## What's Wrong with Poly1305?

Improving Poly1305 through a Systematic Exploration of Design Aspects of Polynomial Hash Functions

Jean Paul Degabriele

Jan Gilcher

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RWC 2024







#### Outline





#### 3 Systematic Benchmarking of Design and Implementations Choices



#### $\Delta$ -Universal Hash in Practice

• **Definition:** Given  $z \in \mathcal{T}$  and  $M \neq M' \in \mathcal{M}$ ,

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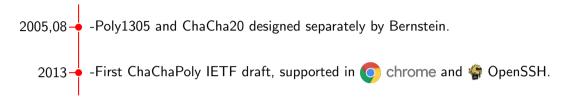
- Various practical applications:
  - Data Structures: hash tables [CW79].
  - Message Authentication Codes: UMAC, Badger, Poly1305-AES, GMAC [ISO/IEC 9797-3].
  - ► AEAD: AES-GCM, ChaCha20-Poly1305 [RFC 8446].

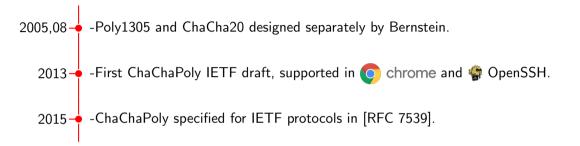
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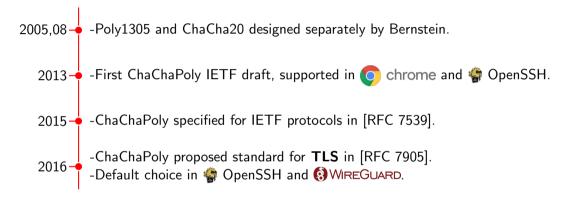
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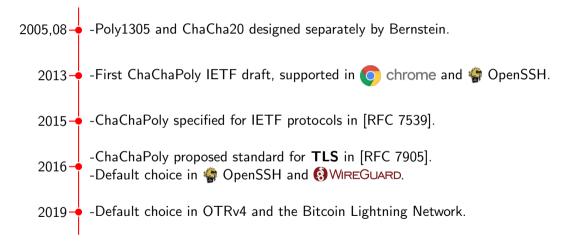
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- Good performance across all architectures without needing specific hardware support.
- Alternative and backup AEAD scheme to AES-GCM.
- Fast adoption even with the predominance of AES-GCM.
- Conservative and simple design, focused on performance with standard AEAD security.

For  $M = M_1 \| \cdots \| M_n$ ,

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- Limited security of ChaChaPoly in the multi-user setting due to Poly1305 [DGGP21].

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# Given today's advancements and applications, would we still converge to this same design?

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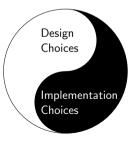
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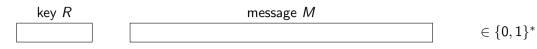
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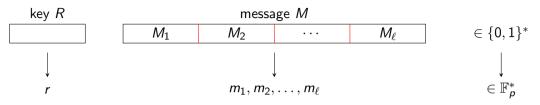
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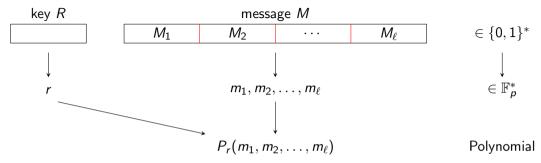
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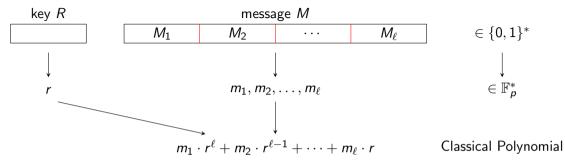
#### Our Exposition [DGGP24]:

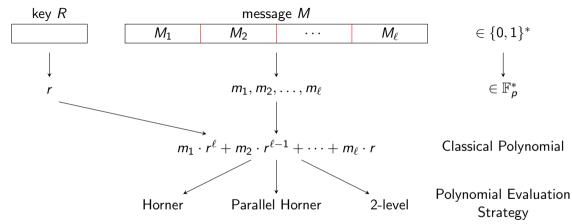


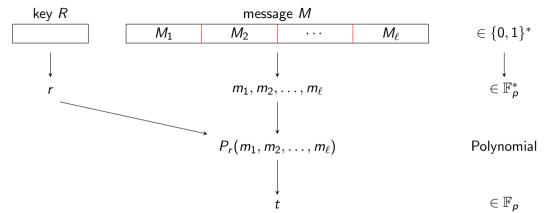


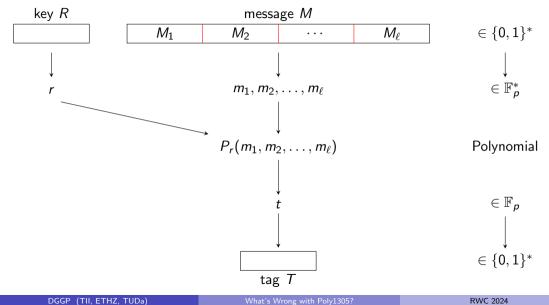








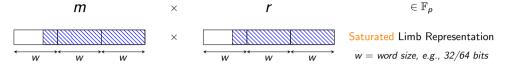


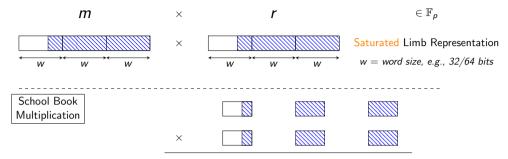


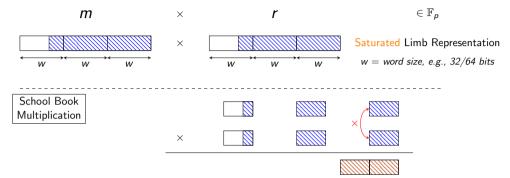
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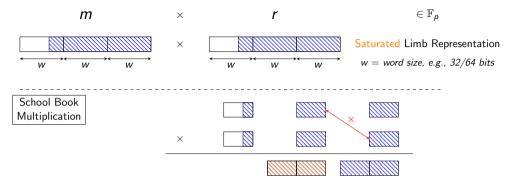
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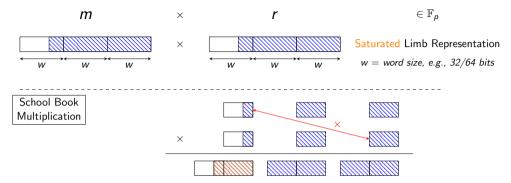
$m$ × $r$ $\in \mathbb{F}_p$
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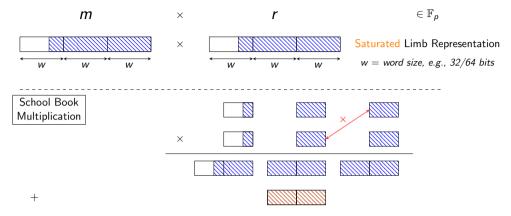


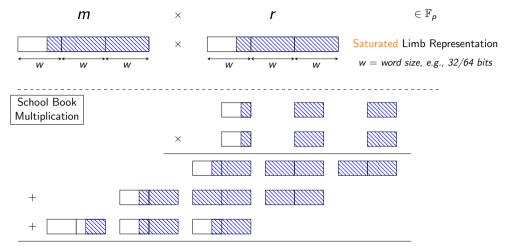


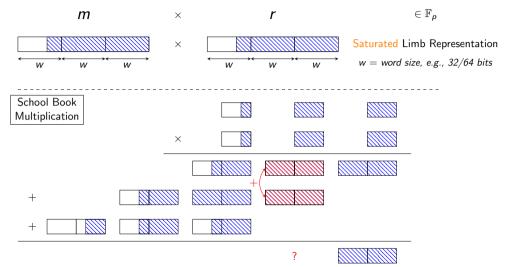




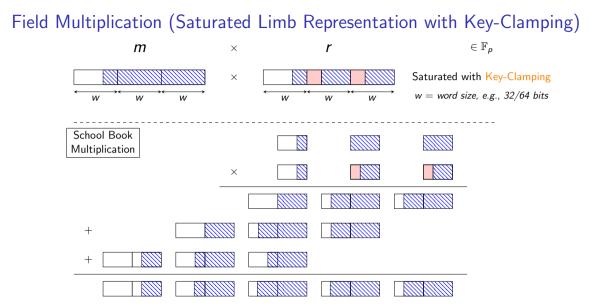








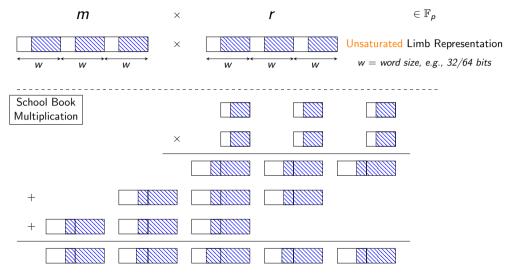
#### Field Multiplication (Saturated Limb Representation with Key-Clamping) $\in \mathbb{F}_{p}$ т Х r Saturated with Key-Clamping Х w = word size, e.g., 32/64 bitsw w w w w w School Book Multiplication × ++



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- We want to be able to understand and test different combinations.
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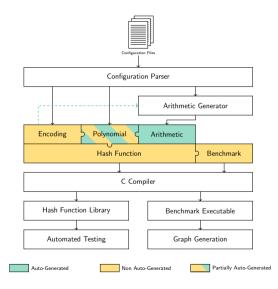
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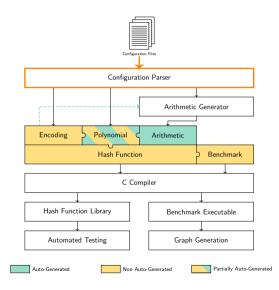
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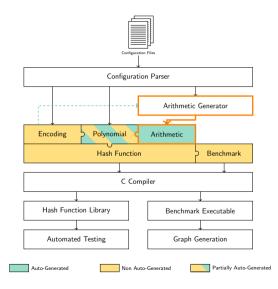
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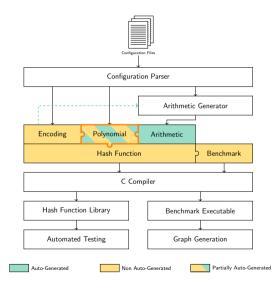
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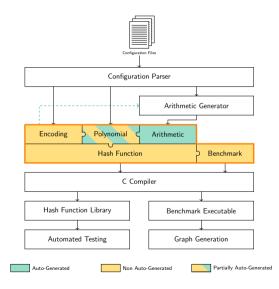
- Modularize!
  - We use our systematization to define modular *configurations*.
- Generic Implementations and Auto-Generation!
  - ▶ Write generic implementations, setting specific parameters at compile time.
  - ► However, fully generic code can lead to bad performance.
  - Where this is likely to occur we automatically generate efficient implementations.

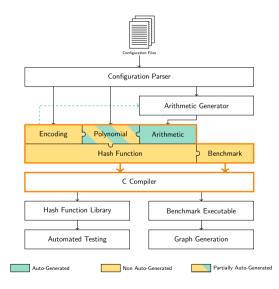


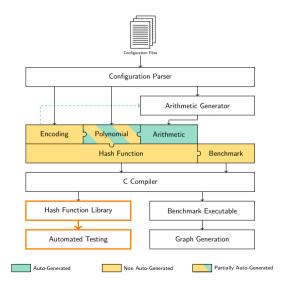


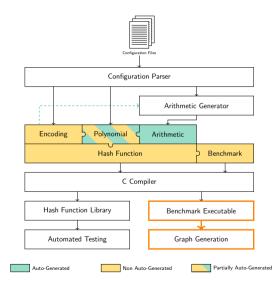












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#### openssl poly1305-x86.pl

[B]esides SSE2 there are floating-point and AVX options; FP is deemed unnecessary, because pre-SSE2 processor are too old to care about, while it's not the fastest option on SSE2-capable ones;

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- Allow various optimization strategies to tune implementations to different hardware.
- But without tailoring the design towards a specific implementation.
  - Don't design for FPUs!

- No clamping to support FPU implementations as these are not worth the security loss.
- Stick with Classical Polynomial over  $\mathbb{F}_p$ . Pack limbs as full as we can.
- Designs allow: Delayed reduction, 2-level polynomial evaluation, exploiting CPU parallelism.

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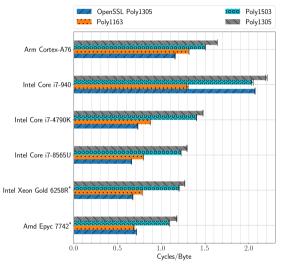
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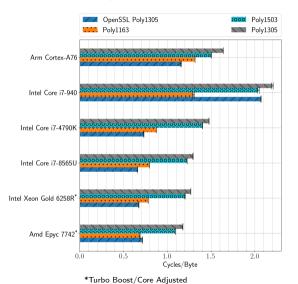
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Resulting Hash function:	Poly1163	Poly1503
DGGP (TII, ETHZ, TUDa)	What's Wrong with Poly13	05? RWC 2024

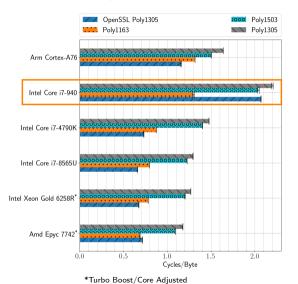


\*Turbo Boost/Core Adjusted



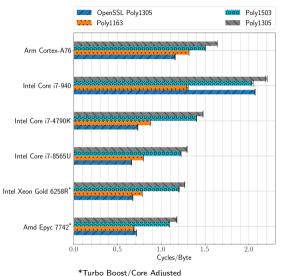
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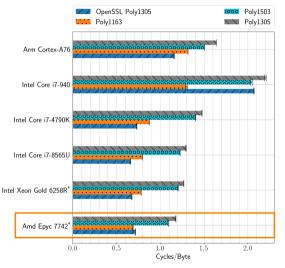
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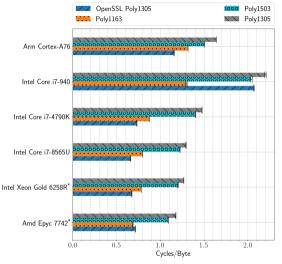
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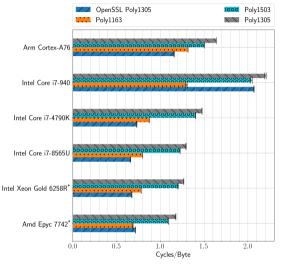
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- Poly1503: Replacement for Poly1305 with 34 bits of extra security  $(103 \rightarrow 137)$  at similar performance.

### Where to Find More Details

SoK on Polynomial Hash:



#### Code of Polynomial Hash Framework:



https://doi.ieeecomputersociety.org/ 10.1109/SP54263.2024.00132 https://github.com/jangilcher/polyno mial\_hashing\_framework

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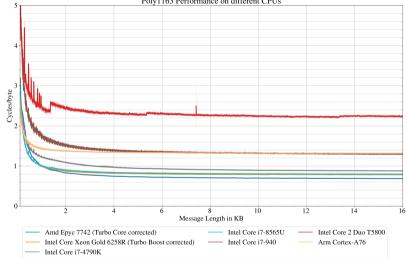
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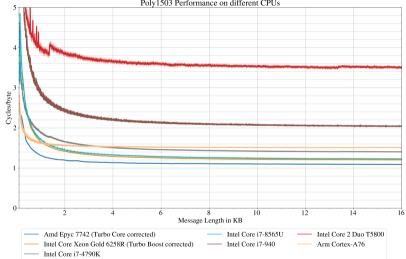
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#### Benchmarks: Poly1163



Poly1163 Performance on different CPUs

#### Benchmarks: Poly1503



Poly1503 Performance on different CPUs