Reach Restricted Reactive Program Obfuscation

And its Application to MA-ABE

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Some Surprising Results

Laconic Oblivious Transfer from DDH [CDG+17]

Identity Based Encryption (IBE) from DDH [DG17a, DG17b]

2-round MPC from 2-round OT [BL17, GS17]

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What is Obfuscation?

Program Obfuscation:

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Program Obfuscation:

- keeping secrets in a program
- even against an adversary that captures the entire computer on which it is run
- without any trusted hardware

Subtle to formalize

Obfuscation Results

Different notions of Obfuscation

- Virtual Black-Box Obfuscation [BGI+01]
- Indistinguishability Obfuscation (iO) [BGI+01, JLS20]
- Average Case Obfuscation [HRsV07]
- Virtual Grey Box Obfuscation [BCTKP14]
- Differing Inputs Obfuscation [ABG+13] and public-coin DiO [IPS14]

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Require strong assumptions in general (if not impossible)

- Obfuscation achievable from standard assumptions, when programs are sampled in a customized fashion:
  - Obfuscation for Re-Encryption [HRsV07] - from DDH
  - Obfuscation for Evasive Functions [BBC+13] - from DDH variant
  - Obfuscation for Compute-and-Compare functions [WZ17] - from LWE
  - **In this work:** For programs sampled interactively, enforcing a restriction on what information the adversary has about its contents.

A Motivating Example

A program that contains a message and an encryption public-key PK.

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Can be turned into a valid notion of obfuscation:

Interactive sampling of the program: (PK,SK) pairs are generated secretly. Each SK can be published fully, or not revealed at all, as requested by the adversary.

Simulation of Obfuscated Program: If SK published, an adversary is allowed to learn the message — from which a valid ciphertext can be constructed. If SK not published, a random ciphertext is a valid simulation of the real ciphertext.

Conversely, such an obfuscation yields PKE.

Our Goal

A definition that formalizes similar seemingly naïve ideas of obfuscation

Example: IBE as Obfuscation
Ciphertext is the obfuscation of the following program:

Hardwired: message m, identity id, a signature verification key VK

On input σ: if σ is a valid signature on id w.r.t. VK, output m.

Issue a decryption key for id by simply signing id

Reactive Program

Let Σ, M be the space of states and messages respectively.

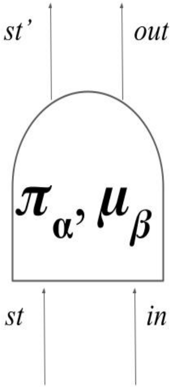
Reactive Program $P = (\pi_\alpha, \mu_\beta)$**:**

Transition function $\pi_\alpha : \Sigma \times \mathcal{X} \to \Sigma$
Message function $\mu_\beta : \Sigma \to M$
(for some hardwired secrets α, β)

Evaluating a Reactive Program:

$P(\mathsf{st}, in)$:

$$\mathsf{st}' = \pi_\alpha(\mathsf{st}, in)$$
$$out = \mu_\beta(\mathsf{st}')$$



Reach-Restricted Reactive Program

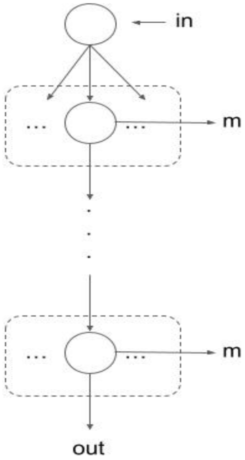
We require a partition of the state space,
$\Sigma = \Sigma_1 \cup \cdots \cup \Sigma_n$

A typical example will have:

- $O(\kappa)$ parts
- $2^{O(\kappa)}$ states in each part.

where κ is the security parameter.

The parts should form a tree.



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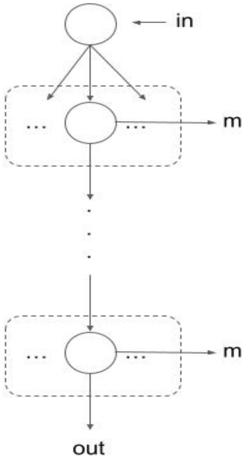
Reach-Restriction:

Adversary can find inputs that take the program to at most one state in each part of the partition

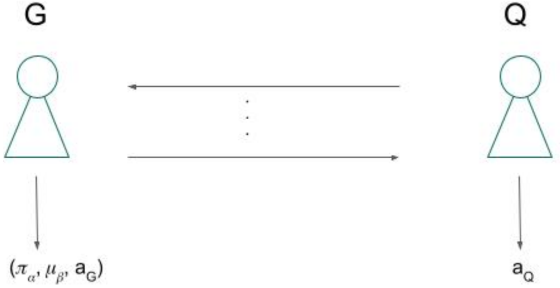
When the program is "sampled properly"



Interactive Sampling

Rules for sampling a program formalized as a class of
Reactive Program Generators

- A generator G interacts with an adversary Q
- Outputs a reactive program (π_α, μ_β).
  Also auxiliary information a_G, a_Q produced



Formalizing Reach Restriction

To which all states can an adversary Q take a reactive program generated by a generator G (even given $(\pi_\alpha, \mu_\beta, \mathrm{a}_G;\ \mathrm{a}_Q)$)?

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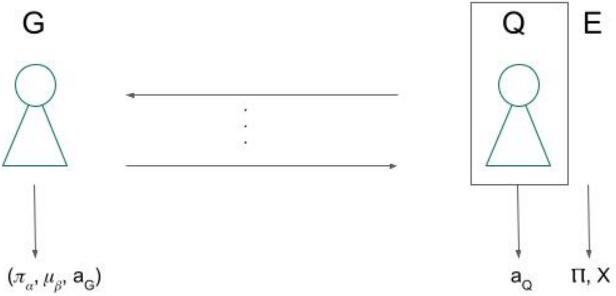
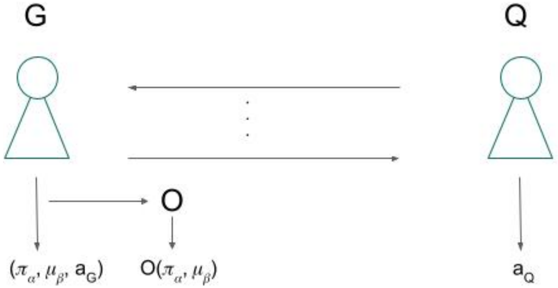
To which all states can an adversary Q take a reactive program generated by a generator G (even given $(\pi_\alpha, \mu_\beta, \mathrm{a}_G;\ \mathrm{a}_Q)$)?

- An extractor E can output all such states.
  - Encoded as an (idealized) reactive program Π and input sequences X for it, s.t. reachable states in π_α are reached in Π using X.
- Can have at most one state in each part in the state-space partition.



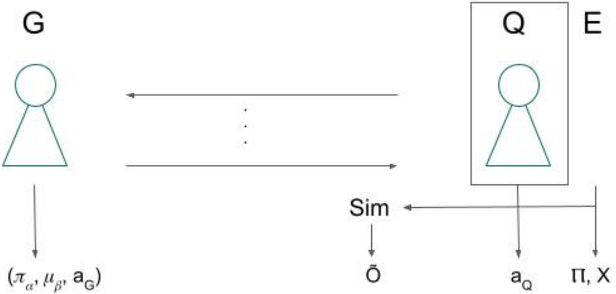
The Real World:

- G interacts with Q
- output of interaction: $(\pi_\alpha, \mu_\beta, \mathrm{a}_G;\ \mathrm{a}_Q)$
- Obfuscator $\mathcal{O}$ outputs $\mathcal{O}(\pi_\alpha, \mu_\beta)$



The Ideal World:

- G interacts with Q
- output of interaction: $(\pi_\alpha, \mu_\beta, \mathrm{a}_G;\ \mathrm{a}_Q)$, E outputs Π, X
- $\mathrm{Sim}\Big(\Pi, X, \{\mu_\beta(\mathrm{st}) \mid \mathrm{st} \in \Pi(X)\}\Big)$ outputs $\tilde{\mathcal{O}}$



Defining R3PO Security

A Strong Simulation-Based Definition

$\mathcal{O}$ is an R3PO scheme for $\mathcal{G}$ w.r.t. a class of adversaries $\mathcal{Q}$ if, $\forall G \in \mathcal{G}$ and $Q \in \mathcal{Q}$, there exists a simulator Sim s.t. Real World is indistinguishable from Ideal World

$$\left\{\mathcal{O}(\pi_\alpha, \mu_\beta),\ \mathrm{a}_G,\ \mathrm{a}_Q\right\} \approx \left\{\mathsf{Sim}\Big(\Pi, X, \{\mu_\beta(\mathsf{st}) \mid \mathsf{st} \in \Pi(X)\}\Big),\ \mathrm{a}_G,\ \mathrm{a}_Q\right\}$$

Example 1: Commitment Opening R3PO

Commitment Scheme

- $\mathsf{gen}(1^\kappa) \to \mathsf{crs}$
- $\mathsf{commit}(\mathsf{crs}, m) \to (c, d)$
- $\mathsf{open}(\mathsf{crs}, c, d) \to m$

Properties Required:

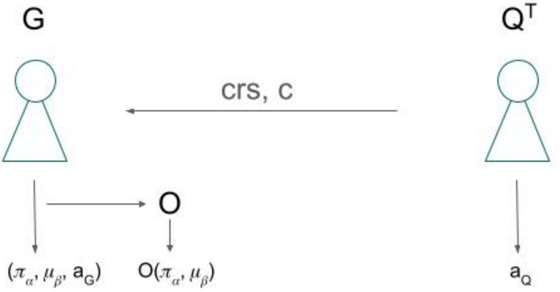
1 Computational Hiding: commitment c does not reveal message m.

2 Computational Binding: commitment c can be opened to at most a single message m. Further, there exists an extractor $\mathcal{E}$ that can extract this m.

Example 1: Commitment Opening R3PO

Interaction:

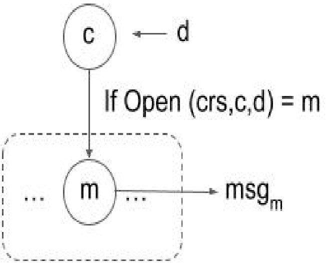
- Q^T gets *crs* from T and sends *crs*, c to G.



Example 1: Commitment Opening R3PO

Reactive Program:

$$\pi(\mathsf{st}_c^1, d) = \begin{cases} \mathsf{st}_m^2, \text{ if open}(\mathsf{crs}, c, d) = m \\ \perp, \text{ else} \end{cases}$$



⋆ Interested in adversaries of the form Q^T that sample crs honestly.

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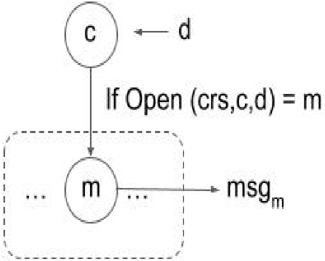

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Theorem 1 (Informally)

If the DDH assumption holds, there exists a Commitment scheme and a R3PO for Commitment-Opening.

Example 2: Signature-Checking

Signature Scheme

- $\mathsf{gen}(1^\kappa) \to (\mathsf{vk}, \mathsf{sk})$
- $\mathsf{sign}(\mathsf{sk}, m) \to \tau$
- $\mathsf{verify}(\mathsf{vk}, m, \tau) \to \{0, 1\}$

Properties Required:

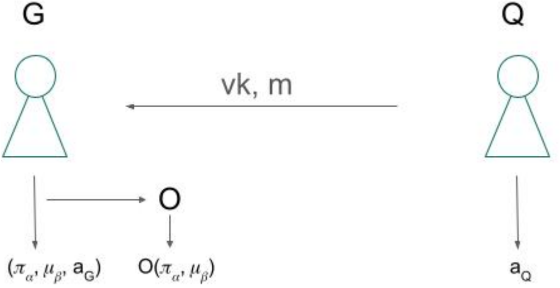
1 Correctness.

2 Unforgeability: without sk, hard to forge signature on a new message.

Example 2: Signature-Checking R3PO

Interaction:

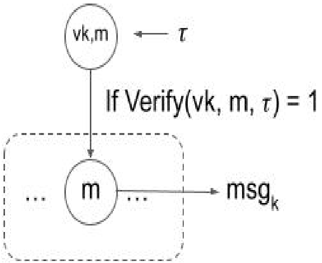
- Q sends vk, m to G.



Example 2: Signature-Checking R3PO

Reactive Program:

$$\pi(\mathsf{st}^1_{\mathsf{vk},m}, \tau) = \begin{cases} \mathsf{st}^2_m, \text{ if verify}(\mathsf{vk}, m, \tau) = 1 \\ \perp, \text{ else} \end{cases}$$



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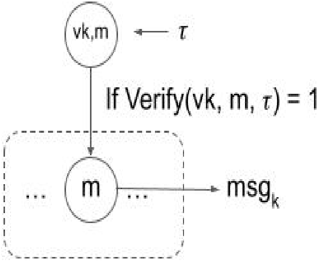


Theorem 2 (Informally)

If the DDH assumption holds, there exists a Signature scheme and a R3PO for Signature-Checking.

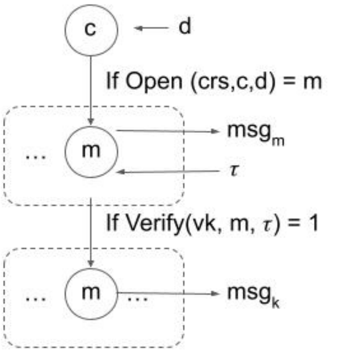
Towards R3PO of Larger Reactive Programs

Can we combine R3PO for Commitment Opening and Signature Checking?

Reactive Program:

$$\pi(\mathsf{st}_c^1, d) = \begin{cases} \mathsf{st}_m^2, \text{ if } \mathsf{open}(\mathsf{crs}, c, d) = m \\ \perp, \text{ else} \end{cases}$$

$$\pi(\mathsf{st}_m^2, \tau) = \begin{cases} \mathsf{st}_m^3, \text{ if } \mathsf{verify}(\mathsf{vk}, m, \tau) = 1 \\ \perp, \text{ else} \end{cases}$$



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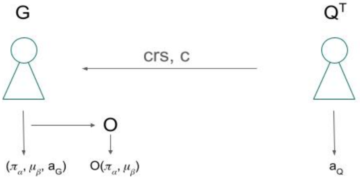

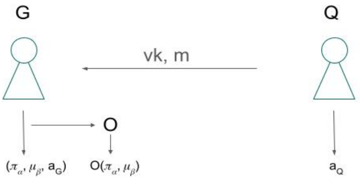

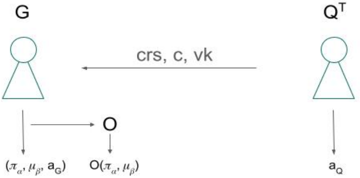

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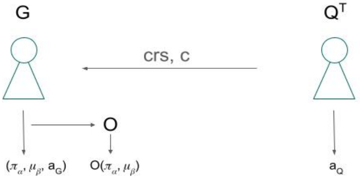
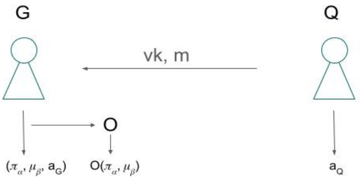
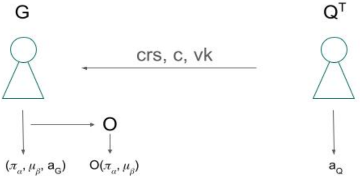




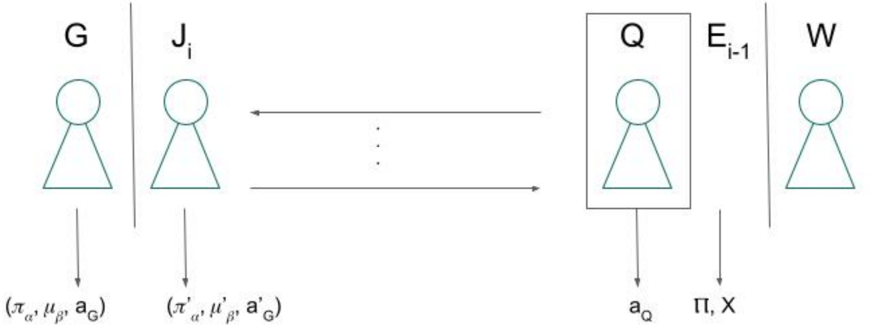


Need to be careful in handling the interaction!

Decomposition Property

We say that a generator class $\mathcal{G}$ **decomposes** to a generator class $\mathcal{G}_i$ (at partition i) if the following bi-simulations are indistinguishable.

- In the interaction between $G \in \mathcal{G}$ and $Q \in \mathcal{Q}$, for all $(i-1)$-partial reach extractors E_{i-1}, there exists J_i, W s.t. J_i outputs $(\pi'_\alpha, \mu'_\beta, \mathrm{a}'_G)$.



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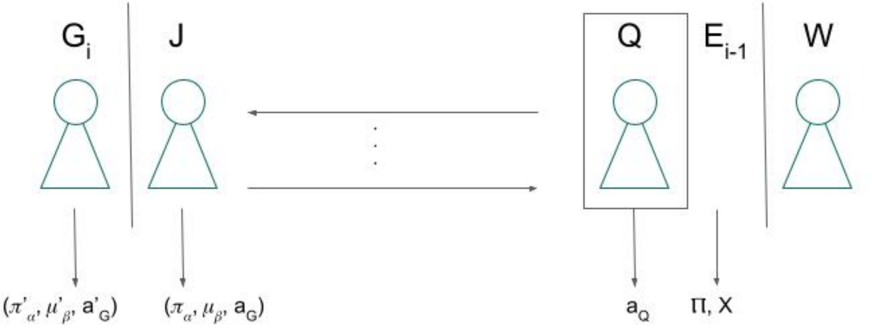

Composition Theorem

Theorem 3 (Informally)

Let $\mathcal{G}_1, \ldots, \mathcal{G}_n$ be generator classes with R3PO schemes $\mathcal{O}_1, \ldots, \mathcal{O}_n$. If a generator class $\mathcal{G}$ decomposes to $\mathcal{G}_i$ at each partition $i \in [n]$, then there exists a R3PO scheme for $\mathcal{G}$.

The proof uses garbled-circuit chaining.

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Corollary:

If there exists a R3PO for commitment-opening, then:

- there exists a R3PO for sequence of commitment-openings
- there exists a 2-round MPC protocol secure against semi-honest dishonest majority corruption [BL17, GS17]

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Corollary:

If there exists a R3PO for signature-checking, then:

- there exists a R3PO for sequence of signature-checkings
- there exists an adaptive-secure IBE scheme [DG17a, DG17b]

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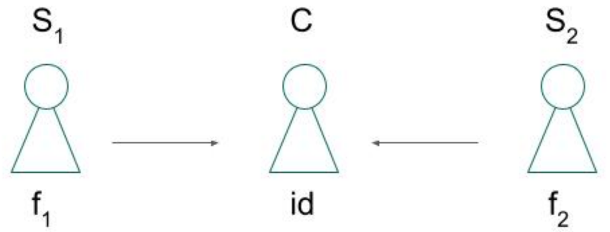
Corollary:

If there exists a R3PO for commitment-opening and a R3PO for signature-checking, then:

- there exists a R3PO for commitment-opening followed by signature checking.
- if there exists an ABE scheme, there exists a "private" MA-ABE scheme (our work).

Prior Works for MA-ABE

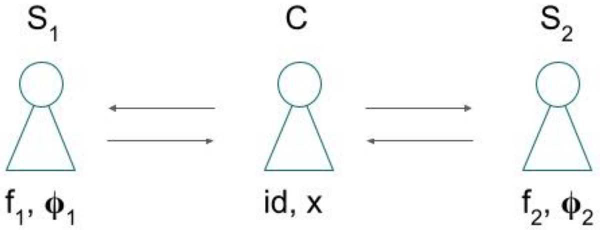


Global ID Model [Cha07, LW10]:

- Each client has a global id
- Only interaction: servers send credentials for id to a client
- Current results rely on the Random Oracle Model: E.g., [DKW20] for DNF formulae under the LWE assumption.

Private MA-ABE: A client can privately decide on the attributes it wants to acquire, as long as it conforms to the servers' policy. Client can send a message to each server first.

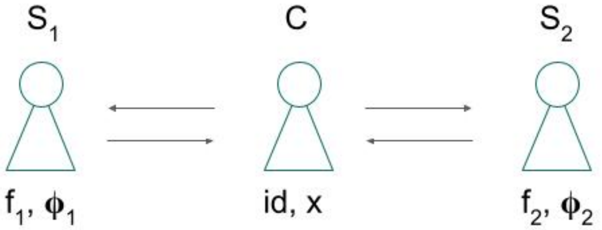
Attribute Verification



2-round protocol for Attribute Verification:

- Decentralized setup of Servers: publish global public keys.
- Round 1: C sends a request to S_1 and S_2.
- Round 2: Servers S_1 and S_2 send response to C.

Attribute Verification

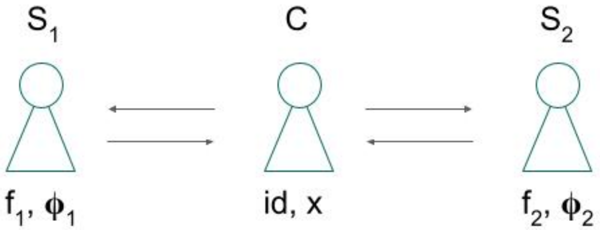


Completeness:

if $\Phi_1(\text{id}, x) = 1$ and $\Phi_2(\text{id}, x) = 1$, then C gets $f_1(\text{id}, x)$, $f_2(\text{id}, x)$.

Hiding: Server S_b learns nothing about (id, x) and Φ_{1-b}.

Attribute Verification



Solution using R3PO:

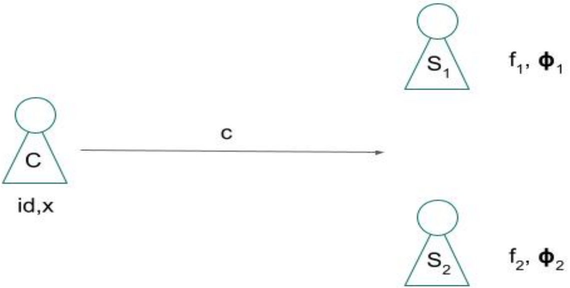
Use commitment to hide (id, x).

Use signatures to give proof of verification.

Attribute Verification

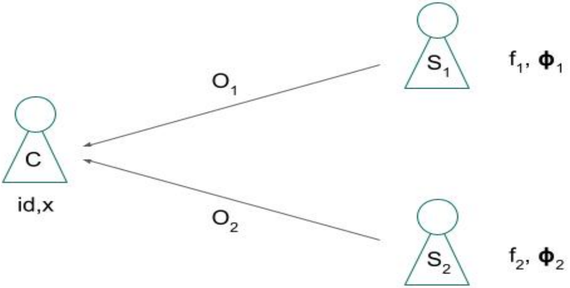
Round 1:

- Client C computes $(c, d) \leftarrow \text{commit}(\text{crs}, \text{id})$ and sends c to servers S_1, S_2.



Round 2**:**

- Server S_1 sends $\mathcal{O}_1$ to Client C.
- Server S_2 sends $\mathcal{O}_2$ to Client C.



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- Server S_2 sends $\mathcal{O}_2$ to Client C.

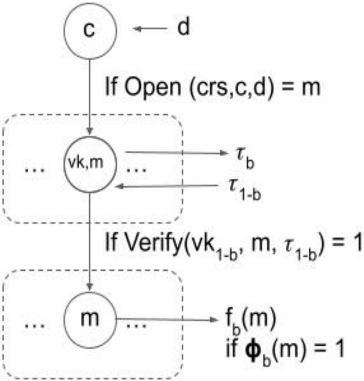
where, each $\mathcal{O}_i$ is R3PO of program with:

transition function π:

$$\pi(\mathsf{st}_c^1, d) = \mathsf{st}_m^2, \text{ if } \mathsf{open}(\mathsf{crs}, c, d) = m$$
$$\pi(\mathsf{st}_m^2, \tau) = \mathsf{st}_m^3, \text{ if } \mathsf{verify}(\mathsf{vk}_{1-b}, m, \tau_{1-b}) =$$

message function μ:

$$\mu_{\mathsf{sk}_b, f_b, \Phi_2}(\mathsf{st}_m^3) = \mathsf{sign}(\mathsf{sk}_b, m)$$
$$\mu_{\mathsf{sk}_b, f_b, \Phi_b}(\mathsf{st}_m^3) = f_b(m), \text{ if } \Phi_b(m) = 1$$



Attribute Verification

Theorem 4

If there exists an R3PO for commitment-opening and signature-checking, then there exists a 2-round Protocol for Attribute Verification.

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If there exists an R3PO for commitment-opening and signature-checking, then there exists a 2-round Protocol for Attribute Verification.

Corollary: Given the following primitives:

- a CP-ABE scheme for general policies
- R3PO for commitment-opening and signature-checking

there exists a Private MA-ABE scheme for general policies.

In Conclusion

- R3PO: Obfuscation of interactively sampled programs
- A library of R3PO instantiations from standard assumptions:
  - Commitment-Opening
  - Signature-Checking
    - Can optionally restrict to a message prefix.
  - Hash-Checking
- A composition theorem to build R3PO for larger program classes.
  - Encapsulates Garbled Circuit Chaining technique
- As an application, we construct Private MA-ABE
- **Open Directions:** More applications, capturing more constructions (e.g., Garbled RAM), adding more features (e.g., blindness)

Thank You

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