Partial Sums Meet FFT: Improved Attack on 6-Round AES

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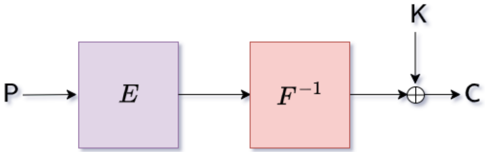
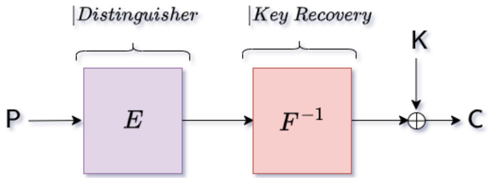
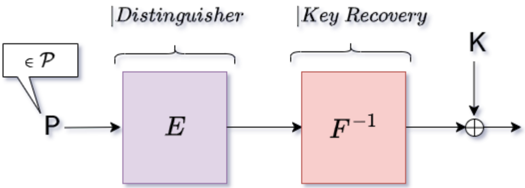
Cryptanalysis Perspective



Cryptanalysis Perspective



Key Recovery



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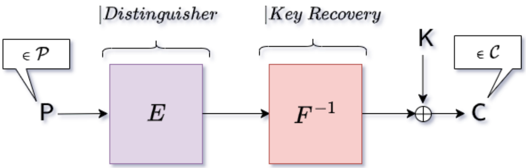

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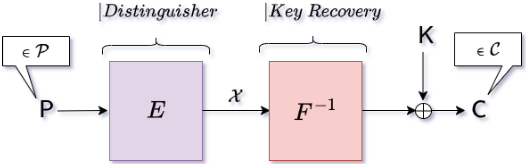
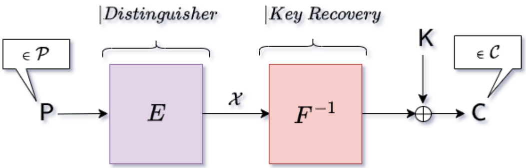
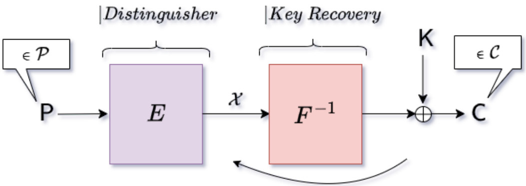




$$\bigoplus_{X \in \mathcal{X}} X = \bigoplus_{P \in \mathcal{P}} E(P) = 0$$

Integral/Zero-Sum Distinguisher

Key Recovery



$$\bigoplus_{X \in \mathcal{X}} X = \bigoplus_{P \in \mathcal{P}} E(P) = 0 = \bigoplus_{C \in \mathcal{C}} F(C \oplus K), \text{ For the right key } K$$

Integral/Zero-Sum Distinguisher

Key Recovery Programme

procedure FOO($\mathcal{C} \subseteq \{0,1\}^m$ of size 2^m)

for $\mathsf{K} \in \{0,1\}^m$ **do**

$\mathbf{S} = \mathbf{0}$

for $\mathsf{C} \in \mathcal{C}$ **do**

$\mathbf{S} = \mathbf{S} \oplus F(\mathsf{C} \oplus \mathsf{K})$

if $\mathbf{S} \neq \mathbf{0}$ **then**

Discard K

Key Recovery Programme

procedure $\text{FOO}(\mathcal{C} \subseteq \{0,1\}^m$ of size $2^m)$
    for $\mathsf{K} \in \{0,1\}^m$ **do**
        $\mathbf{S} = \mathbf{0}$
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            Discard K

procedure $\text{BAR}(\mathcal{C} \subseteq \{0,1\}^m$ of size $2^m)$
    for $\mathsf{K} \in \{0,1\}^m$ **do**
        $\mathbf{S} = \mathbf{0}$
        for $\mathsf{C} \in \{0,1\}^m$ **do**
            $\mathbf{S} = \mathbf{S} \oplus F(\mathsf{C} \oplus \mathsf{K}) G_{\mathcal{C}}(\mathsf{C})$
        if $\mathbf{S} \neq \mathbf{0}$ **then**
            Discard K

Key Recovery Programme

procedure $\textsc{Foo}(\mathcal{C} \subseteq \{0,1\}^m$ of size $2^m)$
  for $K \in \{0,1\}^m$ **do**
    $\mathbf{S} = \mathbf{0}$
    for $C \in \mathcal{C}$ **do**
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procedure $\textsc{Bar}(\mathcal{C} \subseteq \{0,1\}^m$ of size $2^m)$
  for $K \in \{0,1\}^m$ **do**
    $\mathbf{S} = \mathbf{0}$
    for $C \in \{0,1\}^m$ **do**
      $\mathbf{S} = \mathbf{S} \oplus F(C \oplus K) G_{\mathcal{C}}(C)$
    if $\mathbf{S} \neq \mathbf{0}$ **then**
      Discard K

$$G_{\mathcal{C}}(C) = \begin{cases} 1, & \text{if occurrences of } C \text{ is odd in } \mathcal{C} \\ 0, & \text{if occurrences of } C \text{ is even in } \mathcal{C} \end{cases}$$

Key Recovery Programme

procedure $\text{FOO}(\mathcal{C} \subseteq \{0,1\}^m$ of size $2^m)$
  for K $\in \{0,1\}^m$ **do**
    S $=$ **0**
    for C $\in \mathcal{C}$ **do**
      S $=$ **S** $\oplus F(\mathsf{C} \oplus \mathsf{K})$
    if S $\neq$ 0 then
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procedure $\text{BAR}(\mathcal{C} \subseteq \{0,1\}^m$ of size $2^m)$
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    for C $\in \{0,1\}^m$ **do**
      S $=$ **S** $\oplus F(\mathsf{C} \oplus \mathsf{K}) G_{\mathcal{C}}(\mathsf{C})$
    if S $\neq$ 0 then
      Discard K

BAR $\equiv$ Convolution for each K,

$$F * G(\mathsf{K}) = \bigoplus_{\mathsf{C} \in \{0,1\}^m} F(\mathsf{C} \oplus \mathsf{K}) G(\mathsf{C})$$

The BAR matrix

$$F * G(K) = \bigoplus_{C \in \{0,1\}^m} F(C \oplus K) G(C)$$

$$\begin{bmatrix} F(0\oplus 0) & F(0\oplus 1) & F(0\oplus 2) & F(0\oplus 3) & F(0\oplus 4) & F(0\oplus 5) & F(0\oplus 6) & F(0\oplus 7) \\ F(1\oplus 0) & F(1\oplus 1) & F(1\oplus 2) & F(1\oplus 3) & F(1\oplus 4) & F(1\oplus 5) & F(1\oplus 6) & F(1\oplus 7) \\ F(2\oplus 0) & F(2\oplus 1) & F(2\oplus 2) & F(2\oplus 3) & F(2\oplus 4) & F(2\oplus 5) & F(2\oplus 6) & F(2\oplus 7) \\ F(3\oplus 0) & F(3\oplus 1) & F(3\oplus 2) & F(3\oplus 3) & F(3\oplus 4) & F(3\oplus 5) & F(3\oplus 6) & F(3\oplus 7) \\ F(4\oplus 0) & F(4\oplus 1) & F(4\oplus 2) & F(4\oplus 3) & F(4\oplus 4) & F(4\oplus 5) & F(4\oplus 6) & F(4\oplus 7) \\ F(5\oplus 0) & F(5\oplus 1) & F(5\oplus 2) & F(5\oplus 3) & F(5\oplus 4) & F(5\oplus 5) & F(5\oplus 6) & F(5\oplus 7) \\ F(6\oplus 0) & F(6\oplus 1) & F(6\oplus 2) & F(6\oplus 3) & F(6\oplus 4) & F(6\oplus 5) & F(6\oplus 6) & F(6\oplus 7) \\ F(7\oplus 0) & F(7\oplus 1) & F(7\oplus 2) & F(7\oplus 3) & F(7\oplus 4) & F(7\oplus 5) & F(7\oplus 6) & F(7\oplus 7) \end{bmatrix} \times \begin{bmatrix} G(0) \\ G(1) \\ G(2) \\ G(3) \\ G(4) \\ G(5) \\ G(6) \\ G(7) \end{bmatrix}$$

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$$F * G(K) = \bigoplus_{C \in \{0,1\}^m} F(C \oplus K) G(C)$$

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The BAR matrix

$$\begin{bmatrix} F(0\oplus 0) & F(0\oplus 1) & F(0\oplus 2) & F(0\oplus 3) & F(0\oplus 4) & F(0\oplus 5) & F(0\oplus 6) & F(0\oplus 7) \\ F(1\oplus 0) & F(1\oplus 1) & F(1\oplus 2) & F(1\oplus 3) & F(1\oplus 4) & F(1\oplus 5) & F(1\oplus 6) & F(1\oplus 7) \\ F(2\oplus 0) & F(2\oplus 1) & F(2\oplus 2) & F(2\oplus 3) & F(2\oplus 4) & F(2\oplus 5) & F(2\oplus 6) & F(2\oplus 7) \\ F(3\oplus 0) & F(3\oplus 1) & F(3\oplus 2) & F(3\oplus 3) & F(3\oplus 4) & F(3\oplus 5) & F(3\oplus 6) & F(3\oplus 7) \\ F(4\oplus 0) & F(4\oplus 1) & F(4\oplus 2) & F(4\oplus 3) & F(4\oplus 4) & F(4\oplus 5) & F(4\oplus 6) & F(4\oplus 7) \\ F(5\oplus 0) & F(5\oplus 1) & F(5\oplus 2) & F(5\oplus 3) & F(5\oplus 4) & F(5\oplus 5) & F(5\oplus 6) & F(5\oplus 7) \\ F(6\oplus 0) & F(6\oplus 1) & F(6\oplus 2) & F(6\oplus 3) & F(6\oplus 4) & F(6\oplus 5) & F(6\oplus 6) & F(6\oplus 7) \\ F(7\oplus 0) & F(7\oplus 1) & F(7\oplus 2) & F(7\oplus 3) & F(7\oplus 4) & F(7\oplus 5) & F(7\oplus 6) & F(7\oplus 7) \end{bmatrix}_{8\times 8}$$

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Properties of BAR matrix $\mathcal{M}_m$

- $\mathcal{M}_m = \frac{1}{2^m}(\mathcal{H}_m \times \Delta \times \mathcal{H}_m)$, $\mathcal{H}_m[i,j] = \frac{1}{2^{m/2}}(-1)^{i \cdot j}$

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$$\mathcal{H}_1 = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \quad \mathcal{H}_2 = \frac{1}{2}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

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$$\text{In General } \mathcal{H}_m = \frac{1}{2^{m/2}}\begin{bmatrix} \mathcal{H}_{m-1} & \mathcal{H}_{m-1} \\ \mathcal{H}_{m-1} & -\mathcal{H}_{m-1} \end{bmatrix}$$

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- FHT: A divide-and-conquer algorithm, complexity $= \mathcal{O}(m2^m)$

Properties of BAR matrix $\mathcal{M}_m$

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Properties of BAR matrix $\mathcal{M}_m$

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$$\begin{aligned}\mathcal{M}_m \times \mathcal{C} &= \frac{1}{2^m}(\mathcal{H}_m \times \Delta \times \mathcal{H}_m) \times \mathcal{C} \\ &= \frac{1}{2^m}(\mathcal{H}_m \times ((\mathcal{H}_m \times \mathcal{M}_m^0) \star (\mathcal{H}_m \times \mathcal{C})))\end{aligned}$$

Properties of BAR matrix $\mathcal{M}_m$

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- Complexity: $2^{2m} \to 4m2^m$

Properties of BAR matrix $\mathcal{M}_m$

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- Complexity: $2^{2m} \rightarrow 4m2^m$
- For $m = 32$: $2^{64} \rightarrow 2^{39}$

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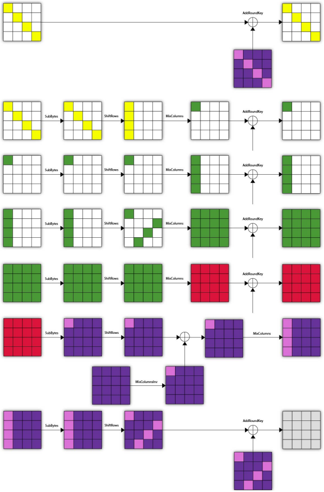
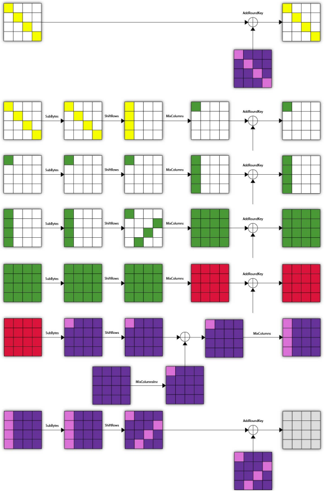
Plan of this Section

1. Motivation
2. Integral attack on AES
3. Partial Sums Meet FFT
4. Results and Conclusion

Integral Attack On AES



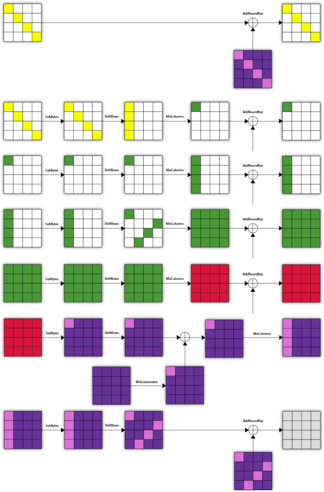
Integral Attack On AES



$$\chi(\mathrm{K}, \mathrm{C}) = S(\mathrm{K}_4 \oplus S_3(\mathrm{K}_3 \oplus \mathrm{C}_3) \oplus S_2(\mathrm{K}_2 \oplus \mathrm{C}_2)$$
$$\oplus S_1(\mathrm{K}_1 \oplus \mathrm{C}_1) \oplus S_0(\mathrm{K}_0 \oplus \mathrm{C}_0))$$

1. Naive Complexity: $c \times 2^{72}$

Integral Attack On AES



$$\chi(\mathrm{K}, \mathrm{C}) = S(\mathrm{K}_4 \oplus S_3(\mathrm{K}_3 \oplus \mathrm{C}_3) \oplus S_2(\mathrm{K}_2 \oplus \mathrm{C}_2) \oplus S_1(\mathrm{K}_1 \oplus \mathrm{C}_1) \oplus S_0(\mathrm{K}_0 \oplus \mathrm{C}_0))$$

1. Naive Complexity: $c \times 2^{72}$
2. FHT [Todo et al. [TA14]]: For each fixed K_4,

$$\chi(\mathrm{K}, \mathrm{C}) = S_{\mathrm{K}_4}(S_3(\mathrm{K}_3 \oplus \mathrm{C}_3) \oplus S_2(\mathrm{K}_2 \oplus \mathrm{C}_2) \oplus S_1(\mathrm{K}_1 \oplus \mathrm{C}_1) \oplus S_0(\mathrm{K}_0 \oplus \mathrm{C}_0))$$

The Problem with Finite Field Arithmetic

- $F * G(k) = \bigoplus_{x} F(x \oplus k)g(x)$ VS $F * G(k) = \sum_{x} F(x \oplus k)g(x)$
- We need functions whose output is an integer and not an element of $\mathbb{F}_2^8$

The Problem with Finite Field Arithmetic

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- Todo et al. [TA14] proposed to consider 8 outputs separately
- $F(\mathsf{K} \oplus \mathsf{C}) = (F^0(\mathsf{K} \oplus \mathsf{C}), ..., F^7(\mathsf{K} \oplus \mathsf{C}))$
- and compute convolution for each F^i separately

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- and compute convolution for each F^i separately
- So we need to run the algorithm for 8 times.

Complexity: Time $c \times 2^8 \times 8 \times 2^{39}$ and Memory $f \times 2^{32}$

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Partial Sum Technique [FKL+00]

$$\chi(K \oplus C) = S(K_4 \oplus S_3(K_3 \oplus C_3) \oplus S_2(K_2 \oplus C_2) \oplus S_1(K_1 \oplus C_1) \oplus S_0(K_0 \oplus C_0)))$$

- Guess (K_1, K_0) and compute $S_1(K_1 \oplus C_1) \oplus S_0(K_0 \oplus C_0)$

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- Guess (K_1, K_0) and compute $S_1(K_1 \oplus C_1) \oplus S_0(K_0 \oplus C_0)$

Declare an empty bit-array A_1 of size 2^{24}

for $c_0, c_1, c_2, c_3 \in \{0,1\}^{32}$ **do**

$a_1 \leftarrow (S_0(C_0 \oplus K_0) \oplus S_1(C_1 \oplus K_1)) G_{\mathcal{C}}(C_0, C_1, C_2, C_3)$

$A_1[a_1, C_2, C_3] \leftarrow A_1[a_1, C_2, C_3] \oplus 1$

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$$\chi(K \oplus C) = S(K_4 \oplus S_3(K_3 \oplus C_3) \oplus S_2(K_2 \oplus C_2) \oplus a_1)$$

Complexity: $2^{16} * 2^{32}$

Partial Sum Technique [FKL+00] at a Glance

$$\mathsf{K}_4 \oplus S_3(\mathsf{K}_3 + \mathsf{C}_3) \oplus S_2(\mathsf{K}_2 + \mathsf{C}_2) \oplus \underbrace{S_1(\mathsf{K}_1 + \mathsf{C}_1) \oplus S_0(\mathsf{K}_0 + \mathsf{C}_0)}_{2^{48}}$$

$$\mathsf{K}_4 \oplus S_3(\mathsf{K}_3 + \mathsf{C}_3) \oplus \underbrace{S_2(\mathsf{K}_2 + \mathsf{C}_2) \oplus A_1}_{2^{48}}$$

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$$\underbrace{\mathsf{K}_4 \oplus A_3}_{2^{48}}$$

Partial Sum Technique [FKL+00] at a Glance

$$K_4 \oplus S_3(K_3 + C_3) \oplus S_2(K_2 + C_2) \oplus \underbrace{S_1(K_1 + C_1) \oplus S_0(K_0 + C_0)}_{2^{48}}$$

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$$K_4 \oplus \underbrace{S_3(K_3 + C_3) \oplus A_2}_{2^{48}}$$

$$\underbrace{K_4 \oplus A_3}_{2^{48}}$$

Complexity: Time $c \times 2^{50}$ and Memory 2^{24}

Plan of this Section

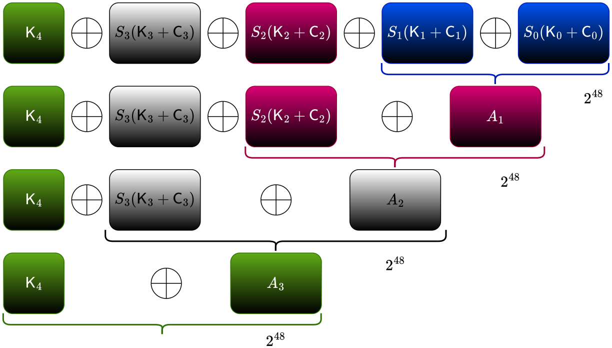
1. Motivation
2. Integral attack on AES
3. Partial Sums Meet FFT
4. Results and Conclusion

Basic Idea

- We follow the general structure of the partial sums attack
- Replace each partial sum with FFT
- However, rearrange the steps to make it FFT compatible
- Rearrange the steps again to reduce memory complexity

Partial Sums Meet FFT

$$\chi(K \oplus C) = S(K_4 \oplus S_3(K_3 \oplus C_3) \oplus S_2(K_2 \oplus C_2) \oplus S_1(K_1 \oplus C_1) \oplus S_0(K_0 \oplus C_0)))$$



Partial Sums Meet FFT

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$$\underbrace{S_1(K_1 + C_1) \bigoplus S_0(K_0 + C_0)}_{A_1}$$

Partial Sums Meet FFT

$$\chi(K \oplus C) = S(K_4 \oplus S_3(K_3 \oplus C_3) \oplus S_2(K_2 \oplus C_2) \oplus S_1(K_1 \oplus C_1) \oplus S_0(K_0 \oplus C_0)))$$

$$\underbrace{\boxed{S_1(K_1 + C_1)} \bigoplus \boxed{S_0(K_0 + C_0)}}_{\boxed{A_1}}$$

$A_1 = [\,]$ of size $2^{16} \times 2^{24}$; ▷ 2^{40} memory

for all $(a_1, C_2, C_3) \in \{0,1\}^{24}$ **do**

  for all $(K_0, K_1) \in \{0,1\}^{16}$ **do**

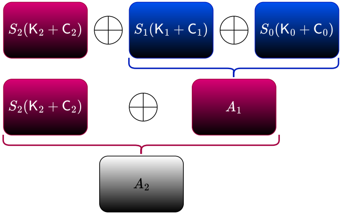
    $A_1[K_0, K_1][a_1, C_2, C_3] \leftarrow \bigoplus_{C_0, C_1} A[C_0, C_1, C_2, C_3] \cdot \mathbb{1}(S_0(C_0 \oplus K_0) \oplus S_1(C_1 \oplus K_1) = a_1)$

Partial Sums Meet FFT

$$S_2(\mathsf{K}_2+\mathsf{C}_2) \oplus \underbrace{S_1(\mathsf{K}_1+\mathsf{C}_1) \oplus S_0(\mathsf{K}_0+\mathsf{C}_0)}_{A_1}$$

$$\underbrace{S_2(\mathsf{K}_2+\mathsf{C}_2) \oplus A_1}_{A_2}$$

Partial Sums Meet FFT



for all $(K_0, K_1) \in \{0,1\}^{16}$ **do**

  $A_2 = [\,]$ of size $2^8 \times 2^{16}$;

  for all C_3 **do**

    for all $(K_2, a_2) \in \{0,1\}^{16}$ **do**

$$A_2[K_2][a_2, C_3] \leftarrow \bigoplus_{a_1, C_2} A_1[K_0, K_1][a_1, C_2, C_3] \cdot \mathbb{1}(a_1 \oplus S_2(C_2 \oplus K_2) = a_2)$$

Partial Sums Meet FFT

for all $(\mathrm{K}_0, \mathrm{K}_1) \in \{0,1\}^{16}$ **do**

$\quad \ldots$

$\quad$ **for all** $k_2 \in \{0,1\}^8$ **do**

$\quad\quad \ldots$

$\quad\quad$ **for all** $k_3 \in \{0,1\}^8$ **do**

$\quad\quad\quad A_4$ of size 2^8;

$\quad\quad\quad$ **for all** $k_4 \in \{0,1\}^8$ **do**

$$A_4[k_4] \leftarrow \bigoplus_{a_3} A_3[k_3][a_3] \cdot S(a_3 \oplus k_4)$$

$\quad\quad\quad$ **for all** $k_4 \in \{0,1\}^8$ **do**

$\quad\quad\quad\quad$ **if** $A_4[k_4] \neq 0$ **then**

$\quad\quad\quad\quad\quad k_0, k_1, k_2, k_3, k_4$ is not a valid key candidate

Complexities of Various Steps

Steps	Time	Memory
1	$2^{24} * (4 * 16 * 2^{16}) = 2^{46}$	2^{40}
2	$2^{16} * (2^{8} * (4 * 16 * 2^{16})) = 2^{46}$	2^{24}
3	$2^{16} * 2^{8} * (4 * 16 * 2^{16}) = 2^{46}$	2^{16}
4	$2^{16} * 2^{8} * 2^{8} * (8 * 4 * 8 * 2^{8}) = 2^{48}$	2^{8}
Total	$2^{48.5}$	2^{40}

Packing Multiple FFT's

for all $(\mathrm{K}_0, \mathrm{K}_1) \in \{0,1\}^{16}$ **do**

$\quad \ldots$

$\quad$ **for all** $k_2 \in \{0,1\}^8$ **do**

$\quad\quad \ldots$

$\quad\quad$ **for all** $k_3 \in \{0,1\}^8$ **do**

$\quad\quad\quad \ldots$

$\quad\quad\quad$ **for all** $k_4 \in \{0,1\}^8$ **do**

$\quad\quad\quad\quad \ldots$

$$A_4[k_4] \leftarrow \bigoplus_{a_3} S(a_3 \oplus k_4) \cdot A_3[k_3][a_3]$$

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Packing Technique

- We assume that the attack is implemented using 64-bit operations in software

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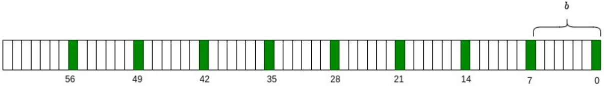

$$\bigoplus_{a_3} S(a_3 \oplus k_4) \cdot A_3[k_3][a_3] = \sum_{a_3} (2^{7b} S^7(\mathsf{K} \oplus \mathsf{C}) + \cdots + S^0(\mathsf{K} \oplus \mathsf{C}))A_3[k_3][a_3]$$

$$= \sum_{a_3} \sum_{j} 2^{jb} S^j(\mathsf{K} \oplus \mathsf{C}) \cdot A_3[k_3][a_3]$$

How large should b be?

- How large should b be so that $S^j(\mathsf{K} \oplus \mathsf{C}) < 2^b \; \forall j$?

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How large should b be?

- How large should b be so that $S^j(\mathsf{K} \oplus \mathsf{C}) < 2^b \ \forall j$?
- Suppose S is a balanced function then each $S^j(\mathsf{K} \oplus \mathsf{C})$ is the sum 128 elements
- Thus each $S^j(\mathsf{K} \oplus \mathsf{C})$ is distributed as $Bin(128, 1/2)$
- Expectation is 64 and Standard deviation $4\sqrt{2}$
- If $b = 7$, Using Chernoff bound, $Pr(S^j(\mathsf{K} \oplus \mathsf{C}) > 2^7)$ is extremely small

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- Thus, $b \leq 7$

Using $b = 7$, we compute 8 convolutions in parallel

Complexity: 2^{45} VS 2^{48}

Complexities of Various Steps

Steps	Time	Memory
1	$2^{46}/7$	2^{40}
2	$2^{46}/7$	2^{24}
3	$2^{46}/7$	2^{16}
4	$2^{48}/8$	2^{8}
Total	$\approx 2^{44}$	2^{40}

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2	$2^{46}/7$	2^{24}
3	$2^{46}/7$	2^{16}
4	$2^{48}/8$	2^{8}
Total	$\approx 2^{44}$	2^{40}

But still we need at least $128GB$ of memory

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Low memory Variant

for all $K_0 \in \{0,1\}^8$ **do**

A_0 of size 2^{32}; $\triangleright$ 2^{32} memory

for all $(C_0, C_1, C_2, C_3) \in \{0,1\}^{32}$ **do**

$a_0 \leftarrow S_0(C_0 \oplus K_0)$

$A_0[a_0, C_1, C_2, C_3] \leftarrow A[C_0, C_1, C_2, C_3]$

A_1 of size $2^8 \times 2^{24}$; $\triangleright$ 2^{32} memory

for all $(C_2, C_3) \in \{0,1\}^{16}$ **do**

for all $(K_1, a_1) \in \{0,1\}^{16}$ **do**

$$A_1[K_1][a_1, C_2, C_3] \leftarrow \bigoplus_{a_0, C_1} A_0[a_0, C_1, C_2, C_3] \cdot \mathbb{1}(a_0 \oplus S_1(C_1 \oplus K_1) = a_1)$$

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A_1 of size $2^8 \times 2^{24}$; $\triangleright$ 2^{32} memory

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Time: $\approx c \times 2^{46}$ and Memory: $0.5\,GB$

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Integral Attack on 6-Round AES

	FHT+Part. Sums	FHT	Part. Sums
AWS Instance	m6i.32xlarge	r6i.32xlarge	m6i.32xlarge
Running Time(m)	48	3120	4859
Total Cost (USD)	5	418	497

In Conclusion: Our attack is 65 times faster and 83 times cheaper

Integral Attack on 6-Round AES

Cipher	Rounds	Data	Time	Technique and Source
AES	6	2^{32} CP	2^{71} Enc.	Square [DKR97]
		$6 \cdot 2^{32}$ CP	2^{52} S-box Eval.	Square & Partial sums [FKL$^+$01]
		2^{71} ACPC	2^{71} Enc.	Boomerang [Bir04]
		2^{33} CP	2^{52} S-box Eval.	Square & Partial sums [Tun12]
		$6 \cdot 2^{32}$ CP	2^{52} Add.	Square & FHT [TA14]
		2^{26} CP	2^{80} Enc.	Mixture Differential [BDK$^+$20]
		2^{55} ACPC	2^{80} Enc.	Retracing Boomerang [DKRS20]
		$2^{79.7}$ ACPC	2^{78} Enc.	Boomeyong [RSP21]
		2^{59} ACPC	2^{61} Enc.	Truncated Boomerang [BL22]
		2^{33} CP	$2^{46.4}$ Add.	Square & Partial sums & FHT

The Improvement Matrix

	AES	Kuznyechik		MISTY1	CLEFIA
Rounds	6	6	7	8 (Full)	12
Improvement Factor	2^5	2^6	2^6	2^3	2^{30}

Thank You for your attention!

Any questions?

Our Attack Without Packing

- Factor of 6 improvement than Todo-Aoki's attack

Our Attack Without Packing

- Factor of 6 improvement than Todo-Aoki's attack
- 16/8 vs. 32 bit addition (Factor of 12 improvement)

Our Attack Without Packing

- Factor of 6 improvement than Todo-Aoki's attack
- 16/8 vs. 32 bit addition (Factor of 12 improvement)
- Factor of 8 improvement than Partial-sum attack

Other Attacks With Packing

- Factor of 20 improvement than Todo-Aoki's attack

Other Attacks With Packing

- Factor of 20 improvement than Todo-Aoki's attack
- Factor of 60 improvement than Partial-sum attack

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