Cryptanalysis Beyond Primitives

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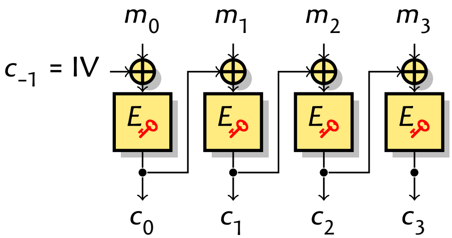
Modes and primitives

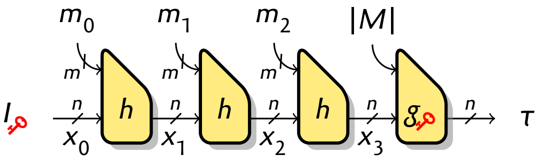
- Primitive with fixed-size inputs, and mode of operation

- Encryption example: CBC-AES



- Mode: CBC
  - Encryption mode
- Primitive: AES block cipher
  - $E : \{0,1\}^\kappa \times \{0,1\}^n \to \{0,1\}^n$
  - $E_{k} : x \mapsto E(k, x)$ permutation

- Authentication example: HMAC-SHA1



- Mode: HMAC
  - Message Authentication Code (MAC)
- Primitive: SHA-1 compression function
  - $h : \{0,1\}^n \times \{0,1\}^m \to \{0,1\}^n$
  - Public function without structural property

Security Analysis

Classical approach

- Security of the protocol (TLS, SSH, ...)
  - Security proofs, assuming security of cryptographic operations
- Security of the modes (HMAC, CBC, ...)
  - Security proofs, assuming security of the primitive
- Security of the primitives (AES, SHA-1, RSA, ...)
  - Studied with cryptanalysis

- Cryptanalysis usually applied inside primitives
- If this talk: cryptanalysis techniques outside the primitive

Cryptanalysis beyond primitives

Generic attacks

- Target the mode itself without using properties of the primitive
- Nice algorithmic problems & mathematical properties

1. Generic attacks define the expected security of a mode
   - Generic collision attack for hash functions and MACs
2. Generic attack are complementary to security proofs
   - Security proofs give lower bound on the security
   - Generic attacks give upper bound on the security
3. Generic attacks are important when the proof does not apply
   - What is the impact of going over the proof limit? (distinguisher or key-recovery)
   - What happens in a different model? (eg with quantum queries)
   - Can there be a mistake in the proof?

The birthday bound

- Security of common modes of operations is limited by collisions
- With an n-bit state, collisions after $2^{n/2}$ blocks

The birthday paradox

- Draw r random values from $\{0,1\}^n$
  - Expected number of collisions is about $r^2/2^{n+1}$
  - Constant probability of having a collision with $r = \Theta(2^{n/2})$
- Variant: Let $\mathcal{A}, \mathcal{B}$ be random subsets of $\{0,1\}^n$
  - Expected number of matches $|\mathcal{A} \cap \mathcal{B}| \approx |\mathcal{A}| \times |\mathcal{B}|/2^n$
  - In particular, $\mathcal{A} \cap \mathcal{B} \neq \emptyset$ with high probability if $|\mathcal{A}| = |\mathcal{B}| = 2^{n/2}$

- Many generic attacks are based on finding collisions between well-chosen sets

Outline

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Generic Attacks Against Hash Functions
- Multicollisions

Generic Attacks Against Hash Combiners

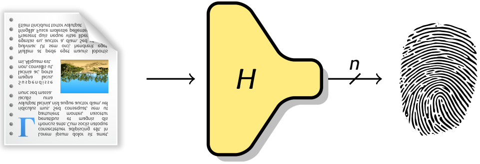
Generic Attacks Against Hash-based MACs

Generic Attacks Against Encryption Modes

Generic Attacks Against MACs in the Quantum Setting

Hash functions

- Public function $H : \{0,1\}^* \to \{0,1\}^n$
- Should behave like a random function
  - No structural property
  - Cryptographic properties without any key!
- Concrete security goals



Preimage attack

Given H and $\bar{X}$, find M s.t. $H(M) = \bar{X}$. Ideal security: 2^n.

Second-preimage attack

Given H and M_1, find $M_2 \neq M_1$ s.t. $H(M_1) = H(M_2)$. Ideal security: 2^n.

Collision attack

Given H, find $M_1 \neq M_2$ s.t. $H(M_1) = H(M_2)$. Ideal security: $2^{n/2}$.

Finding collisions

Collision expected after $\mathcal{O}(2^{n/2})$ evaluations with random inputs

Collision search in practice

- Sort data to avoid quadratic complexity
- Pollard's rho to avoid memory

Pollard's rho

- Given a function $H : \{0,1\}^n \to \{0,1\}^n$, find x, y with $H(x) = H(y)$

1. Iterate H: $x_i = H(x_{i-1})$
2. After roughly $2^{n/2}$ iterations, sequence cycles
3. Detect cycle, locate collision (Floyd, Brent)

- Complexity $\mathcal{O}(2^{n/2})$

Finding collisions

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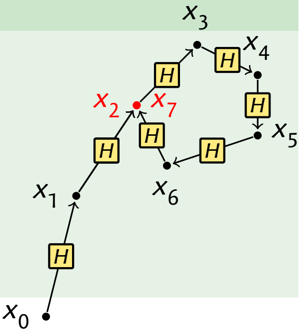
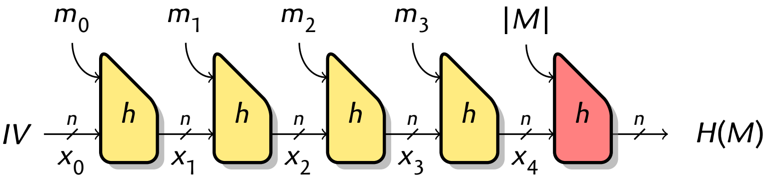


The Merkle-Damgård construction (SHA-1, SHA-2)



- n-bit state, compression function $h : \{0,1\}^n \times \{0,1\}^r \to \{0,1\}^n$
- Finalization using message length (MD strengthening)
- Notation: Iterated compression function h^*
  - $h^*(x, m_1 \parallel m_2 \parallel m_3) = h(h(h(x, m_1), m_2), m_3)$
- Security reduction:
  - Hash collisions imply compression function collision (generic security $2^{n/2}$)
  - Hash preimages imply finalization preimages (generic security 2^n)

Generic attacks on Merkle-Damgård

Many properties "between" collision and preimage broken with birthday complexity, by generic attacks exploiting collisions in smart ways

Multicollision [Joux, Crypto '04]

Find a large set of message $\{M_i\}$ s.t. $\forall i,\ H(M_i) = H(M_0)$ Complexity $\tilde{\mathcal{O}}(2^{n/2})$

Chosen-prefix collision [Stevens, Lenstra & de Weger, EC'07]

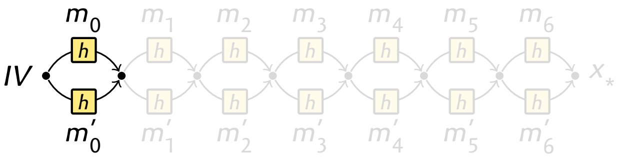
Given challenges C, C', find M, M' s.t. $H(C \parallel M) = H(C' \parallel M')$ Complexity $\mathcal{O}(2^{n/2})$

Diamond structure [Kelsey & Kohno, EC'06]

Given challenges $\{C_i\}$, find $\{M_i\}$ s.t. $\forall i,\ H(C_i \parallel M_i) = H(C_0 \parallel M_0)$ Complexity $\tilde{\mathcal{O}}(\sqrt{|\{C_i\}|2^{n/2}})$

Multicollisions

[Joux, Crypto '04]



1. Find a collision pair m_0 / m'_0 starting from IV
2. Find a collision pair m_1 / m'_1 starting from $x_1 = h^*(m_0)$
3. Repeat t times
4. This yields 2^t messages with the same hash:

$m_0 m_1 m_2 \dots$	$m'_0 m_1 m_2 \dots$	$m_0 m'_1 m_2 \dots$	$m'_0 m'_1 m_2 \dots$
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- Complexity $t \cdot 2^{n/2}$ vs. $\approx 2^{\frac{2^t-1}{2^t} n}$ for a random function

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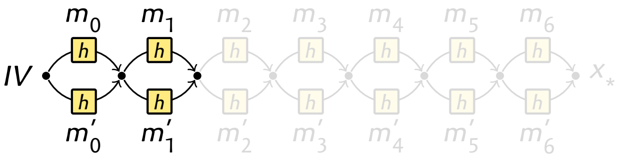

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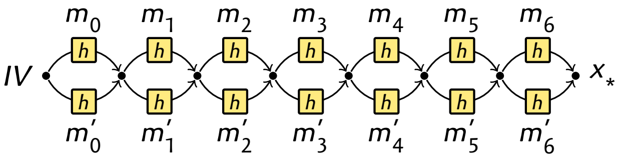

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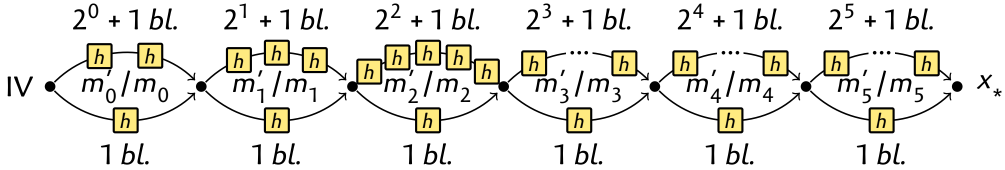
Expandable message

[Kelsey & Schneier, Eurocrypt '05]



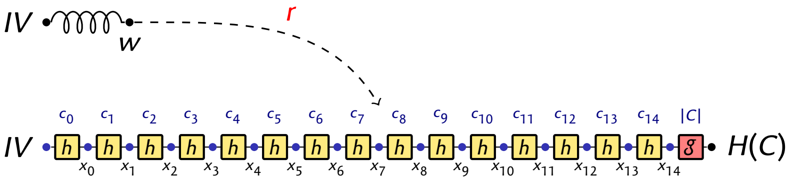
- Multicollision with messages of difference length
  2^t messages of length $t, t+1, \dots t+2^t-1$ blocks
  - Length 0+6: $m_0 m_1 m_2 m_3 m_4 m_5$
  - Length 1+6: $m'_0 m_1 m_2 m_3 m_4 m_5$
  - Length 2+6: $m_0 m'_1 m_2 m_3 m_4 m_5$
  - Length 3+6: $m'_0 m'_1 m_2 m_3 m_4 m_5$
  - ...
  - Length 63+6: $m'_0 m'_1 m'_2 m'_3 m'_4 m'_5$
- Complexity $t \cdot 2^{n/2}$

Second-preimage for long challenges

[Kelsey & Schneier, Eurocrypt '05]

Given a challenge C, find $M \neq C$ with $H(M) = H(C)$ — $\text{len}(C) = 2^s$

0. Build expandable message $\mathcal{M}$ of length 2^s (final state w)
1. Compute internal states $\{x_i\}$ for $H(C)$ (No key: public values)
2. Find r, i with $h(w, r) = x_i$ (Complexity 2^{n-s})
3. Preimage is $\mathcal{M}_{i-1} \parallel r \parallel C[i:]$



- Complexity $2^s + 2^{n-s}$ ($2^{n/2}$ for $s = n/2$)

Outline

Generic Attacks Against Hash Functions

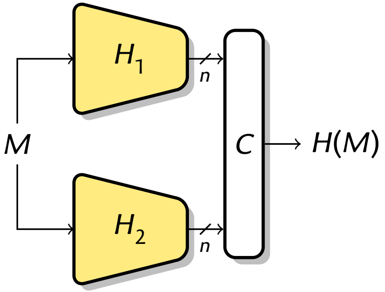
Generic Attacks Against Hash Combiners
- Concatenation combiner
- XOR combiner

Generic Attacks Against Hash-based MACs

Generic Attacks Against Encryption Modes

Generic Attacks Against MACs in the Quantum Setting

Combining two hash functions



"In order to make the PRF as secure as possible, it uses two hash algorithms in a way which should guarantee its security if either algorithm remains secure."
– RFC 2246 (TLS 1.0)

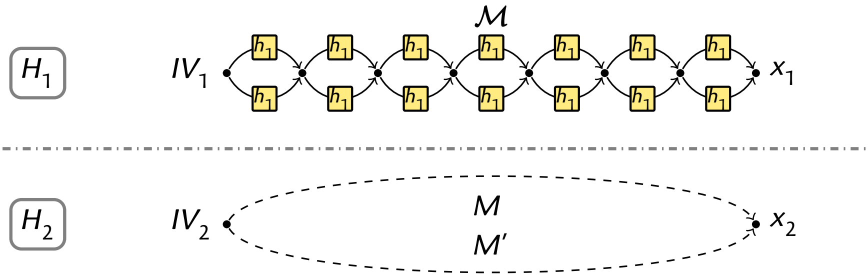
Classical combiners:

- Concatenation: $H_1(M) \| H_2(M)$
- Xor: $H_1(M) \oplus H_2(M)$

"The whole is greater than the sum of its parts"
– Aristotle

Collision attack for $H_1(M) \parallel H_2(M)$

[Joux, C'04]



1. Build a $2^{n/2}$-multicollision for H_1

$$\forall M \in \mathcal{M}, H_1(M) = x_1$$

2. Find $M, M' \in \mathcal{M}$ s.t. $H_2(M) = H_2(M')$

- Complexity $\tilde{\mathcal{O}}(2^{n/2})$ vs. 2^n for a $2n$-bit hash function.

Preimage attack for $H_1(M) \parallel H_2(M)$

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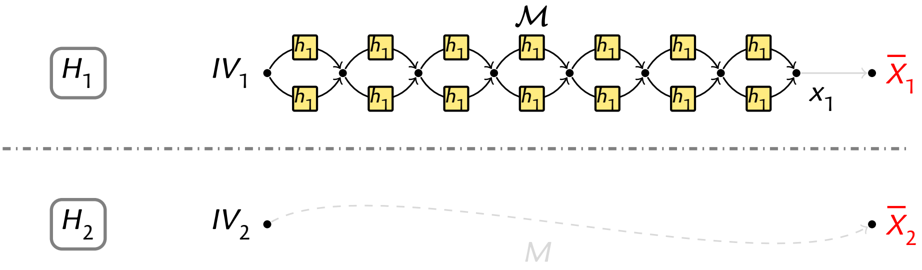

1. Build a 2^n-multicollision for H_1 — $\forall M \in \mathcal{M}, h_1^*(M) = x_1$
2. Find a preimage for H_1: $g(h(x_1, r)) = \overline{X}_1$ — $\forall M \in \mathcal{M}, H_1(M) = \overline{X}_1$
3. Find $M \in \mathcal{M}$ s.t. $H_2(M \parallel r) = \overline{X}_2$

- Complexity $\tilde{O}(2^n)$ vs. 2^{2n} for a $2n$-bit hash function.

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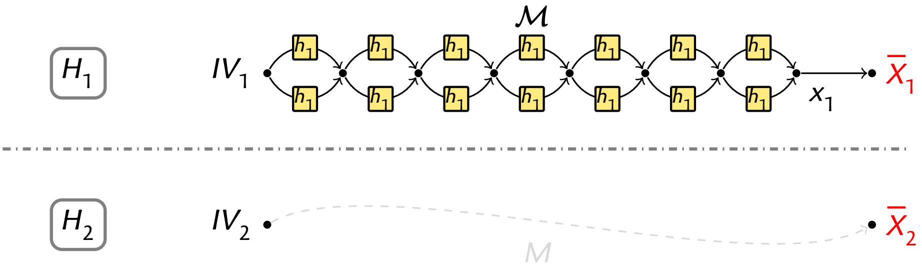

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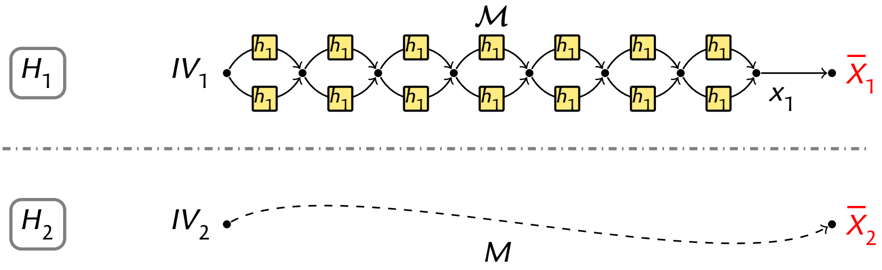
Preimage attack for $H_1(M) \| H_2(M)$ [Joux, C'04]



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Preimage attack for $H_1(M) \parallel H_2(M)$

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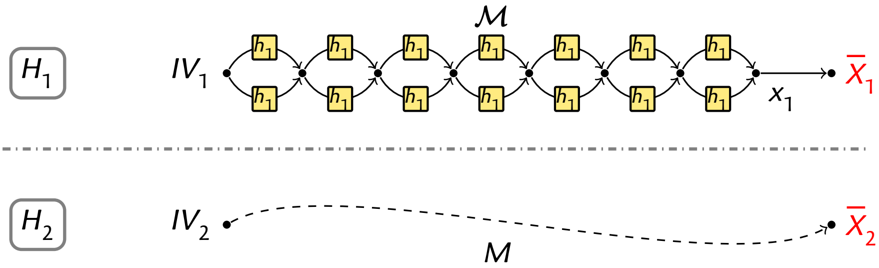

1. Build a 2^n-multicollision for H_1 $\qquad \forall M \in \mathcal{M}, h_1^*(M) = x_1$
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- Complexity $\tilde{\mathcal{O}}(2^n)$ vs. 2^{2n} for a $2n$-bit hash function.

Combining generic and dedicated cryptanalysis

Collision for MD5 ‖ SHA-1

- Using a SHA-1 multicollision (dedicated SHA-1 collision attack)
- Complexity 2^{64} for generic MD5 collision
- ⇒ Complexity $2^{67.6}$ for collision and chosen-prefix collision
- Chosen-prefix collision attack on MD5 ‖ SHA-1 breaks TLS 1.0/1.1 [SLOTH]

Partial preimage for MD5 ‖ SHA-1

- Using a MD5 multicollision (dedicated MD5 collision attack)
- Complexity 2^t for generic partial preimage (fixing t bits)
- ⇒ Complexity $\approx 2^t$ for fixing t bits of MD5 and t bits of SHA-1
- `f5e2024` challenge (rump session)

Generic attacks against combiners

Concatenation combiner

- $H(M) = H_1(M) \| H_2(M)$
- $2n$-bit output
- Generic security: attacks / proofs
  - Collisions: $2^{n/2}$ $2^{n/2}$
  - Preimages: 2^n 2^n
  - Non-ideal: $2^{n/2}$ $2^{n/2}$

XOR combiner

- $H(M) = H_1(M) \oplus H_2(M)$
- n-bit output
- Generic security: attacks / proofs
  - Collisions: $2^{n/2}$ $2^{n/2}$
  - Preimages: ? $2^{n/2}$
  - Non-ideal: $2^{n/2}$ $2^{n/2}$

Surprising result [Joux, C'04]

If H_1 and H_2 are good MD hash functions, $H_1 \| H_2$ is not stronger!

Generic attacks against combiners

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- $H(M) = H_1(M) \parallel H_2(M)$
- $2n$-bit output
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  - Preimages: 2^n / 2^n
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XOR combiner

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Preimage attack against Xor combiner

[L & Wang, EC'15]

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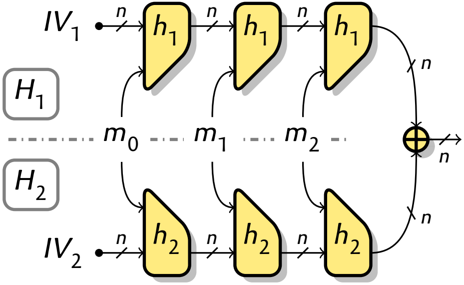

Strategy:

1. Structure to control H_1 and H_2 independently:
   - Sets of states $\mathcal{A} = \{A_j\}$, $\mathcal{B} = \{B_k\}$
   - Set of messages $\{\mathbf{M}_{jk}\}$ with
   $$h_1^*(\mathbf{M}_{jk}) = A_j$$
   $$h_2^*(\mathbf{M}_{jk}) = B_k$$
2. Preimage search for $\overline{X}$:
   - For random blocks r, match $\{g_1(h_1(A_j, r))\}$ and $\{g_2(h_2(B_k, r)) \oplus \overline{X}\}$
   - If there is a match (j, k): Get $\mathbf{M}_{jk}$, preimage is $M = \mathbf{M}_{jk} \parallel r$
   - Complexity $\mathcal{O}(2^n / \min\{|\mathcal{A}|, |\mathcal{B}|\})$

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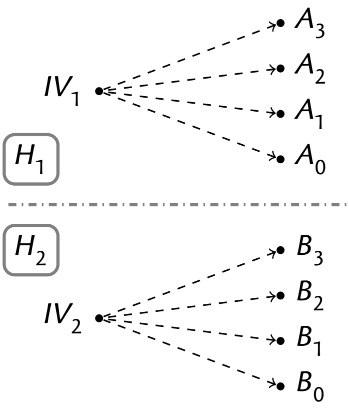

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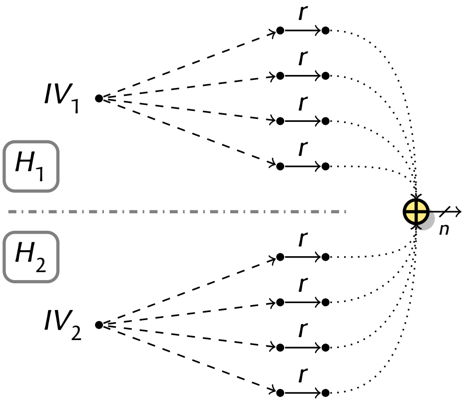

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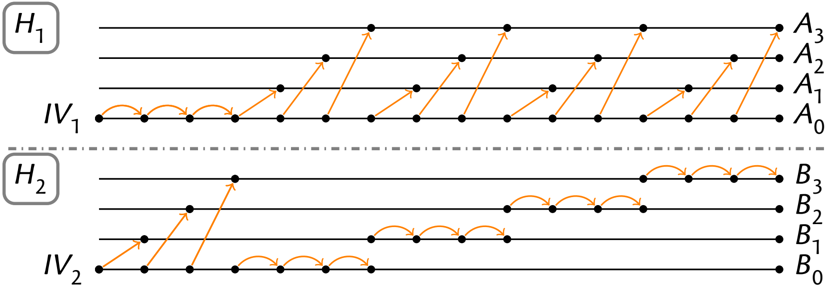
Interchange structure

[L & Wang, EC'15]

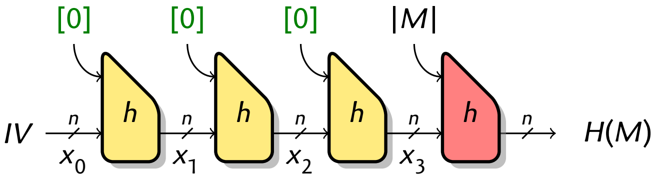
- Interchange structure for a large set of output states



- Complexity $\tilde{\mathcal{O}}(2^{n/2+2t})$ to build a structure with $|\mathcal{A}| = |\mathcal{B}| = 2^t$
- Complexity $\tilde{\mathcal{O}}(2^{5n/6})$ for preimages (tradeoff)

Alternative structure using cycles

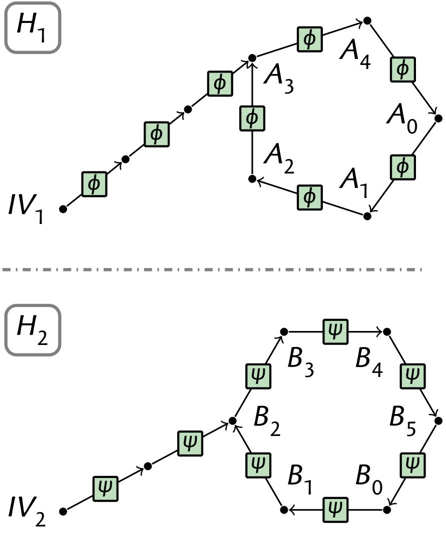
- Alternative presentation of "multicycles" [Bao, Wang, Guo, Gu, C'17]



- Using a long message repeating a fixed block $M = [0]^\lambda$, we iterate fixed functions:

$$\phi : x \mapsto h_1(x, [0])$$
$$\psi : x \mapsto h_2(x, [0])$$

Alternative structure using cycles



- Use cyclic nodes as end-point:
  - $\mathcal{A} = H_1$ cycle, length ℓ_1
  - $\mathcal{B} = H_2$ cycle, length ℓ_2
- With suitable naming, for λ large enough:

$$h_1^*([0]^\lambda) = A_{\lambda \bmod \ell_1} \quad h_2^*([0]^\lambda) = B_{\lambda \bmod \ell_2}$$

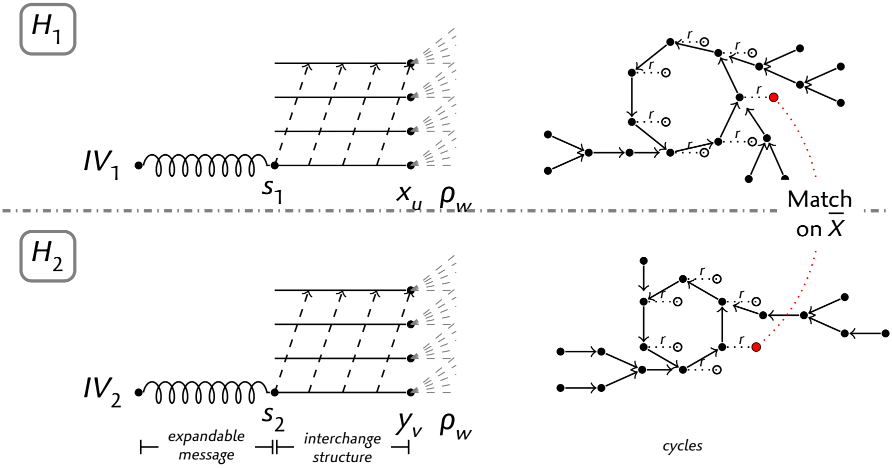
- To reach (A_j, B_k), use Chinese Remainder

$$\begin{cases} h_1^*([0]^\lambda) = A_j \\ h_2^*([0]^\lambda) = B_k \end{cases} \iff \begin{cases} \lambda \bmod \ell_1 = i \\ \lambda \bmod \ell_2 = j \end{cases}$$

  - λ uniformly distributed in range of size $\ell_1 \ell_2$
  - $\Pr[\lambda < 2^t] \approx 2^{n-t}$
- Complexity $\tilde{\mathcal{O}}(2^{3n/4})$ for preimages (tradeoff)

Advanced preimage attack

[ePrint 2024/488]



- Using interchange, small cycles, expandable message
- Complexity $\tilde{\mathcal{O}}(2^{3n/5})$

Summary: Preimage attack for $H_1(M) \oplus H_2(M)$

Interchange structure

- Complexity $\tilde{\mathcal{O}}(2^{5n/6})$ [LW15]
- Works for Merkle-Damgård and HAIFA
  - Finalization function, block counter at each round
- Short messages: length $\tilde{\mathcal{O}}(2^{n/3})$

Using cycles

- Complexity $\tilde{\mathcal{O}}(2^{3n/4})$ (simple)
- Complexity $\tilde{\mathcal{O}}(2^{5n/8})$ [BWGG17]
- Complexity $\tilde{\mathcal{O}}(2^{11n/18})$ [BDGLW20]
- Complexity $\tilde{\mathcal{O}}(2^{3n/5})$ [ePrint 2024/488]
- Works only for Merkle-Damgård mode
  - Finalization function, same function at each step
- Long messages: length $\tilde{\mathcal{O}}(2^{3n/5})$

Generic attacks against combiners

Concatenation combiner

- $H(M) = H_1(M) \parallel H_2(M)$
- $2n$-bit output
- Generic security: attacks / proofs
  - Collisions: $2^{n/2}$ $2^{n/2}$
  - Preimages: 2^{n} 2^{n}
  - Non-ideal: $2^{n/2}$ $2^{n/2}$

XOR combiner

- $H(M) = H_1(M) \oplus H_2(M)$
- n-bit output
- Generic security: attacks / proofs
  - Collisions: $2^{n/2}$ $2^{n/2}$
  - Preimages: $2^{3n/5}$ $2^{n/2}$
  - Non-ideal: $2^{n/2}$ $2^{n/2}$

Surprising result [Joux, C'04]

If H_1 and H_2 are good MD hash functions, $H_1 \parallel H_2$ is not stronger!

Surprising result [L & Wang, EC'15]

If H_1 and H_2 are good MD hash functions, $H_1 \oplus H_2$ is weaker!

Outline

Generic Attacks Against Hash Functions

Generic Attacks Against Hash Combiners

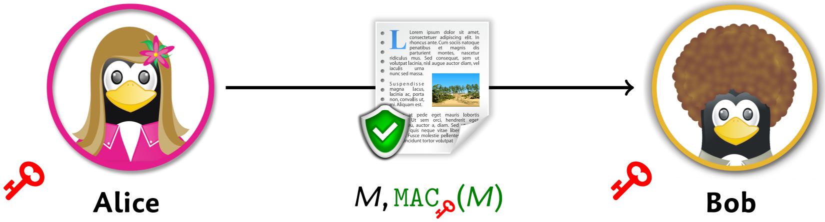
Generic Attacks Against Hash-based MACs

- Generic forgery attack
- Cycle-based attacks

Generic Attacks Against Encryption Modes

Generic Attacks Against MACs in the Quantum Setting

Message Authentication Codes (MAC)



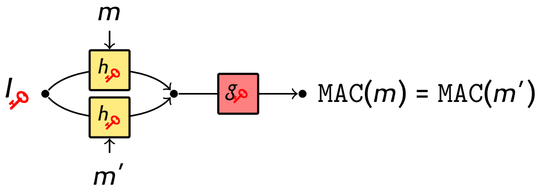
Security Notions

Forgery	Given access to a MAC oracle, forge a valid pair
State recovery	Recover the internal state, for a chosen message
Key recovery	Given access to a MAC oracle, extract the key

Generic forgery attack

[Preneel & van Oorschot '95]

- Consider an iterated deterministic MACs with n-bit state



1. Find internal collisions
   - Query $2^{n/2}$ random short messages
   - 1 internal collision expected, detected in the output
2. Query $t = \mathtt{MAC}(m \parallel c)$
3. $(m' \parallel c, t)$ is a forgery

- Attack motivates MACs with security beyond the birthday bound
  - Non-deterministic: RMAC, Wegman-Carter
  - Larger state: SUM-ECBC, HMAC

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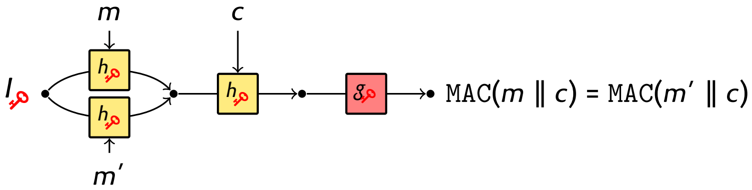

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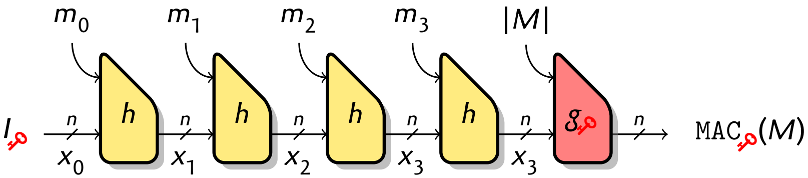
Hash-based MACs



- n-bit chaining value, n-bit MAC
- κ-bit key

we focus on $n = \kappa$

- Key-dependent initial value I
- Unkeyed compression function h
- Key-dependent finalization, with message length g
- Examples: HMAC, envelope MAC, sandwich MAC
- Security proofs up to the birthday bound

Secret-suffix MAC

Definition (Secret-suffix MAC)

$\mathtt{MAC}_k(M) = H(M \parallel k)$

- Birthday security (tight)
- Birthday key-recovery attack [Preneel & van Oorschot '96]
  1. Guess the first key byte as k^*
  2. Find a one-block hash collision (C_0, C_1) with $C_i = M_i \parallel k^*$ (offline)

     C_1 = ??? ⋯ ??? k^* M_1 = ???⋯???

     C_0 = ¿¿¿ ⋯ ¿¿¿ k^* M_0 = ¿¿¿⋯¿¿¿
  3. Query $\mathtt{MAC}(M_1)$ and $\mathtt{MAC}(M_2)$ (online)

     $\mathtt{MAC}(M_1) = H($??? ⋯ ??? k_0 $k_1 k_2 k_3 \dots)$

     $\mathtt{MAC}(M_0) = H($ ¿¿¿ ⋯ ¿¿¿ k_0 $k_1 k_2 k_3 \dots)$
  4. If the MACs are equal, the guess was correct
- Practical attack when using MD5 (*e.g.* APOP) [L'07, Sasaki & al '08]
  - Using cryptanalytic shortcuts

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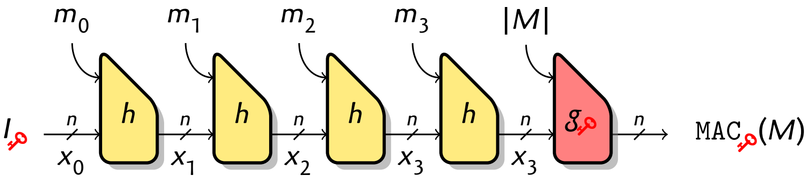
Cycle-based forgery

[L, Peyrin, Wang, AC'13]



- Using a long message repeating a fixed block $M = [0]^\lambda$, we iterate a fixed function

$$\phi : x \mapsto h(x, [0])$$

- Starting point and ending point unknown because of the key

Can we detect properties of ϕ? [Flajolet & Odlyzko, EC'89]

- Largest component size: $\approx 0.76 \times 2^n$
- Largest tree size: $\approx 0.48 \times 2^n$

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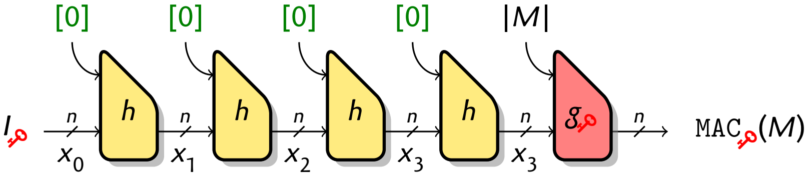

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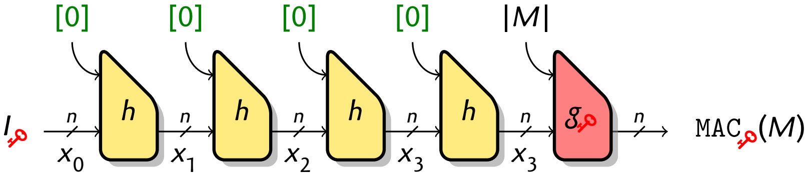

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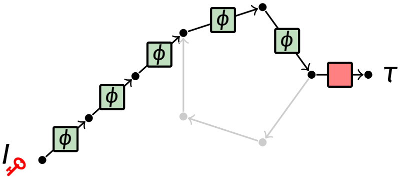
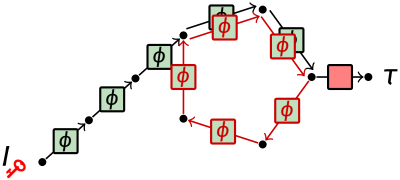
Using the cycle length

1. Offline: find the cycle length ℓ of the main component of ϕ
2. Online: query $t = \mathtt{MAC}([0]^{2^{n/2}})$ and $t' = \mathtt{MAC}([0]^{2^{n/2}+\ell})$



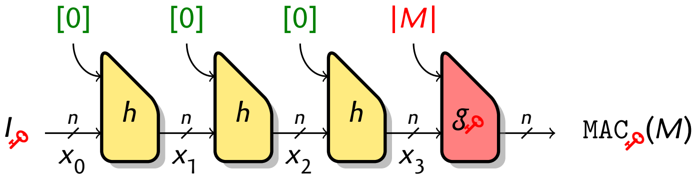


Collision if

- The starting point is in the main component $p = 0.76$
- The cycle is reached with less than $2^{n/2}$ iterations $p \geq 0.5$

Dealing with the message length

- Problem: MACs use msg length.
- Solution: reach the cycle twice
  - Block [1] moves outside graph
- With high proba. $\mathtt{MAC}_k(M_1) = \mathtt{MAC}_k(M_2)$
- Surprisingly powerful: breaks universal hash



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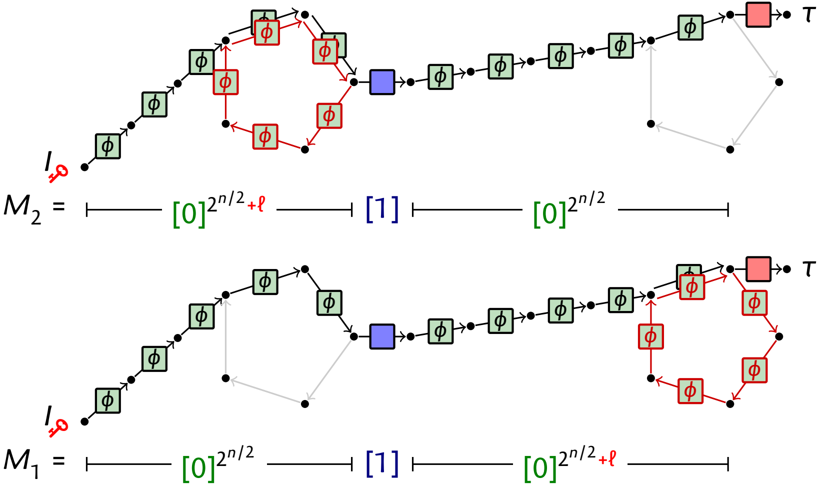

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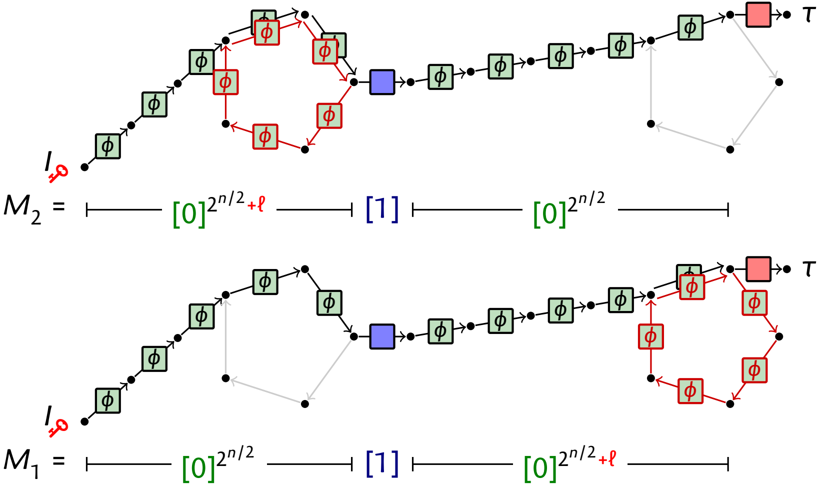
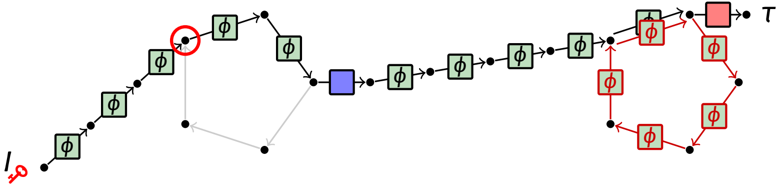


State-recovery attack

- Consider the first cyclic point
- With high probability, root of the giant tree



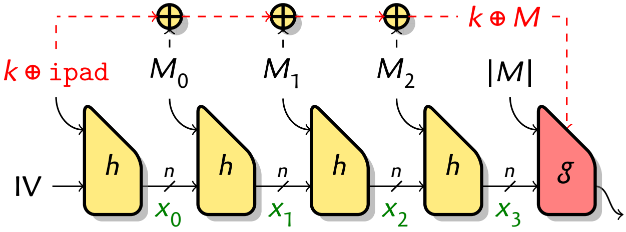
1. Offline: find the root α of the giant tree of ϕ
2. Online: binary search for first cyclic point: smallest z such that

$$\mathtt{MAC}([0]^z \parallel [1] \parallel [0]^{2^{n/2}+\ell}) = \mathtt{MAC}([0]^{z+\ell} \parallel [1] \parallel [0]^{2^{n/2}})$$

- Recover $h^*([0]^z) = \alpha$
- Complexity $\tilde{\mathcal{O}}(2^{n/2})$

Key-recovery attack with a checksum

- Some hash functions have an internal checksum (*e.g.* GOST standards)
- In HMAC, checksum is key-dependent and attacker-controlled



- Related-key attacks on the last block

Surprising result

- The checksum actually makes the hash function weaker!
- HMAC key-recovery attack with complexity $\tilde{O}(2^{3n/4})$

Summary: Cryptanalysis of hash-based MACs

- Security known to be tight (birthday)
- Advanced attacks more powerful than forgeries
  - Using properties of functional graphs, and entropy loss of iteration

- Generic state-recovery attacks
  - Complexity $\tilde{O}(2^{n/2})$ for Merkle-Damgård (tight)
  - Complexity $\tilde{O}(2^{4n/5})$ for HAIFA (not tight)

- Generic key-recovery attack against HMAC with a checksum (HMAC-GOST)
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Outline

Generic Attacks Against Hash Functions

Generic Attacks Against Hash Combiners

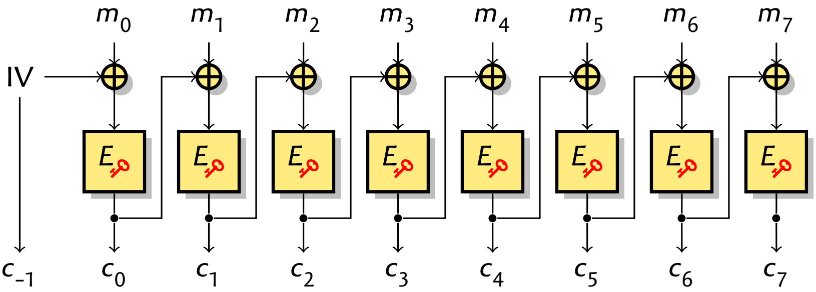
Generic Attacks Against Hash-based MACs

Generic Attacks Against Encryption Modes
- CBC and CTR
- CBC collisions in practice: Sweet32
- Plaintext recovery against CTR

Generic Attacks Against MACs in the Quantum Setting

CBC mode

- CBC is a widely used mode, with birthday security [NIST'80]
- Well known collision attack against CBC



- If $c_i = c_j$, then $c_{i-1} \oplus m_i = c_{j-1} \oplus m_j$
  - $m_i \oplus m_j = c_{i-1} \oplus c_{j-1}$
- Ciphertext collision reveals the xor of two plaintext blocks

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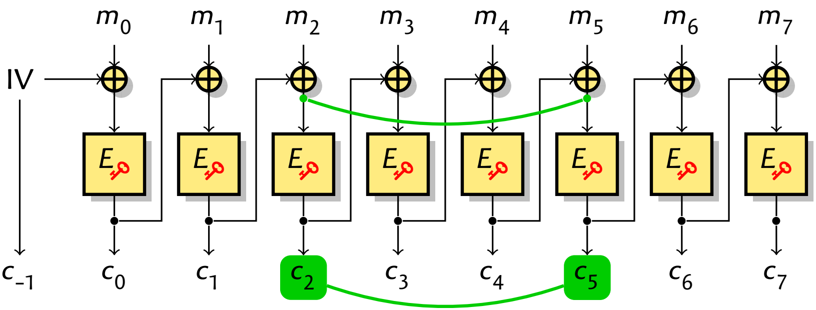

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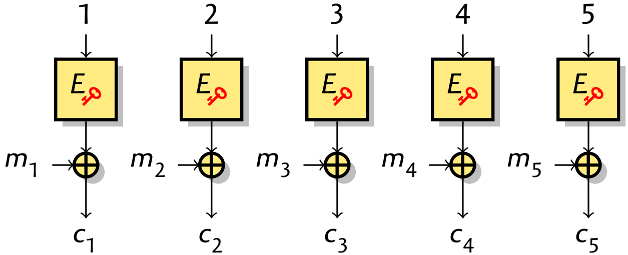
Counter mode (CTR)

- CTR is a more modern mode with birthday security (used in GCM) [NIST'01]
- Well known distinguisher against CTR



- All block-cipher inputs are distinct
- For all $i \neq j$, $m_i \oplus c_i \neq m_j \oplus c_j$
  - $m_i \oplus m_j \neq c_i \oplus c_j$
  - Hard to extract plaintext information from inequations
- Distinguisher: collision after $2^{n/2}$ blocks with random ciphertext

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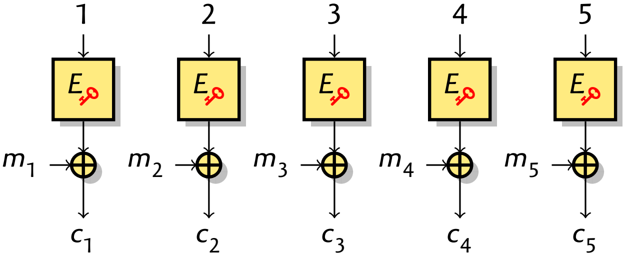

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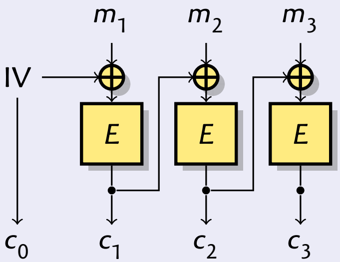
CBC and CTR

CBC mode



- Security proof up to $2^{n/2}$ blocks
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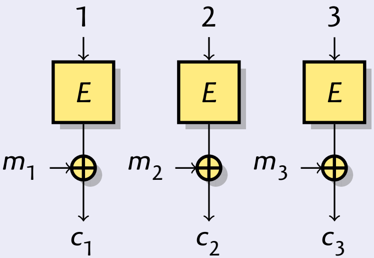
CTR mode



- Security proof up to $2^{n/2}$ blocks
- $m_i \oplus m_j \neq c_i \oplus c_j$ $\forall i, j$
- Distinguisher: Key stream doesn't collide

Birthday security in practice

Block size does matter

- State size is an important security parameter
  - Hash functions and stream ciphers use large state size $n \geq 160$
- Modern block ciphers have a 128-bit block size (*e.g.* AES)
  - 2^{64} blocks correspond to 256 EB
- Block ciphers from the 90's have a 64-bit block size (Blowfish, 3DES)
  - 2^{32} blocks correspond to 32 GB

- In 2016, 64-bit block ciphers were still used in practice
  - Around 1—2% of HTTPS connections used 3DES-CBC in 2015–2017
    - Mandatory support in TLS 1.0 and TLS 1.1
    - Supported for compatibility with old client/server
    - Many servers supported AES but preferred 3DES
  - OpenVPN used Blowfish-CBC by default

Proof-of-concept Attack Demo: Sweet32 [Bhargavan & L, CCS'16]

The BEAST setting

- Man-in-the-browser: chosen plaintext
- Authentication cookie: repeated secret
- Modelled as chosen-prefix secret-suffix oracle:
  $M \mapsto \mathcal{E}(M \parallel S)$ with secret S

worker.js

```
var url = "https://target";
var xhr = new XMLHttpRequest;

while(true) {
  xhr.open("HEAD", url, false);
  xhr.withCredentials = true;
  xhr.send();
  xhr.abort();
}
```

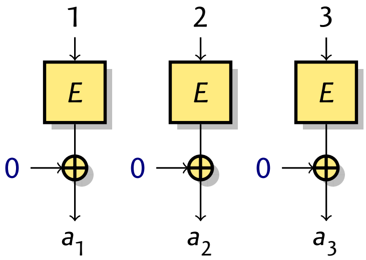
- Wait for collision between blocks from secret cookie and known plaintext

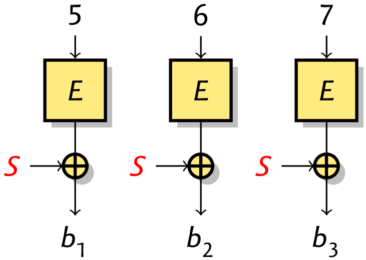
$$\underbrace{\texttt{cookie}}_{\text{unknown}} \oplus \underbrace{\texttt{header}}_{\text{known}} = \underbrace{c_{i-1} \oplus c_{j-1}}_{\text{known}}$$

- Demo with HTTPS traffic with 3DES-CBC
  - Attack successful with ≈ 800 GB of data, collected over 40 hours

Plaintext recovery against CTR

- Collect two kinds of blocks





Chosen plaintext blocks $a_i = E(i)$

Repeated secret $b_j = E(j) \oplus S$

- $\forall i, j,\ a_i \neq S \oplus b_j$
- $\forall i, j,\ S \neq a_i \oplus b_j$

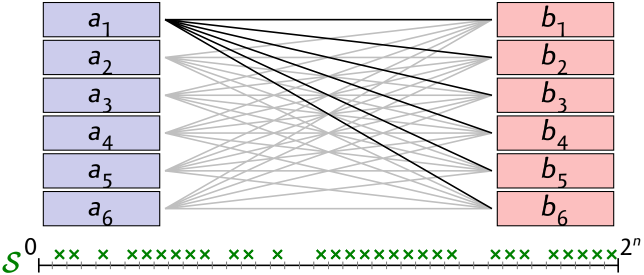
Missing difference problem

Given sets $\mathcal{A}, \mathcal{B} \subset \{0,1\}^n$
Find S such that

$$\forall (a,b) \in \mathcal{A} \times \mathcal{B},\ S \neq a \oplus b$$

Sieving algorithm

[McGrew, FSE'13]



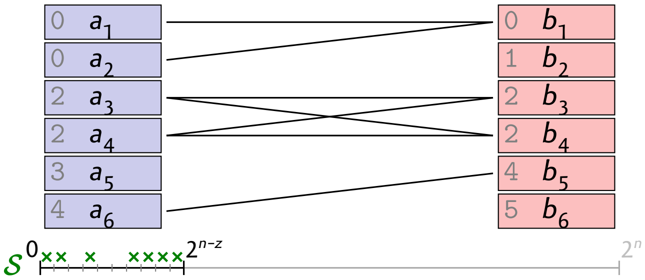
- Compute all $a_i \oplus b_j$, remove from a sieve $\mathcal{S}$

Analysis: Coupon collector problem

- To exclude 2^n candidates S, we need $n \cdot 2^n$ values $a_i \oplus b_j$
  - Lists $\mathcal{A}$ and $\mathcal{B}$ of size $\sqrt{n} \cdot 2^{n/2}$. Complexity: $\tilde{\mathcal{O}}(2^n)$

Known-prefix sieving

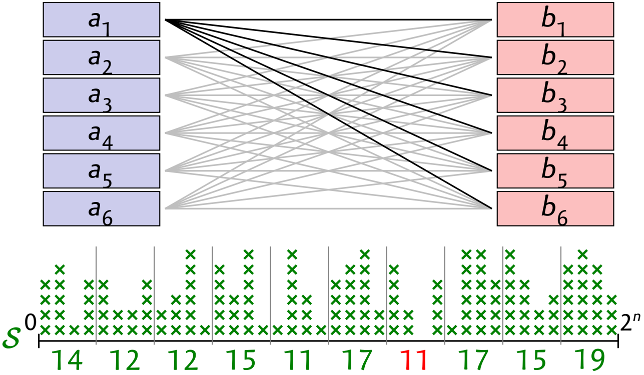
[L & Sibleyras, EC'18]



- Assume S starts with z zero bits
  - Smaller sieve
- Sort lists, consider a_i's and b_j's with matching prefix
- Complexity: $\tilde{\mathcal{O}}(2^{n/2})$ when $z \geq n/2$

Fast-convolution sieving

[L & Sibleyras, EC'18]

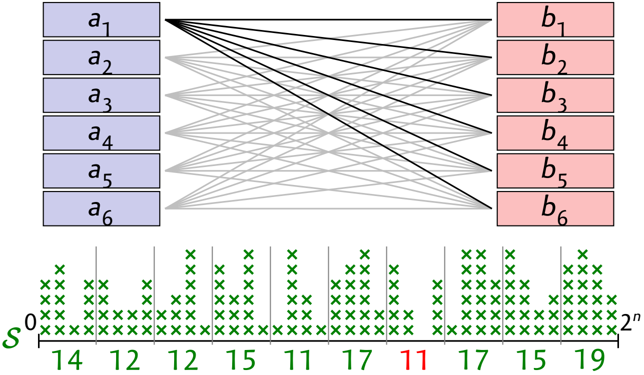


- Use $2^{2n/3}$ queries, sieving with $2^{2n/3}$ buckets of $2^{n/3}$ elements
  - With high probability, smallest bucket corresponds to missing difference

$$\#\left\{(a,b) \in \mathcal{A} \times \mathcal{B} \mid a \oplus b \in \mathcal{X}_u\right\} = \sum_{v \in \{0,1\}^{2n/3}} \#\left\{a \in \mathcal{A} \mid a_{[0..2n/3]} = v\right\} \times \#\left\{b \in \mathcal{B} \mid b_{[0..2n/3]} = u \oplus v\right\}$$

Fast-convolution sieving

[L & Sibleyras, EC'18]



- Use $2^{2n/3}$ queries, sieving with $2^{2n/3}$ buckets of $2^{n/3}$ elements
  - With high probability, smallest bucket corresponds to missing difference
- Sieving can be computed with Fast Walsh-Hadamard transform!
- Complexity: $\tilde{\mathcal{O}}(2^{2n/3})$ for arbitrary S

Application of missing difference algorithms

Application to CTR mode

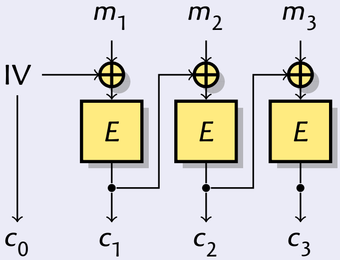
- Message recovery in the BEAST setting
  - Fixed secret encrypted repeatedly
  - Adversary can control the position of a fixed secret
- Target a block with $n/2$ secret bits and $n/2$ known bits
- Message recovery attack with complexity $\tilde{O}(2^{n/2})$ using known-prefix sieving

Applications to Wegman-Carter MAC

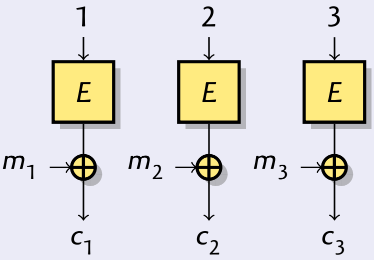
- Recovery of hash key is a missing difference problem
- Complexity $\tilde{O}(2^{2n/3})$ using fast-convolution sieving
- First partial key-recovery below 2^n

Summary: CBC and CTR

CBC mode



CTR mode



- CTR and CBC both leak plaintext data at the birthday bound
- Birthday attacks are practical against 64-bit block ciphers [Sweet32]
- Alternative: beyond-birthday-bound modes (eg. CENC)

Outline

Generic Attacks Against Hash Functions

Generic Attacks Against Hash Combiners

Generic Attacks Against Hash-based MACs

Generic Attacks Against Encryption Modes

Generic Attacks Against MACs in the Quantum Setting
- CBC-MAC
- Simon's algorithm

Expected impact of quantum computers

- Recent progress toward building a large-scale quantum computer
- Some problems can be solved much faster with quantum computers
  - Up to exponential gains
  - But we don't expect to solve all NP problems

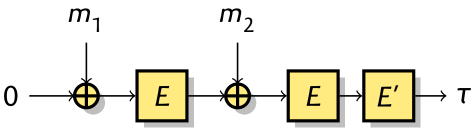
Impact on public-key cryptography

- RSA, DH, ECC broken by Shor's algorithm
  - Breaks factoring and discrete log in polynomial time
  - Large effort to develop quantum-resistant algorithms (*e.g.* NIST)

Impact on symmetric cryptography

- Exhaustive search of κ-bit key in time $2^{\kappa/2}$ with Grover's algorithm
  - Common recommendation: double the key length (AES-256)
  - What is the security of modes of operation in the quantum setting?

CBC-MAC



- One of the earliest MACs, based on CBC encryption mode
- Security proof up to the birthday bound [Bellare, Kilian & Rogaway '94]

Collision attack using two sets of $2^{n/2}$ messages

- $A_x = [0] \parallel x$
- $\mathtt{MAC}(A_x) = E'(E(x \oplus E([0])))$
- $B_y = [1] \parallel y$
- $\mathtt{MAC}(B_y) = E'(E(y \oplus E([1])))$
- $\mathtt{MAC}(A_x) = \mathtt{MAC}(B_y)$ iff $x \oplus E([0]) = y \oplus E([1])$
  - Deduce $\delta = E([0]) \oplus E([1]) = x \oplus y$
  - Produce forgeries: $\mathtt{MAC}([0] \parallel m) = \mathtt{MAC}([1] \parallel m \oplus \delta)$ for all m

Simon's Algorithm

[Simon, SIAM'97]

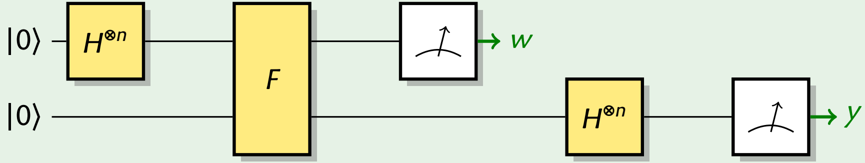
- Quantum algorithm to find collisions with extra structure
- First used in symmetric cryptography by Kuwakado and Morii.

Definition (Simon's problem)

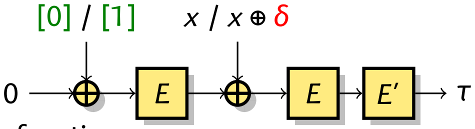
Given $f : \{0,1\}^n \to \{0,1\}^n$ such that there exists $\delta \in \{0,1\}^n$ with $f(x) = f(y) \Leftrightarrow x \oplus y = \delta$ or $x = y$, find δ.

- Classical algorithms require $\mathcal{O}(2^{n/2})$ queries (finding collisions)
- Simon's algorithm require $\mathcal{O}(n)$ quantum queries

One step of Simon's algorithm returns $y \perp \delta$



Quantum attack against CBC-MAC [Kaplan, L, Leverrier, Naya-Plasencia, C'16]



1. Consider the following function:

$$f : \{0,1\} \times \{0,1\}^n \to \{0,1\}^n$$
$$b, x \mapsto \mathtt{MAC}([b] \parallel x) = E'\Big(E\big(x \oplus E([b])\big)\Big)$$

$$f(b,x) = f(b',x') \iff \begin{cases} b = b' \text{ and } x = x' & \text{or} \\ b \neq b' \text{ and } x \oplus x' = E([0]) \oplus E([1]) \end{cases}$$

- f has period $1 \parallel \delta$, with $\delta = E([0]) \oplus E([1])$

2. Use Simon's algorithm to recover $1 \parallel \delta$
3. Produce forgeries: $\mathtt{MAC}([0] \parallel m) = \mathtt{MAC}([1] \parallel m \oplus \delta)$

Summary: Quantum security of modes of operation

Applications of Simon's algorithm

- Generalization breaks most common MAC and AEAD modes
  - CBC-MAC, PMAC, GMAC, GCM, OCB, ... [KLLNP, Crypto'16]
  - ΘCB, LightMAC, LightMAC+... [BLNS, AC'21]
- Corresponds to classical attacks with $2^{n/2}$ queries
  - Query f with $2^{n/2}$ values, look for collisions
- Strong assumption: superposition queries

- Surprising, because common encryption modes are quantum-secure (CBC, CTR) [Unruh, Targhi, Tabia & Anand, PQC'16]
- Some MAC are also quantum-secure: Cascade/HMAC is [Song & Yun, Crypto'17]

Conclusion: Cryptanalysis beyond primitives

Fun research area

- Interesting algorithmic problems for generic attacks
- Concrete attacks with practical impact

- Modes and protocols usually studied with proofs but cryptanalysis is useful
  - Mistakes in proofs
  - Gap between proofs and attacks
  - Different security degradation after the birthday bound
  - Usage when the proof does not apply

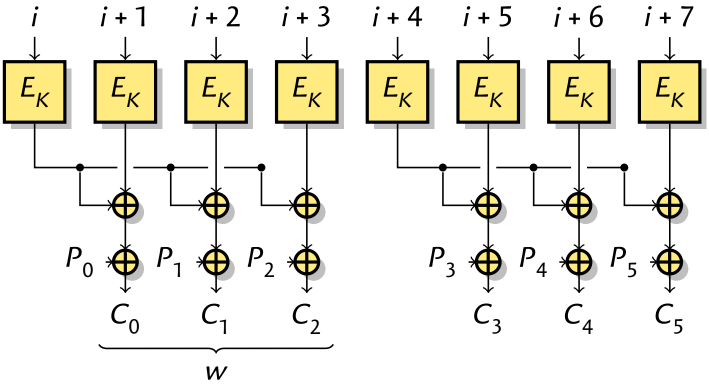
Take away

- Don't assume security above the birthday bound without a proof
- Many generic attacks exploit collisions in smart ways

Chosen-prefix collisions

CENC

[Iwata, FSE '06]



- Variant of the counter mode
- Designed by Iwata [FSE '06]
- Security $2^n/w$ [Iwata, Mennink & Vizár '16]

Attacking key certification

[Stevens, Lenstra & de Weger, EC'07]

Alice

PKI Infrastructure

- Alice generates key
- Asks CA to sign
- Certificate proves ID

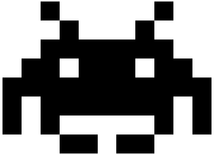
Impersonation attack

1. Bob creates keys s.t. $H(\texttt{Alice}||[key]_A) = H(\texttt{Bob}||[key]_B)$
2. Bob asks CA to certify his key $[key]_B$
3. Bob copies the signature to $[key]_A$, impersonates Alice

Attacking key certification

[Stevens, Lenstra & de Weger, EC'07]

The public key of Alice is:
`Z0t226BvLI05`
`seJ+L6NRaT49`
`0E6p9TY2sW74`



Bob

The public key of Bob is:
`7+zvZNcjdxXx`
`YRfYal4ZFmiY`
`E70hkirqNyfm`

PKI Infrastructure

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Impersonation attack

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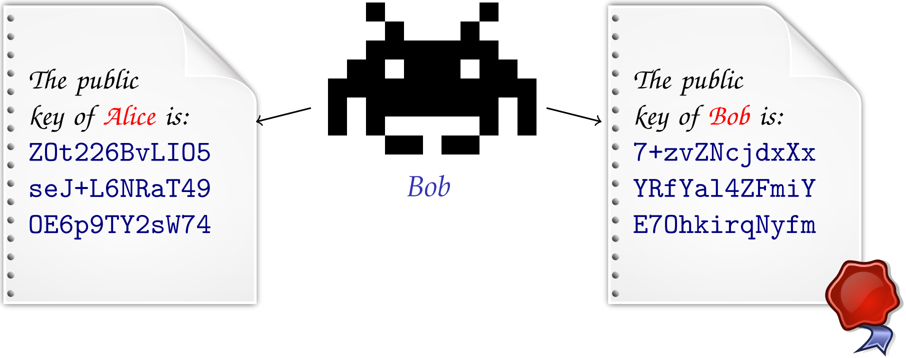

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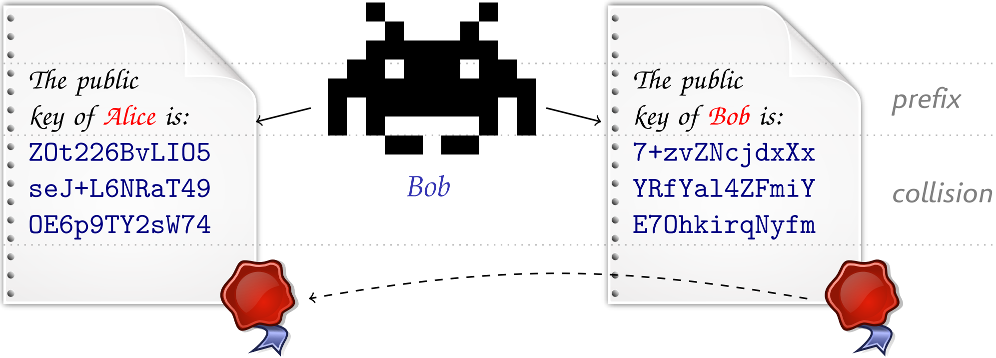

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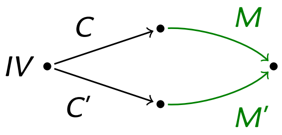
Chosen-prefix collisions

Chosen-prefix collision [Stevens, Lenstra & de Weger, EC'07]

Given challenges C, C',
Find M, M' s.t. $H(C \parallel M) = H(C' \parallel M')$



- Application to certificate
  - C = `Alice`, C' = `Bob`
  - Collision M, M' hidden in the public key

Chosen-prefix collisions

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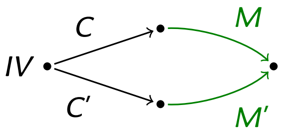

Generic

- Complexity $2^{n/2}$
- Compute $H(C \parallel r)$ and $H(C' \parallel r)$ for $2^{n/2}$ choices r
- Memoryless with helper function

$$\phi(x) = \begin{cases} H(C \parallel x) & \text{if } x \text{ is odd} \\ H(C' \parallel x) & \text{if } x \text{ is even} \end{cases}$$

Dedicated

- More difficult than plain collision
- Still possible using similar techniques

Function	MD5	SHA-1
Hash length	128	160
Collision	2^{16}	$2^{61.6}$
CP collision	$2^{39.1}$	$2^{63.5}$

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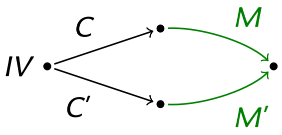

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