Information-Theoretic Multi-Server Client Preprocessing PIR

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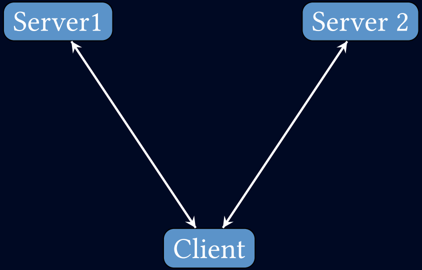
Private Information Retrieval



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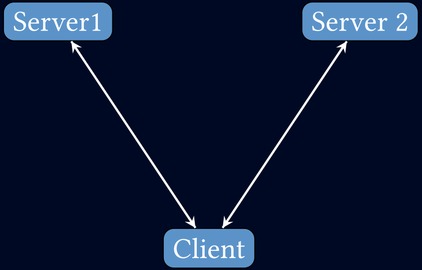

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- client inputs index $i \in \{0, 1, \ldots, n-1\}$

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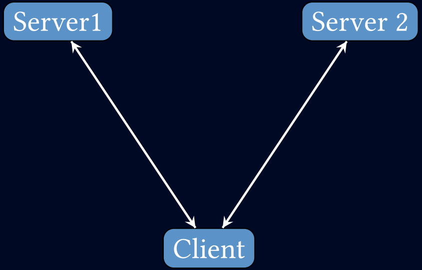

- interactive protocol between client and server(s)
- server(s) input database D of size n
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- Correctness: Client outputs $D[i]$
- Privacy: Server learns nothing about i

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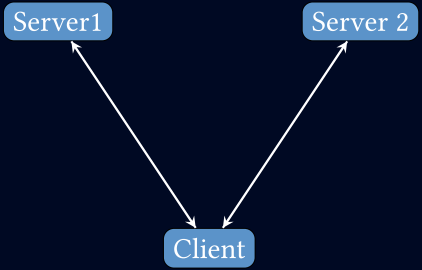

- Traditional goal of PIR - reduce the communication complexity

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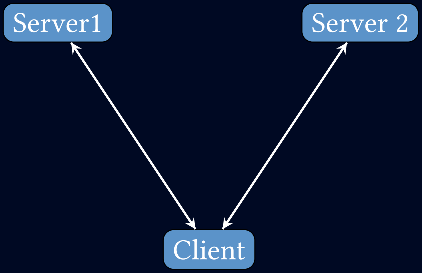

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- [CKGS98] show an $\Omega(n)$ computation lower bound for server complexity

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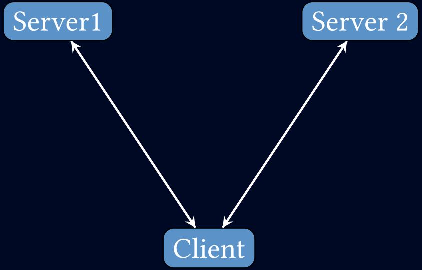

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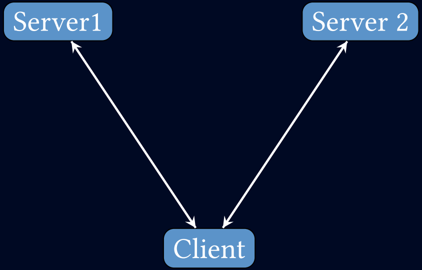
Need for different PIR models to overcome this lower bound!

PIR with Client Preprocessing



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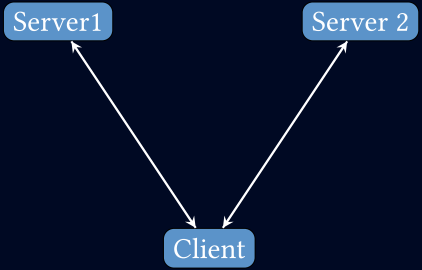

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- expensive offline phase with linear computation overhead
- Client stores a local state

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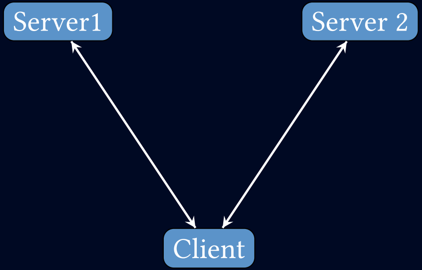

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- expensive offline phase with linear computation overhead
- Client stores a local state
- Sub-linear online phase

Previous Works (PIR with Client Preprocessing)

constructions (# servers)	communication	computation	client storage	assumptions
PRP-PIR [CGK20] (2)	$\widetilde{O}(n^{1/2})$[1]	$\widetilde{O}(\sqrt{n})$	$\widetilde{O}(\sqrt{n})$	OWF

[1] $\widetilde{O}$ notation hides poly log factors

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Does there exist non-trivial protocols for unconditional multi-server PIR with client preprocessing?

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Our Work $(2t)$	$\widetilde{O}(n^{1/2})$	$\widetilde{O}(\sqrt{n})$	$\widetilde{O}(n^{1/2+1/2t})$	–
Set $t = 1/2 \log n$ $(\log n)$	$\widetilde{O}(n^{1/2})$	$\widetilde{O}(\sqrt{n})$	$\widetilde{O}(n^{1/2})$	–

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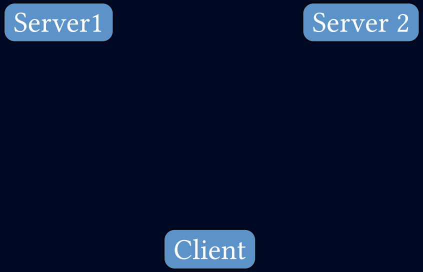
- Punctured key $\hat{k}$ hides x

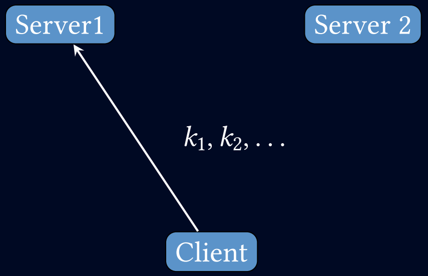
PIR with client preprocessing [CGK20]



PIR with client preprocessing [CGK20]

Offline:

- Client sends $\widetilde{O}(\sqrt{n})$ PPPS keys $k_1, k_2, \ldots$ to server 1



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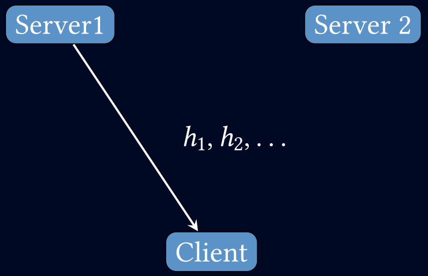

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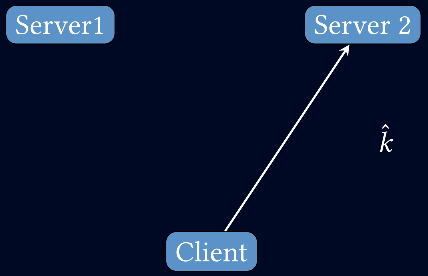

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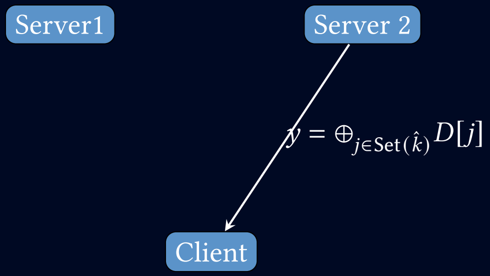

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Server1

Server 2

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Online complexity:

- Client computation: $\widetilde{O}(\sqrt{n})$
- Server computation: $\widetilde{O}(\sqrt{n})$
- bandwidth: $\widetilde{O}(\sqrt{n})$

Key Challenge

- Design PPPS from information-theoretic primitives
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Key insights:

- For PIR correctness, only need each random set to contain indivdual elements from domain with probability $1/\sqrt{n}$
- Multi-server PIR scheme is compatible with a weaker PPPS: puncturing a PPPS key gives multiple disjoint keys/sets

Privately Multi-Puncturable Random Sets (PMPRS)

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(n, t)-**PMPRS** (Gen, Set, Test, Punc
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- $b \leftarrow \mathsf{Test}(k, x)$
- $((S_0, k_0, \mathsf{ind}_0), , \ldots,$
  $(S_{t-1}, k_{t-1}, \mathsf{ind}_{t-1})) \leftarrow \mathsf{Punc}(k, x)$
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Security properties:

- **randomness** For any $x \in [n]$, $Pr(x \in S) = 1/\sqrt{n}$
- **Privacy**: key k_i hides x
- **Correctness**:
  - $S \setminus \{x\} = \dot{\bigcup} S_i$
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Efficient PMPRS: $\widetilde{O}(1)$ Test and $\widetilde{O}(\sqrt{n})$ SerEval complexity; $\widetilde{O}(n^{1/2t})$ key-size

Main Results

Theorem (PMPRS)

There exist (n, t)-PMPRS with key size $\widetilde{O}(n^{1/2t})$, Test and $\mathsf{SerEval}$ complexity $\widetilde{O}(1)$ and $\widetilde{O}(\sqrt{n})$ respectively

Theorem (IT client preprocessing PIR)

(n, t)-PMPRS $\implies$ t server IT PIR with preprocessing with client state size $\widetilde{O}(n^{1/2+1/2t})$, online client/server $\widetilde{O}(\sqrt{n})$ and bandwidth $\widetilde{O}(\sqrt{n})$

$(n, 1/2 \log n)$-**PMPRS** (where n is even power of 2)

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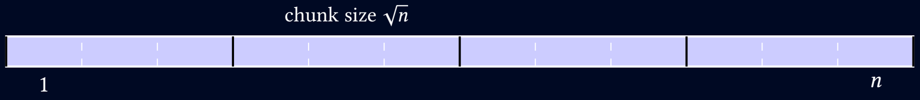
well-partitioned random set

- Contain one random element from each chunk
- chunk $(x) = \lfloor x/\sqrt{n} \rfloor$, offset$(x) = (x \% \sqrt{n})$



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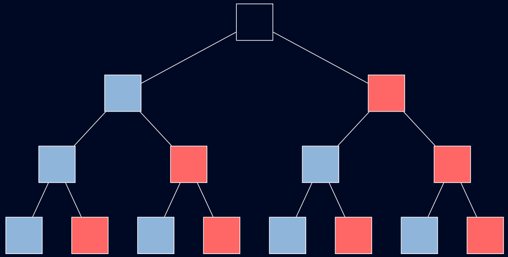

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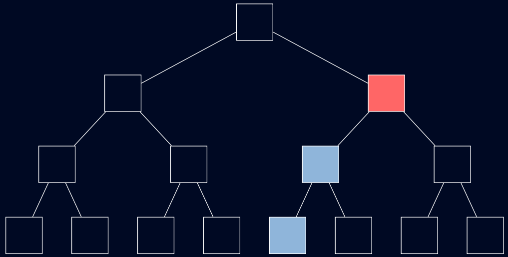

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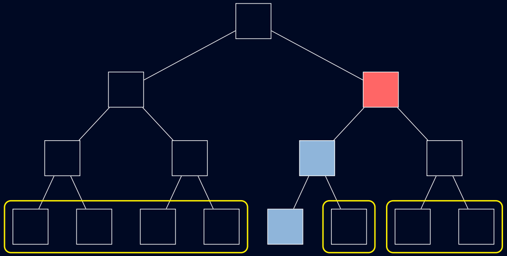
- $\mathrm{Set}(k)$
- $\mathrm{Punc}(k, x)$
  - Each key k_i contains offset values of set S_i
- $\mathrm{SerEval}(D, k_i)$
  - for each guess of chunk indexes of S_i compute database xor bit



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  - General construction based on arbitrary d-ary trees
  - technical challenge: ensuring SerEval cost is $O(\sqrt{n})$
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Parallel Work [ISW24]

- Also study PIR with client preprocessing in IT setting (1 and 2 server setting)
- different techniques and lower bound proof
- Higher complexity for online server cost, offline bandwidth, and other measures

Thanks

eprint: `https://eprint.iacr.org/2024/780.pdf`