# Constant-Size zk-SNARKs in ROM from Falsifiable Assumptions

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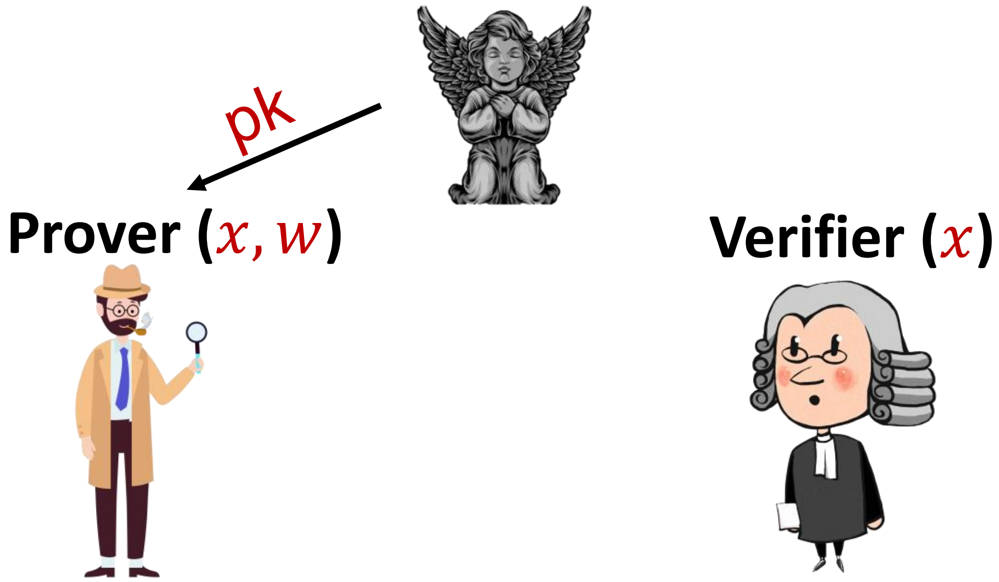

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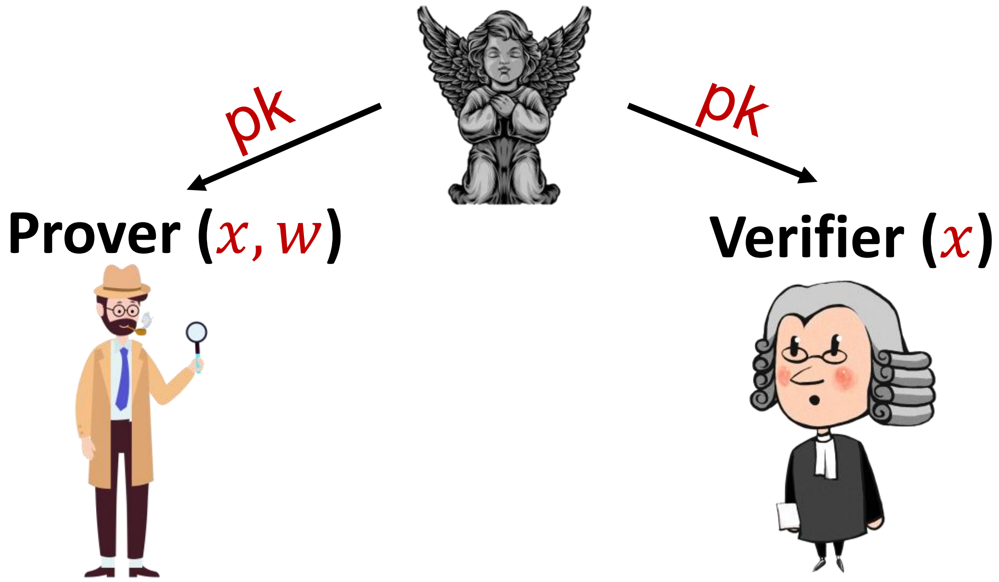

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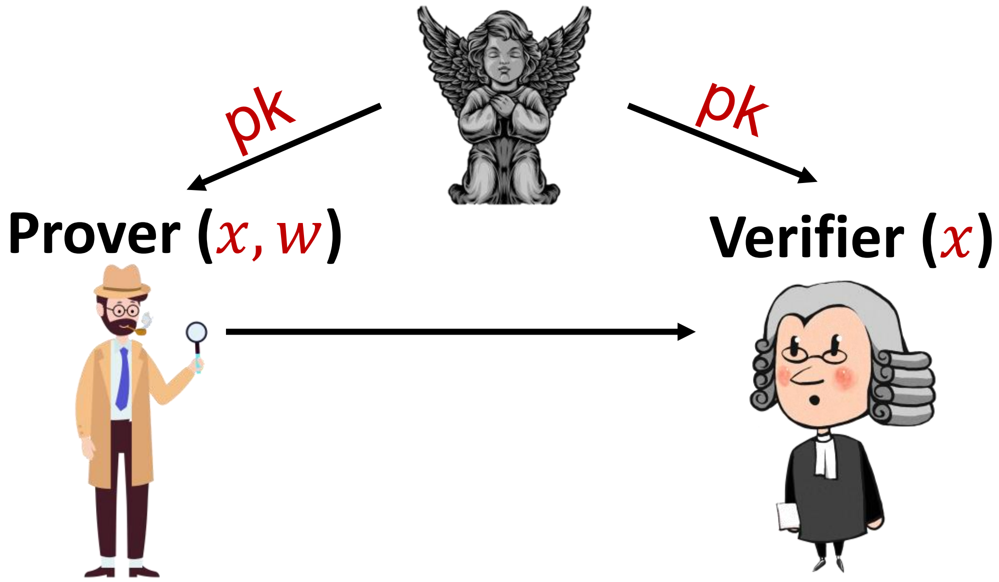

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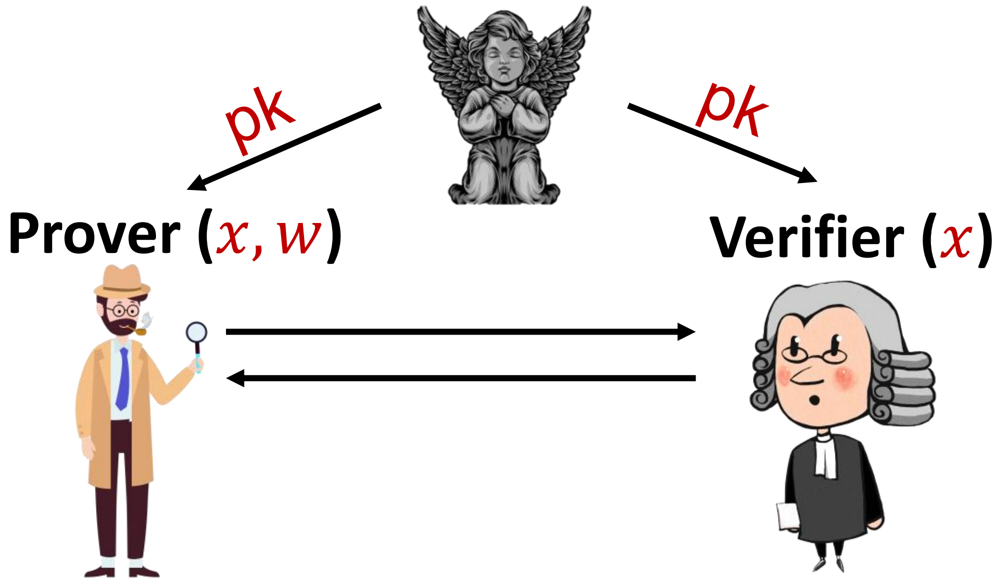

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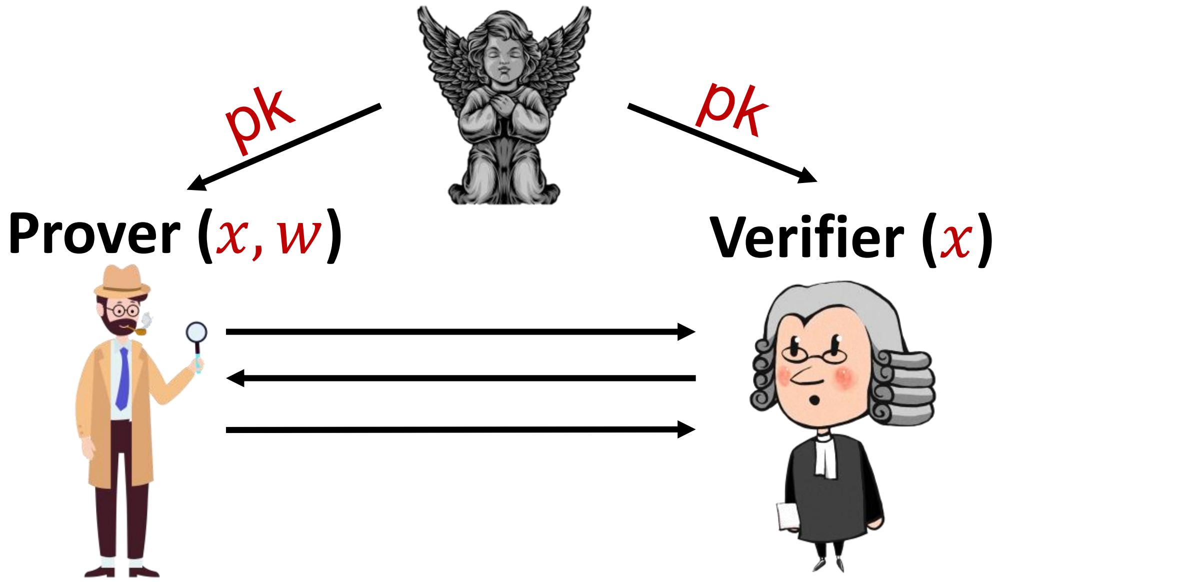

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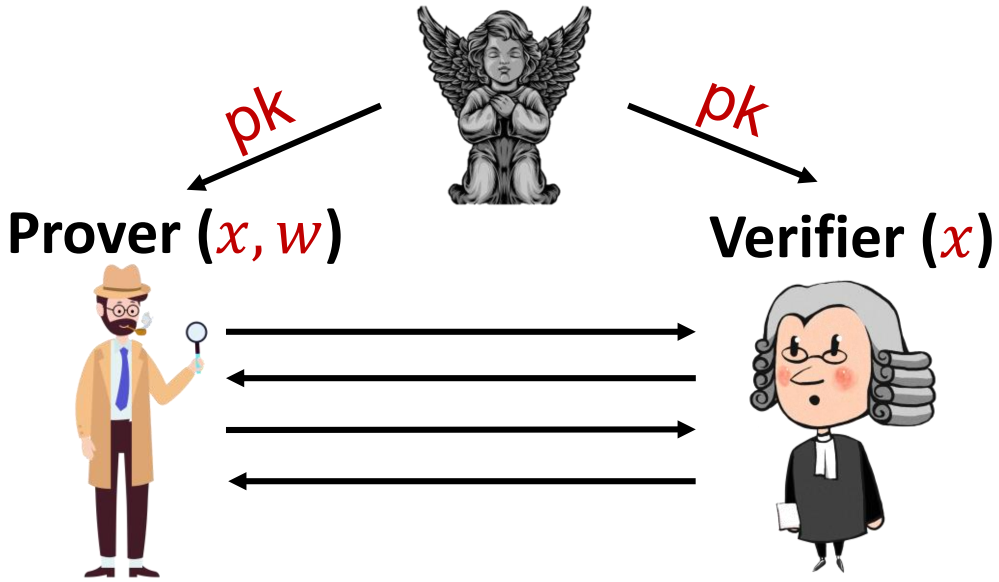

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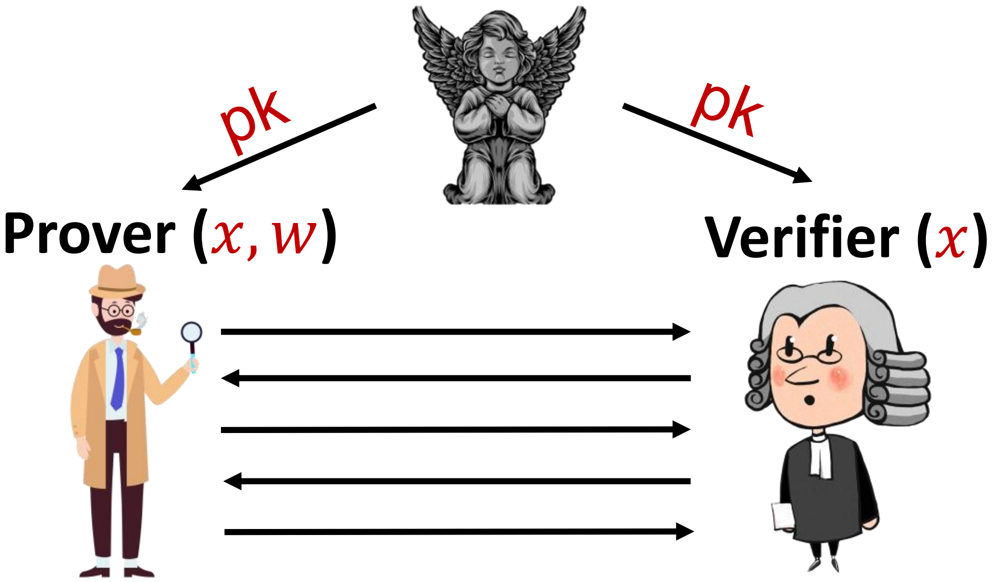

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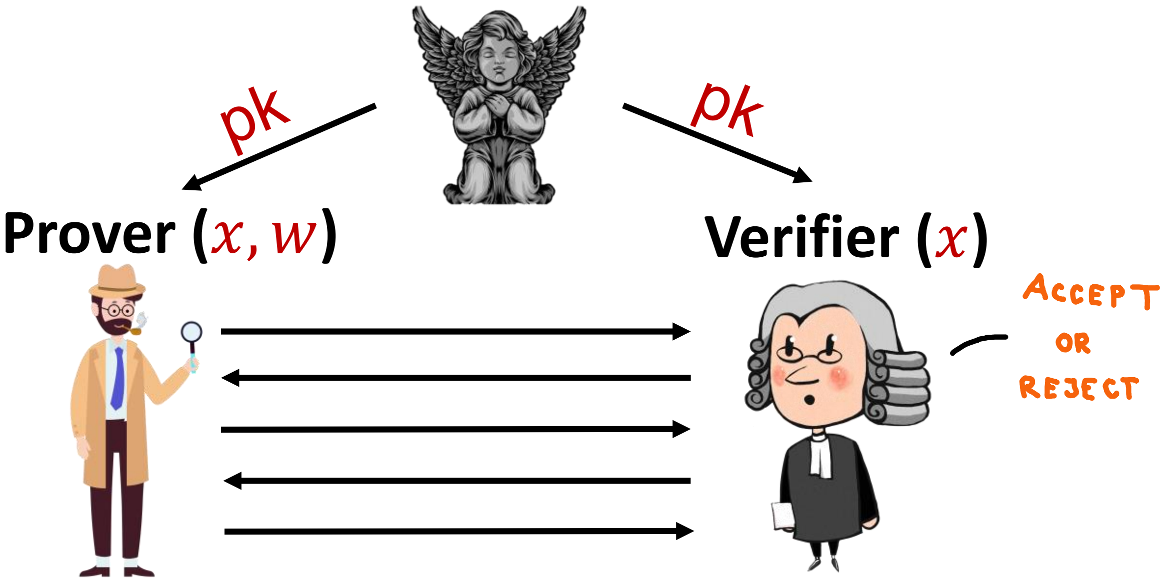

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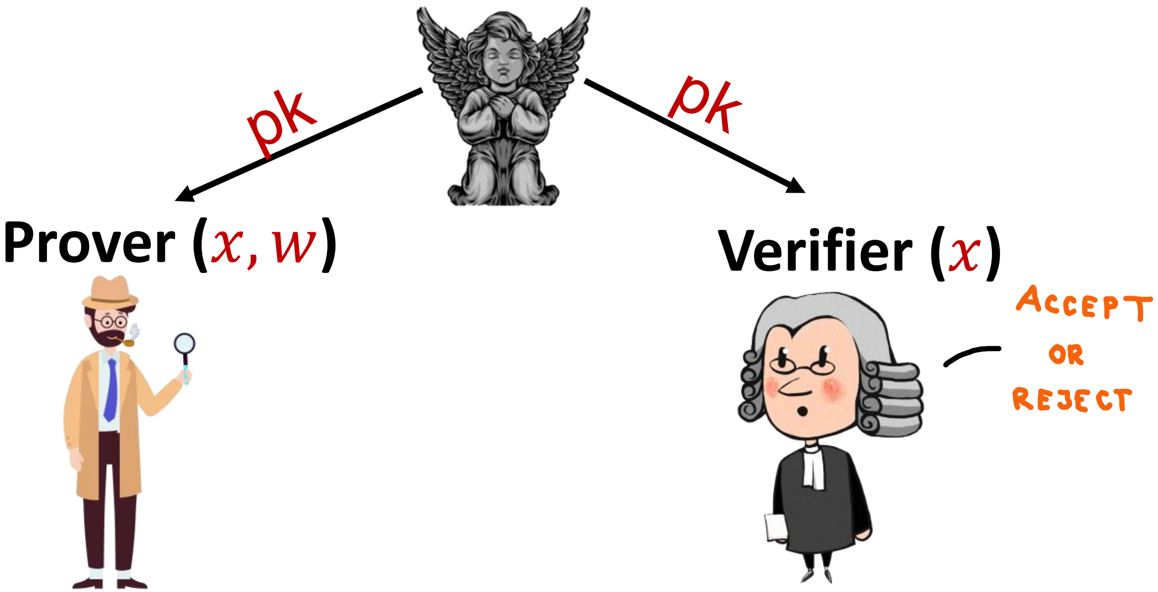

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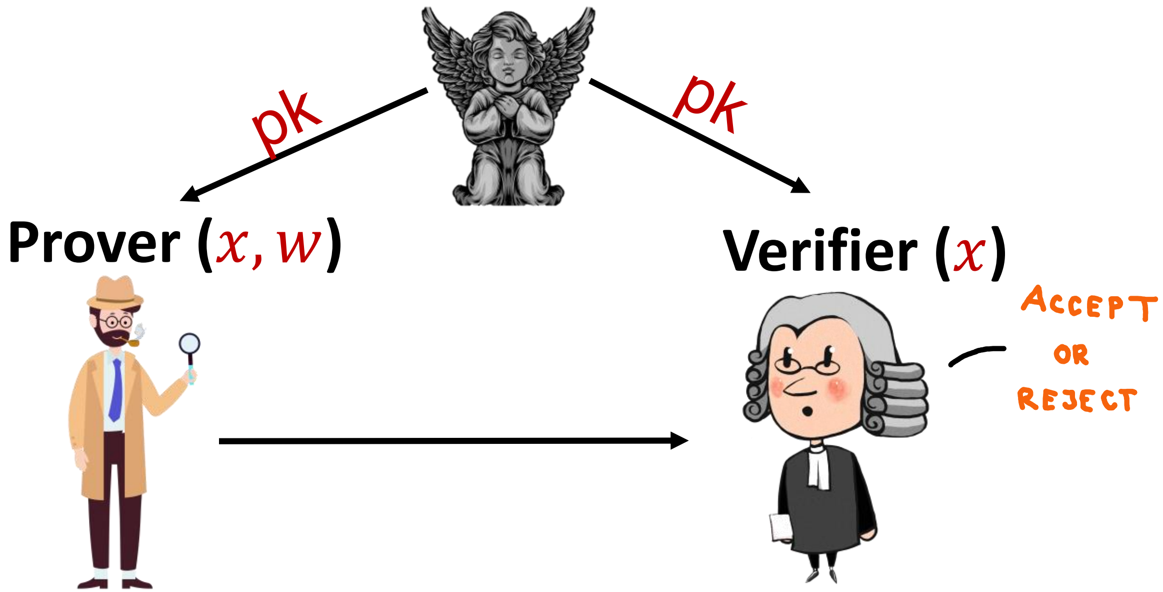
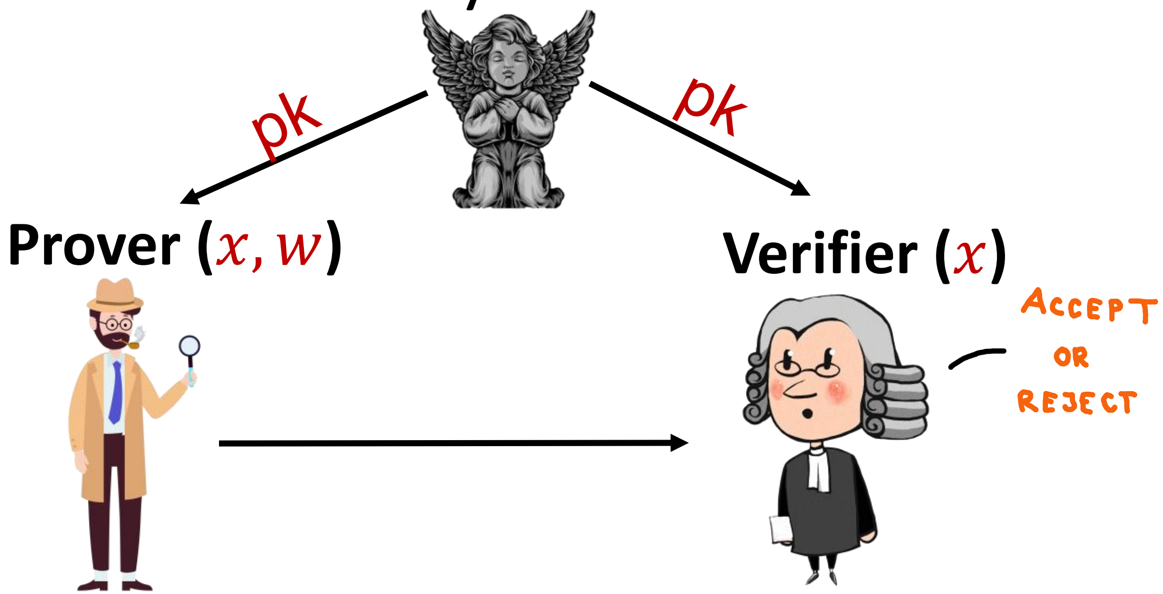
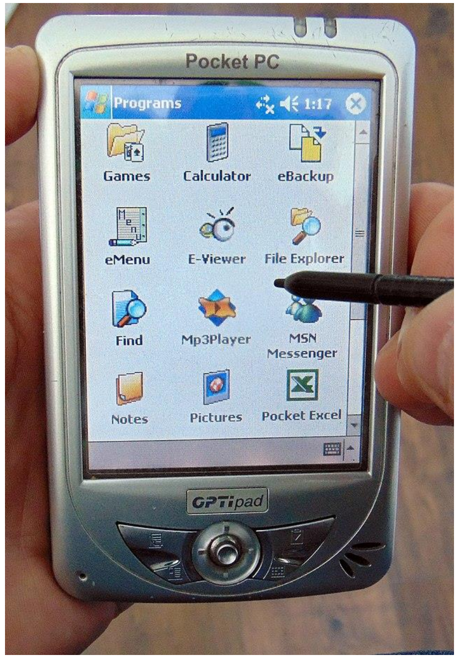


# SNARK

- Succinct Non-interactive Argument of Knowledge
  - **Succinct:** argument size is sublinear in $w$
  - **Non-interactive:** single message from Prover to Verifier
  - **Argument of Knowledge:** prover knows $w$ if verifier accepts (formally: $w$ can be efficiently extracted in the security proof)



# Many Applications



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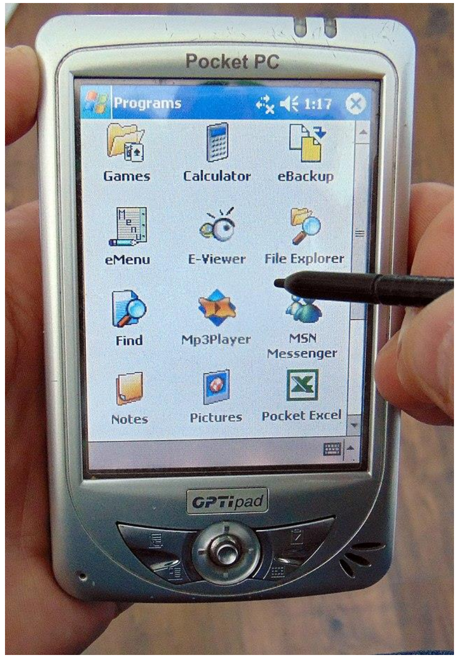
- ❑Verifiable outsourced computation



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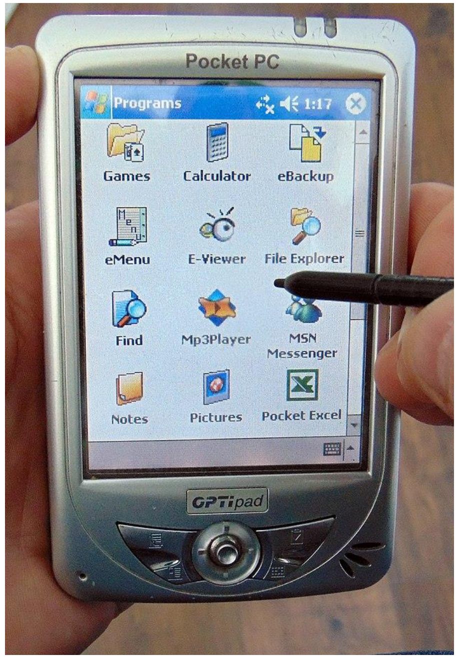

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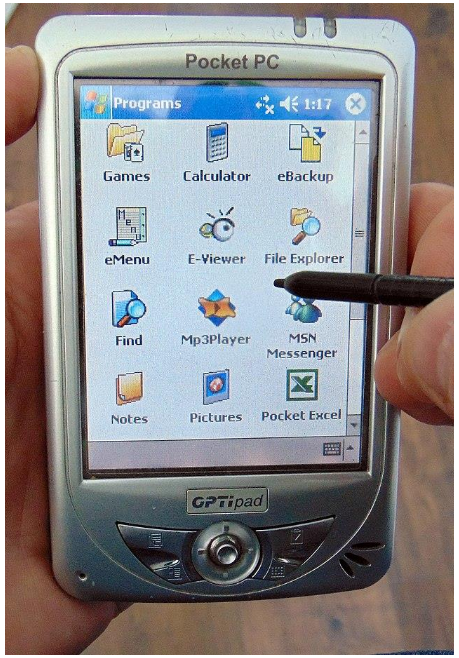

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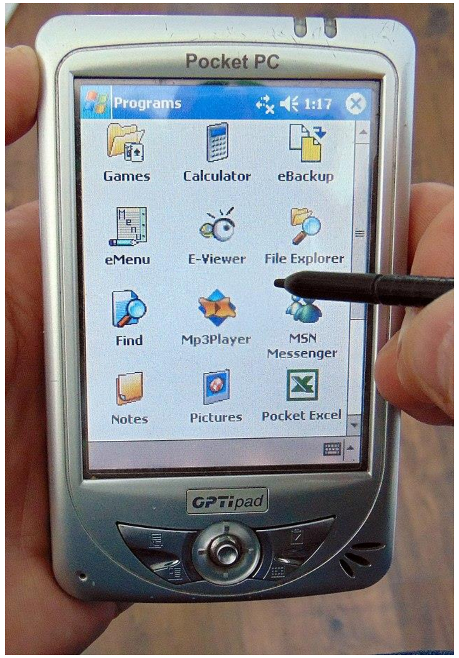

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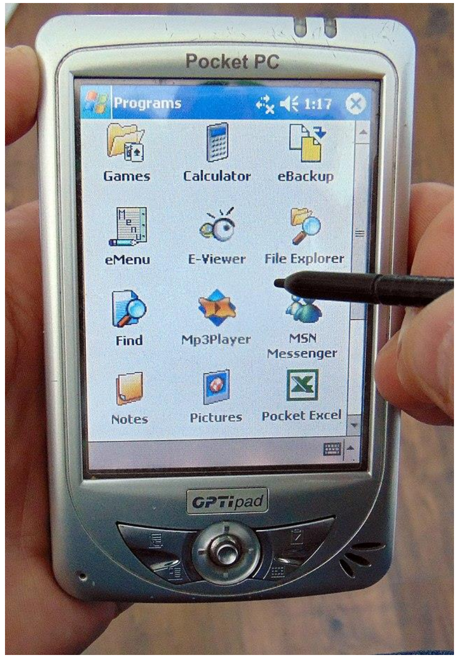

# Recipe for SNARKs

**<u>Ingredient 1</u>**: Polynomial Interactive Oracle Proof (PIOP)

**Prover ($x, w$)**

**Verifier ($x$)**

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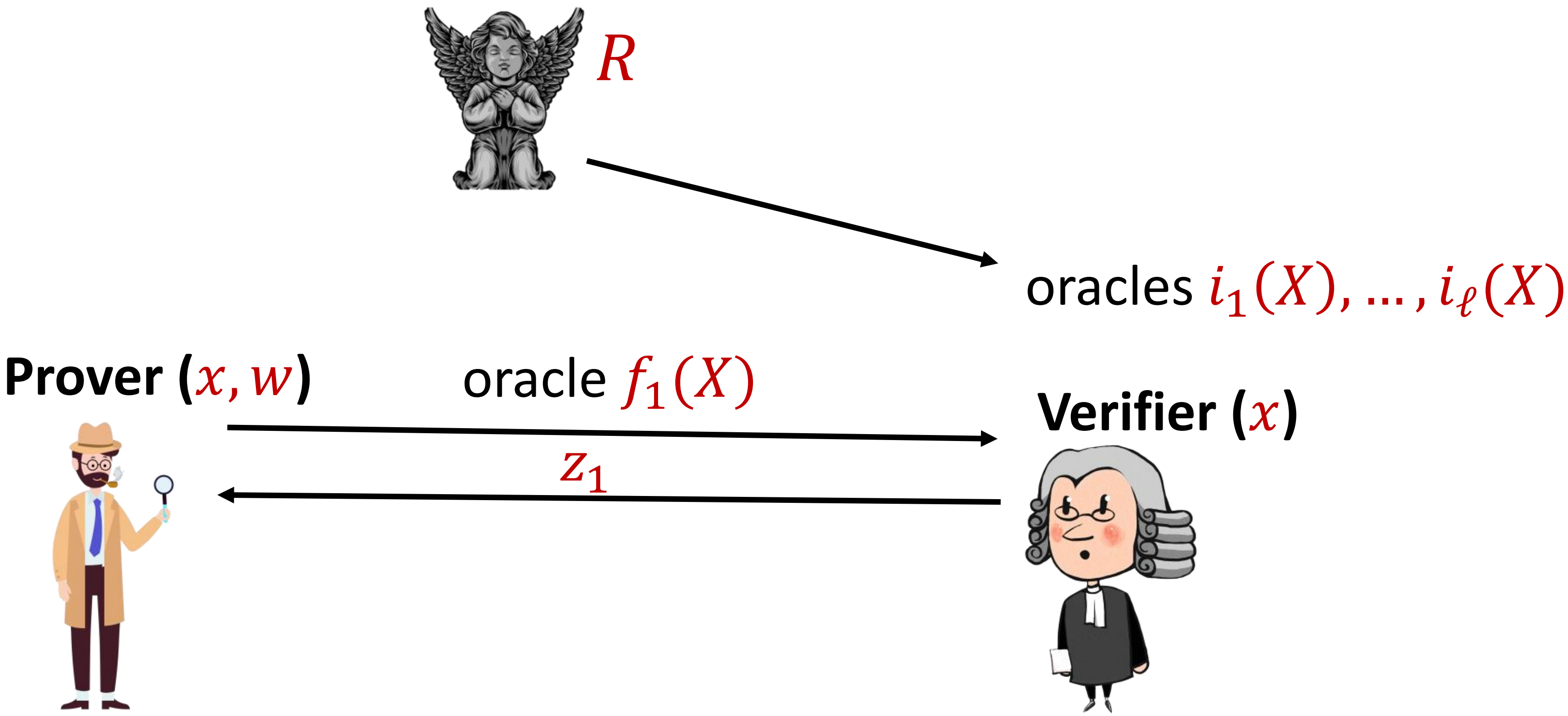

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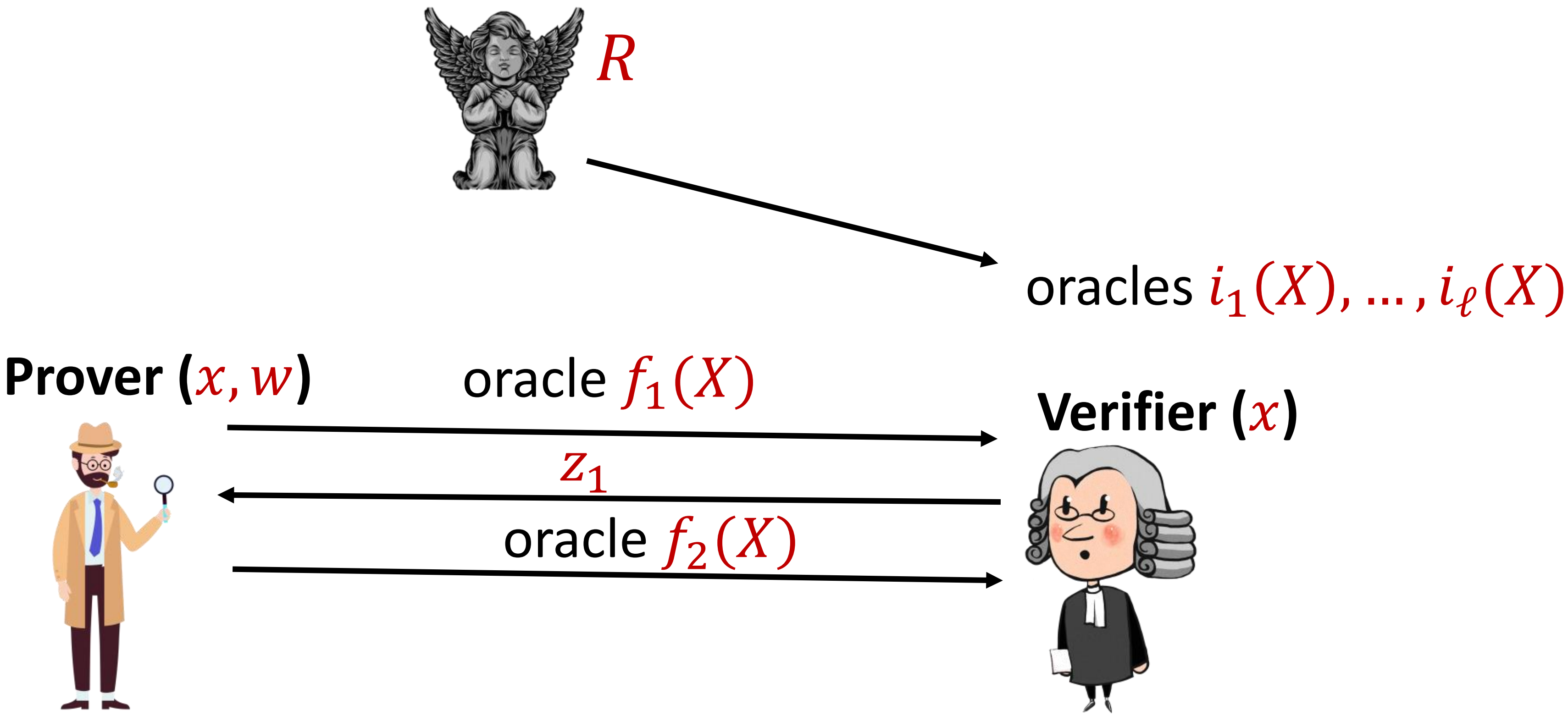

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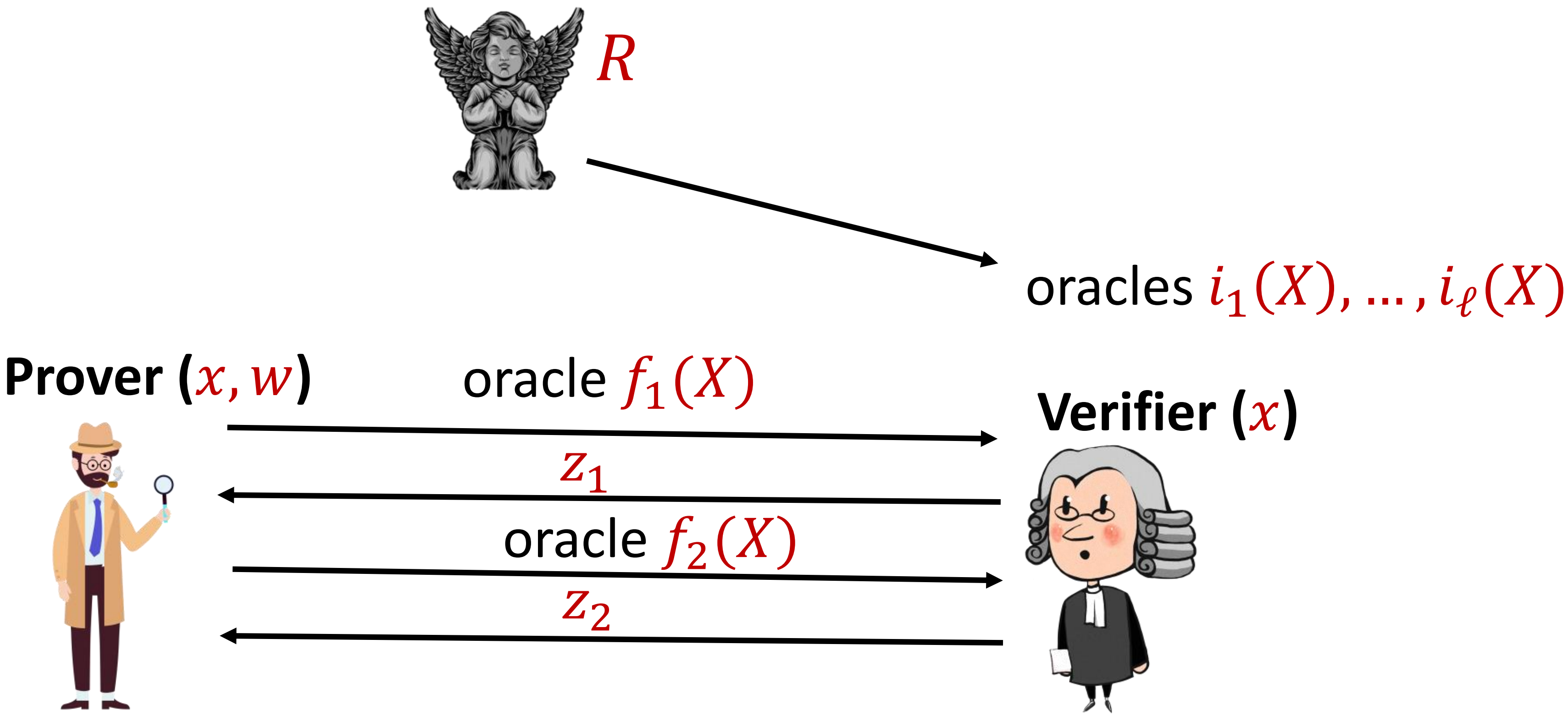

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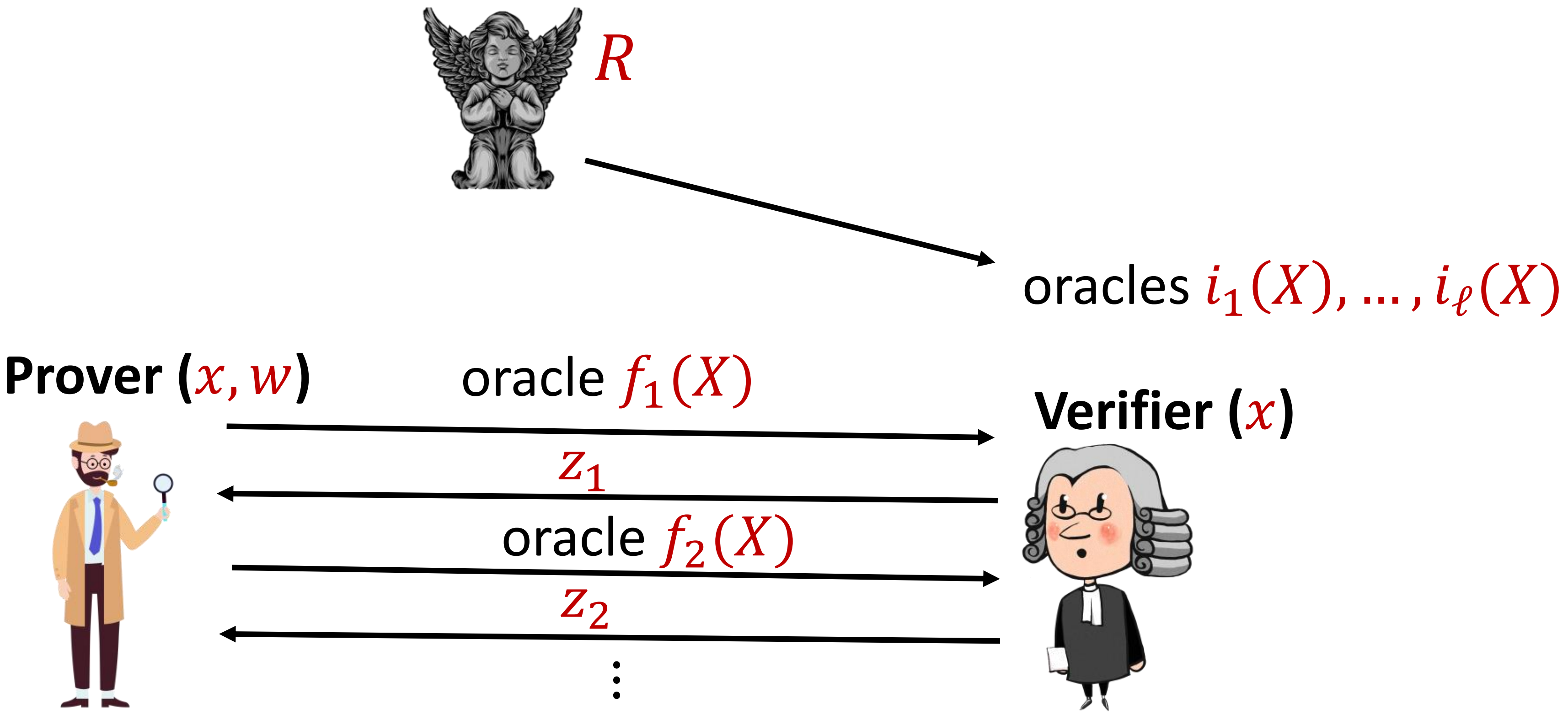

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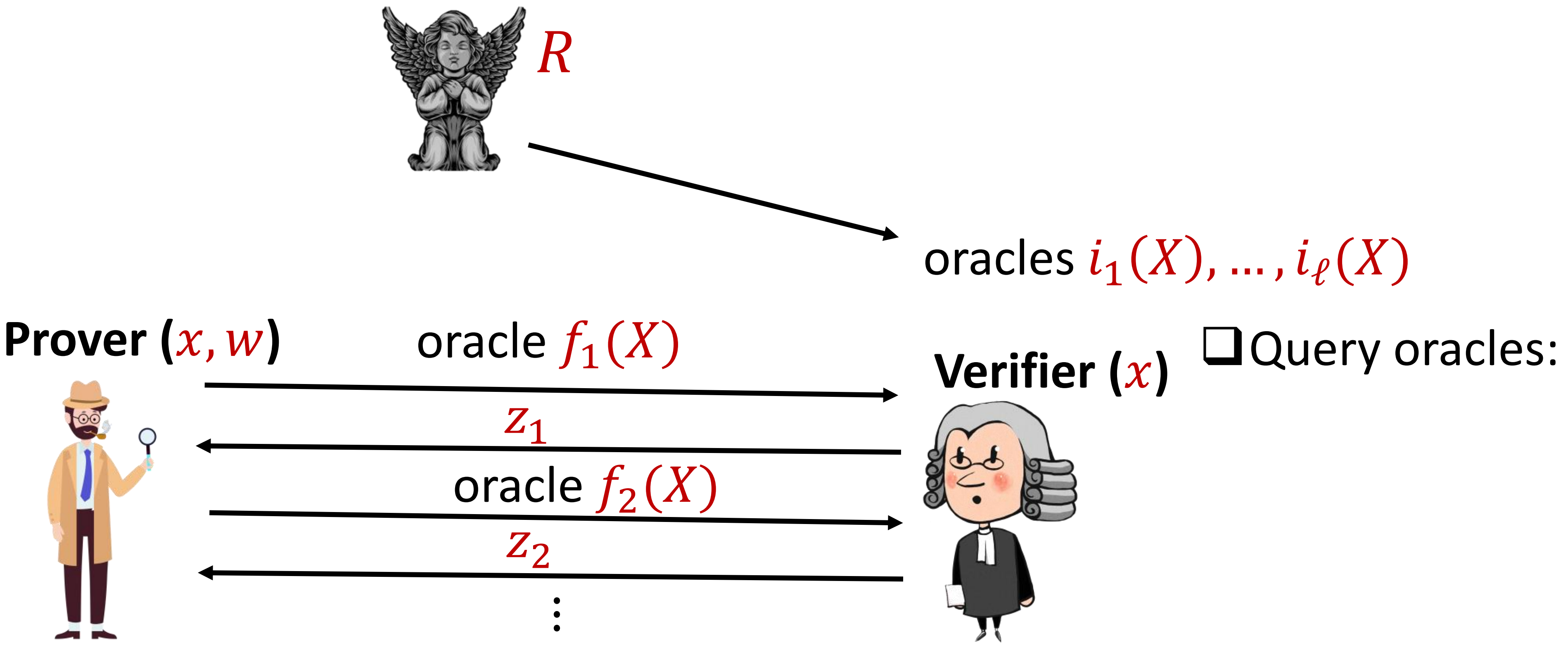

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oracles $i_1(X), \dots, i_\ell(X)$

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oracle $f_1(X)$

$z_1$

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⋮

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- ❑ Query oracles:
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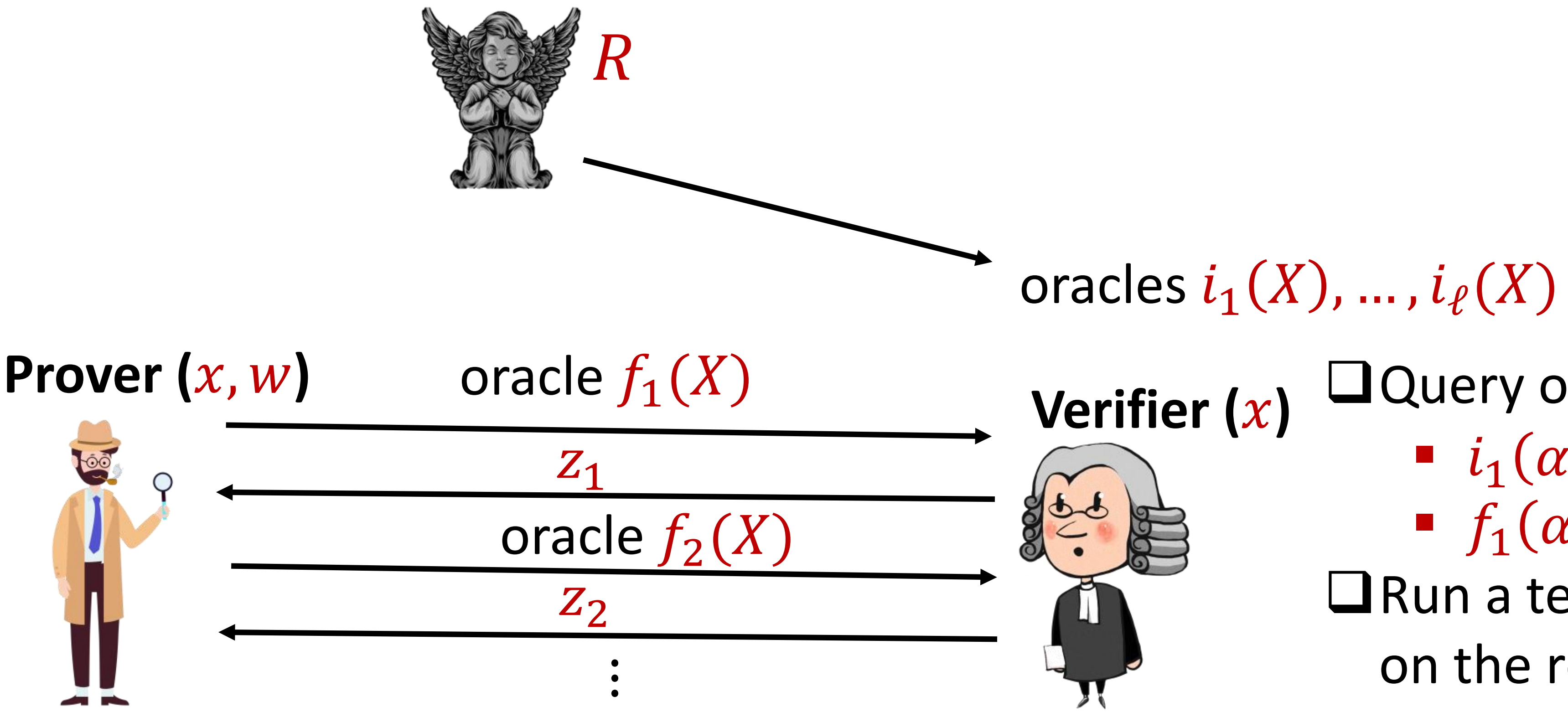


- Query oracles:
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- Run a testing algorithm on the responses

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- PIOP is not succinct
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**Succinctness:** Size of $\mathrm{C}$ and $\pi$ is sublinear in $n$

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Adversary

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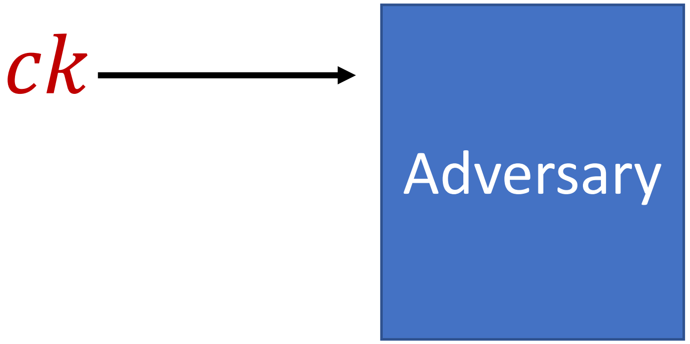

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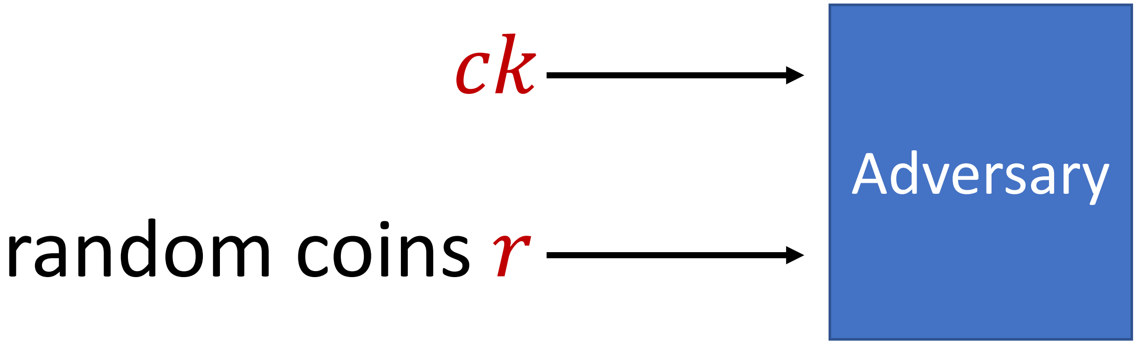

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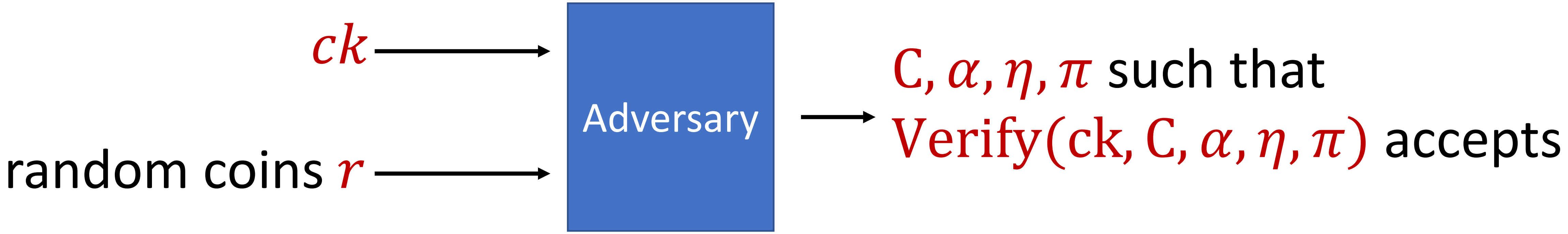

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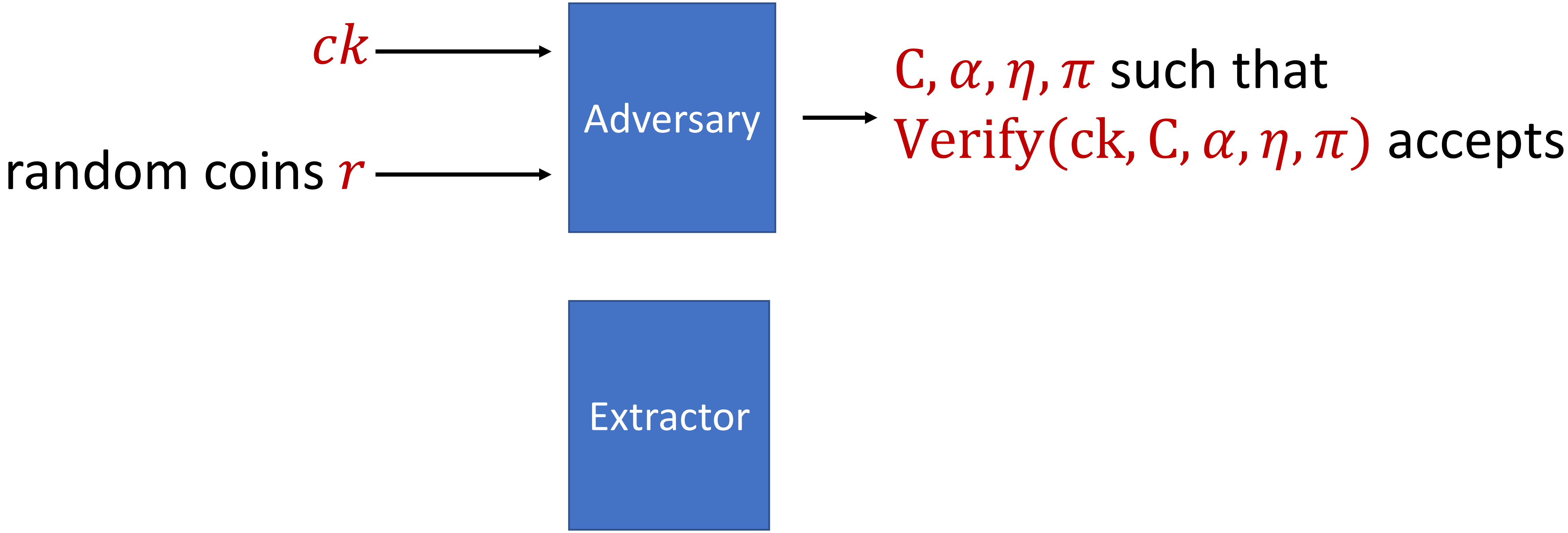

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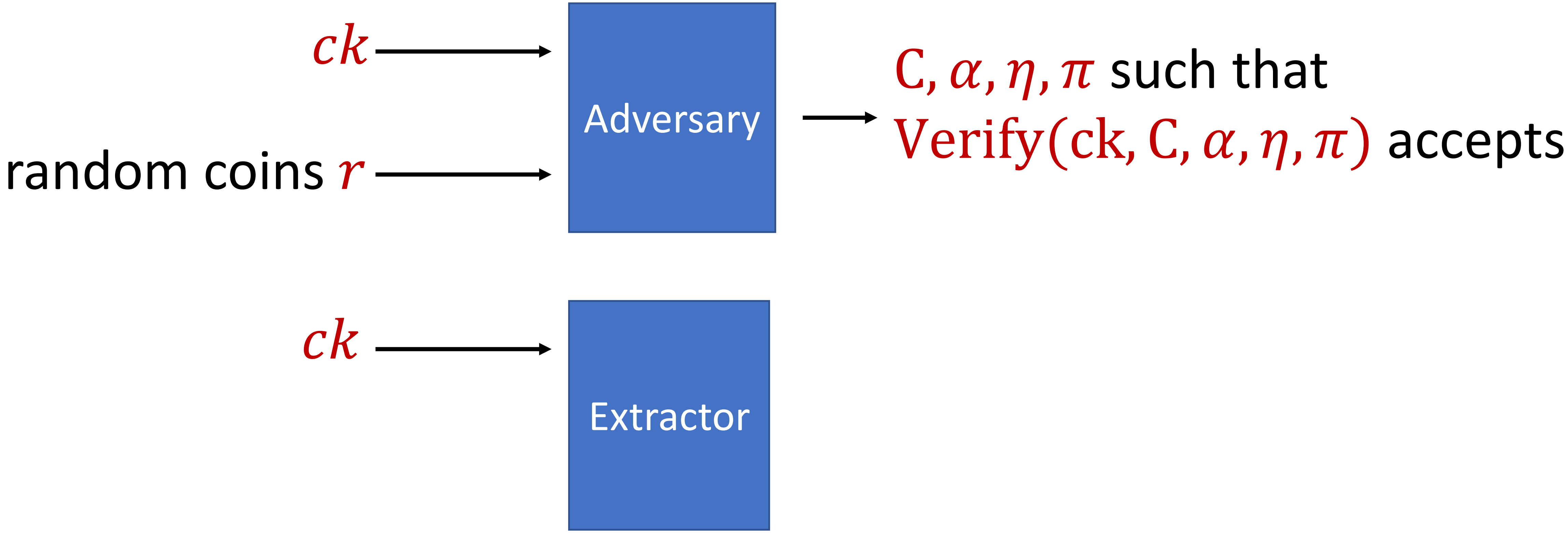

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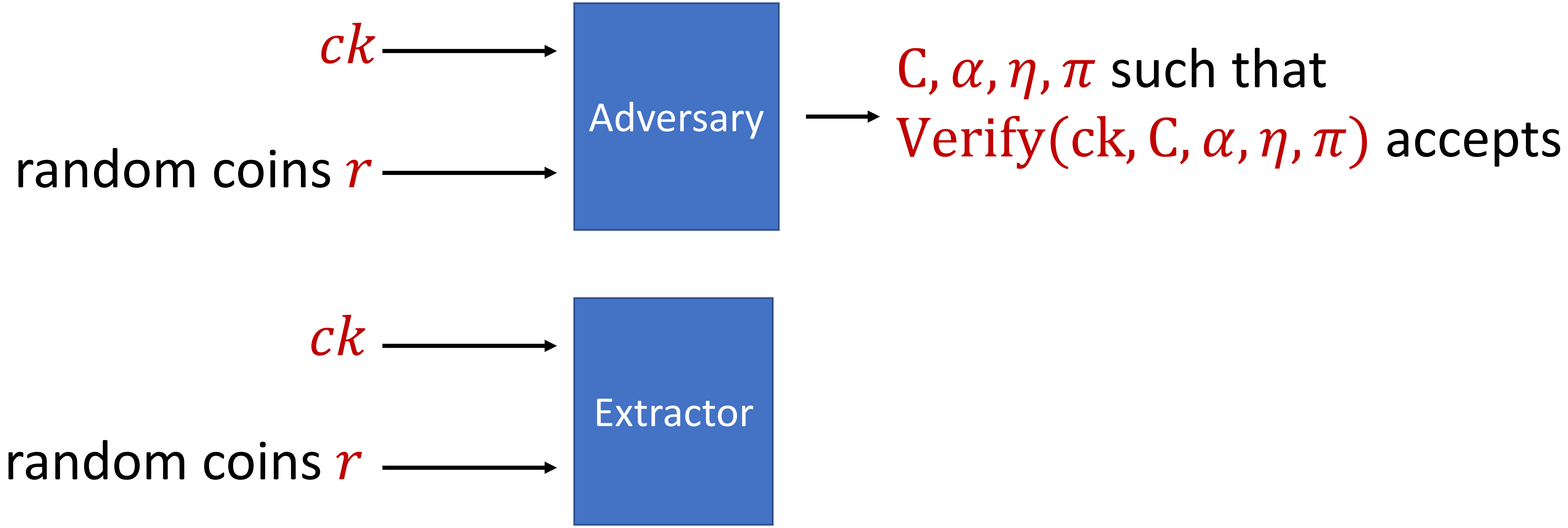

# Non-black-box Extractability

$ck \longrightarrow$ Adversary

random coins $r \longrightarrow$ Adversary

Adversary $\longrightarrow$ $\mathrm{C}, \alpha, \eta, \pi$ such that $\mathrm{Verify}(\mathrm{ck}, \mathrm{C}, \alpha, \eta, \pi)$ accepts

$ck \longrightarrow$ Extractor

random coins $r \longrightarrow$ Extractor

Extractor $\longrightarrow$ $f(X)$ such that $\mathrm{Com}\big(\mathrm{ck}, f(\mathrm{X})\big) = \mathrm{C}$ and $\deg(f) \leq n$

# Mix PIOP and Poly-Com

$R$

**Prover** $(x, w)$

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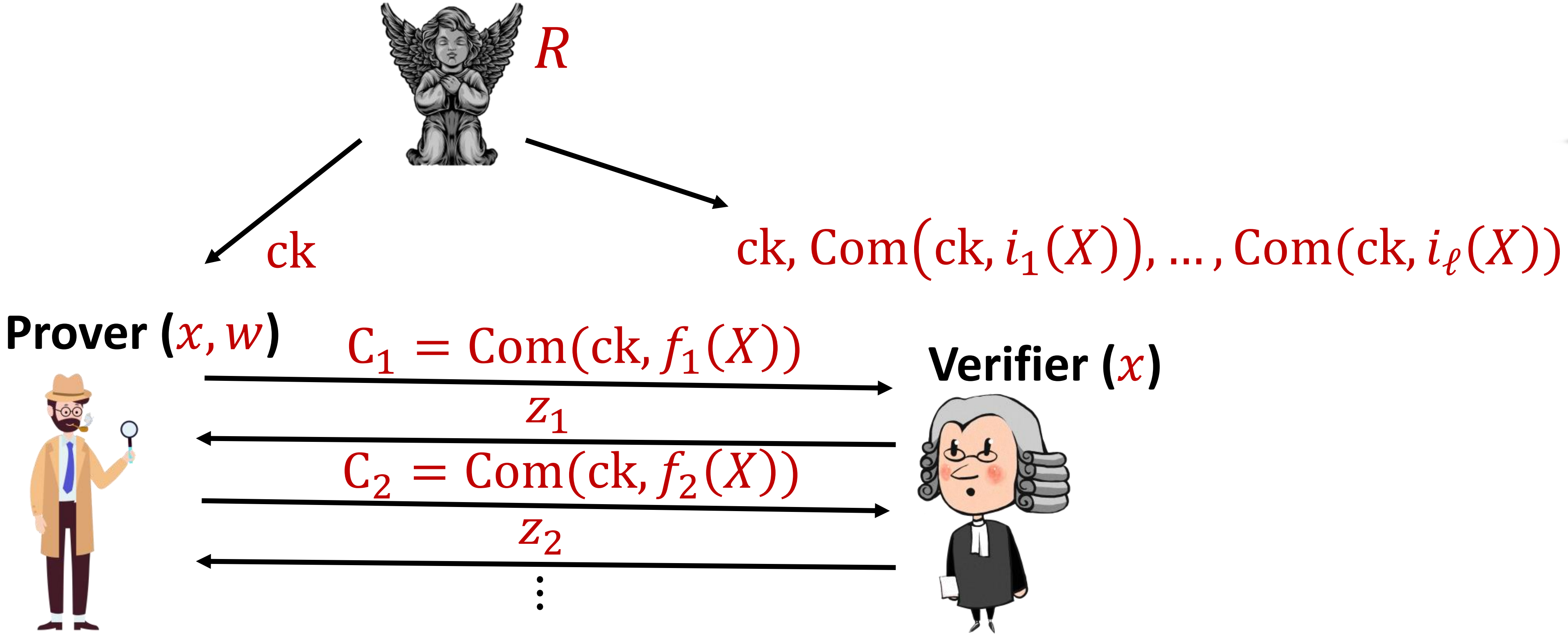

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$\eta_1 = i_1(\alpha_i), \pi_1$

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# Mix PIOP and Poly-Com

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**Ingredient 3**: Fiat-Shamir for non-interactivity

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  - We show extractability with <u>rewinding</u> under a relatively standard <u>falsifiable assumption</u>

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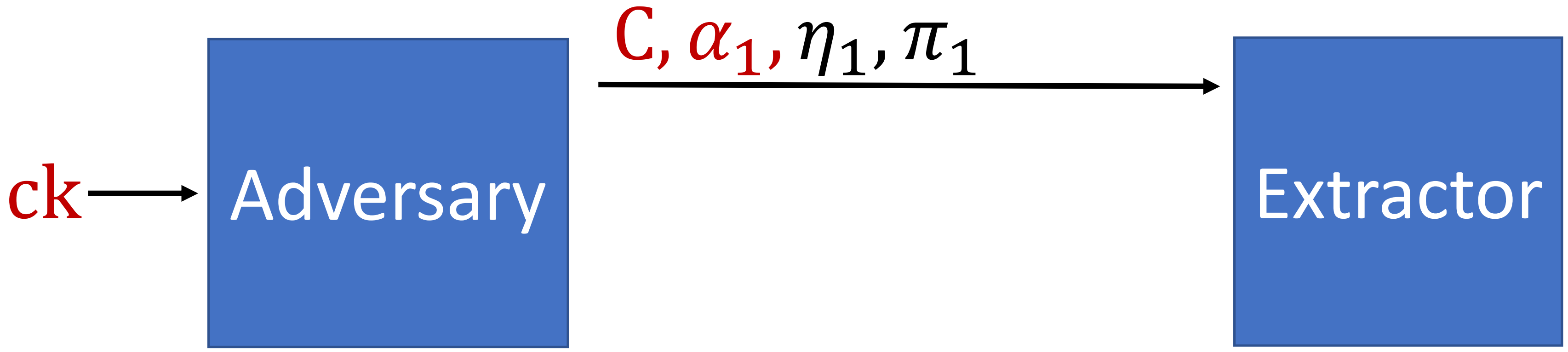

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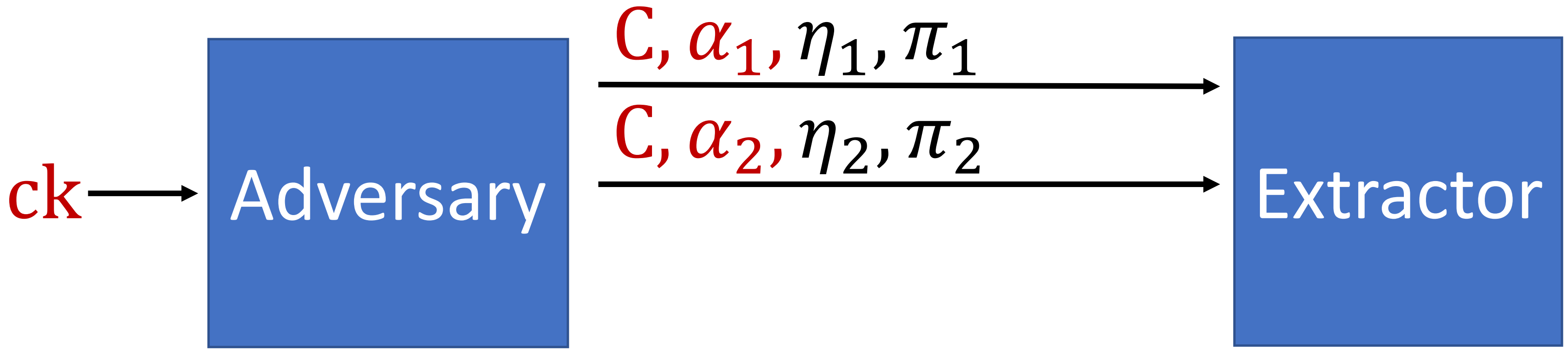

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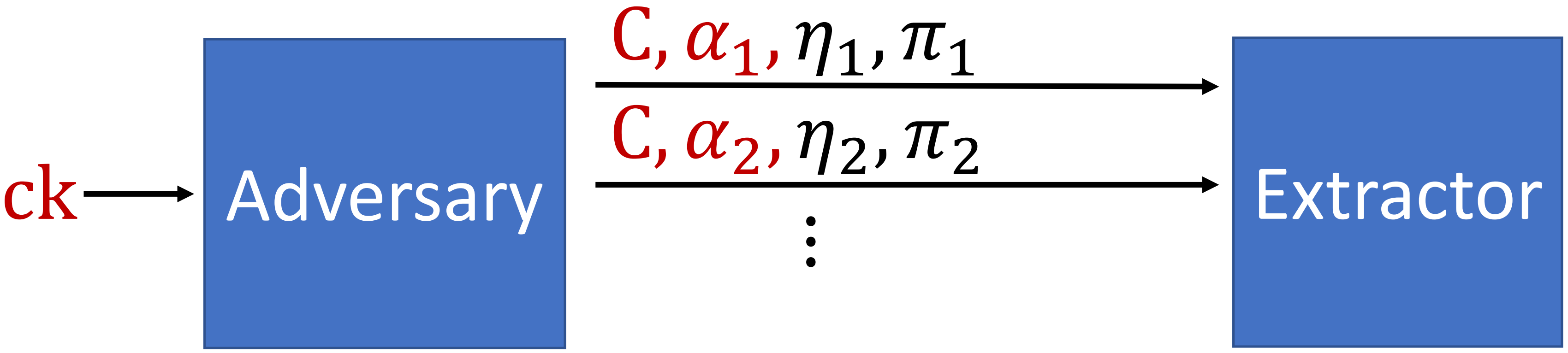

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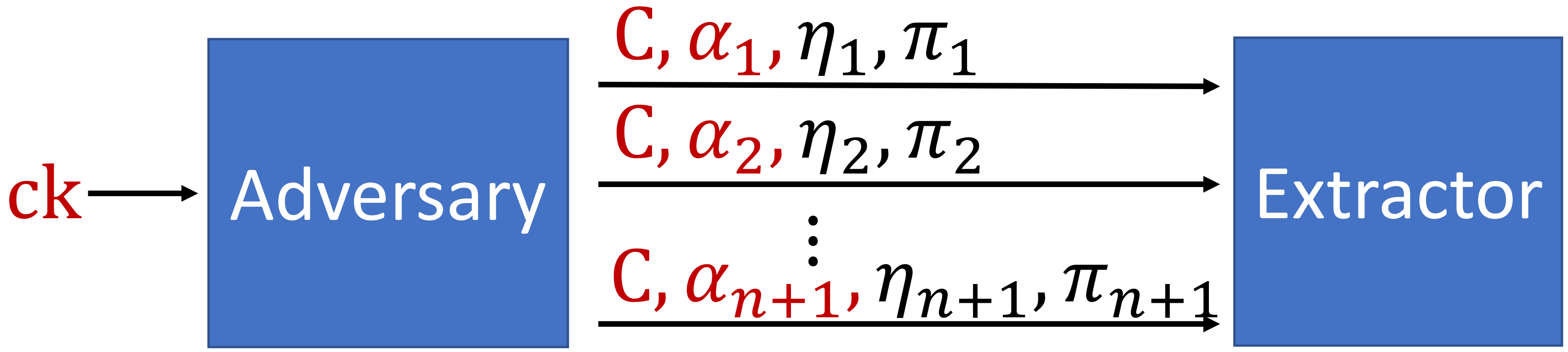

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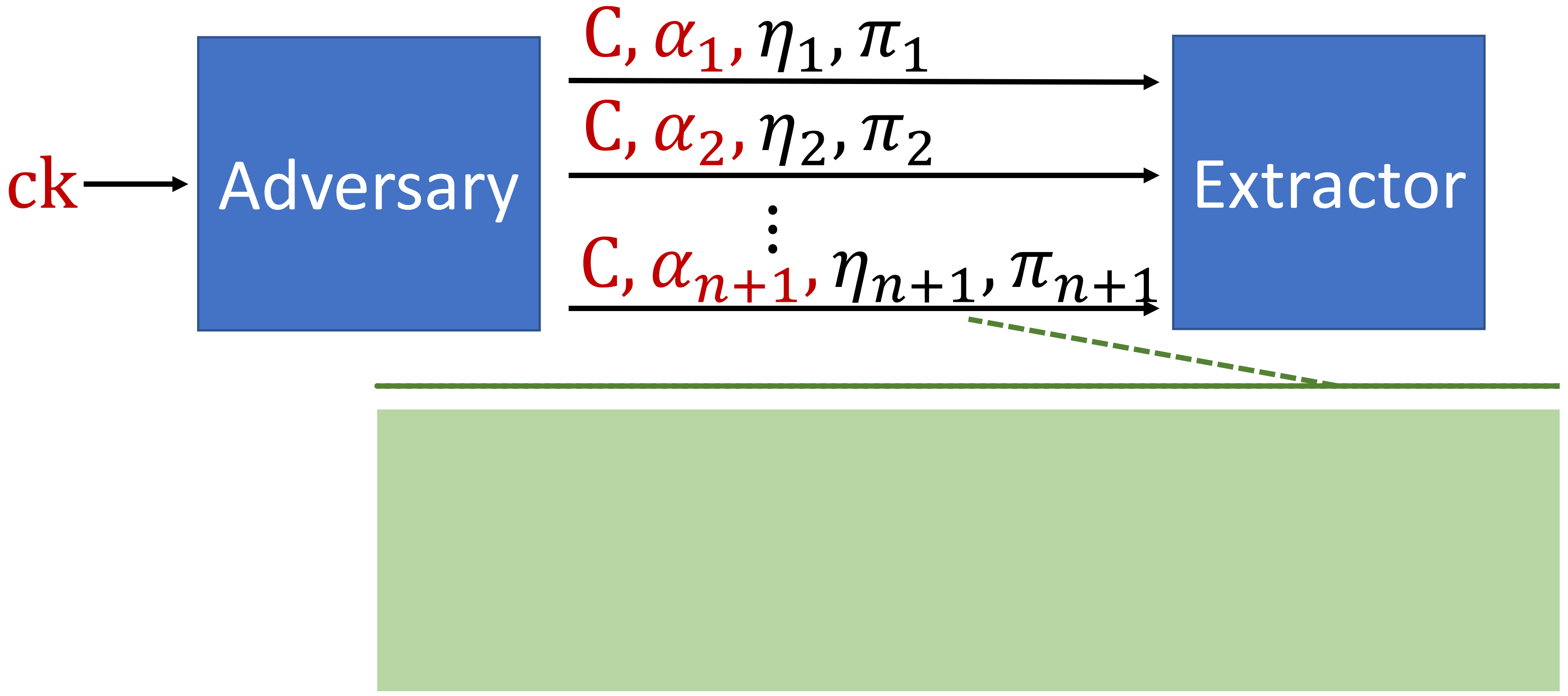

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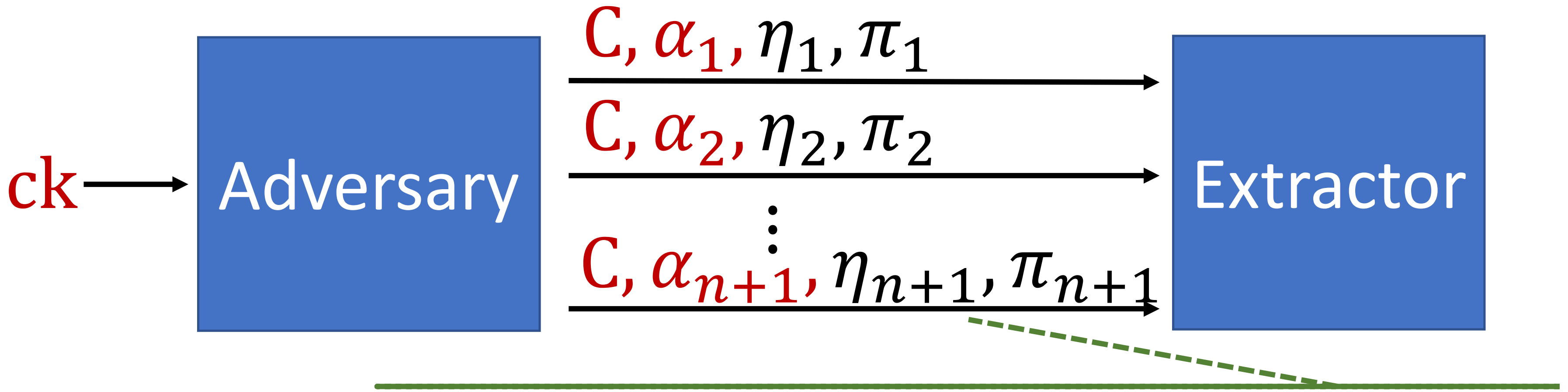


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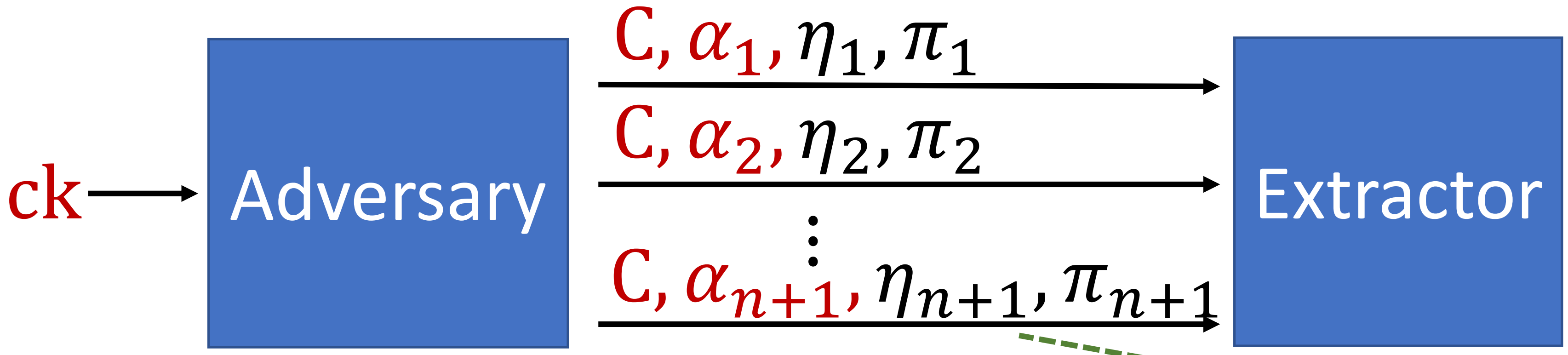


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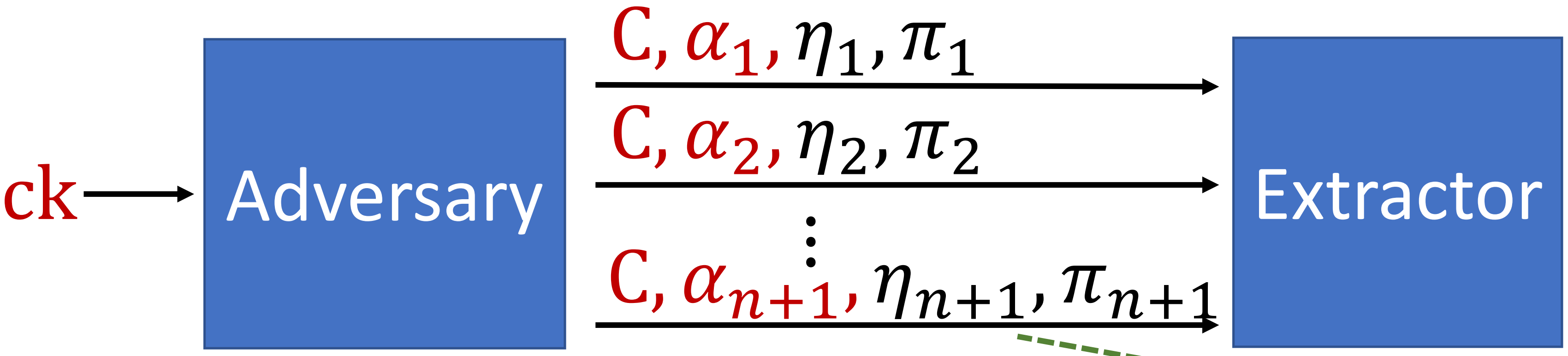


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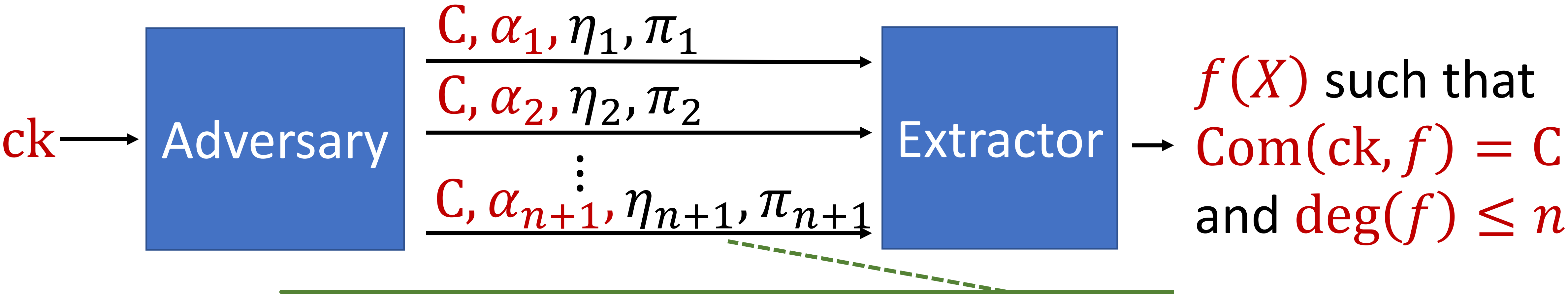


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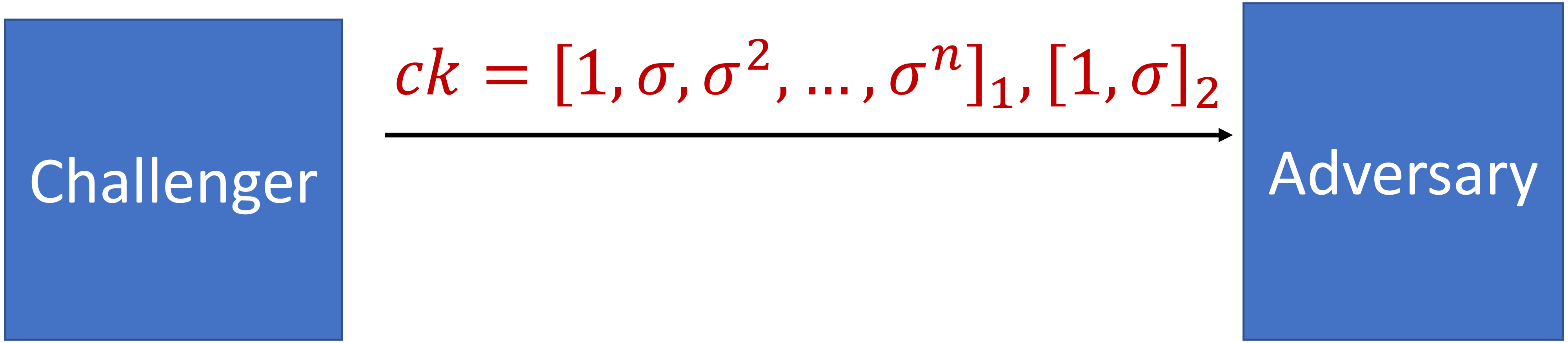

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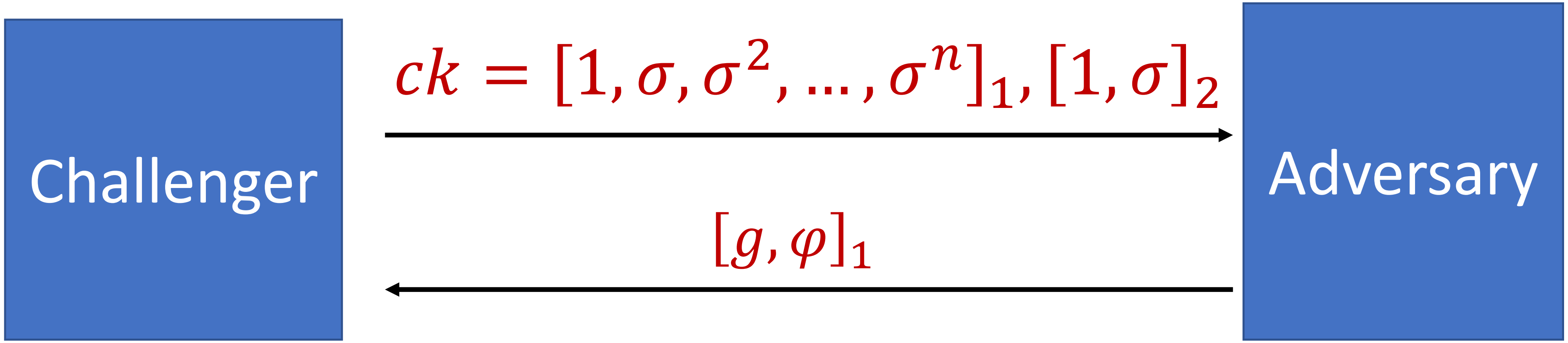

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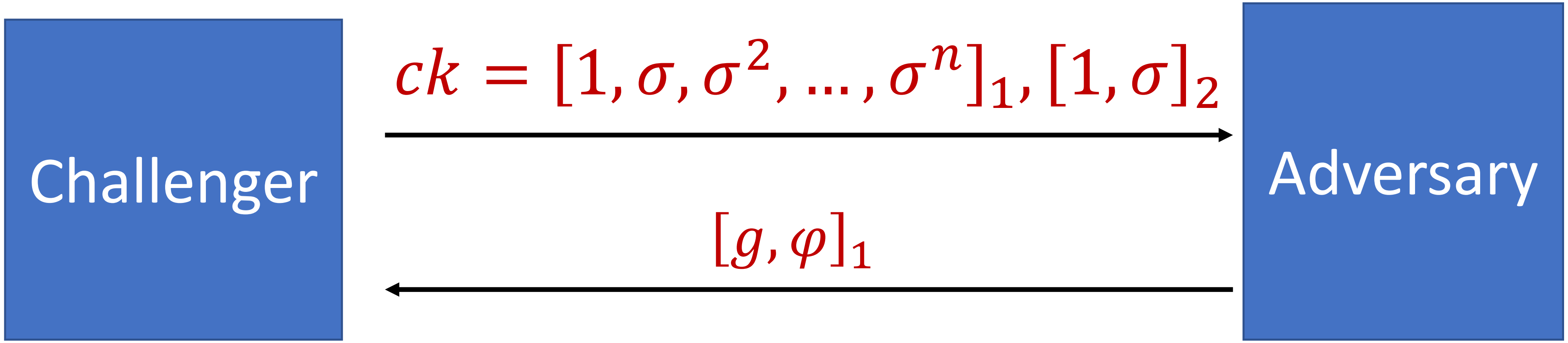


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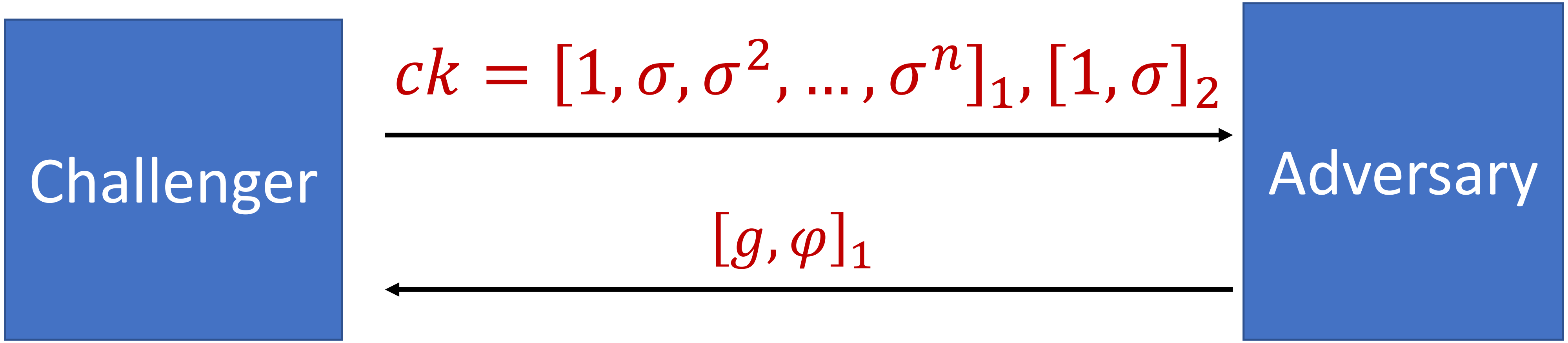


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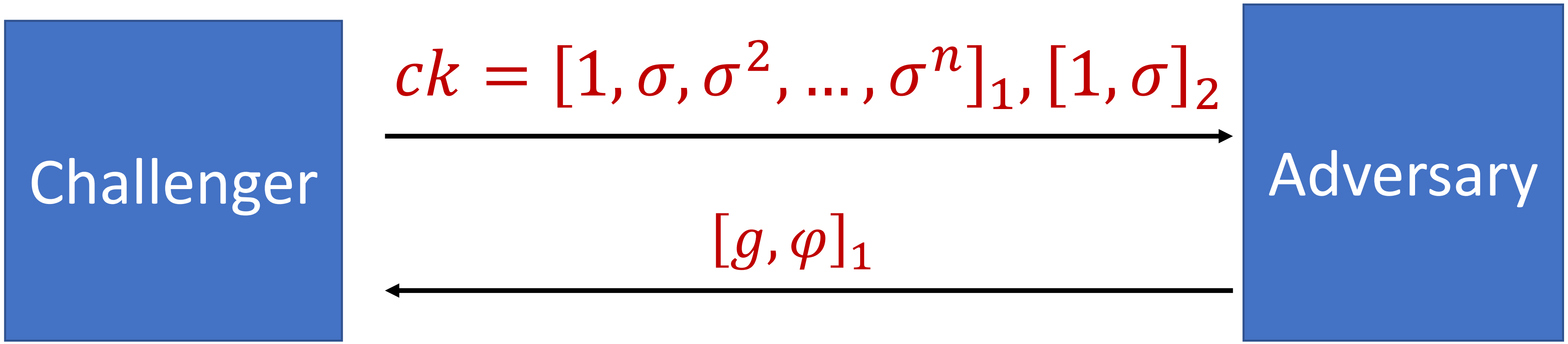


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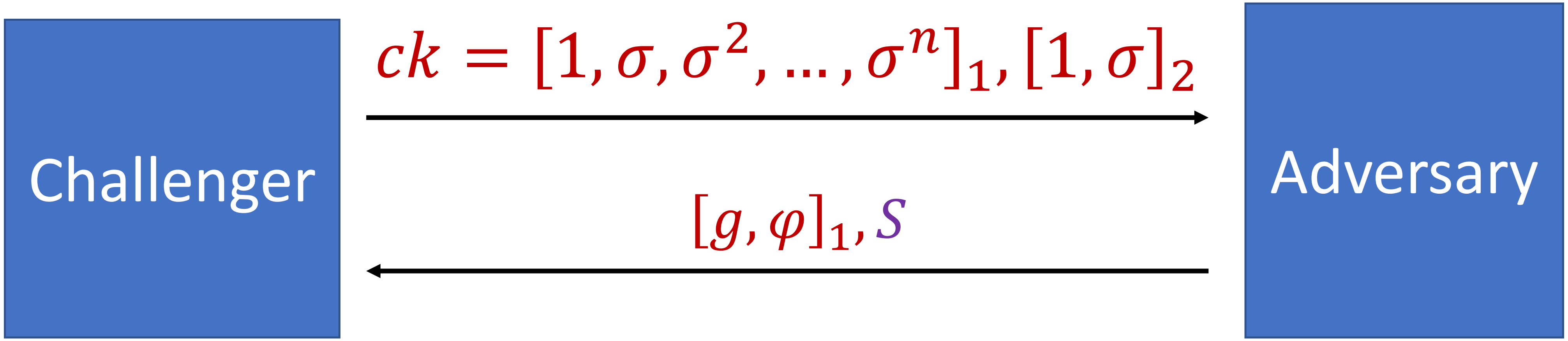

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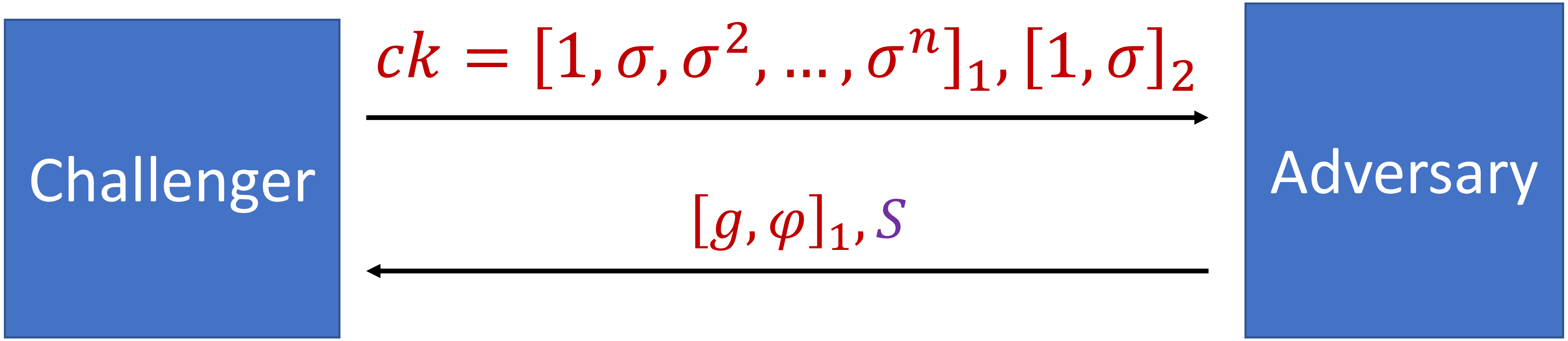


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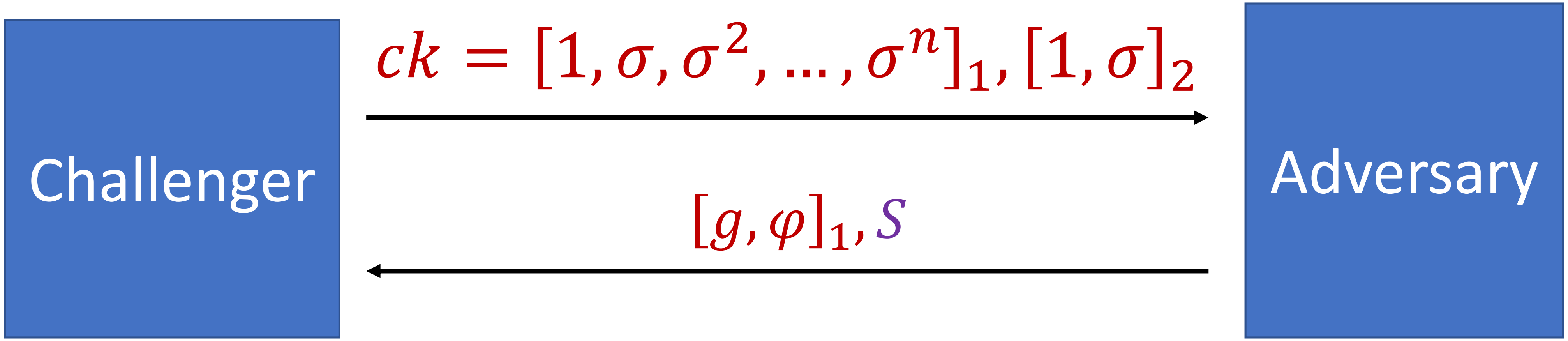


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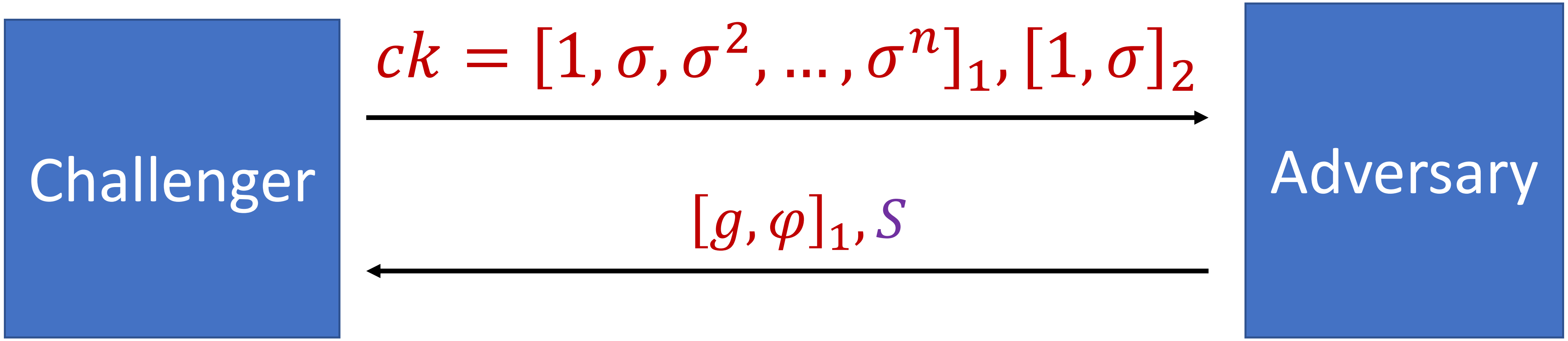


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  - PIOP + black-box extractable polynomial commitment

# More Results

- Special-soundness -> Black-box extractability
  - Rewind the adversary and run with distinct challenges
- Compiler for interactive arguments:
  - PIOP + black-box extractable polynomial commitment
  - Similar to prior compilers

# Consequences

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- KZG is black-box extractable under <u>falsifiable assumption</u>

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- KZG is black-box extractable under falsifiable assumption
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- Constant-size interactive arguments under falsifiable assumption
- Constant-size SNARKs that are secure under falsifiable assumption and random oracle model

Thank you for attention

Questions?