

Ordering Transactions with Bounded Unfairness: Definitions, Complexity and Constructions

<https://eprint.iacr.org/2023/1253>

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1. Introduction

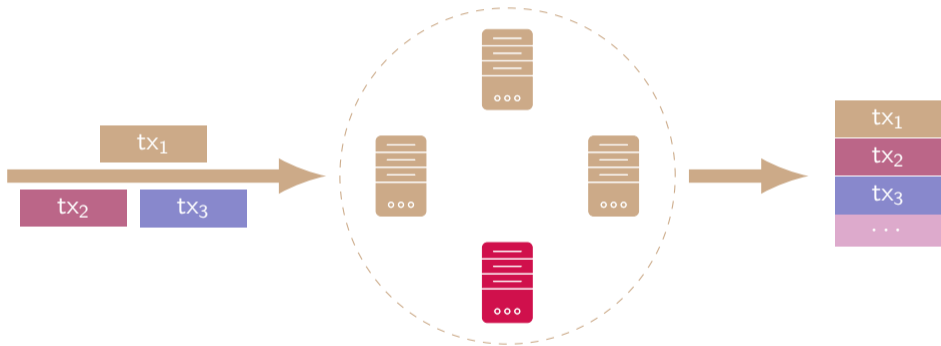
2. Directed-Bandwidth Order Fairness

3. Protocol Overview

4. Fairness vs. Liveness

5. Takeaways

State Machine Replication



State Machine Replication (Cont'd)

- ✓ **Consistency:** Honest parties output the same log (prefix).
- ✓ **Liveness:** New transactions are processed timely.

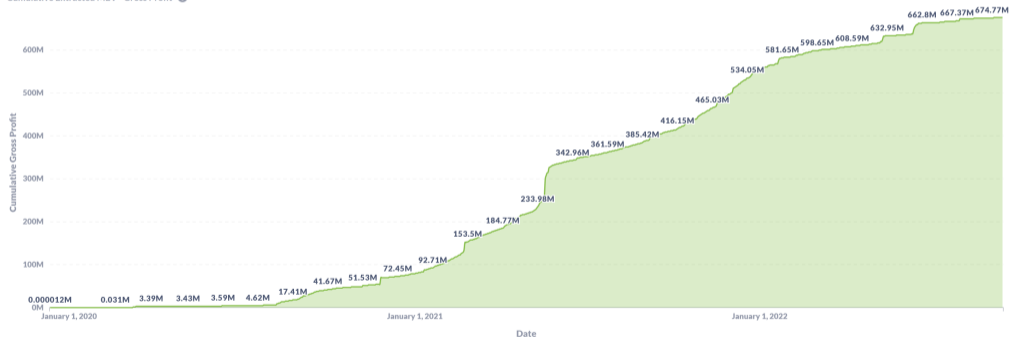
State Machine Replication (Cont'd)

- ✓ **Consistency:** Honest parties output the same log (prefix).
- ✓ **Liveness:** New transactions are processed timely.
- ✗ **Order-fairness:** Emulate the following behavior: A central server processes the commands it receives **sequentially**, in a **First-Come-First-Served** manner.
 - Re-ordering attacks: front-running, sandwich attacks, etc..

Maximal Extractable Value (MEV)

- Total Extracted MEV (Dec 2019 - Sep 2022) : \$675,623,114.¹

Cumulative Extracted MEV - Gross Profit 📄



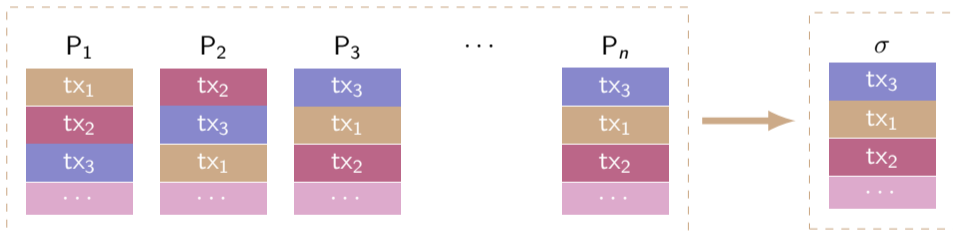
[1] Source: <https://explore.flashbots.net/>

Sender/Receiver Order Fairness

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- **Receiver order fairness:** Order transactions based on the time that they are received by protocol participants.



Condorcet Cycles

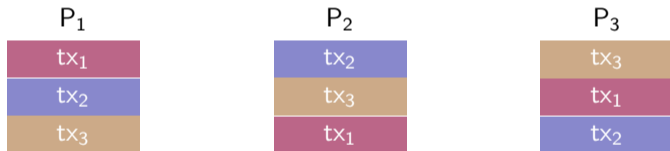
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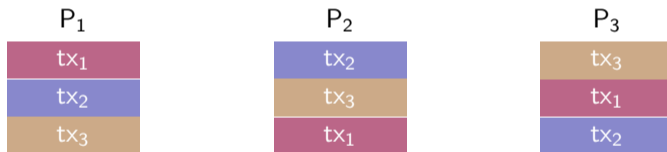
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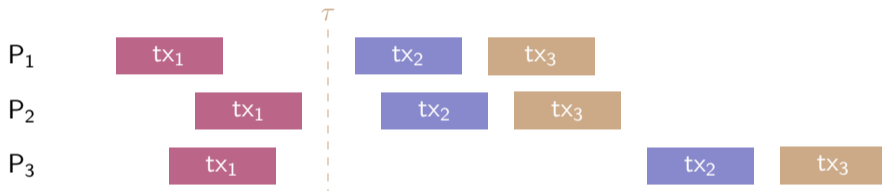
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- $tx_1 \prec tx_2, tx_2 \prec tx_3, tx_3 \prec tx_1$
- Cycles can be chained to arbitrary size.

Timed Order Fairness

- Order $tx \prec tx'$ if their receiving time are sufficiently separated by a time τ . [Zha+20; Kur20]

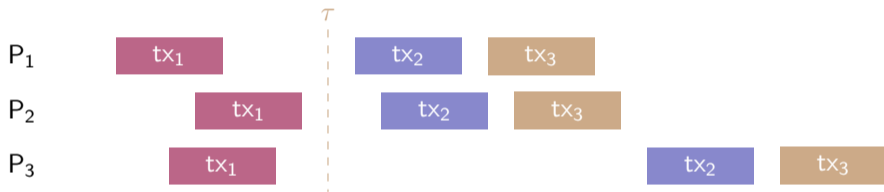


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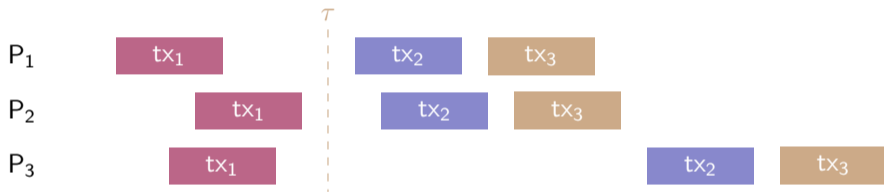
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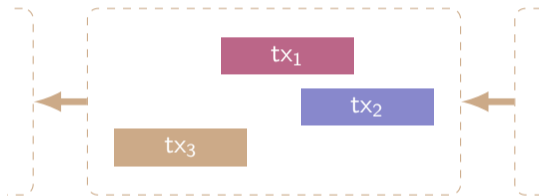
- We have $tx_1 \prec tx_2$ above; yet the order of tx_2 and tx_3 remains unspecified, even if honest parties unanimously saw tx_2 before tx_3 .
- ✗ Give up on ordering many transactions when their dissemination windows overlap.

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Batch Order Fairness

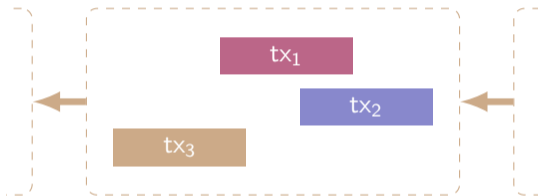
- If Condorcet cycle exists, order them in a “batch”. [Kel+20]



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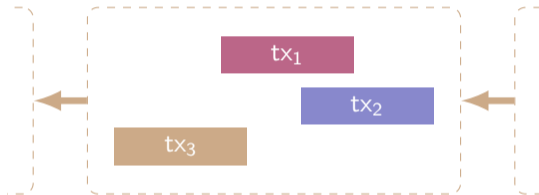


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- ✗ Give up on assigning unique index to each transaction.

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Bounded Unfairness

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- In the context of state machine replication this translates to an important guarantee: Can we minimize the number of **unfair state updates** occurring prior to any given transaction?
 - For example, in DeFi bounded unfairness can minimize the “unfair slippage” when multiple users interact with an AMM.

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Definition ((φ, B)-fair-order). Profile σ is a (φ, B) -fair-order on P_1, \dots, P_n if for all tx, tx' such that $\text{tx} \prec^\varphi \text{tx}'$, it holds that $\sigma(\text{tx}) - \sigma(\text{tx}') \leq B$ where B is a function of $P_1, \dots, P_n, \varphi, \text{tx}$ and tx' .

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 - E.g., $B = |\sigma| - 1$ for some σ, tx, tx' .

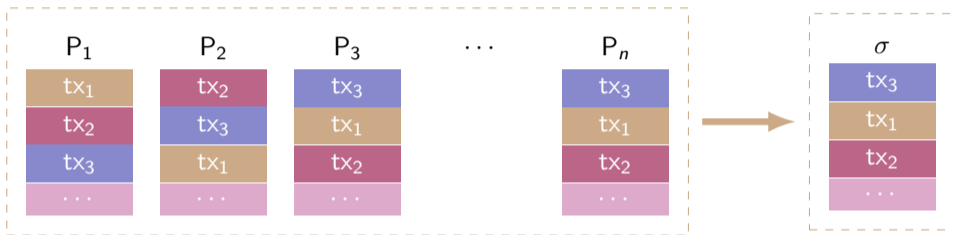
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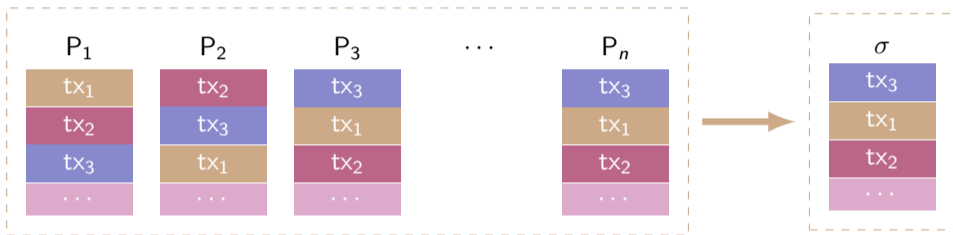
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Question: How to explicitly define the smallest possible function B ?

Transaction Dependency Graphs

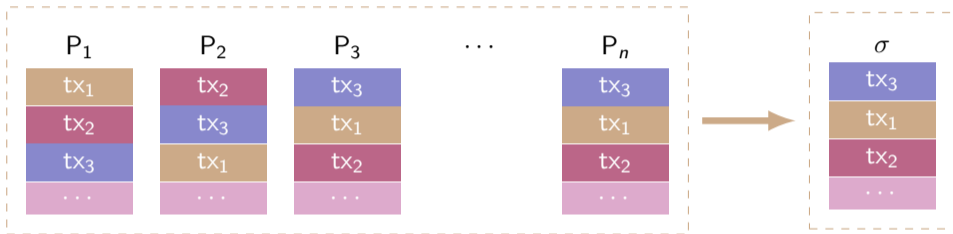


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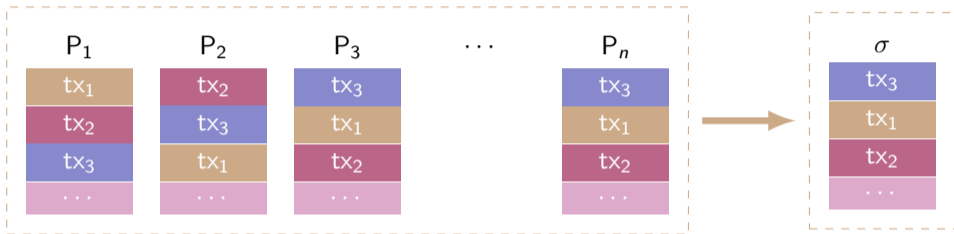
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 - For each transaction, add a node; for $tx \prec^\varphi tx'$, add an edge $tx \rightarrow tx'$.
- Condorcet cycles = Strongly connected components.
- Large $\varphi \implies$ large cycles.

Theorem. $G(\mathcal{P}, \varphi)$ does not contain a cycle of size $k < 1/(1 - \varphi)$.

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 - Backward edges imply that transactions were ordered “unfairly.”

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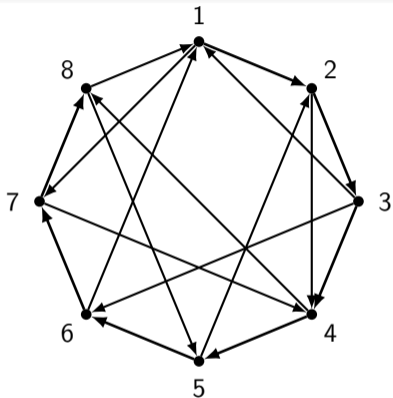
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Definition (Directed Bandwidth). Given a directed graph $G = (V, E)$, DIRECTEDBANDWIDTH asks to find a vertex ordering σ^* such that $\text{DBW}(\sigma^*, G) = \min_{\sigma} \text{DBW}(\sigma, G)$ where

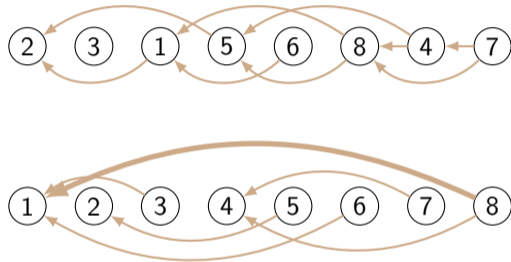
$$\text{DBW}(\sigma, G) = \max_{\substack{(u,v) \in E, \\ \sigma(u) > \sigma(v)}} \sigma(u) - \sigma(v).$$

The directed bandwidth of a graph G is $\text{DBW}(G) = \text{DBW}(\sigma^*, G)$.

Directed Bandwidth Problem (Cont'd)



Dependency graph $G_{\mathcal{P}, \varphi}$



Bandwidth-optimal vertex ordering (above)
and a "bad" ordering (below)

Directed Bandwidth Problem (Cont'd)

Theorem. DIRECTEDBANDWIDTH is NP-hard and NP-hard to approximate within any constant ratio over oriented graphs.

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Theorem. Let \mathbb{G}_n denote the set of all oriented graphs with n vertices. It holds that

$$n - 4 \log n < \max_{G \in \mathbb{G}_n} \text{DBW}(G) < n - \log n/2.$$

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- DIRECTEDBANDWIDTH can be solved trivially in $\mathcal{O}^*(n!)$ time.
- DIRECTEDBANDWIDTH can be solved in $\mathcal{O}^*(2^{|E|} \cdot 3^{|V|})$ time. [Jai+19]

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Directed-Bandwidth Order Fairness

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$$\sigma(tx) - \sigma(tx') \leq \text{DBW}(\text{SCC}(G(\mathcal{P}, \varphi), tx, tx')),$$

where $\text{SCC}(G, tx, tx')$ is a function that outputs an SCC in G that contains both tx, tx' if it exists, and a null graph otherwise.

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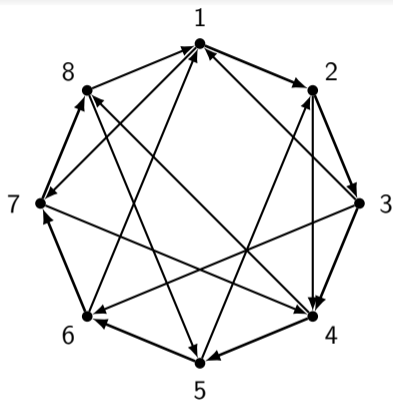
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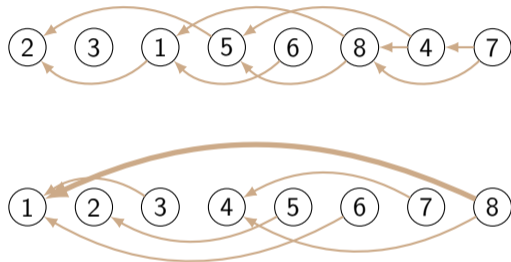
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- $tx \prec^\varphi tx'$ and tx, tx' are not in the same cycle $\implies \sigma(tx) < \sigma(tx')$.
- $tx \prec^\varphi tx'$ and tx, tx' are in the same cycle $C \implies \sigma(tx) - \sigma(tx') < \text{DBW}(C)$.

Directed-Bandwidth Order Fairness (Cont'd)



Dependency graph $G_{\mathcal{P}, \varphi}$



Serialization with bounded unfairness (above)
and a “bad” serialization (below)

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Theorem. *Suppose that a protocol implements (φ, B) -fair-order for a function B . Then for all \mathcal{P} there are tx, tx' with $\text{tx} \prec^\varphi \text{tx}'$, such that B satisfies $B(\mathcal{P}, \varphi, \text{tx}, \text{tx}') \geq \text{DBW}(\text{SCC}(G(\mathcal{P}, \varphi), \text{tx}, \text{tx}'))$.*

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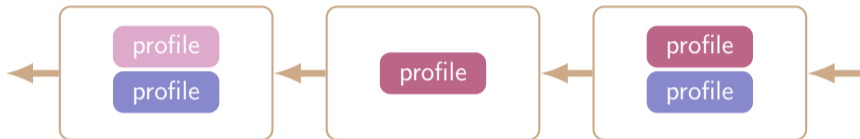
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Protocol Design



- 1 Parties use 2×1 PoW to mine blocks and **transaction profiles** and include valid profiles in the blockchain.
- 2 Build transaction dependency graph using majority preferences in profiles.
- 3 Run `DIRECTEDBANDWIDTH` algorithm on SCCs and serialize transactions.

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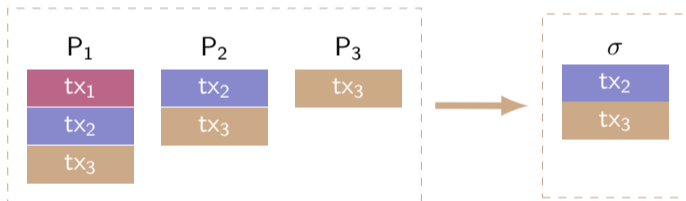
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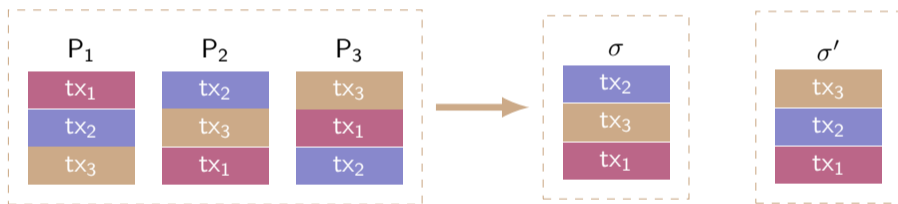
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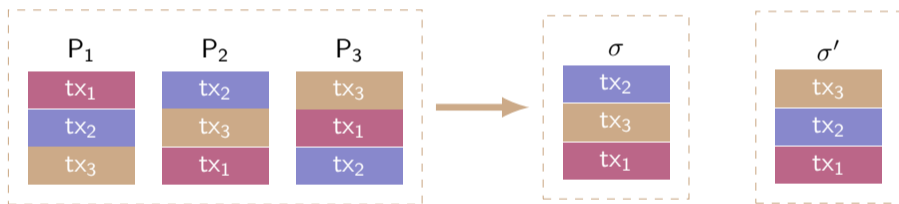
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Theorem. *Suppose the transaction dissemination is asynchronous, there is no protocol that can achieve consistency, liveness and (φ, DBW) -order-fairness.*

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Theorem. *Suppose the transaction dissemination is asynchronous, there is no protocol that can achieve consistency, liveness and (φ, DBW) -order-fairness.*

- We can hope to achieve “weak liveness.”

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- **Definitions:** We define fair transaction serialization with **bounded unfairness**.
- **Complexity:** We analyze the complexity of achieving bounded unfairness.
- **Constructions:** We design a new permissionless blockchain protocol that achieves consistency, weak liveness and bounded unfairness (DBW).
 - We also relax bounded unfairness into a “timed” version that enables the protocol to offer consistency, liveness and bounded unfairness.

Thank You

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References

- [Jai+19] Pallavi Jain, Lawqueen Kanesh, William Lochet, Saket Saurabh, and Roohani Sharma. “Exact and Approximate Digraph Bandwidth”. In: **39th IARCS Annual Conference on Foundations of Software Technology and Theoretical Computer Science, FSTTCS 2019, December 11-13, 2019, Bombay, India**. Ed. by Arkadev Chattopadhyay and Paul Gastin. Vol. 150. LIPIcs. Schloss Dagstuhl - Leibniz-Zentrum für Informatik, 2019, 18:1–18:15.
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