Lower-Bounds on Public-Key Operations in PIR

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CISPA Helmoltz Center for Information Security

University of Waterloo

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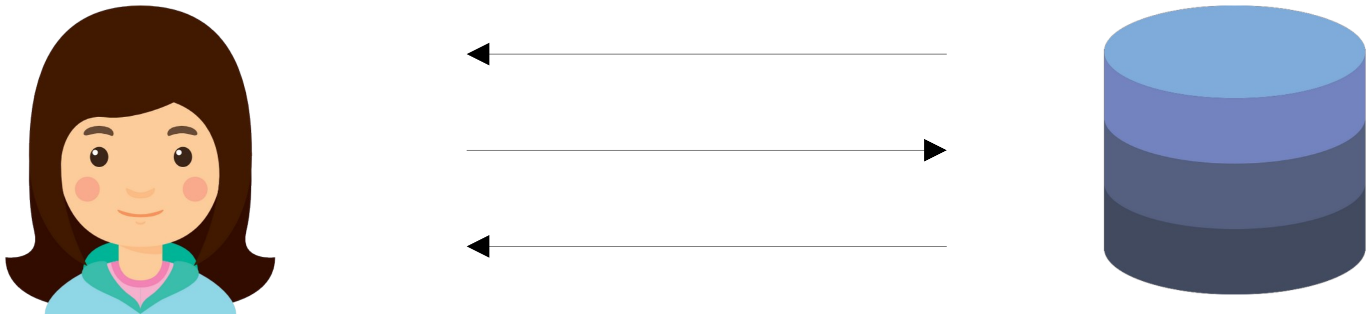
(Non-Preprocessing, Single-Server, Non-Trivial) Private Information Retrieval

$i \in [n]$

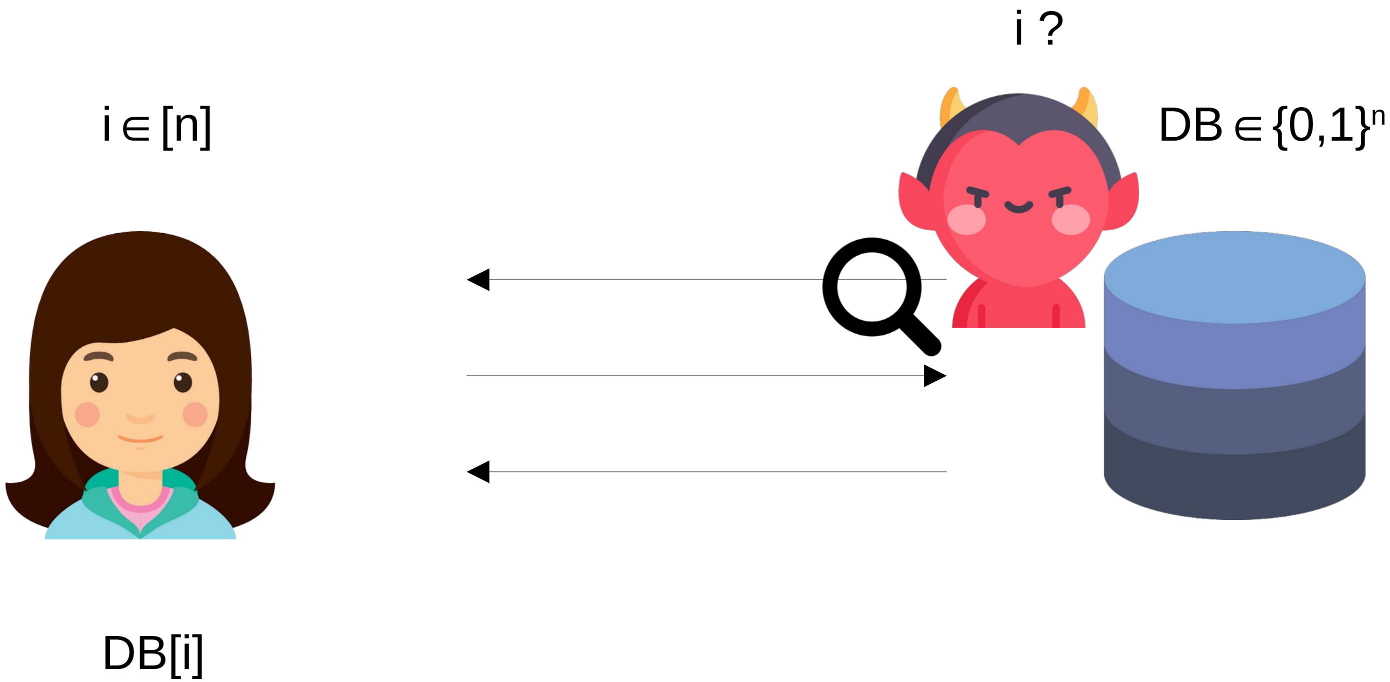
$DB \in \{0,1\}^n$



$DB[i]$

[CGKS95,KO97,DMO00]

(Non-Preprocessing, Single-Server, Non-Trivial) Private Information Retrieval



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[CGKS95,KO97,DMO00]

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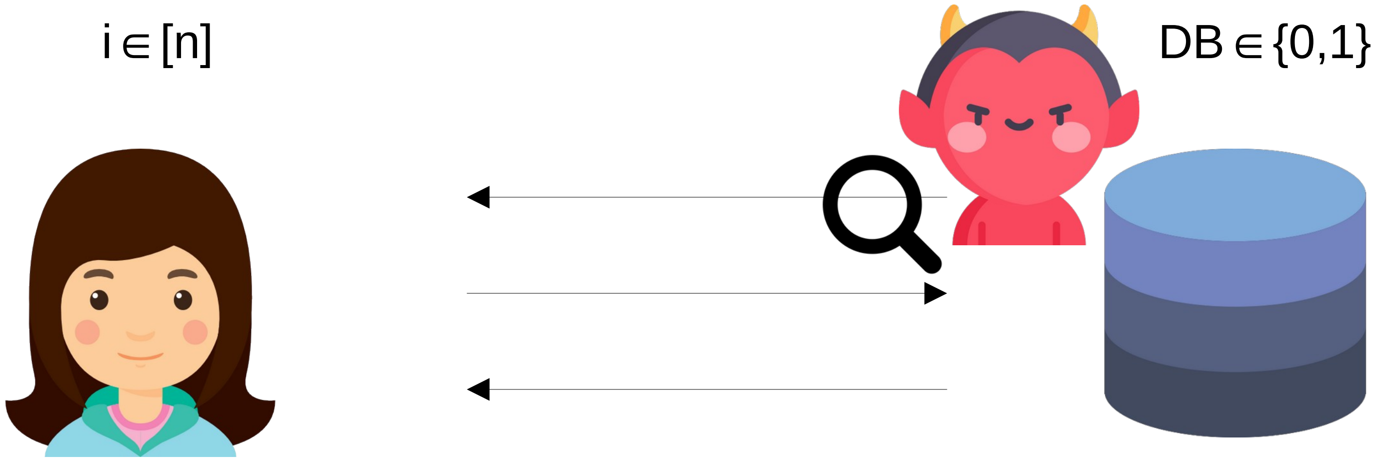

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Communication < (1-ε)n

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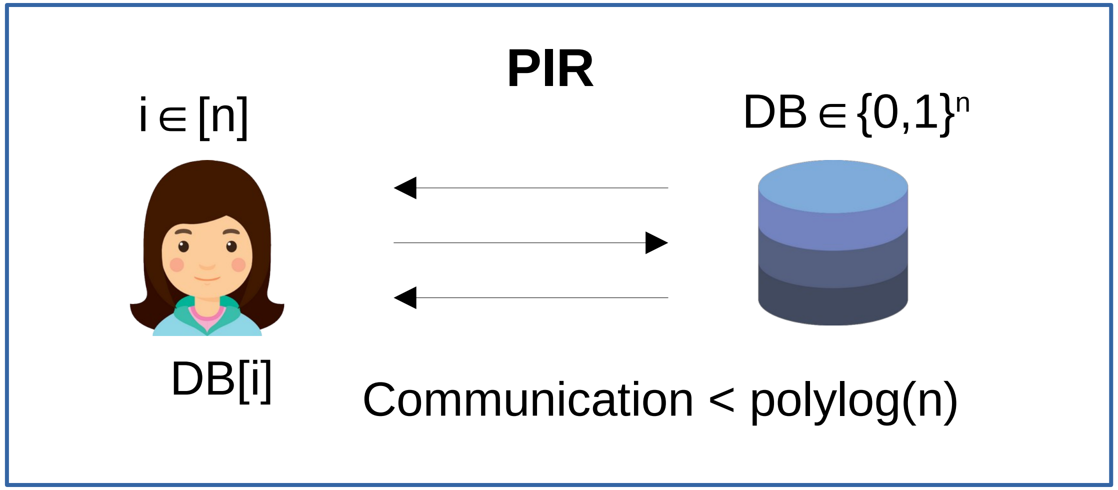

PIR

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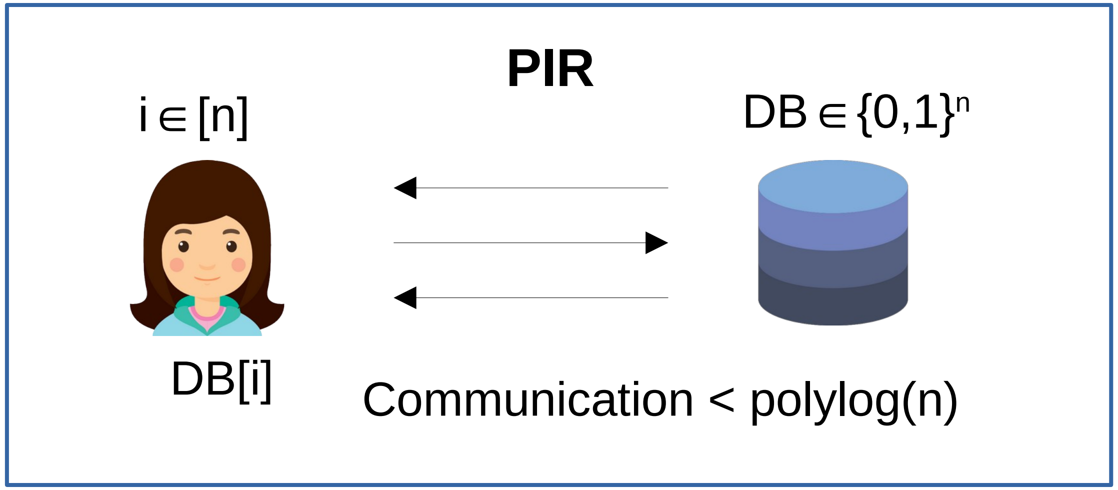

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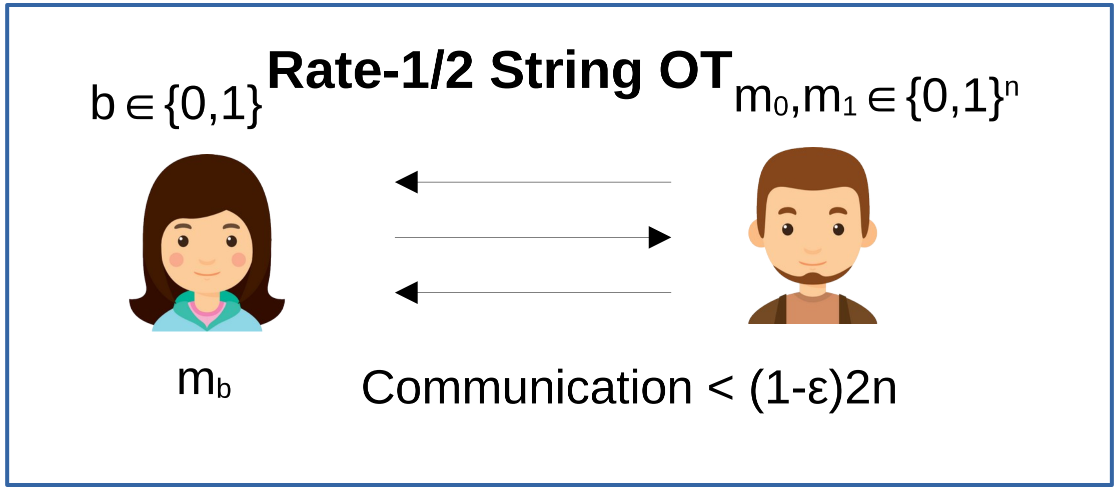

Rate-1/2 String OT

$b \in \{0,1\}$

$m_0, m_1 \in \{0,1\}^n$

m_b

Communication < (1-ε)2n

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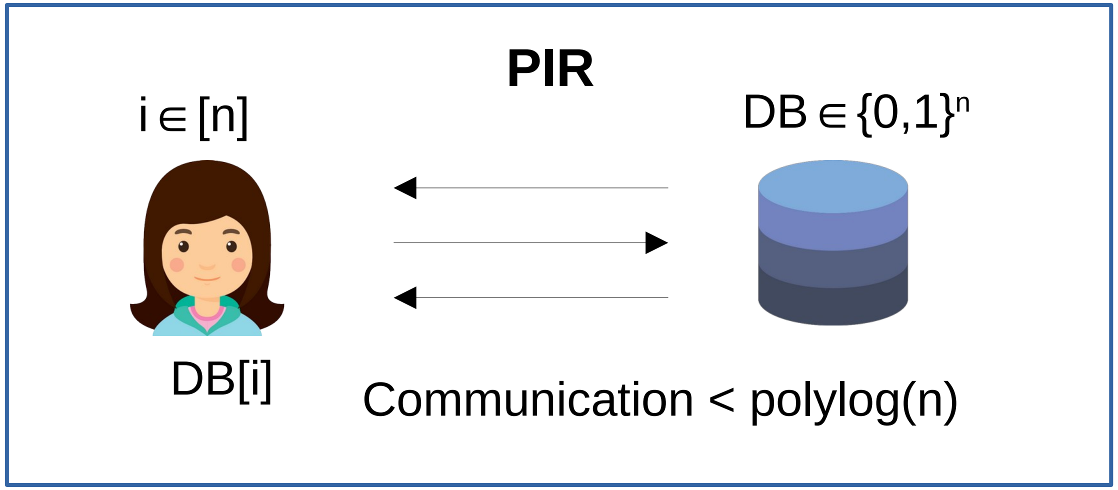

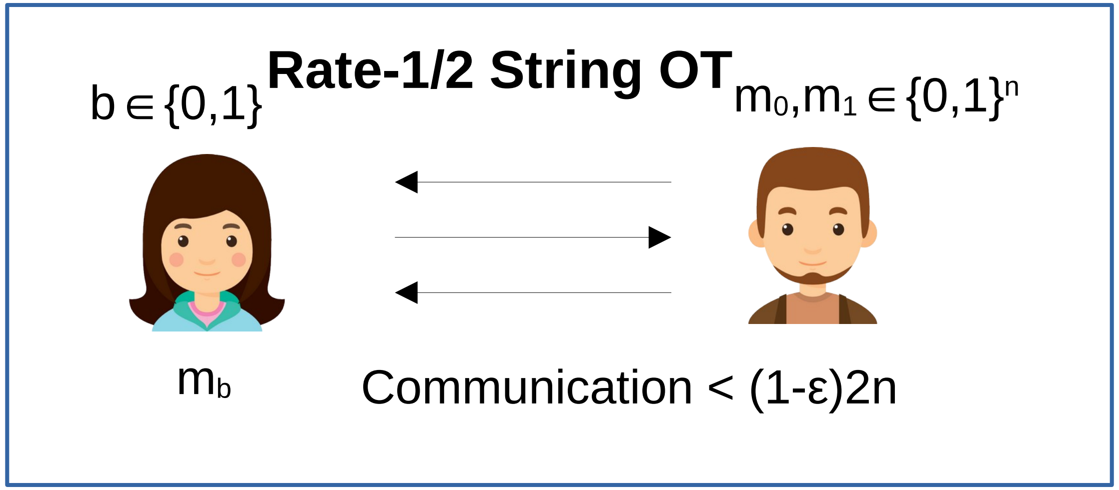

- Non-trivial PIR ⇒ Oblivious Transfer [DMO00]

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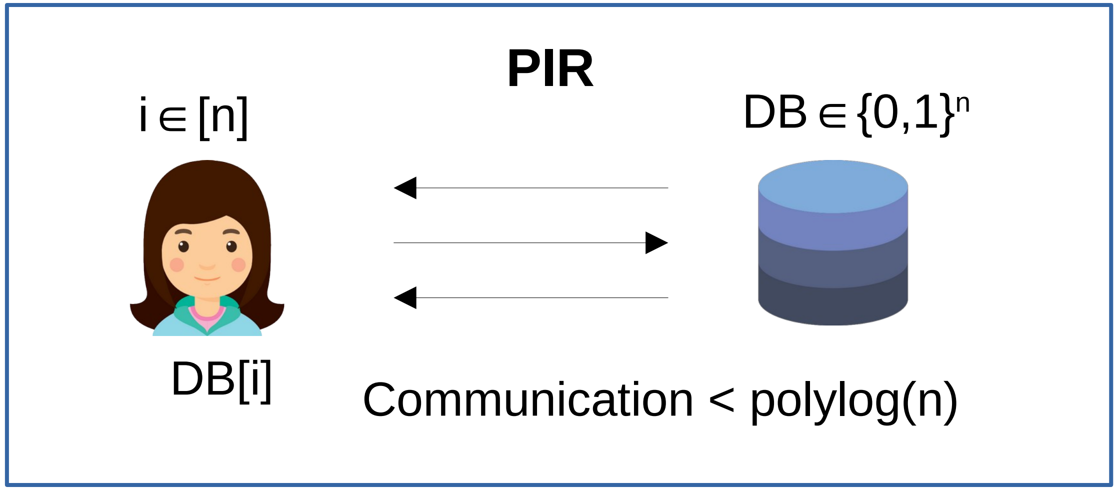

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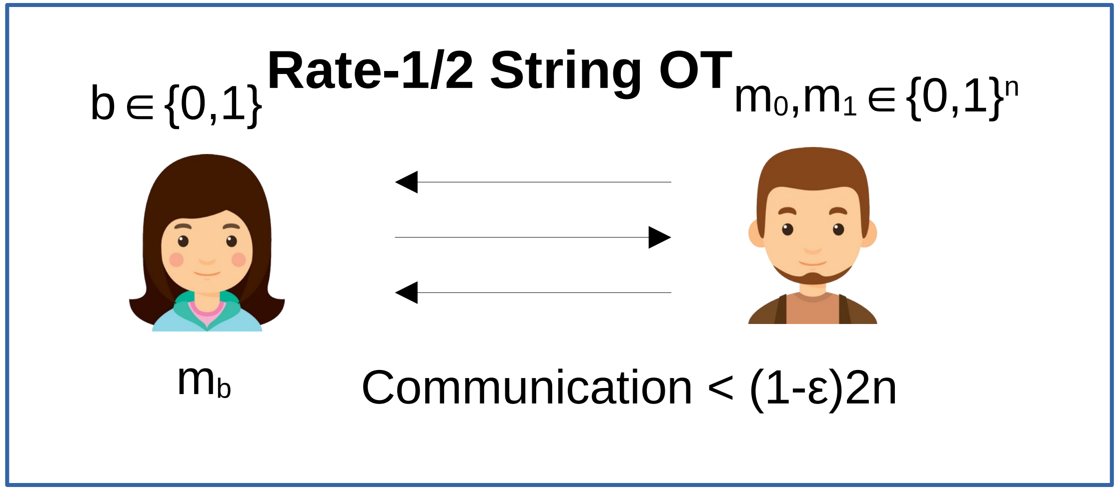

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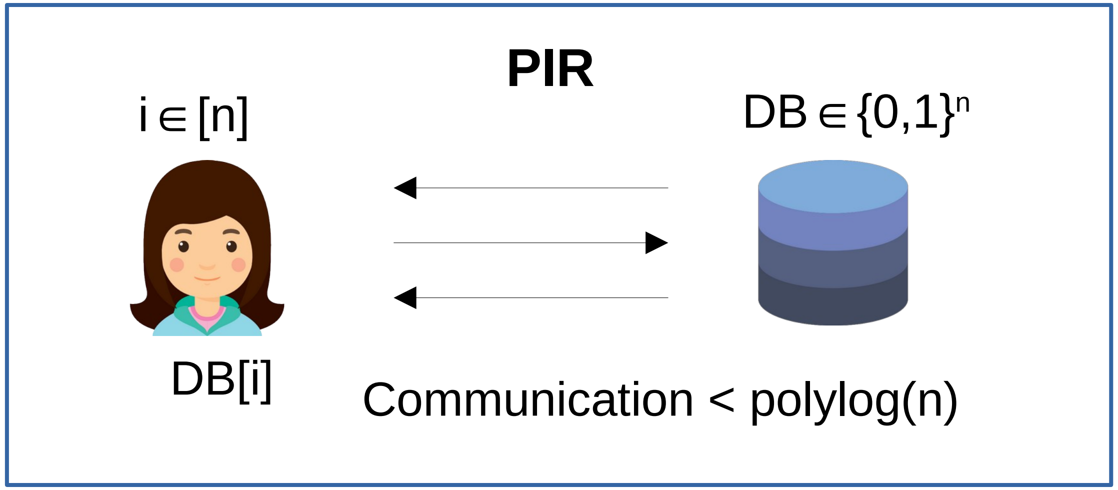

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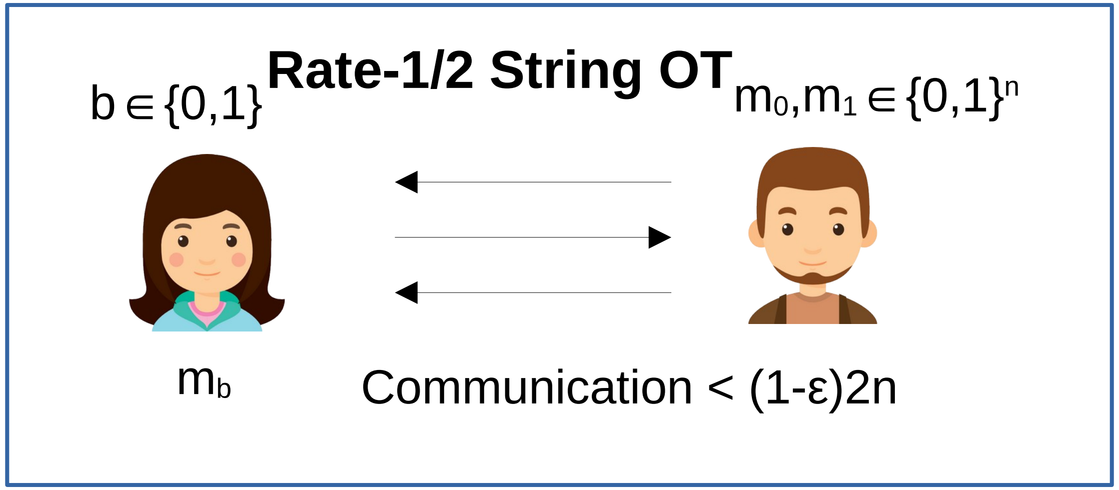

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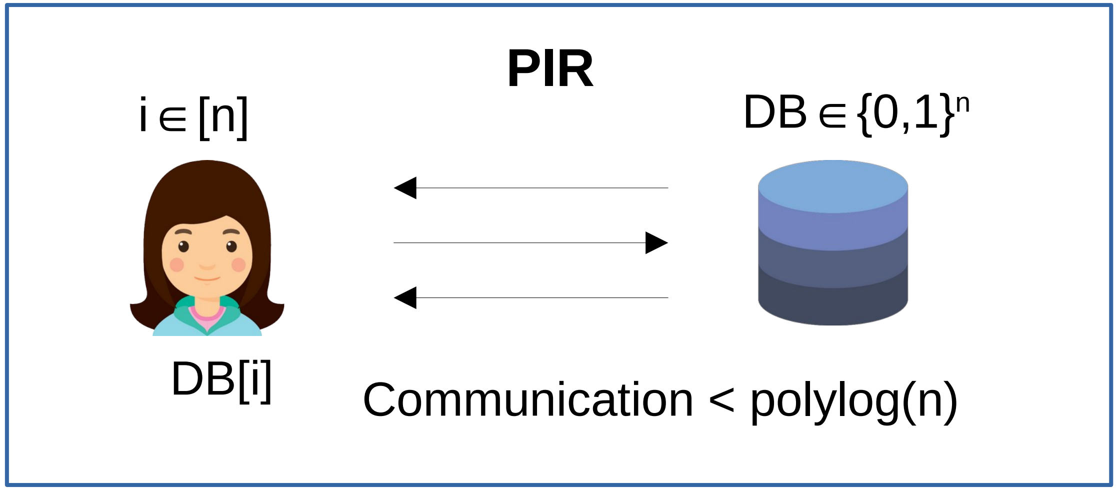

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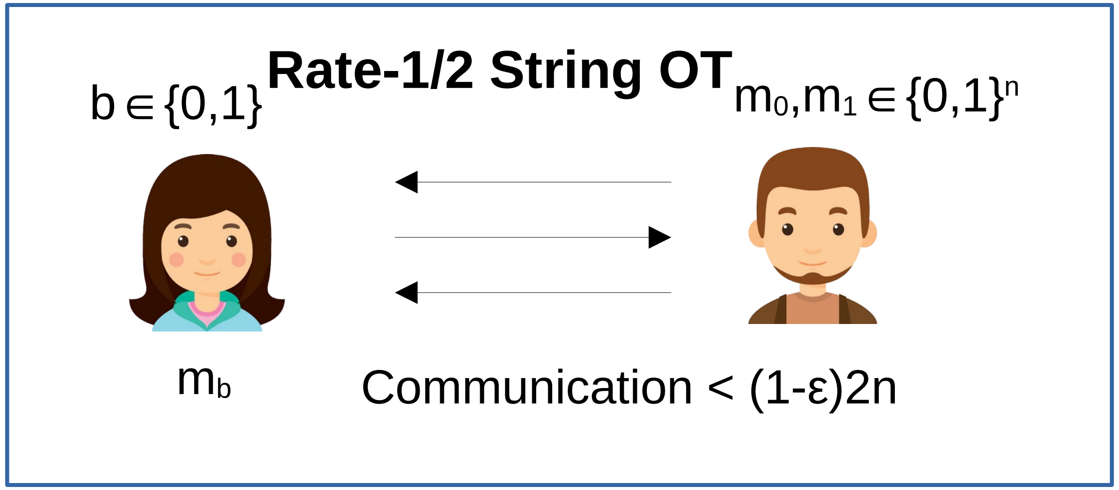

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- Non-trivial PIR ⇒ Oblivious Transfer [DMO00]
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- All known constructions Ω(n) public-key operations

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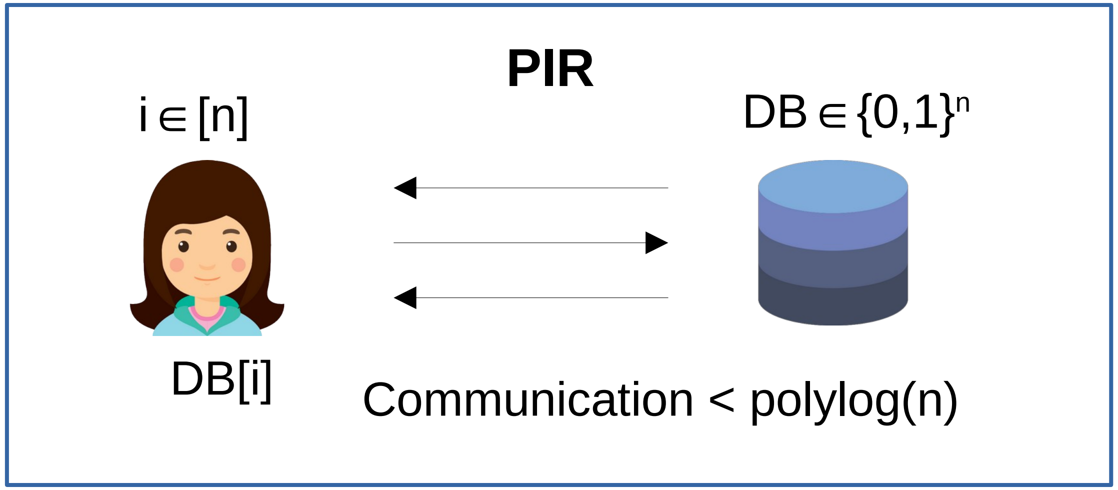

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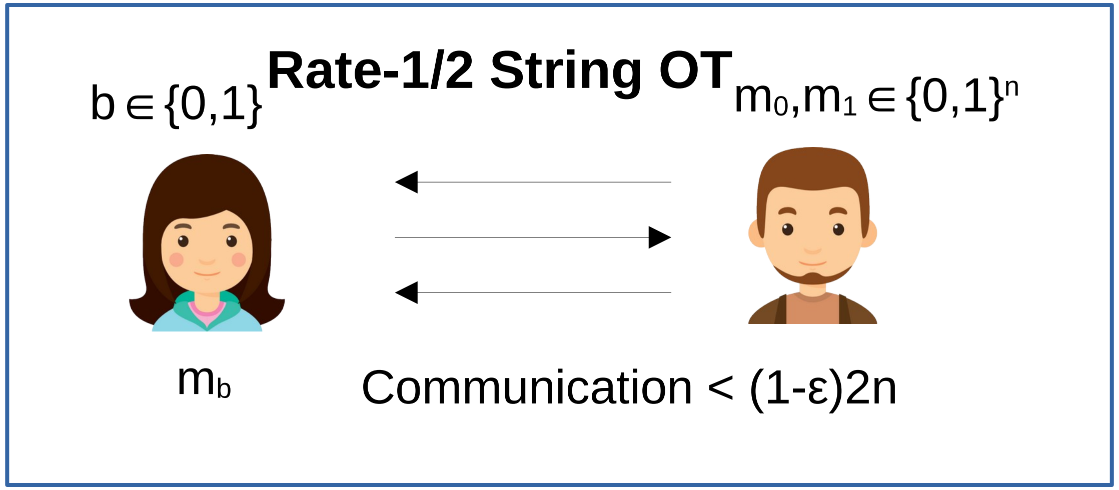

Rate-1/2 String OT

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Communication < $(1-\varepsilon)2n$

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- Requires public-key operations [IR89,GKM+06]
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Are $\Omega(n)$ public-key operations inherent?

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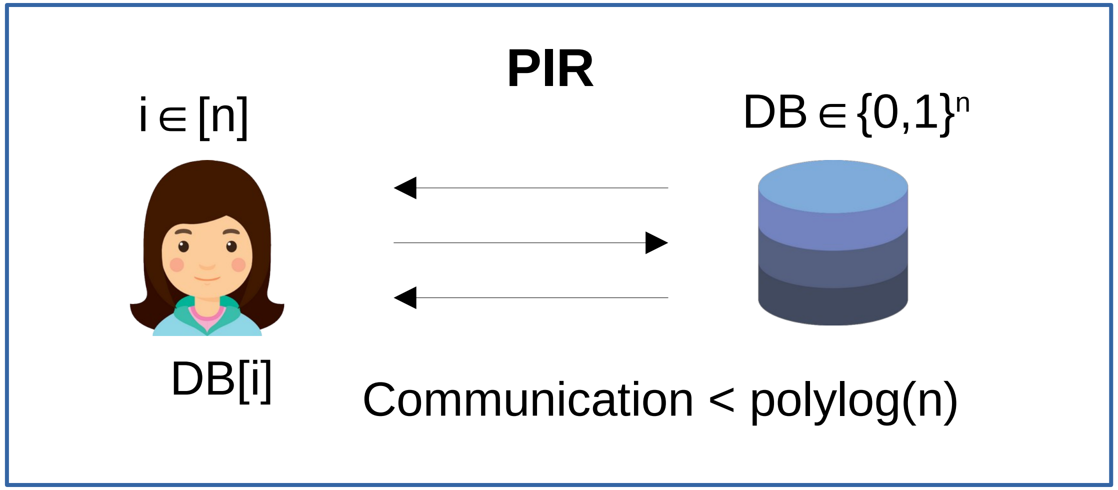

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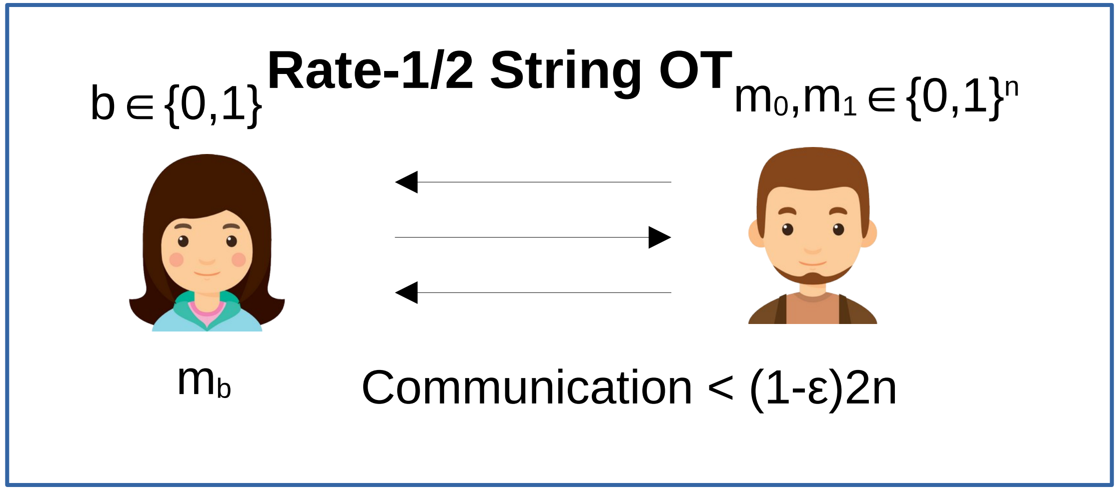

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- Requires Ω(n) server computation [BIM00]
- All known constructions Ω(n) public-key operations

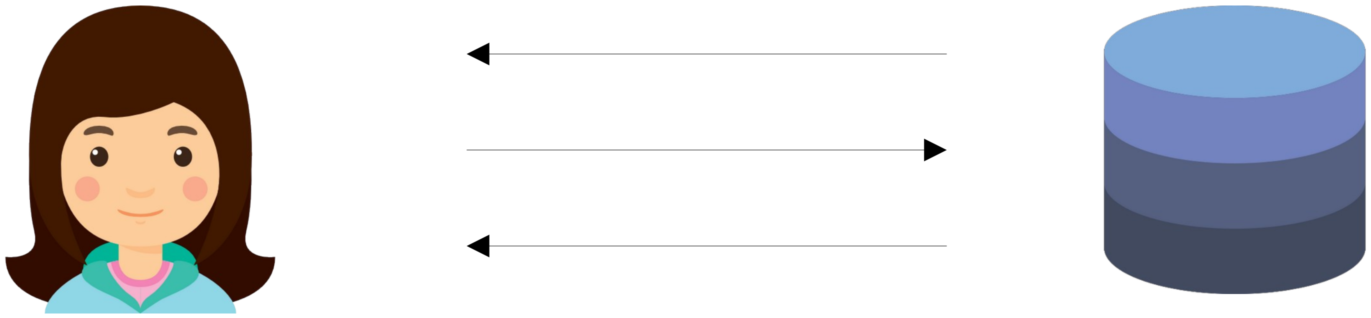
Are Ω(n) public-key operations inherent?

Yes!

PIR in the Generic Group Model

$i \in [n]$

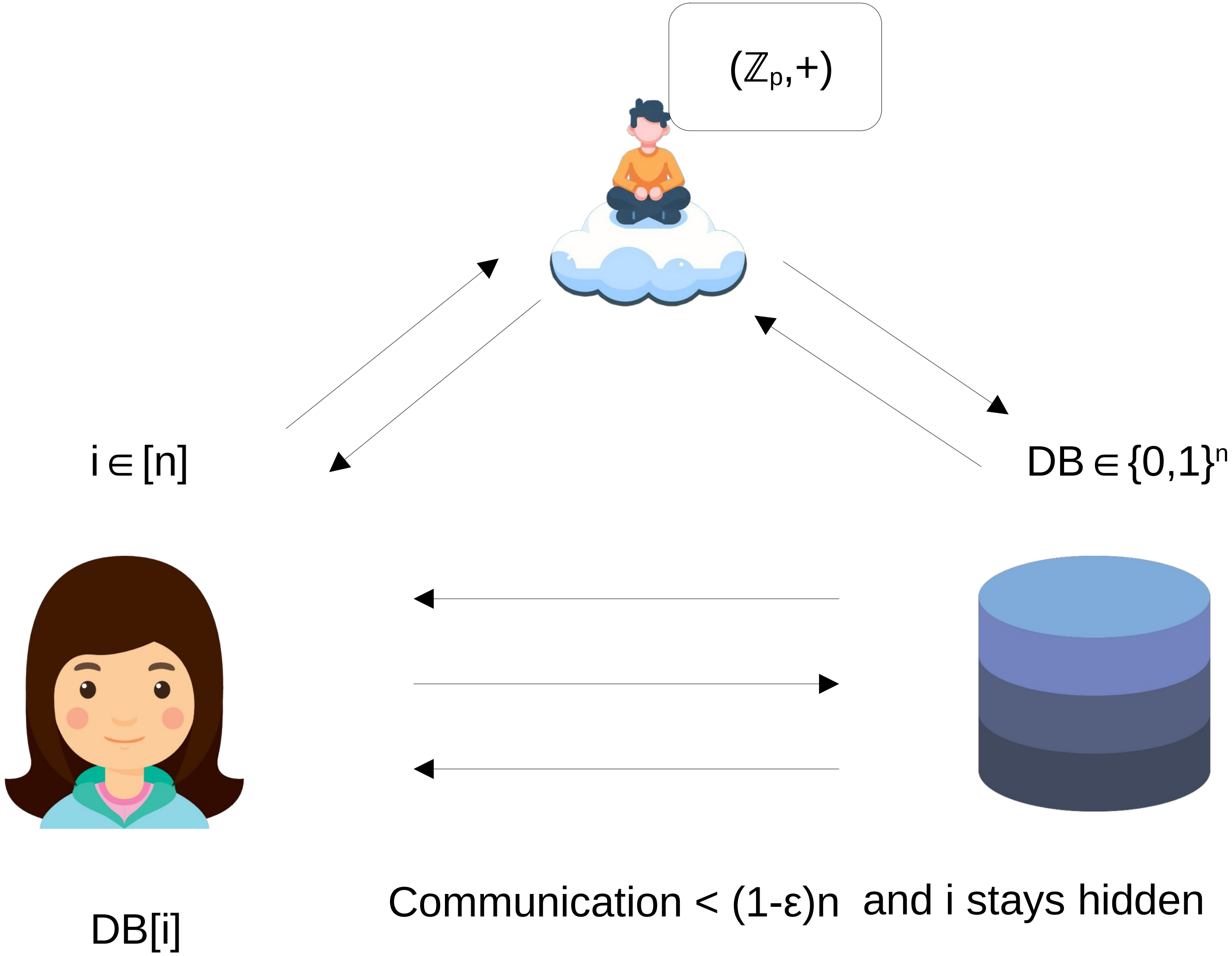
$DB \in \{0,1\}^n$



Communication < $(1-\varepsilon)n$ and i stays hidden

$DB[i]$

PIR in the Generic Group Model



$(\mathbb{Z}_p, +)$

$i \in [n]$

$DB \in \{0,1\}^n$

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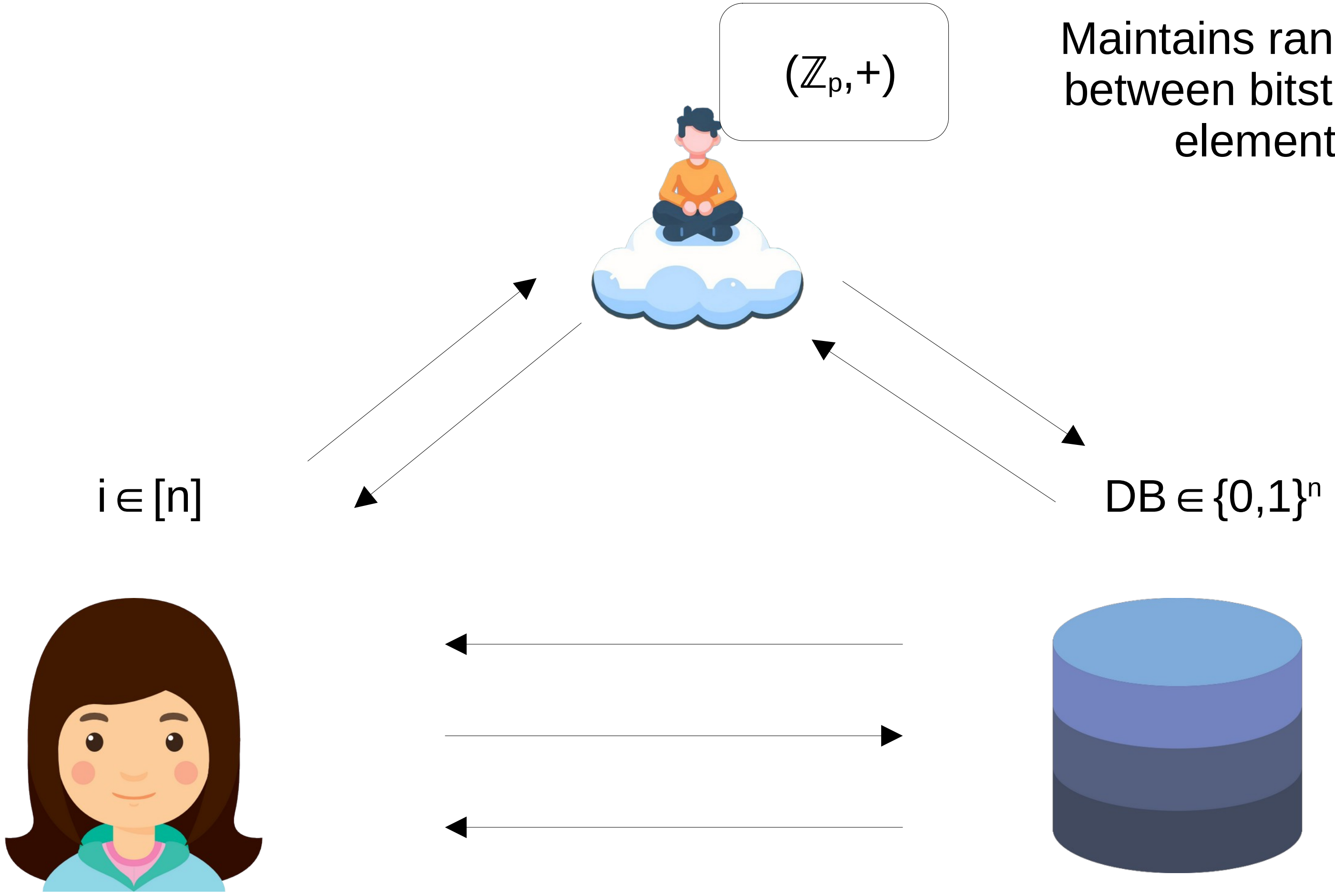
PIR in the Generic Group Model



$(\mathbb{Z}_p,+)$

Maintains random map between bitstrings and elements of

$i \in [n]$

$DB \in \{0,1\}^n$

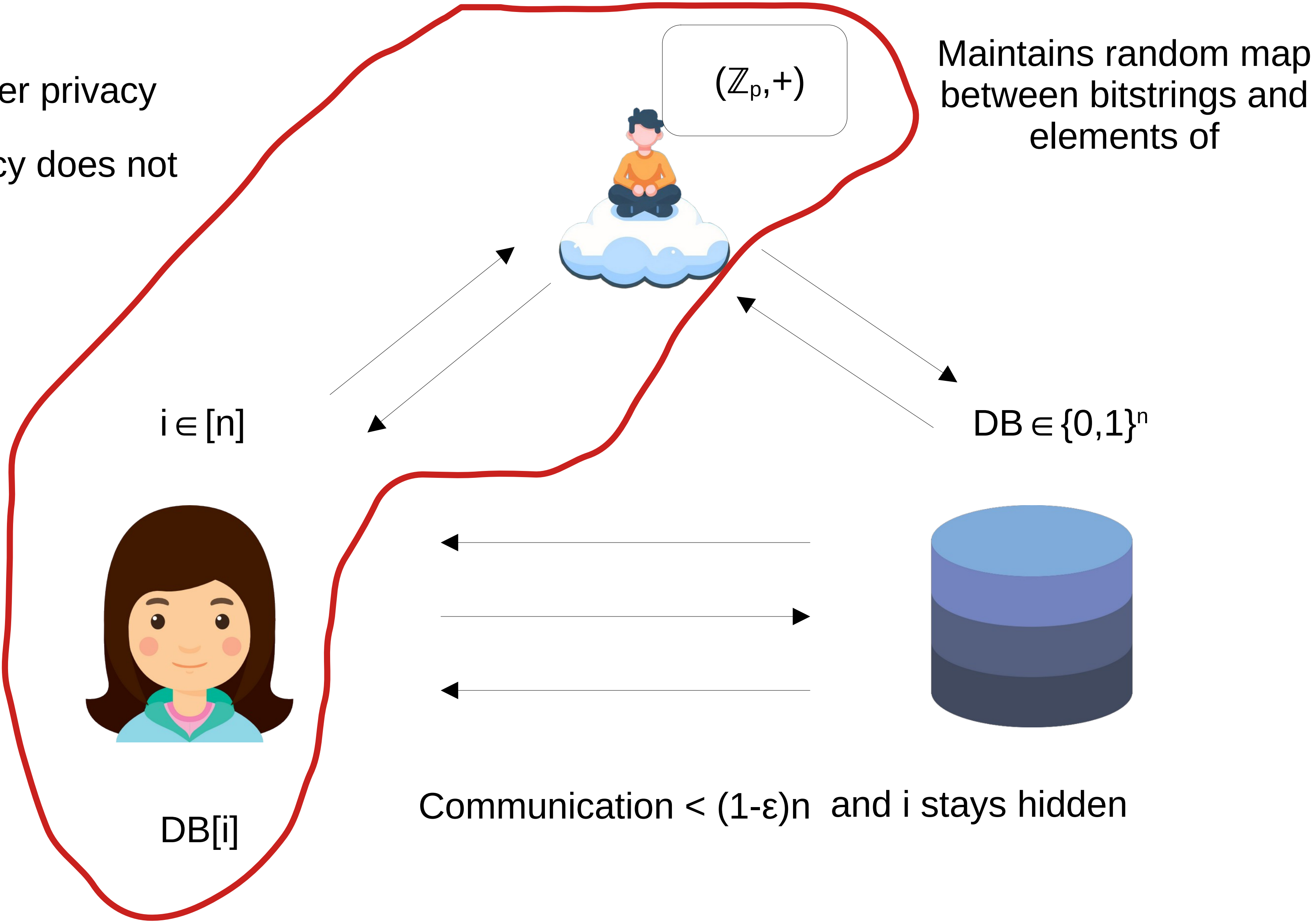
DB[i]

Communication < $(1-\varepsilon)n$ and i stays hidden

PIR in the Generic Group Model

Maintains user privacy

Server privacy does not exist



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Maintains random map between bitstrings and elements of

Communication < $(1-\varepsilon)n$ and i stays hidden

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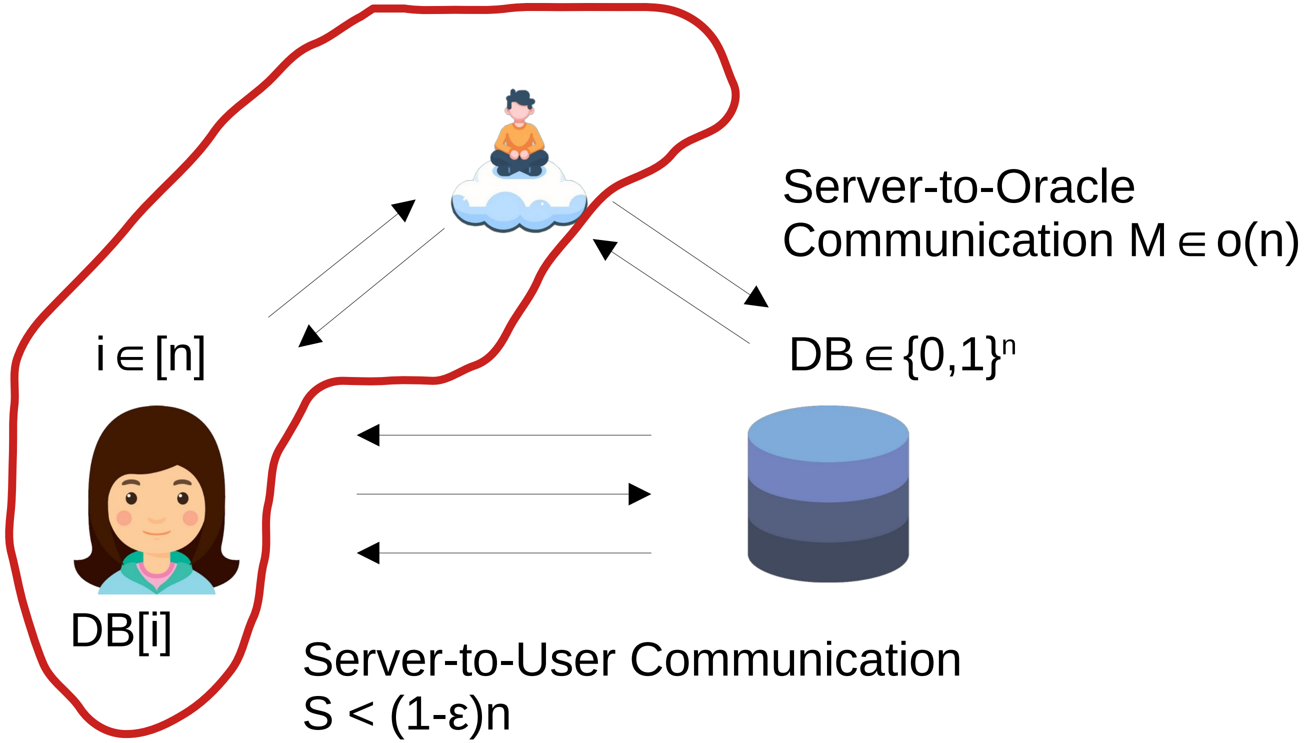

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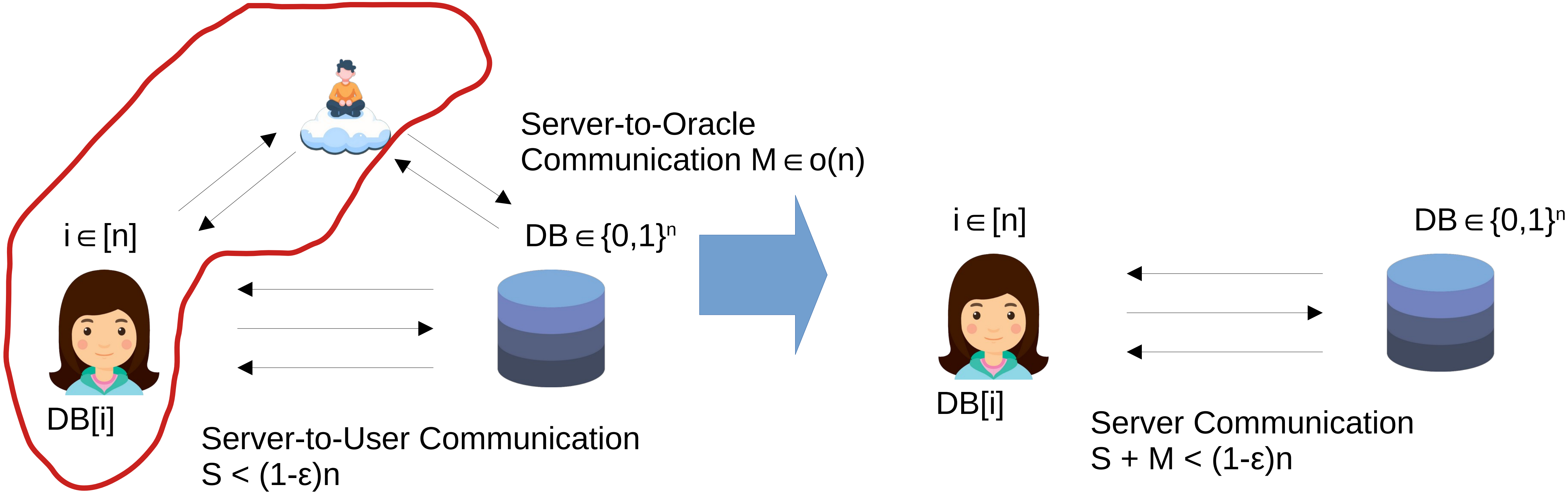


Server-to-Oracle

Communication $M \in o(n)$

$i \in [n]$

$DB \in \{0,1\}^n$

DB[i]

Server-to-User Communication

$S < (1-\varepsilon)n$

$i \in [n]$

$DB \in \{0,1\}^n$

DB[i]

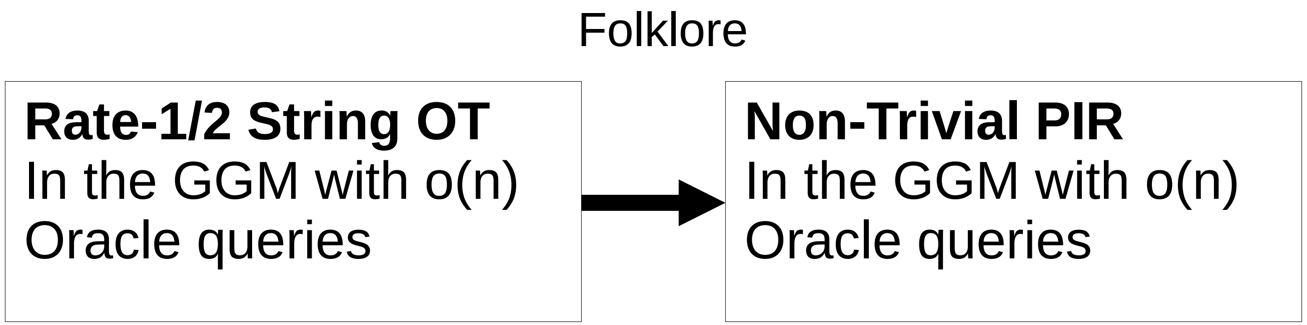
Server Communication

$S + M < (1-\varepsilon)n$

Consequences

Rate-1/2 String OT
In the GGM with o(n)
Oracle queries

Consequences

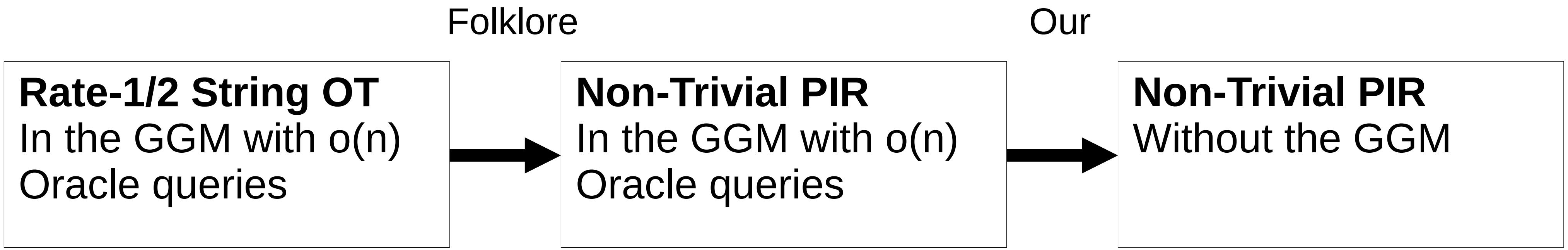


Folklore

Rate-1/2 String OT
In the GGM with o(n)
Oracle queries

Non-Trivial PIR
In the GGM with o(n)
Oracle queries

Consequences



Folklore

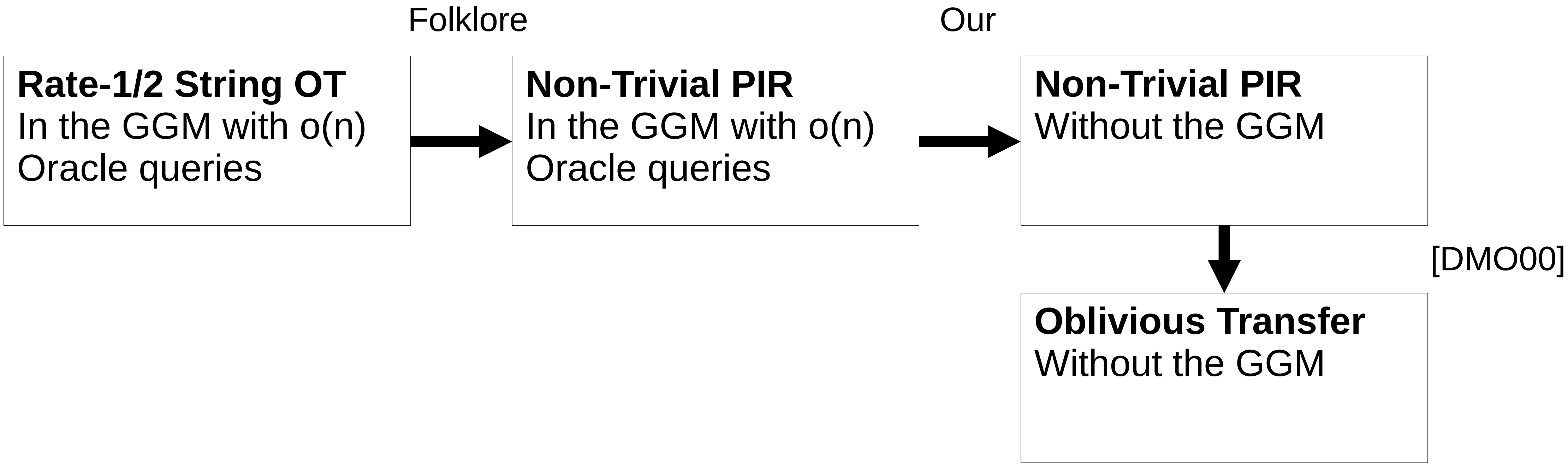
Our

Rate-1/2 String OT
In the GGM with o(n) Oracle queries

Non-Trivial PIR
In the GGM with o(n) Oracle queries

Non-Trivial PIR
Without the GGM

Consequences



Folklore

Our

[DMO00]

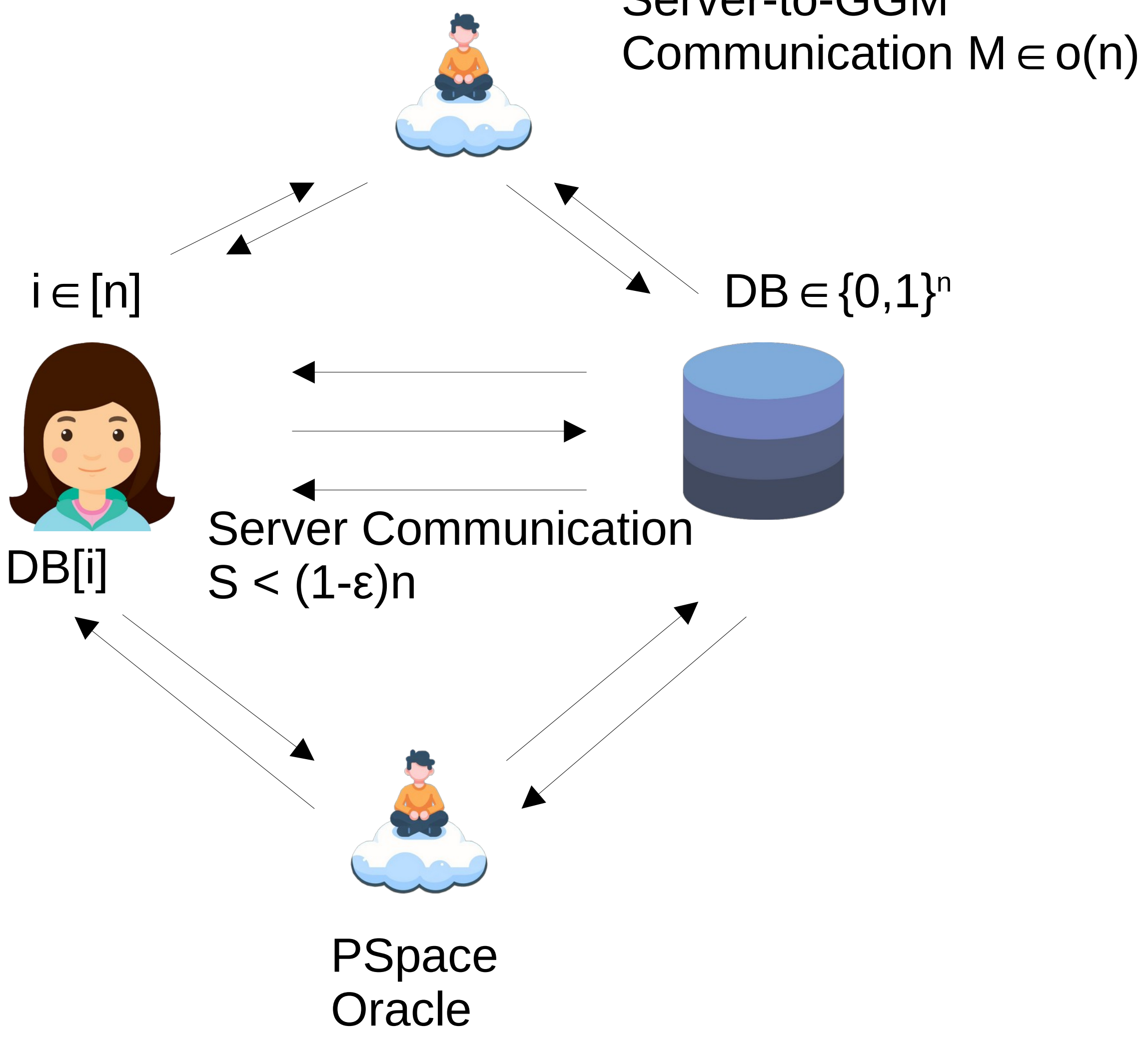
Rate-1/2 String OT
In the GGM with o(n)
Oracle queries

Non-Trivial PIR
In the GGM with o(n)
Oracle queries

Non-Trivial PIR
Without the GGM

Oblivious Transfer
Without the GGM

PIR in the GGM + PSpace



Server-to-GGM
Communication $M \in o(n)$

$i \in [n]$

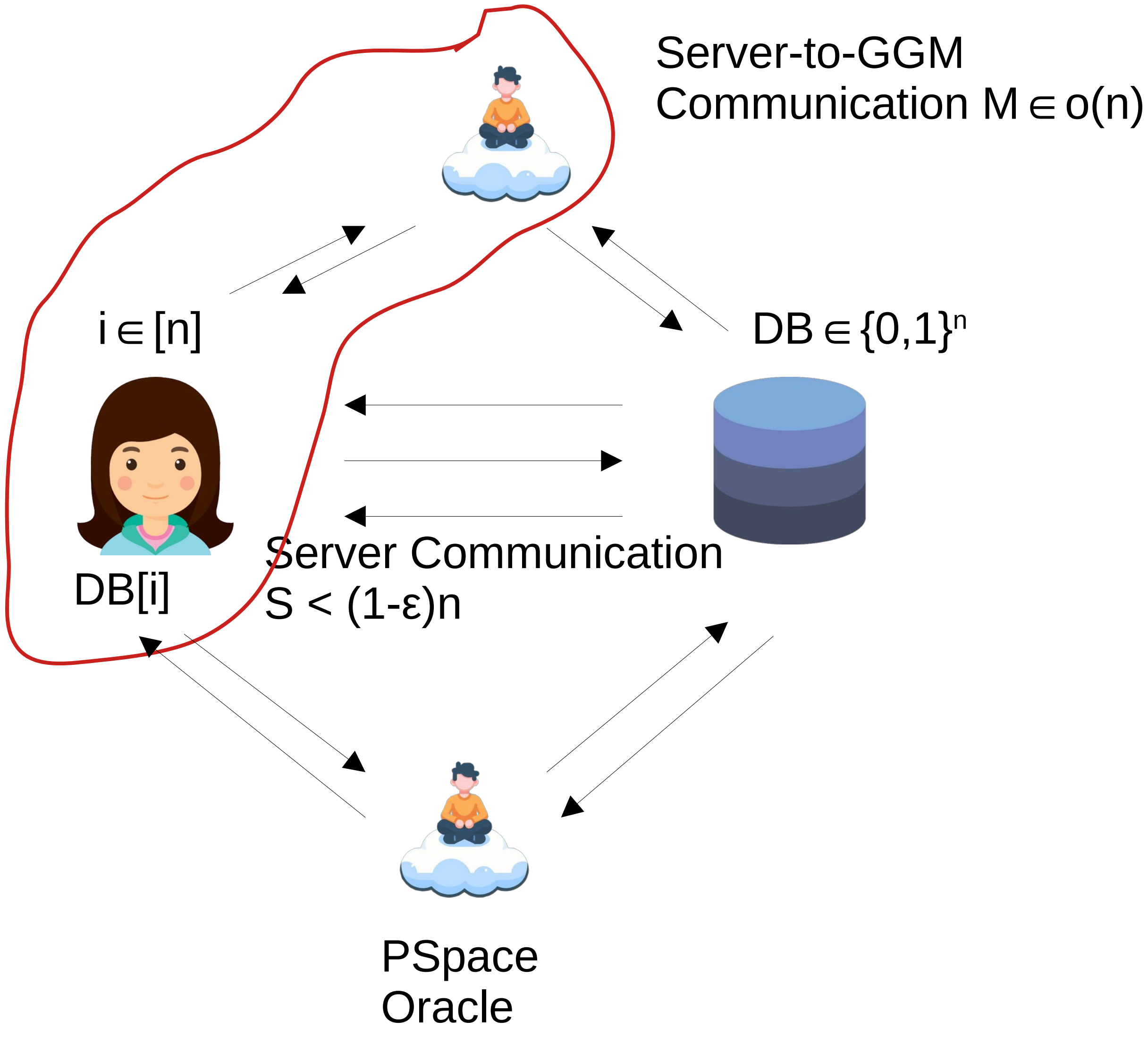
$DB \in \{0,1\}^n$

DB[i]

Server Communication
$S < (1\text{-}\varepsilon)n$

PSpace
Oracle

PIR in the GGM + PSpace



Server-to-GGM
Communication $M \in o(n)$

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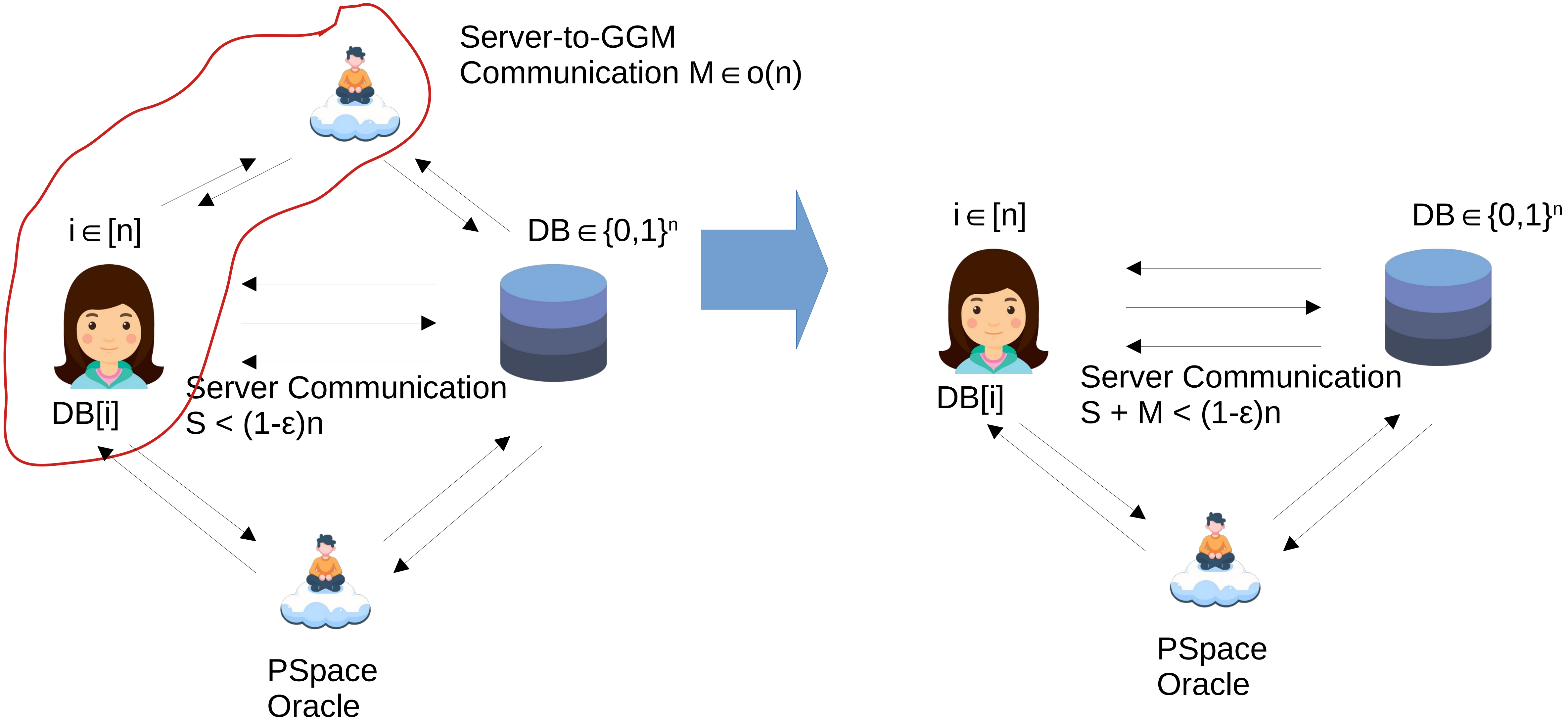
$S < (1\text{-}\varepsilon)n$

PSpace
Oracle

PIR in the GGM + PSpace



Server-to-GGM Communication $M \in o(n)$

$i \in [n]$

$\mathrm{DB} \in \{0,1\}^n$

DB[i]

Server Communication $S < (1\text{-}\varepsilon)n$

PSpace Oracle

$i \in [n]$

$\mathrm{DB} \in \{0,1\}^n$

DB[i]

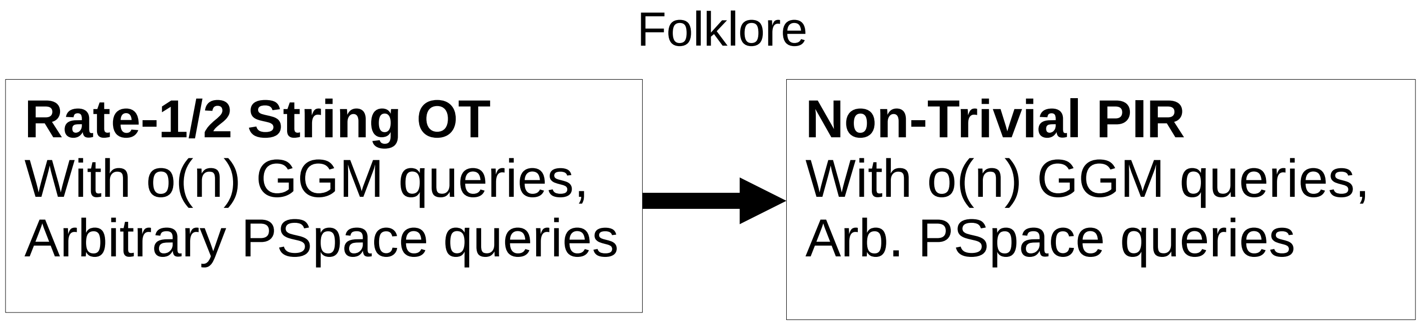
Server Communication $S + M < (1\text{-}\varepsilon)n$

PSpace Oracle

Consequences cont.

Rate-1/2 String OT
With o(n) GGM queries,
Arbitrary PSpace queries

Consequences cont.

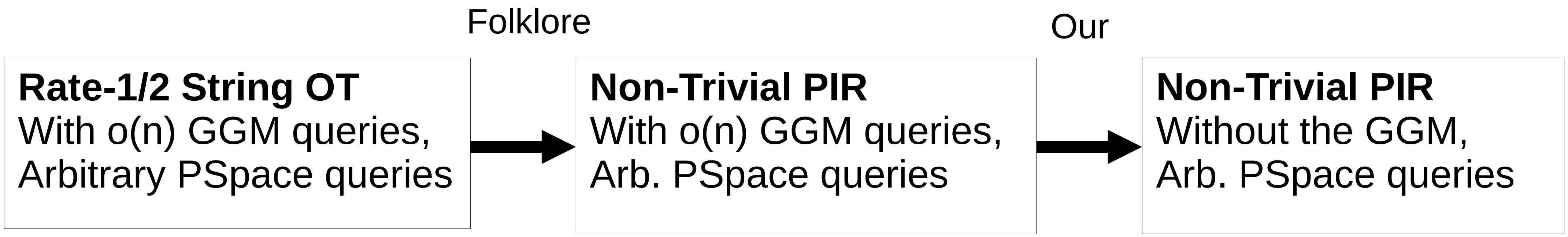


Folklore

Rate-1/2 String OT
With o(n) GGM queries,
Arbitrary PSpace queries

Non-Trivial PIR
With o(n) GGM queries,
Arb. PSpace queries

Consequences cont.



Folklore

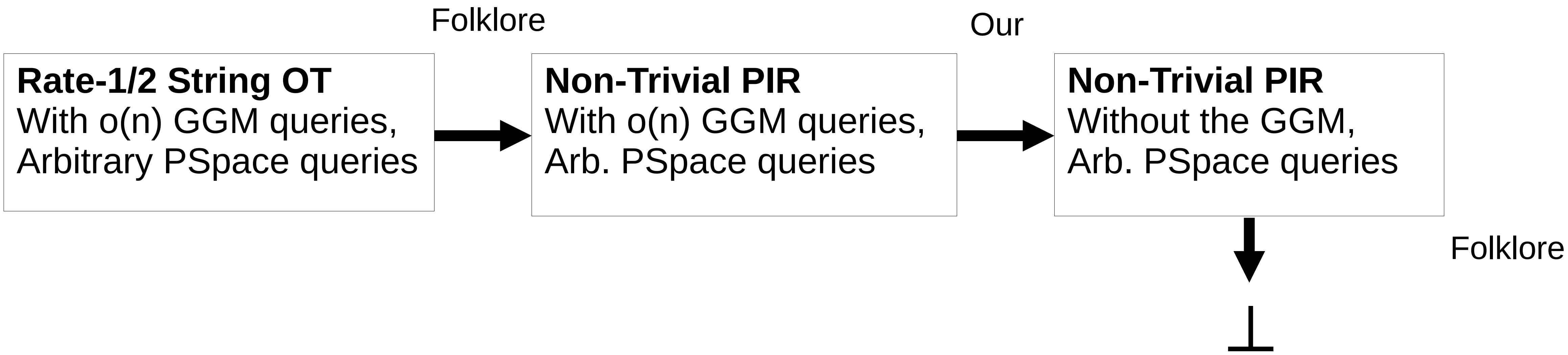
Our

Rate-1/2 String OT
With o(n) GGM queries,
Arbitrary PSpace queries

Non-Trivial PIR
With o(n) GGM queries,
Arb. PSpace queries

Non-Trivial PIR
Without the GGM,
Arb. PSpace queries

Consequences cont.



Folklore

Rate-1/2 String OT
With o(n) GGM queries,
Arbitrary PSpace queries

Our

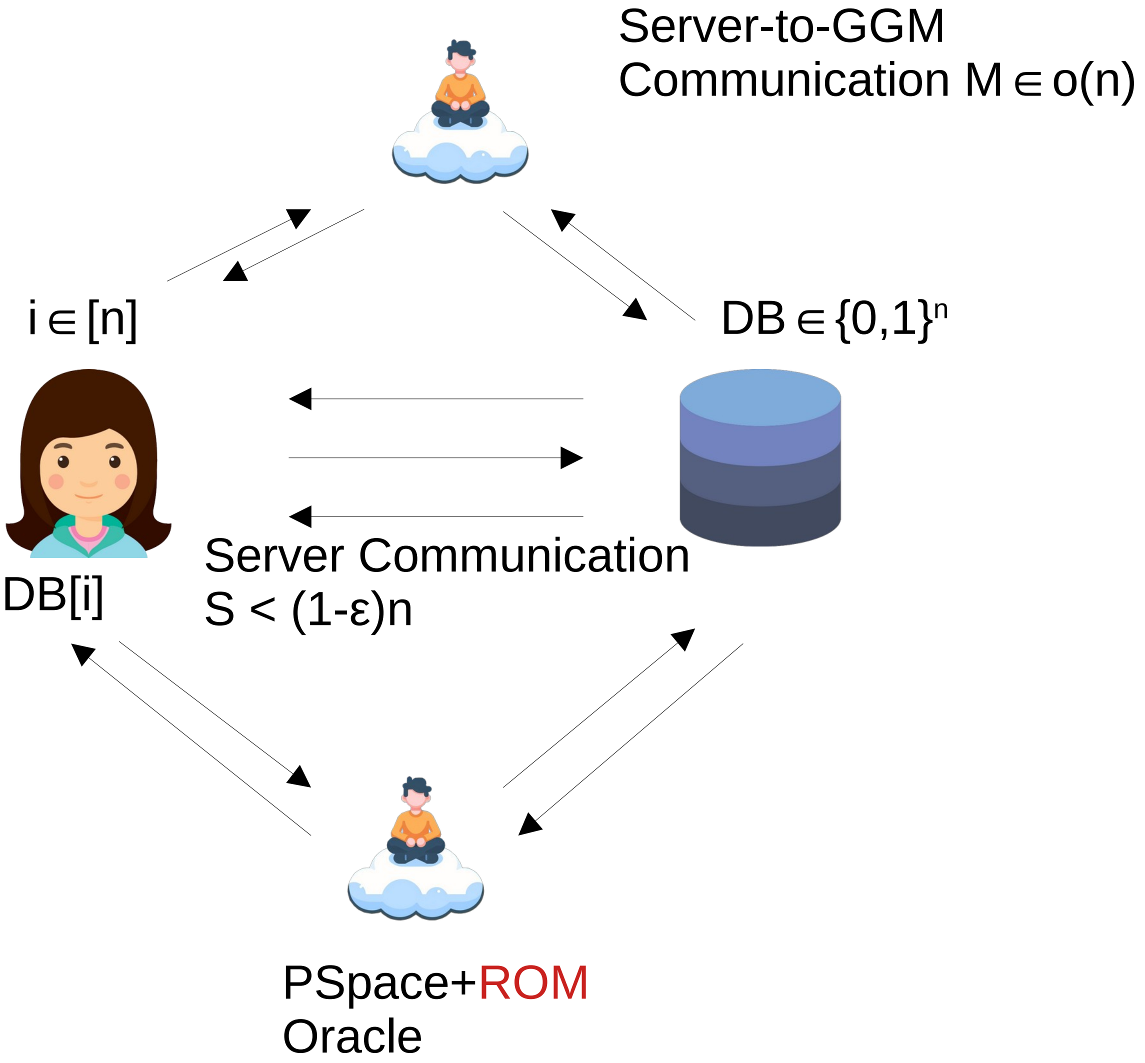
Non-Trivial PIR
With o(n) GGM queries,
Arb. PSpace queries

Non-Trivial PIR
Without the GGM,
Arb. PSpace queries

Folklore

$\perp$

PIR in the GGM + PSpace + ROM



Server-to-GGM
Communication $M \in o(n)$

$i \in [n]$

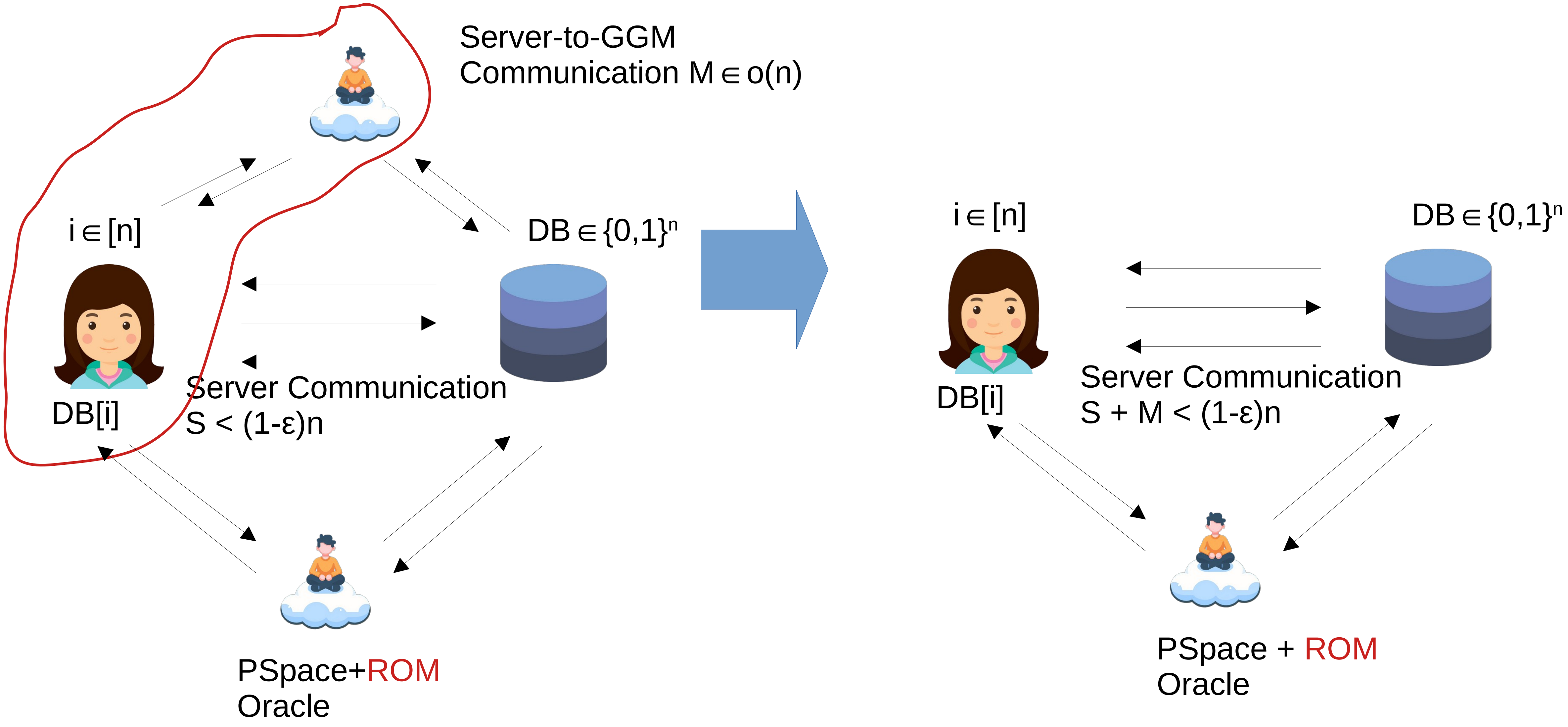
$DB \in \{0,1\}^n$

Server Communication
$S < (1-\varepsilon)n$

DB[i]

PSpace+ROM
Oracle

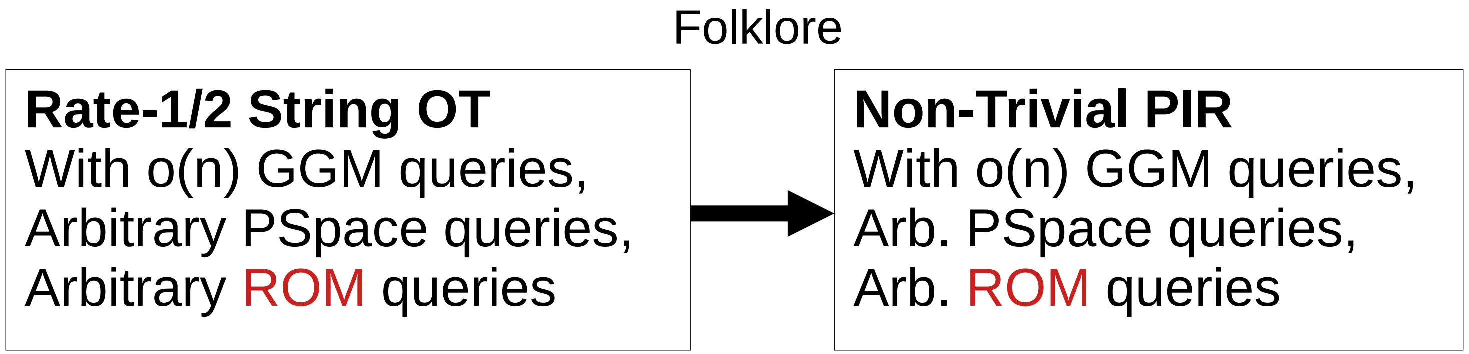
PIR in the GGM + PSpace + ROM



Consequences cont. (2)

Rate-1/2 String OT
With o(n) GGM queries,
Arbitrary PSpace queries,
Arbitrary ROM queries

Consequences cont. (2)

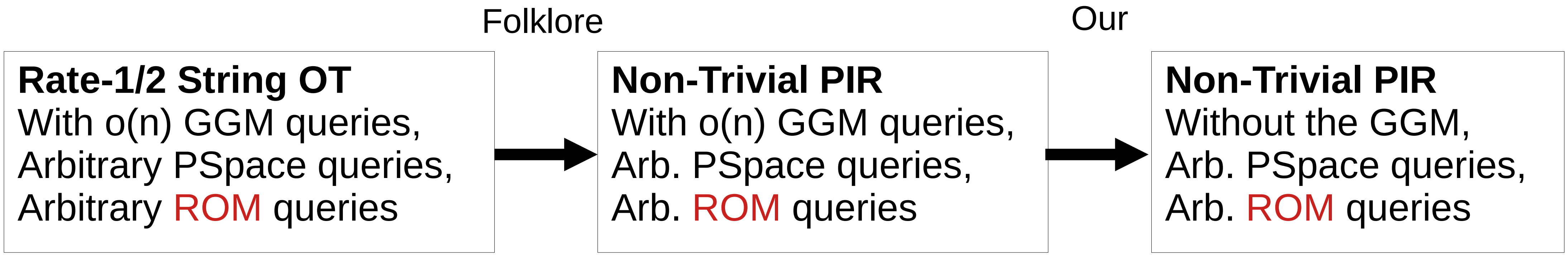


Folklore

Rate-1/2 String OT
With o(n) GGM queries,
Arbitrary PSpace queries,
Arbitrary ROM queries

Non-Trivial PIR
With o(n) GGM queries,
Arb. PSpace queries,
Arb. ROM queries

Consequences cont. (2)



Folklore

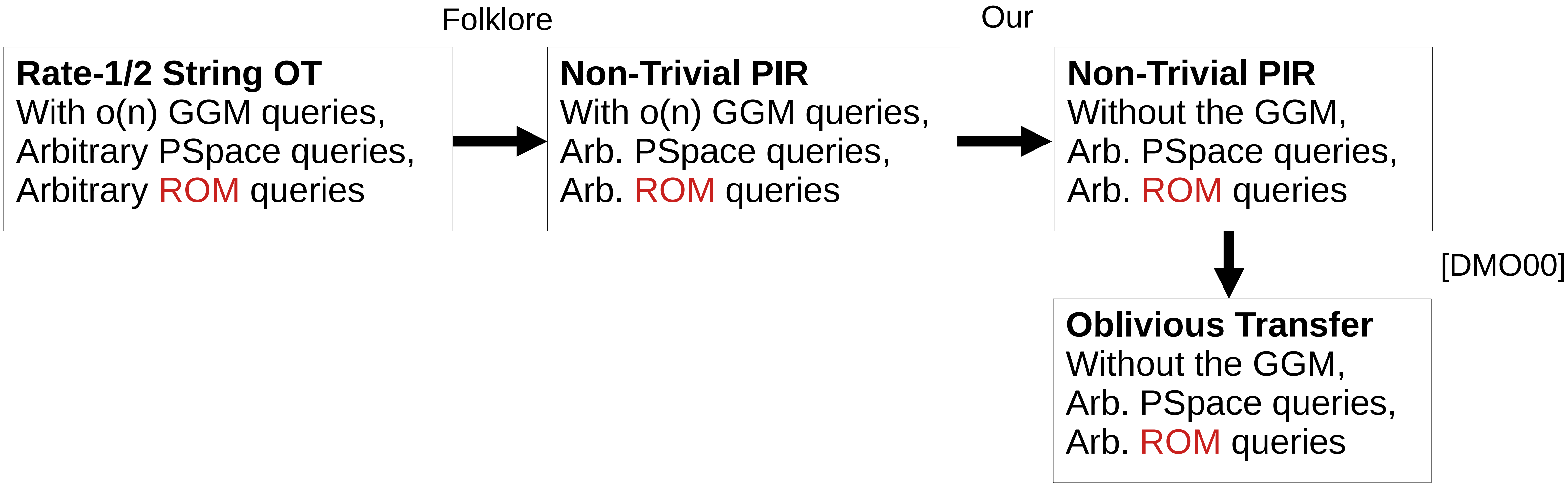
Our

Rate-1/2 String OT
With o(n) GGM queries,
Arbitrary PSpace queries,
Arbitrary ROM queries

Non-Trivial PIR
With o(n) GGM queries,
Arb. PSpace queries,
Arb. ROM queries

Non-Trivial PIR
Without the GGM,
Arb. PSpace queries,
Arb. ROM queries

Consequences cont. (2)



Folklore

Our

[DMO00]

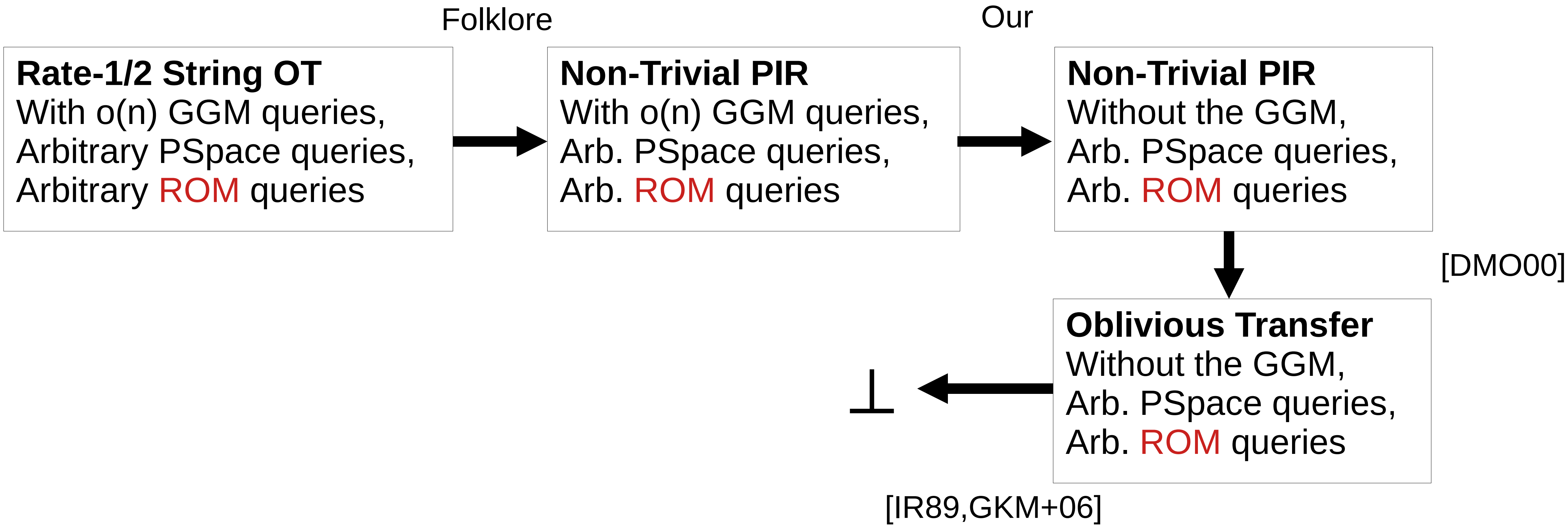
Rate-1/2 String OT
With o(n) GGM queries,
Arbitrary PSpace queries,
Arbitrary ROM queries

Non-Trivial PIR
With o(n) GGM queries,
Arb. PSpace queries,
Arb. ROM queries

Non-Trivial PIR
Without the GGM,
Arb. PSpace queries,
Arb. ROM queries

Oblivious Transfer
Without the GGM,
Arb. PSpace queries,
Arb. ROM queries

Consequences cont. (2)



Folklore

Our

Rate-1/2 String OT
With o(n) GGM queries,
Arbitrary PSpace queries,
Arbitrary ROM queries

Non-Trivial PIR
With o(n) GGM queries,
Arb. PSpace queries,
Arb. ROM queries

Non-Trivial PIR
Without the GGM,
Arb. PSpace queries,
Arb. ROM queries

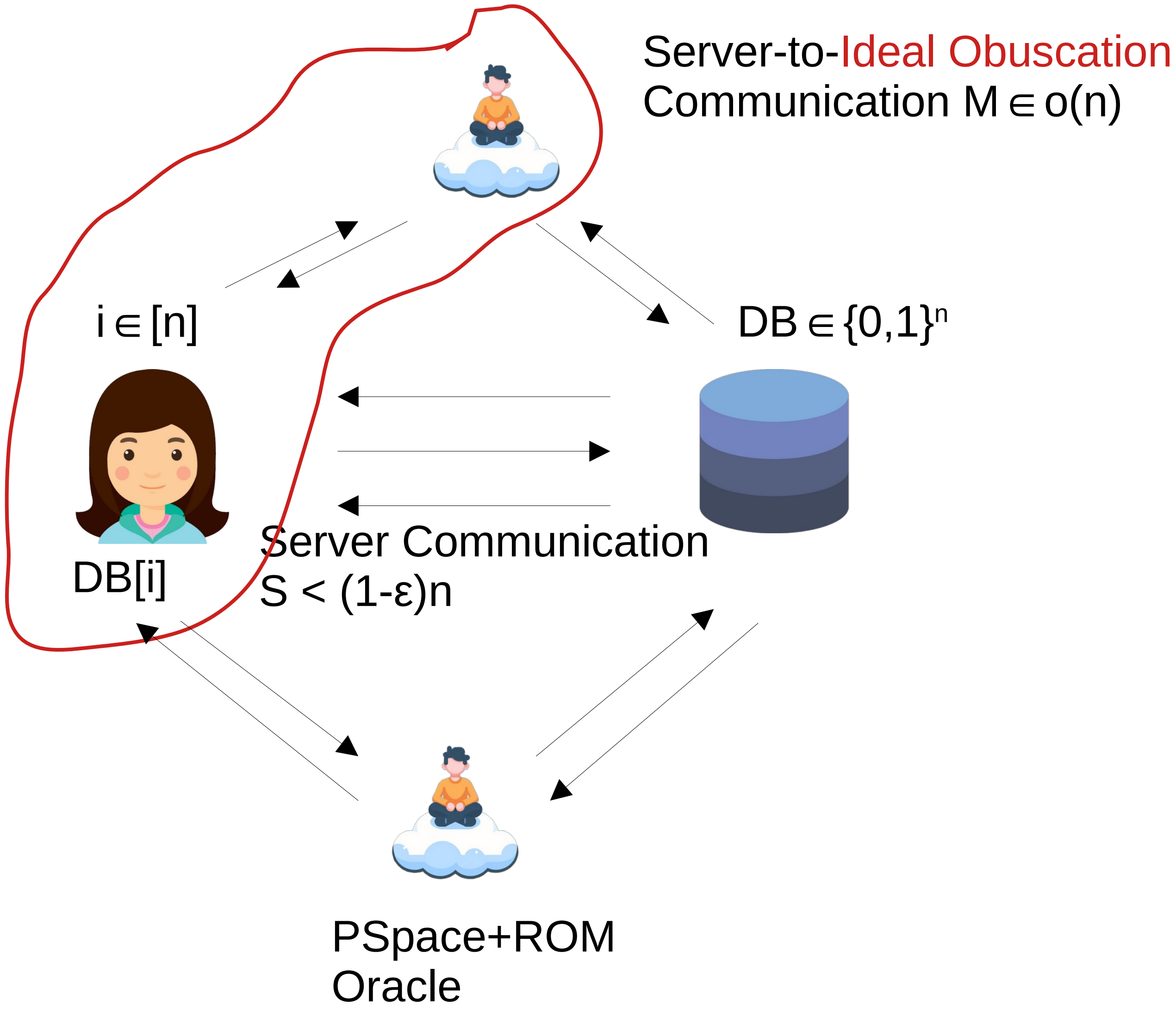
[DMO00]

Oblivious Transfer
Without the GGM,
Arb. PSpace queries,
Arb. ROM queries

$\perp$

[IR89,GKM+06]

PIR in the Simulatable Oracle + PSpace + ROM



Server-to-Ideal Obuscation
Communication $M \in o(n)$

$i \in [n]$

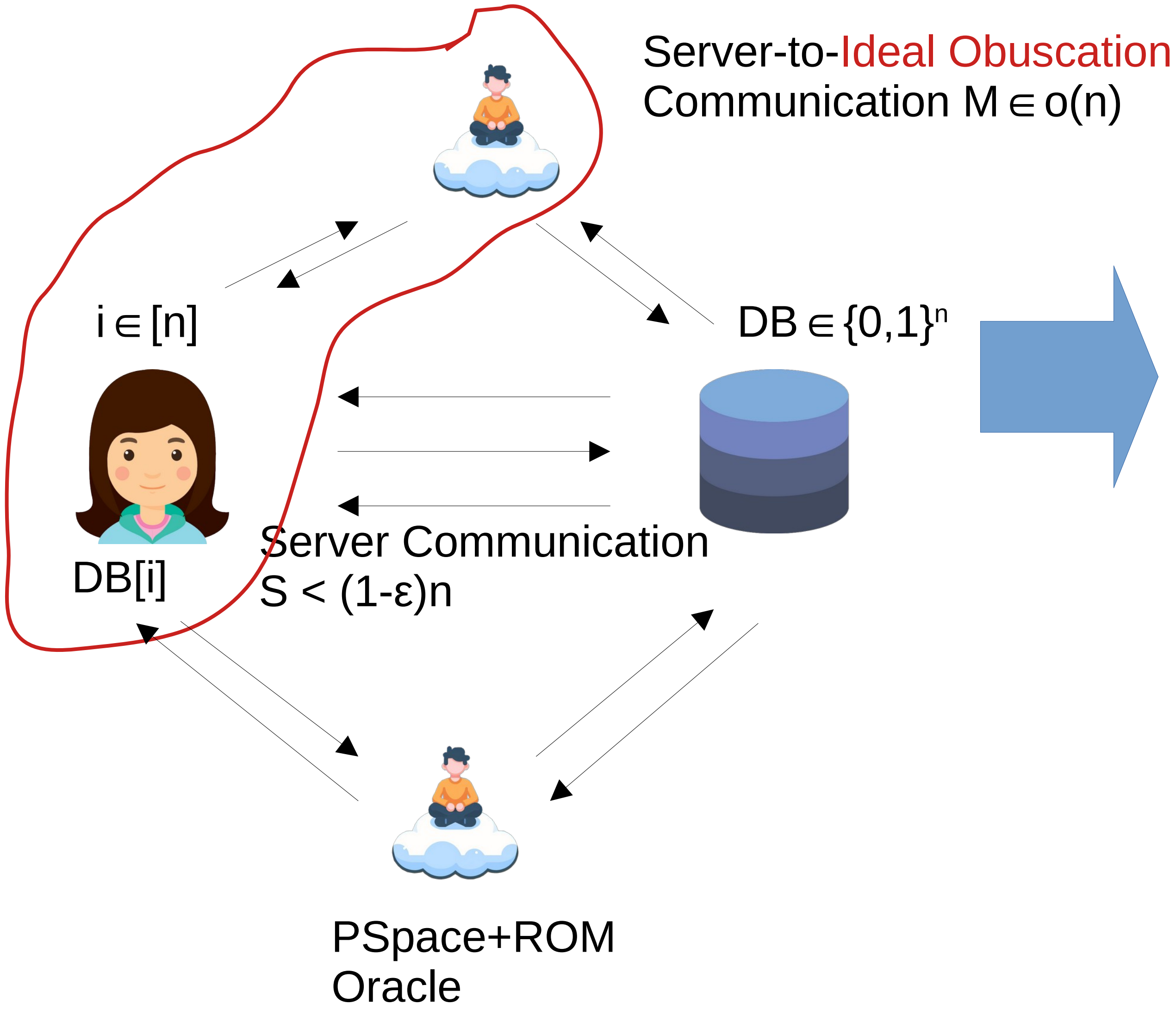
$DB \in \{0,1\}^n$

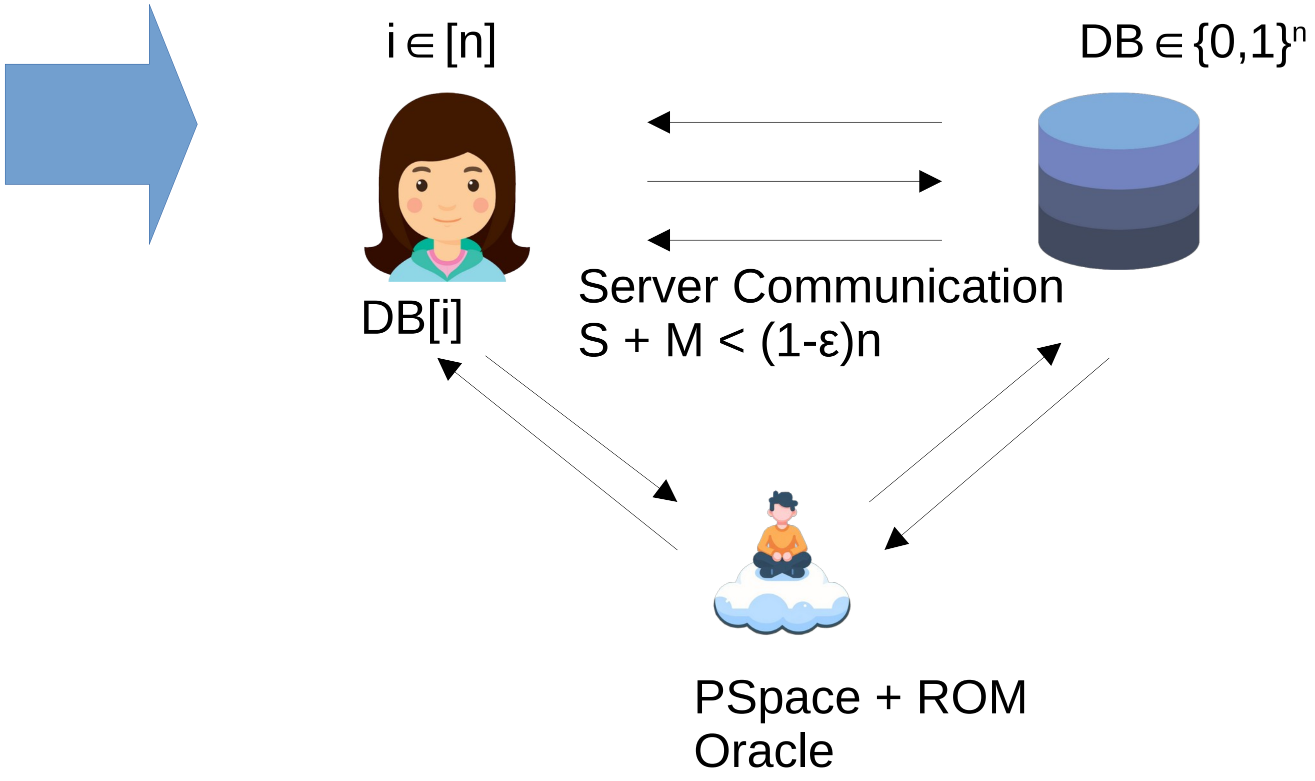
DB[i]

Server Communication
$S < (1-\varepsilon)n$

PSpace+ROM
Oracle

PIR in the Simulatable Oracle + PSpace + ROM

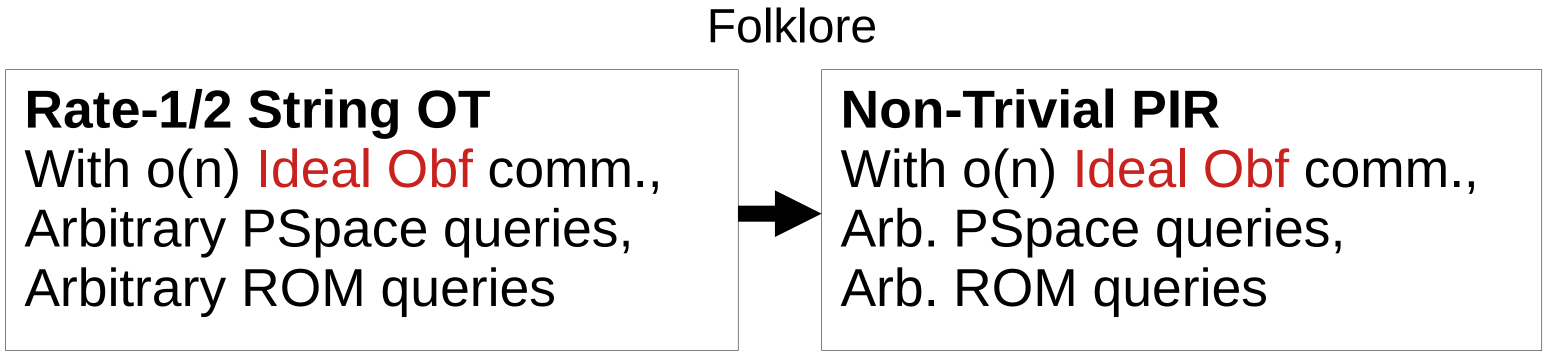




Consequences cont. (3)

Rate-1/2 String OT
With o(n) Ideal Obf comm.,
Arbitrary PSpace queries,
Arbitrary ROM queries

Consequences cont. (3)

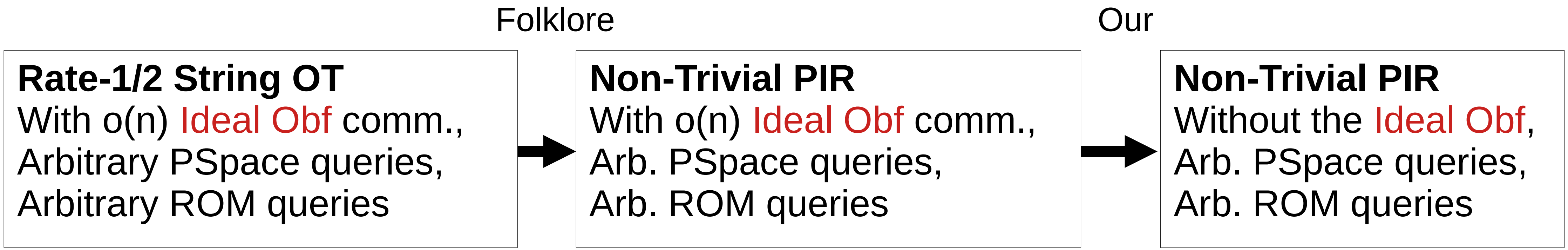


Folklore

Rate-1/2 String OT
With o(n) Ideal Obf comm.,
Arbitrary PSpace queries,
Arbitrary ROM queries

Non-Trivial PIR
With o(n) Ideal Obf comm.,
Arb. PSpace queries,
Arb. ROM queries

Consequences cont. (3)



Folklore

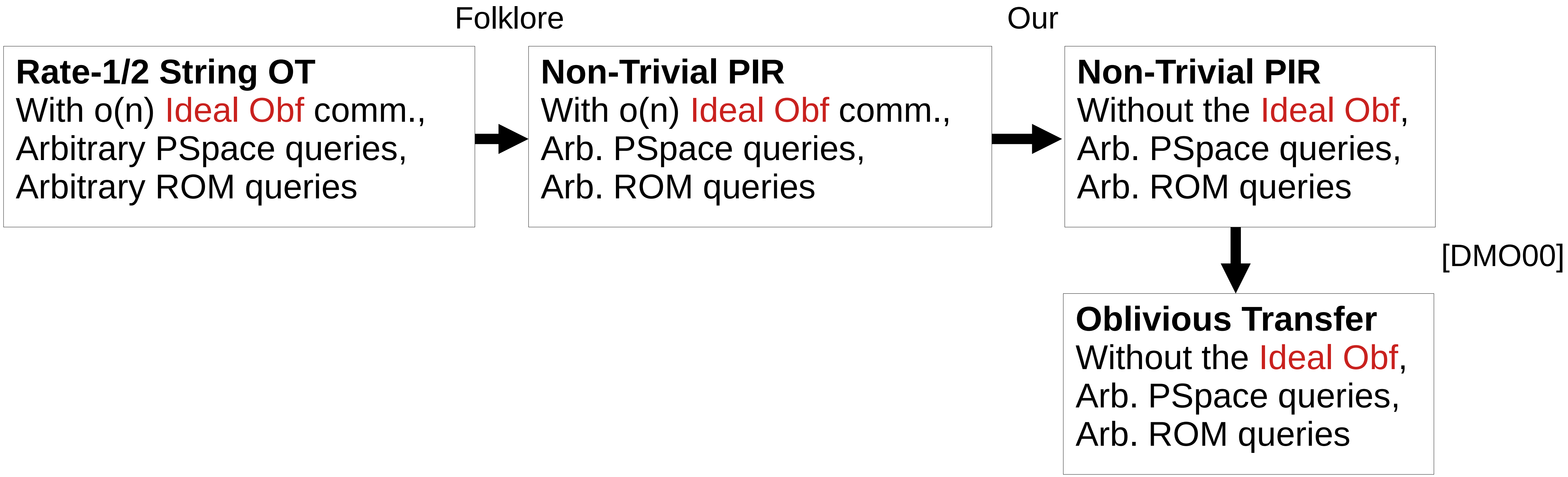
Our

Rate-1/2 String OT
With o(n) Ideal Obf comm.,
Arbitrary PSpace queries,
Arbitrary ROM queries

Non-Trivial PIR
With o(n) Ideal Obf comm.,
Arb. PSpace queries,
Arb. ROM queries

Non-Trivial PIR
Without the Ideal Obf,
Arb. PSpace queries,
Arb. ROM queries

Consequences cont. (3)



Folklore

Our

Rate-1/2 String OT
With o(n) Ideal Obf comm.,
Arbitrary PSpace queries,
Arbitrary ROM queries

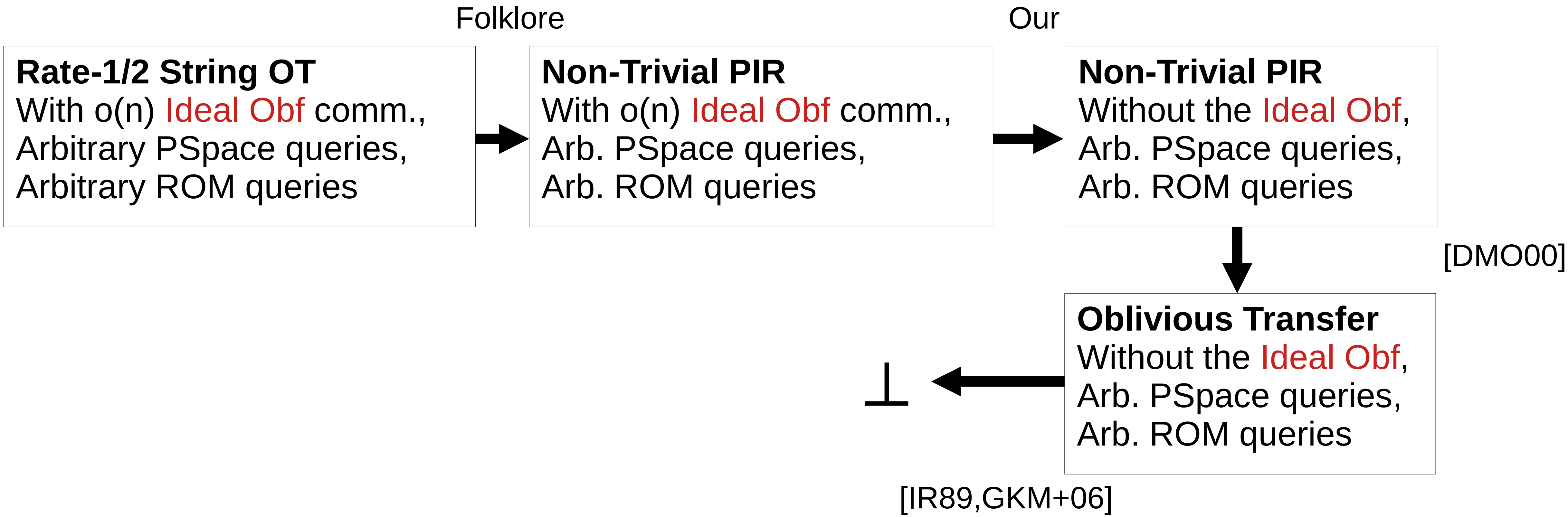
Non-Trivial PIR
With o(n) Ideal Obf comm.,
Arb. PSpace queries,
Arb. ROM queries

Non-Trivial PIR
Without the Ideal Obf,
Arb. PSpace queries,
Arb. ROM queries

[DMO00]

Oblivious Transfer
Without the Ideal Obf,
Arb. PSpace queries,
Arb. ROM queries

Consequences cont. (3)



Folklore

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[DMO00]

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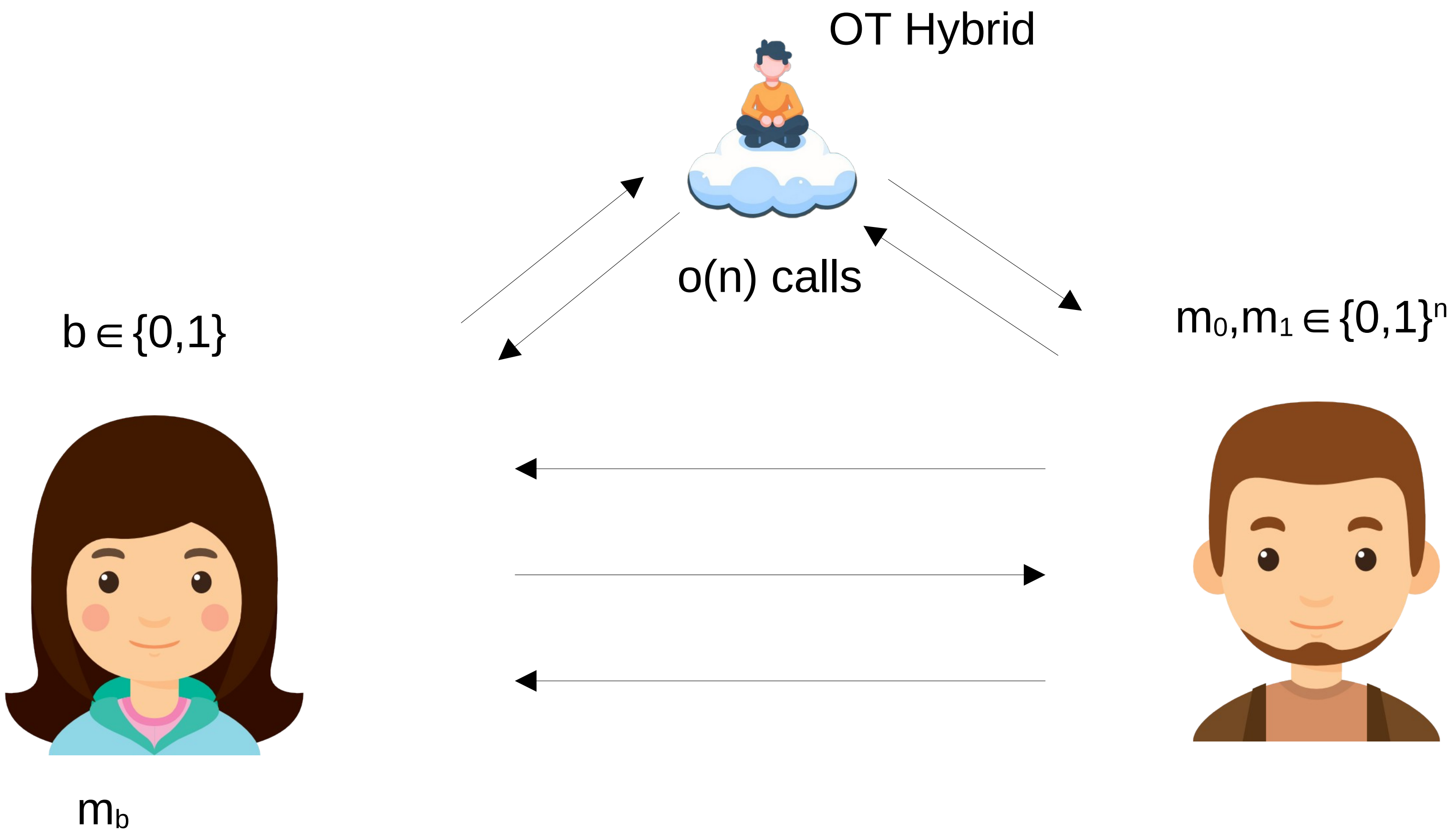
Arb. ROM queries

$\perp$

[IR89,GKM+06]

OT Extensions (Weak Variant)



OT Hybrid

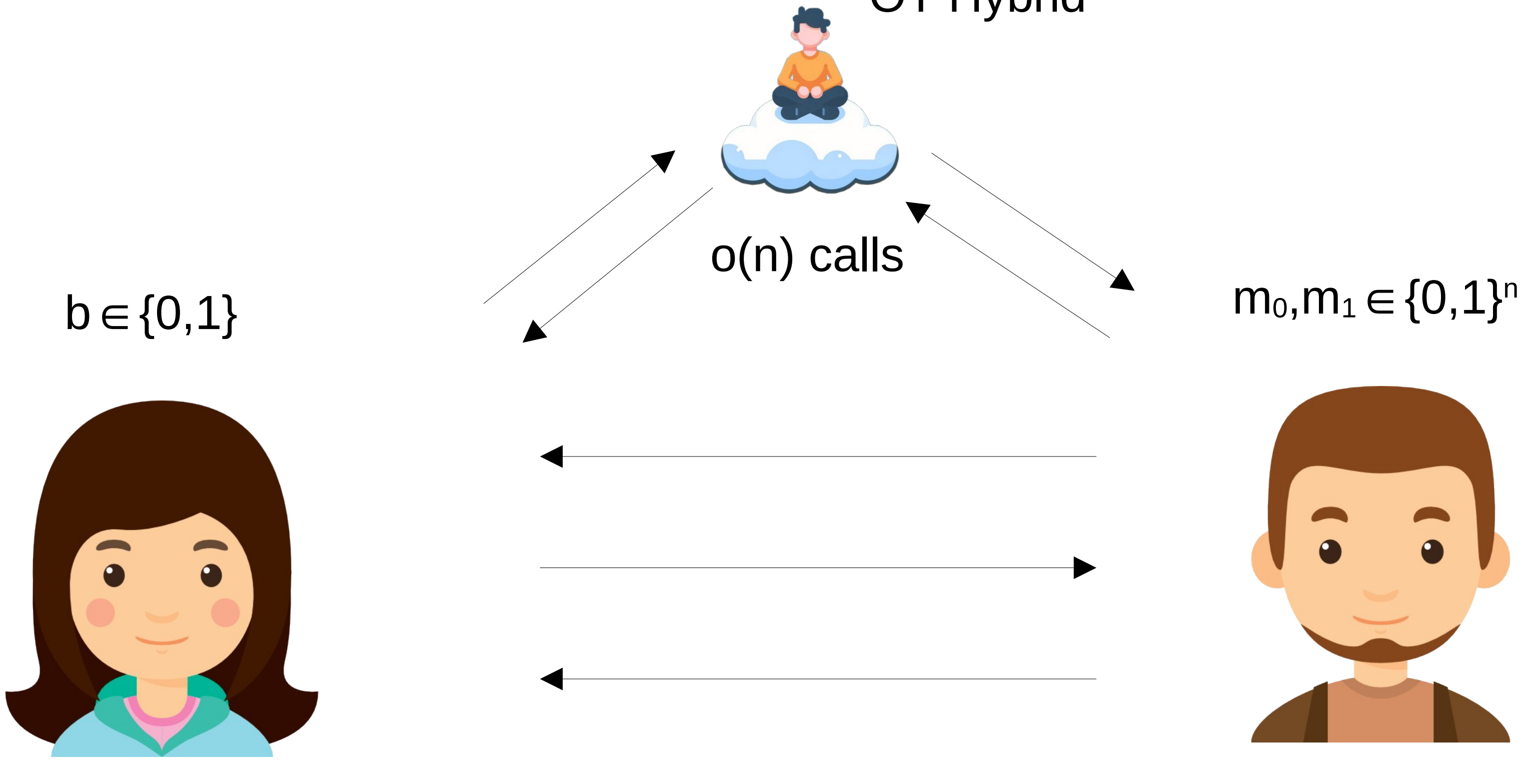
o(n) calls

$b \in \{0,1\}$

$m_0, m_1 \in \{0,1\}^n$

m_b

OT Extensions (Weak Variant)



OT Hybrid

o(n) calls

$b \in \{0,1\}$

$m_0, m_1 \in \{0,1\}^n$

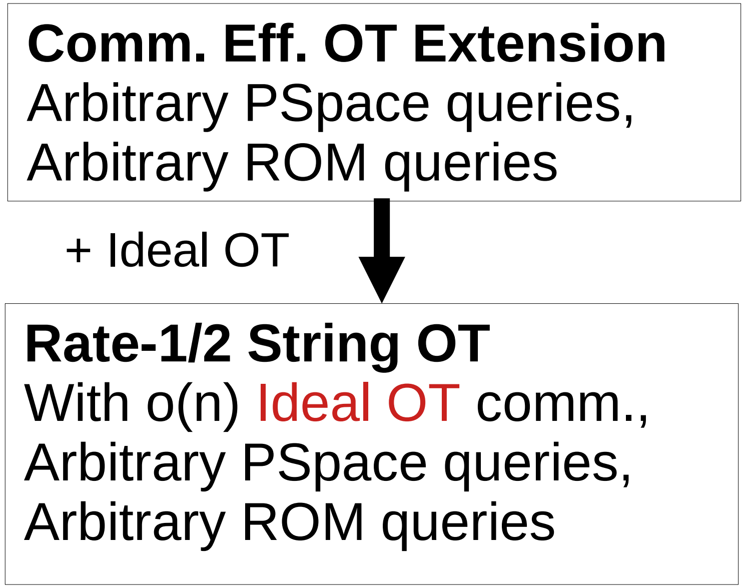
m_b

Communication < $(1-\varepsilon)2n$

OT Extension Consequences

Comm. Eff. OT Extension
Arbitrary PSpace queries,
Arbitrary ROM queries

OT Extension Consequences



Comm. Eff. OT Extension
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+ Ideal OT

Rate-1/2 String OT
With o(n) Ideal OT comm.,
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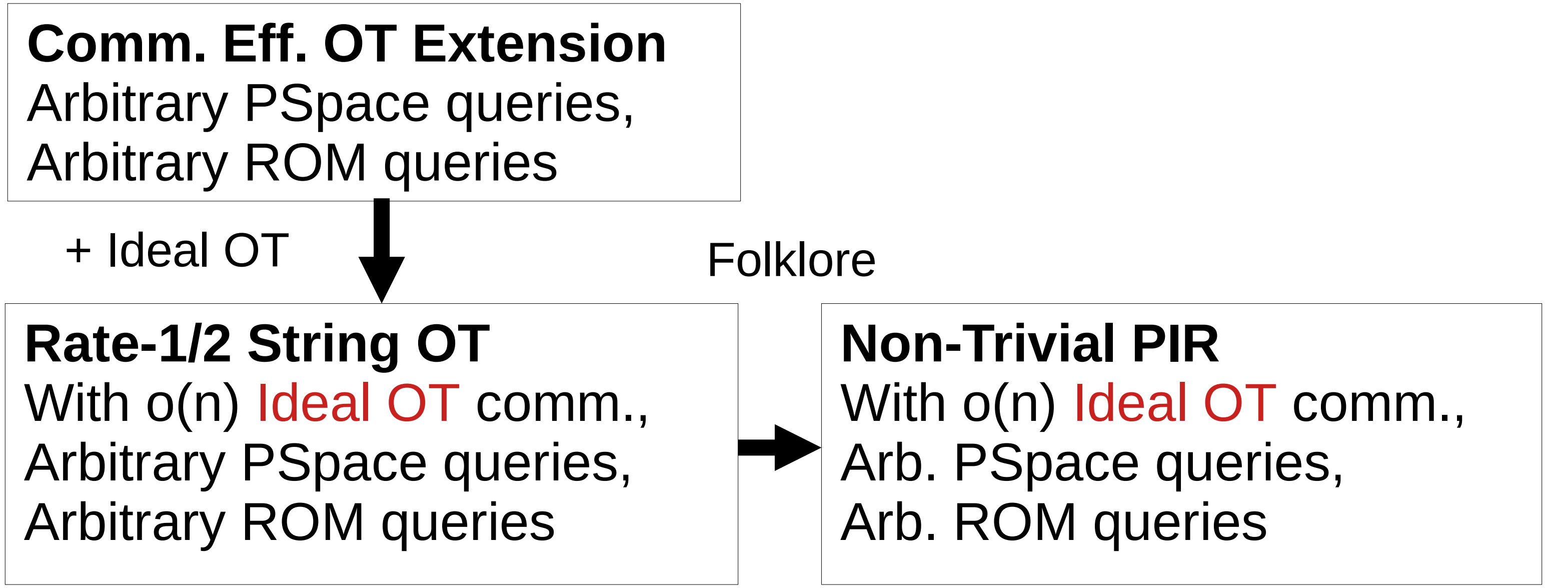
OT Extension Consequences



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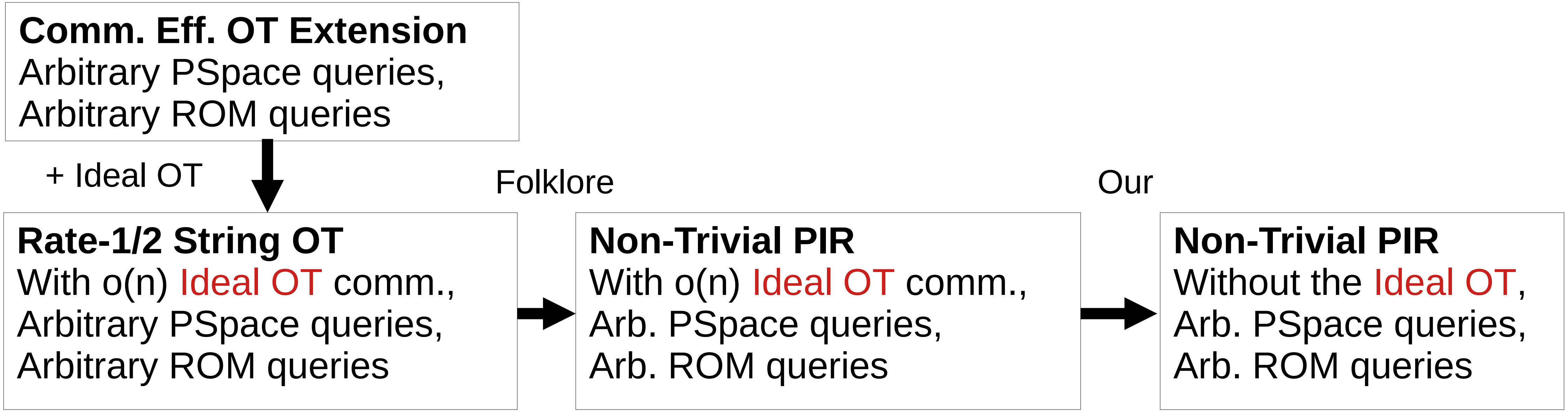

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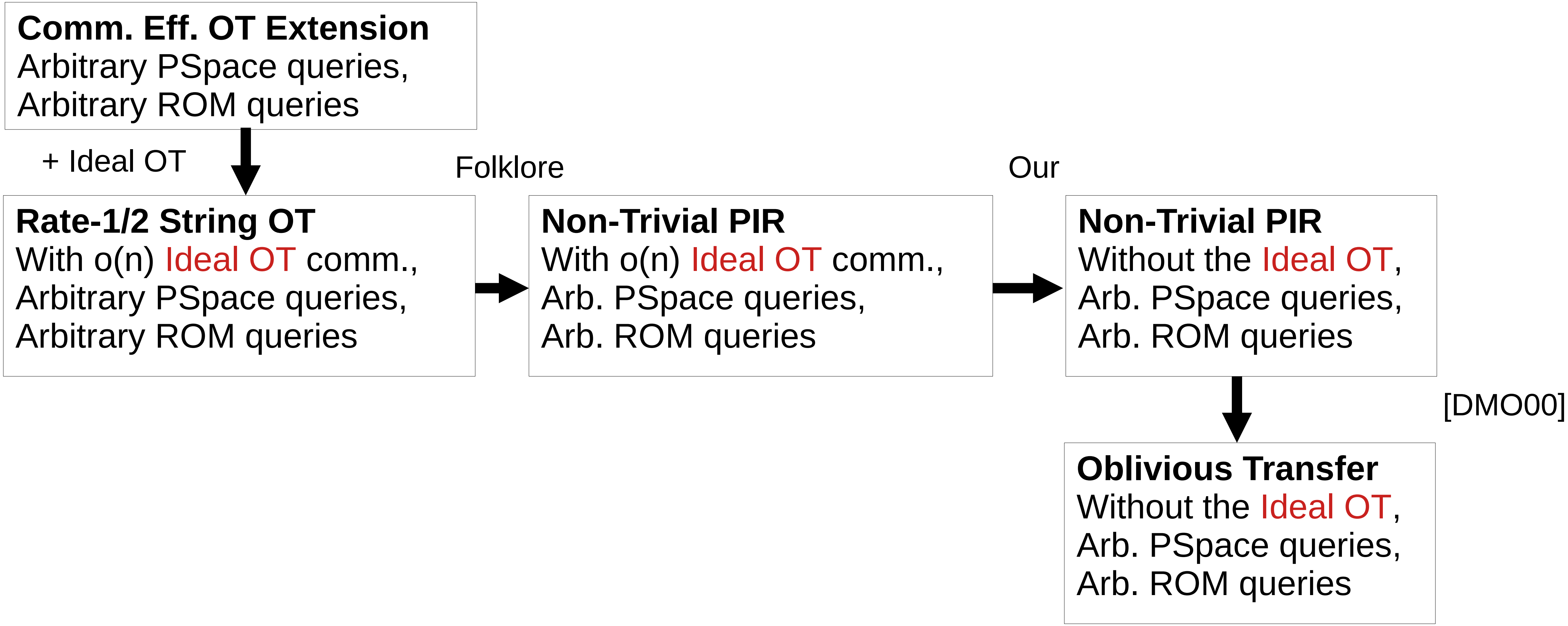
OT Extension Consequences



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[DMO00]

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Without the Ideal OT,
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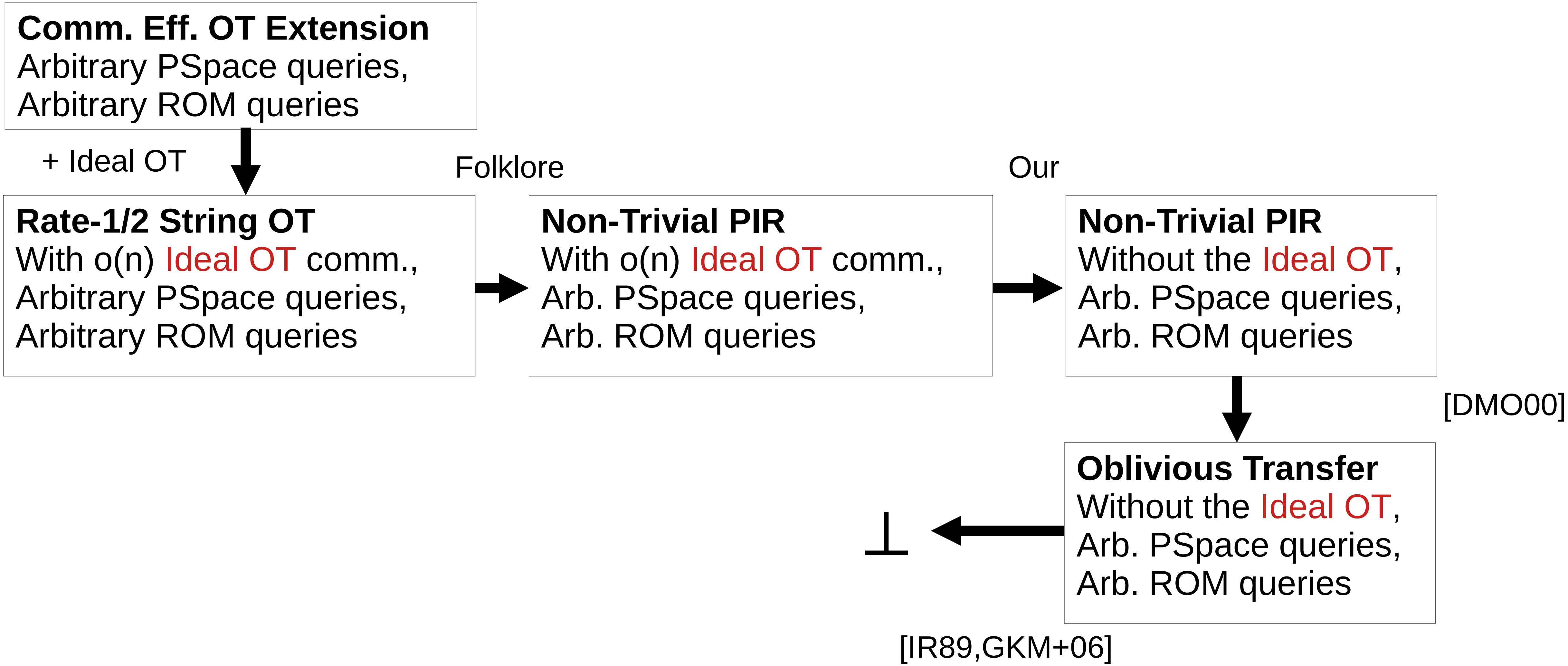
Arb. ROM queries

OT Extension Consequences



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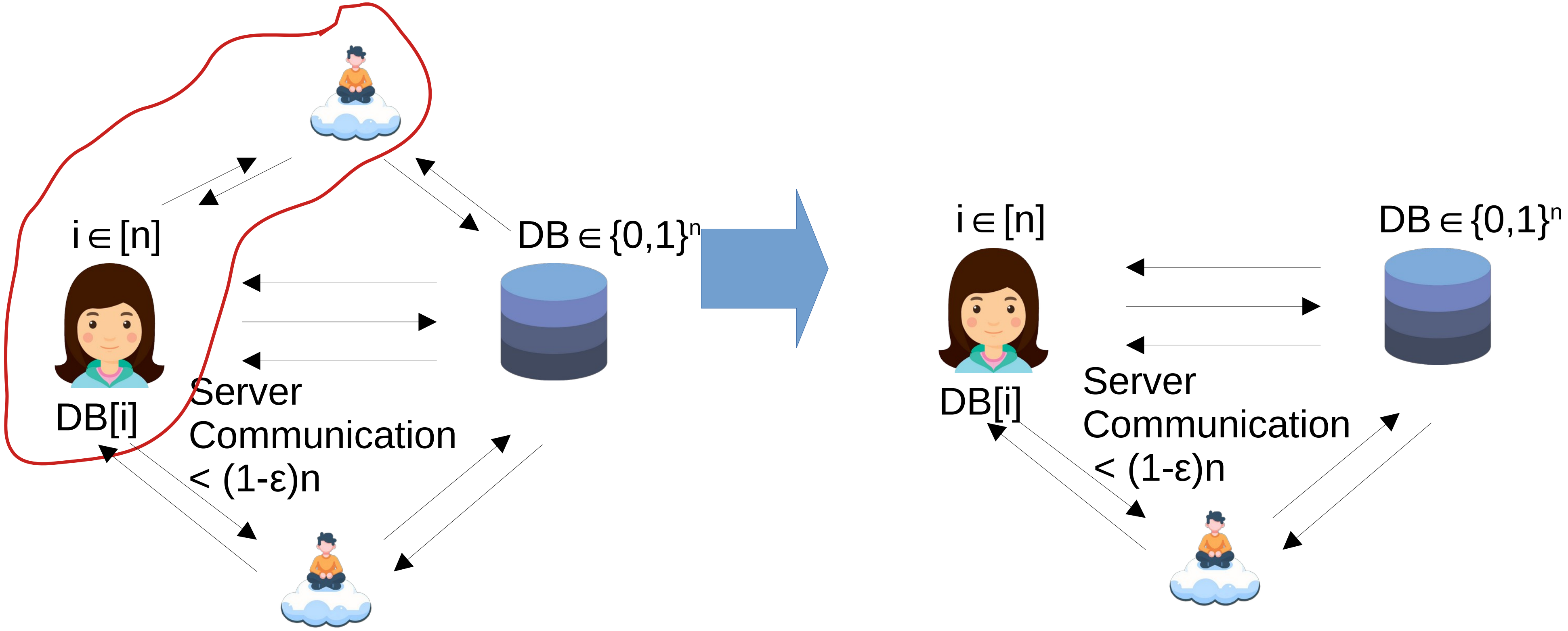
$\perp$

[IR89,GKM+06]

Thank you!


i ∈ [n]
DB ∈ {0,1}ⁿ
DB[i]
Server
Communication
< (1-ε)n
i ∈ [n]
DB ∈ {0,1}ⁿ
DB[i]
Server
Communication
< (1-ε)n