Untangling the Security of Kilian's Protocol: Upper and Lower Bounds

Alessandro Chiesa, Marcel Dall'Agnol, Ziyi Guan, Nick Spooner, Eylon Yogev

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ethereum foundation

Interactive proofs

Interactive proofs

Prover

$P(x, w)$

Interactive proofs

Prover

$P(x, w)$

Verifier

$V(x)$

Interactive proofs

Prover

Is $x \in L$?

Verifier

$P(x, w)$

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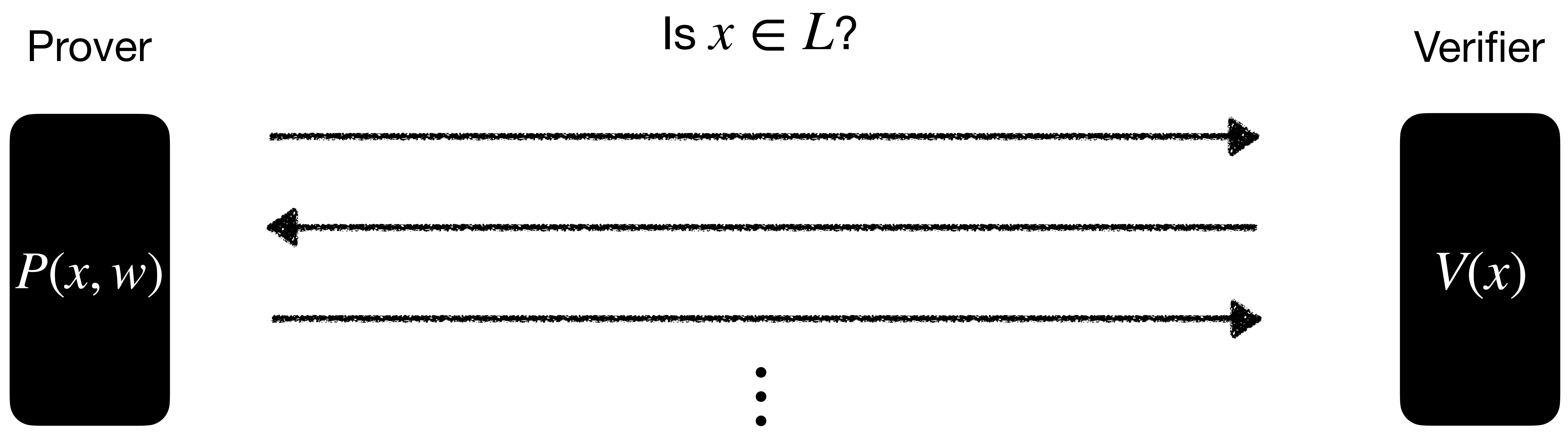
Interactive proofs



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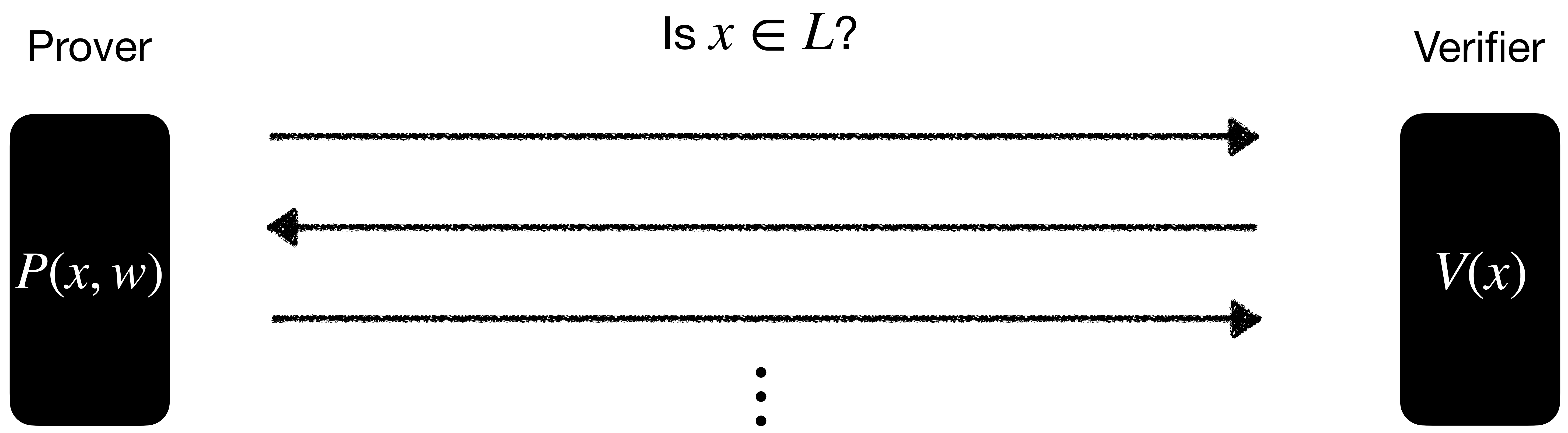

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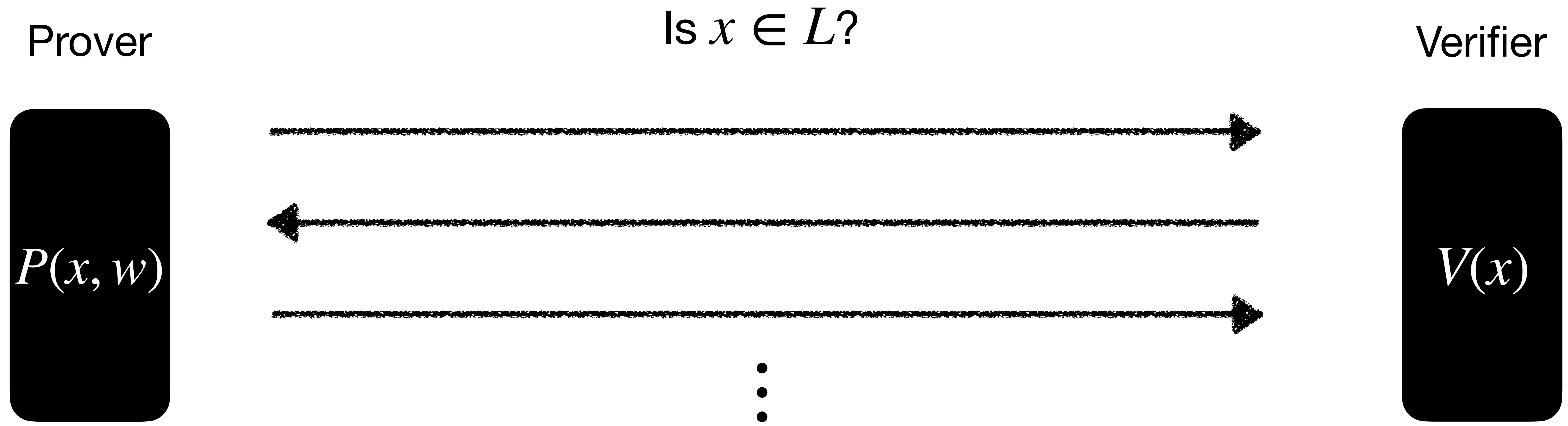

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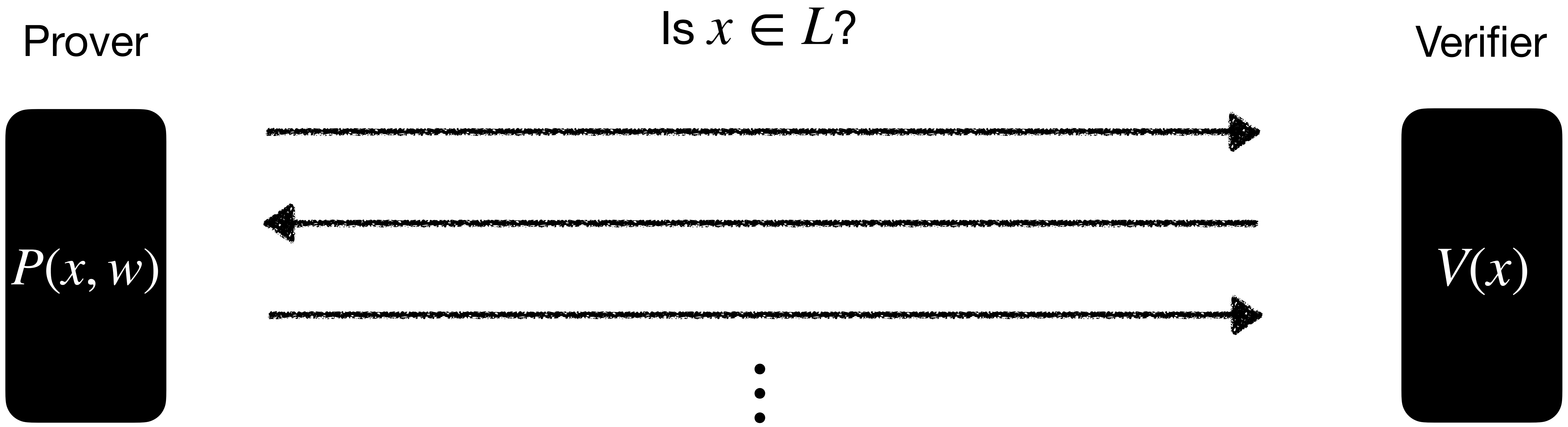

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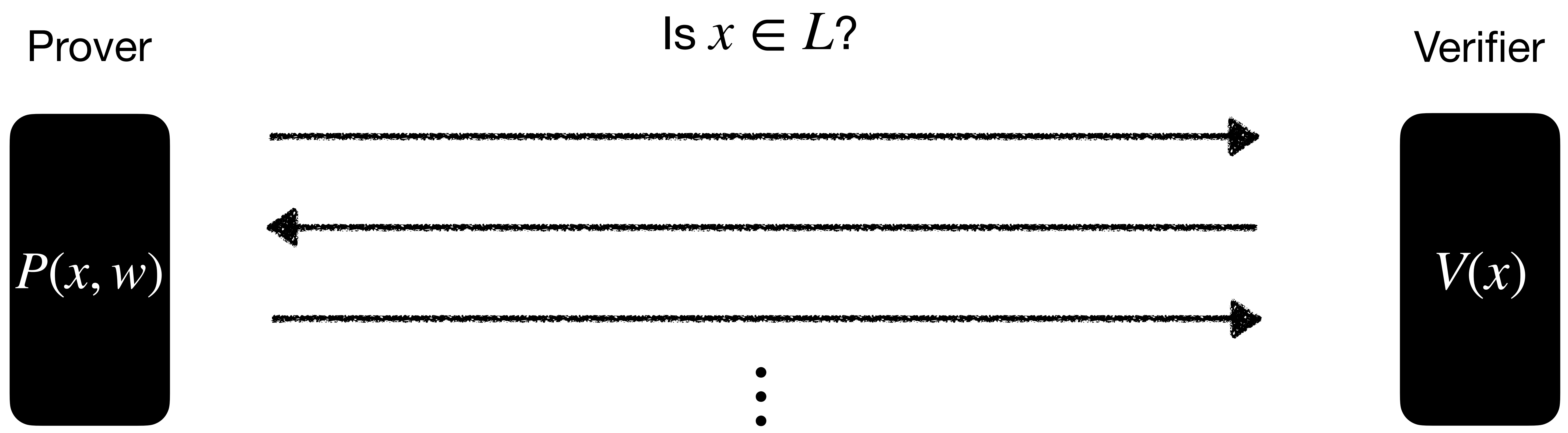

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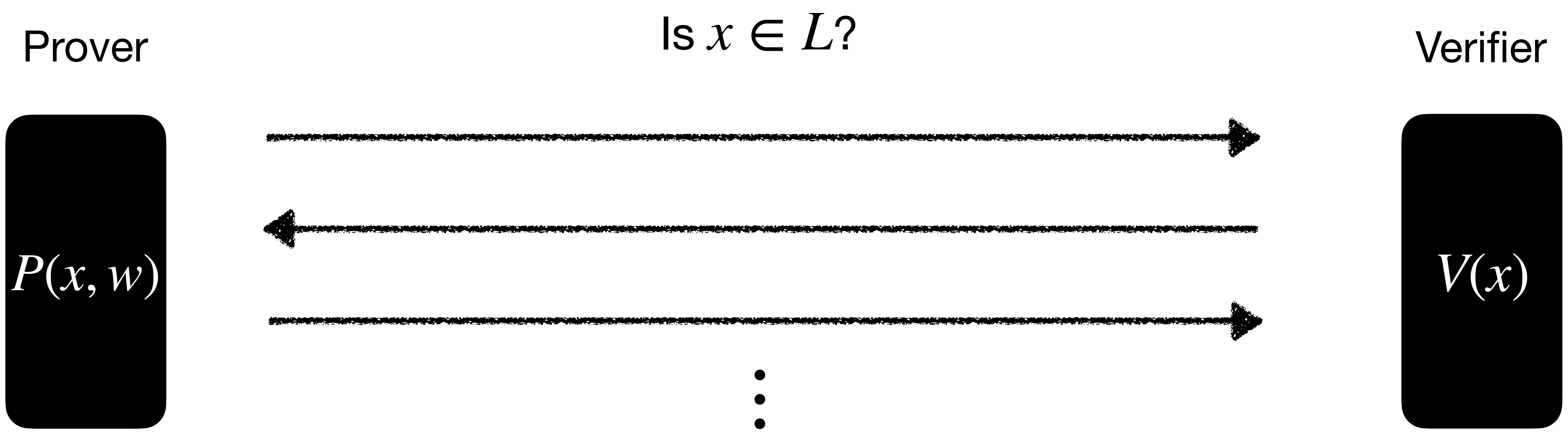

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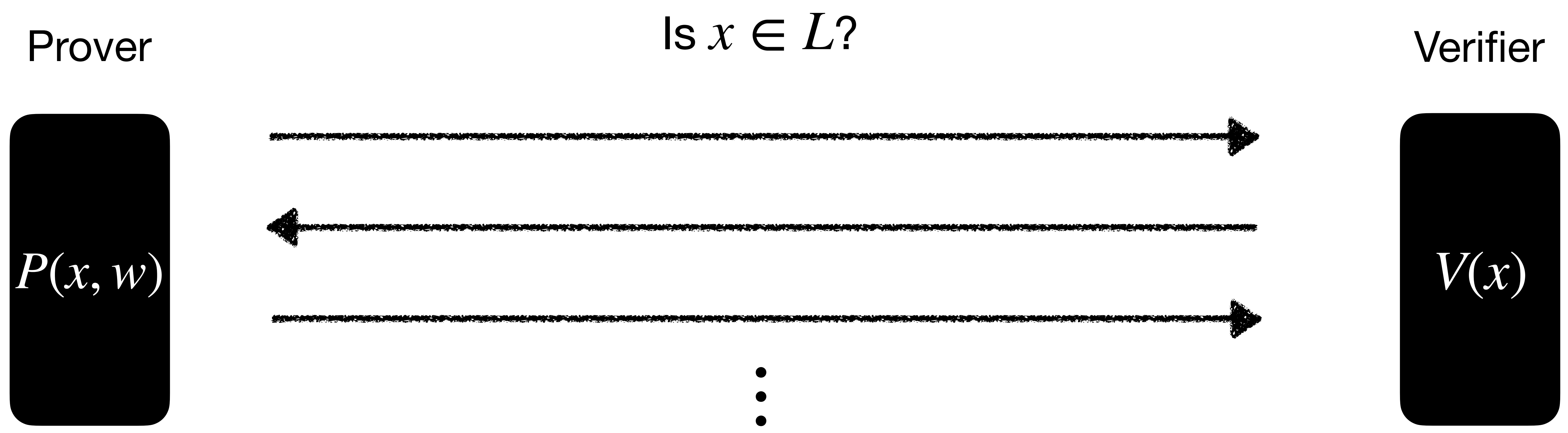

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Limitation: NP-complete languages do not have IPs with cc $\ll |w|$ (or else the language would be easy).

(Indeed, [GH97] proved that, in general, $\mathrm{IP}[\mathrm{cc}] \subseteq \mathrm{BPTIME}[2^{\mathrm{cc}}]$.)

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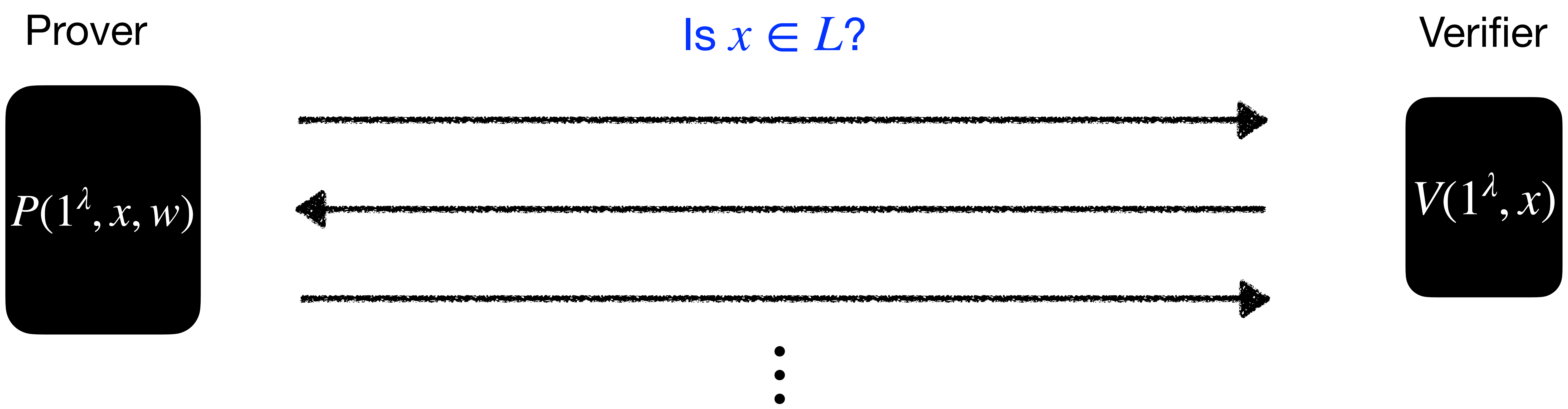

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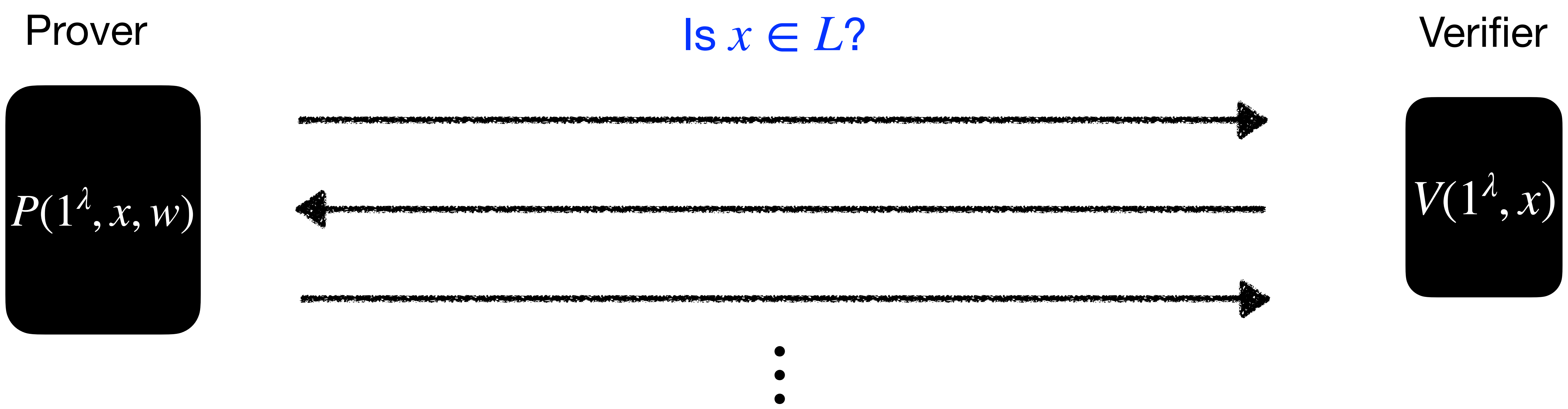

Computational soundness: For every $x \notin L$, security parameter $\lambda \in \mathbb{N}$, and t_{ARG}-bounded adversary $\tilde{P}$,

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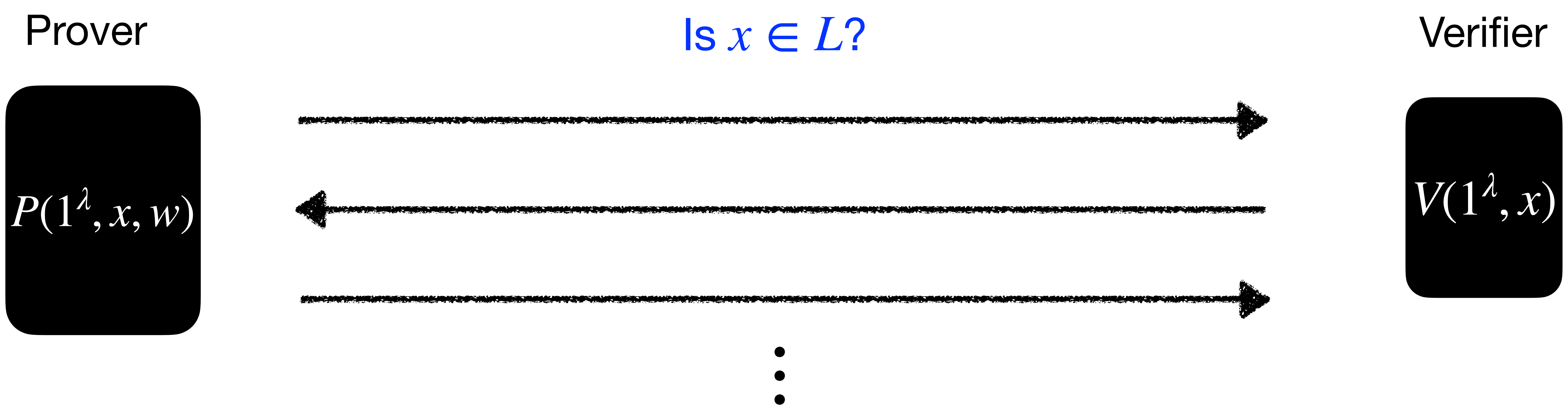

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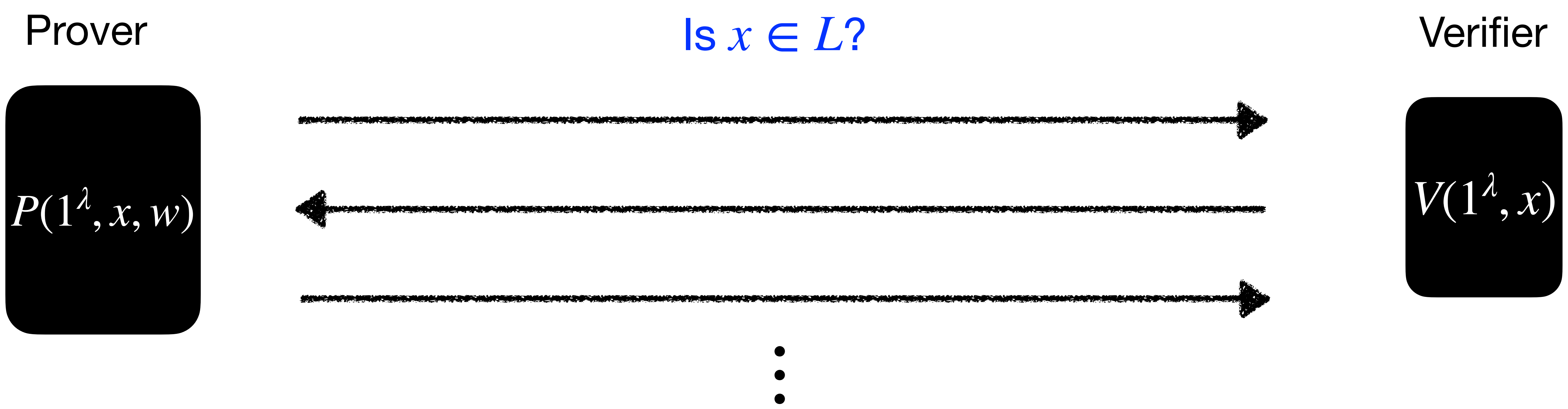

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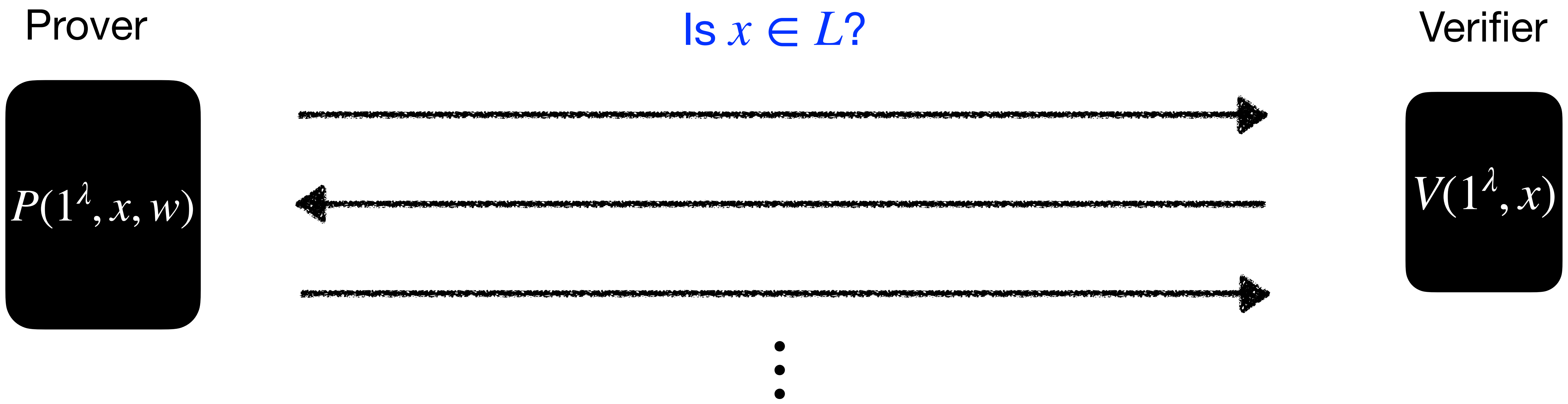

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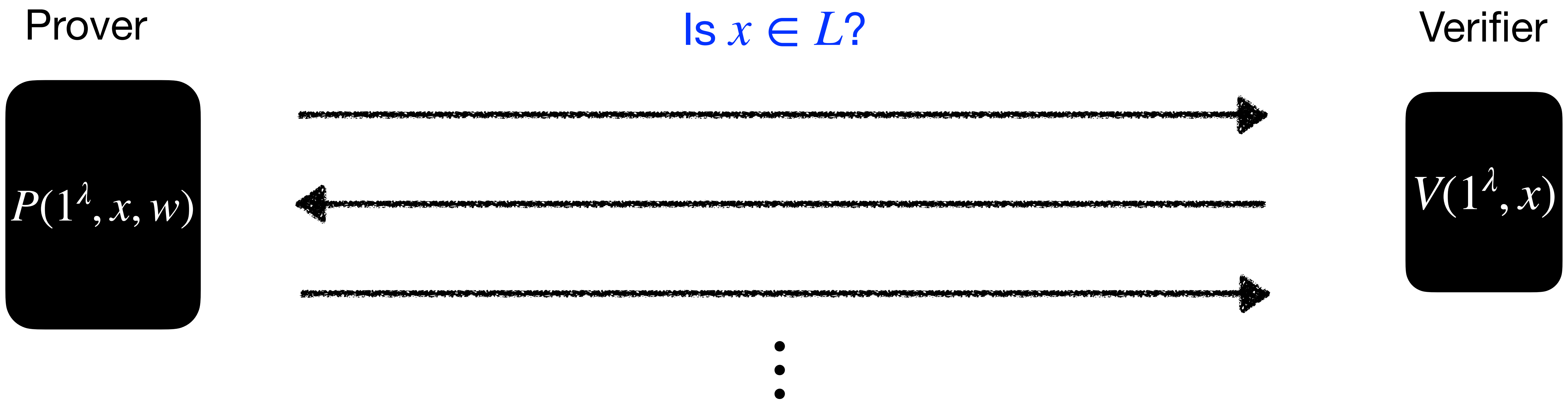

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Further relaxation: Expected-time computational soundness $\epsilon^\star_{\text{ARG}}$ against adversaries with bounded expected running time $t^\star_{\text{ARG}}$.

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Kilian's protocol, the first and simplest succinct argument

Kilian’s protocol

[Kilian92]

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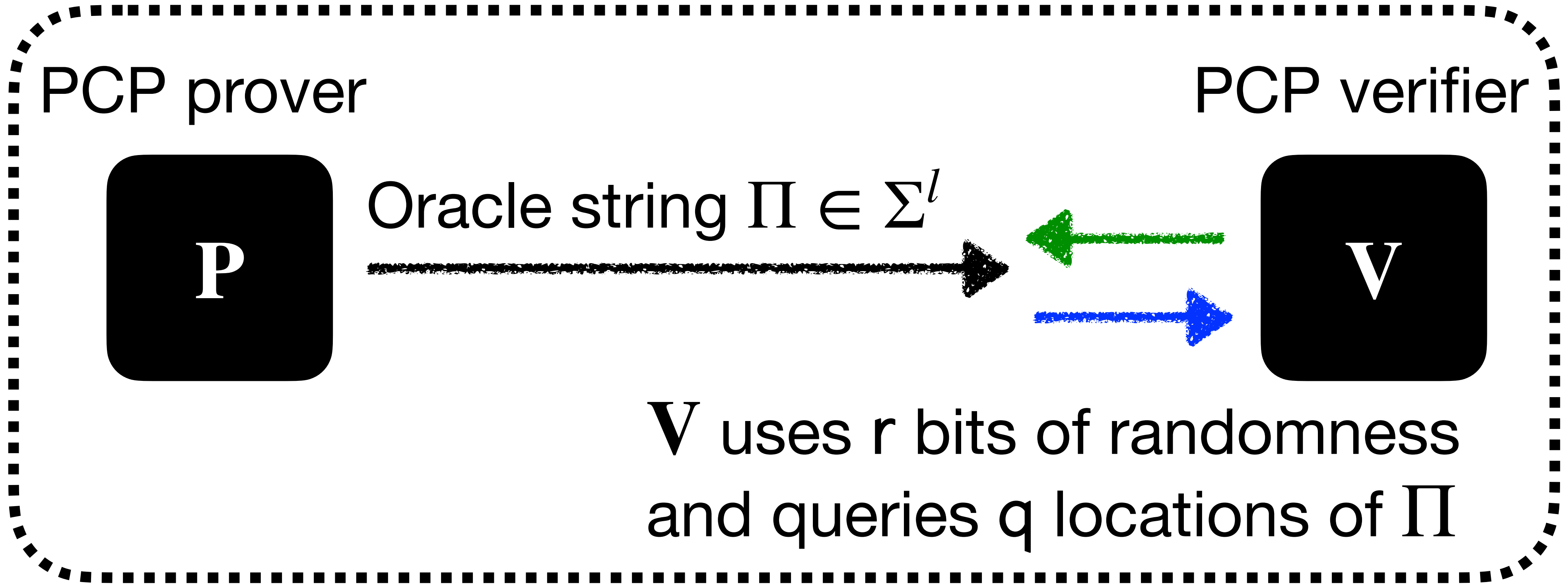

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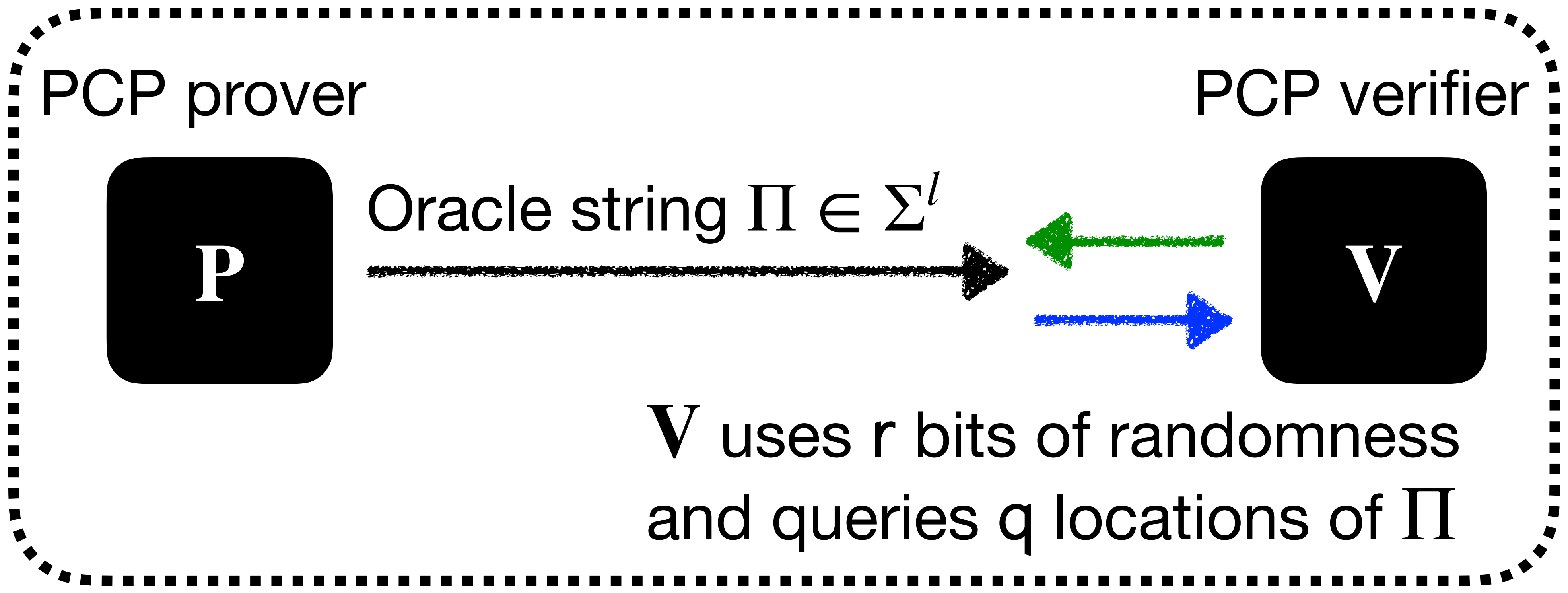

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abstraction for a succinct commitment with local openings (e.g. Merkle tree)

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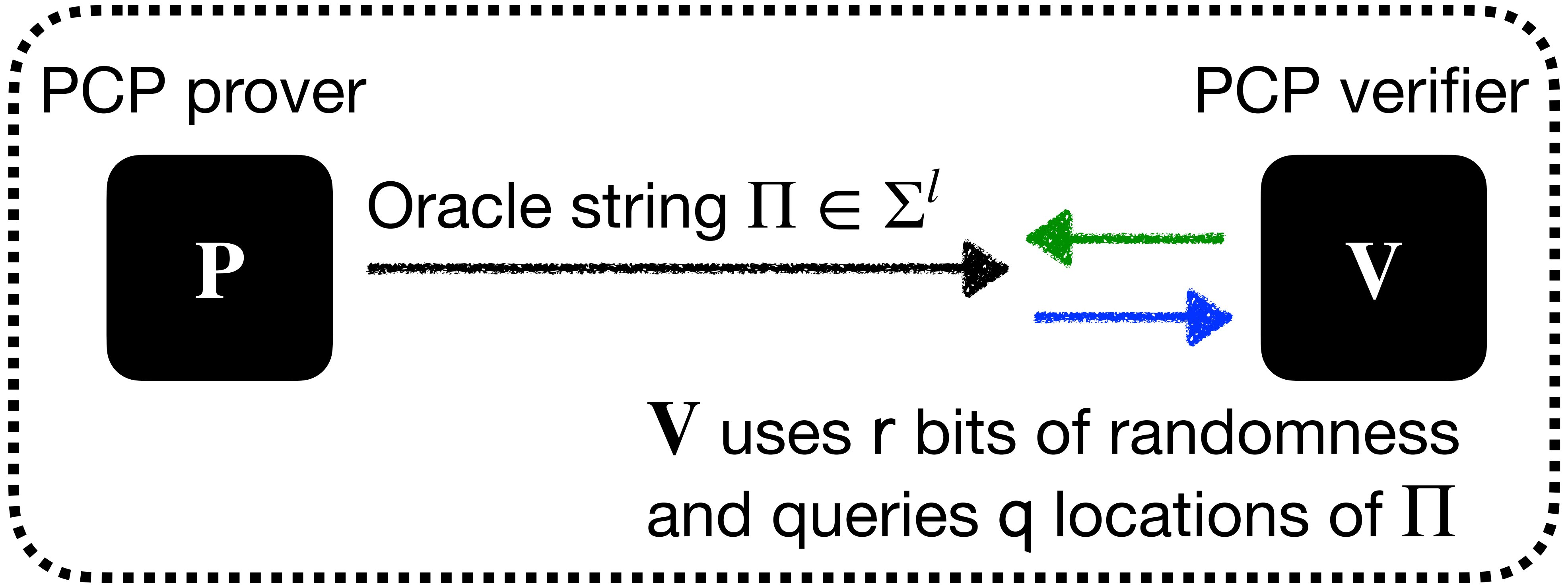

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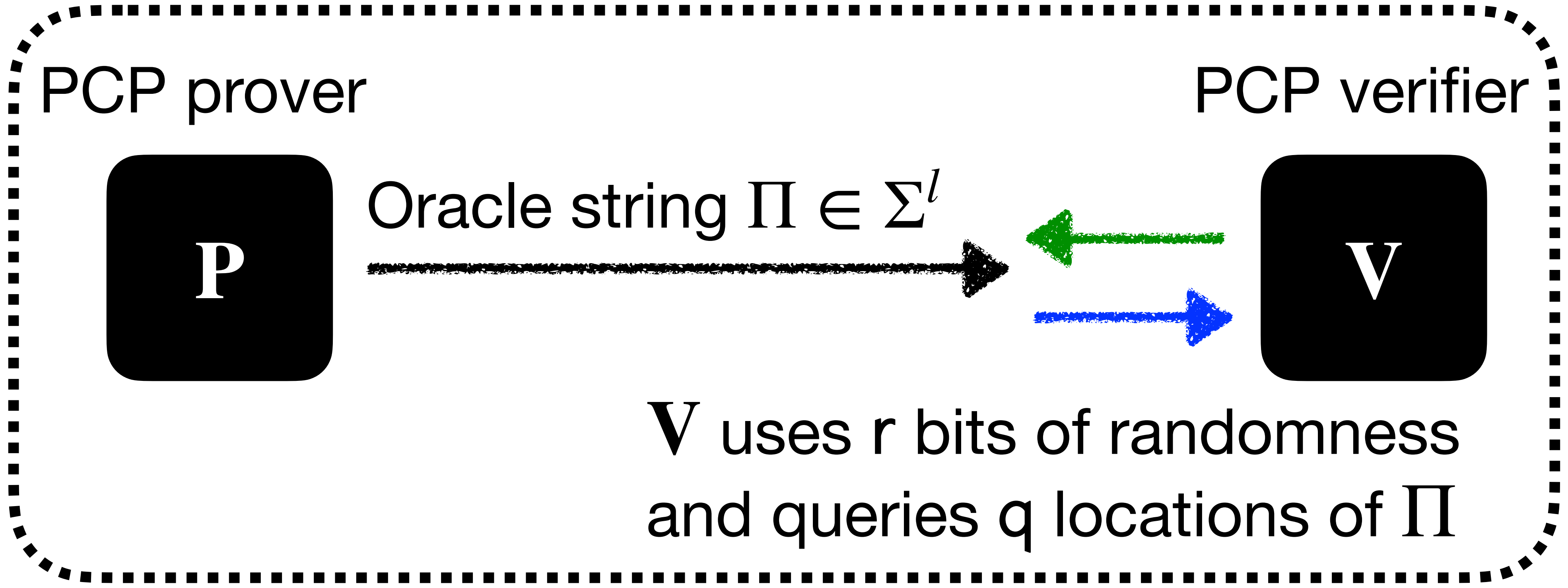

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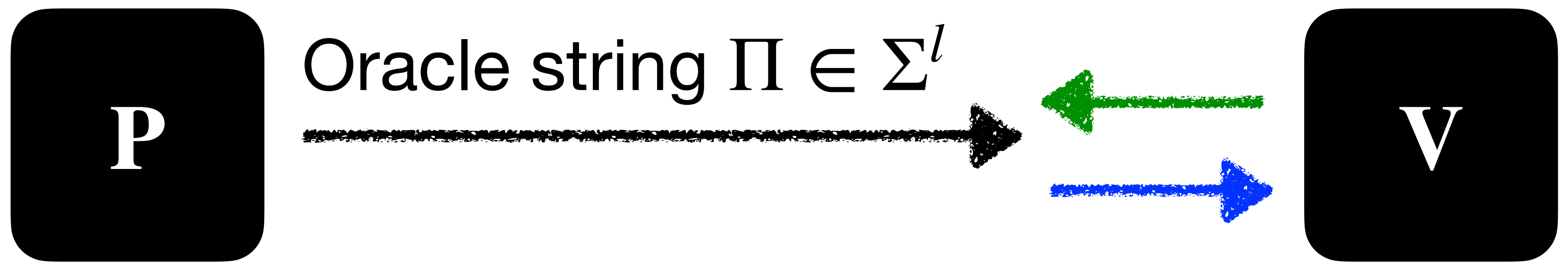
PCP verifier



$\mathbf{V}$ uses r bits of randomness and queries q locations of Π

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Commit phase

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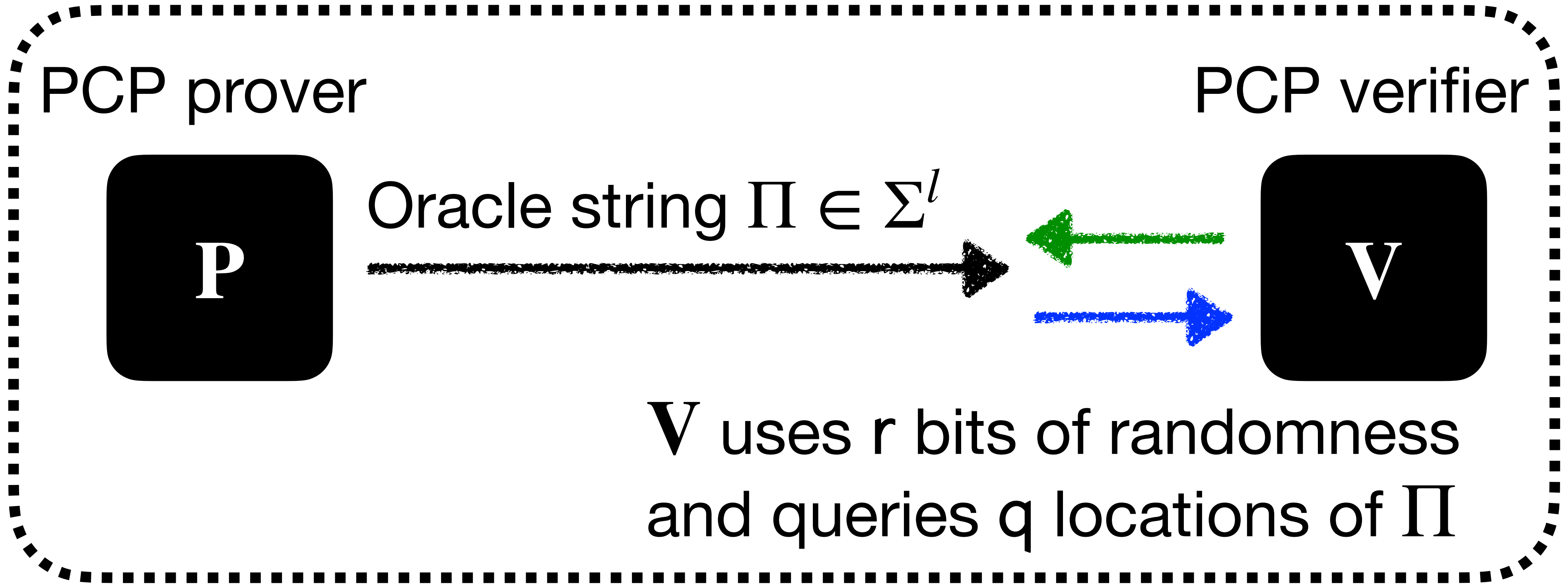

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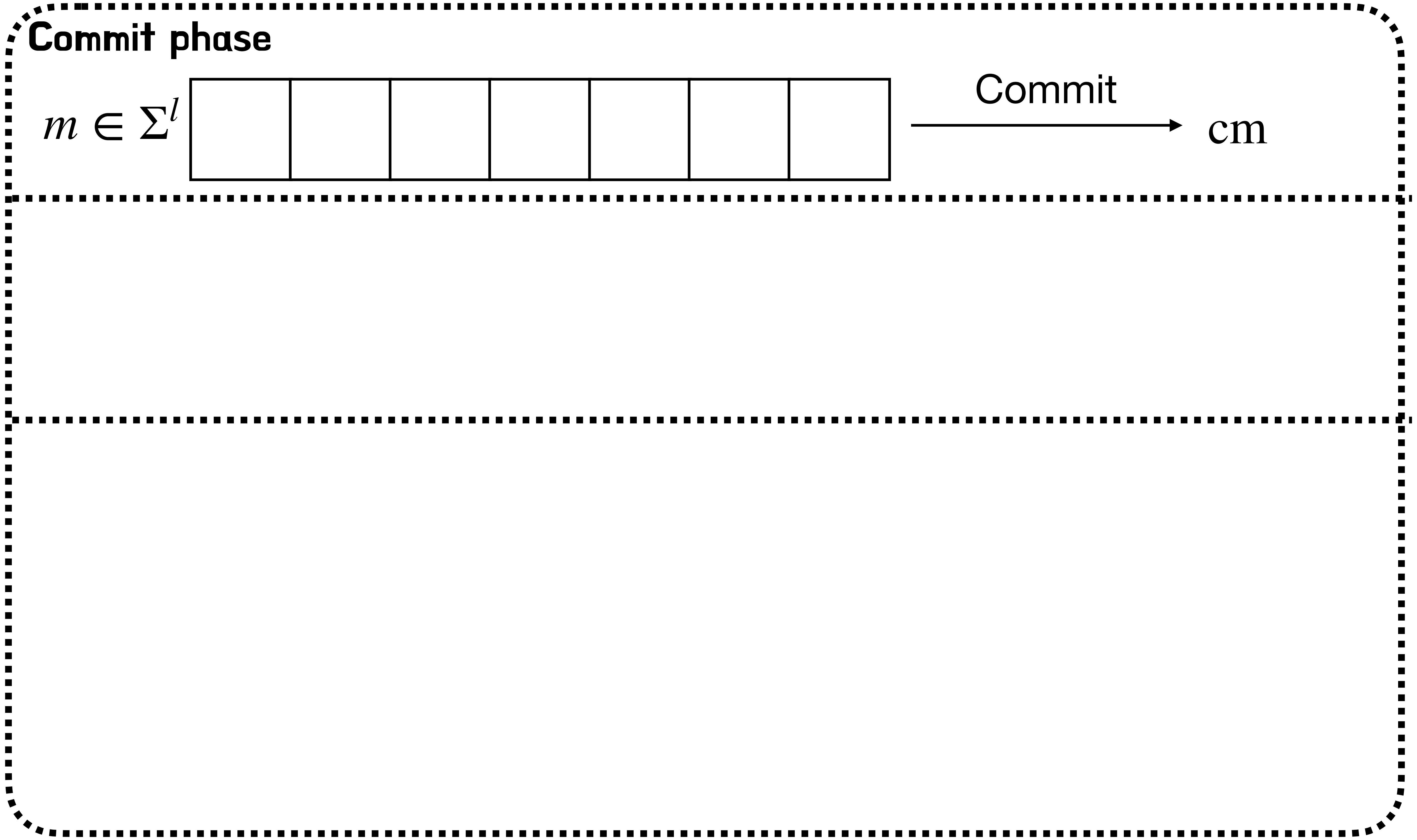

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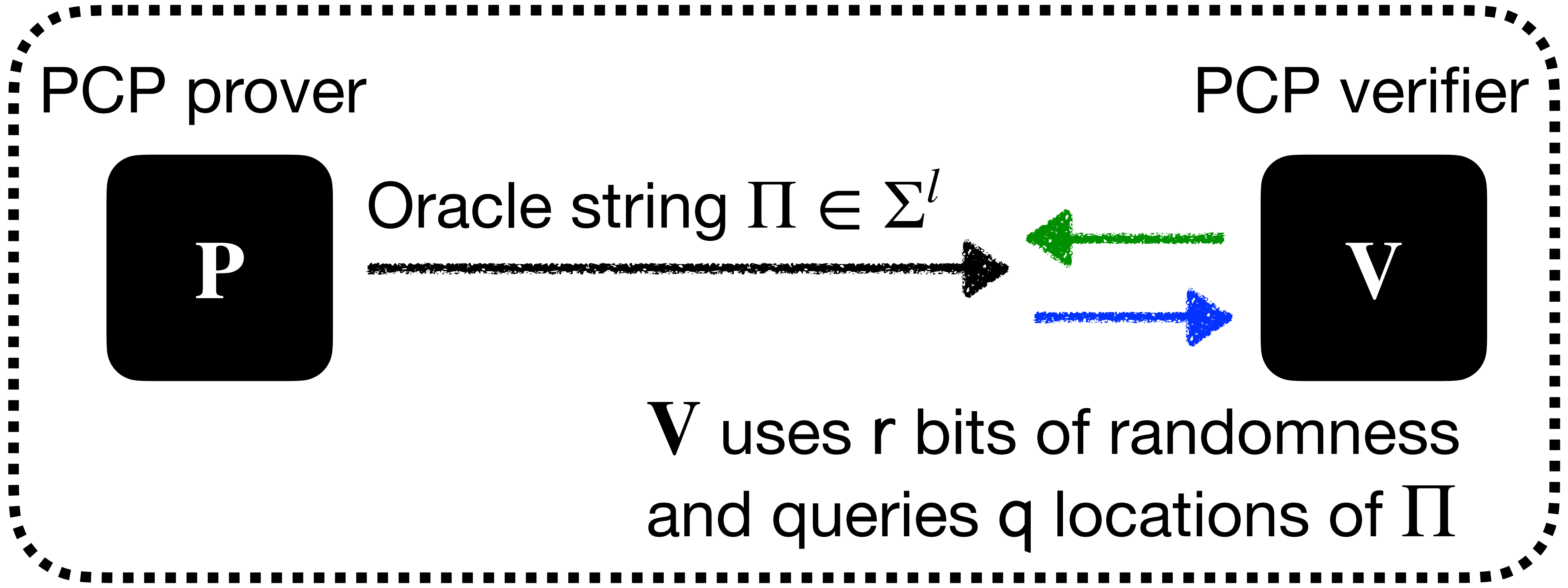

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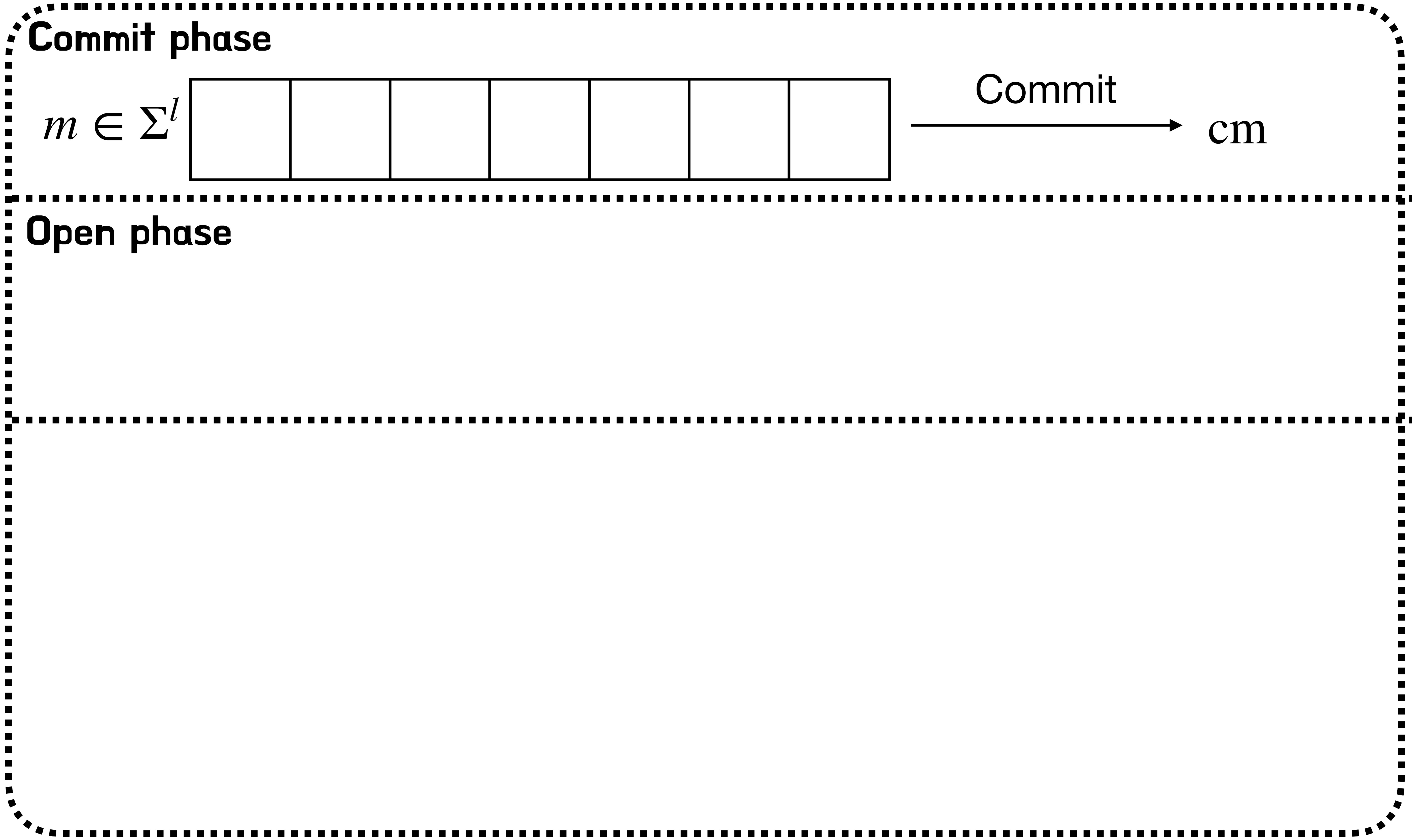

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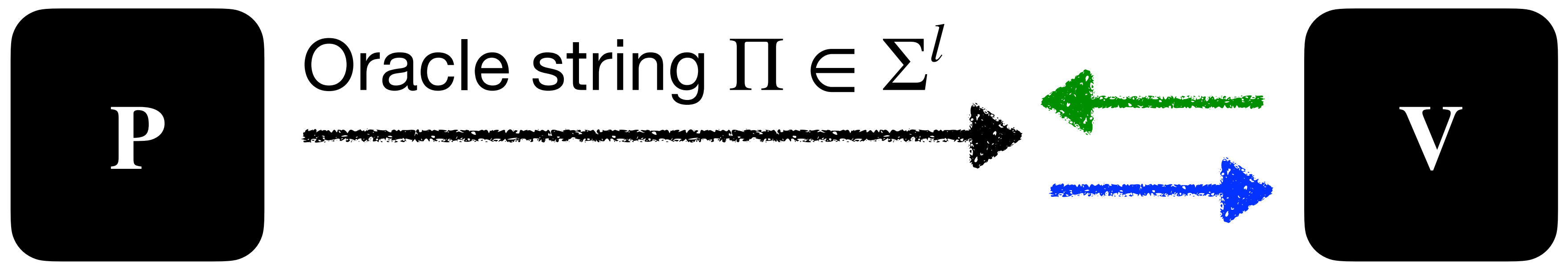
Oracle string $\Pi \in \Sigma^l$



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Commit phase

$m \in \Sigma^l$ — Commit → cm

Open phase

ans

Query set $Q \subseteq [l]$ — Open from cm → pf

If ans $:= m[Q]$ then Check$(\mathrm{cm}, Q, \text{ans}, \text{pf}) = 1$.

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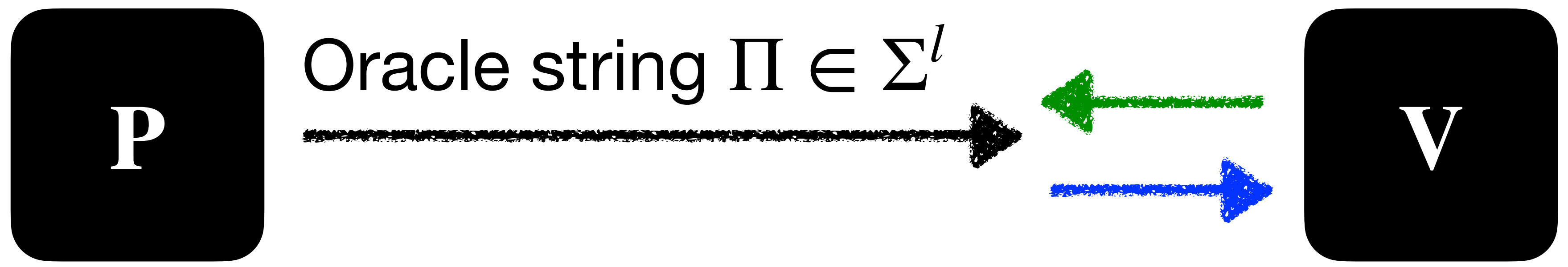

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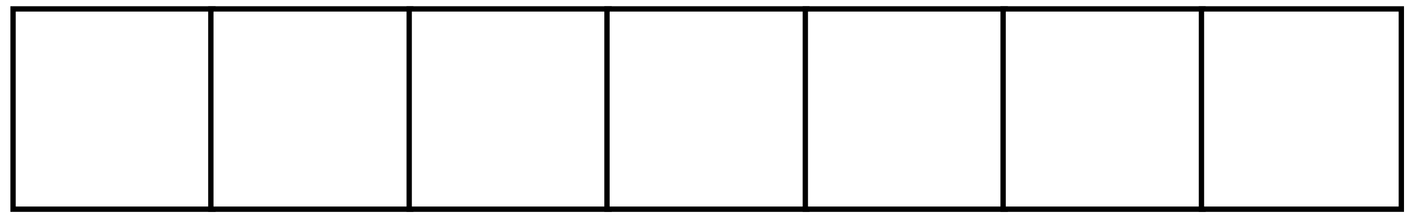

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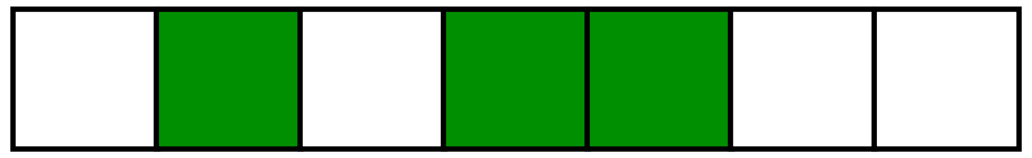

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(Expected-time) position binding

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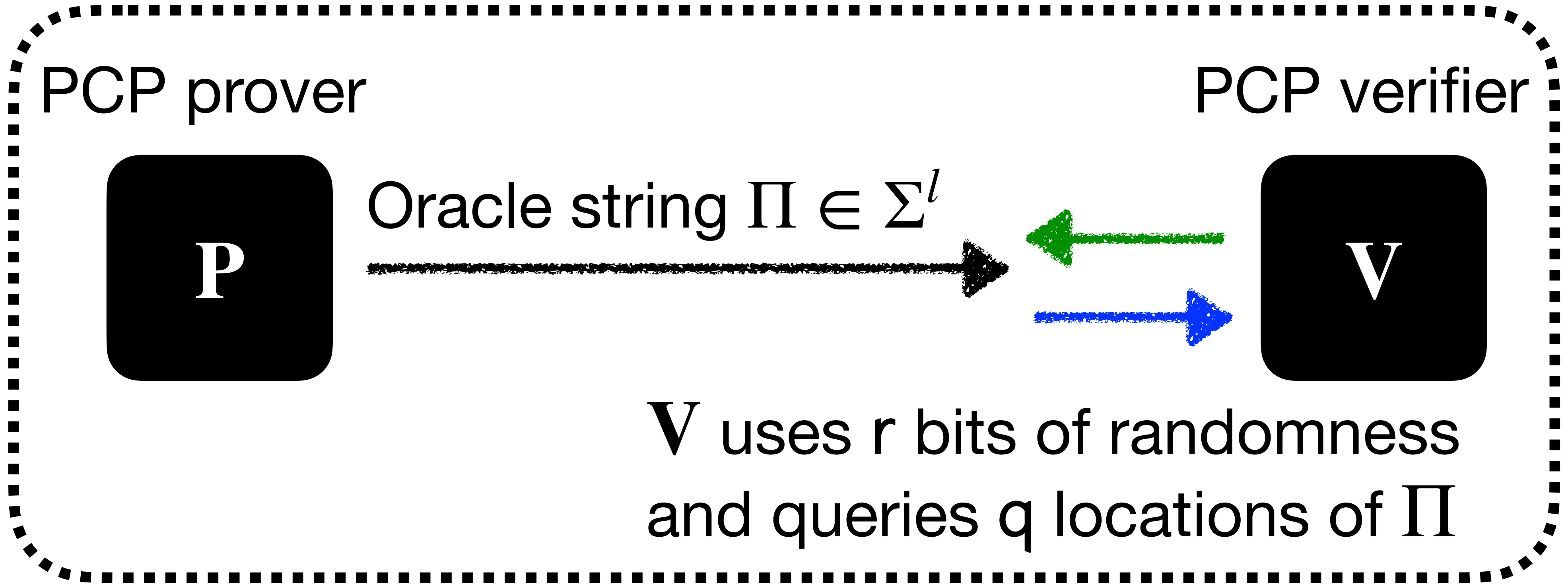

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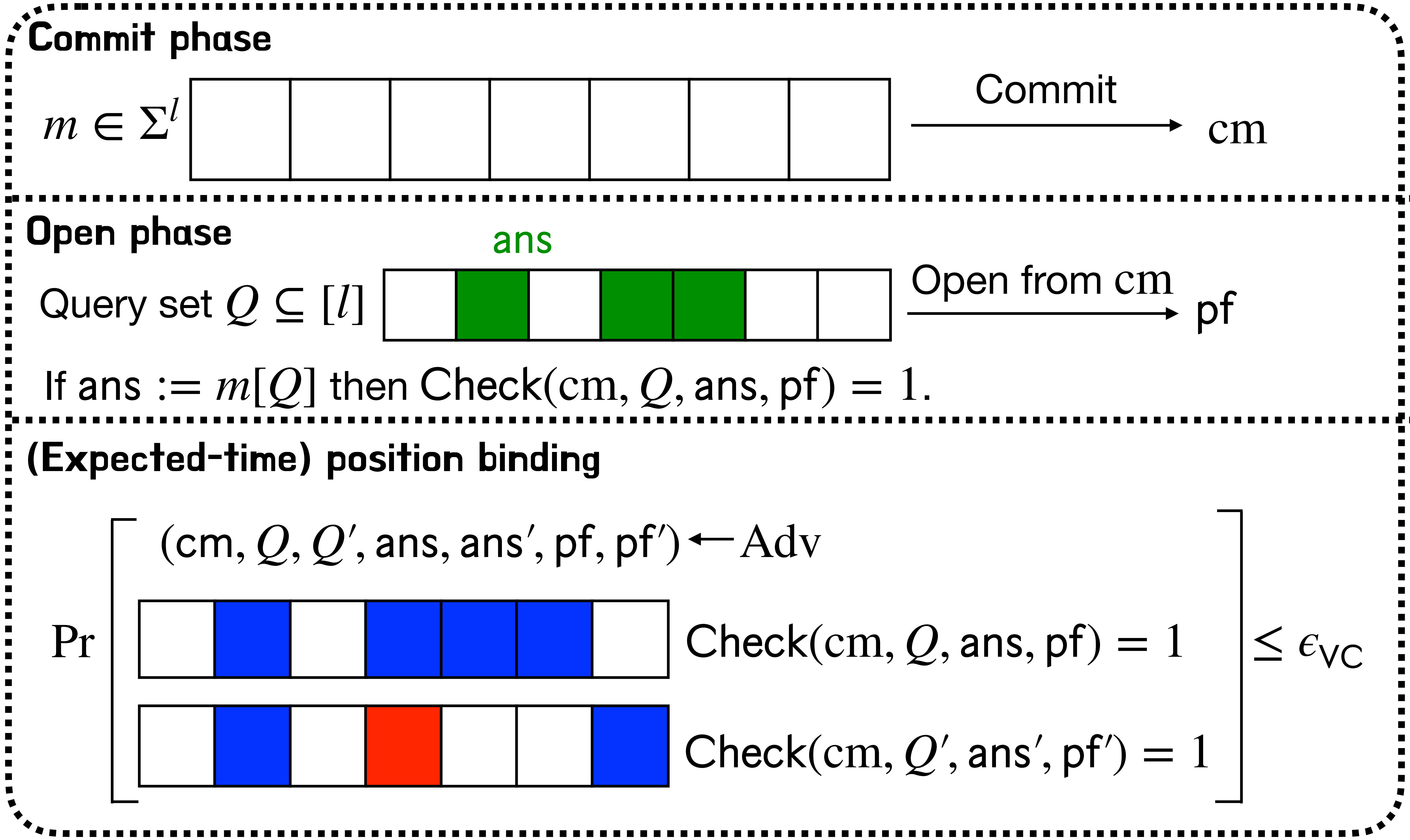

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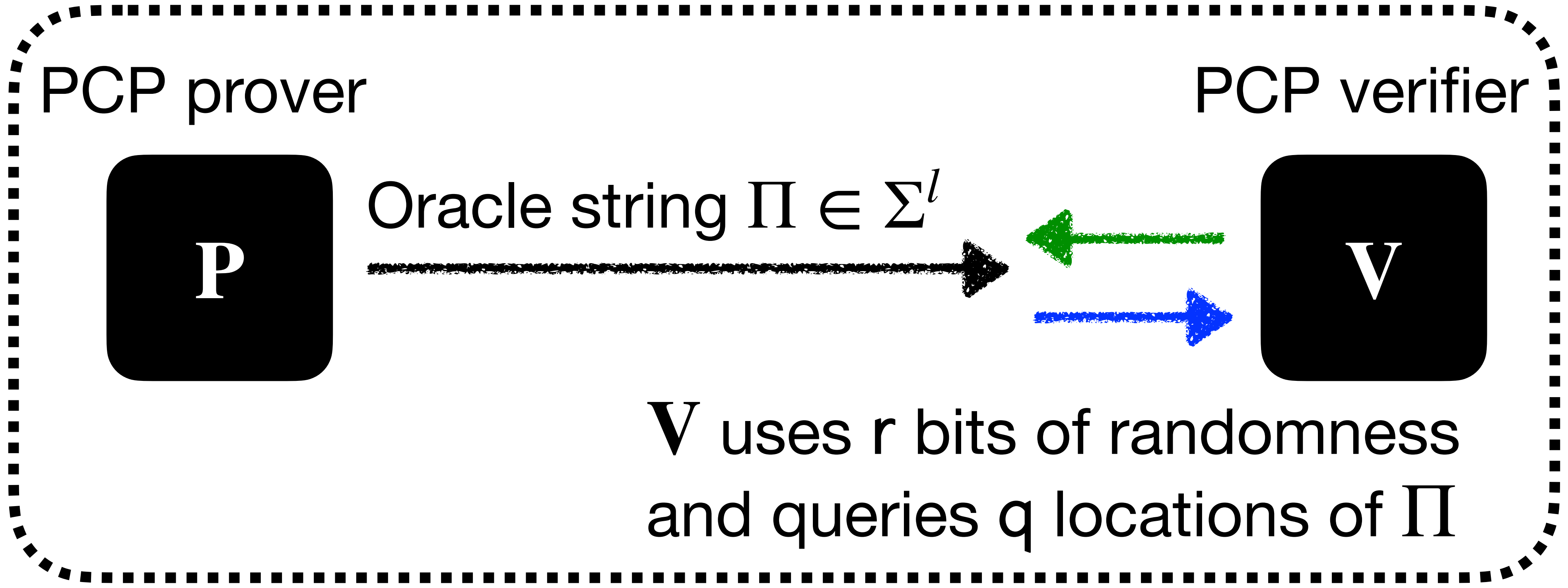

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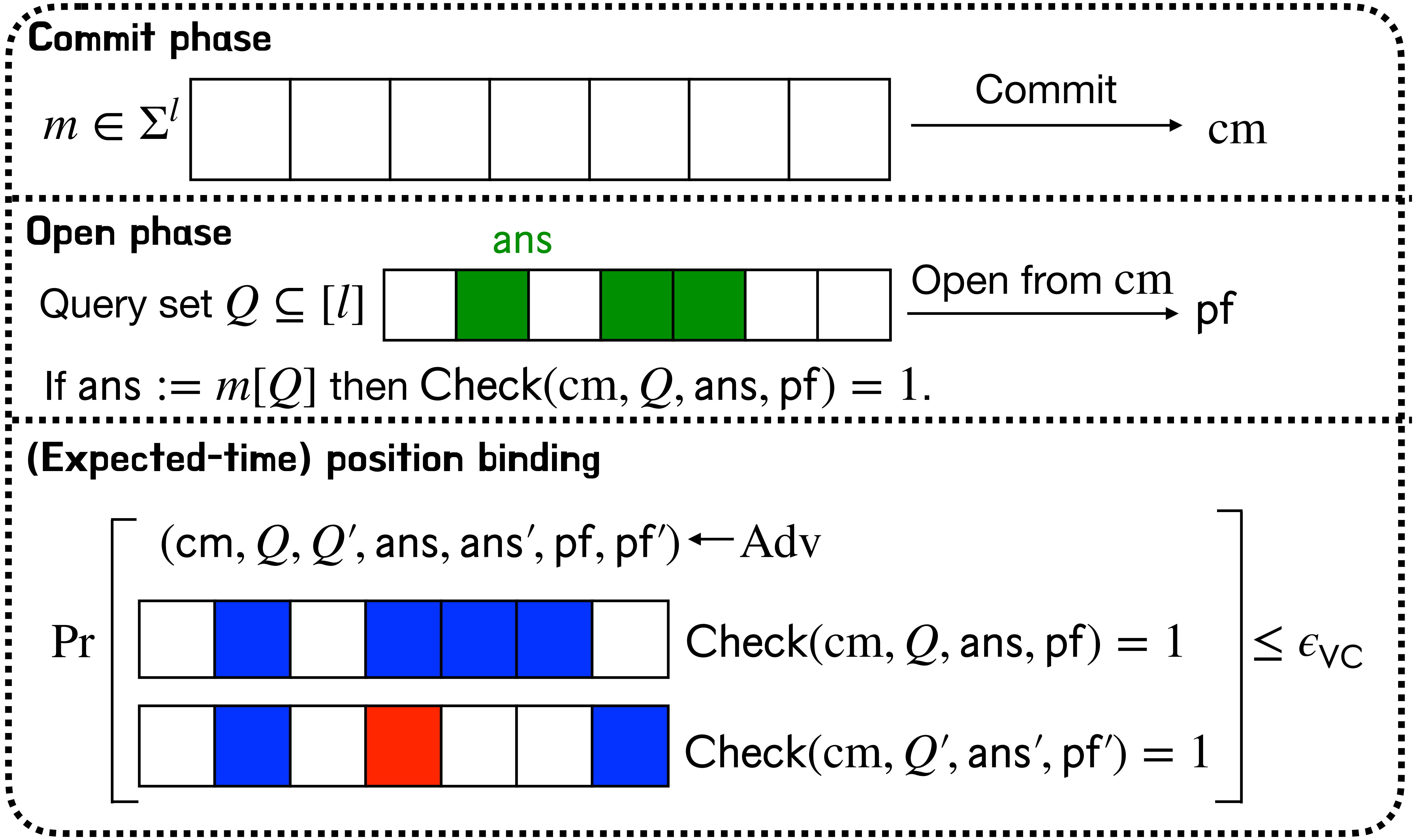

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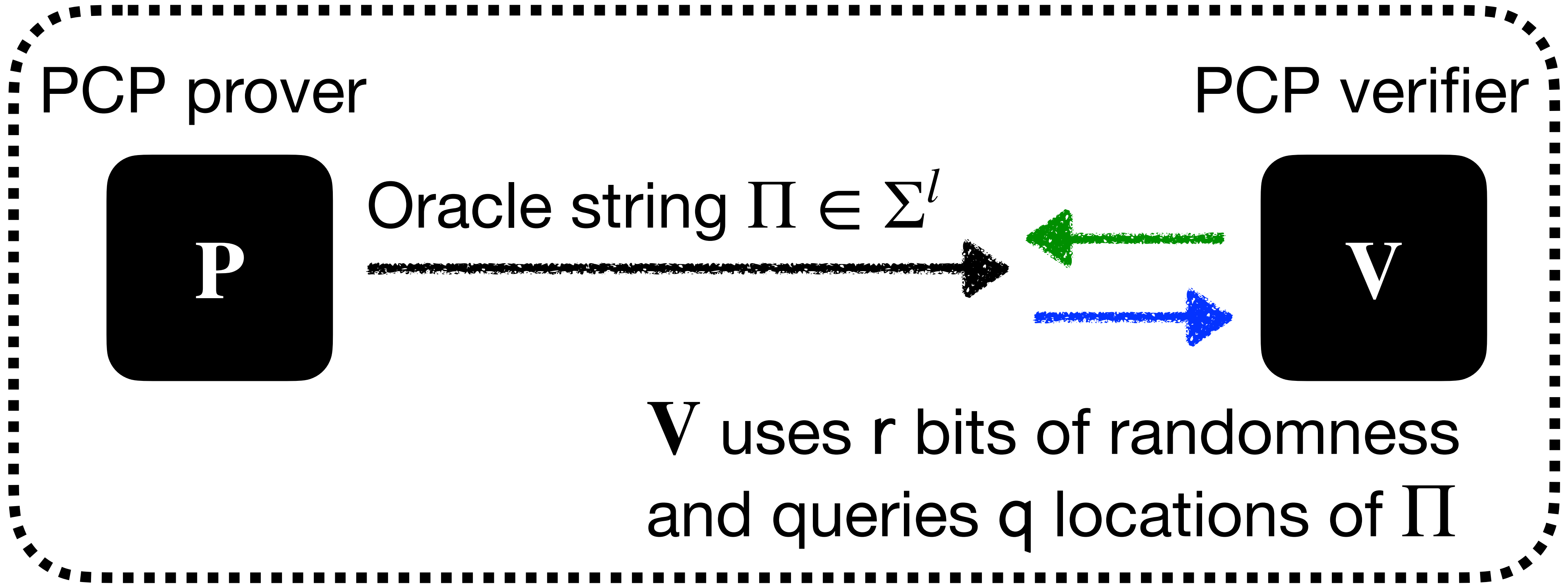

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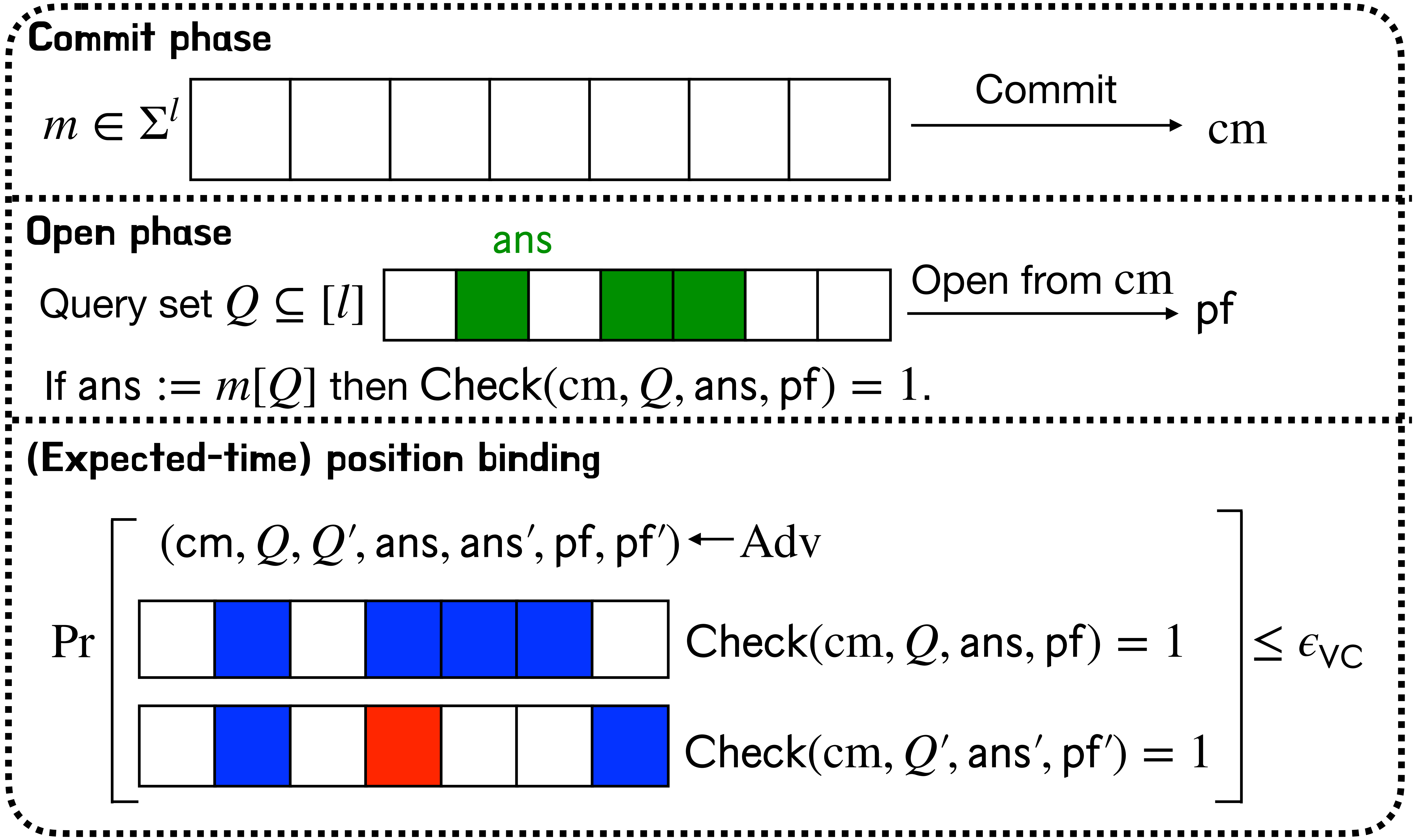

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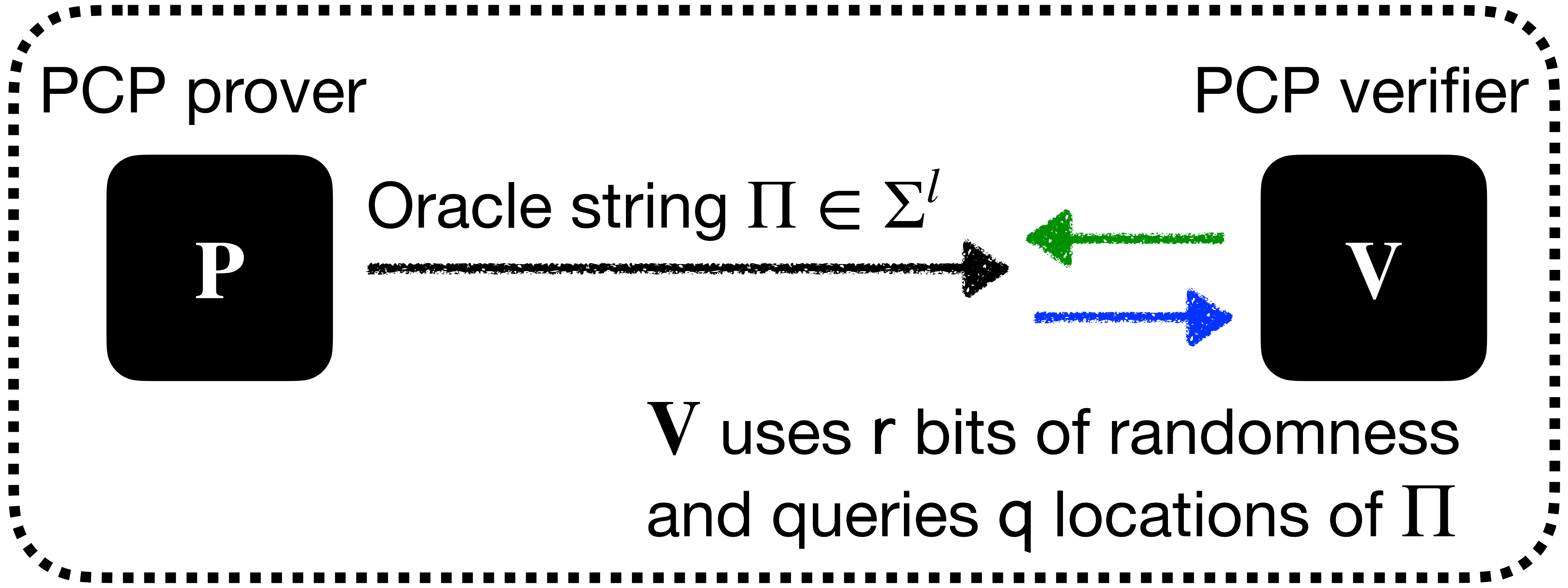

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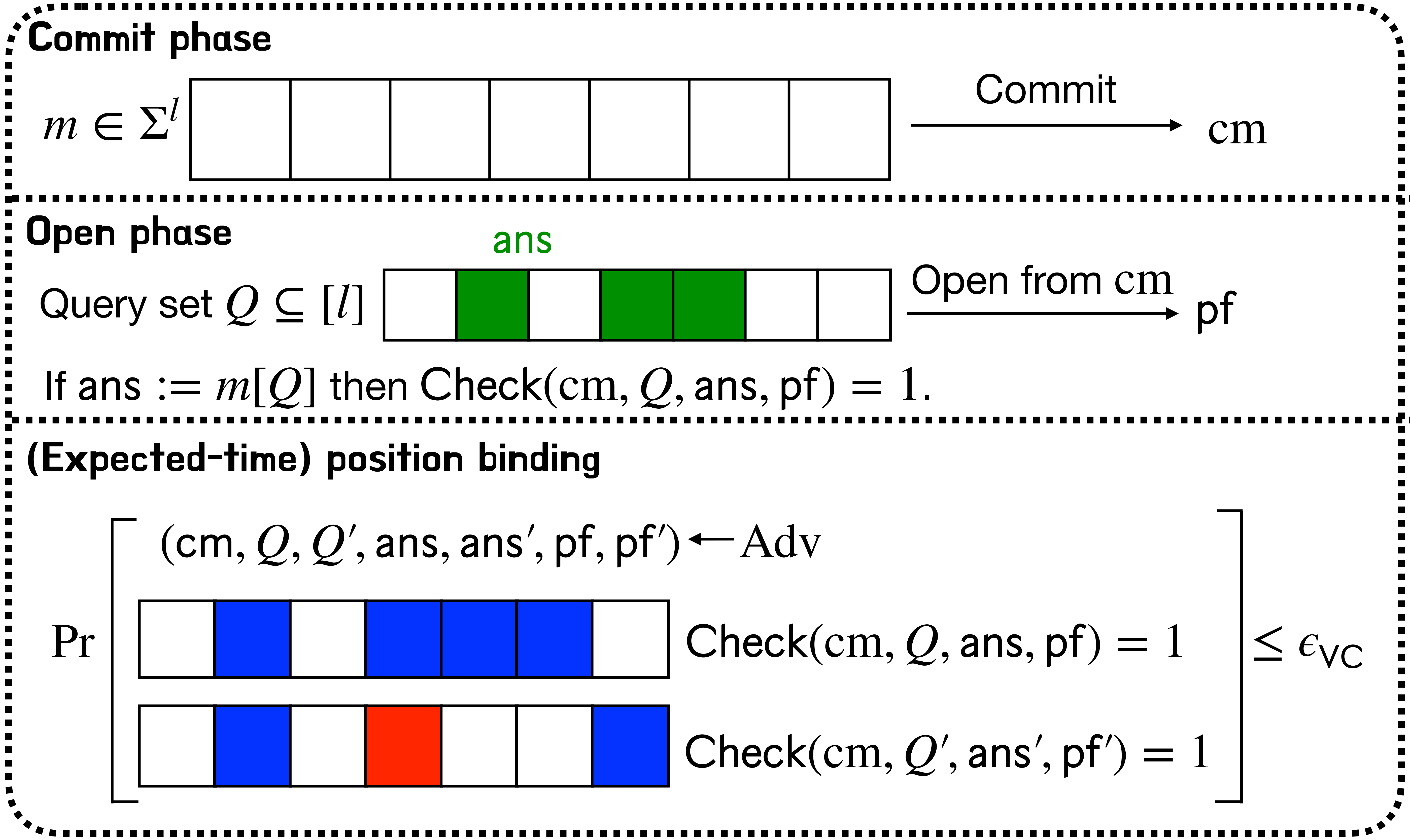

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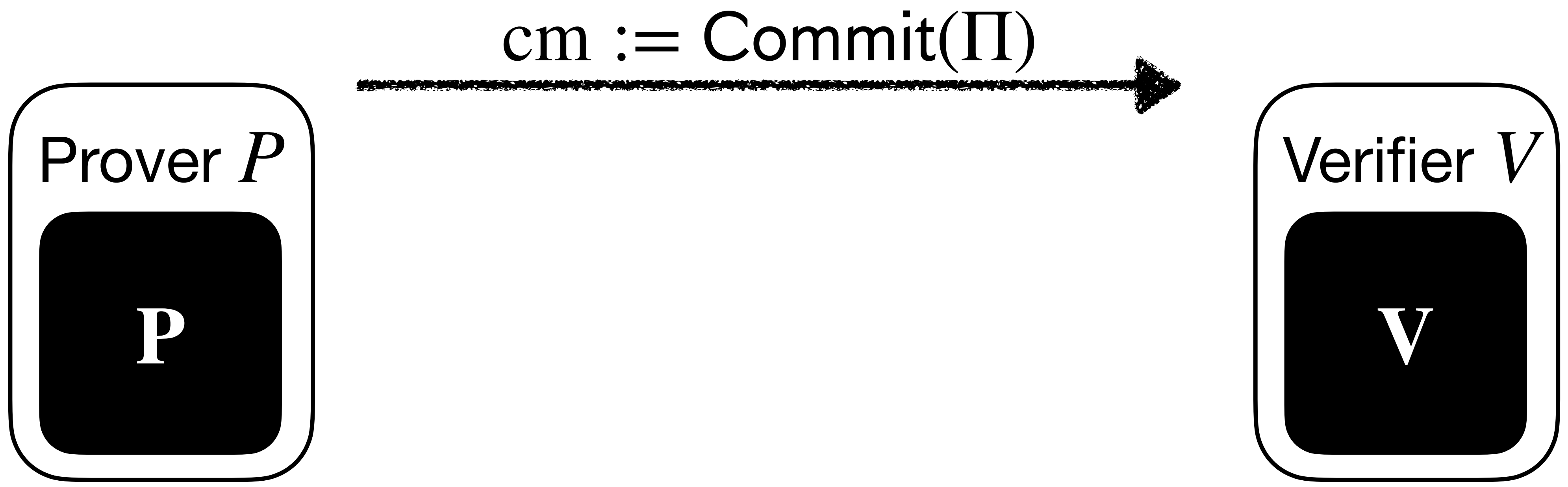

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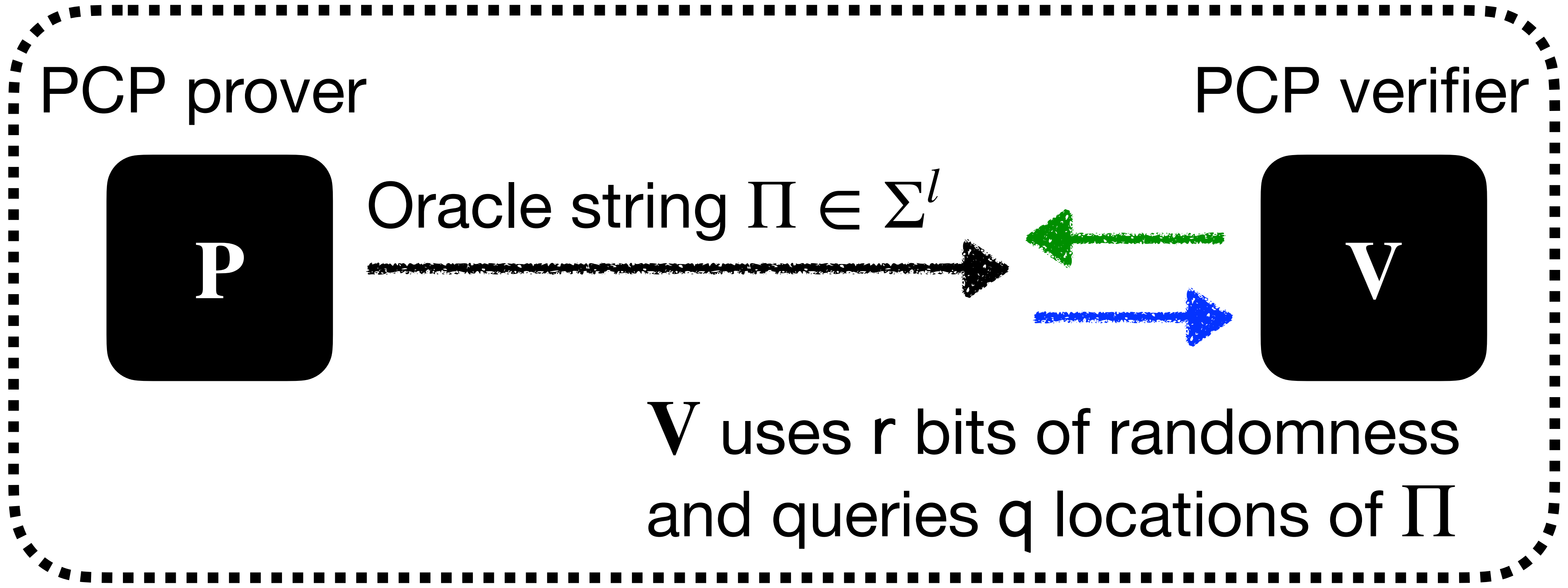

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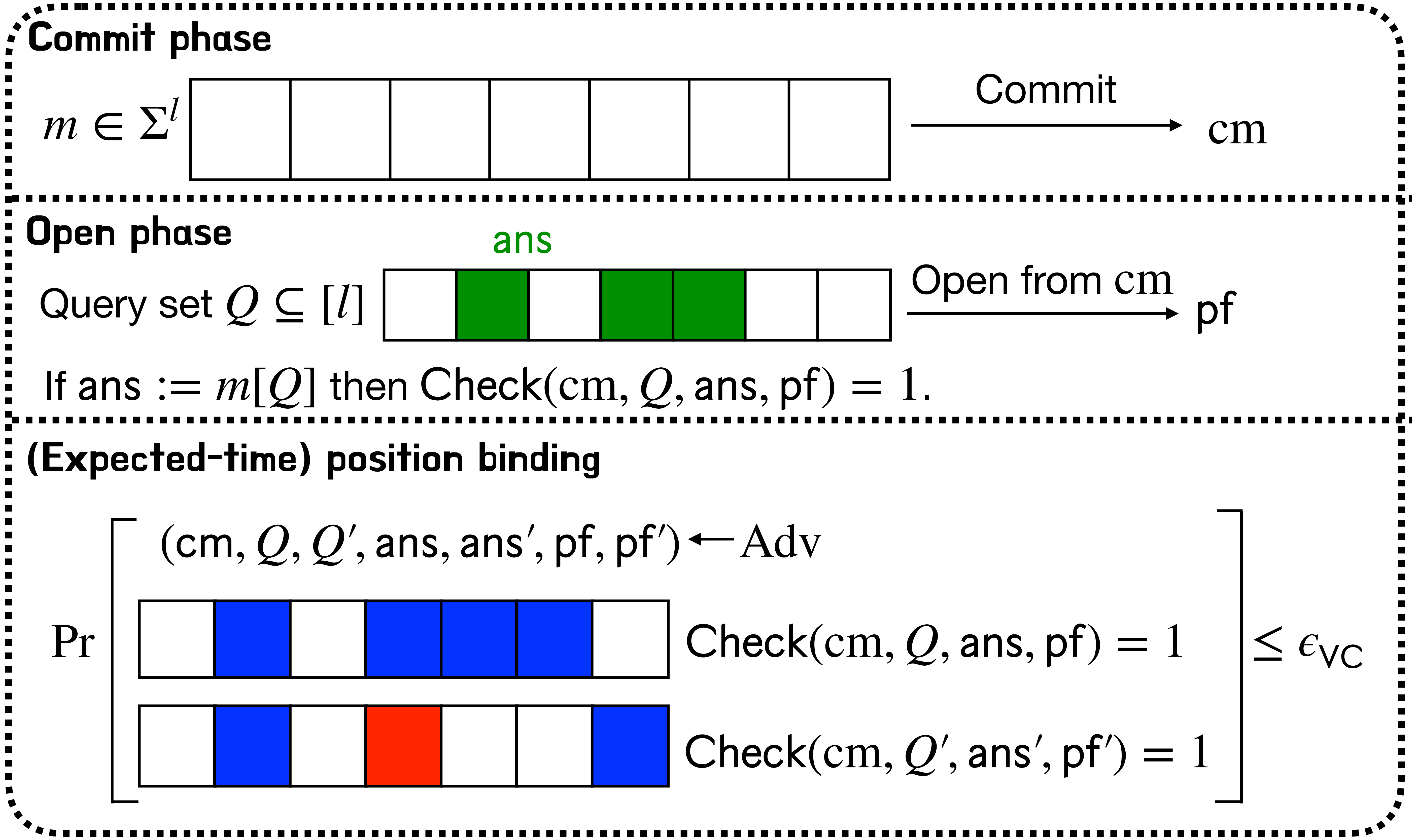

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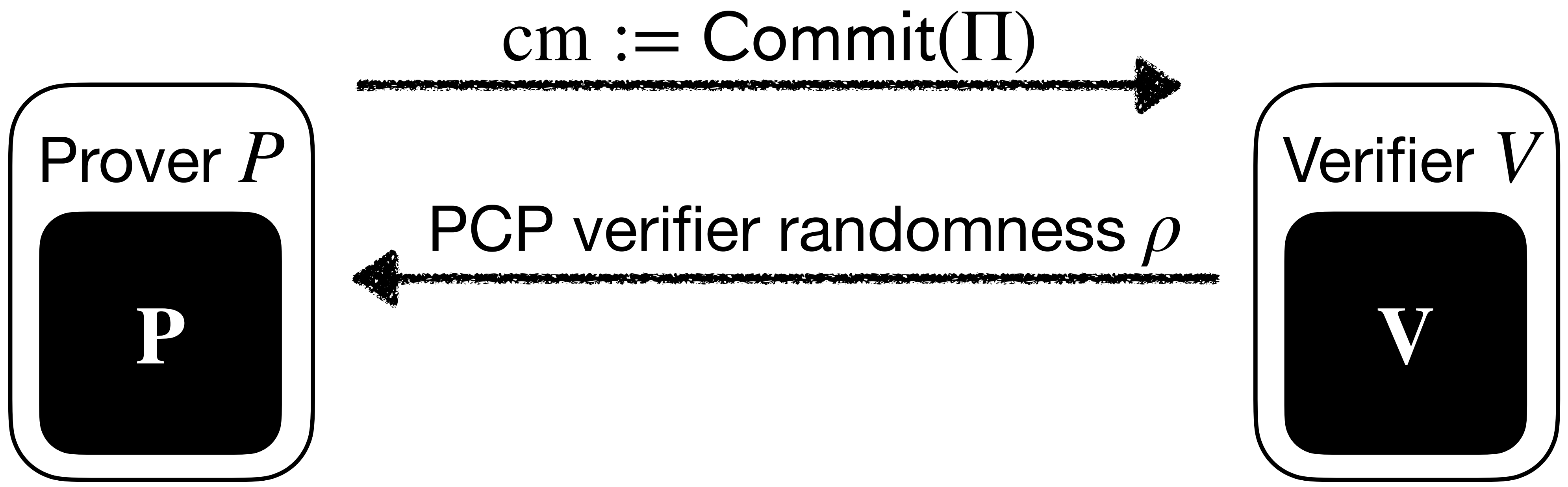

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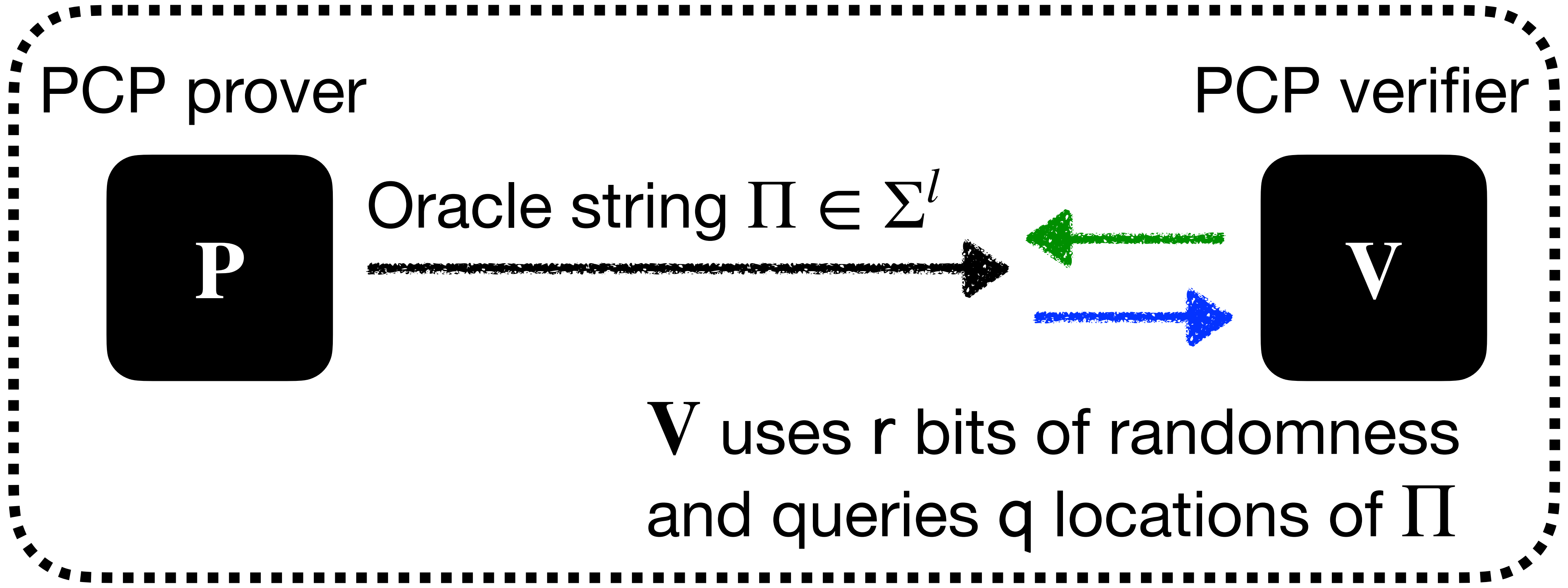

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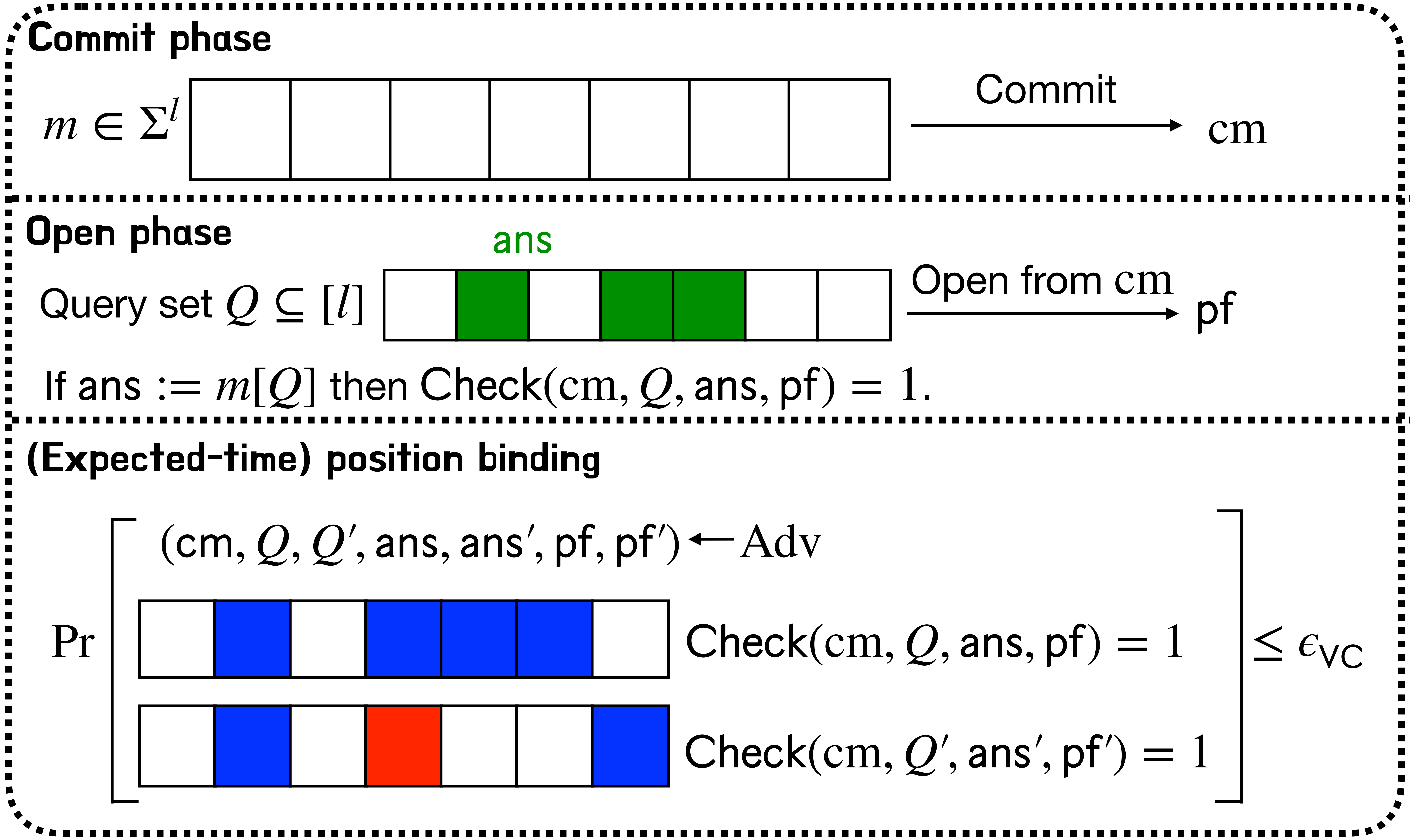

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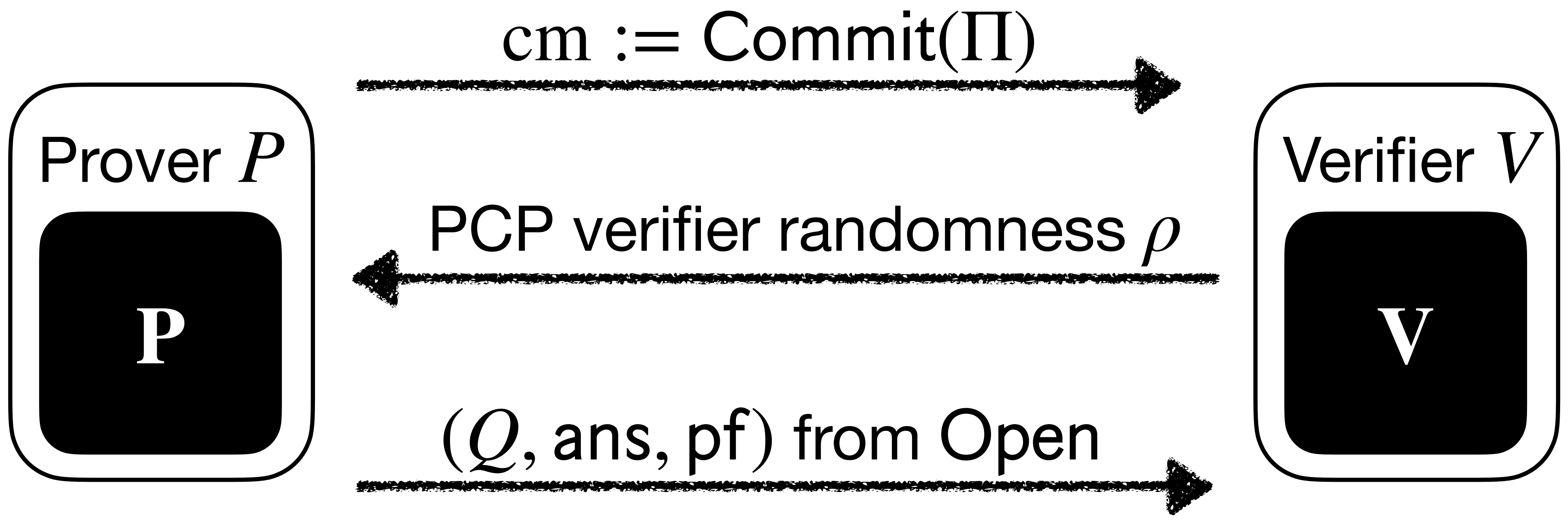

Fundamental question:
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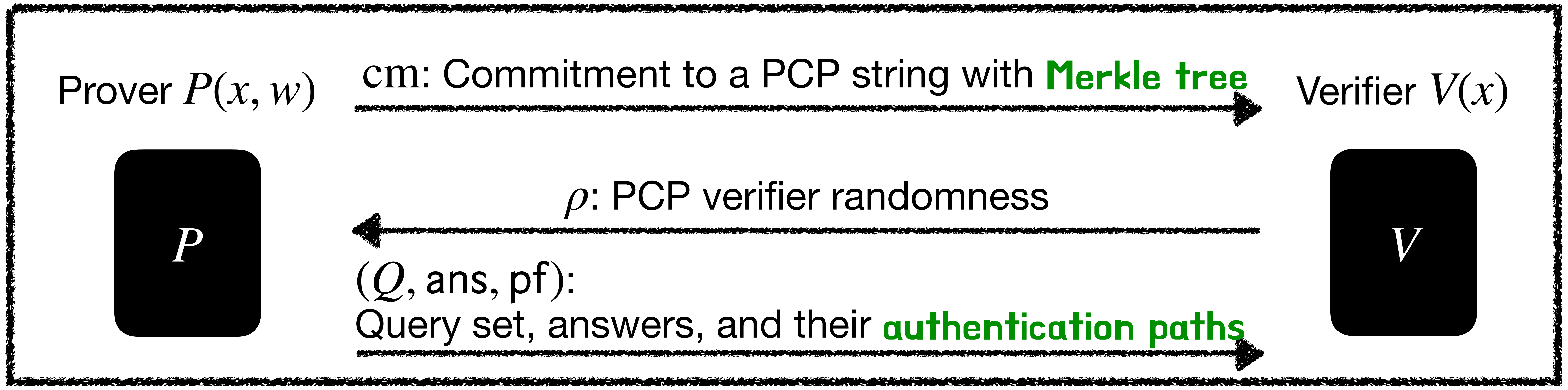

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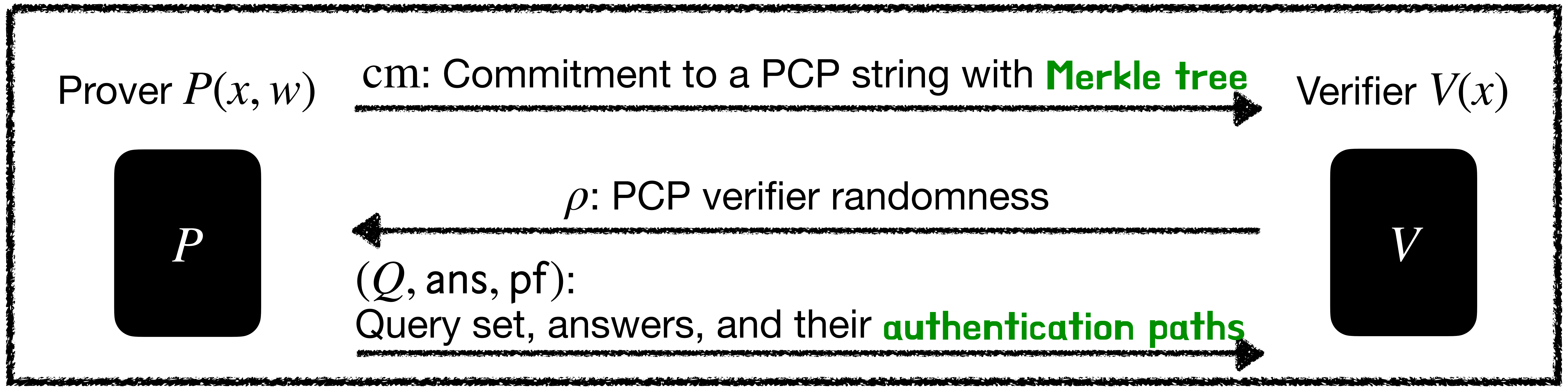

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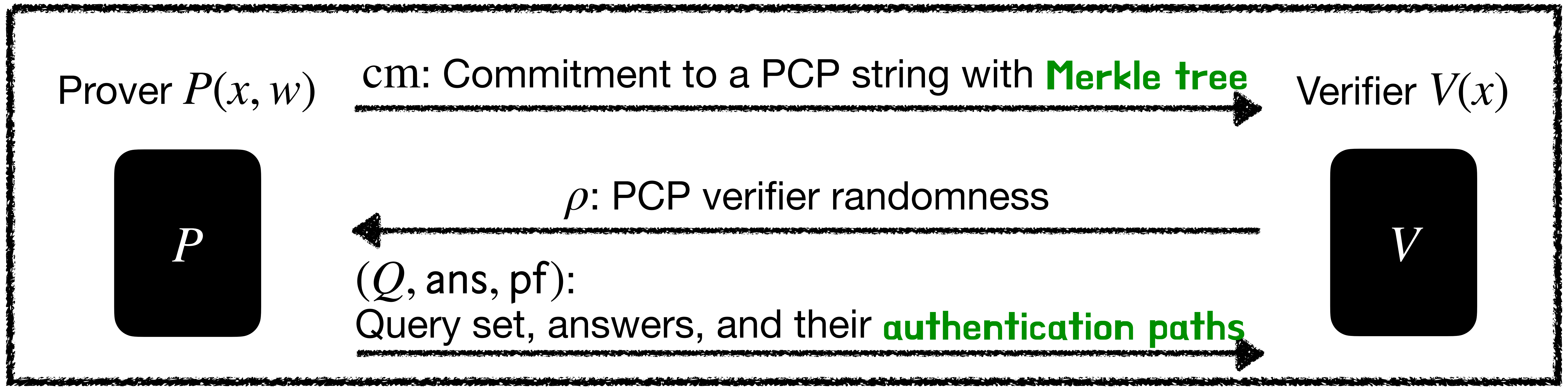

Previously:

- Folklore: well-understood, if ϵ_{PCP} and ϵ_{VC} if negligible, then ϵ_{ARG} is negligible.

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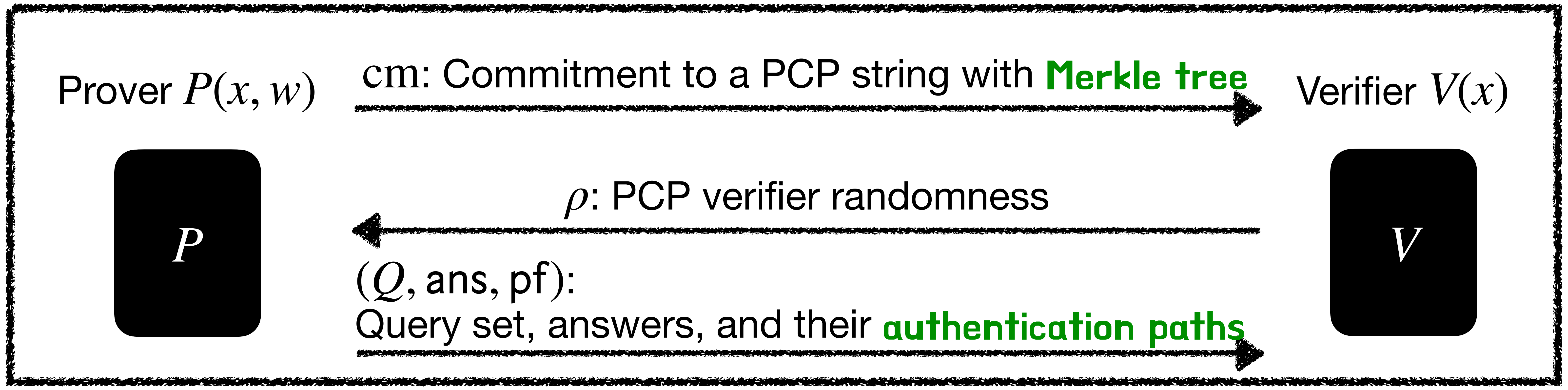

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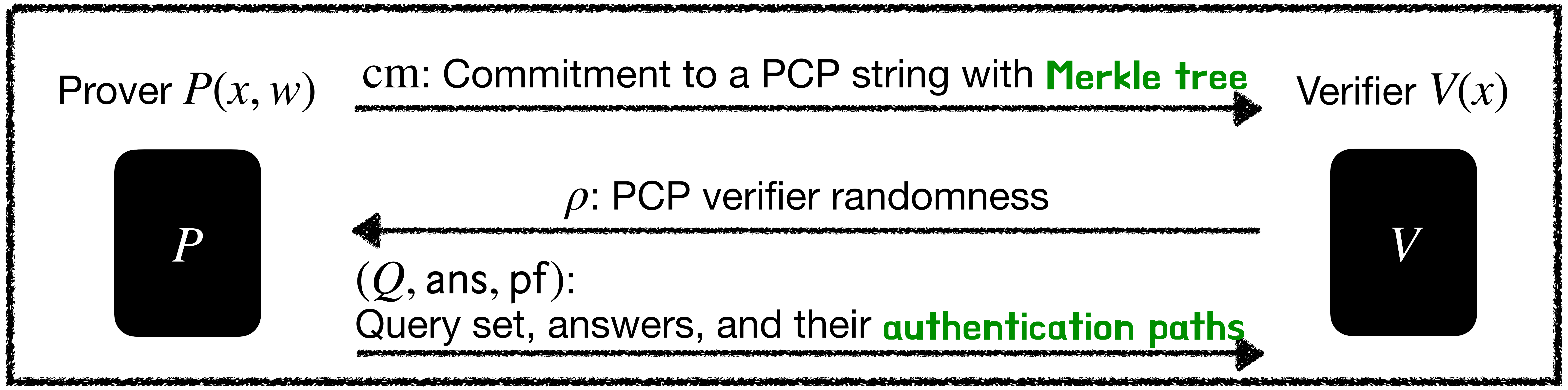

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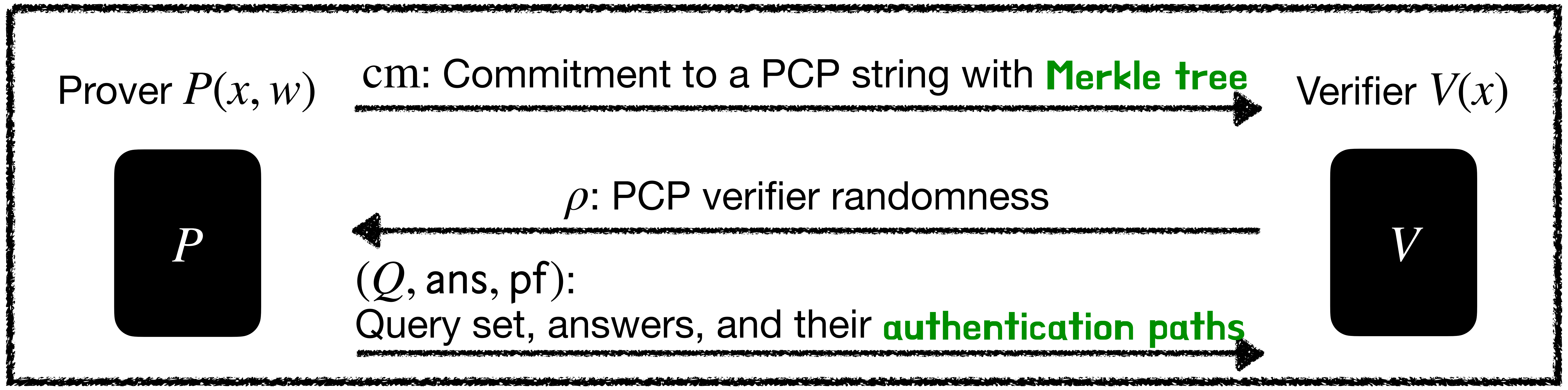

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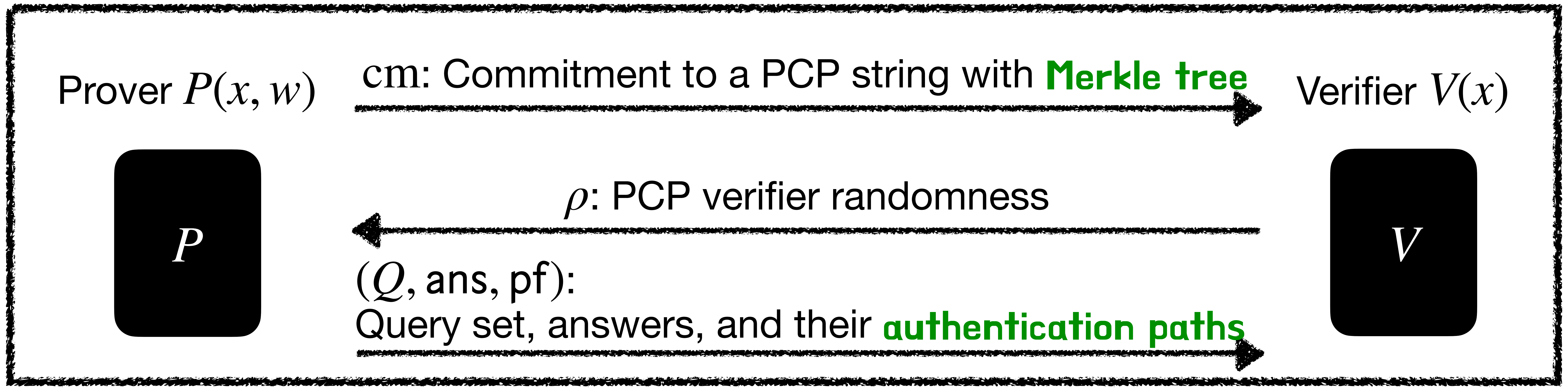

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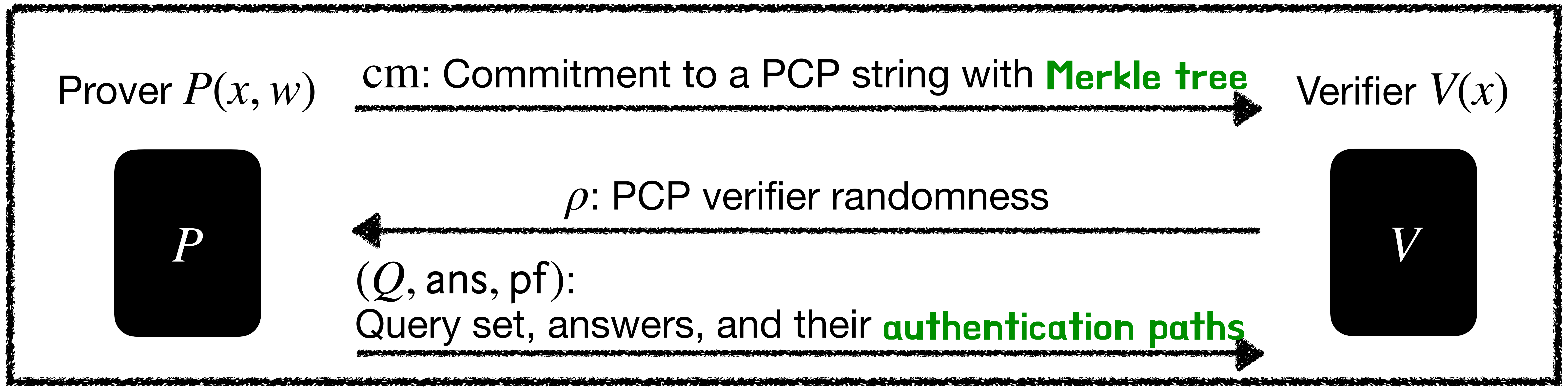

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A similar protocol: Schnorr identification scheme

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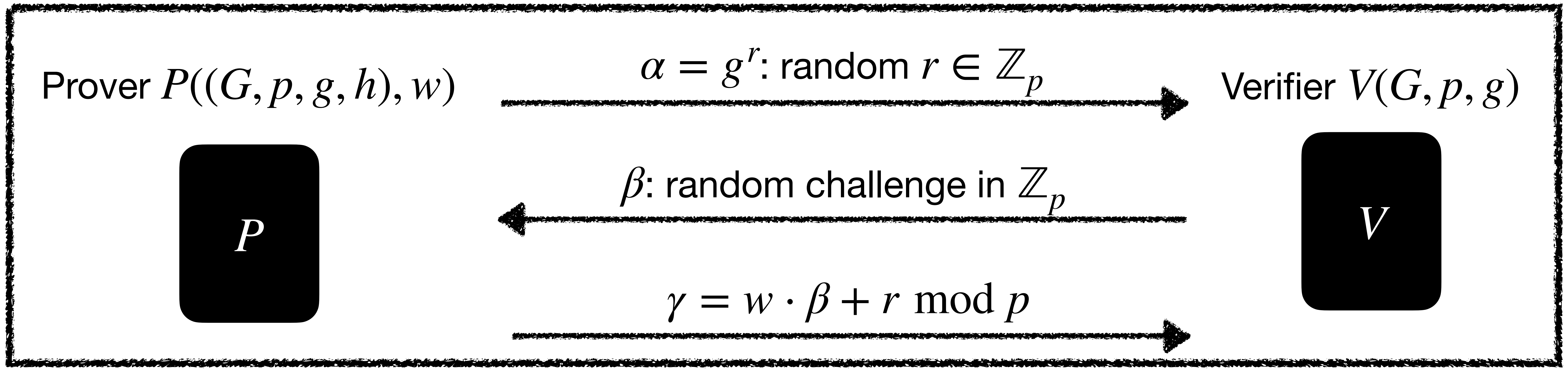

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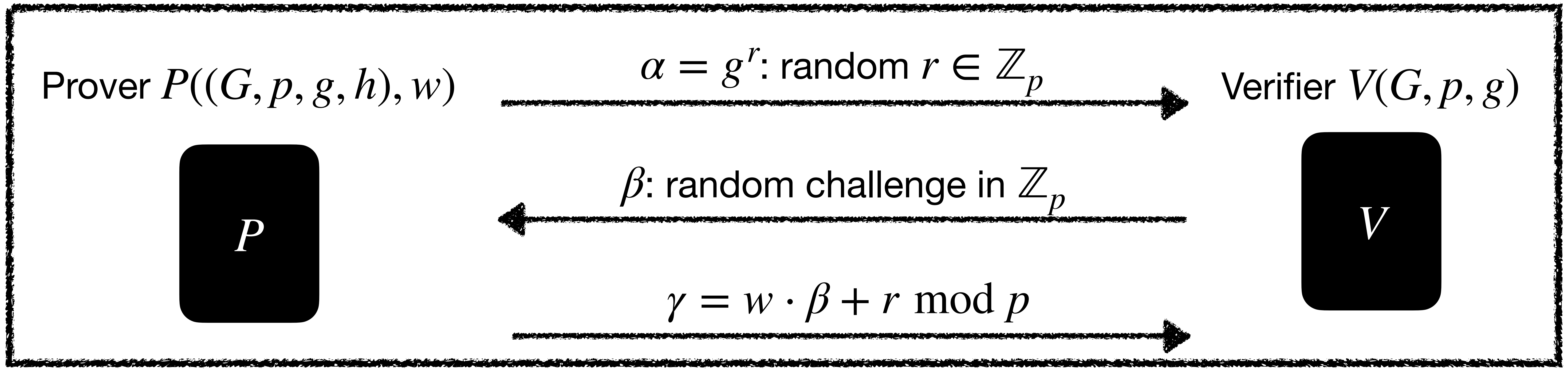

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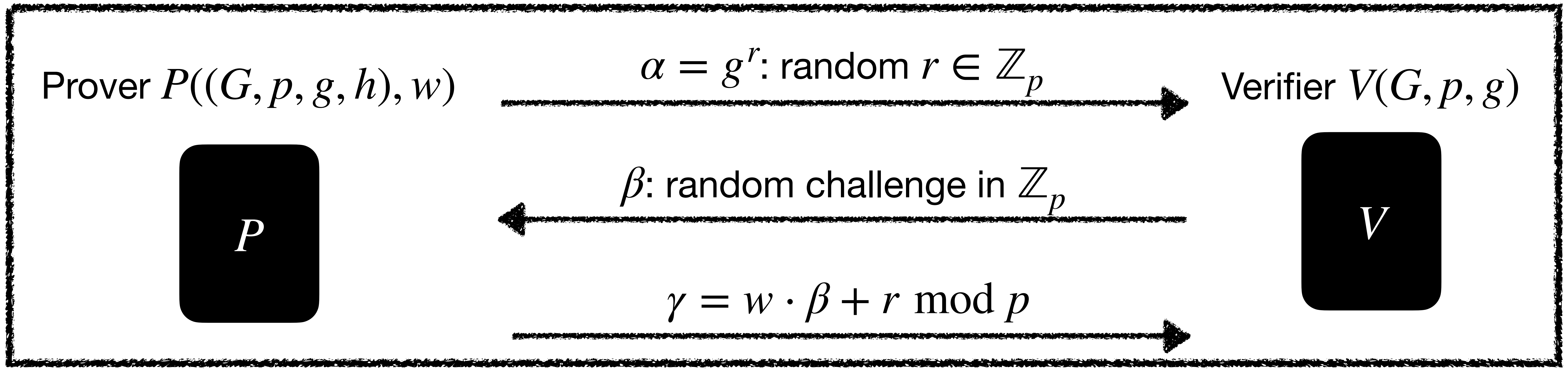

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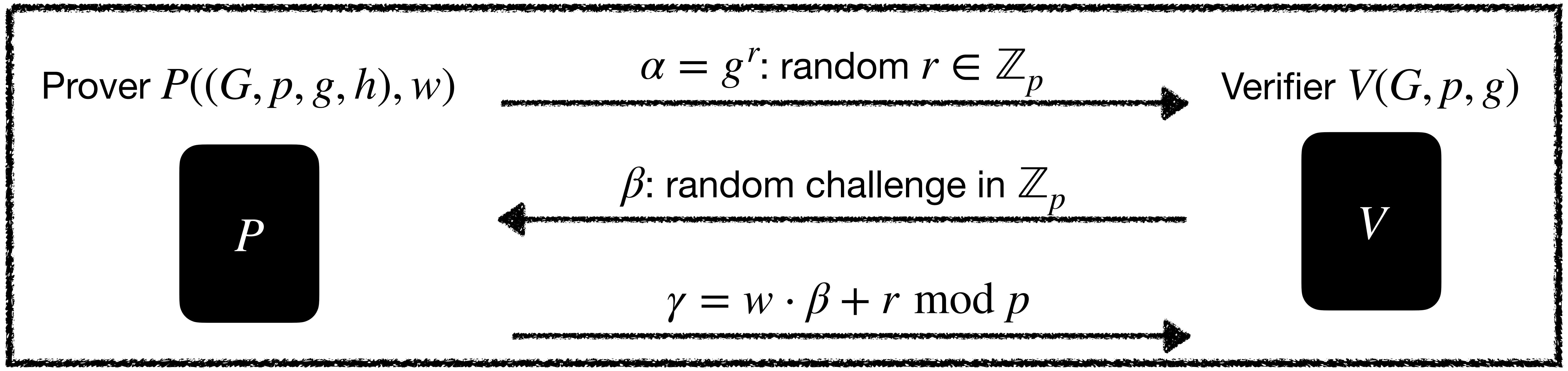

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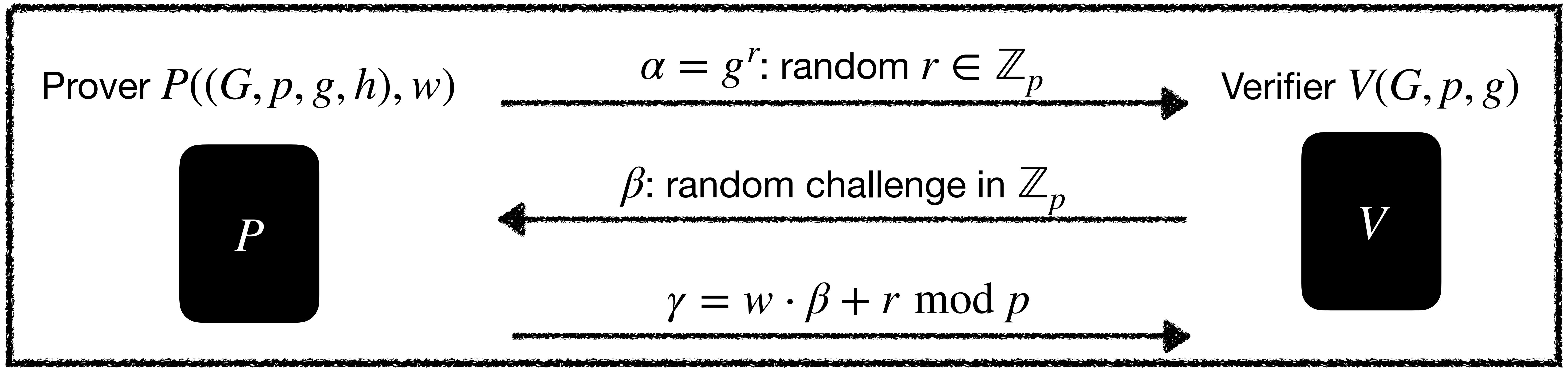

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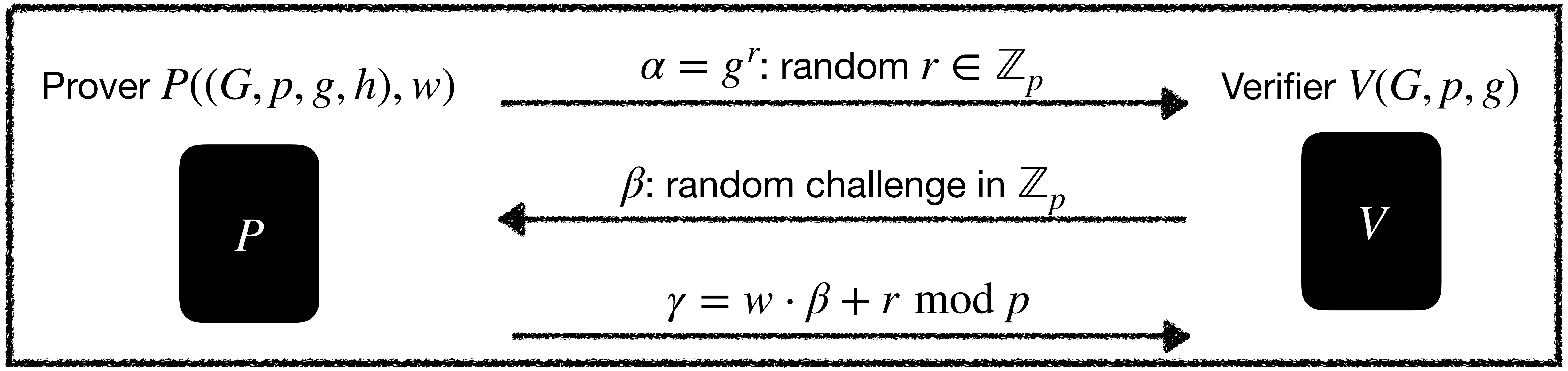

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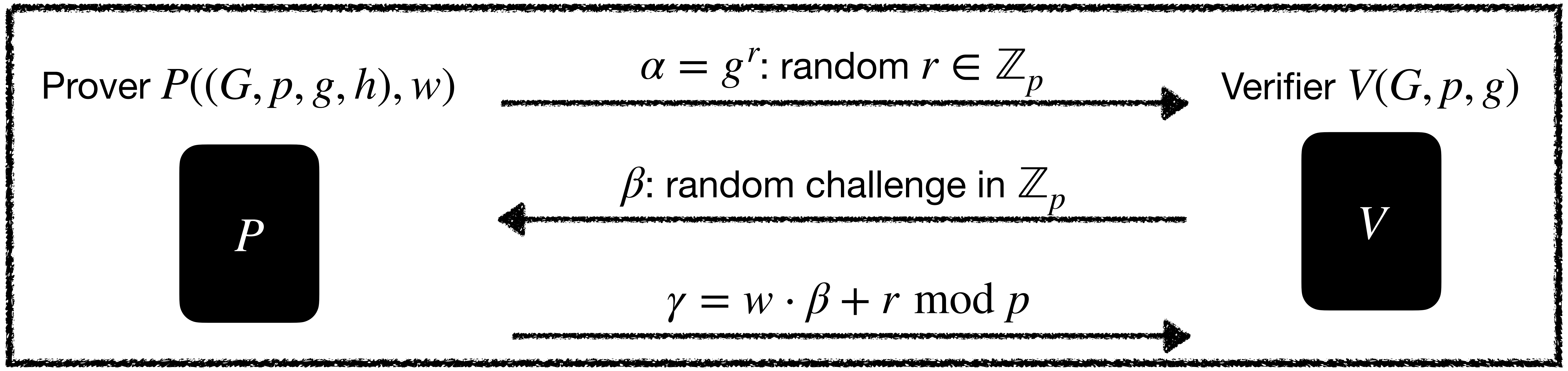

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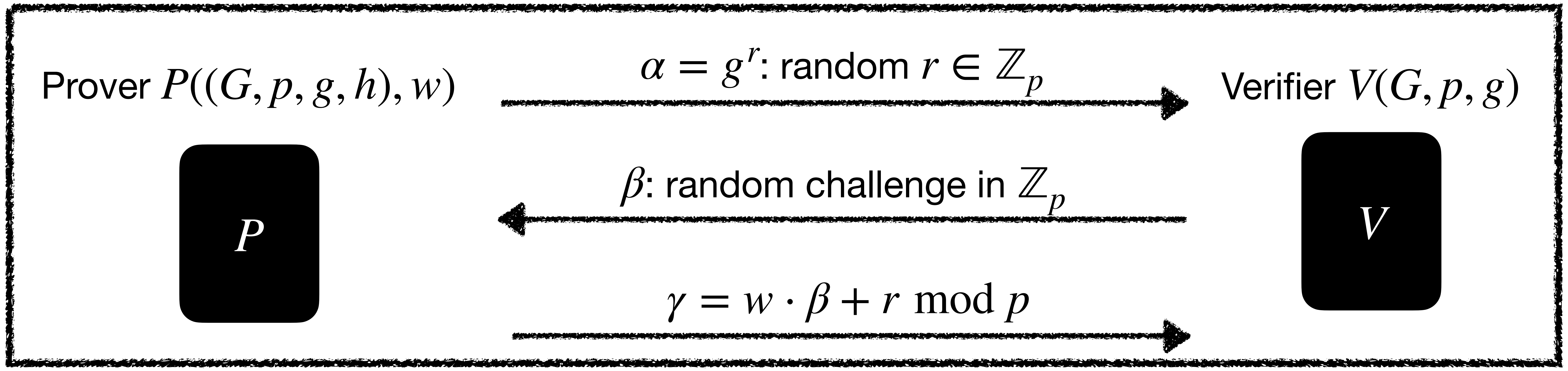

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- proof length l
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How tight are the bounds?

Strict-time setting.

- Setting $\epsilon_{\mathsf{DLOG}}(\lambda, t) \leq O(t^2/2^\lambda)$.
- Best known analysis of the Schnorr identification scheme:

$$\epsilon_{\mathsf{Schnorr}}(\lambda, t_{\mathsf{Schnorr}}) \leq \sqrt{\epsilon_{\mathsf{DLOG}}(\lambda, O(t_{\mathsf{Schnorr}}))} \leq O\left(\sqrt{t_{\mathsf{Schnorr}}^2/2^\lambda}\right).$$

- Our bound:

$$\epsilon_{\mathsf{ARG}}(\lambda, x, t_{\mathsf{ARG}}) \leq 2^{-\lambda} + \epsilon_{\mathsf{DLOG}}(\lambda, t_{\mathsf{ARG}} \cdot l/\epsilon) + \epsilon \leq 2^{-\lambda} + l^{2/3} \cdot O\left(\sqrt[3]{t_{\mathsf{ARG}}^2/2^\lambda}\right).$$

Polynomial gap

Expected-time setting.

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Polylogarithmic gap
Almost tight

Our followup: Quantum Rewinding for IOP-Based Succinct Arguments

Alessandro Chiesa, Marcel Dall'Agnol, Zijing Di, **Ziyi Guan**, Nick Spooner

Quantum Rewinding for IOP-Based Succinct Arguments

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We analyze the post-quantum security of succinct interactive arguments constructed from interactive oracle proofs (IOPs) and vector commitment schemes. We prove that an interactive variant of the BCS transformation is secure in the standard model against quantum adversaries when the vector commitment scheme is collapsing. Our proof builds on and extends prior work on the post-quantum security of Kilians succinct interactive argument, which is instead based on probabilistically checkable proofs (PCPs). We introduce a new quantum rewinding strategy that works across any number of rounds. As a consequence of our results, we obtain standard-model post-quantum secure succinct arguments with the best asymptotic complexity known.

Thank you!

https://eprint.iacr.org/2024/1434

On the price of rewinding

Goal: achieve $\epsilon_{\text{ARG}} = 2^{-40}$ against adversaries of size 2^{60} for Kilian's protocol.

Standard model

$t_{\text{VC}} = O\left(\frac{l}{\epsilon} \cdot t_{\text{ARG}}\right)$

For every $x \notin L$ and $\epsilon > 0$,
$\epsilon_{\text{ARG}}(\lambda, x, t_{\text{ARG}}) \leq \epsilon_{\text{PCP}}(x) + \epsilon_{\text{VC}}(\lambda, l(x), \mathsf{q}(x), t_{\text{VC}}) + \epsilon.$

- Suppose $\epsilon_{\text{PCP}} = 2^{-42}$ with $l = 2^{30}$.
- Suppose $\epsilon_{\text{VC}} = (\lambda, l, \mathsf{q}, t_{\text{VC}}) \leq \frac{t_{\text{VC}}^2}{2^\lambda}$ (achieved by ideal Merkle trees).
- Setting $\epsilon := 2^{-42}$:
  - $t_{\text{VC}} \leq 4 \cdot \frac{2^{30}}{2^{-42}} \cdot t_{\text{ARG}} < 2^{80} \cdot t_{\text{ARG}}$
  - $\epsilon_{\text{VC}} \leq \frac{(2^{80} \cdot t_{\text{ARG}})^2}{2^\lambda} = 2^{160-\lambda} \cdot t_{\text{ARG}}^2 = 2^{280-\lambda}$
- Set $\lambda = 322$ to achieve the desired bound.

Random oracle model

For every $x \notin L$, [CY24]

$\epsilon_{\text{ARG}}(\lambda, x, t_{\text{ARG}}) \leq \epsilon_{\text{PCP}}(x) + \frac{t_{\text{ARG}}^2}{2^\lambda}.$

- Suppose $\epsilon_{\text{PCP}} = 2^{-42}$
- $\epsilon_{\text{VC}} \leq \frac{t_{\text{ARG}}^2}{2^\lambda} = 2^{120-\lambda}$
- Set $\lambda = 162$ to achieve the desired bound.

- If the hash function is assumed ideal then extraction is straightline.
- If the hash function is merely collision-resistant then extraction is rewinding.
These computations illustrate the **PRICE OF REWINDING**.