Diving Deep into the Preimage Security of AES-like Hashing

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27 May 2024

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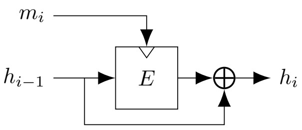
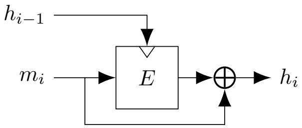
Outline

Construct hash functions based on block ciphers

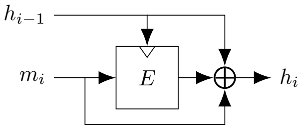
1. Convert an encryption function E to a compression function CF with a PGV mode



Davies-Meyer (DM)

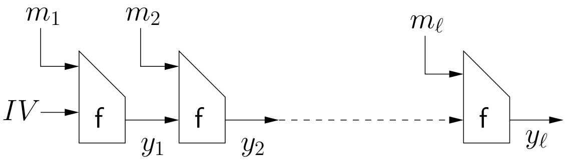


Matyas-Meyer-Oseas (MMO)



Miyaguchi-Preneel (MP)

2. Then iterate the CF following the Merkle-Damgård construction



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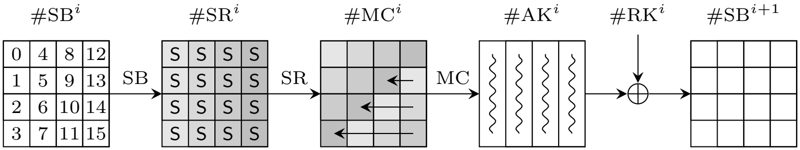
The AES round function

AES is selected by NIST in 2001 from the Rijndael block cipher family.



An encryption state of AES is organized as a $4 * 4$ grid of bytes. An AES round consists of the following operations:

- SubBytes (SB): a non-linear byte-wise substitution (S-box)
- ShiftRows (SR): a cyclic left shift on the *i*-th row by *i* bytes
- MixColumns (MC): a column-wise left multiplication of an MDS matrix
- AddRoundKey (AK): a bitwise XOR of the round key to the state

AES-like Hashing

The outstanding security of AES has inspired many designs.

Hash functions with a compression function **based on** or **similar to** the AES round function is refered to as AES-like hash functions or AES-like hashing.

Examples include:

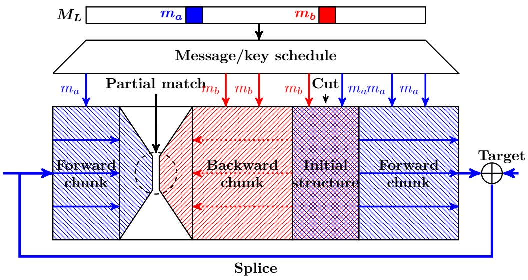
- AES-MMO (standard in the Zigbee protocol suite and ISO/IEC standard)
- Whirlpool (ISO/IEC standard)
- Streebog (ISO/IEC standard)
- Grøstl
- Saturnin
- etc.

Outline

Meet-in-the-Middle attacks

In a series of works, Aoki and Sasaki proposed the Meet-in-the-Middle (MITM) attack on hash functions. An MITM attack is orchestrated as follows:

- For $2^{n-d_b-d_r}$ values of $M_L/\{m_a, m_b\}$:
  - For 2^{d_b} values of m_a, compute forward to the matching point and save in a table T^+;
  - For 2^{d_r} values of m_b, compute backward and save in a table T^-;
  - If find a match between T^+, T^-:
  - Test whether a full match and return the preimage;



Total complexity:

$$2^{(n-d_b-d_r)} \cdot (2^{d_b} + 2^{d_r} + 2^{d_b+d_r-d_M}) \simeq 2^{n-\min(d_b,d_r,d_M)}.$$

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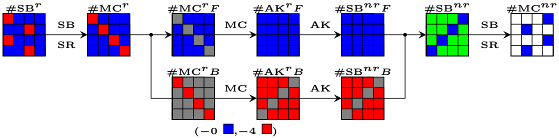
Byte classification in MITM attacks

- a forward neutral byte, represented by ■ (blue)
- a backward neutral byte, represented by ■ (red)
- a constant byte, represented by ■ (grey)
- an unknown byte, represented by □
- a superposition byte, represented by ■ (green), value of which is the sum of its forward and backward neutral components: $v = v^{+} \oplus v^{-}$.
  - a superposition byte is preserved through all **linear operations**
  - after the **nonlinear operations** (*e.g.*, an S-box), a superposition byte becomes unknown



Outline

Exploiting the algebraic structure of AES S-box

At Asiacrypt 2023, Zhang *et al.* observed the non-linear layer of the AES S-box has the following decomposition:

$$x^{254} = (x^{17})^{14} \cdot x^{16}$$

The decomposition has the following properties:

- x^{17} has 15 possible non-zero possible values, 16 in total
- x^{16} is linear, as x^2 over $\mathbb{F}_{2^8}$ is linear

However, the observation did not lead to better results on AES, quoting their words:

This linearizes the non-linear layer of AES, but unfortunately, no attacks better than the current state-of-the-art has been found based on this fact.

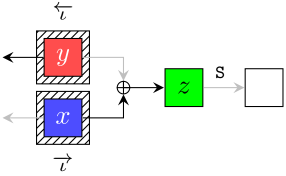
Linearizating AES S-box in MITM attacks

We generalize the observation to the superposition bytes:

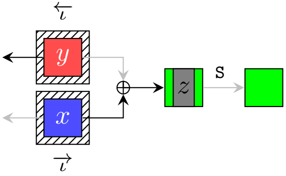
$$v^{254} = (v^{+} + v^{-})^{254} = ((v^{+} + v^{-})^{17})^{14} \cdot (v^{+} + v^{-})^{16}$$
$$= (H(v^{+}, v^{-}))^{14} \cdot ((v^{+})^{16} + (v^{-})^{16})$$

Thus, a superposition byte can be **preserved** after an AES S-box by

- an enumeration over the pool of $|\{H(z) = z^{17} : z \in \mathbb{F}_{2^8}\}| = 16$ hints, and
- an efficient checking, as it only requires local information (unlike GnD)



Orignal superposition



Guessing part of the superposition

Outline

1. Backgound
   - AES-like hashing
   - Meet-in-the-Middle Attack
2. Advanced Techniques for MITM Attacks
   - S-box Linearization (**LIN**)
   - Distributed Initial Structures (**DIS**)
   - Structual Similarity (**SIM**)
3. Applications
   - AES and Rijndael
   - Whirlpool and Streebog
4. Conclusion

Choice of initial states

In previous MITM attacks, one intuitively choose

- a full state $\overleftrightarrow{S}^{\mathtt{ENC}}$ in the encryption function and
- a full state $\overleftrightarrow{S}^{\mathtt{KSA}}$ in the key schedule

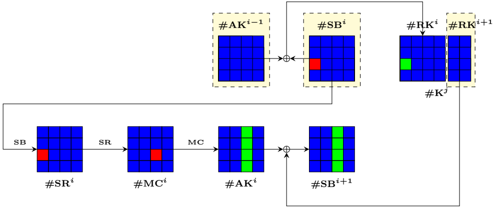
as initial states to allocate ■, ■, or ■ (no superpositions).

In essence, the initial states in MITM attacks are some **independent** intermediate values where we distribute initial DoFs for forward and backward computations.

In this work, we introduce the distributed initial structure (**DIS**), which:

- remove the artificial constraint on initial states
- allow more combinations of the initial states

An example of **DIS** in AES-192



We will distribute initial DoFs in

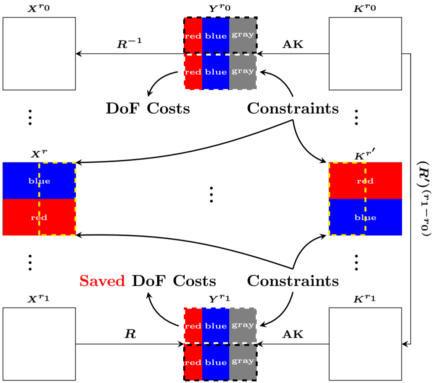
- $\#\mathrm{AK}^{i-1}$
- $\#\mathrm{SB}^{i}$
- the rightmost two columns of $\#\mathrm{K}^{j}$

The effect of **DIS** includes:

- $\#\mathrm{K}^{j}$ can be expressed and used as $\overleftrightarrow{S}^{\mathtt{KSA}}$ for further key schedule propagations
- Thus, superpositions are now allowed in $\overleftrightarrow{S}^{\mathtt{KSA}}$, and
- more superposition information can be preserved in the AES key schedule

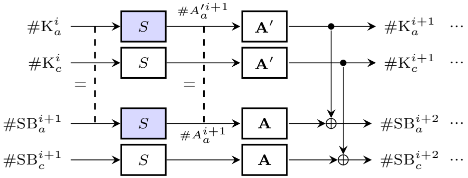
Outline

Related constraints at AddRoundKey



- Previous models consider constraints incurred at AK in different rounds as **independent**
- However, the constraints may be added to the same DoF source in multiple rounds
- In other words, the constraints can be **related**

Exploiting **SIM** in Whirlpool and Streebog



- The encryption and the key schedule share the same round function
- If $\#\mathrm{AK}_a^i$ is 0, then $\#\mathrm{SB}_a^{i+1} = 0 \oplus \#\mathrm{K}_a^i = \#\mathrm{K}_a^i$
- After the same sets of operations, $\#\mathrm{SB}_a^{i+2}$ should be constant after XOR
- Previous models may invoke unnecessary costs to have constants in $\#\mathrm{SB}_a^{i+2}$

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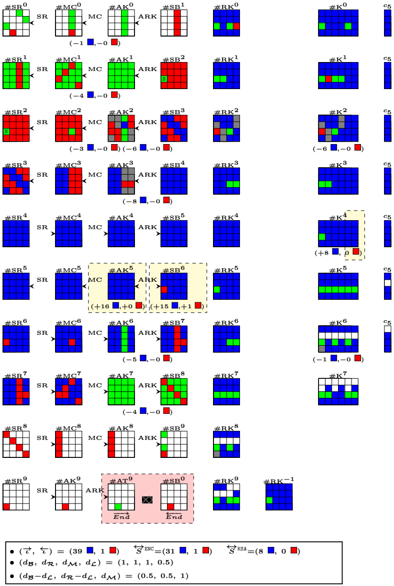
Preimage Attacks

Cipher (target)	#Rounds	T_1 †	T_2 ‡	Memory	Essential technique(s)	References
AES-192 (Hash)	8/12	2^{112}	2^{116}	2^{16}	MITM	[BDGWZ19]
	8/12	2^{100}	2^{115}	2^{96}	**LIN**, **DIS**, BiDir	This work
	9/12	2^{120}	2^{125}	–	MILP	[BDGLSSW21]
	9/12	2^{112}	2^{121}	–	BiDir	[BGST22]
	10/12	2^{124}	2^{127}	2^{124}	**LIN**, **DIS**, BiDir	This work
Rijndael-192/192 (Hash)	9/12	2^{184}	2^{189}	–	BiDir	[Zha23]
	9/12	2^{180}	2^{187}	2^{180}	**LIN**, BiDir	This work
Rijndael-192/256 (Hash)	9/12	2^{168}	2^{181}	–	BiDir	[Zha23]
	10/12	2^{188}	2^{191}	2^{180}	**LIN**, BiDir	This work

† T_1 is the time complexity of the pseudo-preimage attack on compression function.

‡ T_2 is the time complexity of the preimage attack on hash function.



- The **first** 10-round preimage/pseudo-preimage attack on `AES-192`
- Linearizing an S-box from $\#\mathrm{SR}^2$ to $\#\mathrm{SB}^2$
- Distributing initial states to $\#\mathrm{AK}^{i-1}$, $\#\mathrm{SB}^{i}$ and the rightmost two columns of $\#\mathrm{K}^{j}$
- 2^{124} for pseudo-preimage and 2^{127} for preimage
  - different from biclique attacks
  - not reduced to S-box level evaluation

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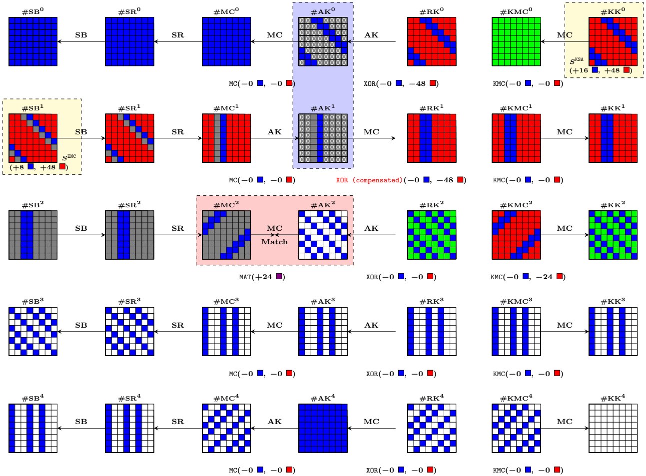
Preimage Attacks						
Cipher (target)	#Rounds	T_1 †	T_2 ‡	Memory	Essential technique(s)	References
Whirlpool (Hash)	5/10	2^{416}	2^{448}	2^{96}	Dedicated method	[SWWW12]
	5/10	2^{352}	2^{433}	2^{160}	BiDir, MulAK	[BGST22]
	5/10	2^{320}	2^{417}	$O(1)$	**SIM**, BiDir	This work
	6/10	2^{448}	2^{481}	2^{256}	Dedicated method, GnD	[SWWW12]
	6/10	2^{440}	2^{477}	2^{192}	GnD	[BGST22]
	6/10	2^{416}	2^{465}	2^{288}	**SIM**, BiDir, GnD	This work
	7/10	2^{480}	2^{497}	2^{128}	GnD, MulAK	[BGST22]
	7.75/10	2^{480}	2^{497}	2^{256}	**SIM**, BiDir, GnD	This work

Collision Attacks					
Cipher (target)	#Rounds	Time	Memory	Essential technique(s)	References
Whirlpool (Hash)	4.5/10	2^{120}	2^{16}	Rebound	[MRST09]
	4.5/10	2^{64}	2^{16}	Rebound	[LMSRR15]
	5/10	2^{120}	2^{64}	Super-SBox	[LMRRS09; GP10]
	5.5/10	2^{184-s}	2^{s}	Rebound	[LMSRR15]
	6/10	2^{228}	2^{228}	Quantum	[HS20]
	6/10	2^{248}	2^{248}	MILP, MITM	[DHSLWH21]
	6/10	2^{240}	2^{240}	New MILP model, MITM	This work
	6.5/10	2^{240}	2^{240}	New MILP model, MITM	This work



- $(\overrightarrow{\iota}, \overleftarrow{\iota})$ = (8+16)=24, (48+0)=48
- $(d_{\mathcal{B}}, d_{\mathcal{R}}, d_{\mathcal{M}}, g_{\mathcal{B}}, g_{\mathcal{R}}, g_{\mathcal{BR}})$ = (+24, +24, +24, +0, +0, +0)
- $(d_{\mathcal{B}}-g_{\mathcal{R}}, d_{\mathcal{R}}-g_{\mathcal{B}}, d_{\mathcal{M}}-g_{\mathcal{B}}-g_{\mathcal{R}}-g_{\mathcal{BR}})$ = (24, 24, 24)

- DoF compensation at round 1
- **Same color** match at round 3
- 2^{32} times (pseudo-preimage) and 2^{16} times (preimage) improvements than previous best attack
- Reduce memory cost to $O(1)$ (compared to previous best attack with 2^{160})

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Preimage Attacks

Cipher (target)	#Rounds	T_1 †	T_2 ‡	Memory	Essential technique(s)	References
Streebog-512 (Compression)	7.5/12	2^{496}	–	2^{64}	Dedicated method	[MLHL15]
	7.5/12	2^{441}	–	2^{192}	GnD, MulAK	[HDSZHW22]
	7.5/12	2^{433}	–	2^{177}	**SIM**, GnD	This work
	8.5/12	2^{481}	–	2^{288}	GnD, MulAK	[HDSZHW22]
	8.5/12	2^{481}	–	2^{129}	**SIM**, GnD	This work
Streebog-512 (Hash)	7.5/12	–	2^{496}	2^{64}	Dedicated method	[MLHL15]
	7.5/12	–	$2^{478.25}$	2^{256}	MITM + Multi-collision¶	[HDSZHW22]
	7.5/12	–	$2^{474.25}$	2^{256}	MITM + Multi-collision	This work
	8.5/12	–	$2^{498.25}$	2^{288}	MITM + Multi-collision	[HDSZHW22]
	8.5/12	–	$2^{498.25}$	2^{256}	MITM + Multi-collision	This work

¶ The attack on the compression function of Streebog is converted into a preimage attack on its hash function using the technique from [AY14].

Conclusion

In this paper, we

- introduced three new advanced techniques into MITM: S-box Linearization (**LIN**), Distributed Initial Structures (**DIS**) and Structual Similarity (**SIM**)
- furnished the MITM framework and constructed more efficient MILP-based model
- found first 10-round MITM preimage/pseudo-preimage attacks on `AES-192` hashing
- improved MITM preimage/pseudo-preimage and/or collision attacks on `Whirlpool` and `Streebog`

For more details, please refer to our paper :)

`https://eprint.iacr.org/2024/300`

TYFL!