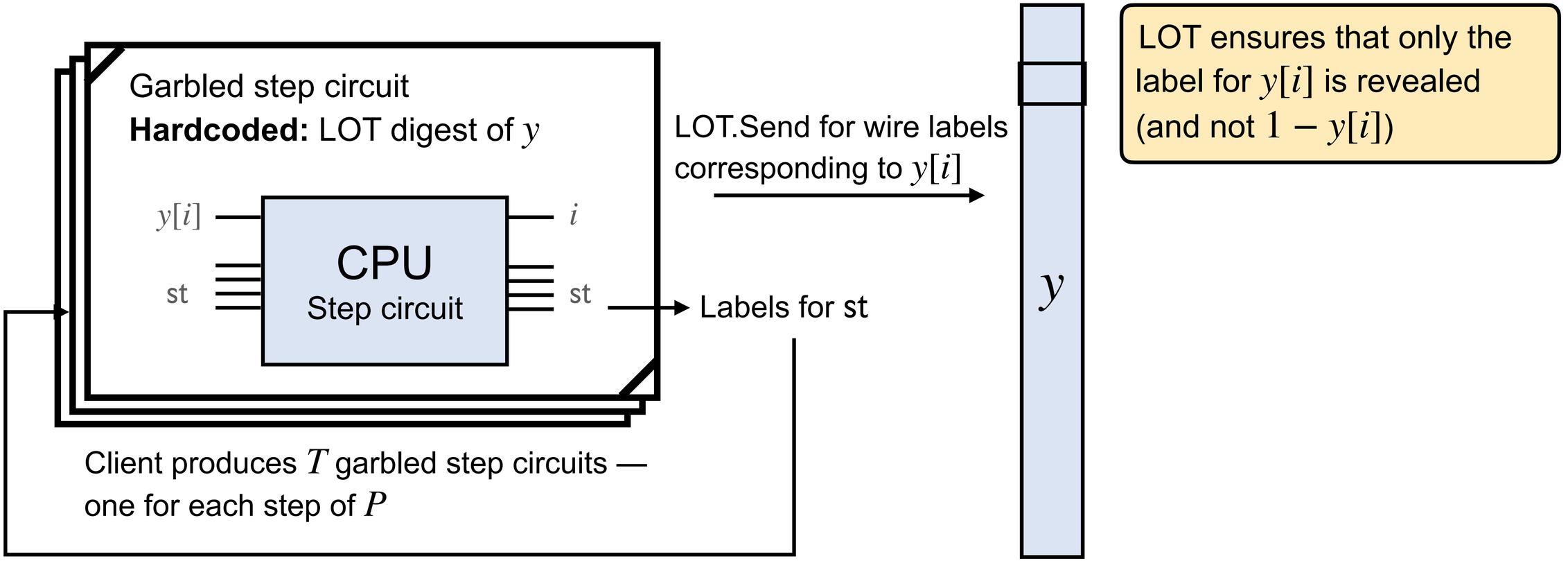

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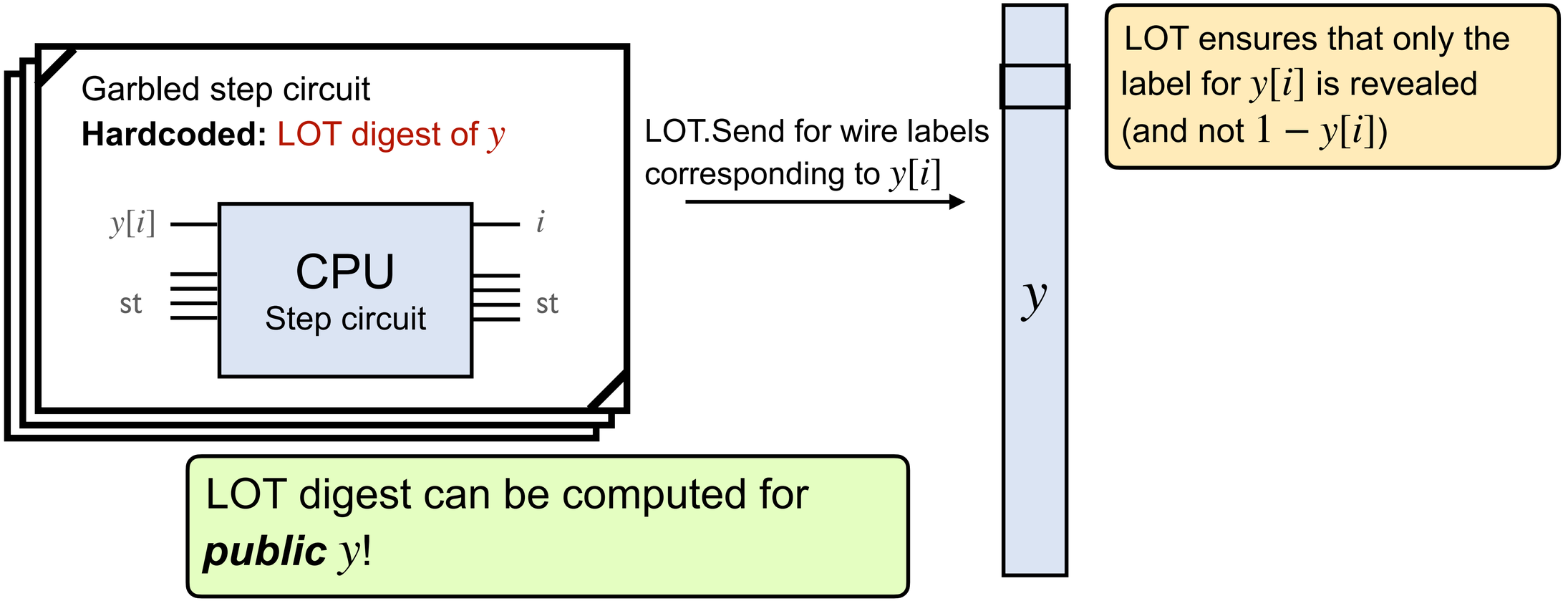
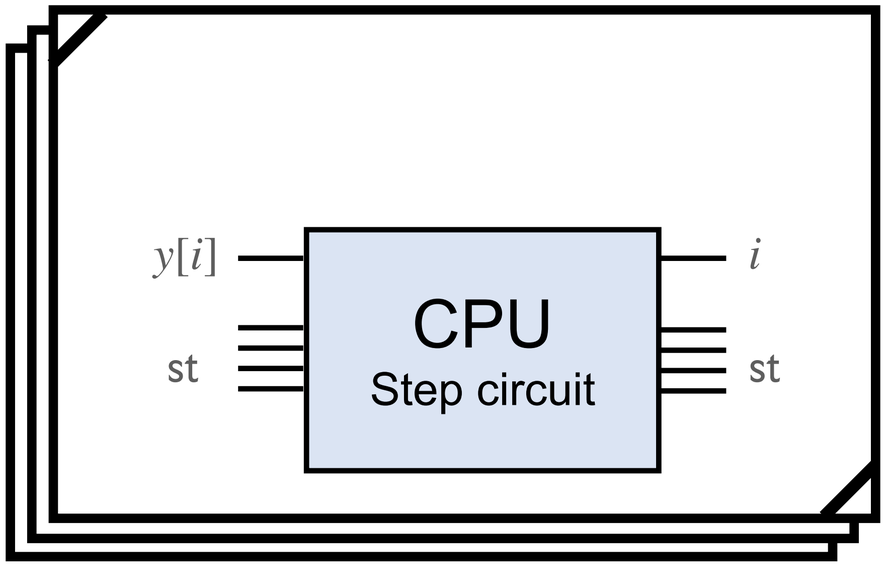
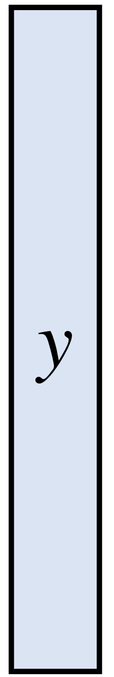


Full security with DEPIR

+ ORAM for the client's database x





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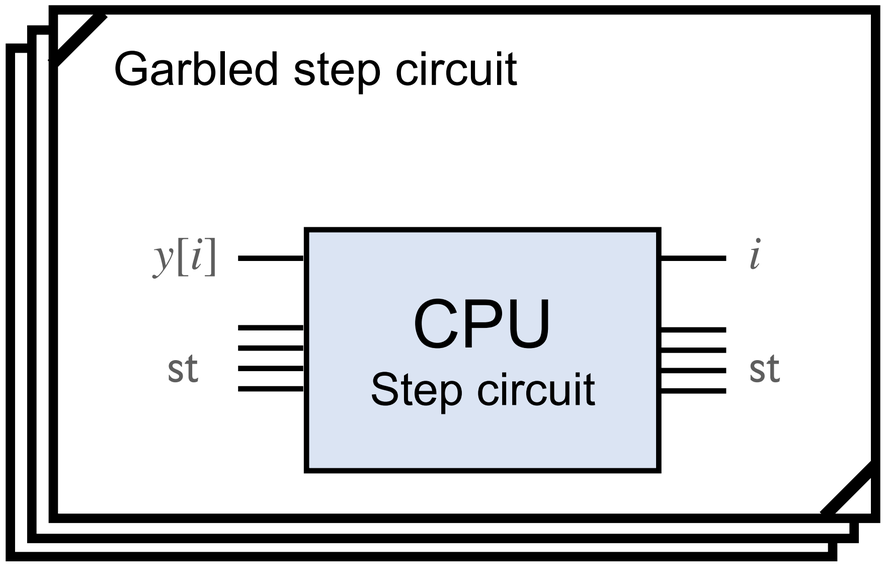

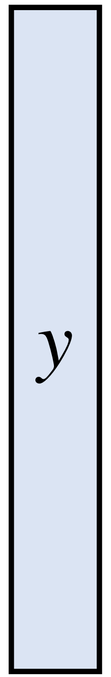

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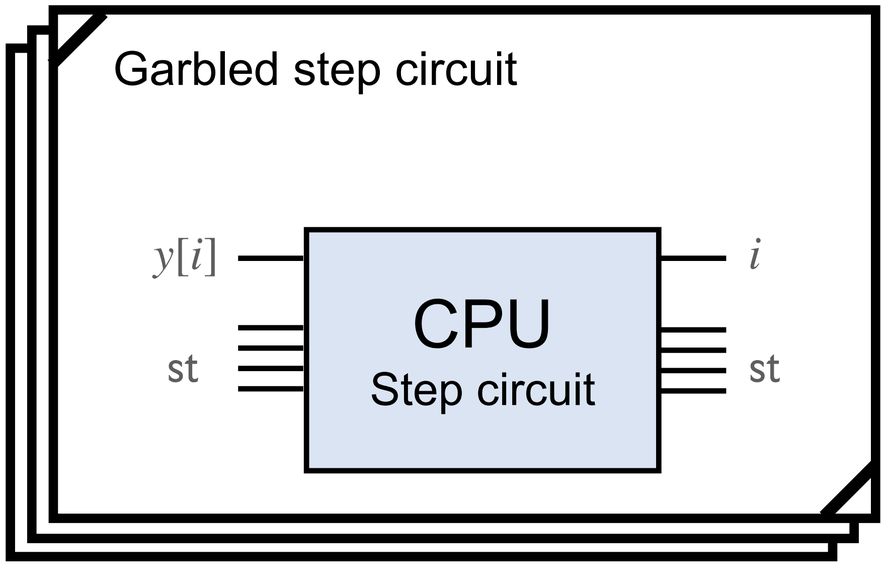

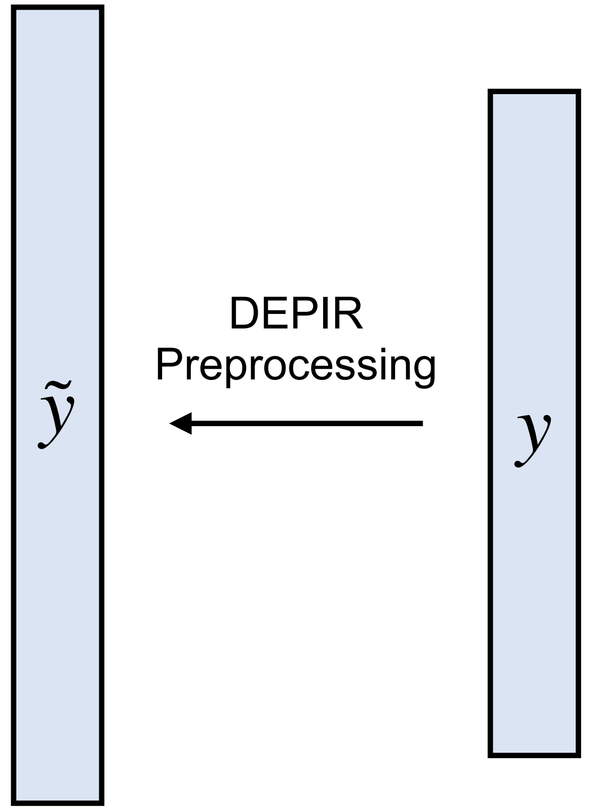

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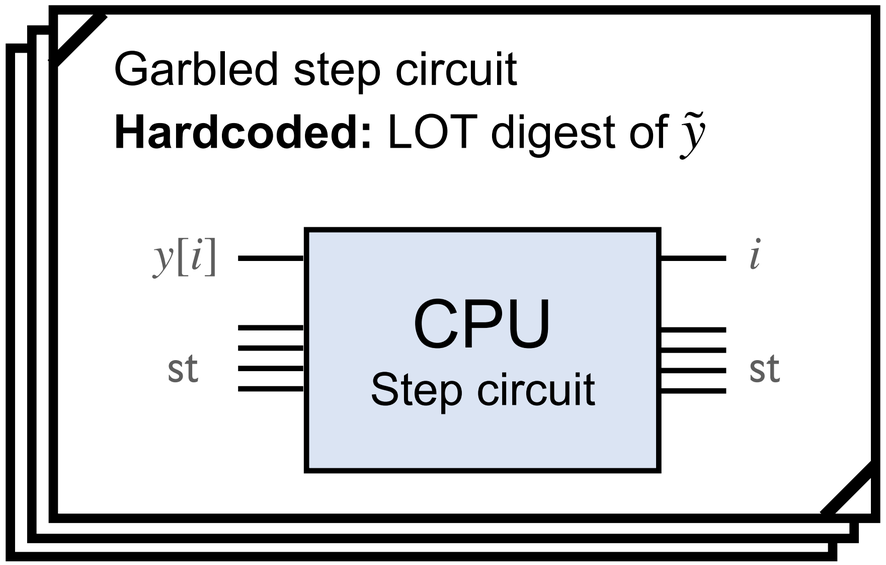

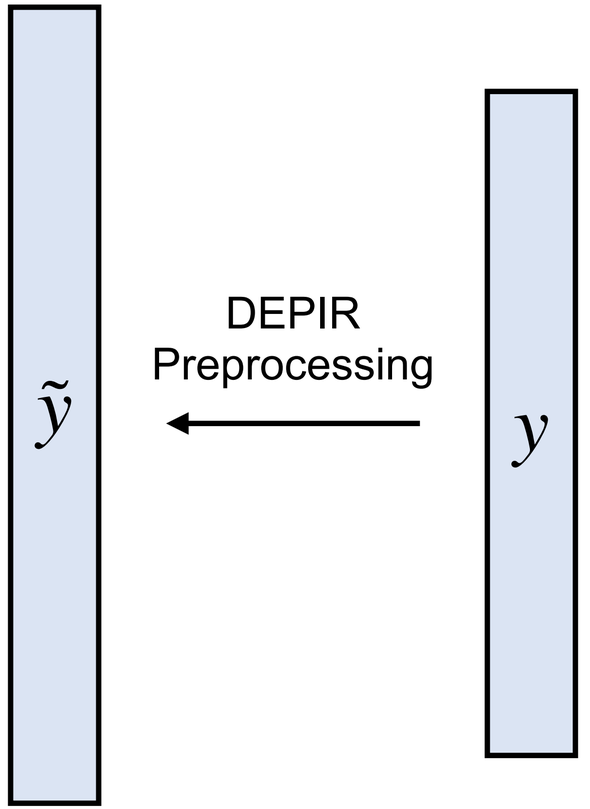

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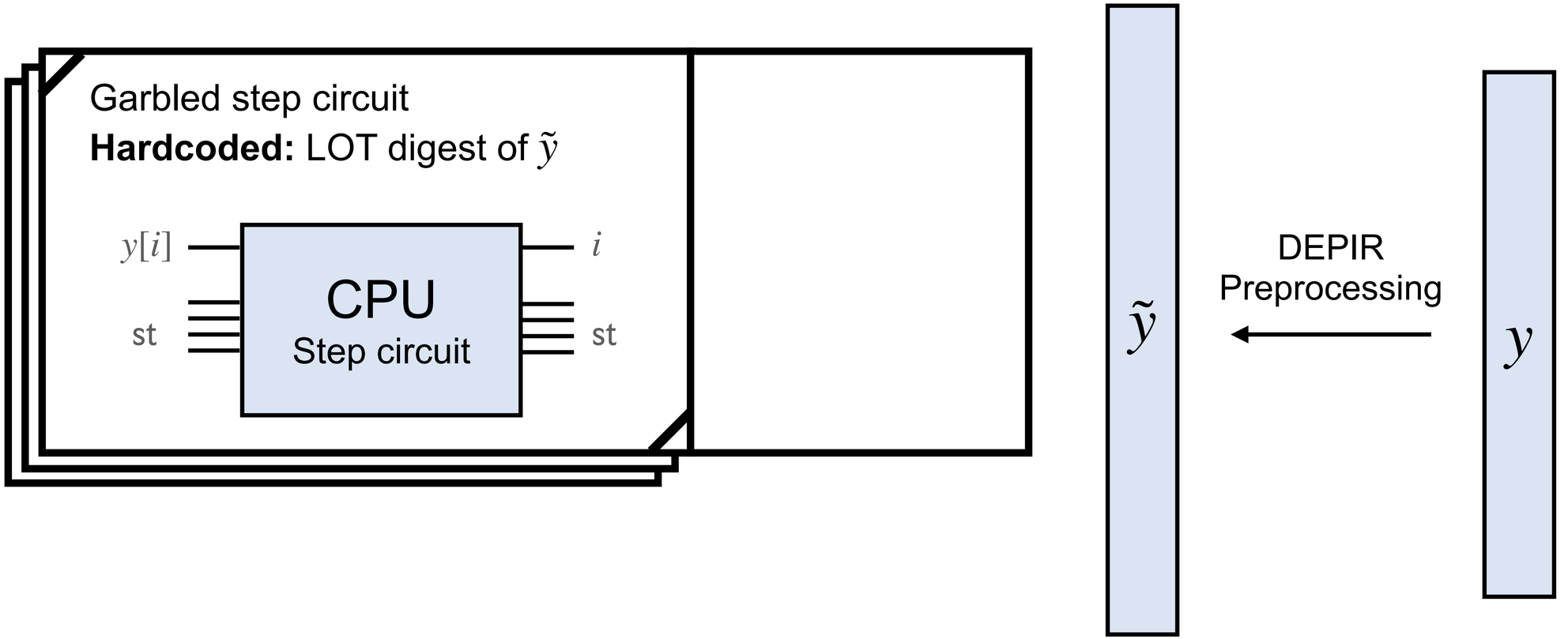

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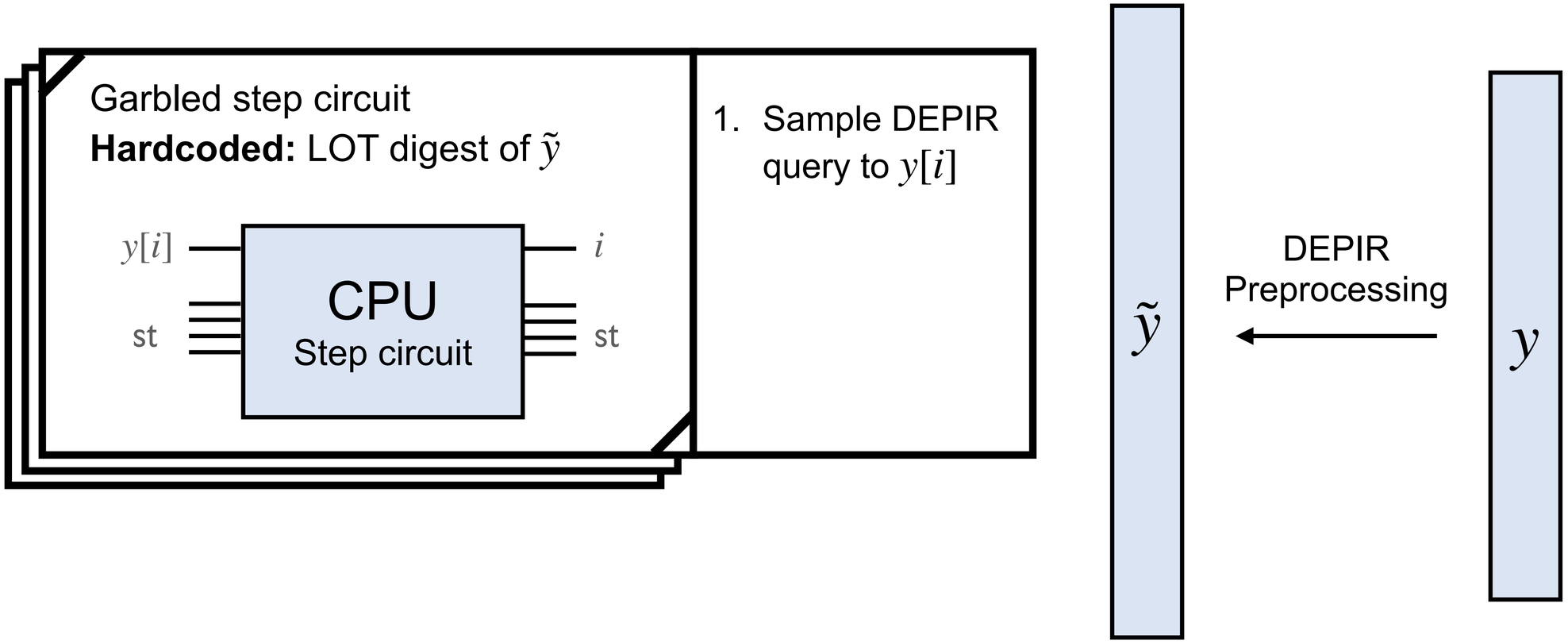

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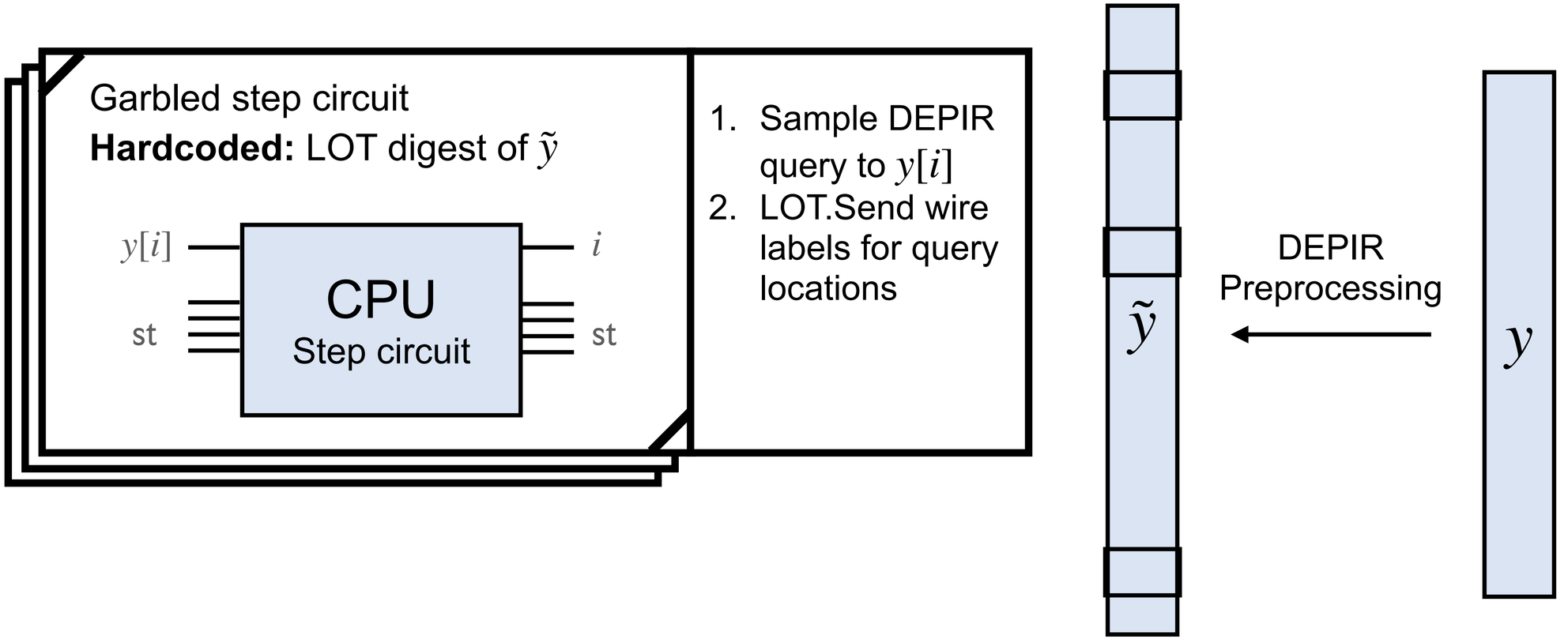

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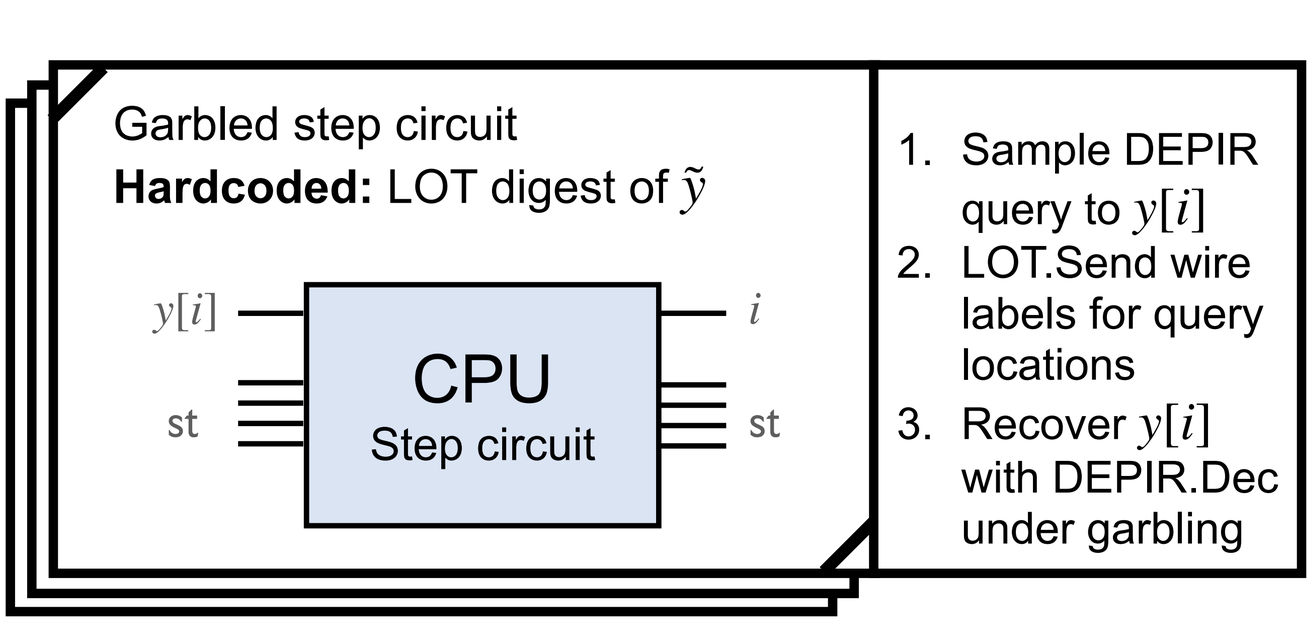
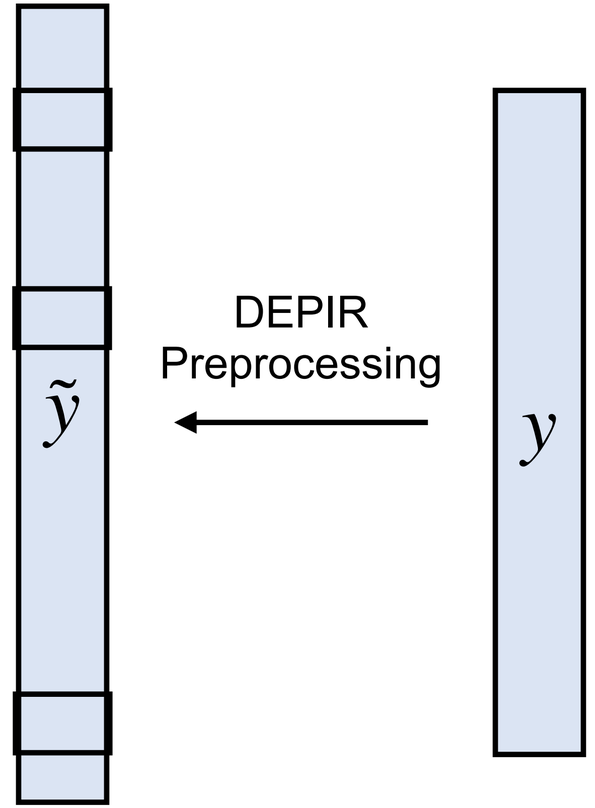
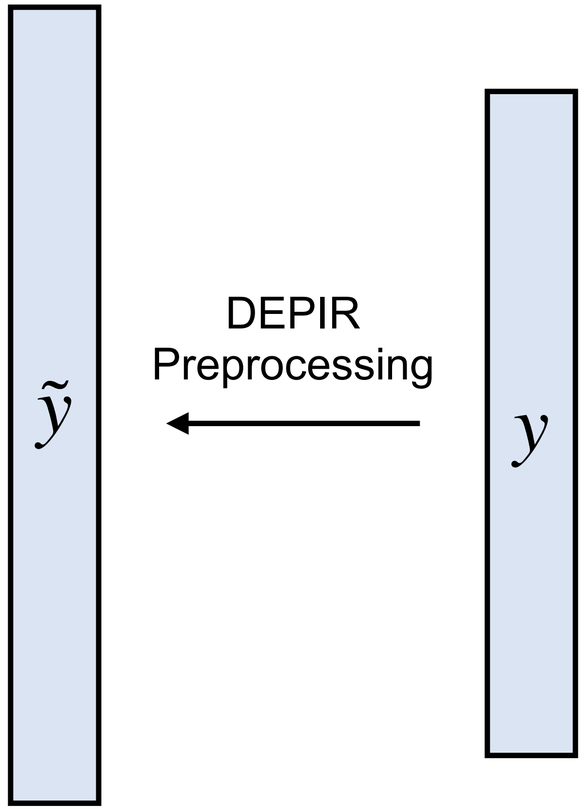




Strong Efficiency with iO



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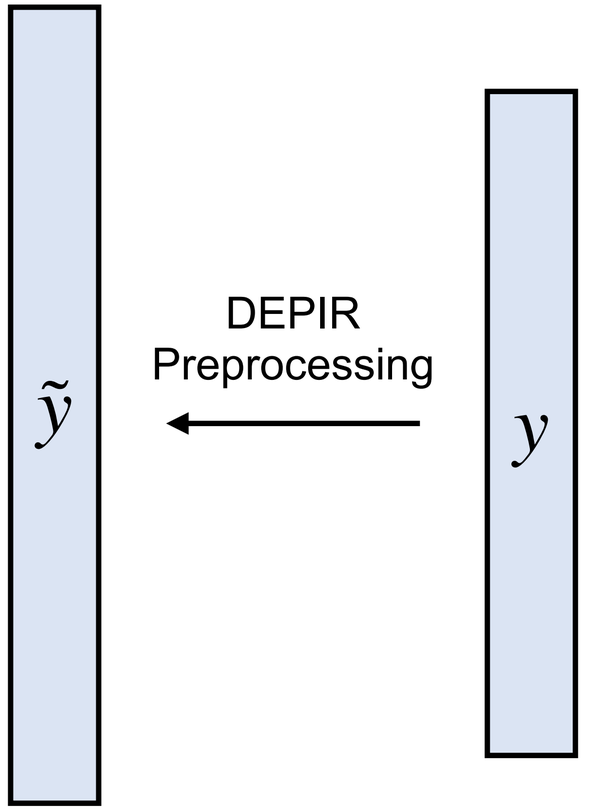

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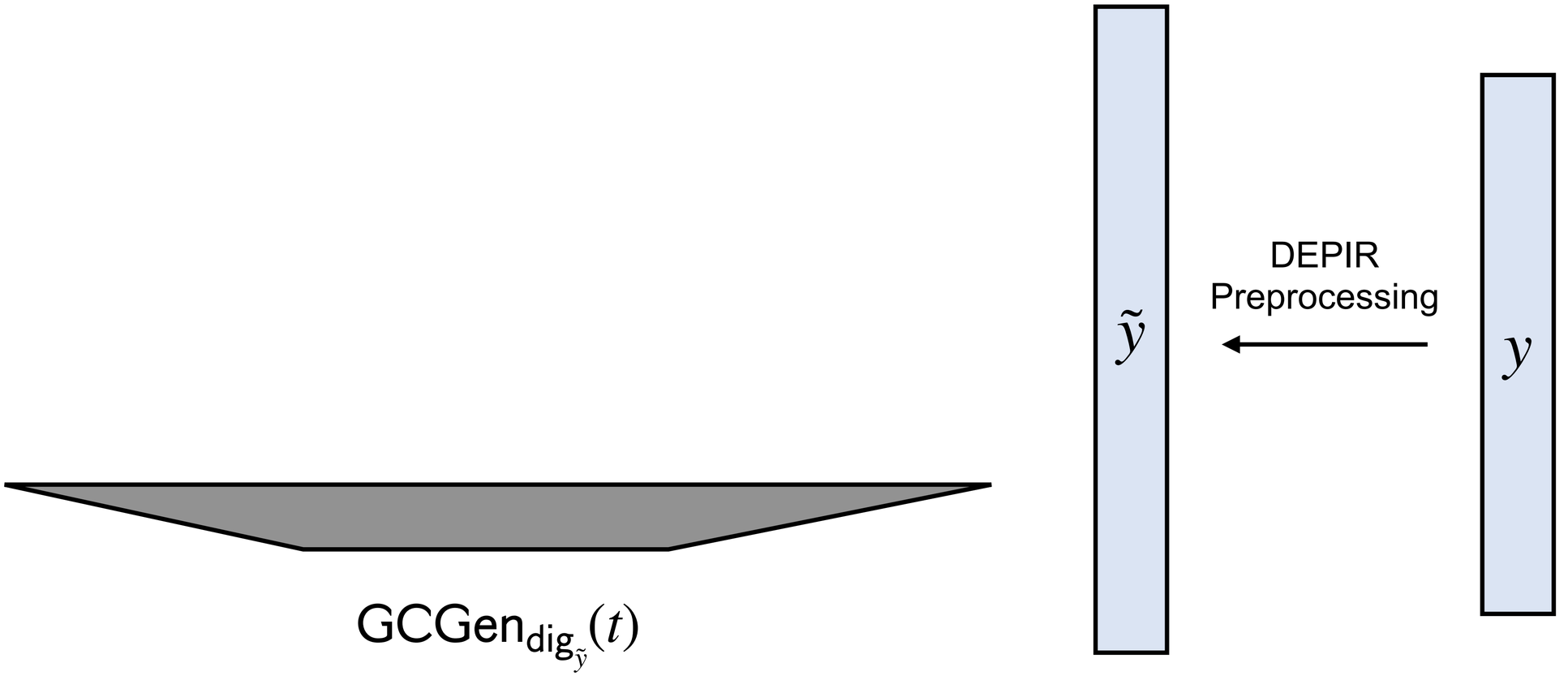

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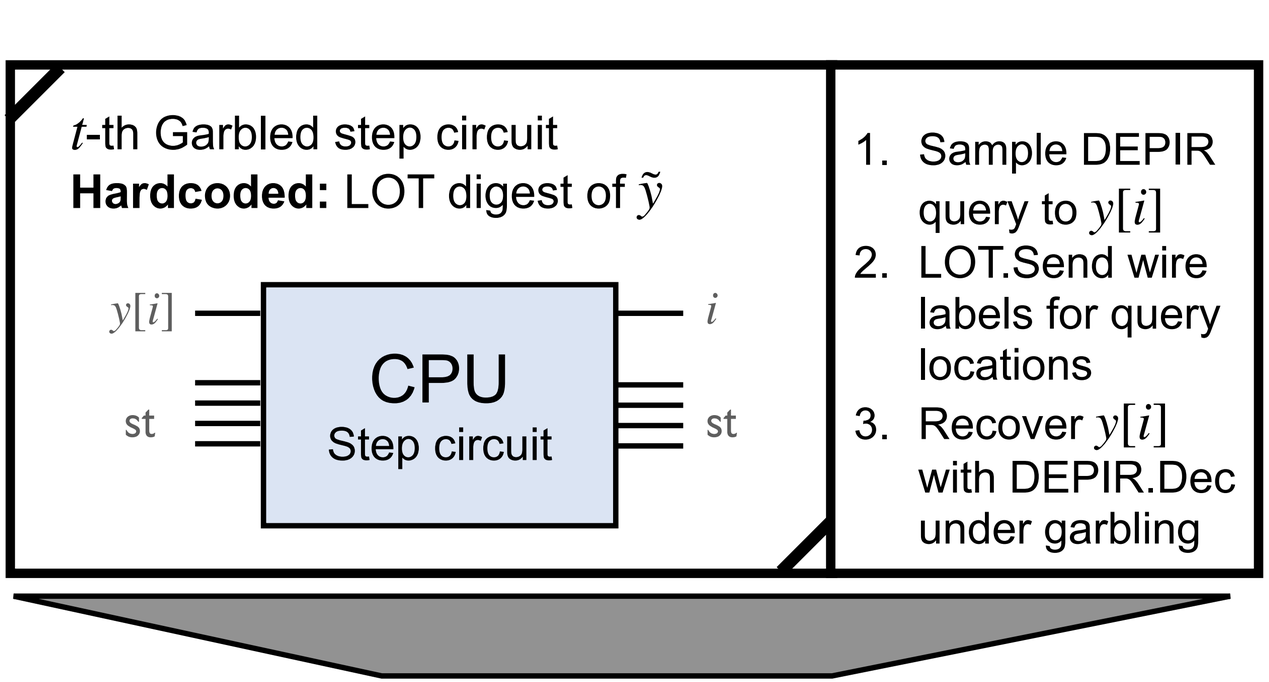
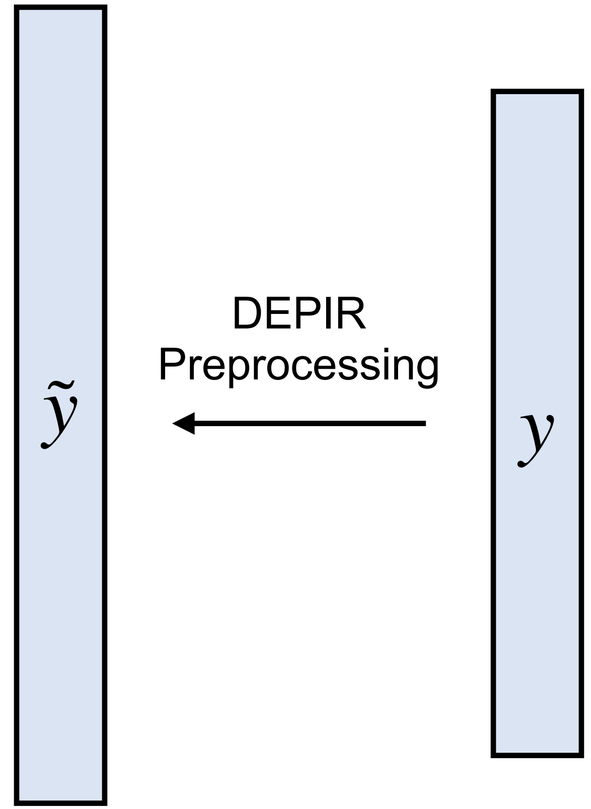


$\mathsf{GCGen}_{\mathsf{dig}_{\tilde{y}}}(t)$



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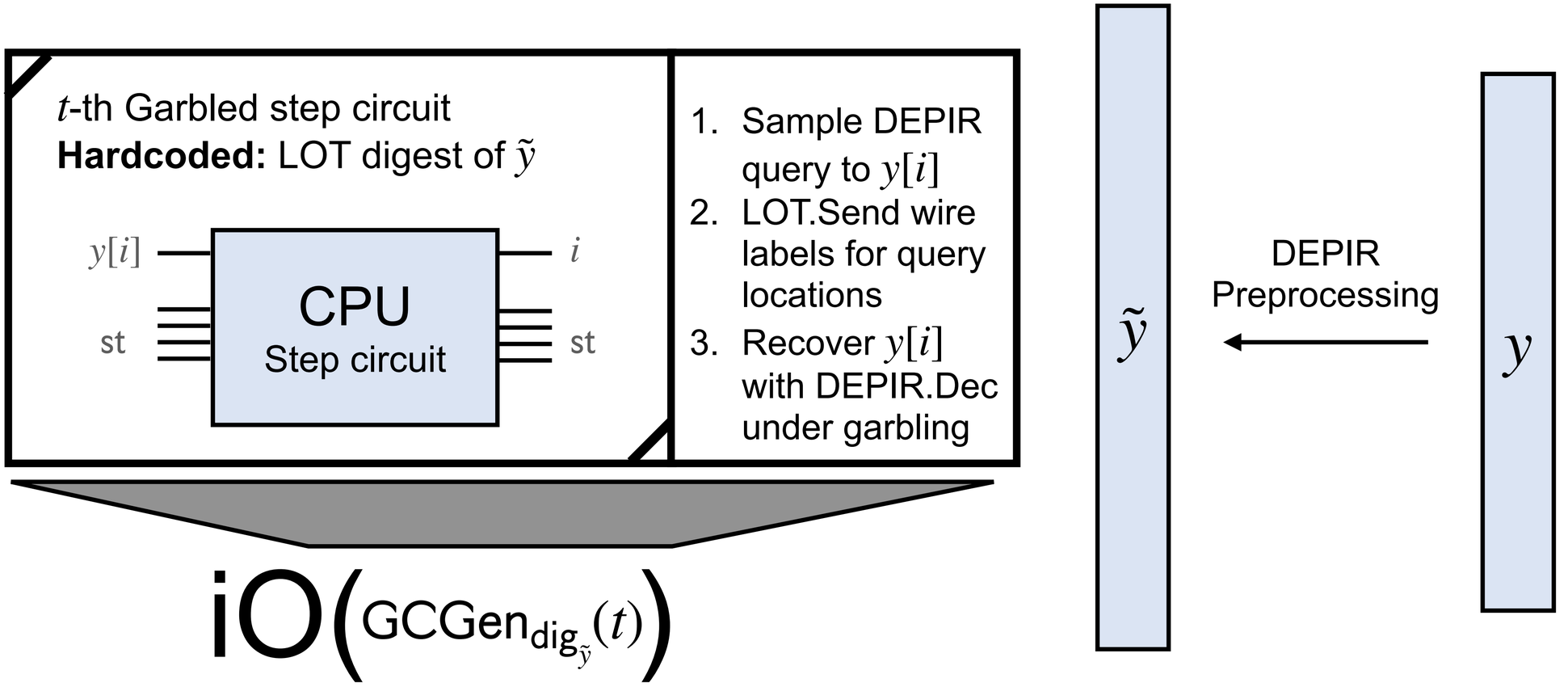

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Thank you!

eprint: 2024/068