Unbiasable Verifiable Random Functions

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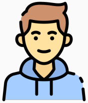
Verifiable Random Functions [MRV99]

VRF.Gen, VRF.Eval, VRF.Vfy



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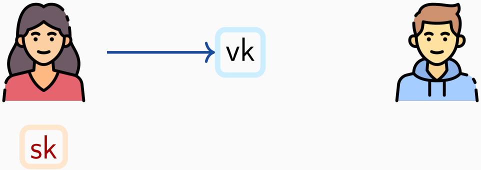

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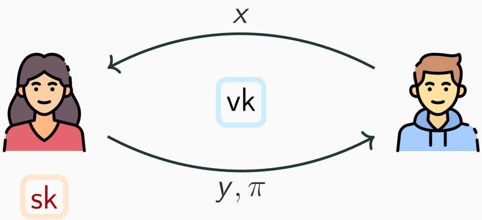

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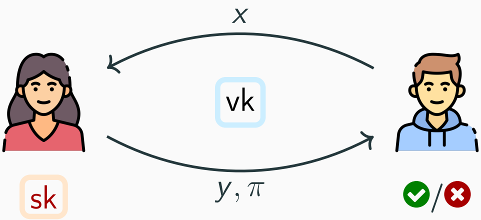

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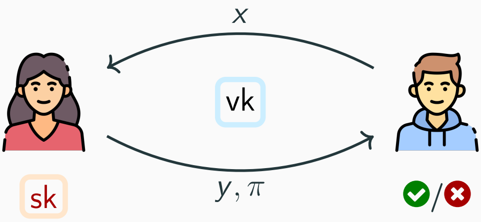
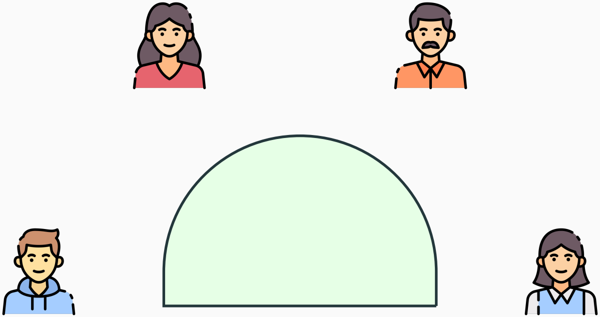


Uniqueness

Pseudorandom

Poker VRF ♡ ♢ ♣ ♠



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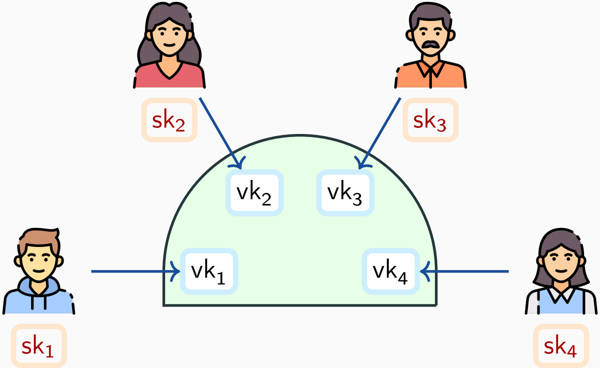

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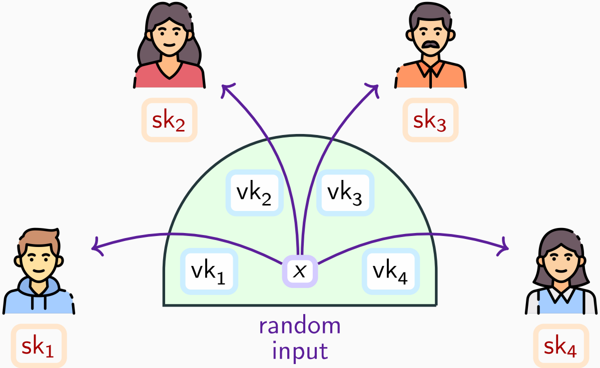

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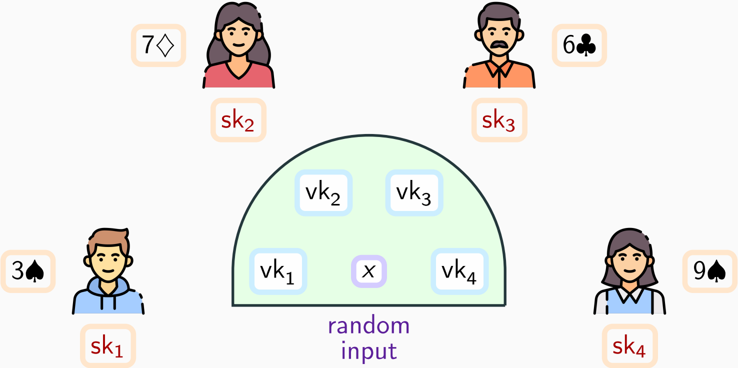

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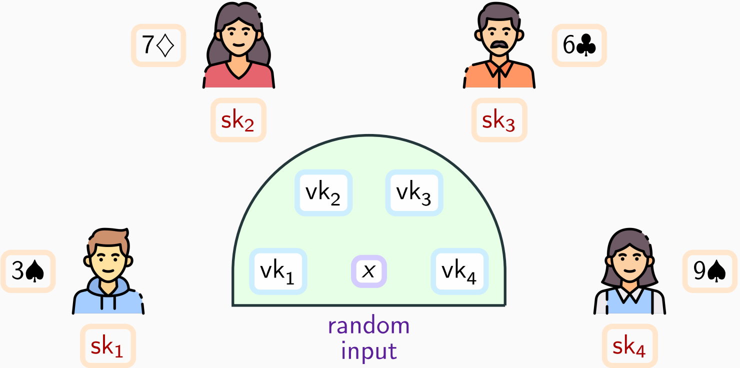
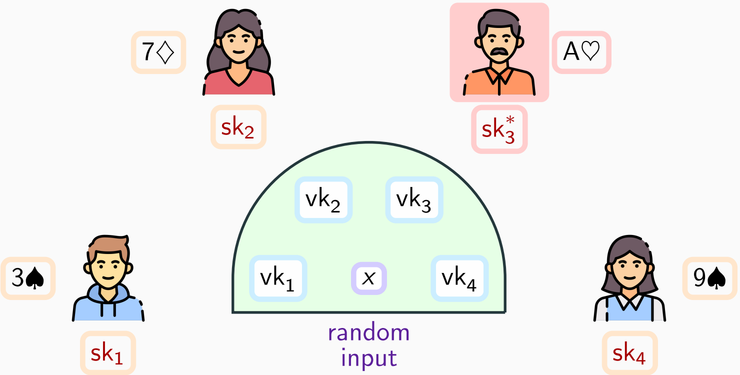


Unpredictability ✓

Fairness ✗

Poker VRF



Unpredictability ✔

Fairness ✖

Poker VRF

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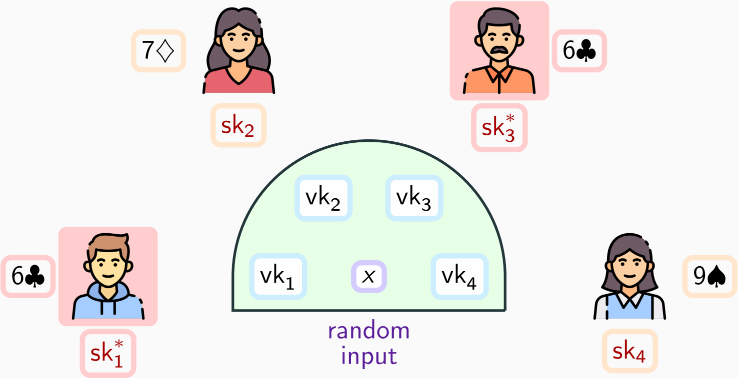


Unpredictability ✓

Fairness ✗

VRF Unbiasability

[EKS+21]

(Weak) Unbiasability

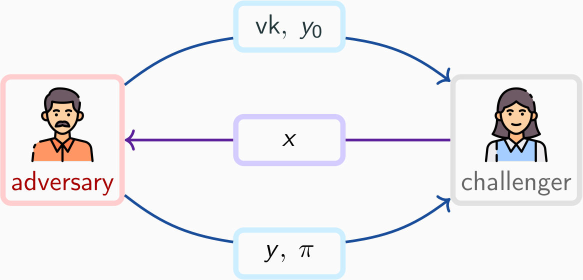
- ✓ Standard model
- ✗ Insufficient for SLE

[DGKR17]

UC-VRF

- ✗ Requires ROM
- ✓ Implies SLE

(Weak) Unbiasability [EKS+21]



Win if $y = y_0$ and π is correct.

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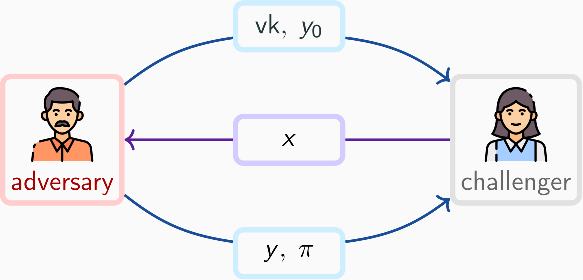

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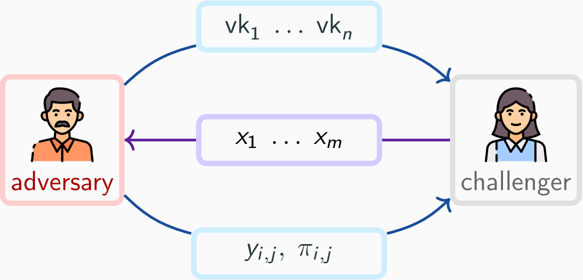
- ✖ Some output bits could be biased
- ✖ Different key's output could be correlated

Unbiasability (First attempt)



Win if $\pi_{i,j}$ are **correct** $y_{i,j}$ are **far** from random.

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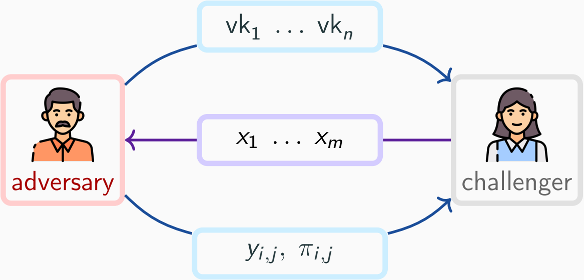

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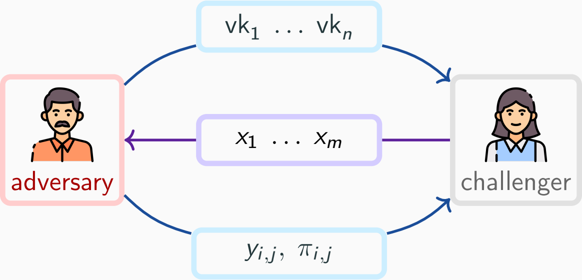
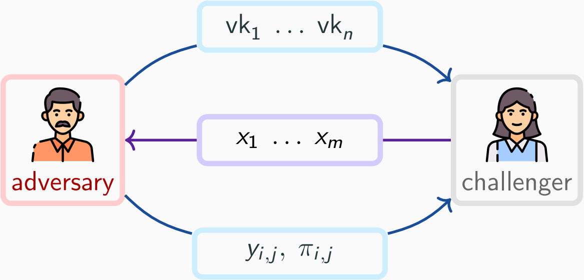
❌ Does not exclude biased vk^* that cannot be evaluated on the full domain

Unbiasability (Second attempt)



1. If $\pi_{i,j}$ is incorrect, **erase** $y_{i,j} \leftarrow \perp$
2. Win if **non-erased** $y_{i,j}$ are **far** from random

Unbiasability (Second attempt)



1. If $\pi_{i,j}$ is incorrect, **erase** $y_{i,j} \leftarrow \perp$
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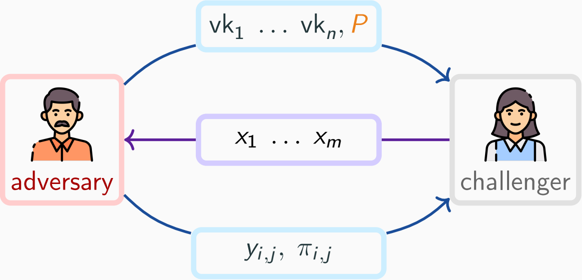
✖ Selective openings are always biased

Unbiasability

[GS24]



1. P is **monotone**[1].
2. If $\pi_{i,j}$ is incorrect, **erase** $y_{i,j} \leftarrow \perp$.
3. Win if $\Pr[P(y) = 1] \gg \Pr[P(z) = 1]$ for a **random** z.

[1] Let x^* be x with some positions erased. Then $P(x^*) \leq P(x)$.

Constructions

Our Constructions

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1. VUF to VRF in the Random Oracle Model.
2. Verifiable Random **Bijection** (VRB) from any VRF + DL.
3. **Unbiasable VRF** from any VRF + DL* + PRF*.

VUF to VRF transform in the ROM

$F_{\mathsf{sk}} : X \to Y$ **V**erifiable **U**npredictable **F**unction with public key vk.

1st Transform: $F^*_{\mathsf{sk}}(x) = H(F_{\mathsf{sk}}(x), x, \mathsf{vk})$.

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- Both are VRF in the ROM.
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    1. $F_{\mathsf{sk}}(x_1) \neq F_{\mathsf{sk}}(x_2)$ for random x_1, x_2.
    2. $F_{\mathsf{sk}_1}(x) \neq F_{\mathsf{sk}_2}(x)$ for random x and $\mathsf{sk}_1 \neq \mathsf{sk}_2$.

Verifiable Random Bijection

$\mathbb{G}$ prime order group where DL is hard.

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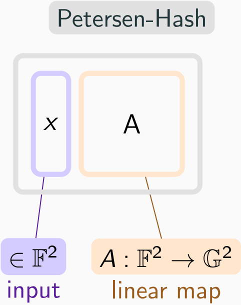

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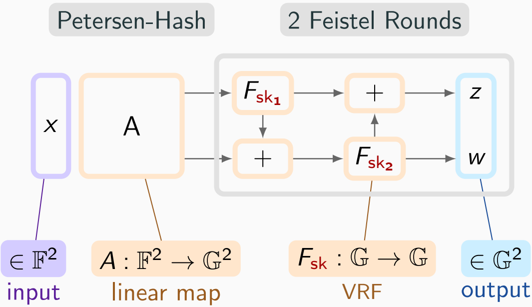

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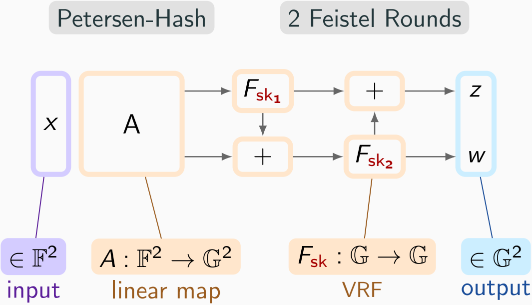

✓ Certified bijection

Verifiable Random Bijection

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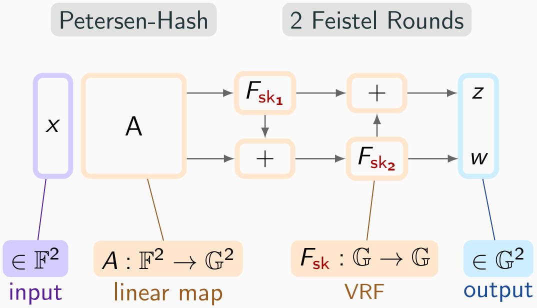

- ✔ Certified bijection
- ✘ Unbiasable only for a single vk!

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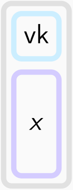

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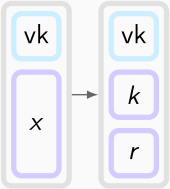

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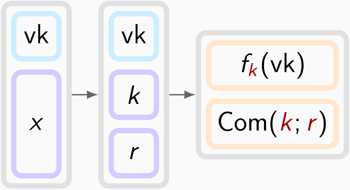

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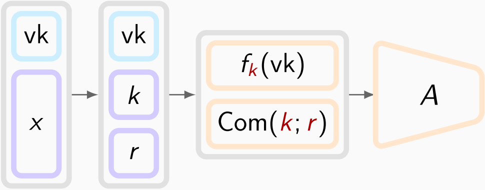

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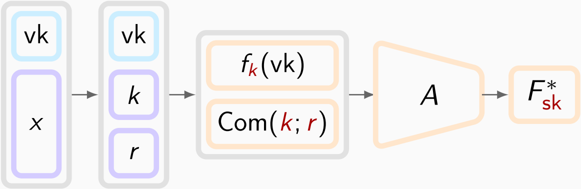

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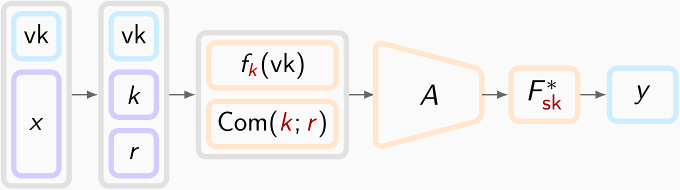

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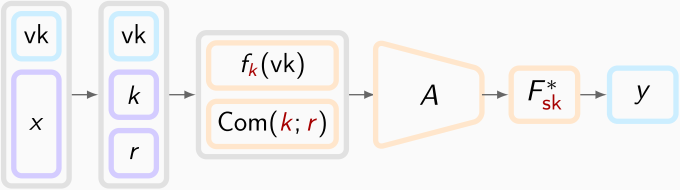

Unbiasable if **DL** and the **PRF** are secure with **preprocessing**.

Conclusions

We provided a new notion of **unbiasability** that is:

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Thanks for your attention!