Cryptanalysis of the Peregrine Lattice-Based Signature Scheme

PKC 2024

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The Cryptanalysis of Peregrine

- Target: Peregrine[1]
    - the first round of the Korean PQC competition candidate in 2023

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  - parallelepiped-learning ⇒ the approximate key found
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- Technique: “parallelepiped-learning” +“lattice decoding”
  - parallelepiped-learning $\Rightarrow$ the approximate key found
  - lattice decoding $\Rightarrow$ fully recovers the secret from the approximations
- Cost: the signature samples required for practical attacks
  - $\approx 25,000$ for the reference implementation
  - ≈ 11 million for the specification version

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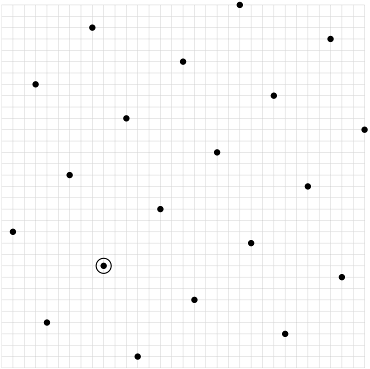
Outline

- Background
- The Peregrine signature scheme
- Learning a hidden transformation
- Practical key recovery attack

Background

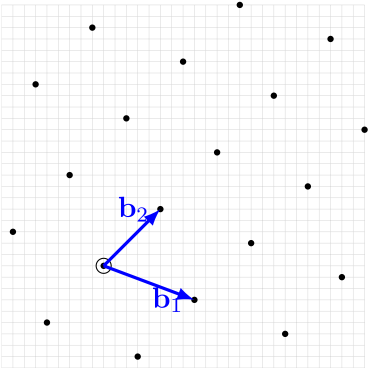
Lattice



Lattice

A lattice $\mathcal{L}$ is a discrete subgroup of $\mathbb{R}^m$.

Lattice



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A lattice is generated by its basis $\mathbf{B} = (\mathbf{b}_1, \cdots, \mathbf{b}_n) \in \mathbb{R}^{m \times n}$, i.e. $\mathcal{L}(\mathbf{B}) = \{\sum_{i=1}^{n} x_i \mathbf{b}_i \mid x_i \in \mathbb{Z}\}$.

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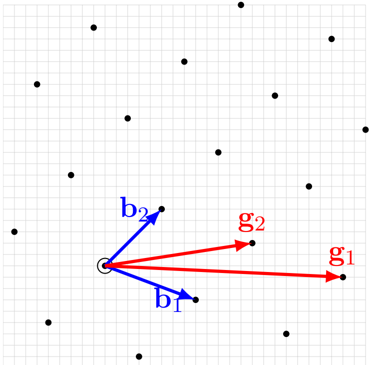

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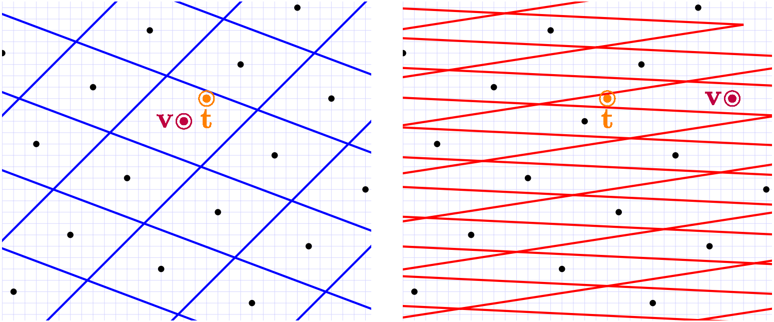
$\mathcal{L}$ has infinitely many bases

$\mathbf{B}$ is good, $\mathbf{G}$ is bad.

Parallelepiped

Each basis defines a parallelepiped $\mathcal{P}(\mathbf{B}) = \left\{\mathbf{xB} \mid \mathbf{x} \in \left[-\frac{1}{2}, \frac{1}{2}\right)^n\right\}$.



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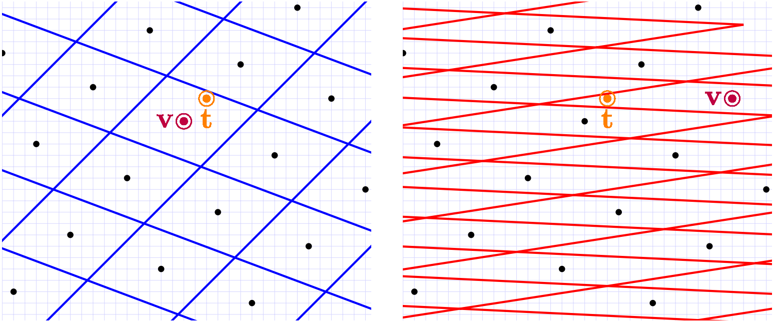

Babai's round-off algorithm outputs $\mathbf{v} \in \mathcal{L}$ such that $\mathbf{v} - \mathbf{t} \in \mathcal{P}$.

Hash-and-sign construction

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- signing: to solve the approximate closest vector problem (CVP)
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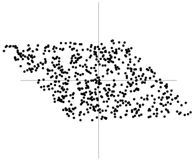
GGH, NTRUSign use deterministic round-off algorithm to solve the CVP.

- $\mathbf{v} - \mathbf{t} \in \mathcal{P}(\mathbf{B})$, the distribution of signatures leaks information of $\mathbf{B}$
- broken by parallelepiped-learning attacks [NR06][2]



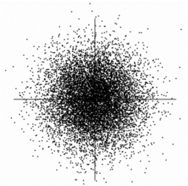
Parallelepiped. Insecure!

[2] [NR06]: Learning a parallelepiped: Cryptanalysis of GGH and NTRU signatures. Nguyen and Regev.

GPV framework

[GPV08][3] presented a provably secure framework.

- deterministic round-off algorithm $\Rightarrow$ trapdoor sampler
- randomizing the rounding with random Gaussian sampling on lattice
- the distribution of signatures is independent of the secret



Gaussian

[3][GPV08]: Trapdoors for Hard Lattices and New Cryptographic Constructions. Gentry, Peikert, Vaikuntanathan.

Falcon

Falcon signature scheme[4]

- selected by NIST for standardization in 2022
- initiated with GPV framework over NTRU lattices
- **advantages:** low bandwidth, good efficiency
- **disadvantages:** complicated, due to Gaussian sampling and floating-point operations

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Designing a simpler and comparably efficient variant of Falcon is a tempting choice!

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The Peregrine signature scheme

Peregrine

Peregrine signature scheme

- one of candidates in the 1st round of the KPQC competition
- the high speed version of Falcon
- Gaussian sampling ✖, centered binomial distribution ✔
- simpler, along with comparable efficiency, easy to mask

Peregrine does not offer a proof of security!!!

The procedure of signing

The signing of Peregrine is in essence the randomized version of Babai's round-off algorithm.

- by adding a binomial vector (J_1, J_2), instead of using Gaussian distribution

Signing

Input: NTRU trapdoor basis $\mathbf{B}$, center $\mathbf{c}$.

Output: random lattice point $\mathbf{s} \in \mathcal{L}(\mathbf{B}) - \mathbf{c}$.

1: $(J_1, J_2) \leftarrow (B_{\mu_1}^{n/2}, B_{\mu_2}^{n/2})$

2: $\mathbf{z} = \lfloor \mathbf{B}^{-1}\mathbf{c} \rceil + (J_1, J_2)$

3: $\mathbf{v} = \mathbf{B}\mathbf{z}$

4: $\mathbf{s} = \mathbf{v} - \mathbf{c}$

5: **return** $\mathbf{s}$

The centered binomial distribution B_μ is defined over $[-\frac{\mu}{2}, \frac{\mu}{2}] \cap \mathbb{Z}$.

Signature distribution

Practical distribution

We have $\begin{pmatrix} s_1 \\ s_2 \end{pmatrix} = \mathbf{B}_{f,g} \cdot \begin{pmatrix} R_1 - J_1 \\ R_2 - J_2 \end{pmatrix}$ where $(R_1, R_2) \sim U([-1/2, 1/2)^n)$.

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- the distribution of (s_1, s_2) is a hidden linear transformation (i.e. $\mathbf{B}_{f,g}$) of a known distribution
- we perform practical key recovery attacks against Peregrine by learning the hidden linear transformation

Secret key leakage

The Peregrine signatures are always in adjacent parallelepipeds, rather than a sole parallelepiped.

Adjacent parallelepipeds ✔

Sole parallelepiped ✖

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Peregrine are also insecure!!!

- the distribution of signatures would leak information of the secret key
- learn the hidden transformation by parallelepiped-learning of [NR06]

Concrete parameters

In this work, we focus on the parameter set of Peregrine–512.

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- **key generation:**
  - in the specification, the coefficients of (f, g) are drawn from B_{26}, and it checks if the Gram–Schmidt norms of $\mathbf{B}_{f,g}$ are less than $1.17\sqrt{q}$
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    - in the reference implementation, this check is commented out
- **the signing:**
    - the specification suggests $\mu_1 = \mu_2 = 26$
    - the reference implementation in effect use $(\mu_1, \mu_2) = (6, 0)$

Learning a hidden transformation

Parallelepiped-learning of [NR06] revisit

Definition 1 (The Hidden Parallelepiped Problem)

Given $\mathbf{B} = (\mathbf{b}_1, \ldots, \mathbf{b}_n) \in \mathrm{GL}_n(\mathbb{R})$ and a certain number of independent parallelepiped samples $\mathbf{y} = \mathbf{B}\mathbf{x}$ with $\mathbf{x} \leftarrow U([-1, 1])$, find an approximation of $\pm\mathbf{b}_i$'s.

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Solving the Hidden Parallelepiped Problem

- the covariance leakage: $\mathbf{K} = \mathbf{B} \cdot \mathbf{Cov}[\mathbf{xx}^t] \cdot \mathbf{B}^t = \mathbf{BB}^t/3$
- the approximate Gram matrix: $\mathbf{K} = 3\mathbf{K} = \mathbf{BB}^t$
- compute factor $\mathbf{L} = \mathbf{P}^t$ such that $\mathbf{K}^{-1} = \mathbf{PP}^t$
- by multiplying $\mathbf{L}$, $\mathbf{C} = \mathbf{LB}$ is orthogonal
- the local minima $\pm\mathbf{c}_i$ can be solved by gradient descent
- by multiplying $\mathbf{L}^{-1}$, the approximation of $\pm\mathbf{b}_i$ found

Hidden Transformation Problem

The Nguyen-Regev parallelepiped-learning attack [NR06] can be extended to more general *Hidden Transformation Problem* (HTP).

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Definition 2 (HTP$_D$)

Let D be a public distribution over $\mathbb{R}^n$. Given a hidden matrix $\mathbf{B} = (\mathbf{b}_1, \ldots, \mathbf{b}_n) \in \mathrm{GL}_n(\mathbb{R})$ and a certain number of independent samples $\mathbf{y} = \mathbf{B}\mathbf{x}$ with $\mathbf{x} \leftarrow D$, find an approximation of $\pm\mathbf{b}_i$'s.

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For Peregrine,

$$D_i = \begin{cases} U([-1/2, 1/2)) + B_{\mu_1} & \text{for } 1 \leq i \leq n/2; \\ U([-1/2, 1/2)) + B_{\mu_2} & \text{for } n/2+1 \leq i \leq n. \end{cases}$$

.

Key recovery algorithm

Our key recovery algorithm

1. distribution deformation
2. gradient descent

Step 1: Distribution deformation

The covariance leakage

- $\mathbf{Cov}[D(\mathbf{B})] = \mathbf{B} \cdot \mathbf{Cov}[D] \cdot \mathbf{B}^t$
- helps to reduce the general HTP to the case in which the covariance leakage is $\mathbf{I}_n$

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The procedure of distribution deformation

- the covariance leakage $\mathbf{K} = \mathbf{Cov}[D(\mathbf{B})]$
- compute $\mathbf{L} = \mathbf{P}^t$ such that $\mathbf{P}\mathbf{P}^t = \mathbf{K}^{-1}$
- $\mathbf{C} = \mathbf{LB}$ such that $\mathbf{Cov}[D(\mathbf{C})] = \mathbf{I}_n$
- $\mathbf{C}$ is orthogonal when $\mathbf{Cov}[D] = \mathbf{I}_n$

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- compute $\mathbf{L} = \mathbf{P}^t$ such that $\mathbf{P}\mathbf{P}^t = \mathbf{K}^{-1}$
- $\mathbf{C} = \mathbf{L}\mathbf{B}$ such that $\mathbf{Cov}[D(\mathbf{C})] = \mathbf{I}_n$
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Distribution deformation reduces the HTP instance regarding $(D, \mathbf{B})$ to the one regarding $(D, \mathbf{C})$ such that $\mathbf{Cov}[D(\mathbf{C})] = \mathbf{I}_n$ and $\mathbf{Cov}[D] = \mathbf{I}_n$.

Step 2: Gradient descent

Let $\alpha_i = \mathbb{E}[z_i^4]$. The fourth moment of $D(\mathbf{C})$ and its gradient:

$$M_{D(\mathbf{C}),4}(\mathbf{w}) = 3\|\mathbf{w}\|^4 - \sum_{i=1}^{n}(3-\alpha_i)\langle \mathbf{c}_i, \mathbf{w}\rangle^4,$$

$$\nabla M_{D(\mathbf{C}),4}(\mathbf{w}) = 12\mathbf{w} - \sum_{i=1}^{n}(12-4\alpha_i)\langle \mathbf{c}_i, \mathbf{w}\rangle^3 \mathbf{c}_i.$$

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Lemma 1

Suppose that $\alpha_i < 3$ for all $1 \le i \le n$, the local minimum of $M_{D(\mathbf{C}),4}(\mathbf{w})$ over all unit vectors $\mathbf{w}$ is obtained at $\pm\mathbf{c}_1, \ldots, \pm\mathbf{c}_n$. There are no other local minima.

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Therefore, the local minima $\mathbf{c}_i$ can be solved by gradient descent[TW20][5].

[5] [TW20]: One bit is all it takes: a devastating timing attack on BLISS's non-constant time sign flips. Tibouchi and Wallet.

The case of [NR06]

In [NR06], $D = U([-1, 1])$, the fourth moment function:

$$M_{D(\mathbf{C}),4}(\mathbf{w}) = \frac{1}{3}\|\mathbf{w}\|^4 - \frac{2}{15}\sum_{i=1}^{n}\langle \mathbf{c}_i, \mathbf{w}\rangle^4,$$

and its gradient:

$$\nabla M_{D(\mathbf{C}),4}(\mathbf{w}) = \frac{4}{3}\mathbf{w} - \frac{8}{15}\sum_{i=1}^{n}\langle \mathbf{c}_i, \mathbf{w}\rangle^3 \mathbf{c}_i.$$

The case of Peregrine

For specification version, $\mu_1 = \mu_2 = 26$:

$$M_{D(\mathbf{C}),4}(\mathbf{w}) = 3\|\mathbf{w}\|^4 - \frac{2346}{31205}\sum_{i=1}^{n}\langle \mathbf{c}_i, \mathbf{w}\rangle^4,$$

$$\nabla M_{D(\mathbf{C}),4}(\mathbf{w}) = 12\mathbf{w} - \frac{9384}{31205}\sum_{i=1}^{n}\langle \mathbf{c}_i, \mathbf{w}\rangle^3 \mathbf{c}_i.$$

For reference implementation version, $(\mu_1, \mu_2) = (6, 0)$:

$$M_{D(\mathbf{C}),4}(\mathbf{w}) = 3\|\mathbf{w}\|^4 - \frac{546}{1805}\sum_{i=1}^{n/2}\langle \mathbf{c}_i, \mathbf{w}\rangle^4 - \frac{6}{5}\sum_{i=n/2+1}^{n}\langle \mathbf{c}_i, \mathbf{w}\rangle^4,$$

$$\nabla M_{D(\mathbf{C}),4}(\mathbf{w}) = 12\mathbf{w} - \frac{2184}{1805}\sum_{i=1}^{n/2}\langle \mathbf{c}_i, \mathbf{w}\rangle^3 \mathbf{c}_i - \frac{24}{5}\sum_{i=n/2+1}^{n}\langle \mathbf{c}_i, \mathbf{w}\rangle^3 \mathbf{c}_i.$$

Practical key recovery attacks

Lattice decoding of [Pre23]

Let $\mathbf{b} = (b^{(1)}, b^{(2)}) \in \mathcal{L}_{\mathsf{NTRU}}$ be the secret vector and $\mathbf{b}' = \left((b')^{(1)}, (b')^{(2)}\right)$ be the approximation of $\mathbf{b}$.

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Prest's decoding technique [Pre23][6]

- Selecting a certain threshold $\varepsilon \in (0, 1/2)$
  - For $\mathbf{e} = \mathbf{b}' - \mathbf{b}$, at least half of the coefficients of $\mathbf{e}$ are in $[-\varepsilon, \varepsilon]$
  - No coefficients of $\mathbf{e}$ in absolute norm exceeds $1 - \varepsilon$

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  - zeros in at least $n/2$ coefficients
  - for NTRU equation, $b^{(1)} + b^{(2)} \cdot h = 0 \bmod q$, then

$$\left\lfloor (b')^{(1)} \right\rceil + \left\lfloor (b')^{(2)} \right\rceil \cdot h = d^{(1)} + d^{(2)} \cdot h \bmod q.$$

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$$\left\lfloor (b')^{(1)} \right\rceil + \left\lfloor (b')^{(2)} \right\rceil \cdot h = d^{(1)} + d^{(2)} \cdot h \bmod q.$$

The secret $\mathbf{b}$ can be fully recovered by solving linear system for $\mathbf{d}$.

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Probability-based guessing strategy

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Lemma 2

Let $b' \sim \mathcal{N}(b, \sigma^2)$ for some unknown integer center b, and known standard deviation σ. Let $x = b' - \lfloor b' \rceil$. The probability that $\lfloor b' \rceil = b$ is given by:

$$\psi_\sigma(x) = \frac{\rho_\sigma(x)}{\rho_\sigma(x + \mathbb{Z})}$$

where we let as usual $\rho_\sigma(t) = \exp\big(-t^2/(2\sigma^2)\big)$.

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The standard deviation is inversely proportional to required signatures N: $\sigma \approx C_\sigma/\sqrt{N}$ and constant C_σ can be derived by curve fitting.

Experimental results

For reference implementation

- signature samples: $\approx 25,000$
- running time: < 0.5 hours

$N \times 10^{-3}$	10	15	20	25	30	35	40	45	50
Instance 1	0	0	0	2	4	5	5	4	5
Instance 2	0	0	1	1	5	3	5	5	5
Instance 3	0	0	2	3	3	4	5	5	5
Instance 4	0	0	0	0	5	5	5	5	5
Instance 5	0	0	0	3	1	5	5	5	5
Instance 6	0	0	0	3	5	5	5	5	5
Instance 7	0	0	0	1	4	4	5	5	5
Instance 8	0	0	0	3	5	3	5	5	5
Instance 9	0	0	0	0	5	5	5	5	5
Instance 10	0	0	0	4	2	5	5	5	5

Experimental results

For the specification version

- signature samples: ≈ 11 million
- running time: < 20 hours

$N \times 10^{-6}$	3	5	7	9	11	13	15	17	20
Instance 1	0	0	0	0	3	5	5	5	5
Instance 2	0	0	0	0	0	4	3	5	5
Instance 3	0	0	0	0	0	5	5	5	5
Instance 4	0	0	0	0	0	2	4	5	5
Instance 5	0	0	0	0	3	3	3	5	5
Instance 6	0	0	0	0	1	5	5	5	5
Instance 7	0	0	0	0	4	3	5	5	5
Instance 8	0	0	0	0	2	3	2	5	5
Instance 9	0	0	0	0	0	5	5	5	5
Instance 10	0	0	0	0	3	3	5	5	5

Conclusion

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- The same attack can be extended to the case of Peregrine-1024

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More efficient countermeasures against statistical attacks need further investigations!

Thank you!