On Proving Equivalence Class Signatures Secure from Non-interactive Assumptions

Balthazar Bauer, Georg Fuchsbauer
and Fabian Regen

PKC, 16 April 24

What is a digital signature scheme?

A *signature scheme* is a triple of p.p.t. algorithms

- Keygen() $\rightarrow (sk, pk)$
- Sign$(sk, m) \rightarrow \sigma$
- Verify$(pk, m, \sigma) \rightarrow 0$ or 1

Equivalence class signatures (EQS) [FHS19]

Defined over group $(\mathbb{G}, p, g)$

Messages space $(\mathbb{G}^*)^2$; partitioned by

$$m \sim m' :\Leftrightarrow \exists\, \mu \in \mathbb{Z}_p^* : m = \mu \cdot m'$$

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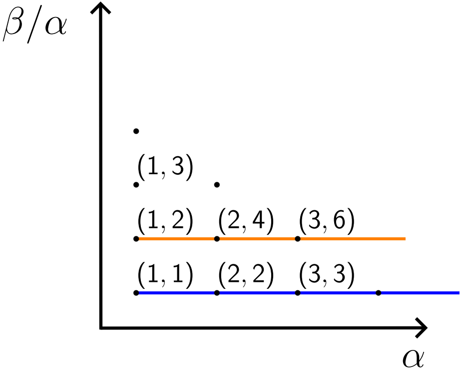

Equivalence classes
for $m = (\alpha \cdot g, \beta \cdot g)$

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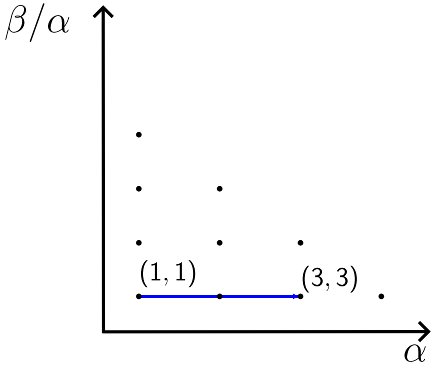
Additional functionality:

Adapt$(pk, m, \sigma, \mu \in \mathbb{Z}_p^*)$: returns signature on $\mu \cdot m$



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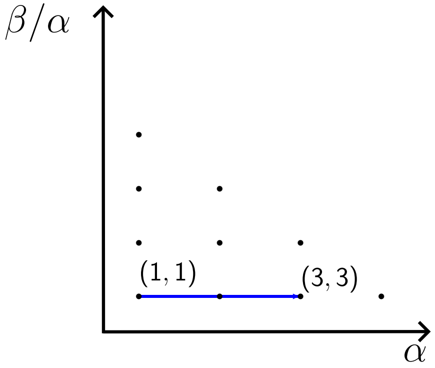

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Game UNF:

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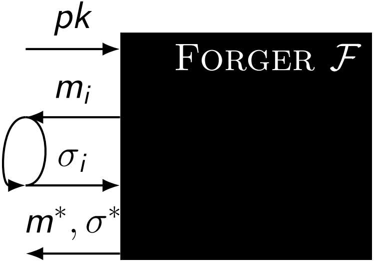

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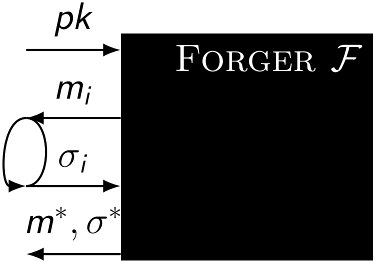

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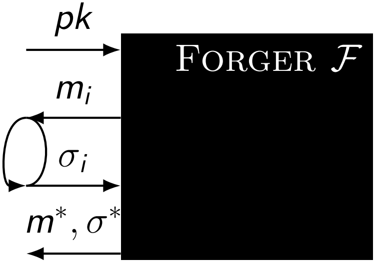

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Scheme *secure* if $\mathsf{Adv}^{\mathsf{UNF}}_{\mathcal{F}} := \Pr[\mathcal{F} \text{ wins}] \approx 0$

Security of EQS

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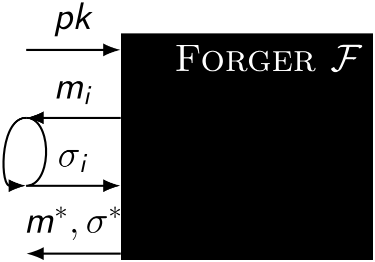

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Anonymity (even against issuer)**:**

- m_i looks random ($\Leftarrow$ class hiding)
- σ_i is random signature on m_i ($\Leftarrow$ Adapt)

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- Verifiably encrypted signatures [HRS15], access-control encryption [FGKO17], sanitizable signatures [BLL+19], incentive systems [BEK+20], mix nets [ST21], anonymous counting tokens [BRS23] ...

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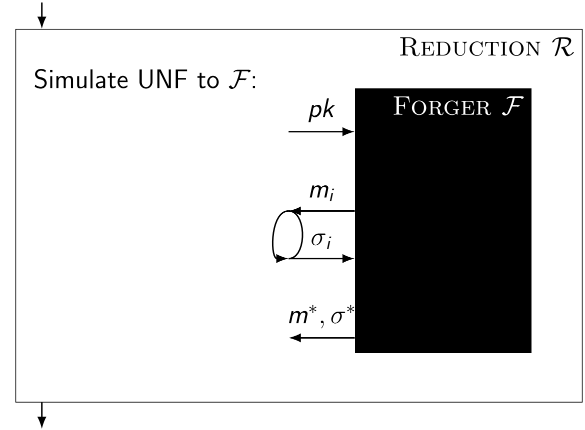
Is there a scheme satisfying the original notion with a proof from a non-interactive assumption?

Security reductions

Reduction $\mathcal{R}$ from computational problem Π to UNF
using adversary $\mathcal{F}$

$\Pi : c$



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Theorem. For any EQS scheme and any Π, no reduction can exist

Proof idea

Simplification: Assume $\mathcal{R}$ partitions $(\mathbb{G}^*)^2$ into **signable** and **exploitable** messages

$S := \{m \mid \mathcal{R} \text{ can answer a signing query for } m\}$

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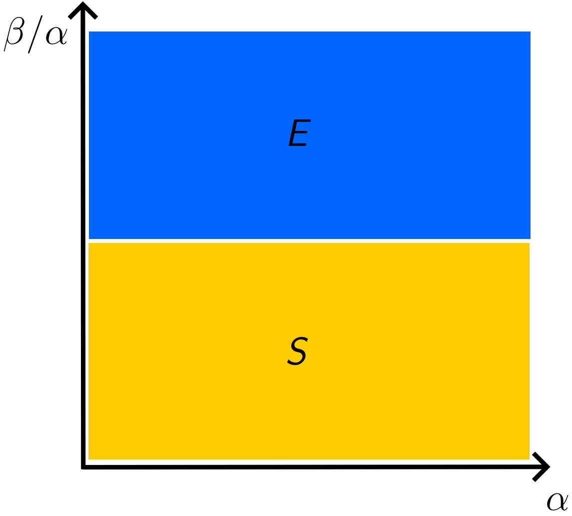
- S and E must both be "big"
- do not intersect

Case 1: E and S do not share classes



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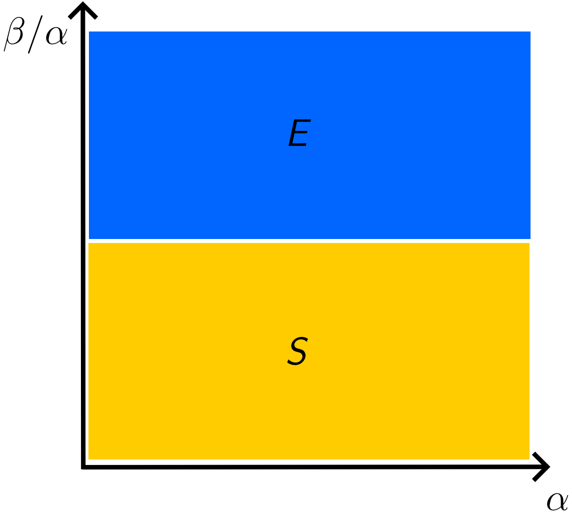

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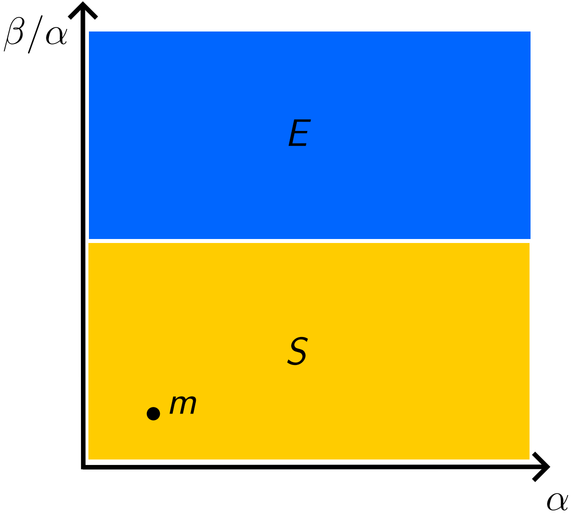

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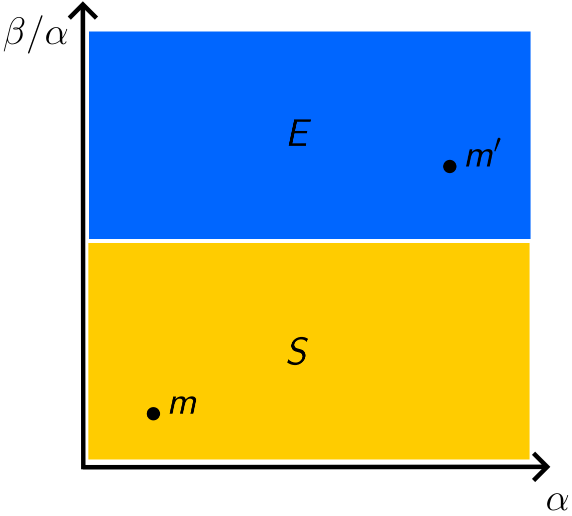

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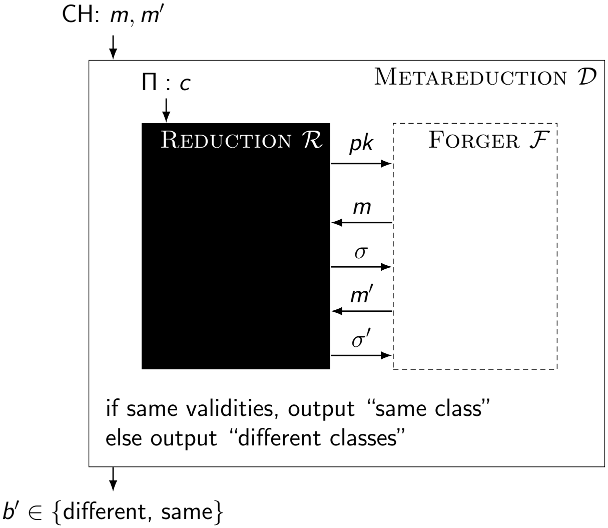
if exactly one message signable then separate classes

Breaking class hiding



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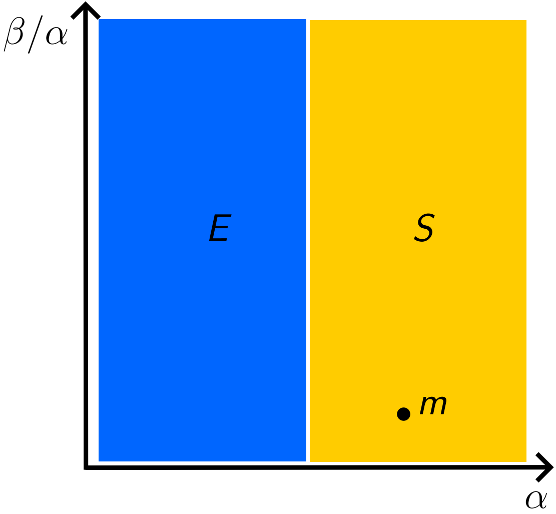

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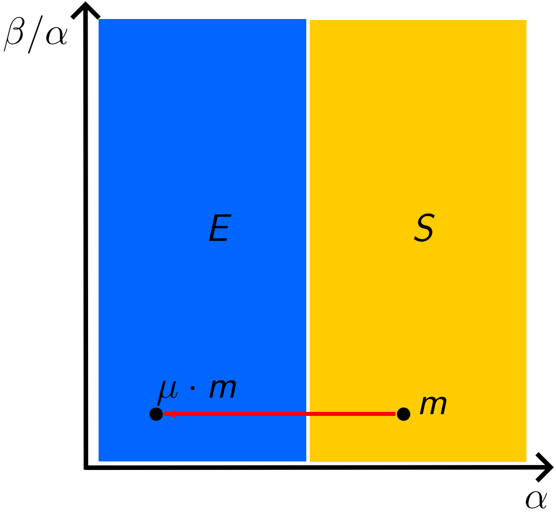

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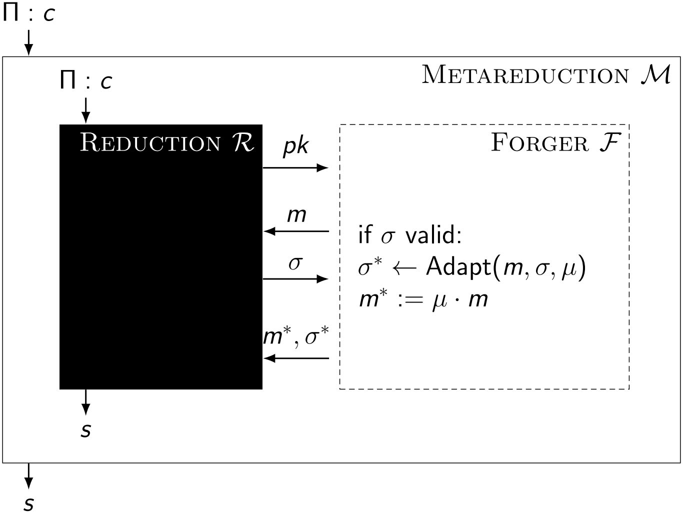

Π : c
METAREDUCTION $\mathcal{M}$
Π : c
REDUCTION $\mathcal{R}$
pk
m
σ
m^*, σ^*
FORGER $\mathcal{F}$
if σ valid:
$\sigma^* \leftarrow \mathsf{Adapt}(m, \sigma, \mu)$
$m^* := \mu \cdot m$
s
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such that

$$\mathsf{Adv}^{\mathrm{CH}}_{\Sigma,\mathcal{D}^{\mathcal{R}}} + \mathsf{Adv}^{\Pi}_{\mathcal{M}^{\mathcal{R}}} + \mathsf{Adv}^{\Pi}_{\mathcal{R}^{\mathcal{F}}} \geq \frac{\phi^5}{384}$$

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Result. Their proofs are flawed[1]

[1] B. Bauer, G. Fuchsbauer, F. Regen: On security proofs of existing equivalence class signature schemes (ia.cr/2024/183)