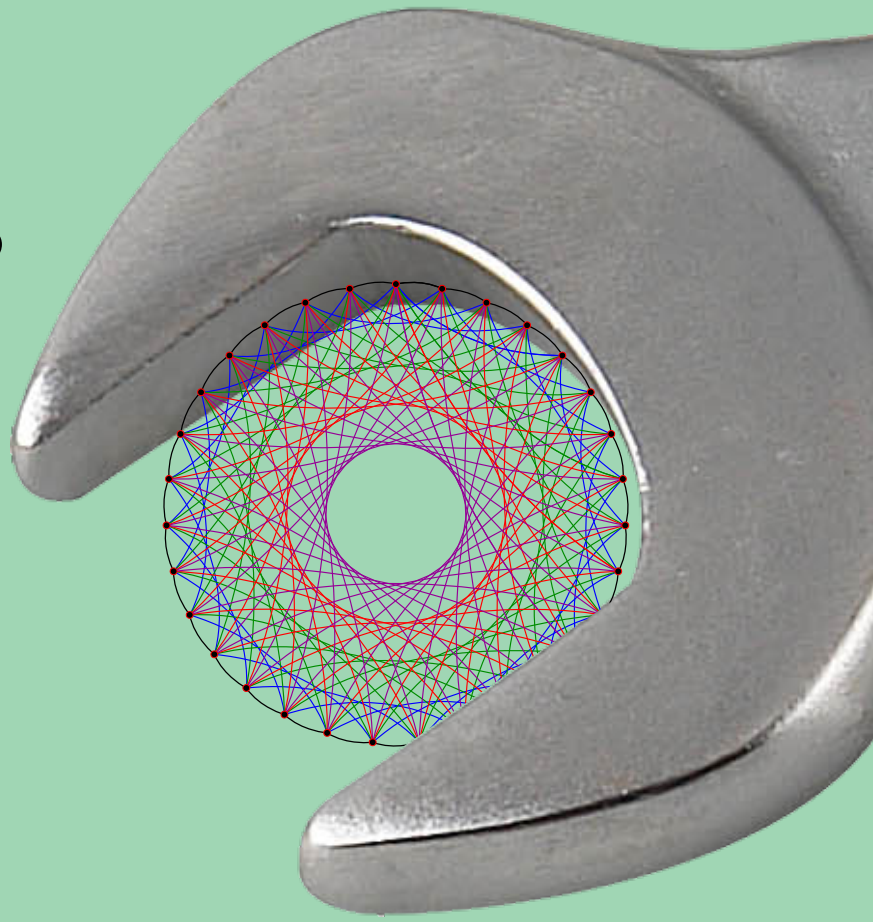
New proof systems and an OPRF from CSIDH



PKC 2024
Sydney, Australia

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Robi Pedersen
COSIC, KU Leuven

Group Action Setting

Group G acting freely and transitively on a set $\mathcal{E}$ by the action

$$\star : G \times \mathcal{E} \to \mathcal{E}$$
$$(\mathfrak{a}, E) \mapsto E' = \mathfrak{a} \star E$$

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Exponent group $\mathcal{G} = \mathbb{Z}/|G|\mathbb{Z}$

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(Scalar) Multiplication

$[a^e]E = \mathfrak{g}^{a^e} \star E$
Exponentiation

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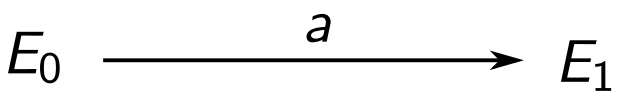
$[ab]E = \mathfrak{g}^{ab} \star E$
(Scalar) Multiplication

$[a^e]E = \mathfrak{g}^{a^e} \star E$
Exponentiation

$[f(a)]E = \mathfrak{g}^{f(a)} \star E$
Polynomial Evaluation

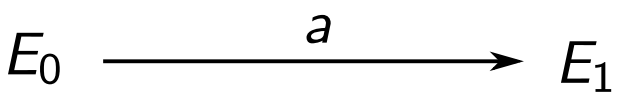
ID protocol



$\mathcal{P}$ $\mathcal{V}$

$$\mathcal{L} = \left\{ ((E_0, E_1), a) : E_1 = [a]E_0 \right\}$$

ID protocol



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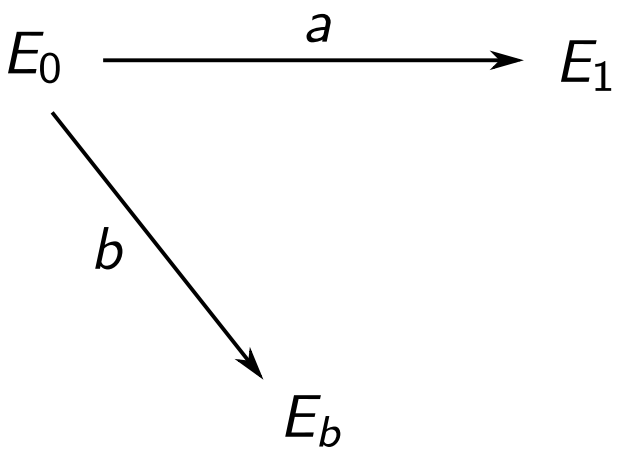

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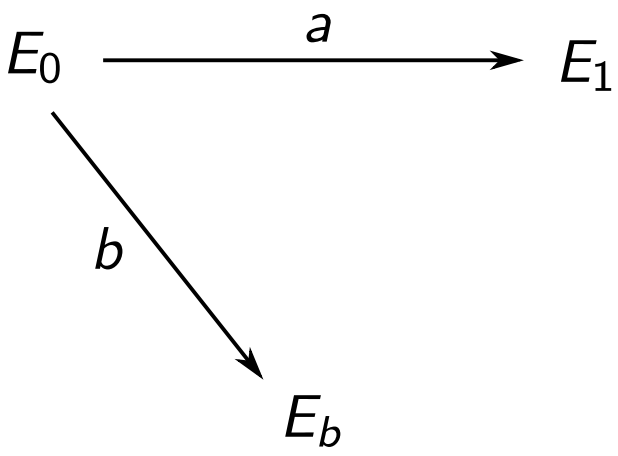

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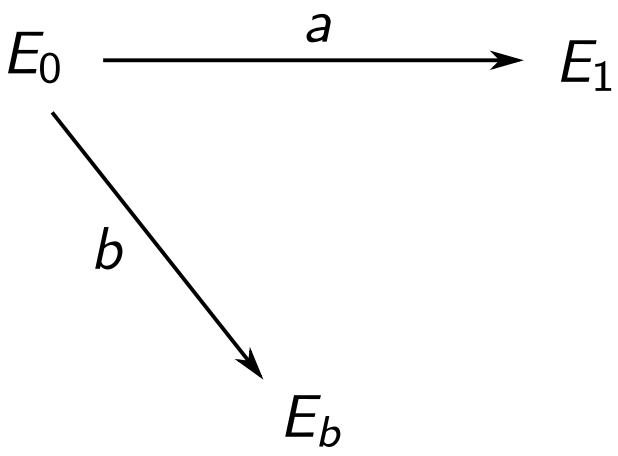

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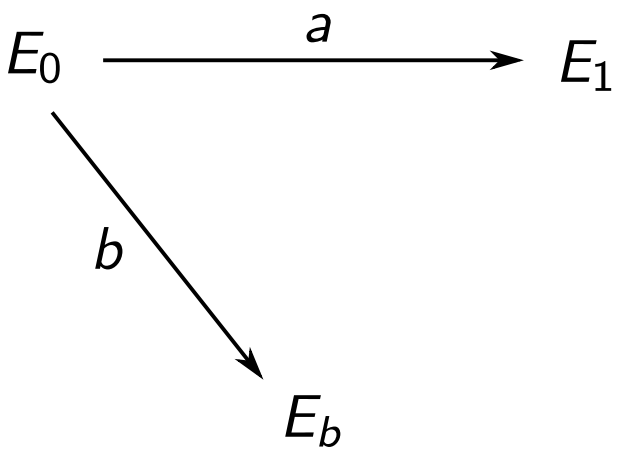

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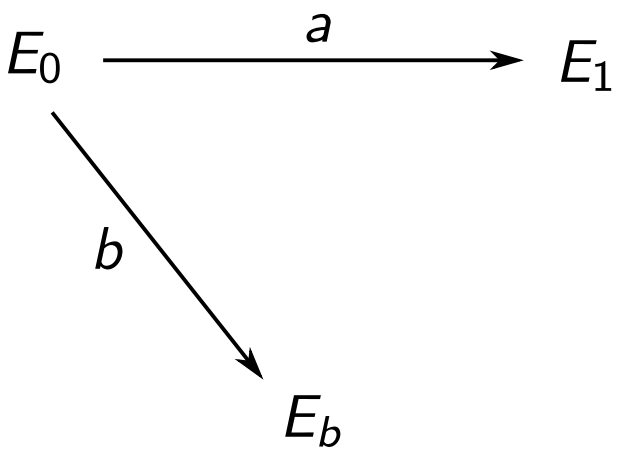

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$r = b - ca$

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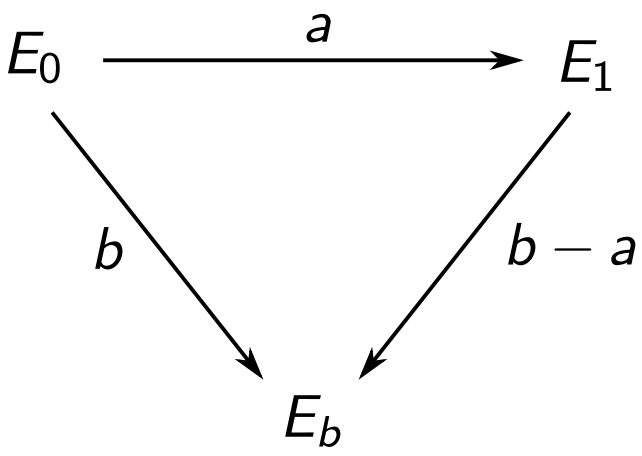

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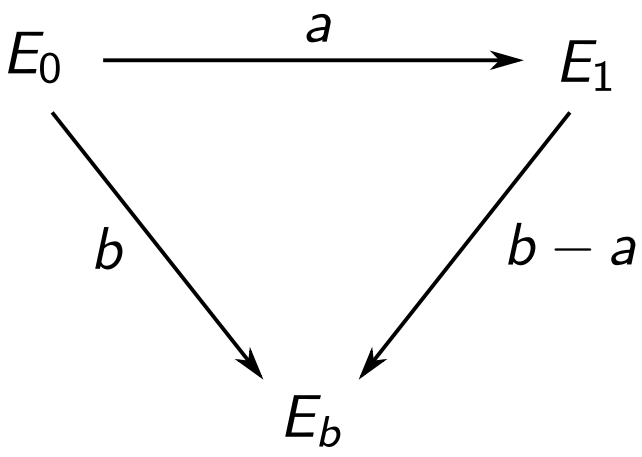

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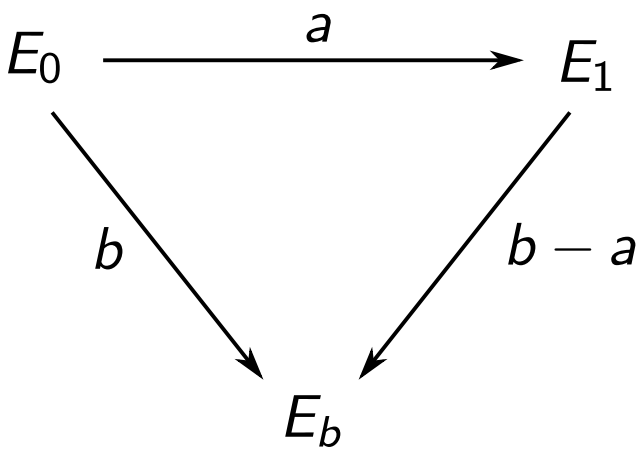

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$c \leftarrow \{0,1\}$

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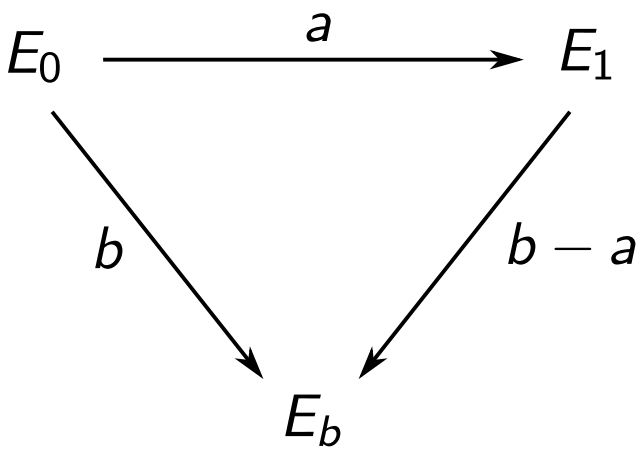
$r = b - ca$

$\xrightarrow{r}$

$[r]E_c \stackrel{?}{=} E_1$

$$\mathcal{L} = \left\{ ((E_0, E_1), a) : E_1 = [a]E_0 \right\}$$

Addition



$\mathcal{P}$ $\mathcal{V}$

$b \leftarrow \mathcal{G}$

$E_b = [b]E_0$

E_b →

$c \leftarrow \{0, 1\}$

← c

$r = b - ca$

r →

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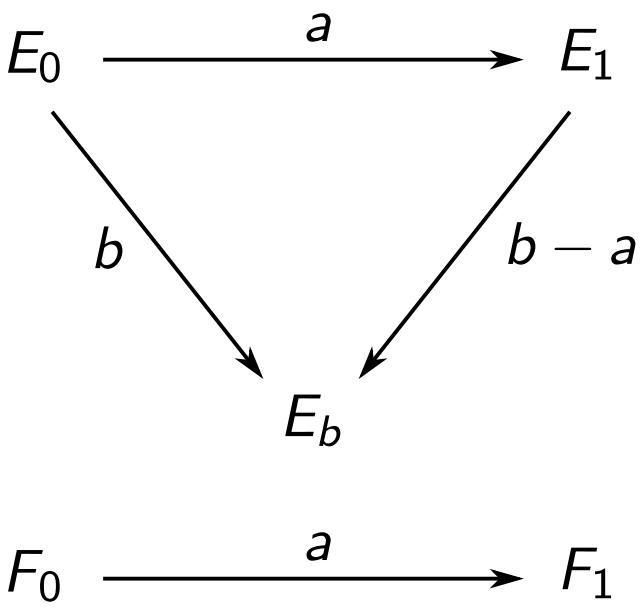
Addition



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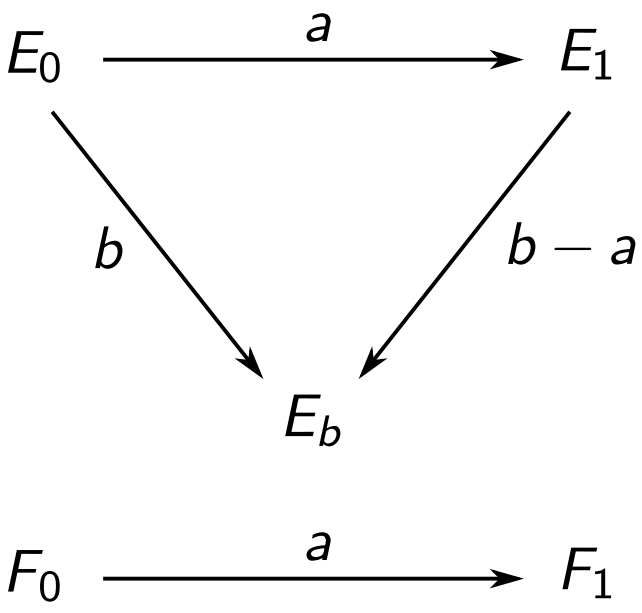

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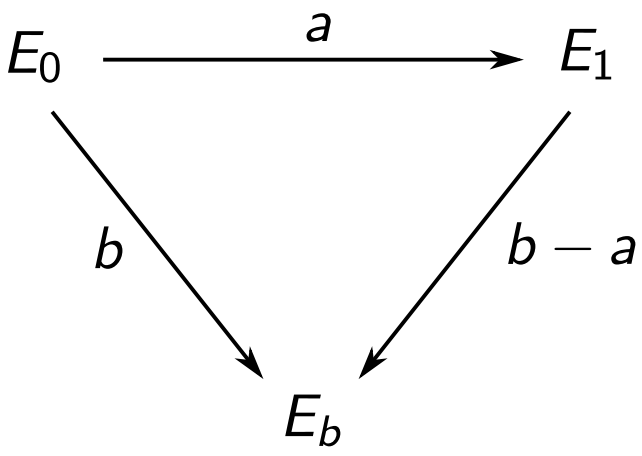
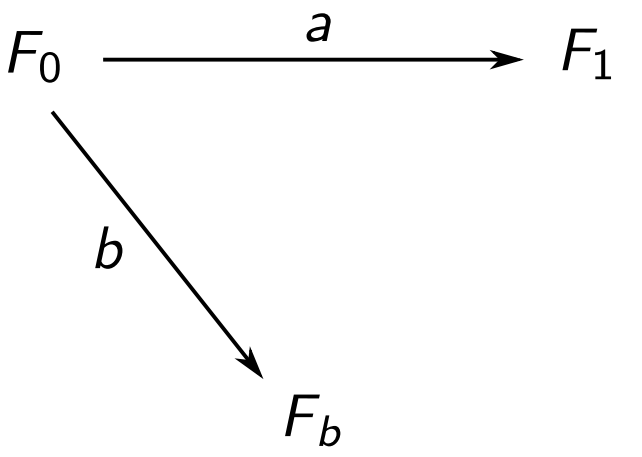
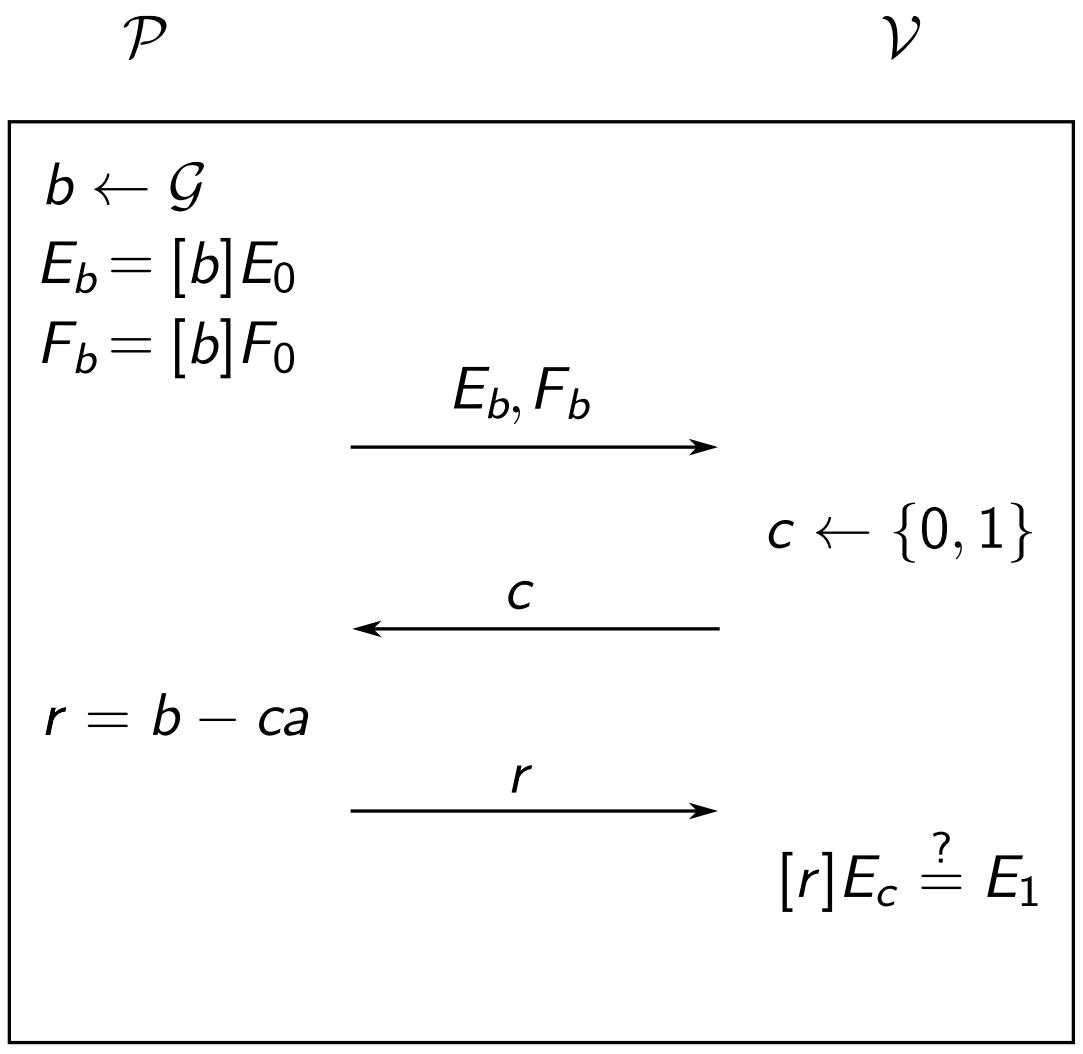
$$\mathcal{L}_2^{\text{Add}} = \left\{ ((E_0, E_1), (F_0, F_1), a) : E_1 = [a]E_0 \wedge F_1 = [a]F_0 \right\}$$

Addition

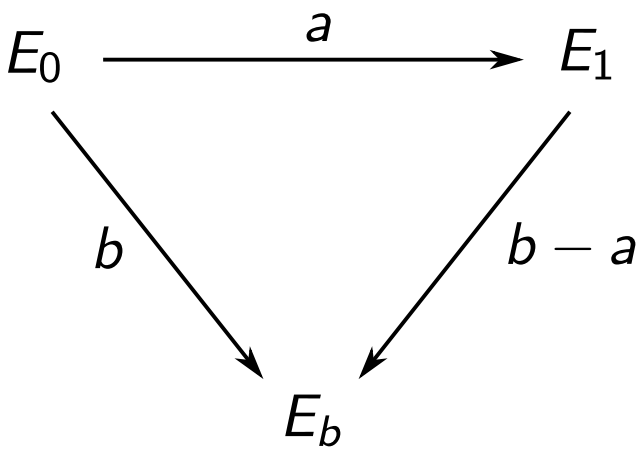


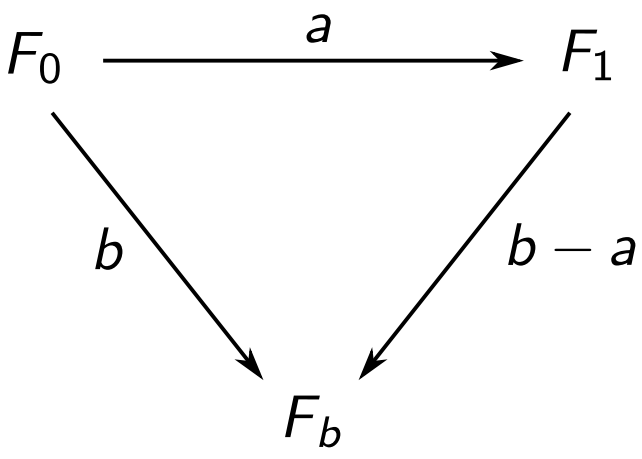


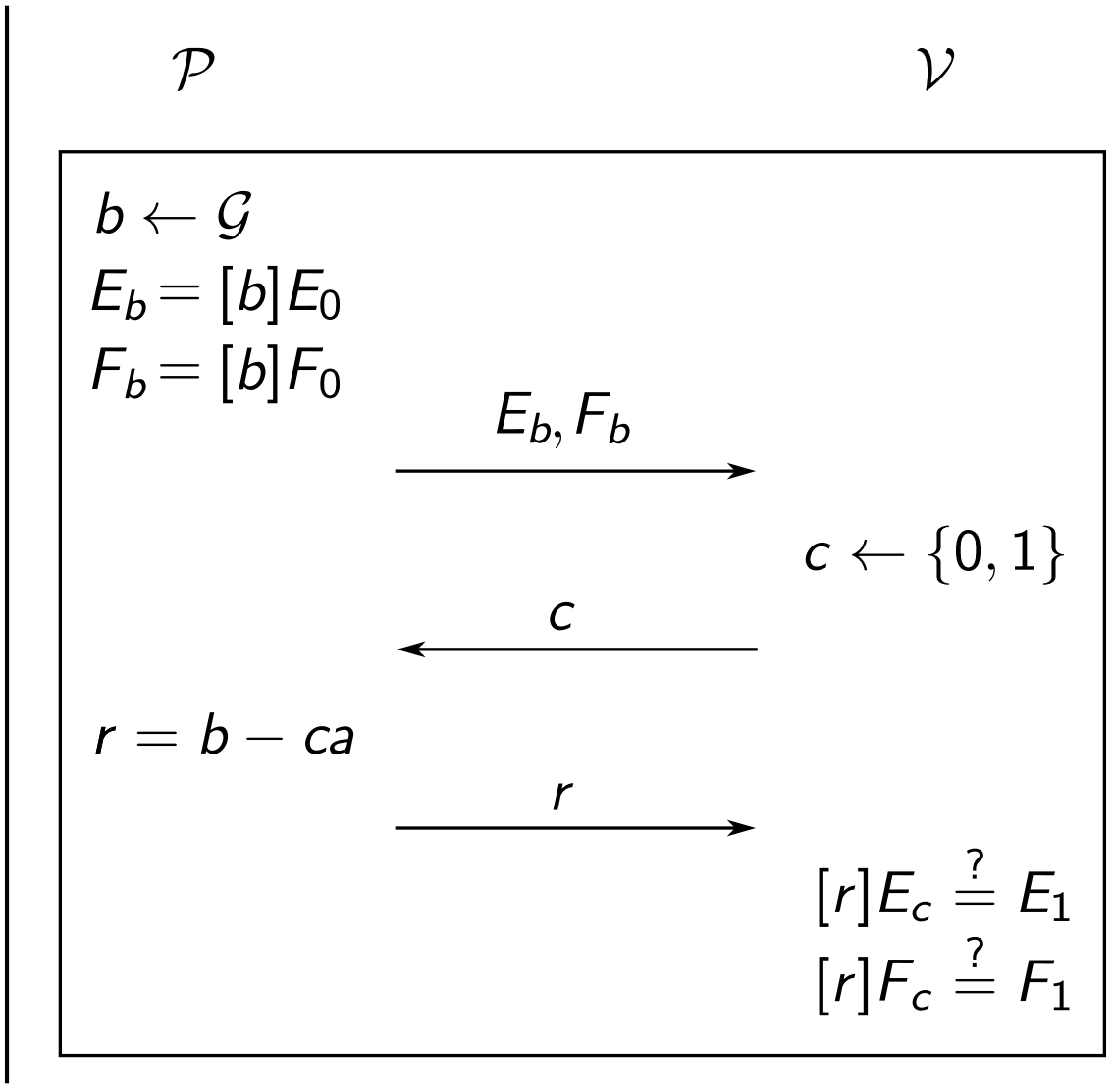


$$\mathcal{L}_2^{\mathsf{Add}} = \left\{ ((E_0, E_1), (F_0, F_1), a) : E_1 = [a]E_0 \wedge F_1 = [a]F_0 \right\}$$

Addition

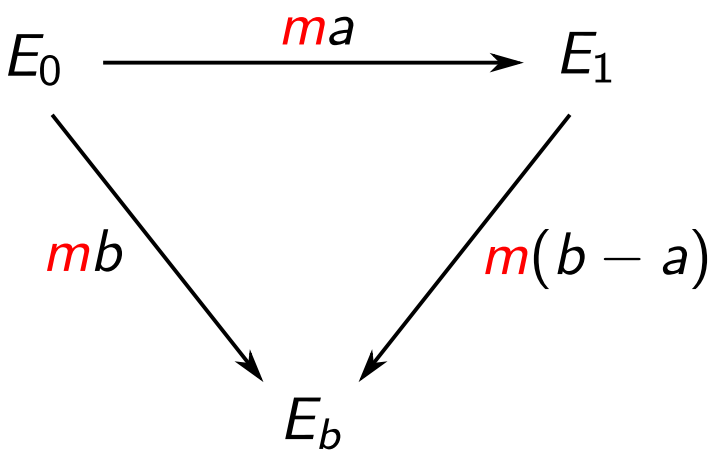


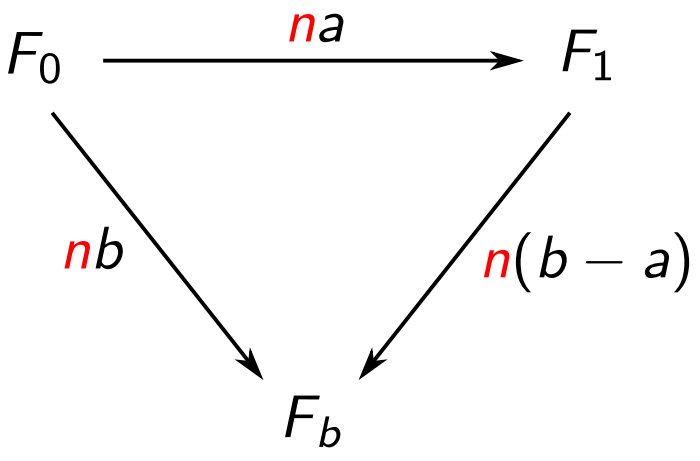


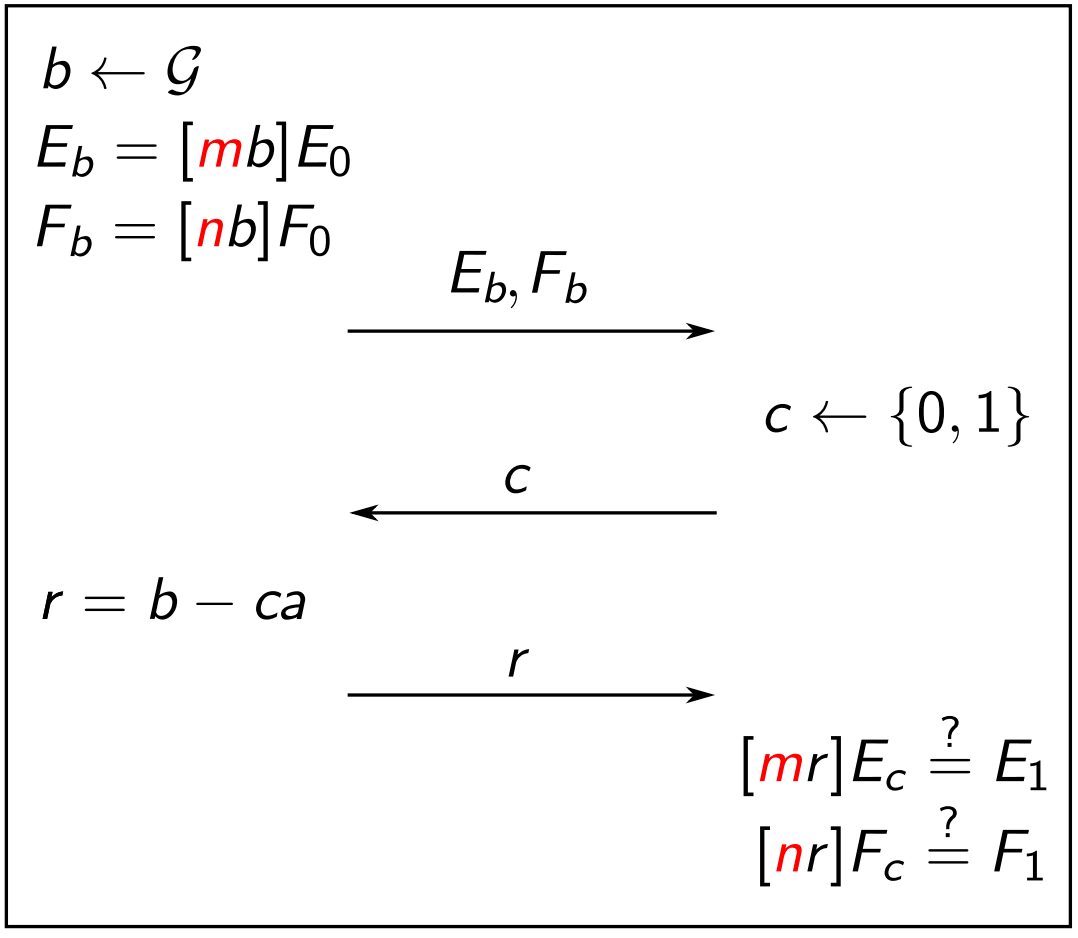


$$\mathcal{L}_2^{\mathsf{Add}} = \left\{ ((E_0, E_1), (F_0, F_1), a) : E_1 = [a]E_0 \wedge F_1 = [a]F_0 \right\}$$

Scalar Multiplication







$$\mathcal{L}_2^{\text{Scal}} = \left\{ ((E_0, E_1, m), (F_0, F_1, n), a) : E_1 = [ma]E_0 \wedge F_1 = [na]F_0 \right\}$$

Languages

$$\mathcal{L}_k^{\mathsf{Add}} = \left\{ \left((E_i, E_i')_{i=1,\dots,k}, a \right) : \bigwedge_{i=1}^{k} E_i' = [a] E_i \right\}$$

$$\mathcal{L}_k^{\mathsf{Scal}} = \left\{ \left((E_i, E_i', c_i)_{i=1,\dots,k}, a \right) : \bigwedge_{i=1}^{k} E_i' = [c_i a] E_i \right\}$$

Languages

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$$\mathcal{L}_k^{\mathsf{Scal}} = \left\{ \left((E_i, E_i', c_i)_{i=1,\dots,k}, a \right) : \bigwedge_{i=1}^{k} E_i' = [c_i a]E_i \right\}$$

$$\mathcal{L}^{\mathsf{Exp}} = \left\{ \left(([a]E, E', e), a \right) : E' = [a^e]E \right\}$$

$$\mathcal{L}^{\mathsf{Mult}} = \left\{ \left(([a]E, [b]E, E'), (a, b) \right) : E' = [ab]E \right\}$$

Languages

$$\mathcal{L}_k^{\mathsf{Add}} = \left\{ \left((E_i, E_i')_{i=1,\ldots,k}, a \right) : \bigwedge_{i=1}^{k} E_i' = [a]E_i \right\}$$

$$\mathcal{L}_k^{\mathsf{Scal}} = \left\{ \left((E_i, E_i', c_i)_{i=1,\ldots,k}, a \right) : \bigwedge_{i=1}^{k} E_i' = [c_i a]E_i \right\}$$

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$$\mathcal{L}^{\mathsf{Mult}} = \left\{ \left(([a]E, [b]E, E'), (a, b) \right) : E' = [ab]E \right\}$$

Addition $\equiv$ Scalar Mult. $\leq$ Exponentiation $\equiv$ Multiplication $\leq$ Key Recovery

A proof for $\mathcal{L}^{\mathsf{Mult}}$

$[ab]E_0$

A proof for $\mathcal{L}^{\mathsf{Mult}}$

$x, y \leftarrow \mathcal{G}$

$\alpha = a - x$

$\beta = b - y$

$[ab]E_0$

A proof for $\mathcal{L}^{\mathsf{Mult}}$

$x, y \leftarrow \mathcal{G}$

$\alpha = a - x$

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$$[ab]E_0 = [(\alpha + x)(\beta + y)]E_0$$

A proof for $\mathcal{L}^{\mathsf{Mult}}$

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$\alpha = a - x$

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$$[ab]E_0 = [(\alpha + x)(\beta + y)]E_0 = [\alpha\beta][\alpha y][\beta x][xy]E_0$$

A proof for $\mathcal{L}^{\mathsf{Mult}}$

$x, y \leftarrow \mathcal{G}$

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public

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public

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$$\pi_1 : \Big((E_0, [y]E_0, 1), (F, [\alpha y]F, \alpha), y\Big) \in \mathcal{L}_2^{\mathsf{Scal}}$$

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$$\pi_1 : \Big((E_0, [y]E_0, 1), (F, [\alpha y]F, \alpha), y\Big) \in \mathcal{L}_2^{\mathsf{Scal}}$$

$$\pi_2 : \Big((E_0, [x]E_0, 1), (F', [\beta x]F', \beta), x\Big) \in \mathcal{L}_2^{\mathsf{Scal}}$$

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Trusted Setup $\mathcal{T} : (x, y, M_x = [x]E_0, M_y = [y]E_0, M_z = [xy]E_0)$

A proof for $\mathcal{L}^{\mathsf{Mult}}$

$x, y \leftarrow \mathcal{G}$

$\alpha = a - x$

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public

$$[ab]E_0 = [(\alpha + x)(\beta + y)]E_0 = [\alpha\beta][\alpha y][\beta x]M_z$$

$$\pi_1 : \Big((E_0,\ M_y\ ,1), (F, [\alpha y]F, \alpha), y\Big) \in \mathcal{L}_2^{\mathsf{Scal}}$$

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$(\pi_1, \pi_2, \alpha, \beta)$

Trusted Setup $\mathcal{T} : (x, y, M_x = [x]E_0, M_y = [y]E_0, M_z = [xy]E_0)$

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$\beta = b - y$

public

$$[ab]E_0 = [(\alpha + x)(\beta + y)]E_0 = [\alpha\beta][\alpha y][\beta x]M_z$$

$$\pi_1 : \Big((E_0,\ M_y\ ,1), (F, [\alpha y]F, \alpha), y\Big) \in \mathcal{L}_2^{\mathsf{Scal}}$$

$$\pi_2 : \Big((E_0,\ M_x\ ,1), (F', [\beta x]F', \beta), x\Big) \in \mathcal{L}_2^{\mathsf{Scal}}$$

$(\pi_1, \pi_2, \alpha, \beta)$

$$[\alpha]M_x \stackrel{?}{=} E_a \wedge [\beta]M_y \stackrel{?}{=} E_b \wedge \mathsf{Verify}(\pi_1) \wedge \mathsf{Verify}(\pi_2)$$

Trusted Setup $\mathcal{T} : (x, y, M_x = [x]E_0, M_y = [y]E_0, M_z = [xy]E_0)$

A proof for $\mathcal{L}^{\mathsf{Mult}}$ using MPC-in-the-head

$$\mathcal{T} : \Big((x_i, y_i, z_i)_{i \in \{1,\ldots,n\}}, M_x, M_y \Big)$$

A proof for $\mathcal{L}^{\text{Mult}}$ using MPC-in-the-head

$$\mathcal{T} : \Big((x_i, y_i, z_i)_{i \in \{1,\ldots,n\}}, M_x, M_y \Big)$$

$$\sum_{i=1}^{n} x_i = x, \quad \sum_{i=1}^{n} y_i = y, \quad \sum_{i=1}^{n} z_i = xy$$

$$M_x = [x]E_0, \quad M_y = [y]E_0$$

A proof for $\mathcal{L}^{\text{Mult}}$ using MPC-in-the-head

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$$\mathcal{T} : \Big((x_i, y_i, z_i)_{i\in\{1,\ldots,n\}}, M_x, M_y\Big)$$

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$$M_x = [x]E_0, \quad M_y = [y]E_0$$

$\alpha = a - x$ $\beta = b - y$

$\mathcal{P}_1$:

$\mathcal{P}_2$:

$\mathcal{P}_3$:

..

$\mathcal{P}_{n-1}$:

$\mathcal{P}_n$:

A proof for $\mathcal{L}^{\text{Mult}}$ using MPC-in-the-head

$$\mathcal{T} : \Big((x_i, y_i, z_i)_{i\in\{1,\ldots,n\}}, M_x, M_y\Big)$$

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$\alpha = a - x$ $\quad \beta = b - y$

$\mathcal{P}_1 : (\alpha, \beta, x_1, y_1, z_1)$

$\mathcal{P}_2 :$

$\mathcal{P}_3 :$

$..$

$\mathcal{P}_{n-1} :$

$\mathcal{P}_n :$

A proof for $\mathcal{L}^{\text{Mult}}$ using MPC-in-the-head

$$\mathcal{T} : \Big((x_i, y_i, z_i)_{i\in\{1,\ldots,n\}}, M_x, M_y\Big)$$

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$\mathcal{P}_2 : (\alpha, \beta, x_2, y_2, z_2)$

$\mathcal{P}_3 :$

$..$

$\mathcal{P}_{n-1} :$

$\mathcal{P}_n :$

A proof for $\mathcal{L}^{\text{Mult}}$ using MPC-in-the-head

$$\mathcal{T} : \Big((x_i, y_i, z_i)_{i \in \{1,\ldots,n\}}, M_x, M_y \Big)$$

$$\sum_{i=1}^{n} x_i = x, \quad \sum_{i=1}^{n} y_i = y, \quad \sum_{i=1}^{n} z_i = xy$$

$$M_x = [x]E_0, \quad M_y = [y]E_0$$

$\alpha = a - x$ $\quad \beta = b - y$

$\mathcal{P}_1 : (\alpha, \beta, x_1, y_1, z_1)$

$\mathcal{P}_2 : (\alpha, \beta, x_2, y_2, z_2)$

$\mathcal{P}_3 : (\alpha, \beta, x_3, y_3, z_3)$

$\ldots$

$\mathcal{P}_{n-1} :$

$\mathcal{P}_n :$

A proof for $\mathcal{L}^{\text{Mult}}$ using MPC-in-the-head

$$\mathcal{T} : \Big((x_i, y_i, z_i)_{i \in \{1,\ldots,n\}}, M_x, M_y \Big)$$

$$\sum_{i=1}^{n} x_i = x, \quad \sum_{i=1}^{n} y_i = y, \quad \sum_{i=1}^{n} z_i = xy$$

$$M_x = [x]E_0, \quad M_y = [y]E_0$$

$\alpha = a - x$ $\quad \beta = b - y$

$\mathcal{P}_1 : (\alpha, \beta, x_1, y_1, z_1)$

$\mathcal{P}_2 : (\alpha, \beta, x_2, y_2, z_2)$

$\mathcal{P}_3 : (\alpha, \beta, x_3, y_3, z_3)$

$\ldots$

$\mathcal{P}_{n-1} : (\alpha, \beta, x_{n-1}, y_{n-1}, z_{n-1})$

$\mathcal{P}_n : (\alpha, \beta, x_n, y_n, z_n)$

A proof for $\mathcal{L}^{\text{Mult}}$ using MPC-in-the-head

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$\alpha = a - x$ $\beta = b - y$ E_0

$\mathcal{P}_1 : (\alpha, \beta, x_1, y_1, z_1)$

$\mathcal{P}_2 : (\alpha, \beta, x_2, y_2, z_2)$

$\mathcal{P}_3 : (\alpha, \beta, x_3, y_3, z_3)$

$\ldots$

$\mathcal{P}_{n-1} : (\alpha, \beta, x_{n-1}, y_{n-1}, z_{n-1})$

$\mathcal{P}_n : (\alpha, \beta, x_n, y_n, z_n)$

A proof for $\mathcal{L}^{\text{Mult}}$ using MPC-in-the-head

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$$M_x = [x]E_0, \quad M_y = [y]E_0$$

$\alpha = a - x$ $\beta = b - y$ E_0

$\mathcal{P}_1 : (\alpha, \beta, x_1, y_1, z_1)$ $E_1 = [\beta x_1 + \alpha y_1 + z_1]E_0$

$\mathcal{P}_2 : (\alpha, \beta, x_2, y_2, z_2)$

$\mathcal{P}_3 : (\alpha, \beta, x_3, y_3, z_3)$

$\ldots$

$\mathcal{P}_{n-1} : (\alpha, \beta, x_{n-1}, y_{n-1}, z_{n-1})$

$\mathcal{P}_n : (\alpha, \beta, x_n, y_n, z_n)$

A proof for $\mathcal{L}^{\text{Mult}}$ using MPC-in-the-head

$$\mathcal{T} : \Big((x_i, y_i, z_i)_{i \in \{1,\dots,n\}}, M_x, M_y \Big)$$

$$\sum_{i=1}^{n} x_i = x, \quad \sum_{i=1}^{n} y_i = y, \quad \sum_{i=1}^{n} z_i = xy$$

$$M_x = [x]E_0, \quad M_y = [y]E_0$$

$\alpha = a - x$ $\quad \beta = b - y$ $\quad E_0$

$\mathcal{P}_1 : (\alpha, \beta, x_1, y_1, z_1)$ $\quad E_1 = [\beta x_1 + \alpha y_1 + z_1]E_0$

$\mathcal{P}_2 : (\alpha, \beta, x_2, y_2, z_2)$ $\quad E_2 = [\beta x_2 + \alpha y_2 + z_2]E_1$

$\mathcal{P}_3 : (\alpha, \beta, x_3, y_3, z_3)$

$\dots$

$\mathcal{P}_{n-1} : (\alpha, \beta, x_{n-1}, y_{n-1}, z_{n-1})$

$\mathcal{P}_n : (\alpha, \beta, x_n, y_n, z_n)$

A proof for $\mathcal{L}^{\text{Mult}}$ using MPC-in-the-head

$$\mathcal{T} : \Big((x_i, y_i, z_i)_{i \in \{1,\dots,n\}}, M_x, M_y \Big)$$

$$\sum_{i=1}^{n} x_i = x, \quad \sum_{i=1}^{n} y_i = y, \quad \sum_{i=1}^{n} z_i = xy$$

$$M_x = [x]E_0, \quad M_y = [y]E_0$$

$\alpha = a - x$ $\quad \beta = b - y$ $\quad E_0$

$\mathcal{P}_1 : (\alpha, \beta, x_1, y_1, z_1)$ $\quad E_1 = [\beta x_1 + \alpha y_1 + z_1]E_0$

$\mathcal{P}_2 : (\alpha, \beta, x_2, y_2, z_2)$ $\quad E_2 = [\beta x_2 + \alpha y_2 + z_2]E_1$

$\mathcal{P}_3 : (\alpha, \beta, x_3, y_3, z_3)$ $\quad E_3 = [\beta x_3 + \alpha y_3 + z_3]E_2$

$\dots$ $\quad \dots$

$\mathcal{P}_{n-1} : (\alpha, \beta, x_{n-1}, y_{n-1}, z_{n-1})$

$\mathcal{P}_n : (\alpha, \beta, x_n, y_n, z_n)$

A proof for $\mathcal{L}^{\text{Mult}}$ using MPC-in-the-head

$$\mathcal{T} : \Big((x_i, y_i, z_i)_{i\in\{1,\ldots,n\}}, M_x, M_y\Big)$$

$$\sum_{i=1}^{n} x_i = x, \quad \sum_{i=1}^{n} y_i = y, \quad \sum_{i=1}^{n} z_i = xy$$

$$M_x = [x]E_0, \quad M_y = [y]E_0$$

$\alpha = a - x$ $\quad \beta = b - y$

E_0

$\mathcal{P}_1 : (\alpha, \beta, x_1, y_1, z_1)$	$E_1 = [\beta x_1 + \alpha y_1 + z_1]E_0$
$\mathcal{P}_2 : (\alpha, \beta, x_2, y_2, z_2)$	$E_2 = [\beta x_2 + \alpha y_2 + z_2]E_1$
$\mathcal{P}_3 : (\alpha, \beta, x_3, y_3, z_3)$	$E_3 = [\beta x_3 + \alpha y_3 + z_3]E_2$
$\ldots$	$\ldots$
$\mathcal{P}_{n-1} : (\alpha, \beta, x_{n-1}, y_{n-1}, z_{n-1})$	$E_{n-1} = [\beta x_{n-1} + \alpha y_{n-1} + z_2]E_{n-2}$
$\mathcal{P}_n : (\alpha, \beta, x_n, y_n, z_n)$	$E_n = [\alpha\beta + \beta x_n + \alpha y_n + z_n]E_{n-1}$

A proof for $\mathcal{L}^{\text{Mult}}$ using MPC-in-the-head

$$\mathcal{T} : \Big((x_i, y_i, z_i)_{i\in\{1,\dots,n\}}, M_x, M_y\Big)$$

$$\sum_{i=1}^{n} x_i = x, \quad \sum_{i=1}^{n} y_i = y, \quad \sum_{i=1}^{n} z_i = xy$$

$$M_x = [x]E_0, \quad M_y = [y]E_0$$

$\alpha = a - x$ $\quad \beta = b - y$

E_0

$\mathcal{P}_1 : (\alpha, \beta, x_1, y_1, z_1)$ $\quad E_1 = [\beta x_1 + \alpha y_1 + z_1]E_0$

$\mathcal{P}_2 : (\alpha, \beta, x_2, y_2, z_2)$ $\quad E_2 = [\beta x_2 + \alpha y_2 + z_2]E_1$

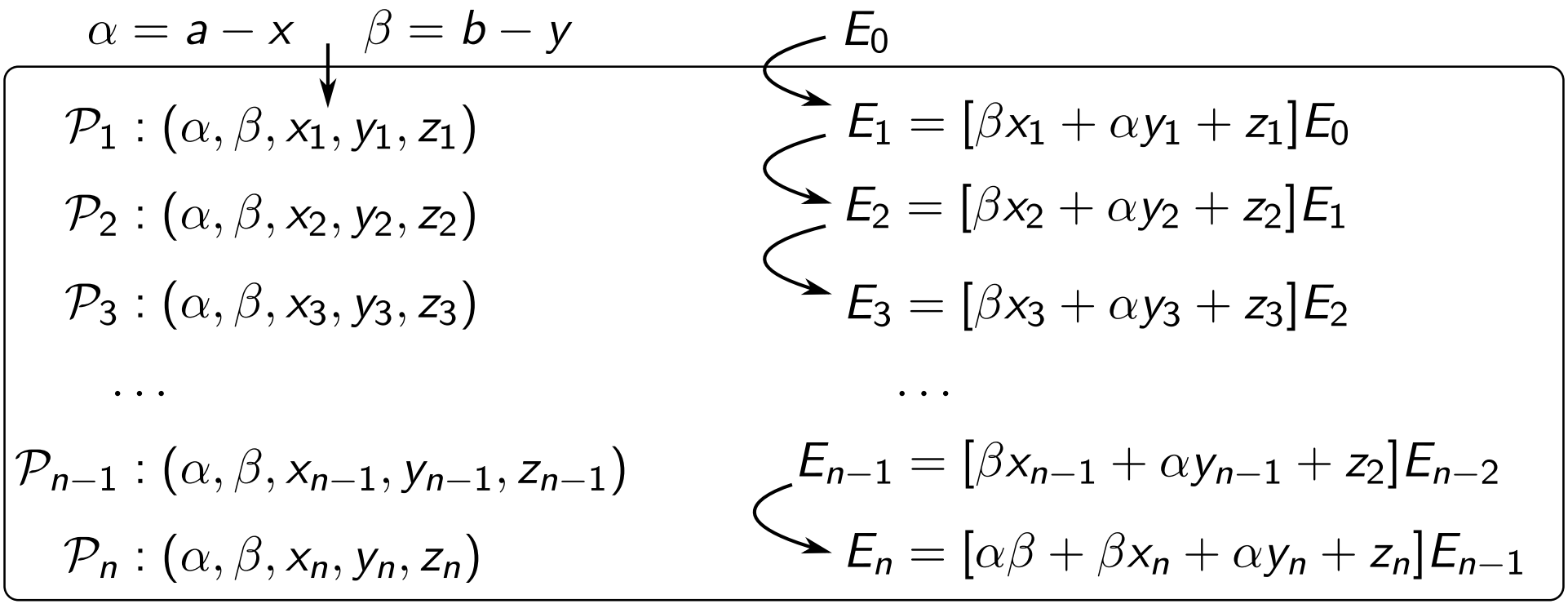
$\mathcal{P}_3 : (\alpha, \beta, x_3, y_3, z_3)$ $\quad E_3 = [\beta x_3 + \alpha y_3 + z_3]E_2$

$\dots$ $\quad \dots$

$\mathcal{P}_{n-1} : (\alpha, \beta, x_{n-1}, y_{n-1}, z_{n-1})$ $\quad E_{n-1} = [\beta x_{n-1} + \alpha y_{n-1} + z_2]E_{n-2}$

$\mathcal{P}_n : (\alpha, \beta, x_n, y_n, z_n)$ $\quad E_n = [\alpha\beta + \beta x_n + \alpha y_n + z_n]E_{n-1}$

A proof for $\mathcal{L}^{\text{Mult}}$ using MPC-in-the-head



A proof for $\mathcal{L}^{\text{Mult}}$ using MPC-in-the-head

$$T_1 : \Big((x_i, y_i, z_i)_{i\in\{1,\dots,n\}}, M_x, M_y\Big)_1 \qquad T_2 : \Big((x_i, y_i, z_i)_{i\in\{1,\dots,n\}}, M_x, M_y\Big)_2$$

$$T_3 : \Big((x_i, y_i, z_i)_{i\in\{1,\dots,n\}}, M_x, M_y\Big)_3 \qquad \dots$$

$$T_{m-1} : \Big((x_i, y_i, z_i)_{i\in\{1,\dots,n\}}, M_x, M_y\Big)_{m-1} \qquad T_m : \Big((x_i, y_i, z_i)_{i\in\{1,\dots,n\}}, M_x, M_y\Big)_m$$

E_0

$\mathcal{P}_1$

$\mathcal{P}_2$:

$\mathcal{P}_3$:

$\dots$

$\mathcal{P}_{n-1}$:

$\mathcal{P}_n$:

A proof for $\mathcal{L}^{\mathsf{Mult}}$ using MPC-in-the-head

$$T_1 : \Big((x_i, y_i, z_i)_{i\in\{1,\ldots,n\}}, M_x, M_y\Big)_1 \qquad T_2 : \Big((x_i, y_i, z_i)_{i\in\{1,\ldots,n\}}, M_x, M_y\Big)_2$$

$$T_3 : \Big((x_i, y_i, z_i)_{i\in\{1,\ldots,n\}}, M_x, M_y\Big)_3 \qquad \ldots$$

$$T_{m-1} : \Big((x_i, y_i, z_i)_{i\in\{1,\ldots,n\}}, M_x, M_y\Big)_{m-1} \qquad T_m : \Big((x_i, y_i, z_i)_{i\in\{1,\ldots,n\}}, M_x, M_y\Big)_m$$

E_0

$\mathcal{P}_1$

$\mathcal{P}_2$:

$\mathcal{P}_3$:

$\ldots$

$\mathcal{P}_{n-1}$:

$\mathcal{P}_n$:

A proof for $\mathcal{L}^{\text{Mult}}$ using MPC-in-the-head

$T_1 : \left((x_i, y_i, z_i)_{i\in\{1,\dots,n\}}, M_x, M_y\right)_1$ $\quad$ $T_2 : \left((x_i, y_i, z_i)_{i\in\{1,\dots,n\}}, M_x, M_y\right)_2$

$T_3 : \left((x_i, y_i, z_i)_{i\in\{1,\dots,n\}}, M_x, M_y\right)_3$ $\quad$ $\dots$

$T_{m-1} : \left((x_i, y_i, z_i)_{i\in\{1,\dots,n\}}, M_x, M_y\right)_{m-1}$ $\quad$ $T_m : \left((x_i, y_i, z_i)_{i\in\{1,\dots,n\}}, M_x, M_y\right)_m$

$\alpha = a - x$ $\quad$ $\beta = b - y$ $\quad$ E_0

$\mathcal{P}_1$:

$\mathcal{P}_2$:

$\mathcal{P}_3$:

$\dots$

$\mathcal{P}_{n-1}$:

$\mathcal{P}_n$:

A proof for $\mathcal{L}^{\text{Mult}}$ using MPC-in-the-head

$T_1 : \left((x_i, y_i, z_i)_{i\in\{1,\dots,n\}}, M_x, M_y\right)_1$ $\quad$ $T_2 : \left((x_i, y_i, z_i)_{i\in\{1,\dots,n\}}, M_x, M_y\right)_2$

$T_3 : \left((x_i, y_i, z_i)_{i\in\{1,\dots,n\}}, M_x, M_y\right)_3$ $\quad$ $\dots$

$T_{m-1} : \left((x_i, y_i, z_i)_{i\in\{1,\dots,n\}}, M_x, M_y\right)_{m-1}$ $\quad$ $T_m : \left((x_i, y_i, z_i)_{i\in\{1,\dots,n\}}, M_x, M_y\right)_m$

$\alpha = a - x$ $\quad$ $\beta = b - y$

$\mathcal{P}_1 : (\alpha, \beta, x_1, y_1, z_1)$ $\quad$ E_0

$\mathcal{P}_2 : (\alpha, \beta, x_2, y_2, z_2)$ $\quad$ $E_1 = [\beta x_1 + \alpha y_1 + z_1]E_0$

$\mathcal{P}_3 : (\alpha, \beta, x_3, y_3, z_3)$ $\quad$ $E_2 = [\beta x_2 + \alpha y_2 + z_2]E_1$

$\dots$ $\quad$ $E_3 = [\beta x_3 + \alpha y_3 + z_3]E_2$

$\dots$

$\mathcal{P}_{n-1} : (\alpha, \beta, x_{n-1}, y_{n-1}, z_{n-1})$ $\quad$ $E_{n-1} = [\beta x_{n-1} + \alpha y_{n-1} + z_2]E_{n-2}$

$\mathcal{P}_n : (\alpha, \beta, x_n, y_n, z_n)$ $\quad$ $E_n = [\alpha\beta + \beta x_n + \alpha y_n + z_n]E_{n-1}$

A proof for $\mathcal{L}^{\text{Mult}}$ using MPC-in-the-head

$T_1 : \left((x_i, y_i, z_i)_{i\in\{1,\dots,n\}}, M_x, M_y\right)_1$ $\quad$ $T_2 : \left((x_i, y_i, z_i)_{i\in\{1,\dots,n\}}, M_x, M_y\right)_2$

$T_3 : \left((x_i, y_i, z_i)_{i\in\{1,\dots,n\}}, M_x, M_y\right)_3$ $\quad$ $\dots$

$T_{m-1} : \left((x_i, y_i, z_i)_{i\in\{1,\dots,n\}}, M_x, M_y\right)_{m-1}$ $\quad$ $T_m : \left((x_i, y_i, z_i)_{i\in\{1,\dots,n\}}, M_x, M_y\right)_m$

$\alpha = a - x$ $\quad$ $\beta = b - y$

$\mathcal{P}_1 : (\alpha, \beta, x_1, y_1, z_1)$ $\quad$ E_0

$\mathcal{P}_2 : (\alpha, \beta, x_2, y_2, z_2)$ $\quad$ $E_1 = [\beta x_1 + \alpha y_1 + z_1]E_0$

$\mathcal{P}_3 : (\alpha, \beta, x_3, y_3, z_3)$ $\quad$ $E_2 = [\beta x_2 + \alpha y_2 + z_2]E_1$

$\dots$ $\quad$ $E_3 = [\beta x_3 + \alpha y_3 + z_3]E_2$

$\mathcal{P}_{n-1} : (\alpha, \beta, x_{n-1}, y_{n-1}, z_{n-1})$ $\quad$ $\dots$

$\mathcal{P}_n : (\alpha, \beta, x_n, y_n, z_n)$ $\quad$ $E_{n-1} = [\beta x_{n-1} + \alpha y_{n-1} + z_2]E_{n-2}$

$E_n = [\alpha\beta + \beta x_n + \alpha y_n + z_n]E_{n-1}$

Costs

	Prover cost	Verifier cost	Trusted Party	Statement type

ZK : standard zero-knowledge proof

ITH : MPC-in-the-Head proof

$\mathcal{T}$: Trusted Setup , *cnc* : Cut-and-Choose

Costs

	Prover cost	Verifier cost	Trusted Party	Statement type
Scal.ZK$_{k=1}$	λ	λ	No	$(E, [s]E)$

ZK : standard zero-knowledge proof

ITH : MPC-in-the-Head proof

$\mathcal{T}$: Trusted Setup , *cnc* : Cut-and-Choose

Costs

	Prover cost	Verifier cost	Trusted Party	Statement type
Scal.ZK$_{k=1}$	λ	λ	No	$(E, [s]E)$
Scal.ZK$_{k=2}$	2λ	2λ	No	$(E, [a]E, [b]E, [a+b]E)$

ZK : standard zero-knowledge proof

ITH : MPC-in-the-Head proof

$\mathcal{T}$: Trusted Setup , *cnc* : Cut-and-Choose

Costs

	Prover cost	Verifier cost	Trusted Party	Statement type
Scal.ZK$_{k=1}$	λ	λ	No	$(E, [s]E)$
Scal.ZK$_{k=2}$	2λ	2λ	No	$(E, [a]E, [b]E, [a+b]E)$
				$(E, [a]E, [b]E, [ab]E)$

ZK : standard zero-knowledge proof

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$\mathcal{T}$: Trusted Setup , *cnc* : Cut-and-Choose

Costs

	Prover cost	Verifier cost	Trusted Party	Statement type
Scal.ZK$_{k=1}$	λ	λ	No	$(E, [s]E)$
Scal.ZK$_{k=2}$	2λ	2λ	No	$(E, [a]E, [b]E, [a+b]E)$
Mult.ZK	4λ	4λ	Yes	$(E, [a]E, [b]E, [ab]E)$
Exp.ZK	$\lfloor \log_2 e \rfloor 6\lambda$	$\lfloor \log_2 e \rfloor 6\lambda$	Yes	$(e, [a]E, [a^e]E)$

ZK : standard zero-knowledge proof

ITH : MPC-in-the-Head proof

$\mathcal{T}$: Trusted Setup , *cnc* : Cut-and-Choose

Costs

	Prover cost	Verifier cost	Trusted Party	Statement type
Scal.ZK$_{k=1}$	λ	λ	No	$(E, [s]E)$
Scal.ZK$_{k=2}$	2λ	2λ	No	$(E, [a]E, [b]E, [a+b]E)$
Mult.ZK	4λ	4λ	Yes	
MultITH$_{\mathcal{T}}$	1.89λ	2.52λ	Yes	$(E, [a]E, [b]E, [ab]E)$
Exp.ZK	$\lfloor \log_2 e \rfloor 6\lambda$	$\lfloor \log_2 e \rfloor 6\lambda$	Yes	
ExpITH$_{\mathcal{T}}$	1.89λ	1.89λ	Yes	$(e, [a]E, [a^e]E)$

ZK : standard zero-knowledge proof

ITH : MPC-in-the-Head proof

$\mathcal{T}$: Trusted Setup , *cnc* : Cut-and-Choose

Costs

	Prover cost	Verifier cost	Trusted Party	Statement type
Scal.ZK$_{k=1}$	λ	λ	No	$(E, [s]E)$
Scal.ZK$_{k=2}$	2λ	2λ	No	$(E, [a]E, [b]E, [a+b]E)$
Mult.ZK	4λ	4λ	Yes	$(E, [a]E, [b]E, [ab]E)$
MultITH$_{\mathcal{T}}$	1.89λ	2.52λ	Yes	
MultITH$_{\text{cnc}}$	4.96λ	4.25λ	No	
Exp.ZK	$\lfloor \log_2 e \rfloor 6\lambda$	$\lfloor \log_2 e \rfloor 6\lambda$	Yes	$(e, [a]E, [a^e]E)$
ExpITH$_{\mathcal{T}}$	1.89λ	1.89λ	Yes	
ExpITH$_{cnc}$	3.52λ	3.52λ	No	

ZK : standard zero-knowledge proof

ITH : MPC-in-the-Head proof

$\mathcal{T}$: Trusted Setup , *cnc* : Cut-and-Choose

Applications: (Verifiable) oblivious pseudo-random function

$\mathcal{S}$

key $k = (f_0, f_1, \ldots, f_d) \in \mathcal{G}^{d+1}$

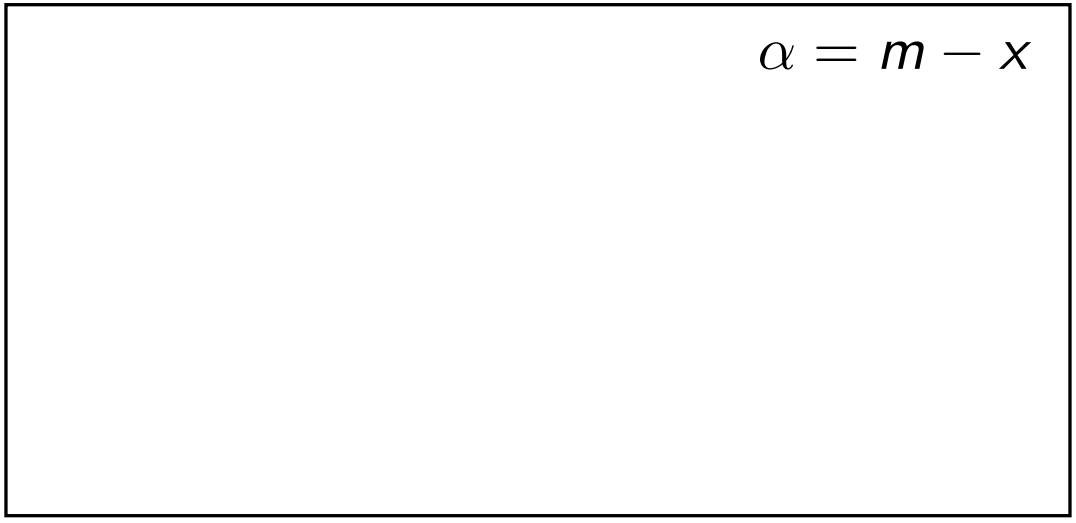
$\mathcal{C}$

input $m \in \mathcal{G}$

Applications: (Verifiable) oblivious pseudo-random function

$\mathcal{S}$ $\qquad\qquad$ $\mathcal{C}$

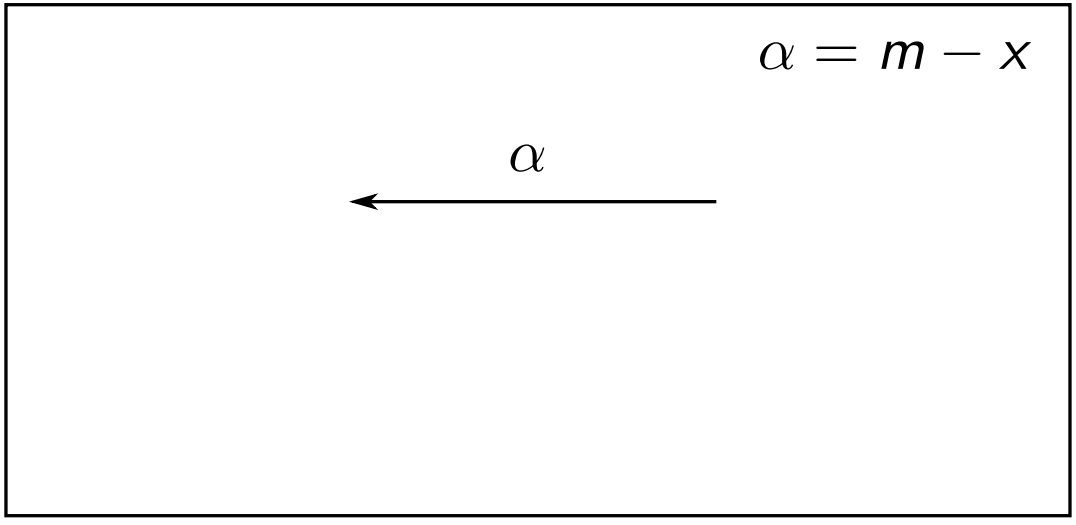
key $k = (f_0, f_1, \ldots, f_d) \in \mathcal{G}^{d+1}$ $\qquad$ input $m \in \mathcal{G}$



Applications: (Verifiable) oblivious pseudo-random function

$\mathcal{S}$ $\qquad\qquad$ $\mathcal{C}$

key $k = (f_0, f_1, \ldots, f_d) \in \mathcal{G}^{d+1}$ $\qquad$ input $m \in \mathcal{G}$



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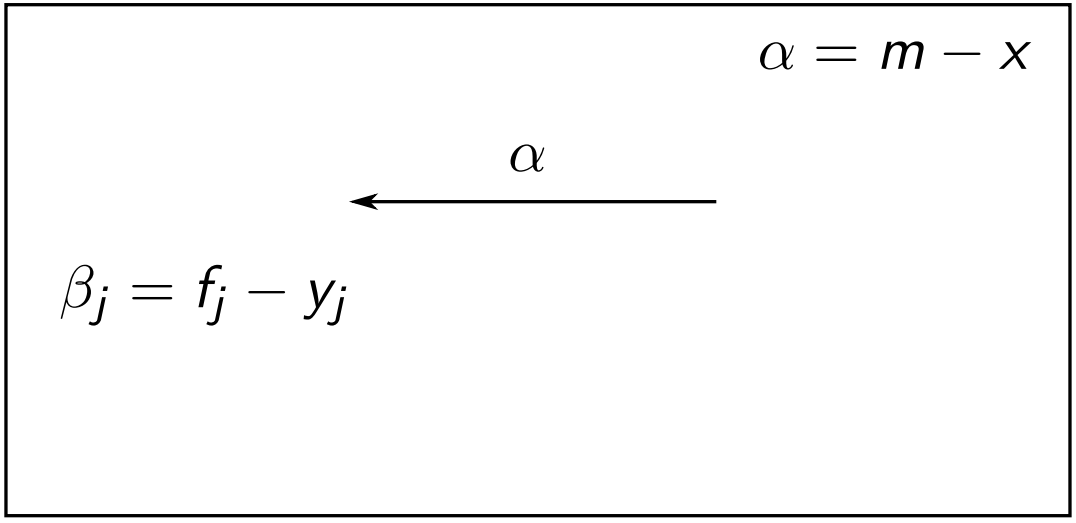

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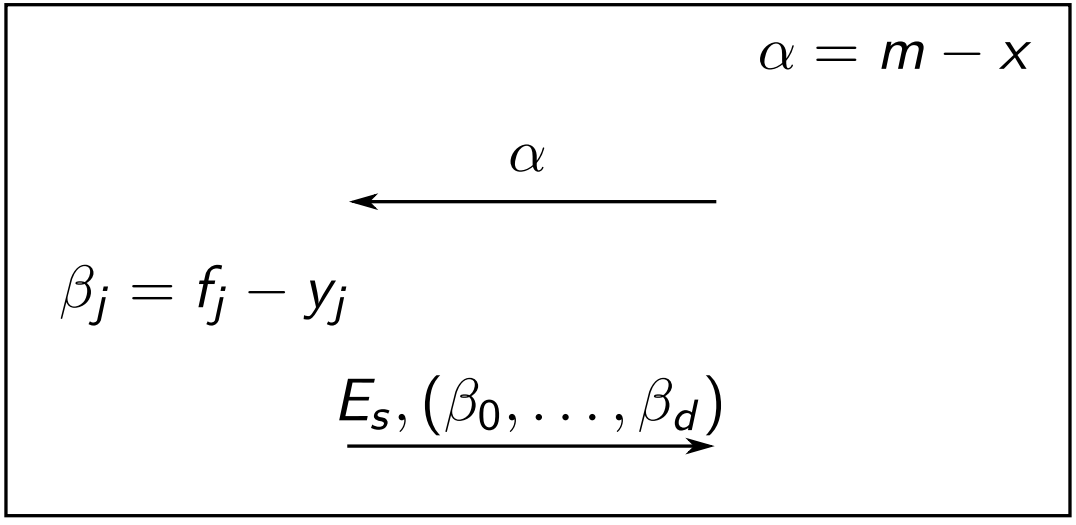

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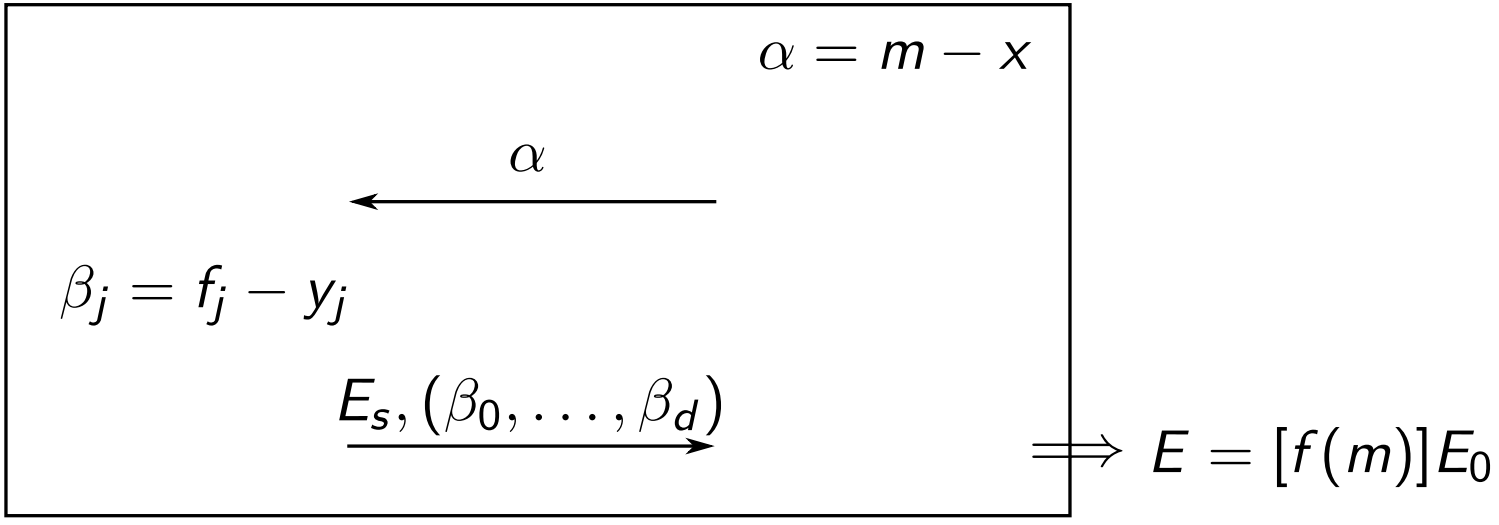

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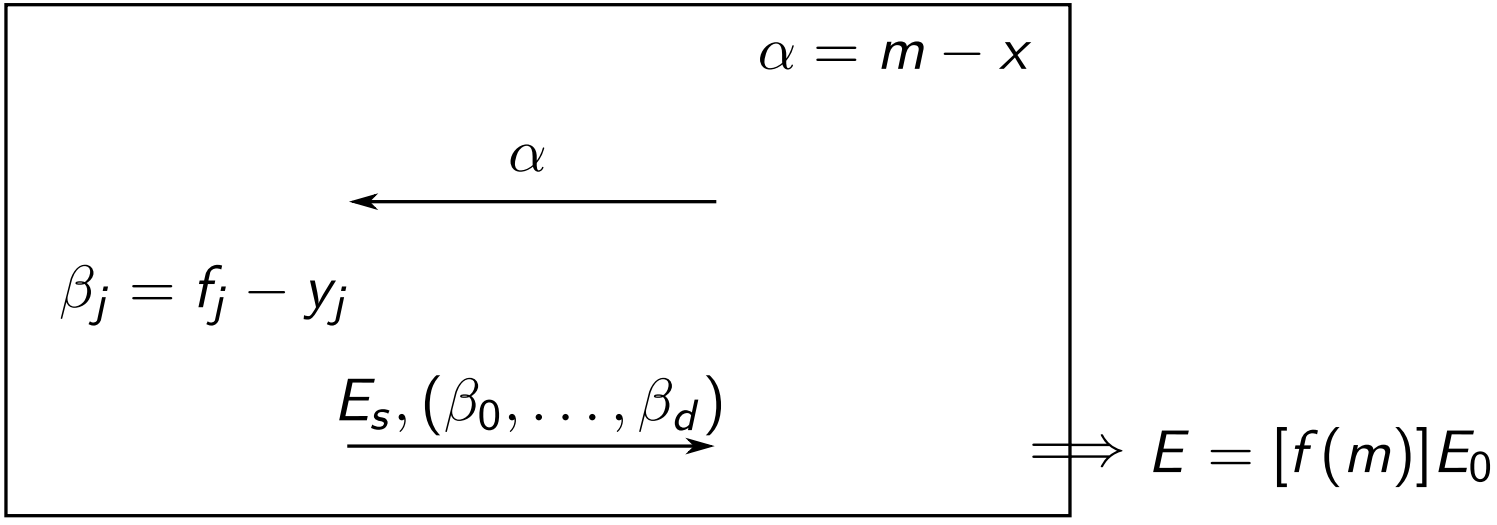

- $\deg f \geq 2$

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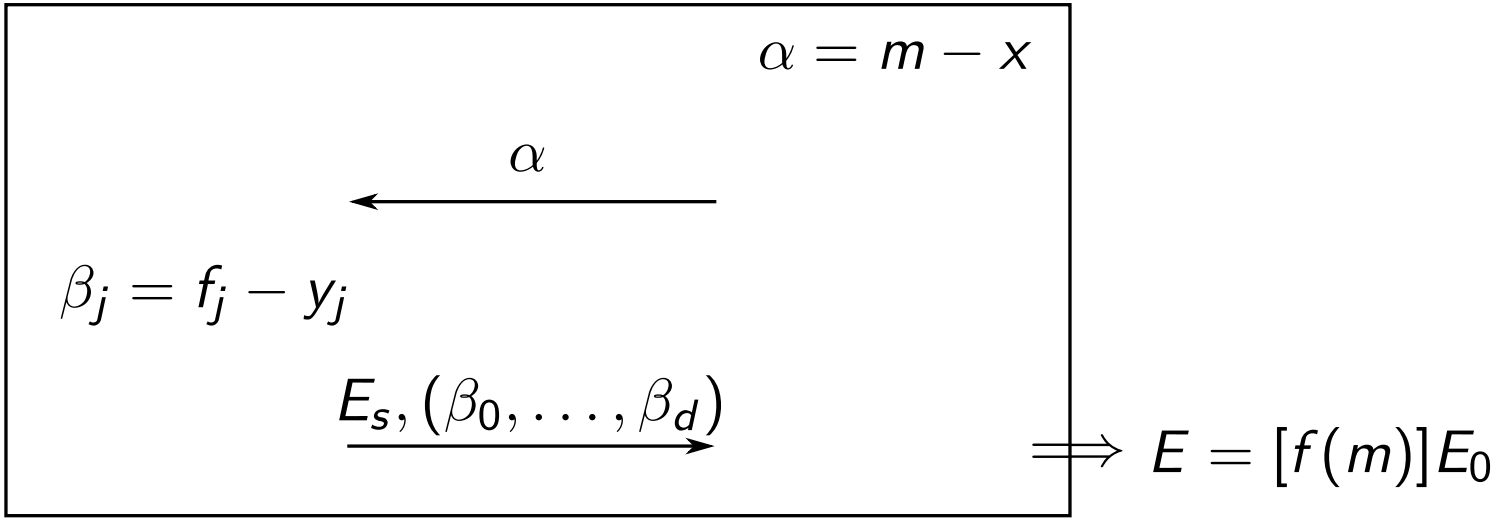

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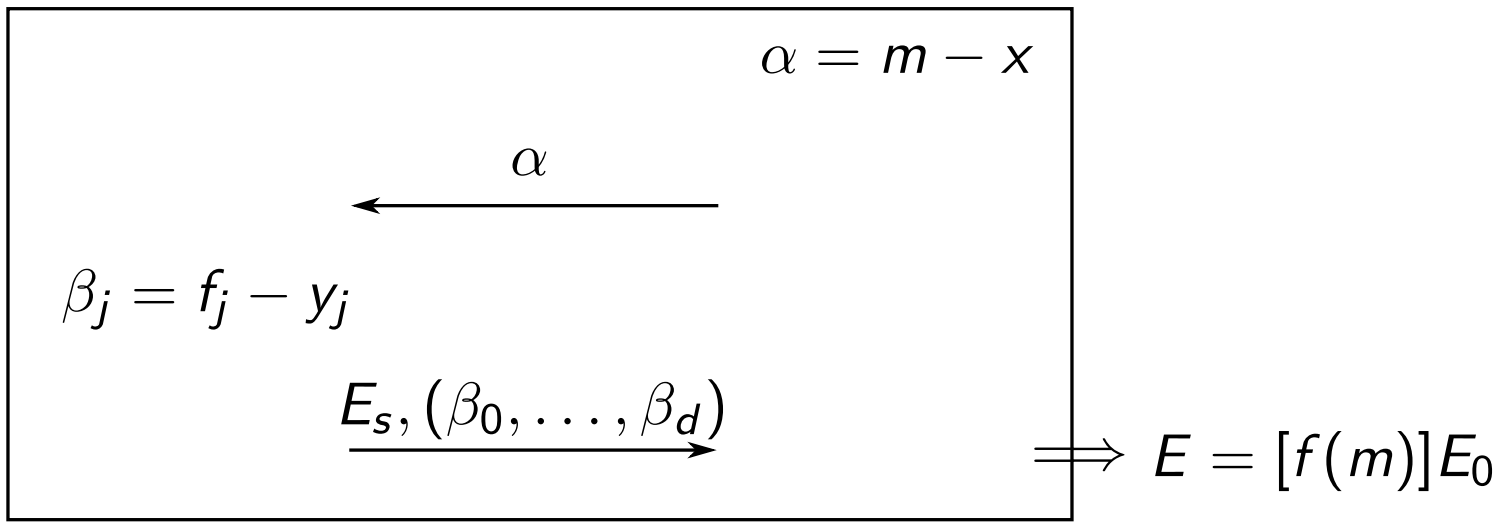

- $\deg f \geq 2$
- f has uniform output
- f is collision-resistant
- Given oracle access to $[f(m)]E_0 \leftarrow \mathcal{O}(m)$, find $(m^*, [f(m^*)]E_0)$ with m^* not yet queried.

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Source	Malicious Client	Verifiable	Number of Rounds	Approx. Computational cost	Approx. Communication cost	No Trusted Setup	Without Class Group

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[1]	X	X	$2\lambda + 2$	3λ	$3\lambda \log p$	✓	✓
[2]	X	X	2	5λ	$2\lambda(\log p + \lambda)$	X	X

[1]: Lena Heimberger, Tobias Hennerbichler, Fredrik Meisingseth, Sebastian Ramacher, Christian Rechberger: *OPRFs from Isogenies: Designs and Analysis*. Cryptology ePrint Archive, Paper 2023/369 (2023).

[2]: Dan Boneh, Dmitry Kogan, Katharine Woo: *Oblivious Pseudorandom Functions from Isogenies*. In Moriai, S., Wang, H. (eds.) ASIACRYPT 2020, Part II. LNCS, vol. 12492, pp. 520-550. Springer, Heidelberg (Dec 2020).

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$d = 2$	✓	X	2	2	$6 \log p$	X*	X
$d = 3$	✓	X	2	2	$9 \log p$	X*	X

*: Trusted setup can be removed for one-time cost of 5λ.

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$d = 3$	✓	X	2	2	$9 \log p$	X*	X
$d = 2$	✓	✓	2	8λ	$2\lambda \log p$	X	X
$d = 3$	✓	✓	2	14λ	$3.5\lambda \log p$	X	X

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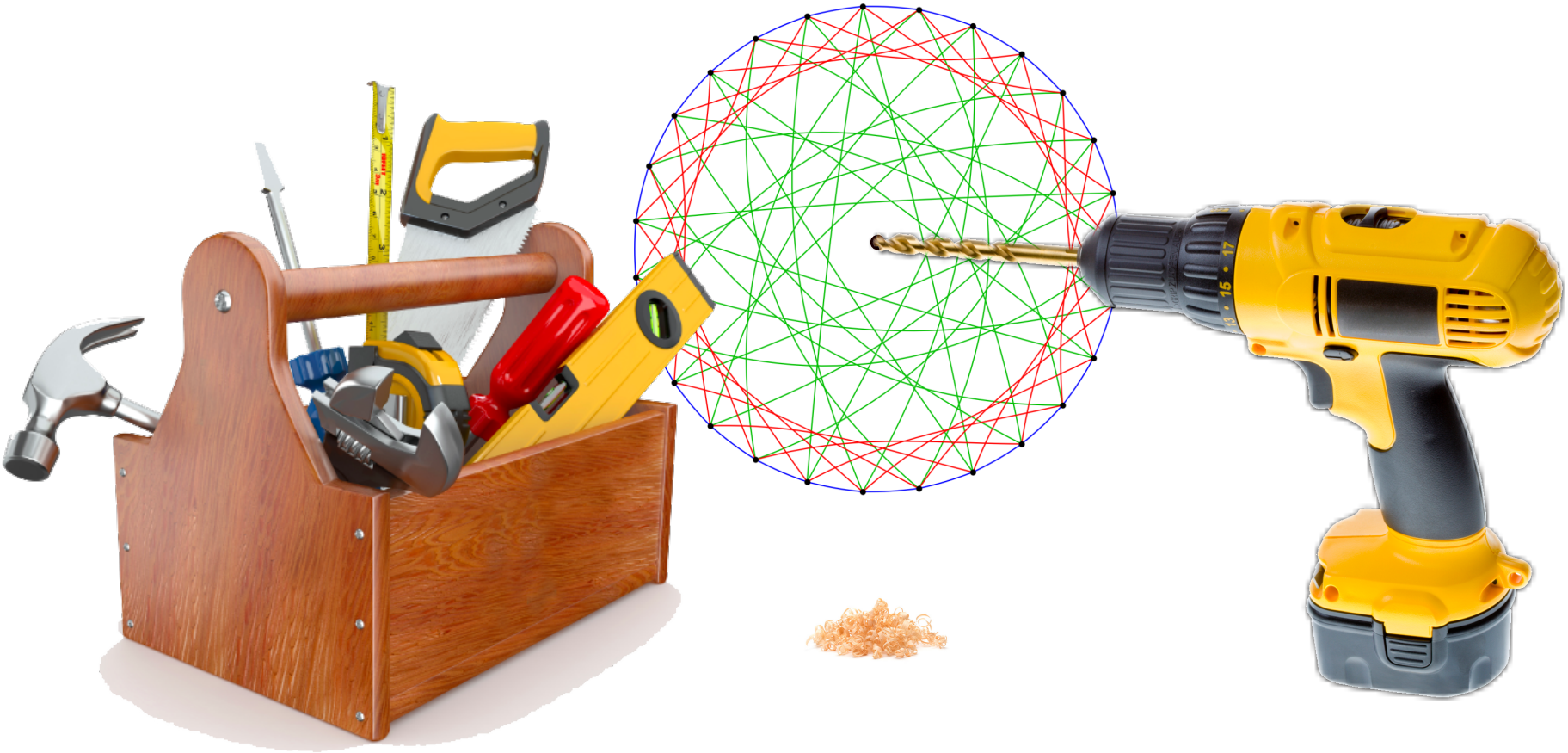
Thank you!



The OPRF

$$B = (x, \{y_j\}^{j\in[d]}, (\widetilde{z_C}, \widetilde{z_S}), \{(z_C^{(j,k)}, z_S^{(j,k)})\}_{k\in[j-1]}^{j\in[d]}),$$

$$z_C^{(j,k)} + z_S^{(j,k)} = y_j x^k \quad \text{and} \quad \widetilde{z_C} + \widetilde{z_S} = \sum_{j=1}^{d} y_j x^j .$$

Client: $\alpha = m - x$ and send it to the server.

Server: For $j = 1, \ldots, d$, compute $\beta_j = f_j - y_j$, then compute

$$E_S = \left[f_0 + \widetilde{z_S} + \sum_{j=1}^{d} f_j \alpha^j + \sum_{j=1}^{d} \sum_{k=1}^{j-1} \binom{j}{k} \alpha^{j-k} z_S^{(j,k)} \right] E_0 .$$

Send $(\beta_1, \ldots, \beta_d)$ and E_S it to the client.

Client: Return

$$\left[\widetilde{z_C} + \sum_{j=1}^{d} \beta_j x^j + \sum_{j=1}^{d} \sum_{k=1}^{j-1} \binom{j}{k} \alpha^{j-k} \left(\beta_j x^k + z_C^{(j,k)} \right) \right] E_S .$$