# Asymptotically Optimal Message Dissemination with Applications to Blockchains

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*Søren Eller Thomsen,* Partisia

# The flooding functionality

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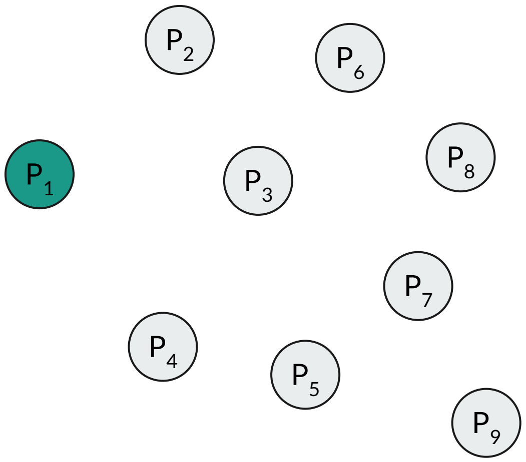


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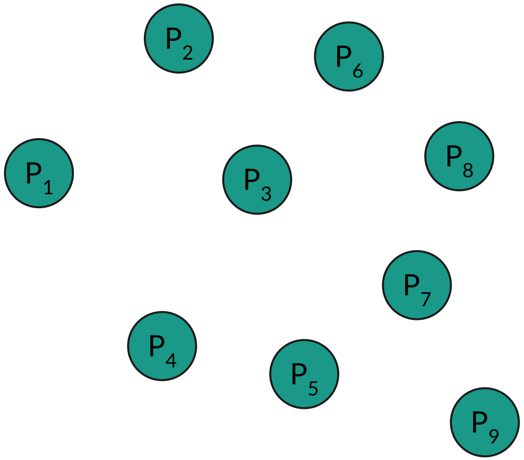


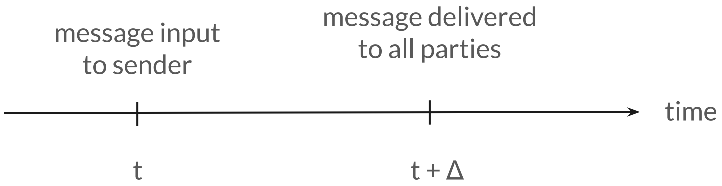

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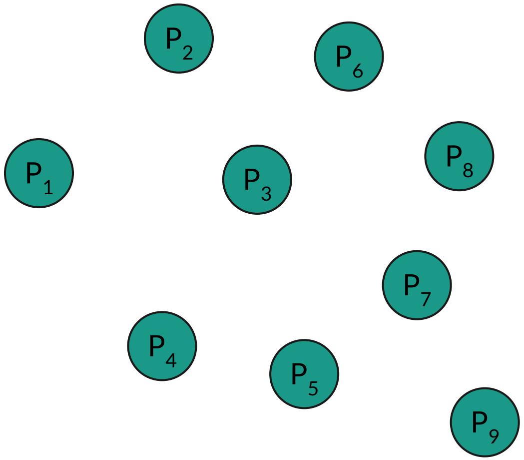
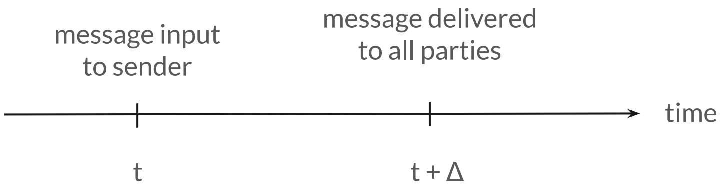



Probability that not all parties have received the message by time $t + \Delta$ must be negligible in security parameter



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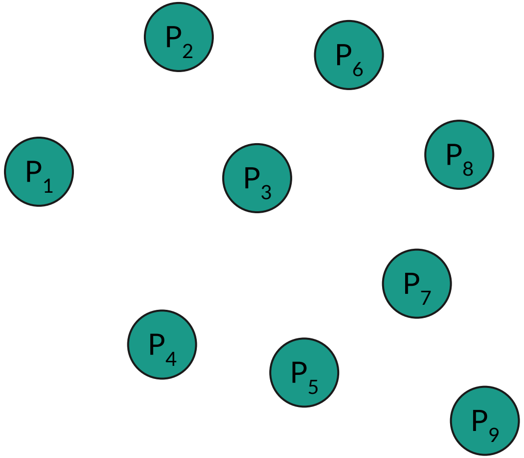


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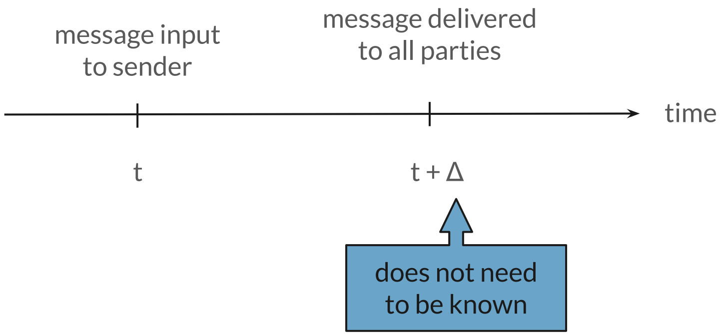
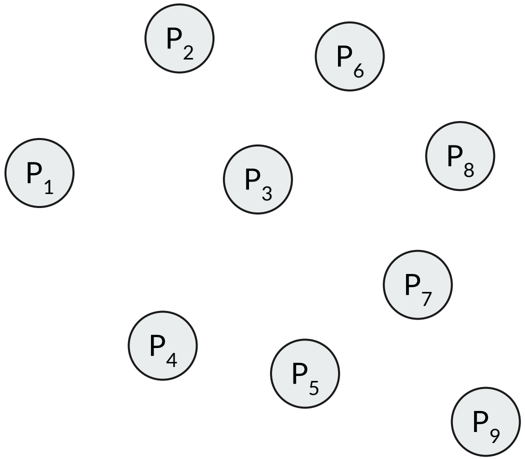


# Adversary

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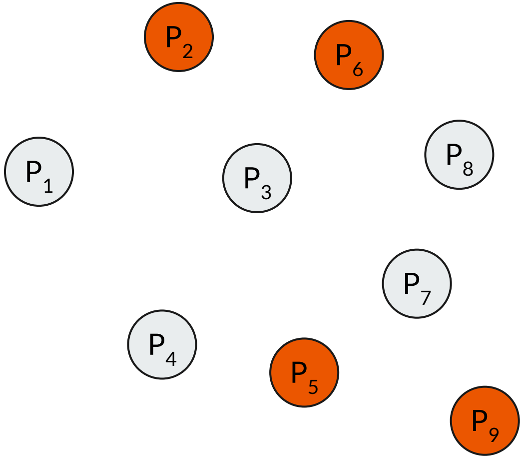
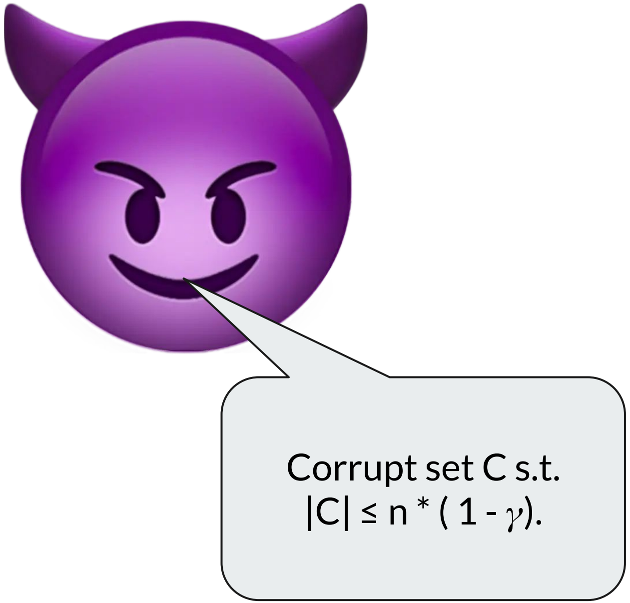
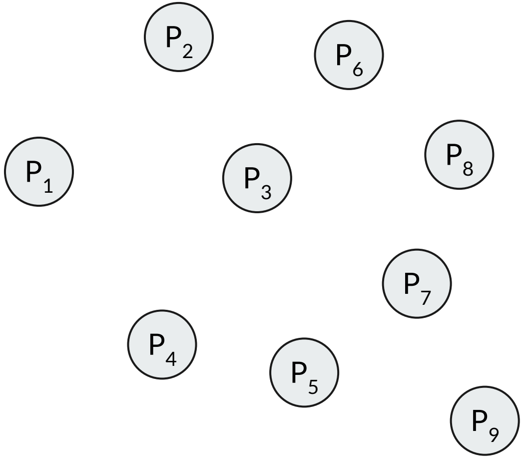


# Flooding protocols




**State of the art**

Each party forwards the message to a random subset of parties of (expected) size $\kappa$.

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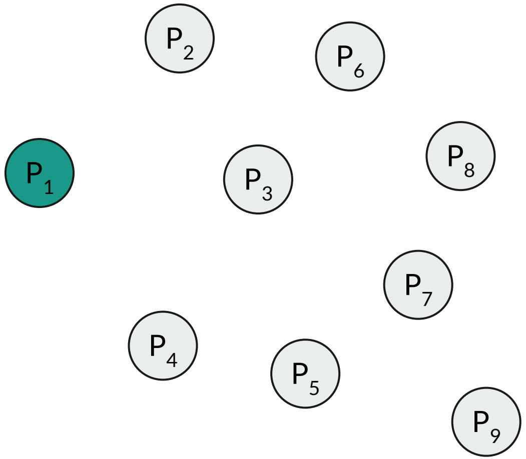


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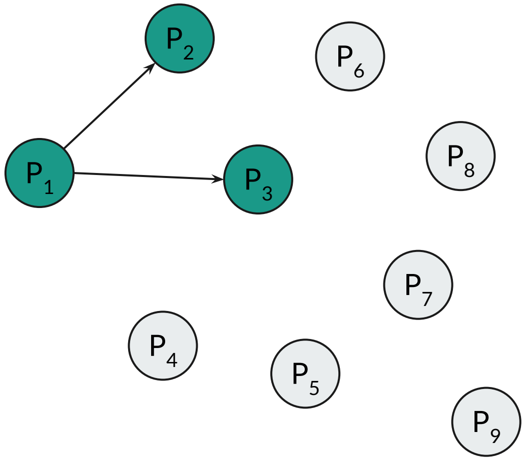


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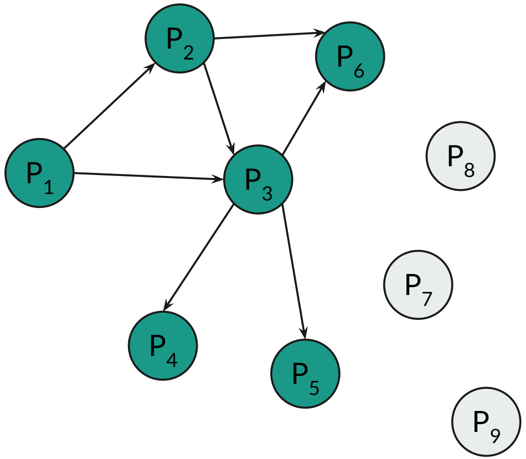


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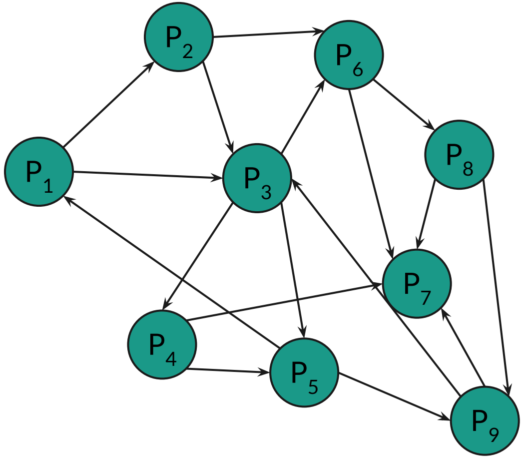


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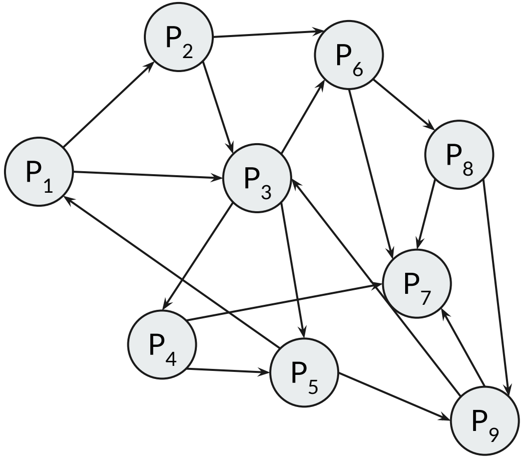


1. Max number of neighbors
2. Diameter
3. Max per party communication

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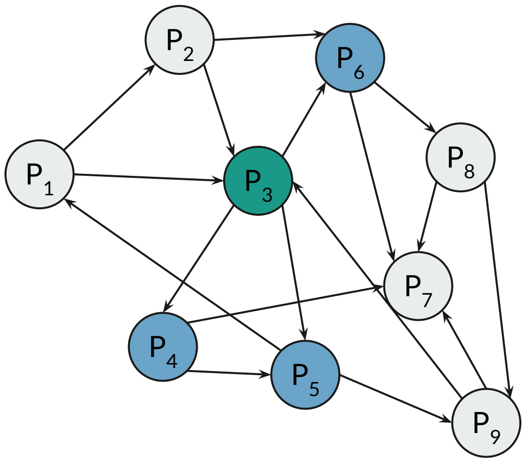


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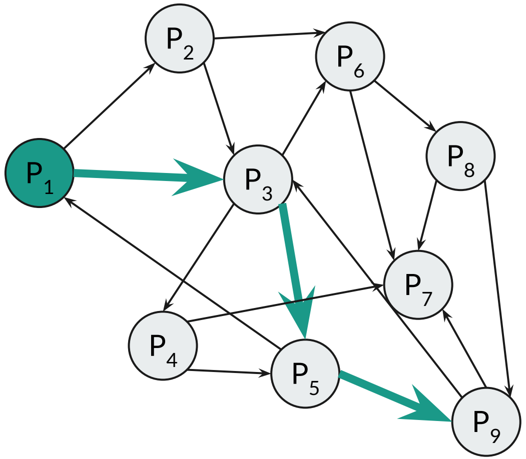


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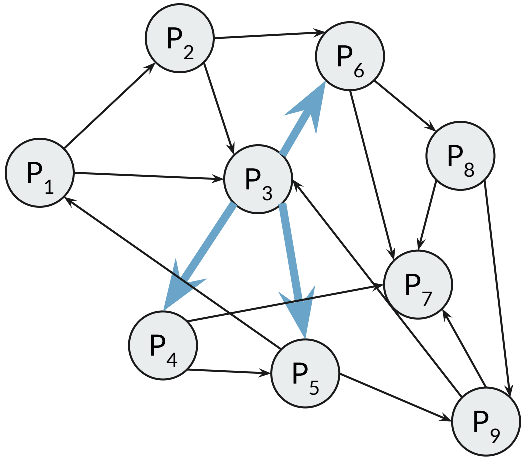
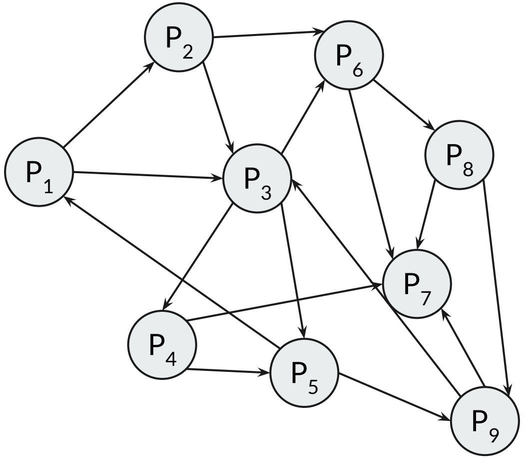



1. Max number of neighbors
2. Diameter
3. **Max per-party communication**

# State of the art flooding (in presence of 😈)




Each party forwards the message to a random subset of parties of (expected) size $\kappa$. [MNT22, LMMRT22]

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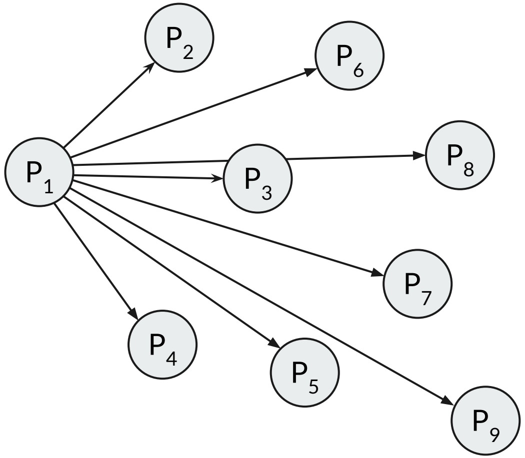
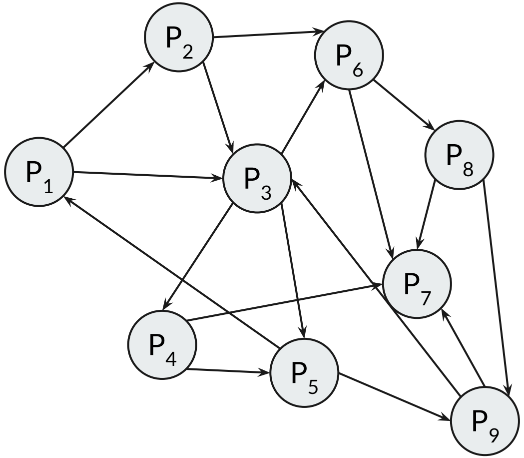



Sender forwards the message to all parties.

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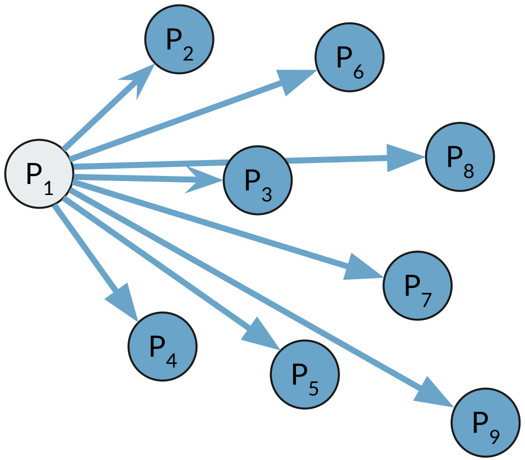


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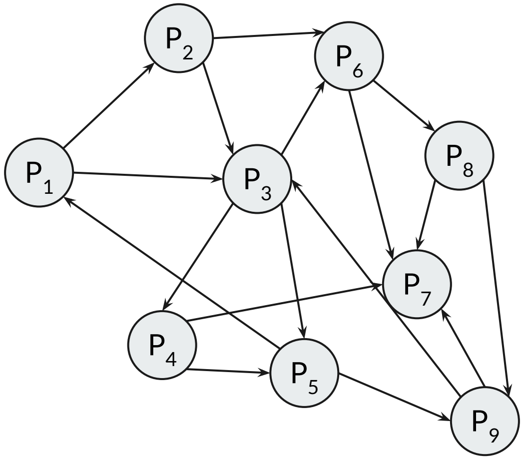


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# Our results: Two asymptotically optimal protocols

| Protocol | Max neighbors | Max per-party communication | Diameter |
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| [MNT22, LMMRT22] | $O(\gamma^{-1} \cdot (log(n) + \kappa))$ | $O(l \cdot \gamma^{-1} \cdot (log(n) + \kappa))$ | $O(log(n))$ |

*n* = number of parties.
$\kappa$ = security parameter.
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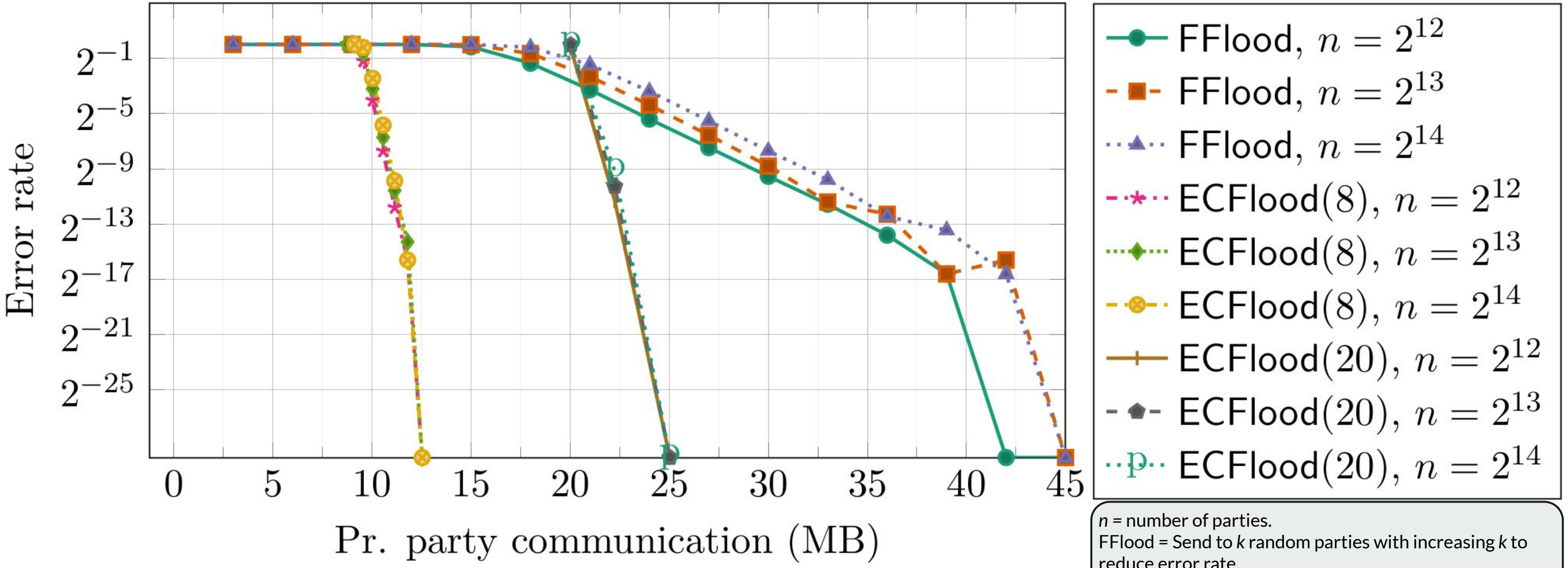
# Efficiency evaluation of ECFlood

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Necessary for Erdős–Rényi flooding protocols shown in [KMG03] and necessary for fanout type flooding shown in [LMMRT22]

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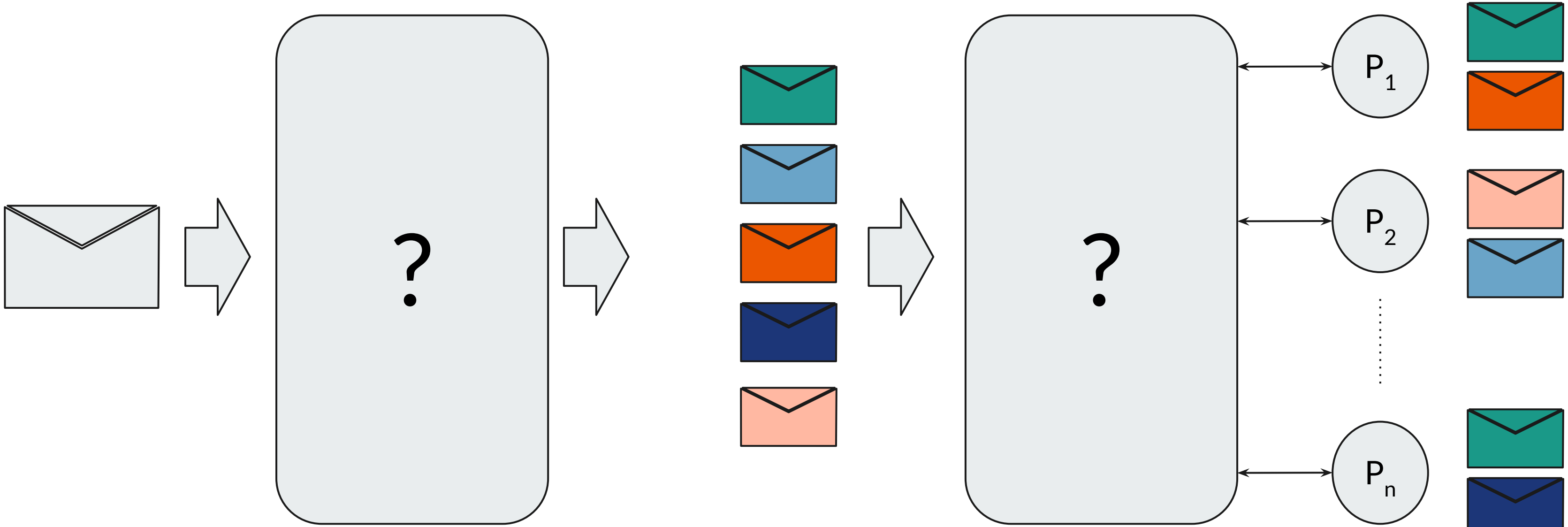

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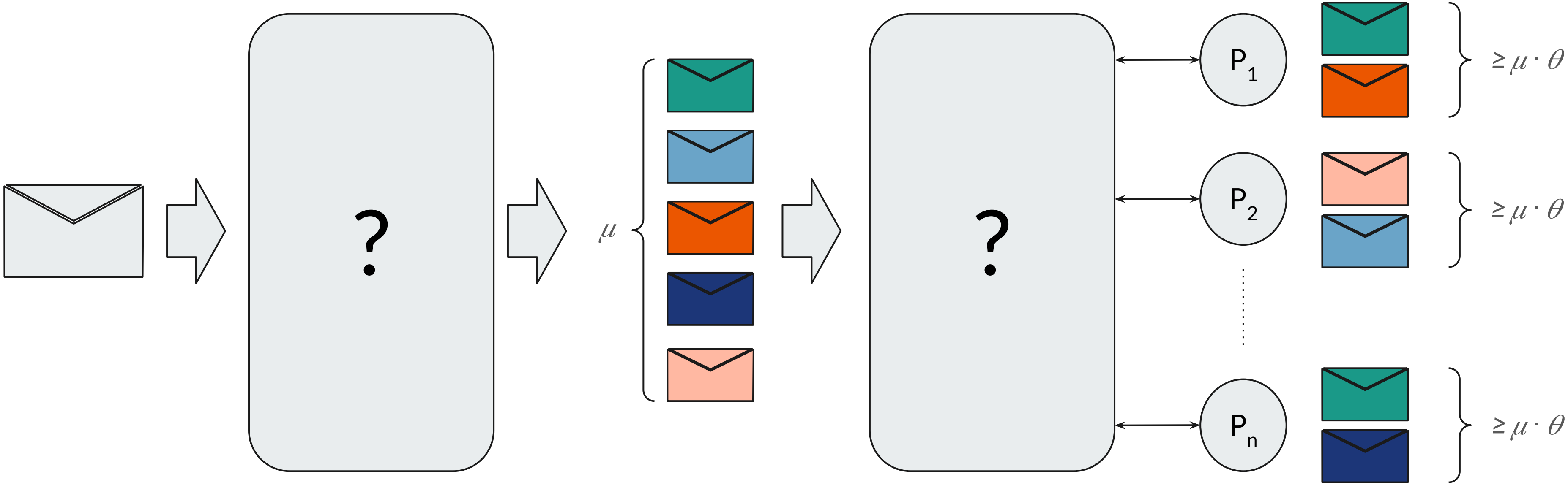

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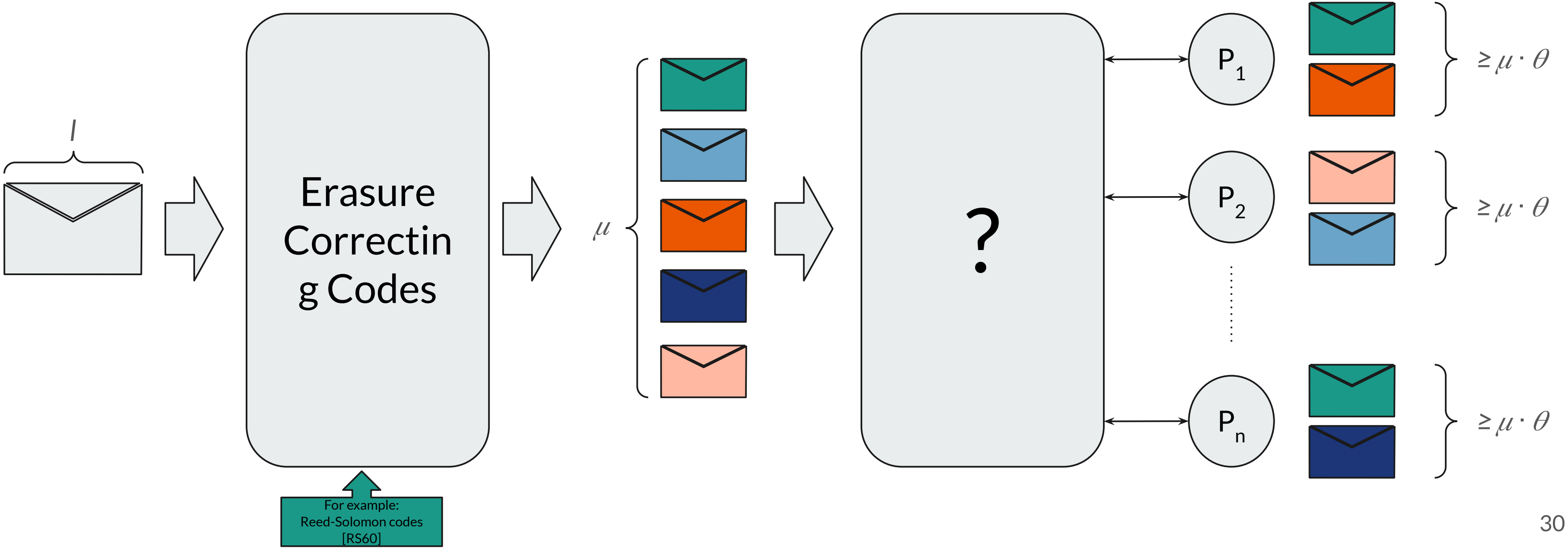

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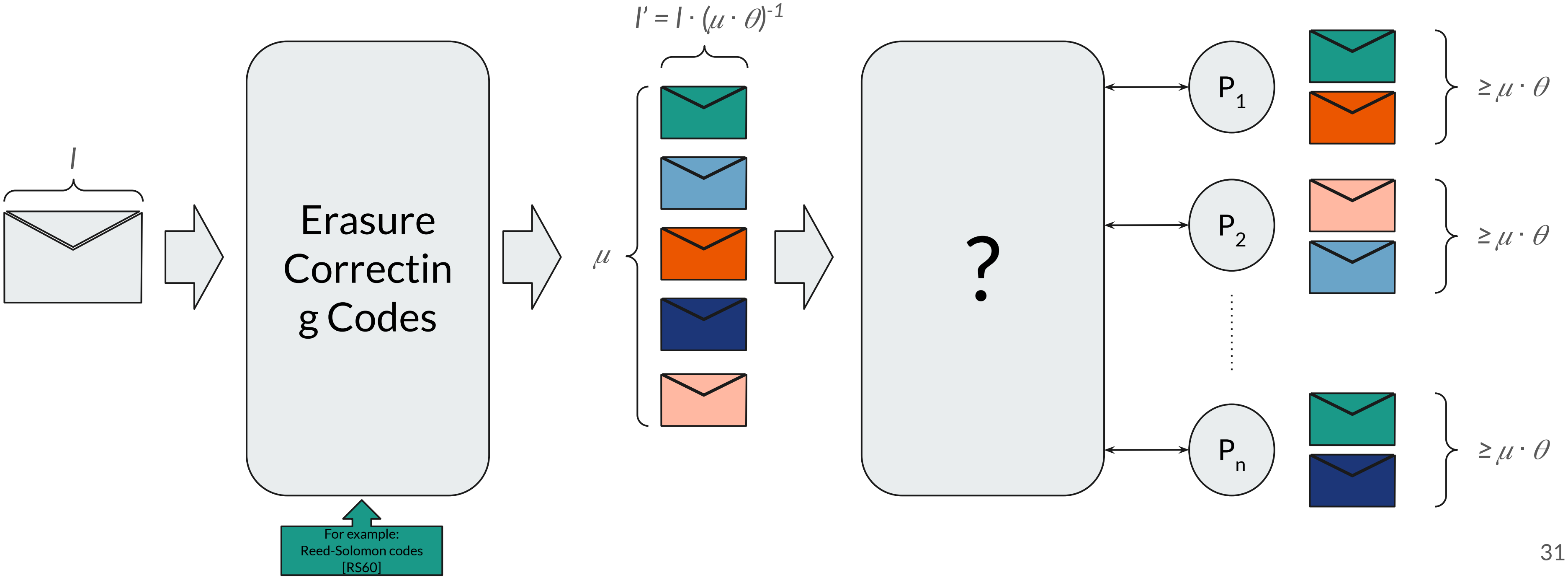

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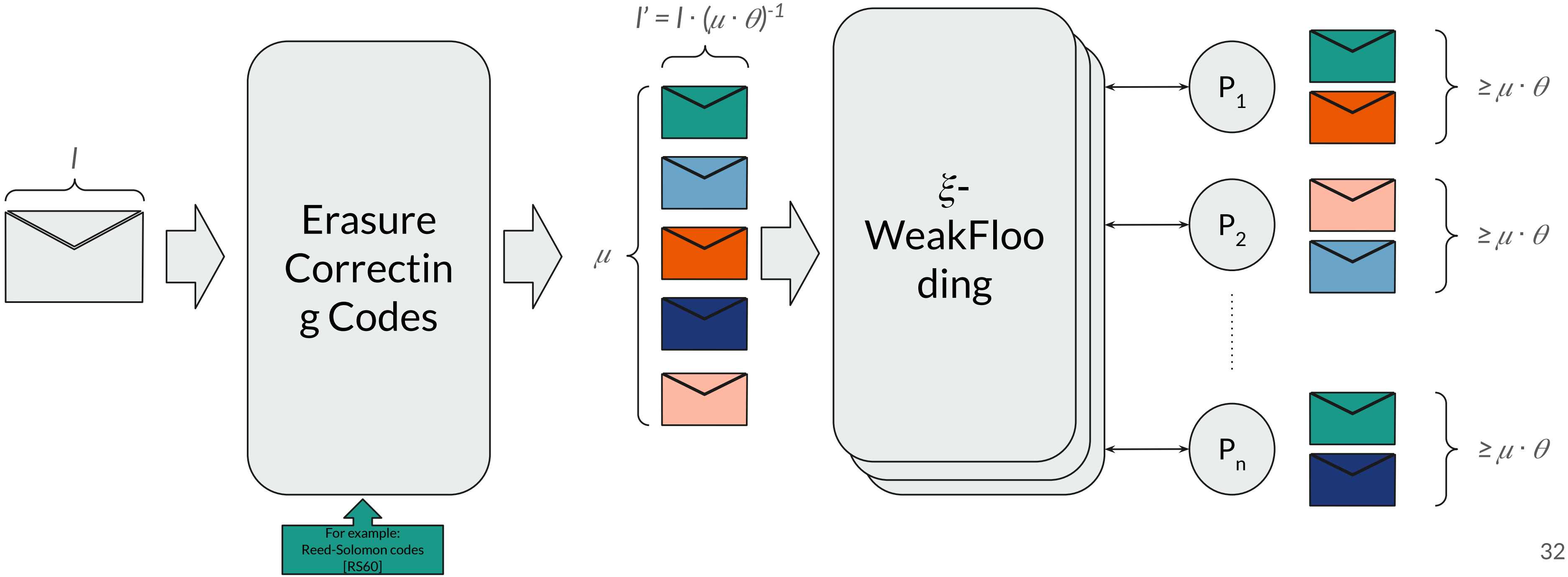

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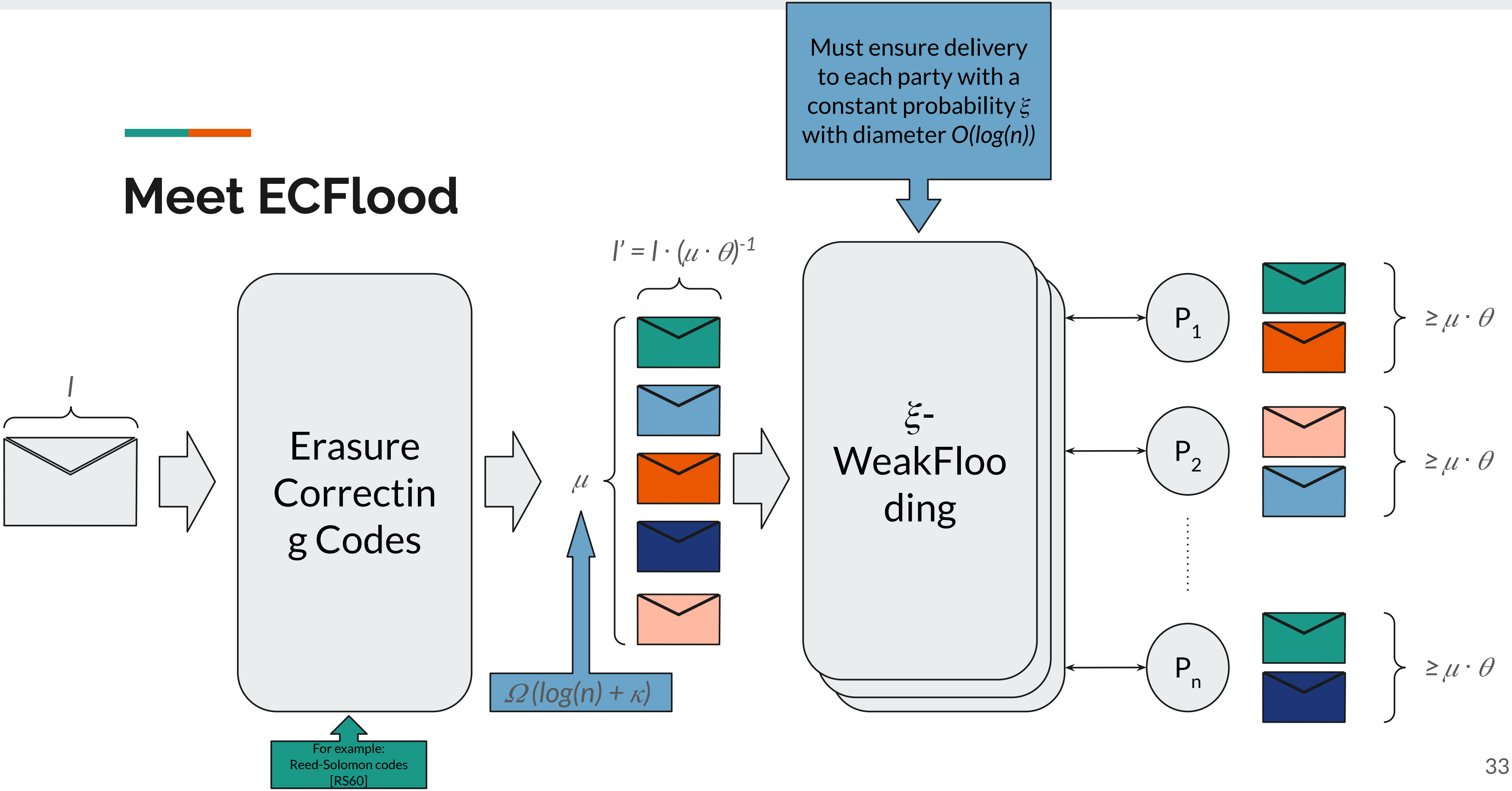

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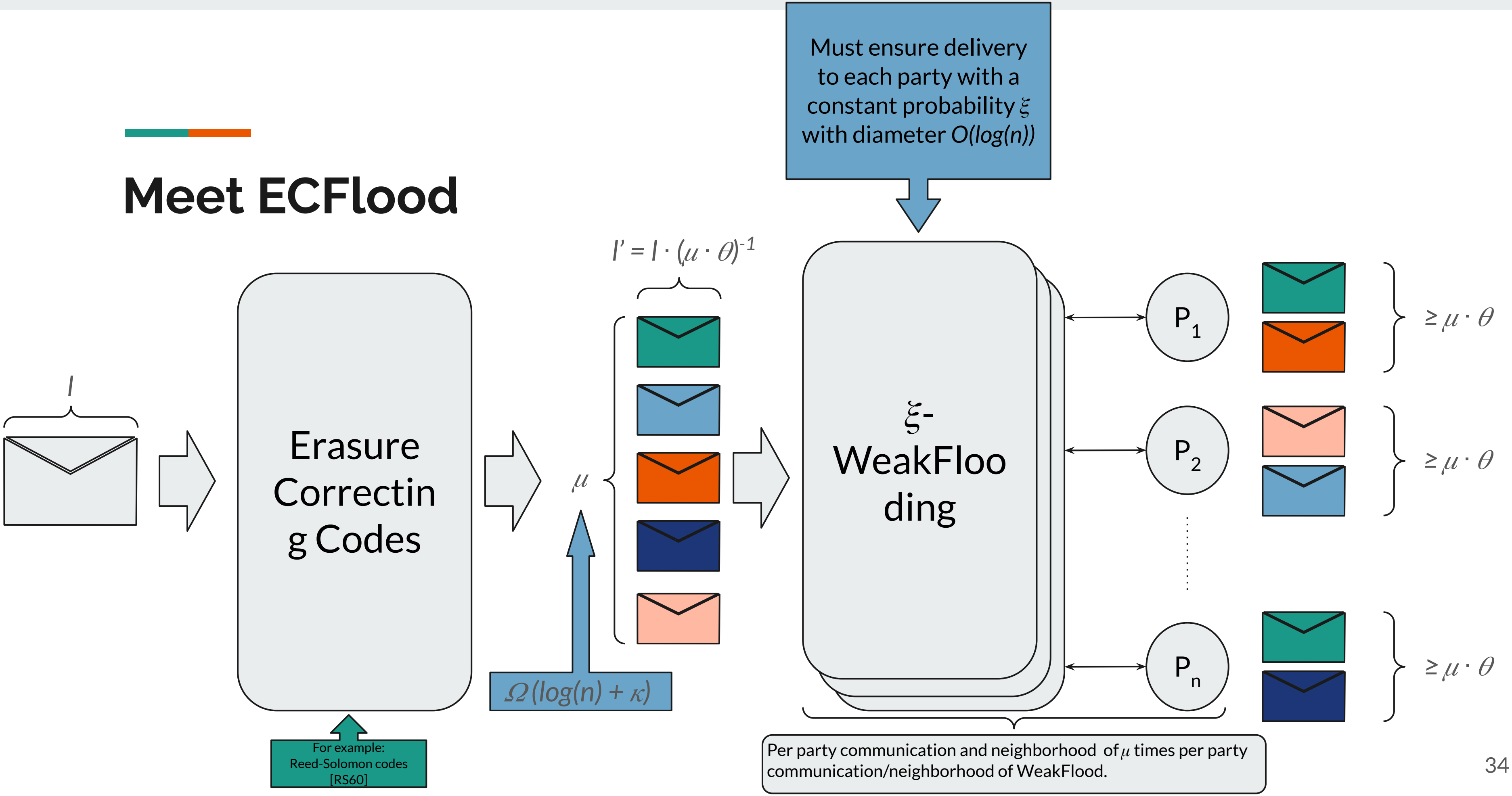

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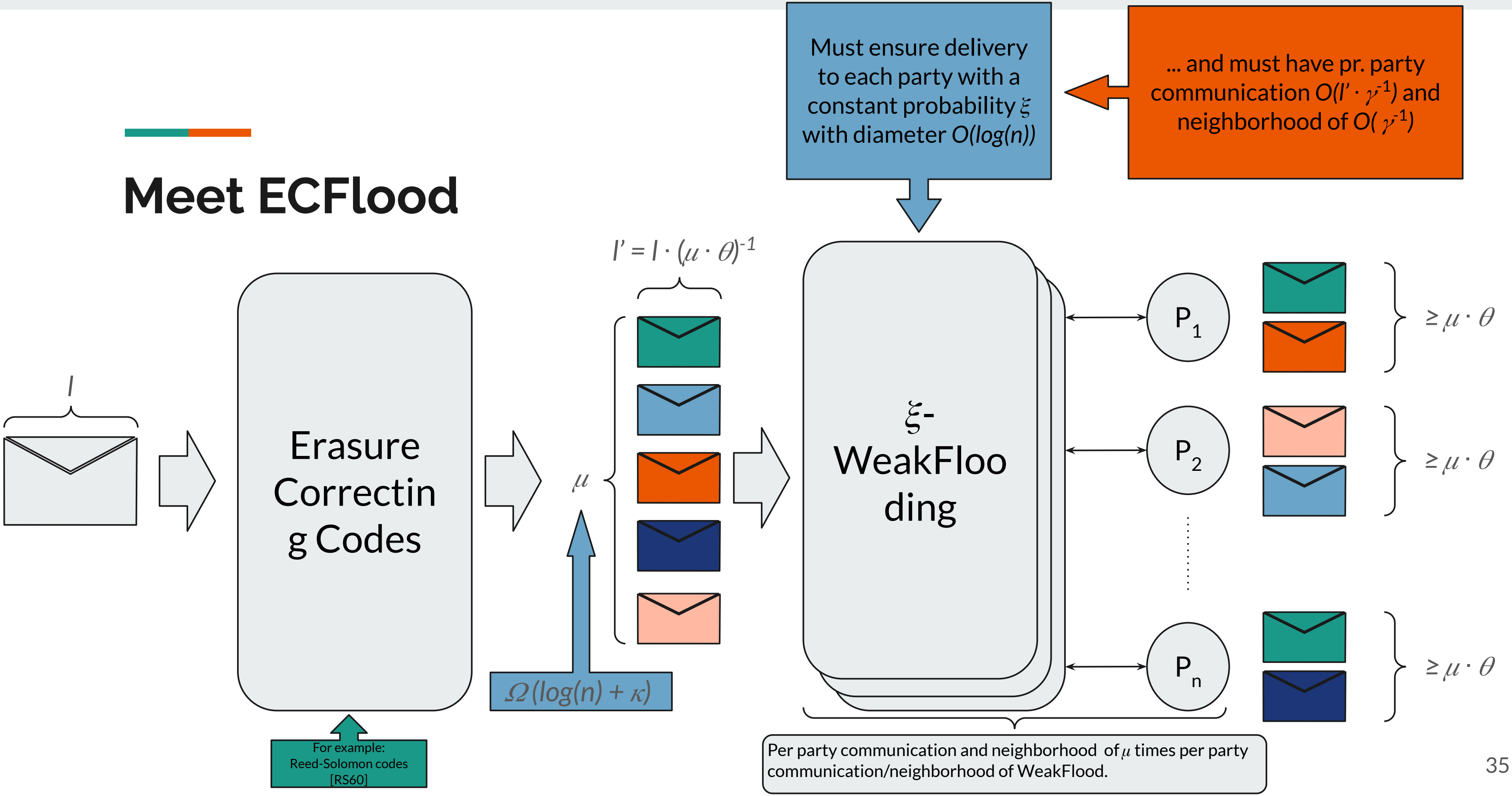

# Requirements for a WeakFlooding protocol

1. Must ensure delivery with diameter $O(\log(n))$ to each party with constant probability
2. Must have $O(\gamma^{-1})$ neighborhoods
3. Must have per party communication of $O(l' \cdot \gamma^{-1})$ for messages of length $l'$.

# So… Any candidates for a $\xi$-WeakFlooding protocol?

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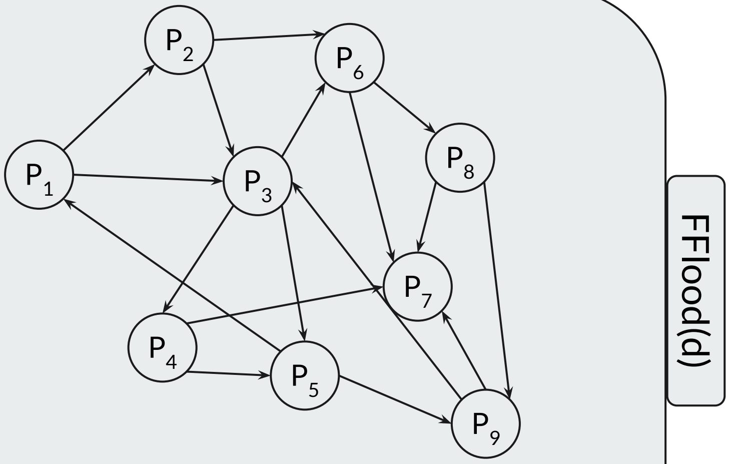
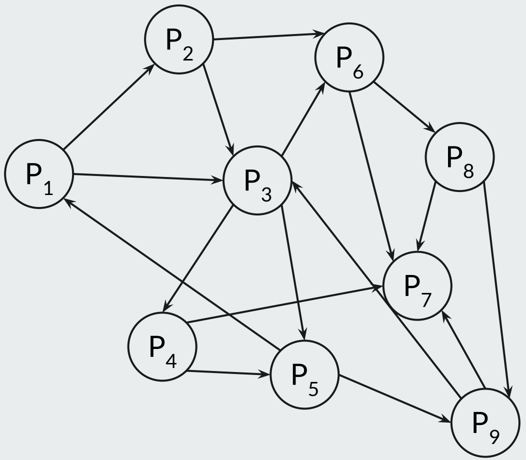



Each party forwards the message to a random subset of parties of size d.

# So… Any candidates for a $\xi$-WeakFlooding protocol?

For $d = O(\gamma^{-1})$, FFlood($d$):

1. Ensures delivery with diameter $O(log(n))$ to each party with constant probability ✅
2. Has $d$ neighborhoods ✅
3. Has per party communication of $O(l' \cdot d)$ for messages of length $l'$ ✅

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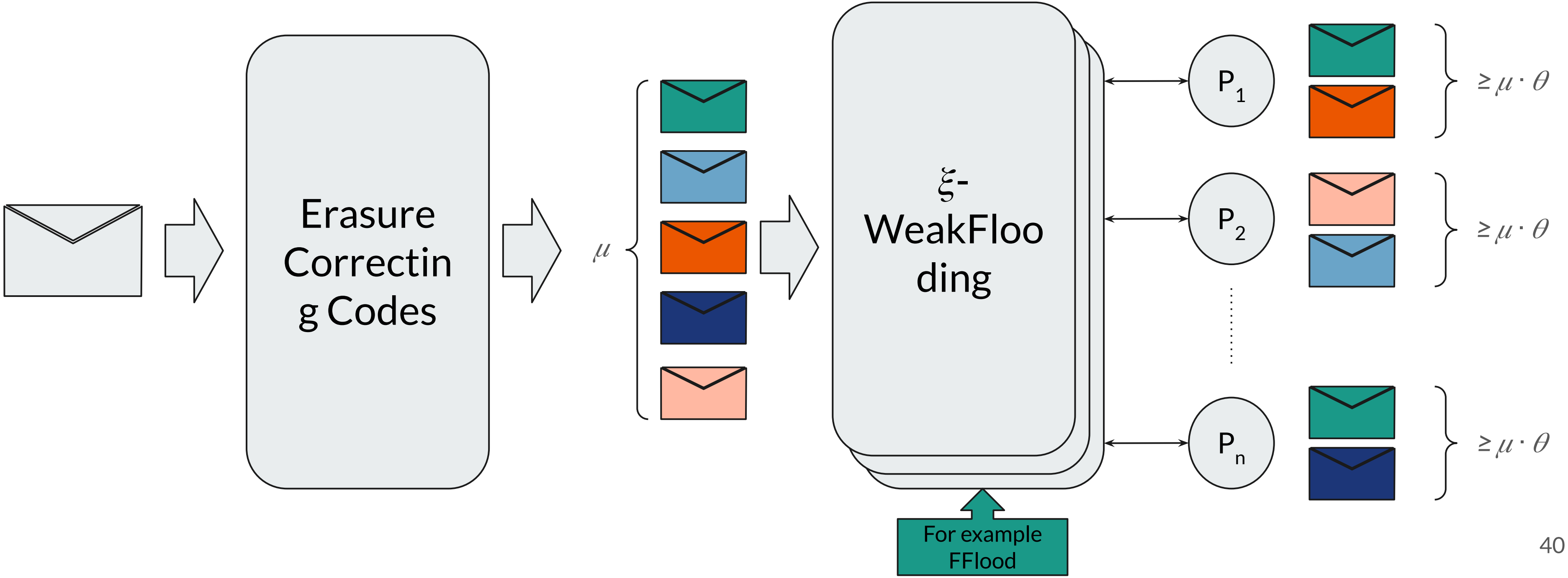
FFlood(d)

# ECFlood


Erasure Correctin g Codes
$\mu$
$\xi$-WeakFlooding
For example FFlood
$P_1$
$P_2$
$P_n$
$\geq \mu \cdot \theta$
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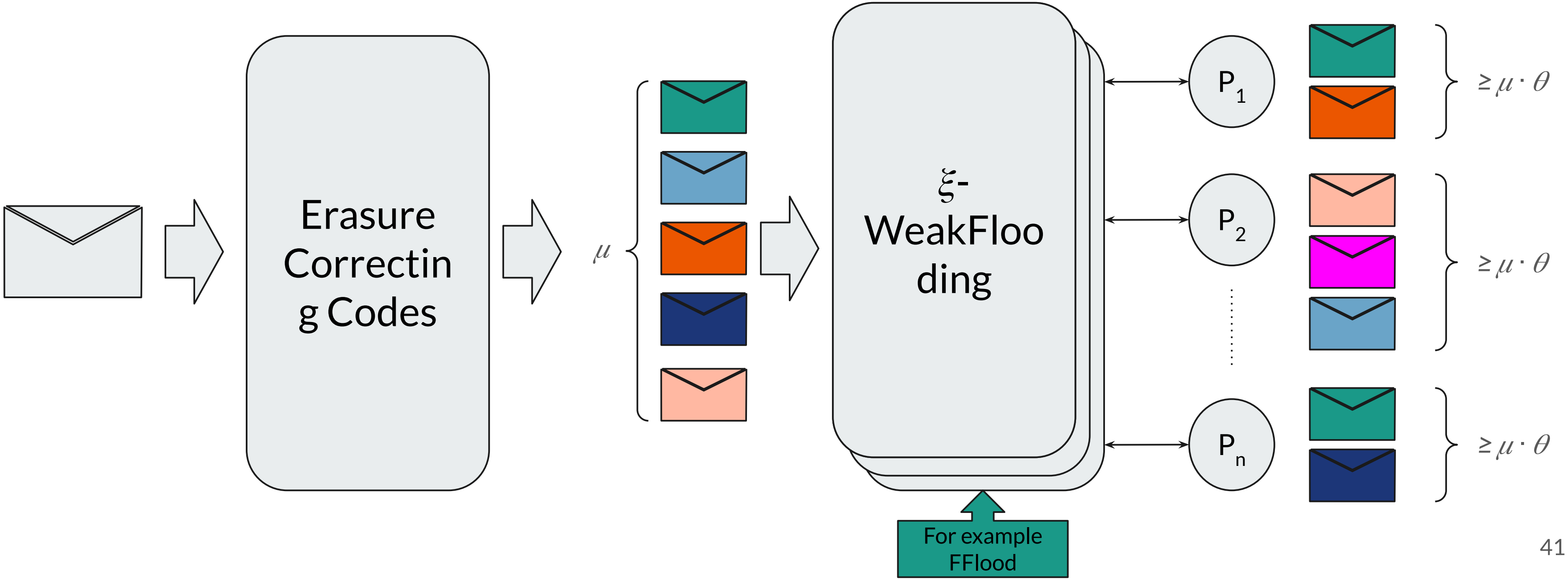

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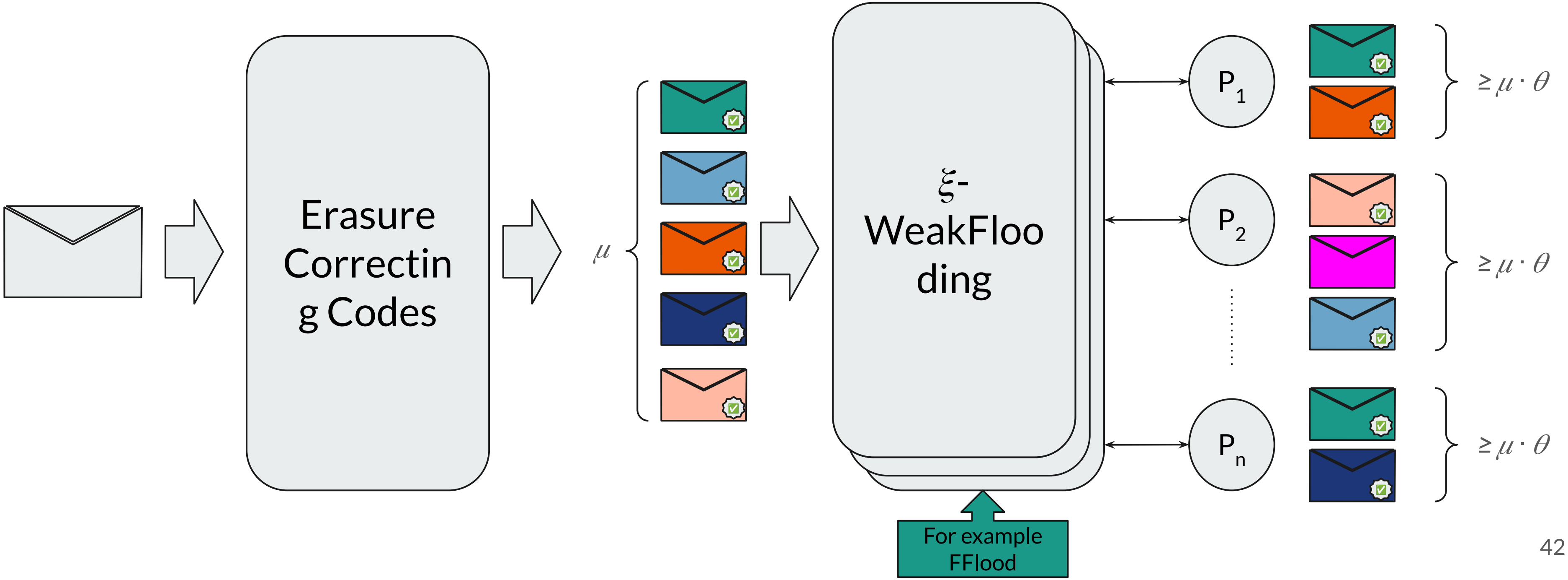

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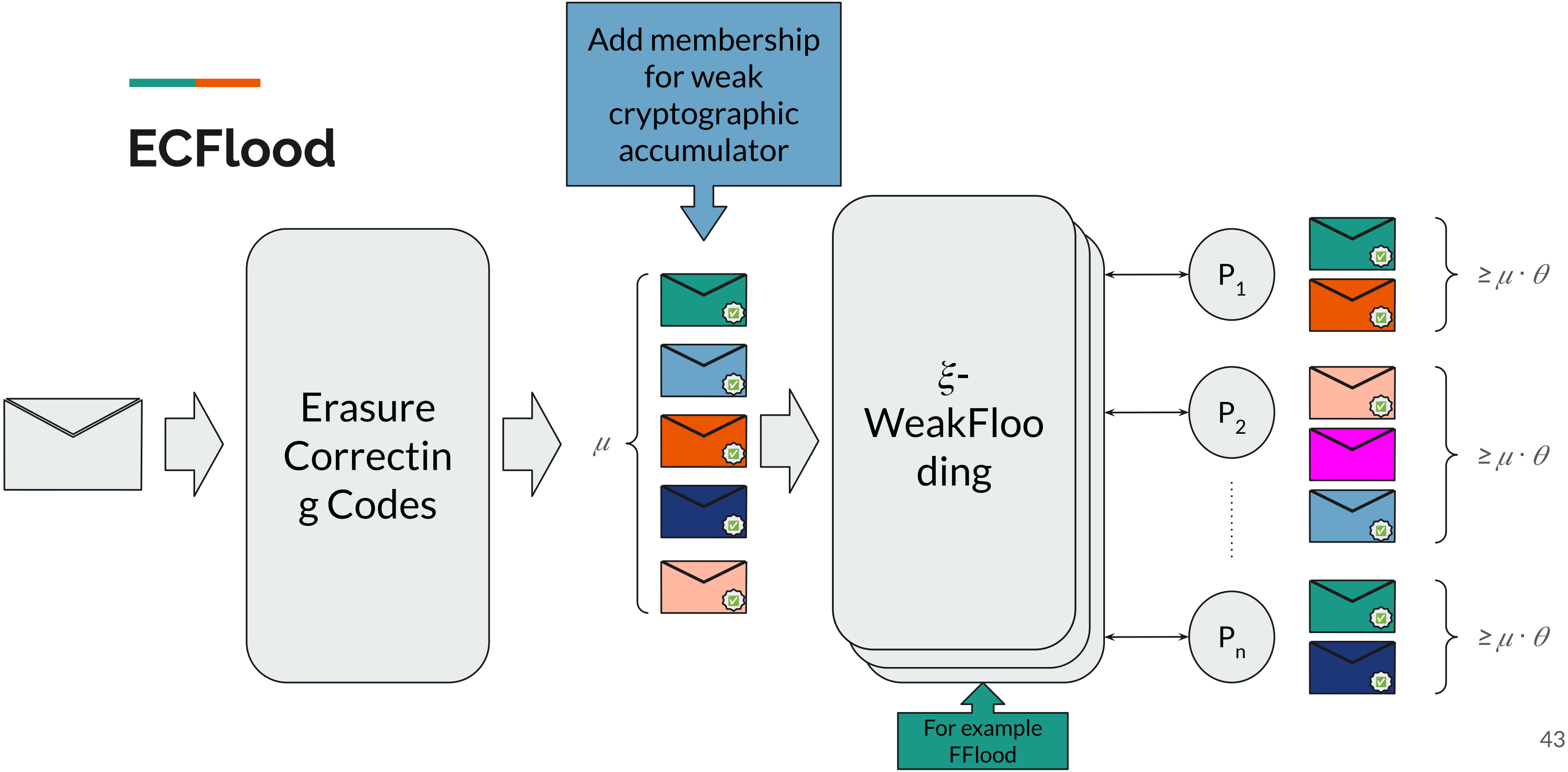
Add membership for weak cryptographic accumulator
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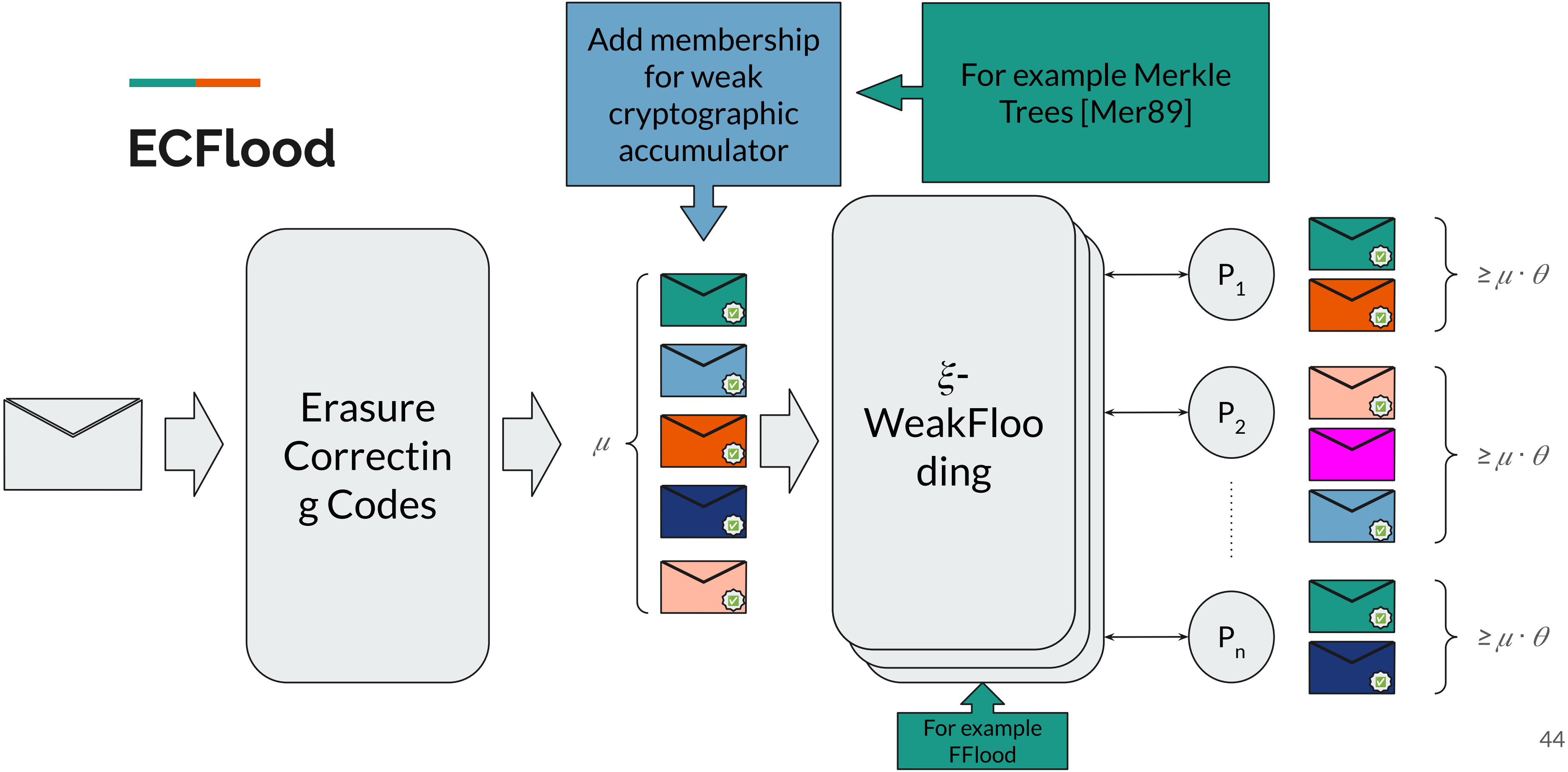

Add membership for weak cryptographic accumulator
For example Merkle Trees [Mer89]
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For example FFlood
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$P_2$
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# How long should the message be?

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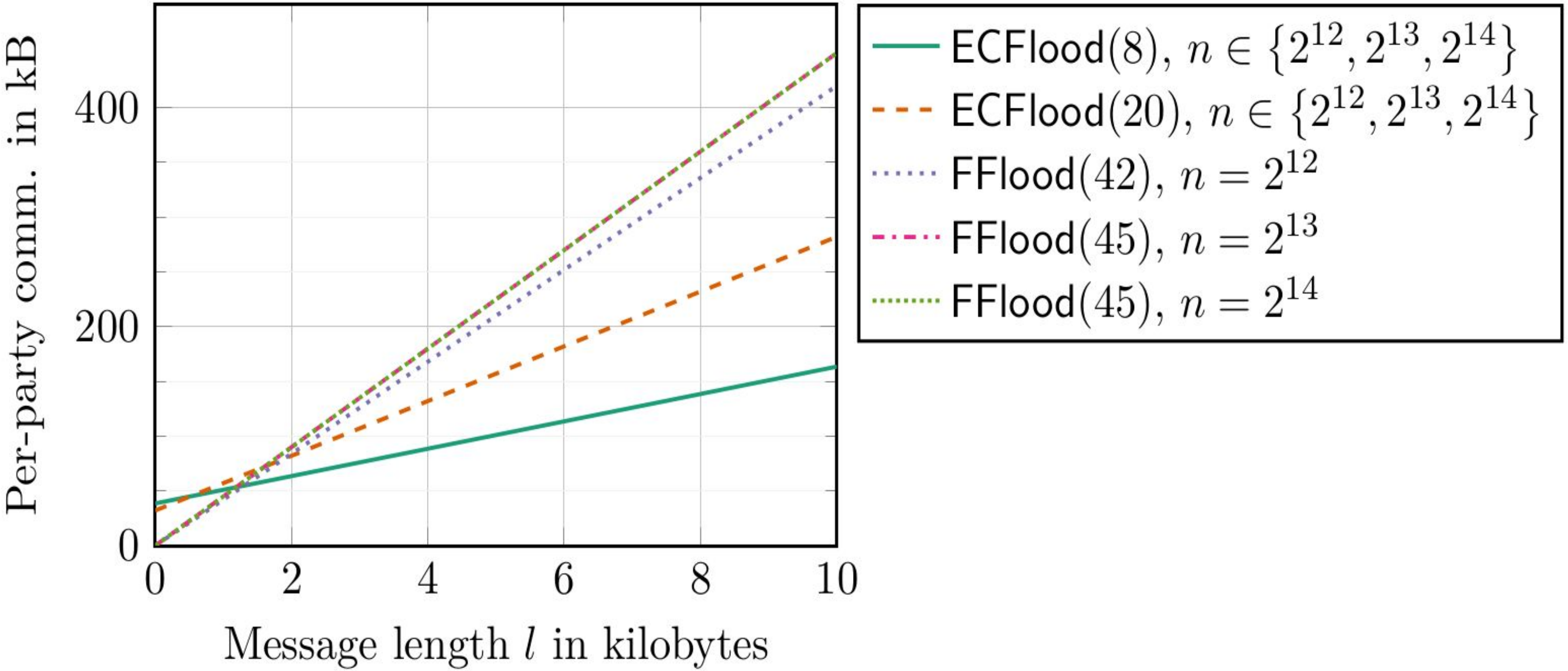

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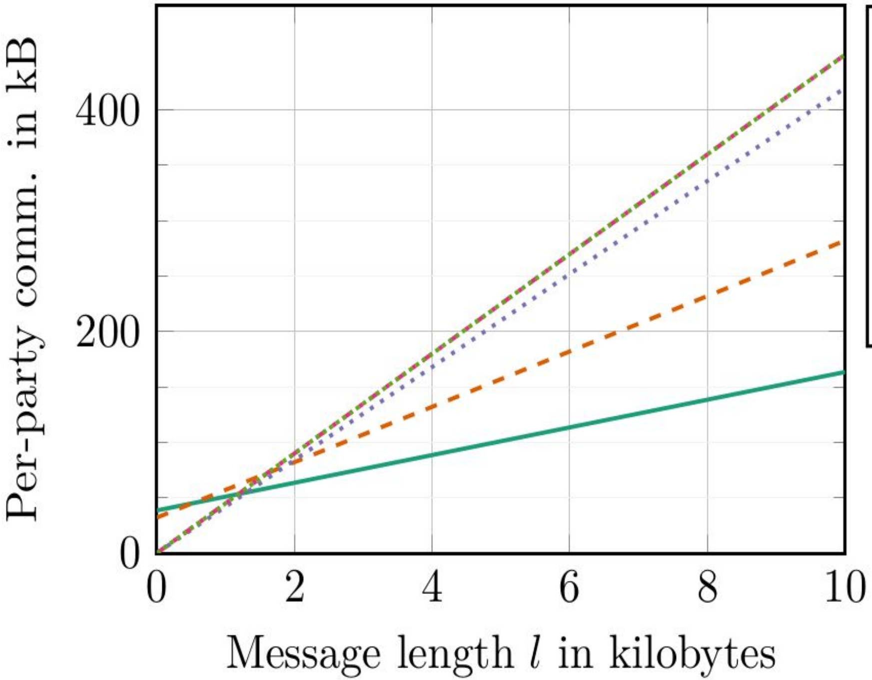


Optimal for:

$l = \Omega((\log(n) + \kappa) \cdot (\log(\log(n)) + \kappa))$

# Per-party communication lower bound

*Theorem:* Any flooding protocol must have max per party communication $\Omega(l \cdot \gamma^{-1})$.

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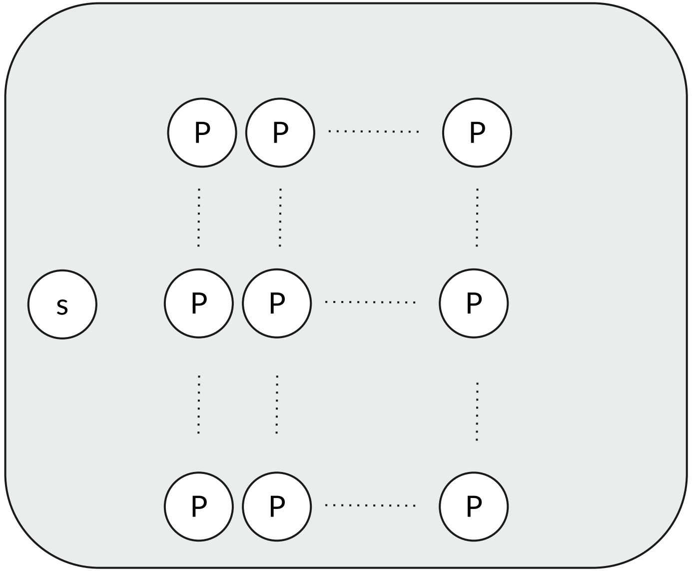
# Optimality of $O(l \cdot \gamma^{-1})$ per party communication



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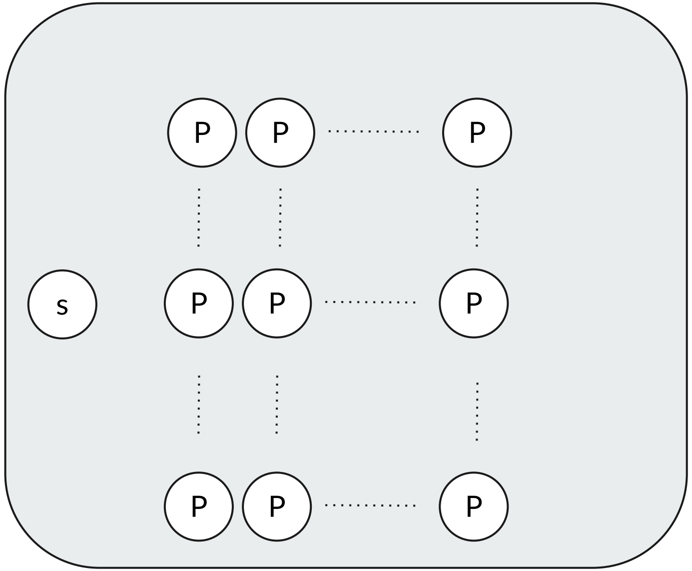


😈 strategy:

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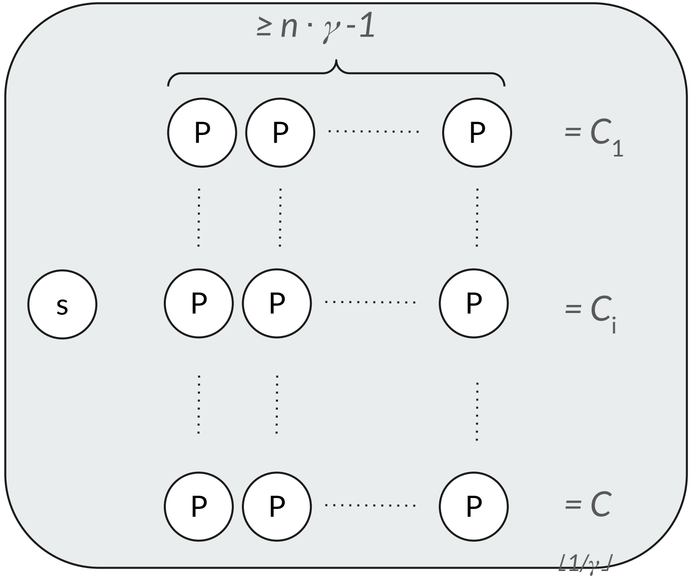


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1. **Divide parties into sets of size ≈ $n \cdot \gamma$.**

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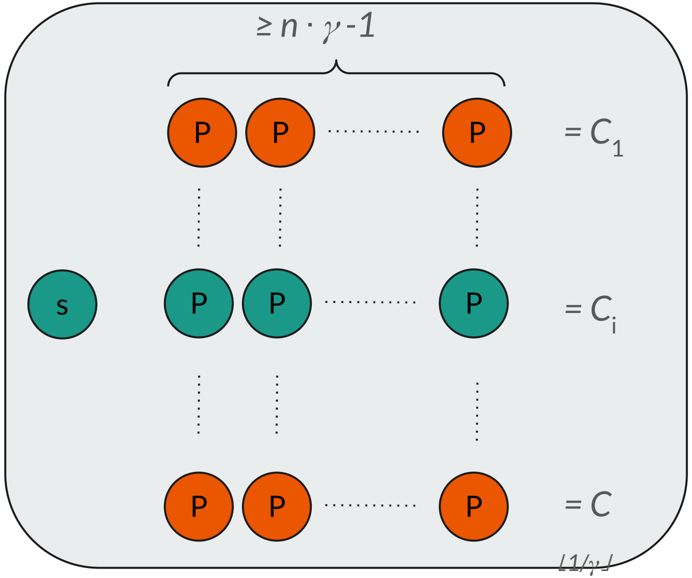


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2. **Choose random *i* and corrupt everyone but sender and $C_i$.**

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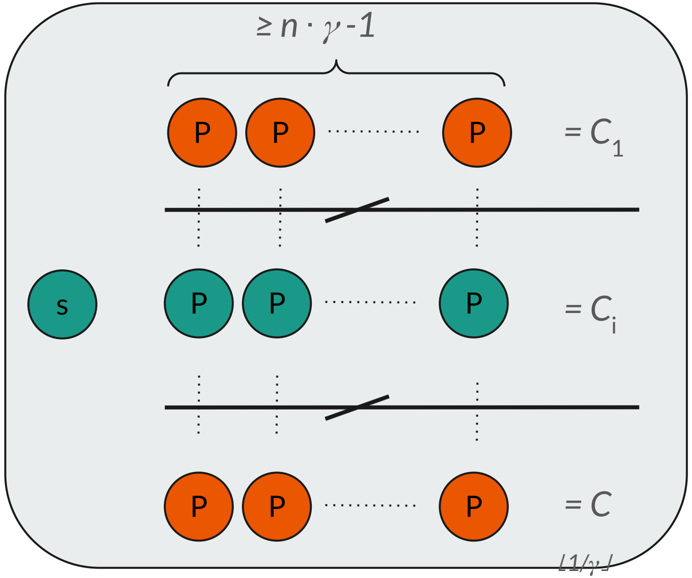


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1. Divide parties into sets of size ≈ $n \cdot \gamma$.
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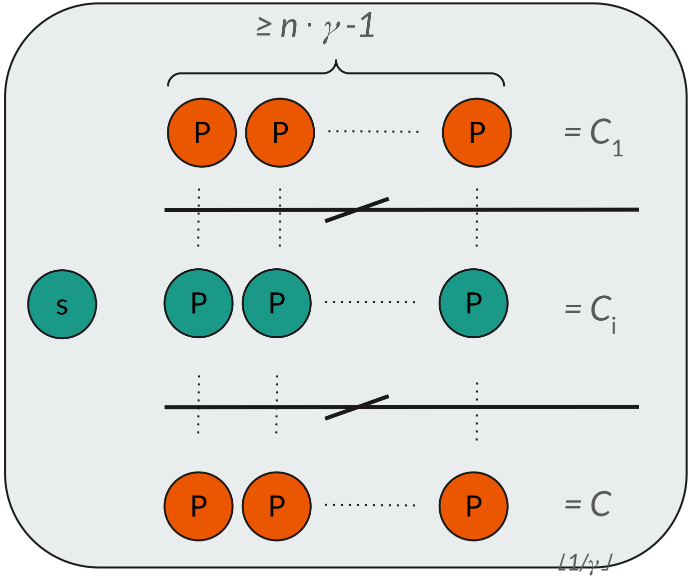
3. **No dishonest cliques communicates with other cliques.**

# Optimality of $O(l \cdot \gamma^{-1})$ per party communication



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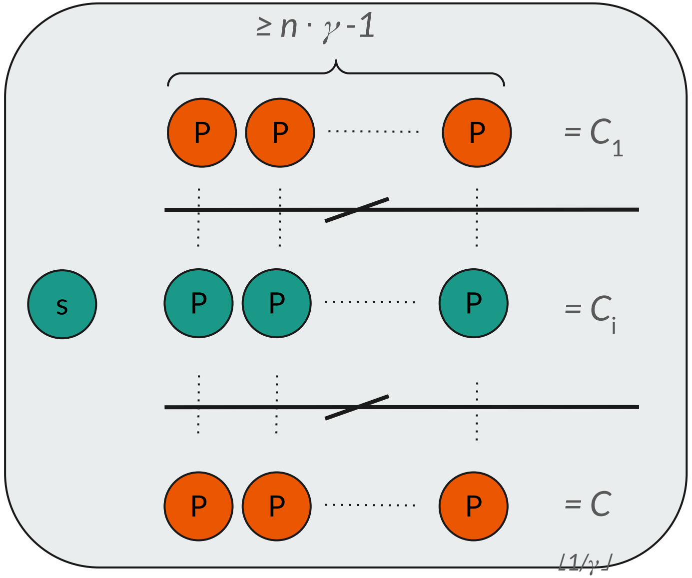


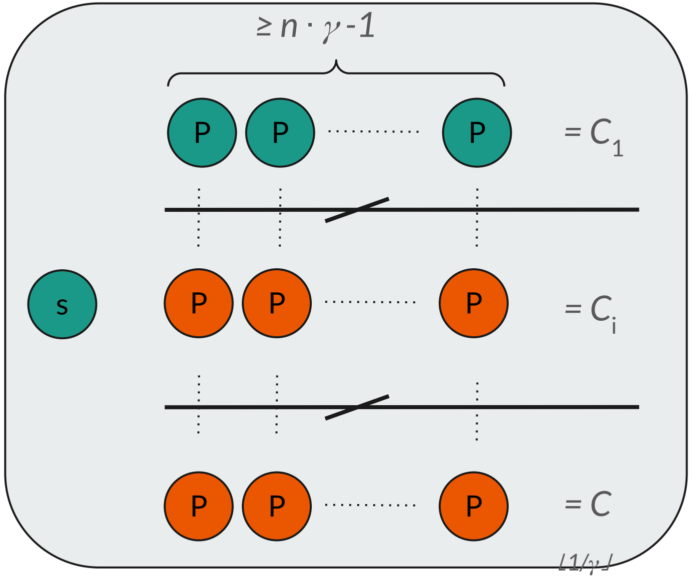

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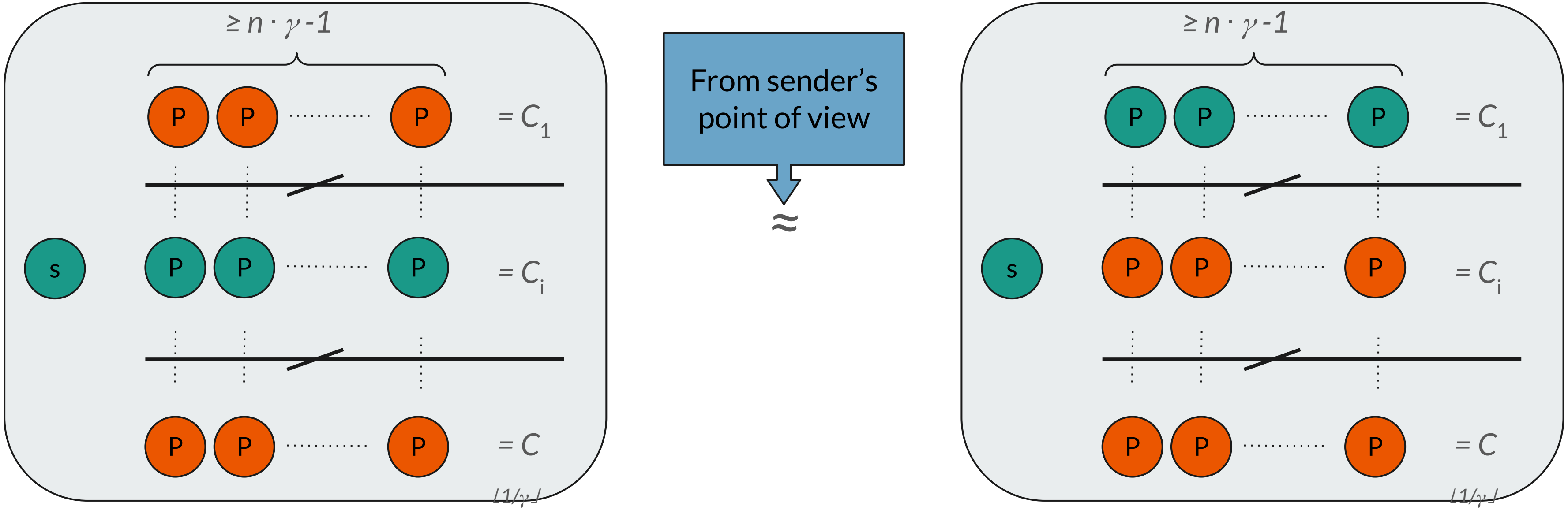

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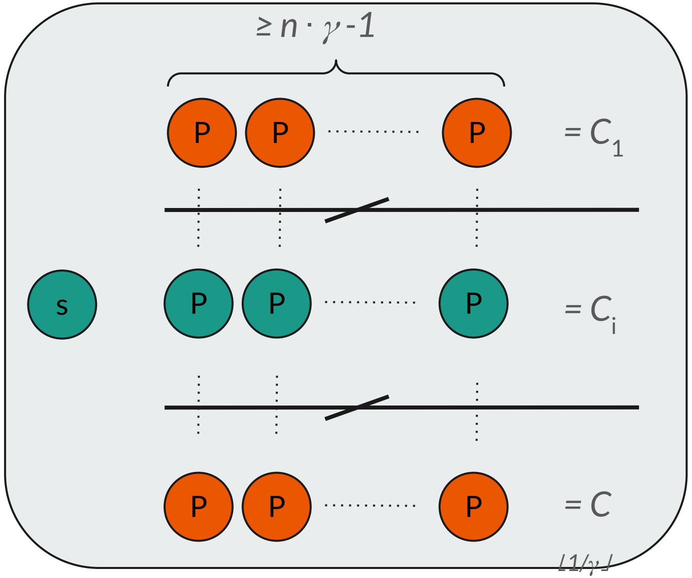
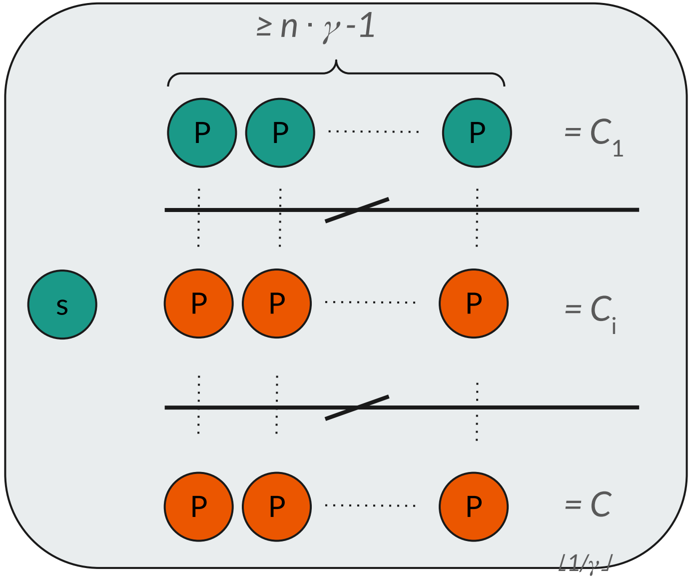



From sender's point of view

≈

Each $C_i$ must receive $l$ bits from the sender with overwhelming probability.



# Also in the paper

*Theorem:* Property-based flooding implies UC-flooding.

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*Theorem:* Property-based flooding implies UC-flooding.

Constructive proof by emulating parties as [LMMRT22]

*Theorem:* Secure protocol in the non-weighted setting *implies* another protocol that is secure in the weighted setting.

# Conclusion

In this talk:

1. Presented ECFlood: A flooding protocol with a logarithmic neighborhood, a logarithmic diameter, and only $O(l \cdot \gamma^{-1})$ per party communication.
2. Presented simulations showing practical advantages over existing approaches.
3. Shown the optimality of a $O(l \cdot \gamma^{-1})$ per party communication.

Details and additional results in the full version of the paper: https://eprint.iacr.org/2022/1723

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Partisia

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