



Automating the Search for Cryptanalytic Attacks


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- Leuven · 26 Mar 2024

> <https://iaik.tugraz.at>



Outline

 Motivation

 Finding Distinguishers with MILP/SAT Solvers

- Mixed-Integer Linear Programming (MILP)
- Boolean Satisfiability and Constraint Programming (SAT/SMT, CP)

 Dedicated Algorithms

 Optimized Key Recovery Attacks

 Frameworks

Motivation



Differential Cryptanalysis [BS90]

Method



Attack Goals



collision,
forgery



key recovery

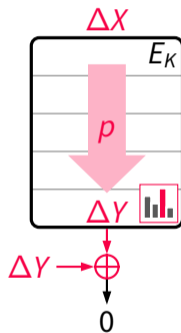
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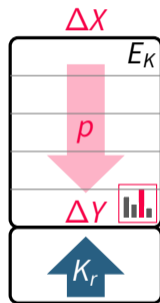
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Linear Cryptanalysis [Mat93]

Method

Linear mask α



Linear mask β

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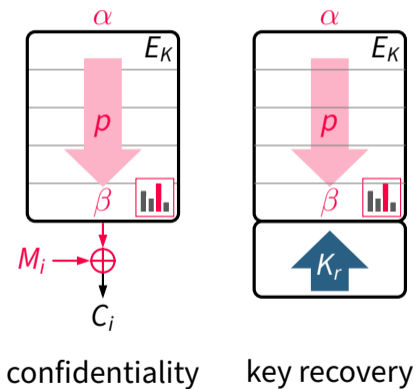


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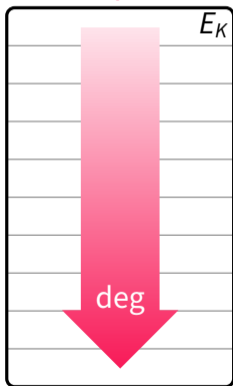
Attack Goals



Integral Cryptanalysis [Lai94; Knu94; KW02]

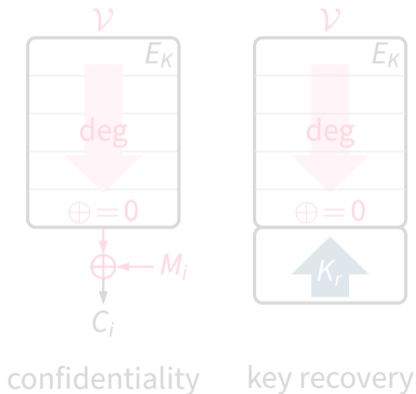
Method

Cube space \mathcal{V}



Zero-sum $\oplus=0$

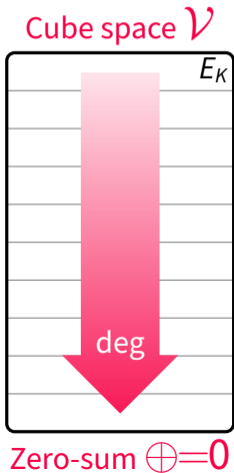
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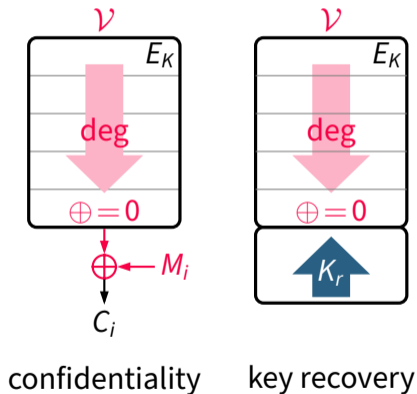
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How to Find Distinguishers



By hand

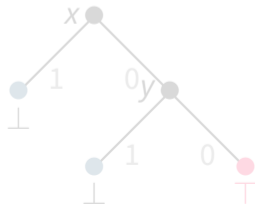
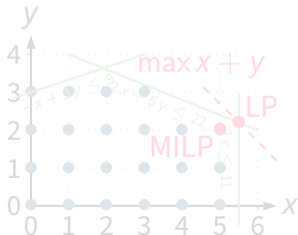


Dedicated solvers



General-purpose solvers:

- SAT/SMT (Boolean **SAT**isfiability/**Sat.** Modulo Theories)
- MILP (**Mixed Integer Linear Programming**)
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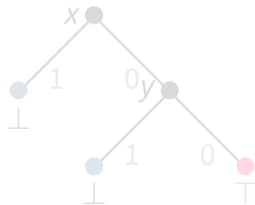
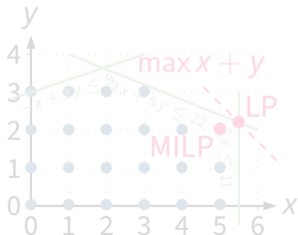


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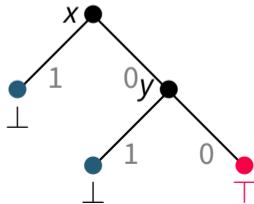
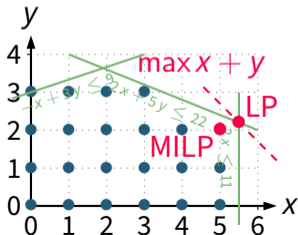


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- CP (**C**onstraint **P**rogramming)



Finding Distinguishers with MILP/SAT Solvers




Basic Approach

 Model constraints that characterize correct characteristics/solutions

- Coarse-grained: truncated patterns (which S-boxes are active?)
- Fine-grained: precise differences/masks

 Model cost (if applicable)

 Express the search goal: any one / all / best / good solution(s)?

Mixed-Integer Linear Program (MILP)

Linear Programming (LP) is a method to solve optimization problems

- on the real-valued, positive decision variables $x \in \mathbb{R}^d, x \geq 0$
- with a linear objective function (min or max) $f(x) = c^T x = \sum_{i=1}^d c_i x_i$
- under J linear constraints (s.t.) $Ax \leq b$, i.e., $\sum_{i=1}^d a_{ji} x_i \leq b_j$ for $1 \leq j \leq J$:

$$\max_{x \in \mathbb{R}^d} \{c^T x \mid Ax \leq b \wedge x \geq 0\}$$

Mixed-Integer Linear Programming (MILP) allows some of the decision variables to be constrained to integer values: $x \in \mathbb{Z}^i \times \mathbb{R}^{d-i}$.

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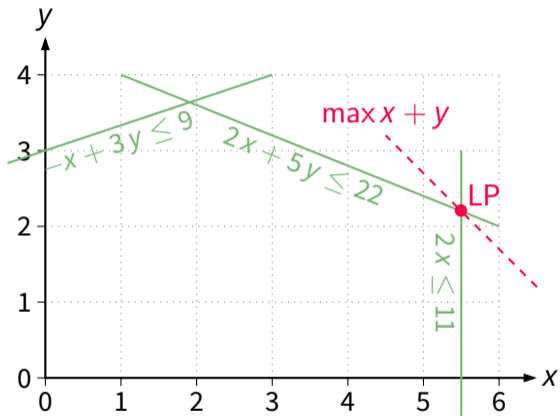
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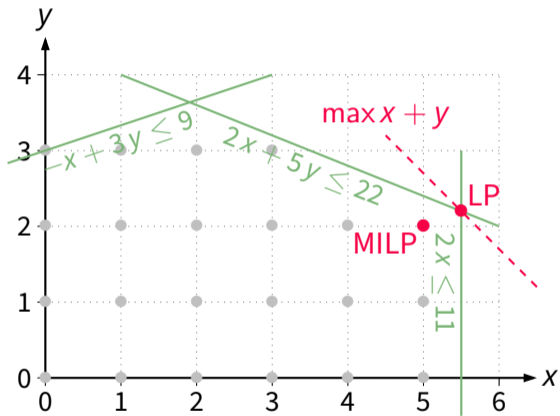
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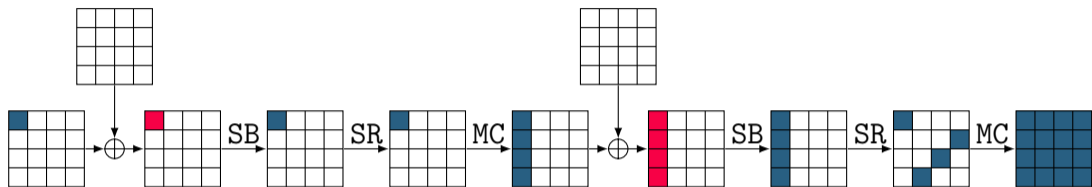
LP vs. MILP



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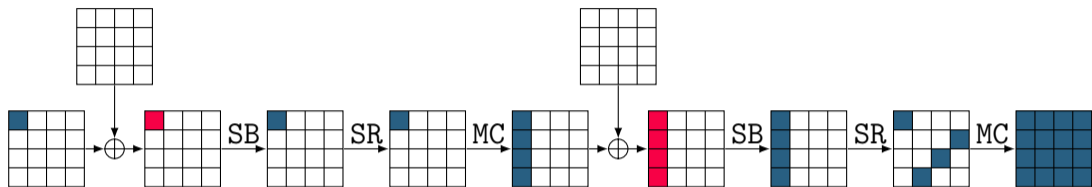
MILP – Example application: AES [MWGP11; WW11]



Variables: 1 binary variable per state byte (active/inactive)

- AddRoundKey: input = output
- SubBytes: input = output, cost = sum(inputs)
- ShiftRows: variable renaming
- MixColumns: for each active column: $\text{sum}(\text{inputs}) + \text{sum}(\text{outputs}) \geq 5 (= \mathcal{B})$

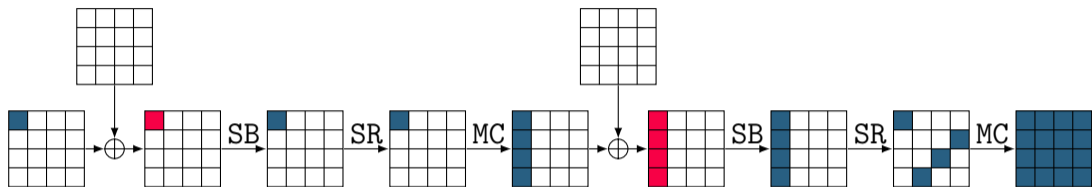
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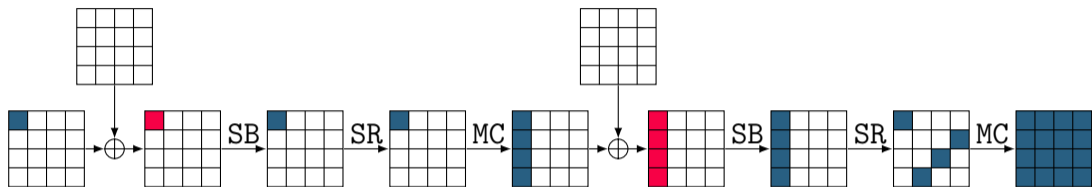
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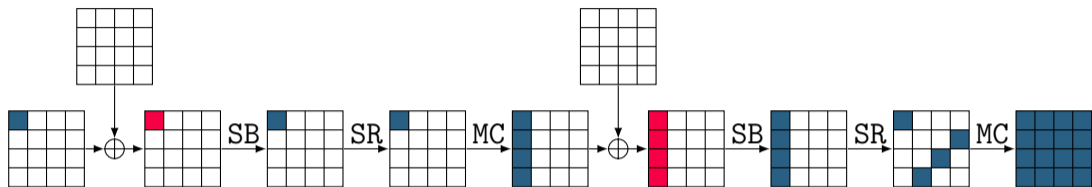
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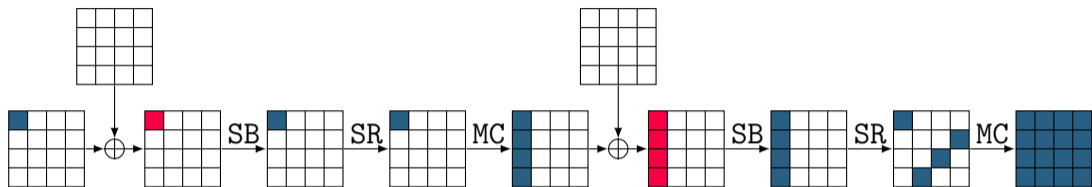
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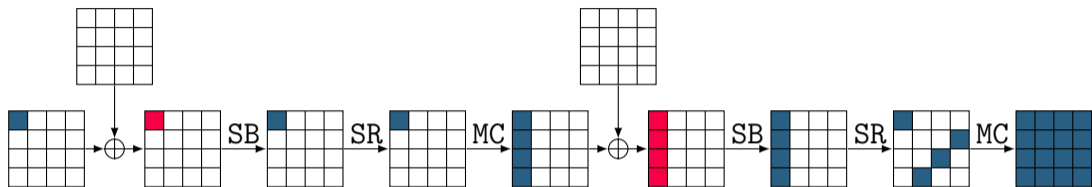
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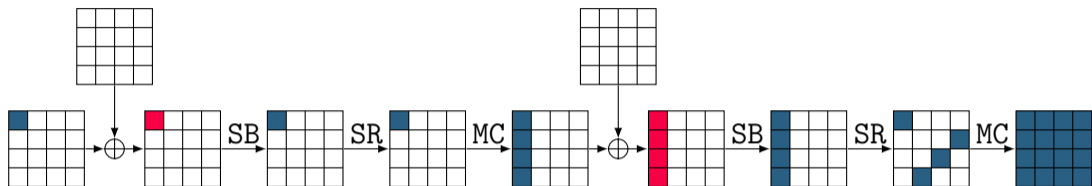
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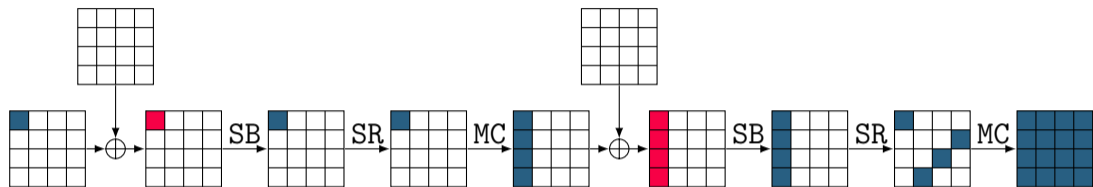
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Variables:

- $S_{r,i,j} \in \{0, 1\}$: Is S-box in row i , column j in round r active?
- $M_{r,j} \in \{0, 1\}$: Is MixColumns j in round r active?

Linear Program:

$$\min \sum_{r,i,j} S_{r,i,j} \quad (\text{Min \# active S-boxes})$$

$$\text{s.t. } \mathcal{B} \cdot M_{r,j} \leq \sum_i S_{r,i,(i+j)\%4} + \sum_i S_{r+1,i,j} \leq 8 \cdot M_{r,j} \quad (\text{For each MixColumns})$$

$$\sum_{i,j} S_{0,i,j} \geq 1 \quad (\text{Non-triviality})$$

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MILP – Example application: AES – Code in sagemath

```
#!/usr/bin/env sage
rounds = range(4)
p = MixedIntegerLinearProgram(maximization=False)
S = p.new_variable(name='sbox', binary=True)
M = p.new_variable(name='mcol', binary=True)

for r in rounds:
    for j in [0..3]:
        activecells = sum(S[r,i,(i+j)%4] for i in [0..3]) \
            + sum(S[r+1,i,j] for i in [0..3])
        p.add_constraint(5*M[r,j] <= activecells <= 8*M[r,j])
p.add_constraint(sum(S[0,i,j] for i in [0..3] for j in [0..3]) >= 1)

p.set_objective(sum(S[r,i,j] for r in rounds for i in [0..3] \
                    for j in [0..3]))
p.solve()
print(p.get_objective_value(), p.get_values(S))
```

MILP – Advanced models

- Modeling more complex relations of “allowed transitions” accurately
 - Activity patterns for linear layers: XOR, near-MDS matrices [ABD+23], ...
 - DDT/LAT for bitwise S-box models [SHW+14b; SHW+14a], ARX [FWG+16]
- Need to translate **vertex representation** into **half-space representation**

$$\text{XOR : } \left\{ \begin{array}{l} \text{inputs } \square, \square \rightarrow \text{output } \square \\ \text{inputs } \square, \blacksquare \rightarrow \text{output } \blacksquare \\ \text{inputs } \blacksquare, \square \rightarrow \text{output } \blacksquare \\ \text{inputs } \blacksquare, \blacksquare \rightarrow \text{output } \square \text{ or } \blacksquare \end{array} \right. \rightarrow \left\{ \begin{array}{l} l_1 + l_2 \geq 0 \\ l_1 + 0 \geq l_2 \\ l_2 + 0 \geq l_1 \end{array} \right.$$

- For large tables, this becomes very heavy (e.g., 8-bit S-boxes [AST+17; SW23])

SAT/SMT/CP – Different Levels of Convenience

- 1 SAT (Satisfiability) Solvers:** Find valid solution or prove unsatisfiability of CNF

$$\bigwedge_i \bigvee_j l_{ij} \quad \text{with literals } l_{ij} \in \{v_{ij}, \neg v_{ij}\}$$

Example solvers: MiniSAT, lingeling, and a myriad others

- 2 SMT (Sat. Modulo Theories) Solvers:** Accept a more general grammar including bitvector operations such as integer addition.

Example solvers: STP (“Simple Theorem Prover”), ...

- 3 CP (Constraint Programming) Solvers:** Accept an even more general grammar (depends on solver).

Example solvers/frameworks: MiniZinc, Z3, Choco, ...

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SAT/SMT/CP for Finding Distinguishers [MP13; Köl14]

- ➔ Solves a **constraint satisfaction problem**, may not be optimal
 - “Emulate” optimization: “is there a solution better than $X, X + 1, X + 2, \dots$?”
- ⊕ Useful to find valid solutions under some constraints
 - Finding characteristics that follow a given truncated pattern
 - Finding solutions for other crypto problems (preimage, ...)
- ⊖ Not so efficient for some more complex problems
 - Not so good for modelling a **cost** sum or optimization [EME22]
 - Not perfectly **parallelizable**

Dedicated Algorithms



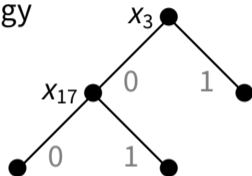
Dedicated Tools for Hash Functions: Examples

- **SHA-1:** HashClash
🔗 <https://github.com/cr-marcstevens/hashclash>
- **SHA-2:** nldtool
🔗 <https://github.com/iaikkrypto/nldtool>
- **SHA-3:** KeccakTools
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Dedicated Guess-and-Determine Search

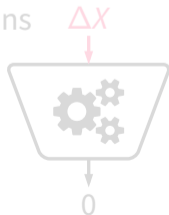
- **Guess-and-Determine Search** is a general search strategy

- Traverse search tree to find a valid solution
- SAT solvers use it on CNF level
- This is an example on small (differential) circuits



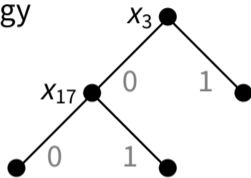
- `nldtool`: Automated search for characteristics and solutions

- Hash collision search
- Application example: SHA-2 [MNS11; MNS13; DEM15]

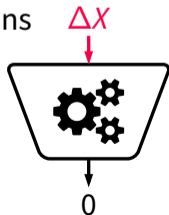


Dedicated Guess-and-Determine Search

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- **nldtool**: Automated search for characteristics and solutions
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Guess-and-Determine Search Algorithm

while there are undetermined bits **do**

Decision (Guessing)

- 1** Pick an undetermined bit
- 2** Constrain this bit

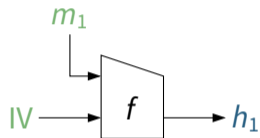
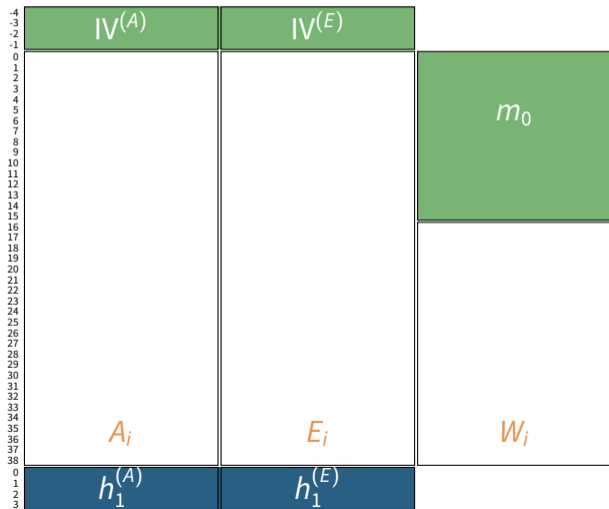
Deduction (Propagating)

- 3** Propagate the new information to other variables and equations
- 4** **if** no inconsistency is detected, goto step 1

Correction (Backtracking)

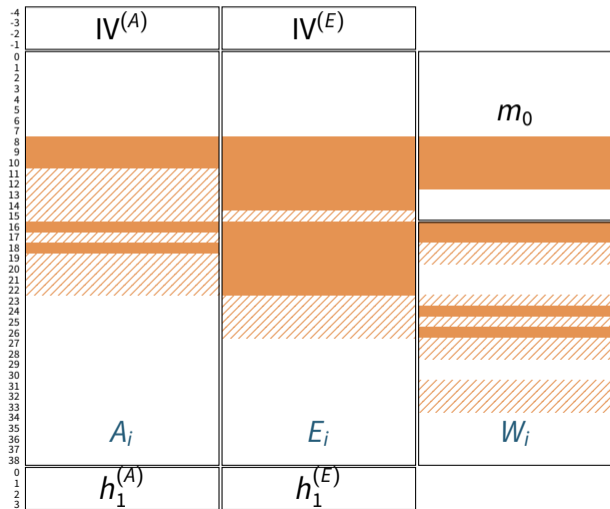
- 5** **if** possible, apply a different constraint to this bit, goto step 3
- 6** **else** undo guesses until this critical bit can be resolved

Example: Semi-Free-Start Collision for 39-step SHA-512



- Shows state words A_i , E_i , W_i
- Inputs IV , m_1
- Output h_1

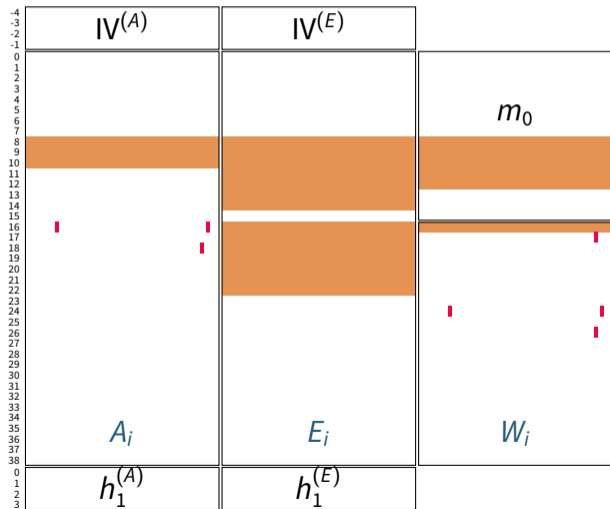
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Starting point:

0. “Local Collision” with few active message words
- Active words with differences [?]
 - ▨ No differences [-] (cancellation required)
 - No differences [-]

Example: Semi-Free-Start Collision for 39-step SHA-512



Search strategy:

1. Fix high-probability parts
2. Fix signed differences
3. Find message pair

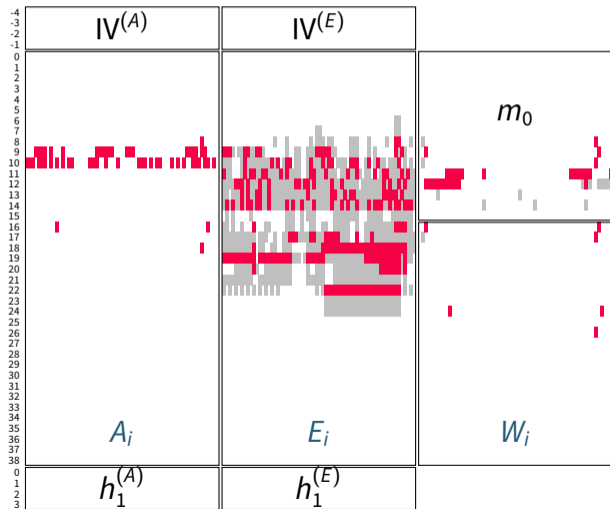
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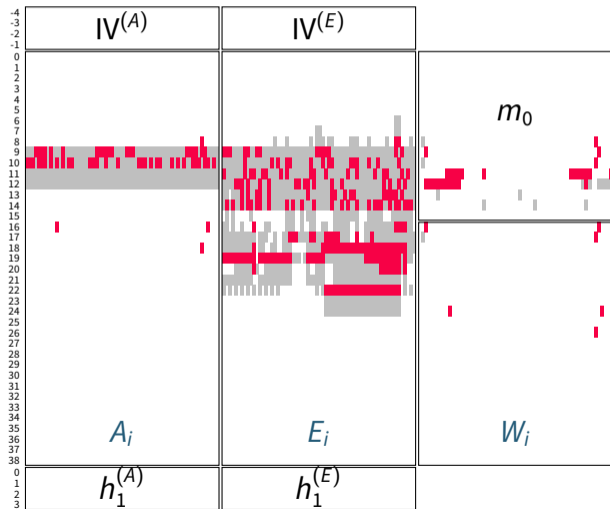
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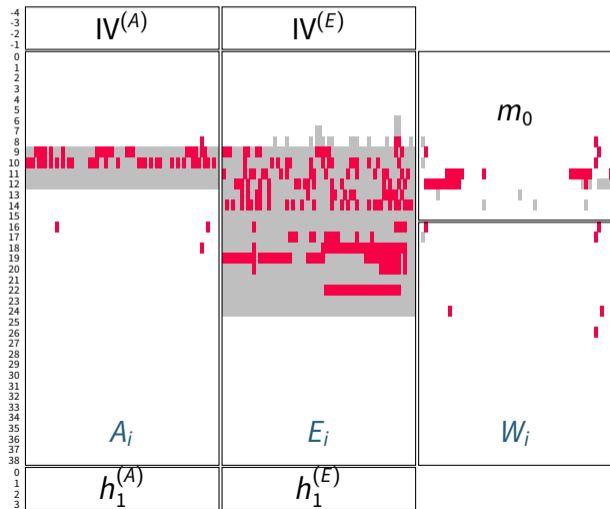
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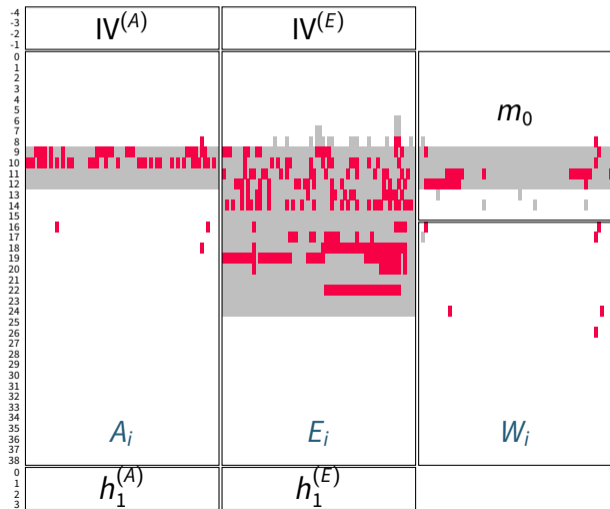
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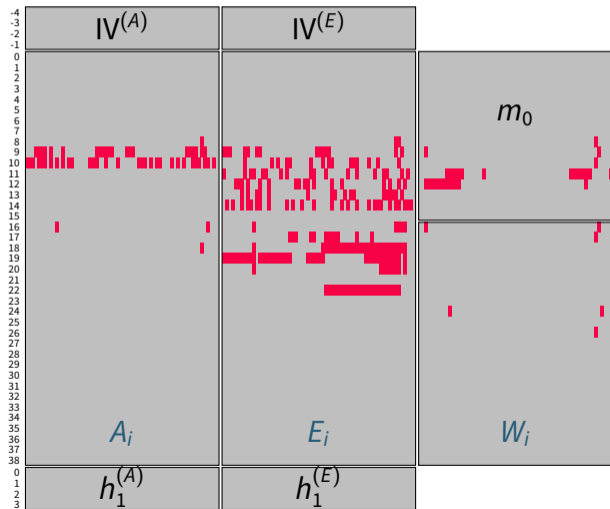
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Optimized Key Recovery Attacks



The Need for Tools

- Key recovery has long been ignored
- Fewer choices to make for the attacker

...but ...

- Optimizations involve choices and tradeoffs
- Precise evaluation is tedious
- “Optimal” distinguisher doesn’t guarantee optimal attack

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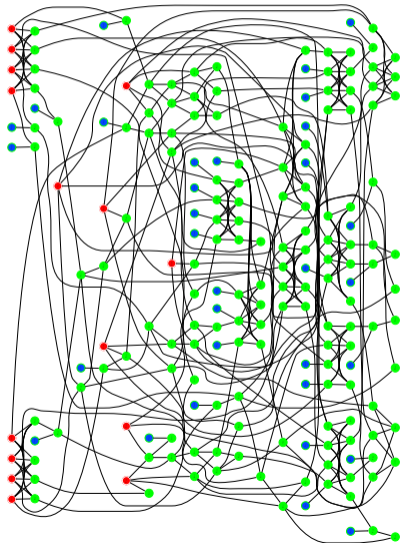
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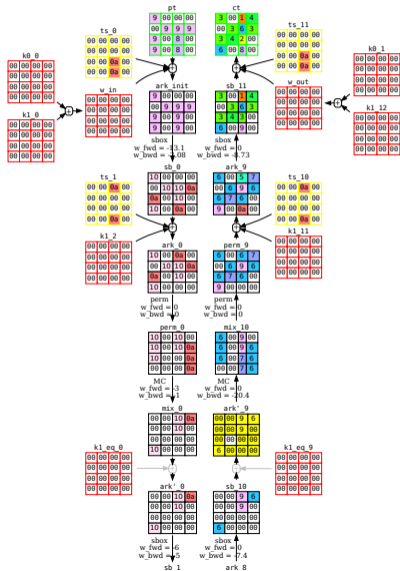
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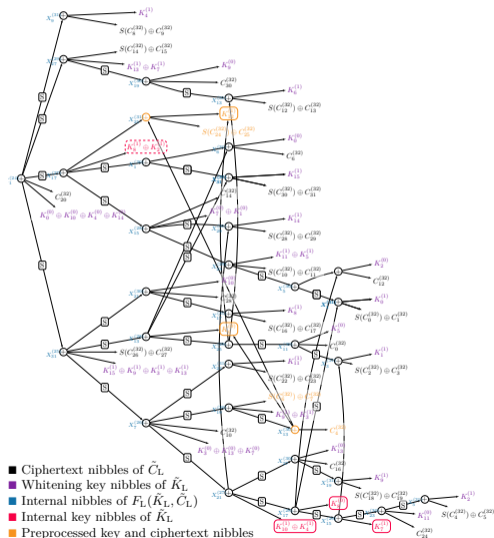
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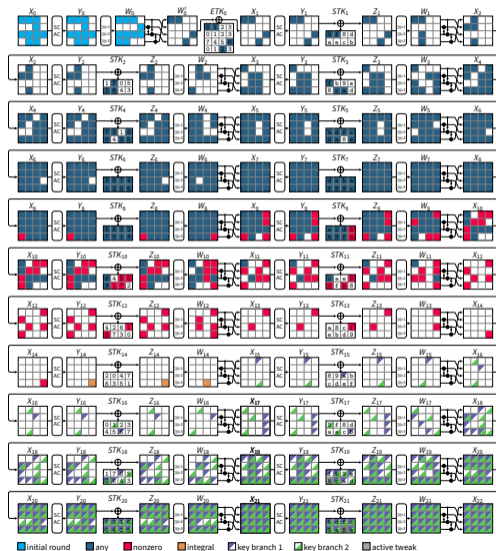
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Why Optimizing Full Key Recovery Attacks is Challenging

- 🔗 Preferably use a **joint model** for distinguisher and key recovery
 - Only works for satisfiability-based distinguishers
- 📊 **Complexity formulas** are often complicated
 - Mix of polynomial/exponential terms; simplified assumptions
- 👣 **Multi-step** processes lead to heavy models
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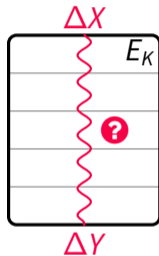
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Some distinguishers are based on the **non-existence** of a valid characteristic:

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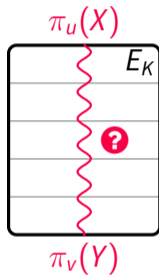


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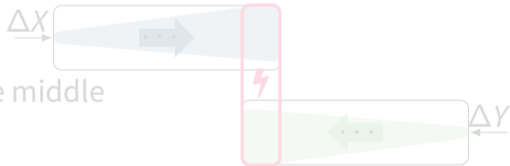
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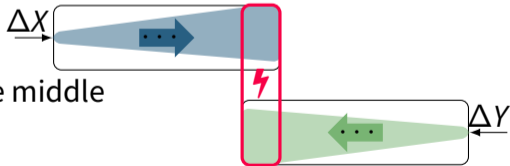
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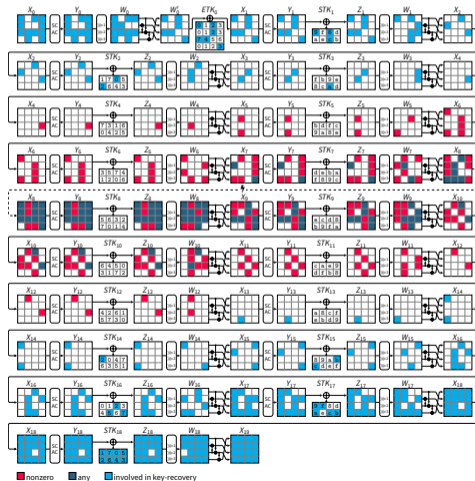
Example: Finding Full ID/ZC/Integral Attacks [HSE23]

- Impossible-differential (ID) attacks
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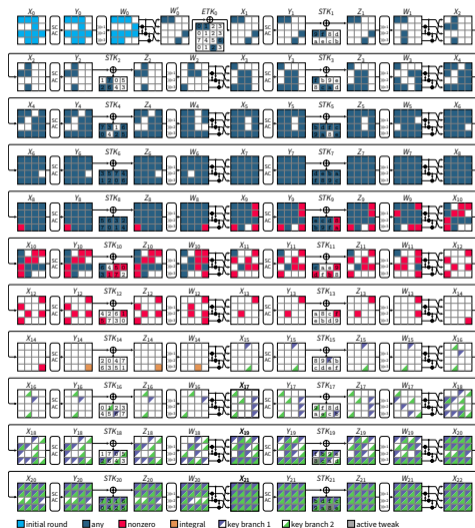
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Frameworks



What exactly is a “Framework”?

Judging from paper titles, we have a plethora of frameworks, but ...

- ❓ Generality & Applicability
- ❓ Reuseability & Extensibility
- ❓ Maintainability & Verifiability

Frameworks: Examples

 **CryptoSMT** [Köl14; AK18]  <https://github.com/kste/cryptosmt>

- Differential/linear trails, clustering, key/preimage recovery, ...; based on SMT (STP, Boolector, CryptoMiniSat)

 **CASCADA** [RR22]  <https://github.com/ranea/CASCADA>

- Differential/linear trails, impossible differentials/zero-correlation, ...; based on SMT

 **CLAASP** [BGG+23]  <https://github.com/Crypto-TII/claasp>

- All of the above, neural tests; supports many solvers

Frameworks: Challenges

- **Cipher representation**
 - Based on building blocks? As a DAG? Software/hardware code?
- **Efficiency vs. precision**
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Conclusion

⚙️ Tools and solvers can help find attacks and derive bounds

🔍 Very active area with many open challenges

- More efficient and precise models
- Application to other design paradigms and attack techniques
- Modeling full attacks (not just the distinguisher)
- Frameworks and reuseability

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