On the Black-Box Complexity of Private-Key IPFE

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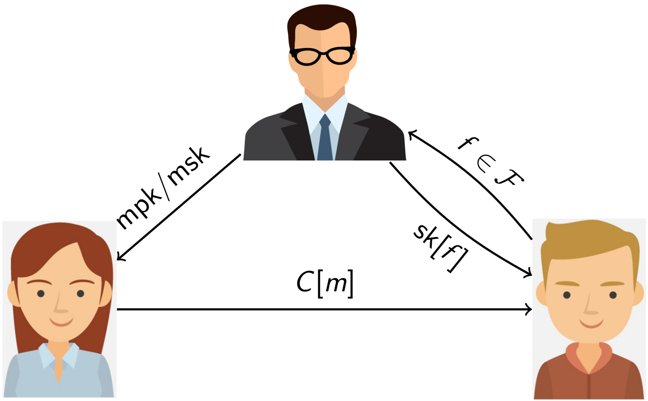
2024-12-05

Functional Encryption [SW05, O'N10, BSW11]

$$(\mathsf{mpk}, \mathsf{msk}) \leftarrow \mathsf{MKGen}(1^\kappa)$$
$$\mathsf{sk}[f] \leftarrow \mathsf{KGen}(\mathsf{msk}, f)$$



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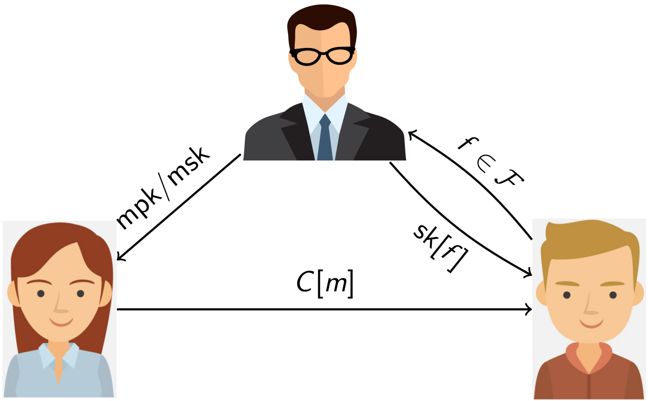

$\mathcal{A}$ gets

- mpk or Enc oracle,
- $C(m^*)$,
- $\mathsf{sk}[f_1], \ldots, \mathsf{sk}[f_n]$

and learns only $f_1(m^*), \ldots, f_n(m^*)$.

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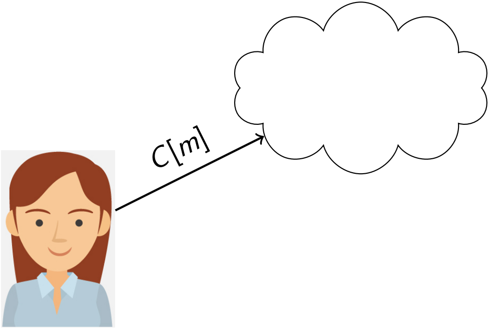
Application of sk-FE



$$\mathsf{msk} \leftarrow \mathsf{MKGen}(1^\kappa)$$
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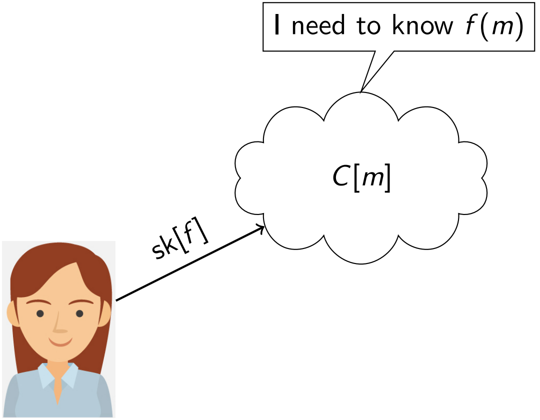
Application of sk-FE

I need to know $f(m)$

$C[m]$

msk

Application of sk-FE



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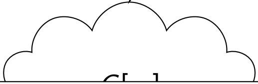
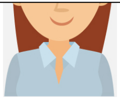


Is sk-FE easier to build than pk-FE?



msk

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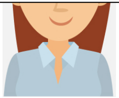
Application of sk-FE

I need to know $f(m)$

Is sk-FE easier to build than pk-FE?
What are the minimal assumptions for sk-FE?



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Bounded-collusion

There is an a-priori bound on the number of functional secret keys that $\mathcal{A}$ can get.

$$\text{IND-CPA secure PKE} \iff \text{bounded-collusion pk-FE [SS10, GVW12, AV19]}$$

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$$\text{IND-CPA secure PKE} \Longleftrightarrow \text{bounded-collusion pk-FE [SS10, GVW12, AV19]}$$

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pk vs. sk

For general functions: sk-FE $\Longleftrightarrow$ pk-FE ($\Longleftrightarrow$ iO)[BV15, AJ15]

pk vs. sk

$$\text{For general functions: sk-FE} \iff \text{pk-FE } (\iff \text{iO})\text{[BV15, AJ15]}$$

$$\text{sk additively HE} \iff \text{pk additively HE [Rot11]}$$

$$\text{IND-CPA PKE} + \text{circular SKE} \implies \text{KDM-secure PKE [KM20]}$$

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IND-CPA PKE $+$ circular SKE $\Longrightarrow$ KDM-secure PKE [KM20]

What about FE for specific functionalities?

Functionalities

	Point functions (IBE) $f_{\mathrm{id}'}(\mathrm{id}, m) = \begin{cases} m & \text{if } id = id' \\ \perp & \text{otherwise} \end{cases}$	
pk	✓ pairings, lattices, and more ✗ generic groups [PRV12, SS21, Zha22] ✗ TDP (BB use) [BPR$^+$08]	
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Predicate encryption (PE): FE for “all-or-nothing” functionalities

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\begin{tikzcd}
\text{Generic groups} \arrow[r, "{[\text{PRV12, SS21, Zha22}]}", "/" marking] & \text{pk-IBE} \arrow[r, "{[\text{GKLM12}]}", "/" marking] & \text{pk-PE for threshold predicates} \\
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Previous FE impossibility results

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- Only for predicate encryption

Our result

Theorem

There exists no unbounded-collusion sk-IPFE in the random oracle model.*

*against unbounded adversaries that can make only polynomially many ROM queries

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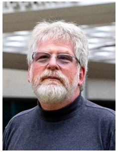
$\Downarrow$ [IR89]



Impagliazzo

Rudich

There exists no unbounded-collusion black-box construction of sk-IPFE from OWFs/CRHFs

Proof overview

Goal: Learn $\langle \mathbf{v}^\star, \mathbf{w} \rangle$

Given: $C[\mathbf{w}] \xleftarrow{\$} \mathsf{Enc}^{O}(\mathsf{msk}, \mathbf{w})$, $\mathsf{sk}[\mathbf{v}_i] \xleftarrow{\$} \mathsf{KGen}^{O}(\mathsf{msk}, \mathbf{v}_i)$ with $\mathbf{v}^\star \notin \mathrm{Span}\left(\mathbf{v}_1, \ldots, \mathbf{v}_t\right)$

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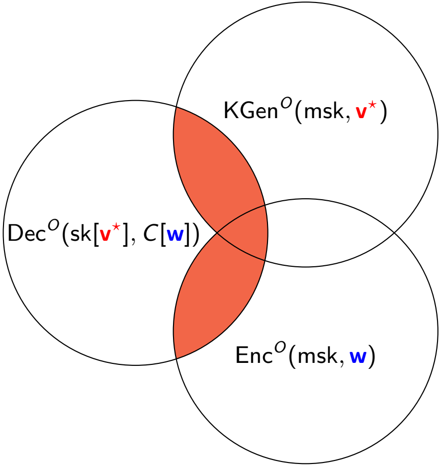

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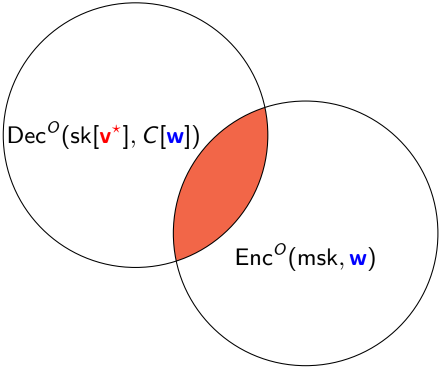

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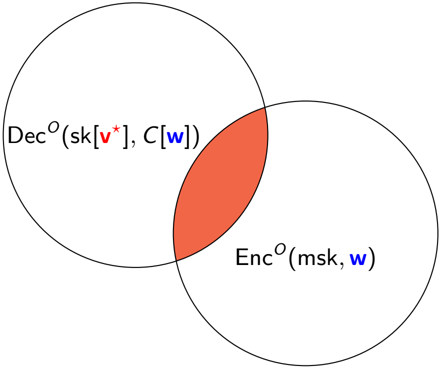

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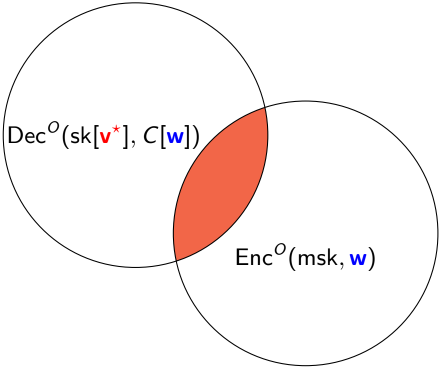

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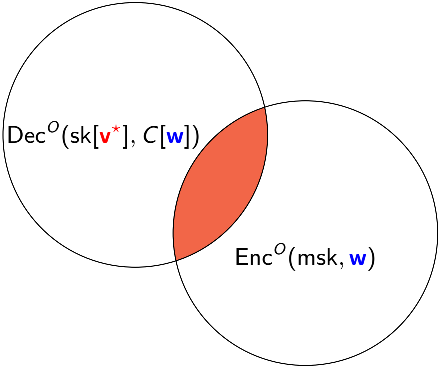

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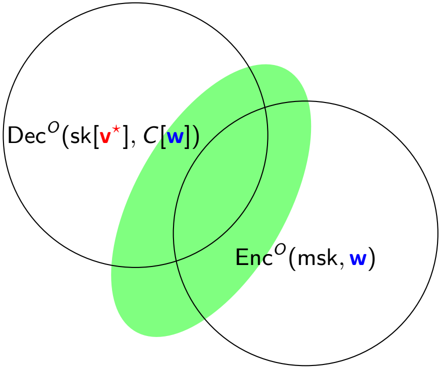


The Combinatorial Lemma

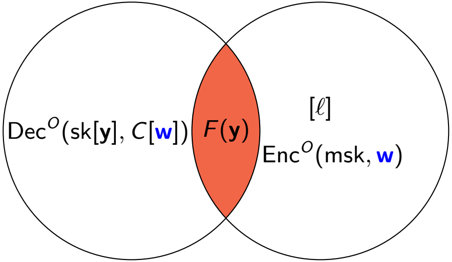
Lemma

For every $F : \mathbb{Z}_q^n \to 2^{[\ell]}$ with polynomial ℓ there exists a polynomial t such that with overwhelming probability

$$F(\mathbf{y}^*) \subseteq \bigcup_{i=1}^{t} F(\mathbf{y}_i),$$

for $\mathbf{y}^*, \mathbf{y}_1, \ldots, \mathbf{y}_t \leftarrow \mathbb{Z}_q^n$ subject to $\mathbf{y}^* \notin \mathrm{Span}(\mathbf{y}_1, \ldots, \mathbf{y}_t)$.

Intuition:



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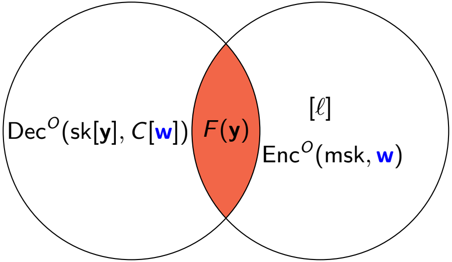
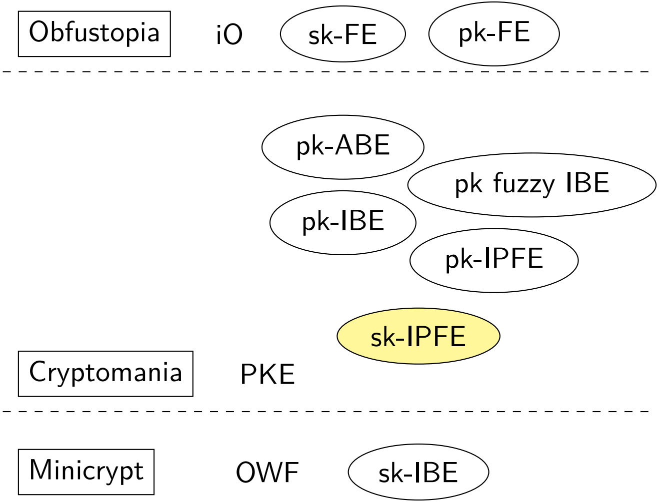


Proofs:

	$q = 2$	arb. prime q
Existential variant (weaker)	Double-counting	
Average-case variant (this)	Fourier	Cauchy-Schwarz

Conclusion & Open problems



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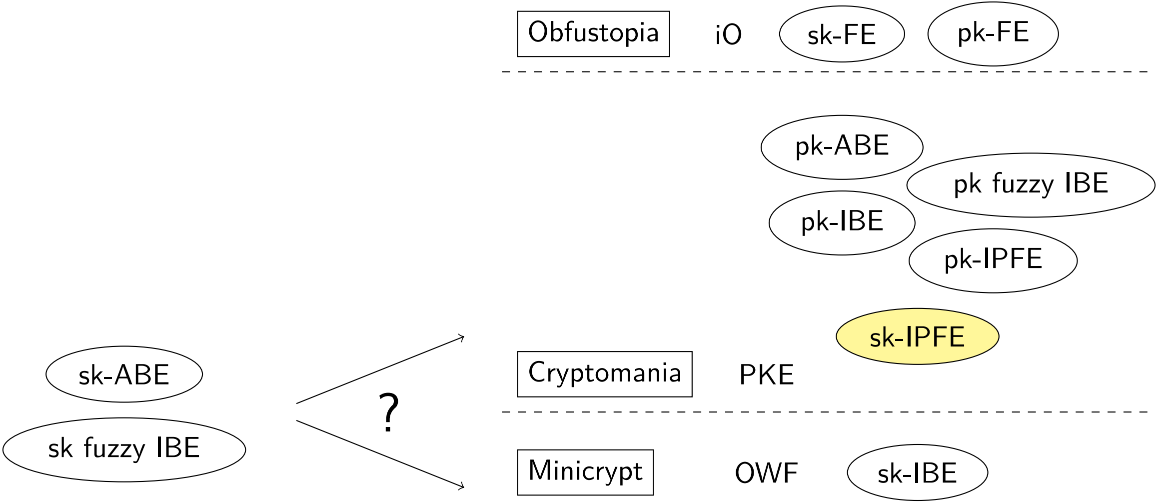

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Pictures

Alice, Bob, and other faces: freepik.com
Devil face: vecteezy.com
Russell Impagliazzo: quantamagazine.org
Steven Rudich: commons.wikimedia.org, Andrej Bauer, CC BY-SA 2.5 SI

Our result (more detail)

Theorem

There exists no unbounded-collusion sk-IPFE for dimension n and modulus q if

- $n \geqslant 3$
- *and q^n is super-polynomial in the security parameter*

in the random oracle model.*

*against unbounded adversaries that can make only polynomially many ROM queries

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- sk-IPFE for $n = 1$ is just plain SKE
- if q^n is polynomial, then we can use $\mathsf{Enc}^{O}(\mathsf{msk}, \mathbf{x}) = (\mathsf{SKE.Enc}^{O}(\mathsf{PRF}(\mathbf{y}), \langle \mathbf{x}, \mathbf{y} \rangle))_{\mathbf{y} \in \mathbb{Z}_q^n}$

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- bounded-collusion sk-IPFE can be built from OWF [AV19]

Our security game

Challenger		Adversary
$\mathsf{msk} \leftarrow \mathsf{MKGen}(1^\kappa)$		
$b \xleftarrow{\$} \{0,1\}$		
	$\leftarrow$ $\mathbf{v}^\star, \mathbf{v}_1, \ldots, \mathbf{v}_t$ with $\mathbf{v}^\star \notin \mathsf{Span}(\mathbf{v}_1, \ldots, \mathbf{v}_t)$	$t = \mathsf{poly}(\kappa)$
$\mathsf{sk}[\mathbf{v}_i] \leftarrow \mathsf{KGen}^{O}(\mathsf{msk}, \mathbf{v}_i)$ for all $i \in [t]$	$\rightarrow$ $\mathsf{sk}[\mathbf{v}_1], \ldots, \mathsf{sk}[\mathbf{v}_t]$	
$\mathbf{w}_i \xleftarrow{\$} \mathbb{Z}_q^n$		
$C[\mathbf{w}_i] \xleftarrow{\$} \mathsf{Enc}^{O}(\mathsf{msk}, \mathbf{w}_i)$	$\leftarrow$	$\eta = \mathsf{poly}'(\kappa)$ queries
If $b = 0$: $m_i^* := \langle \mathbf{w}_i, \mathbf{v}^\star \rangle$	$\rightarrow$ $C[\mathbf{w}_i], m_i^*$	
If $b = 1$: $m_i^* \xleftarrow{\$} \mathbb{Z}_q$		
	$\leftarrow$ b'	
$b \stackrel{?}{=} b'$		

Proof overview (complete)

$\mathcal{A}$ brute-force searches for sk[$\mathbf{v}^\star$]

- Check if $\mathsf{Dec}^{O}(\mathsf{sk}[\mathbf{v}^\star], C[\mathbf{w}_i]) = m_i$ for all $i \in [\eta]$
- If such a secret key is found, return $b' = 0$, otherwise $b' = 1$

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- Check if $\mathsf{Dec}^O(\mathsf{sk}[\mathbf{v}^\star], C[\mathbf{w}_i]) = m_i$ for all $i \in [\eta]$
  - Self-simulate all ROM queries consistently!
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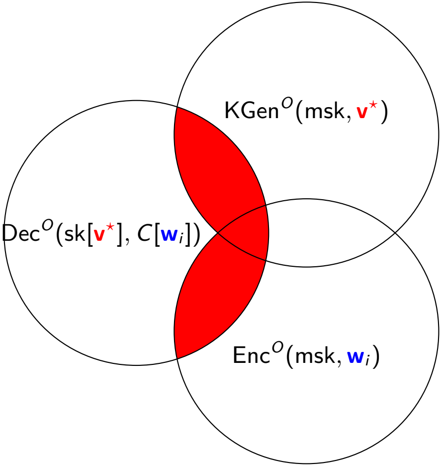

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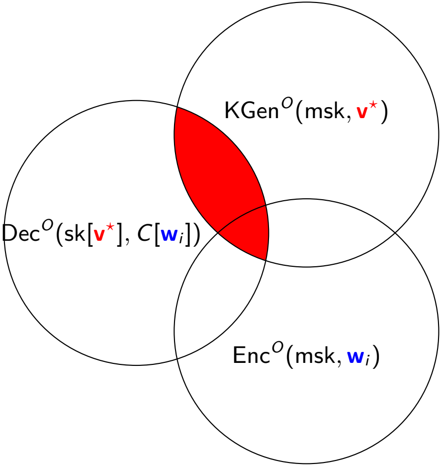

Proof overview (complete)

$\mathcal{A}$ brute-force searches for $\mathsf{sk}[\mathbf{v}^\star]$ and RO Q-A pairs made during $\mathsf{KGen}^O(\mathsf{msk}, \mathbf{v}^\star)$

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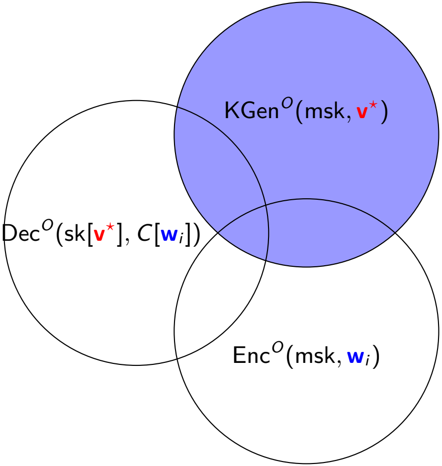
- If such a secret key is found, return $b' = 0$, otherwise $b' = 1$



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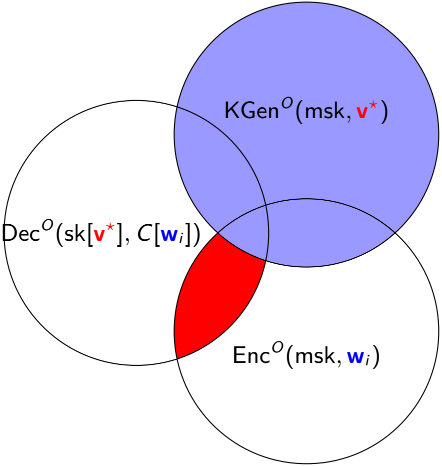

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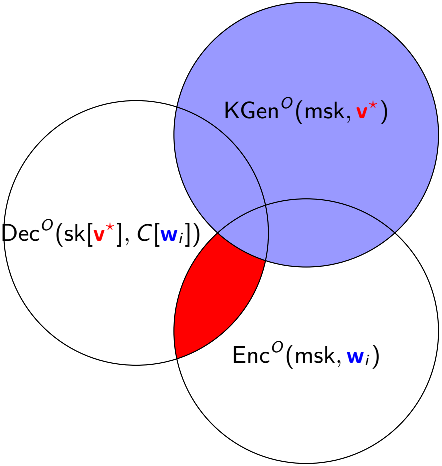
  - ROM queries made during $\mathsf{Enc}^O(\mathsf{msk}, \mathbf{w}_i)$:
    - Decrypt $C[\mathbf{w}_i]$ with many known secret keys $\mathsf{sk}[\mathbf{v}_i]$ using the real RO.
- If such a secret key is found, return $b' = 0$, otherwise $b' = 1$



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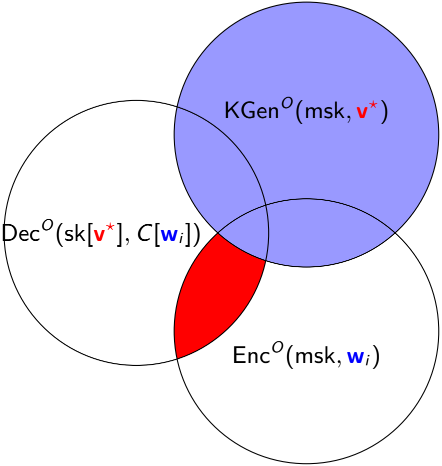

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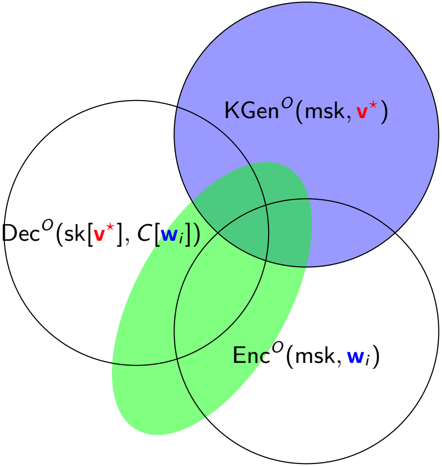

The Combinatorial Lemma (formal)

Lemma

Fix $n = n(\kappa)$ *and suppose* $q^{-n} \in \text{negl}(\kappa)$ *and* $n \geqslant 3$. *Let* $\ell = \text{poly}(\kappa)$, q *be a prime number and* $F : \mathbb{Z}_q^n \to 2^{[\ell]}$. *Fix a constant* c. *Then, there exists a polynomial* $t = t(\kappa)$ *such that with probability at least* $1 - \kappa^{-c}$

$$F(\mathbf{y}^*) \subseteq \bigcup_{i=1}^{t} F(\mathbf{y}_i),$$

where $\mathbf{y}^* \leftarrow \mathbb{Z}_q^n$, *and we sample a random* $(n-1)$*-dimensional subspace* V *subject to* $\mathbf{y}^* \notin V$ *and we sample* $\mathbf{y}_1, \ldots, \mathbf{y}_t$ *all uniformly at random from* V.