Probabilistic Hash-and-Sign with Retry in the Quantum Random Oracle Model

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Table of Contents

1. Background

2. New Security Proof

3. Applications

4. Summary

Table of Contents

1. Background

2. New Security Proof

3. Applications

4. Summary

Hash-and-sign in NIST PQC

- In 2023, NIST issued additional call for digital signatures for diversity of portfolio.
- Hash-and-sign is adopted in many candidates (14 out of 40).

Hash-and-sign signatures among additional candidates.

Lattice	**Multivariate**	**Code**
HAWK, HuFu, Squirrels	3WISE, HPPC, MAYO, PROV, QR-UOV, SNOVA, TUOV, UOV, VOX	Enhanced pqsigRM, Wave

Hash-and-sign signatures are promissing candidates.

Hash-and-sign

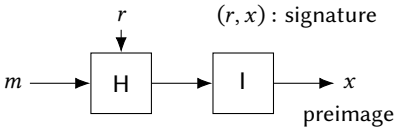
Key generation

$(\mathsf{F}, \mathsf{I}) \leftarrow \mathsf{Gen}(1^\lambda)$

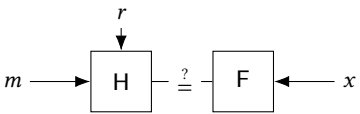
F : hard-to-invert function = verification key

I : trapdoor of F = secret key

Signature generation

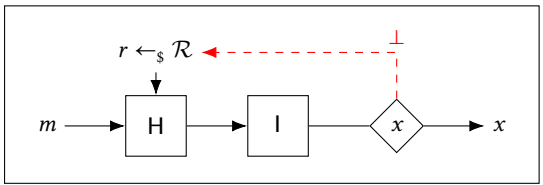


Signature verification



Variations:

- Deterministic hash-and-sign : r is null.
- Probabilistic hash-and-sign : r is random.
- Probabilistic hash-and-sign with retry :

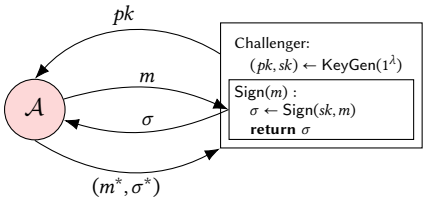


Easier to build signature, harder to prove security.

e.g. surjection is not required.

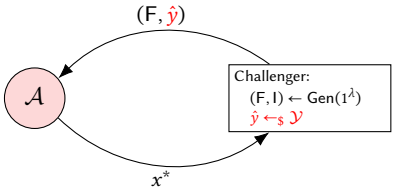
Security Definition

EUF-CMA



$\epsilon_{\mathsf{cma}} = \Pr[\mathsf{Vrfy}(m^*, \sigma^*) = \top \wedge (m^* \text{ was not queried})]$
(EUF-NMA: no signing query)

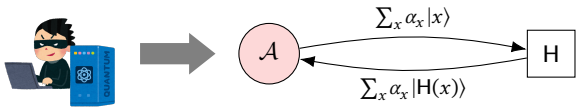
Non-invertibility (INV)



$\epsilon_{\mathsf{inv}} = \Pr[\mathsf{F}(x^*) = \hat{y}]$
(One-wayness: $\hat{y} = \mathsf{F}(x)$ for $x \leftarrow_{\$} \mathcal{X}$)

Provable security requires INV⇒EUF-CMA or CR⇒EUF-CMA.
(CR: Collision Resistance)

(Quantum) Random Oracle Model



Real-world QROM

- INV ⇒ EUF-CMA or CR ⇒ EUF-CMA is proven in (Q)ROM.
- Since QROM models adversary implementing hash function in quantum computer, post-quantum signatures must be provable secure in QROM.

Secure signatures in ROM are not always secure in QROM [YZ22].

[YZ22] Yamanaka, Zhandry (FOCS2022)

Provable Security of Hash-and-sign

Summary on provable security of hash-and-sign

Paradigm	Assumption	Examples	Reduction (ROM)	Reduction (QROM)
deterministic	collision-resistance	Falcon (GPV)	tight [GPV08]	tight [BDH+13]
probabilistic (without retry)	non-invertibility	Wave, MAYO	poly loss [CD20][Beu21]	poly loss [YZ21] (PSF* is required.)
probabilistic with retry	non-invertibility	PROV, QR-UOV	poly loss [SSH11] (Proof is flawed.)	–

*Preimage Sampleable Function: Trapdoor function with some conditions, e.g., F is uniform and surjection.

[GPV08] Gentry, Peikert, Vaikuntanathan (STOC2008)

[BDH+13] Boneh et al. (ASIACRYPT2011)

[YZ21] Yamakawa, Zhandry (EUROCRYPT2021)

[CD20] Chailloux, Debris-Alazard (PKC 2020)

[Beu21] Beullens (SAC2021)

[SSH11] Sakumoto, Shirai, Hiwatari (PQCRYPTO2011)

Table of Contents

1. Background

2. New Security Proof

3. Applications

4. Summary

Overview of New Security Proof

INV⇒EUF-CMA for probabilistic hash-and-sign with retry in QROM

(EUF-NMA ⇒EUF-CMA + INV ⇒EUF-NMA)

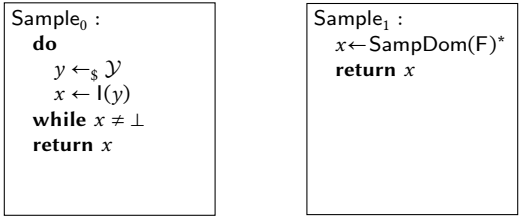
$$\epsilon_{\mathsf{cma}} \leq (2q_{\mathsf{qro}}+1)^2\epsilon_{\mathsf{inv}} + \epsilon_{\mathsf{ps}} + \frac{3}{2}q'_{\mathsf{sign}}\sqrt{\frac{q'_{\mathsf{sign}}+q_{\mathsf{qro}}+1}{2^{|r|}}} + 2(q_{\mathsf{qro}}+2)\sqrt{\frac{q'_{\mathsf{sign}}-q_{\mathsf{sign}}}{2^{|r|}}}$$

q_{qro} : # QRO queries.
q_{sign} : # signing queries.
q'_{sign} : # retries of r.

ϵ_{cma} : advantage for EUF-CMA.
ϵ_{inv} : advantage for non-invertibility.
ϵ_{ps} : explain later.

To make ϵ_{ps} negligible, outputs of trapdoor must be simulatable.

Simulatability of Trapdoor

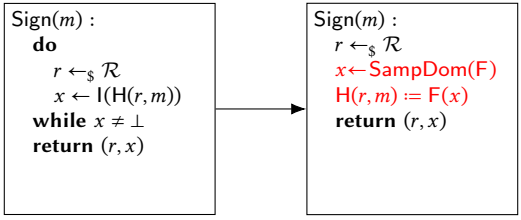


* SampDom : function to simulate preimange-sampling.

- ϵ_{ps} : advantage to distinguish Sample_0 from Sample_1.
- "Negligible ϵ_{ps}" is weakened condition of PSF and computational bound is applicable.

Some trapdoor functions are not PSF but satisfy negligible ϵ_{ps}.

Flaw of Proof in ROM [SSH11]

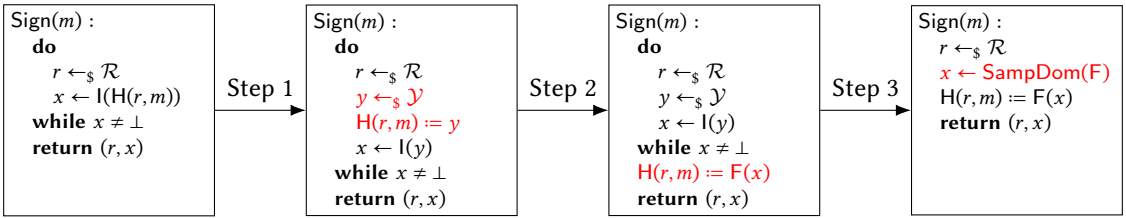


Modification of signing oracle in [SSH11]

- Assuming negligible ϵ_{ps}, signing oracle is simulated by (re)programming $\mathsf{H}(r, m) := \mathsf{F}(x)$.
- Since $\mathsf{F}(x)$ is not uniform, H is no longer random function.

[SSH11] Sakumoto, Shirai, Hiwatari (PQCRYPTO2011)

EUF-NMA ⇒ EUF-CMA in QROM



Modification of signing oracle in our proof

Step 1 : Tight adaptive reprogramming [GHHM21] enables reprogramming H.
Step 2 : Semi-classical O2H [AHU19] cancels reprogramming during retries.
Step 3 : Negligible ϵ_{ps} enables simulation without trapdoor.

[GHHM21] Grilo, Hövelmanns, Hülsing, Majenz (ASIACRYPT2021)
[AHU19] Ambainis, Hamburg, Unruh (CRYPTO2019)

INV ⇒ EUF-NMA in QROM

$\mathcal{A}_{\mathsf{inv}}(\mathsf{F}, \hat{y})$:
  $\mathsf{H} \leftarrow_{\$} \mathcal{Y}^{\mathcal{R}\times\mathcal{M}}$
  $j \leftarrow_{\$} [q_{\mathsf{qro}}]$
  run $\mathcal{A}_{\mathsf{nma}}^{|\mathsf{H}\rangle}(\mathsf{F})$ until j-th query
  measure j-th query as (r', m')
  $\mathsf{H}(r', m') := \hat{y}$
  run $\mathcal{A}_{\mathsf{nma}}^{|\mathsf{H}\rangle}(\mathsf{F})$ until end
  $(m^*, r^*, x^*) \leftarrow \mathcal{A}_{\mathsf{nma}}^{|\mathsf{H}\rangle}(\mathsf{F})$
  return x^*

INV adversary simulating EUF-NMA adversary

Measure-and-reprogram technique [DFM20] results in $\epsilon_{\mathsf{nma}} \leq (2q_{\mathsf{qro}} + 1)^2 \epsilon_{\mathsf{inv}}$.

[DFM20] Don, Fehr, Majenz (CRYPTO2020)

Table of Contents

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3. Applications

4. Summary

Applications of New Security Proof

Complete 🤗

Updated summary on provable security of hash-and-sign

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probabilistic with retry	non-invertibility	PROV, QR-UOV*	poly loss	poly loss

* PROV and QR-UOV adopt our proof.

[GPV08] Gentry, Peikert, Vaikuntanathan (STOC2008)

[BDH+13] Boneh et al. (ASIACRYPT2011)

[CD20] Chailloux, Debris-Alazard (PKC 2020)

[Beu21] Beullens (SAC2021)

Table of Contents

1. Background

2. New Security Proof

3. Applications

4. Summary

Summary

- First security proof for probabilistic hash-and-sign with retry in (Q)ROM.
- Our proof has wide application → Big impact on NIST PQC standardization.
- Our proof is extended to multi-key security (M-EUF-CMA security).